INTERNATIONAL TRANSMISSION

OF STABILIZATION POLICIES
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by

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ABSTRACT

The interdependence of economic policies under flexible exchange rates has (finally) come to be accepted as a fact. As world markets have evolved from national markets into an internationally efficient system, the ability of governments to isolate their economies from international disturbances has decreased dramatically. That is, in today's highly integrated world economic system, economic authorities must come to terms with the economic interdependence of the system.

This study consists of three essays on open economy macroeconomics. In each of these essays we focus our attention on different channels through which the effects of stabilization policies are transmitted internationally.

In the first essay we construct an "extended" small open economy model (the rest of the world is explicitly modelled, but unlike the two country models the small country has no impact on the rest of the world), which recognizes the interdependence of rest of the world variables that are considered exogenous by the small country. We use this model in order to investigate the properties of five monetary policy rules (i.e., a fixed rate of monetary growth rule, a fixed nominal exchange rate rule, a nominal income rule, an inflation rate rule, an inflation rate rule,
and a fixed real exchange rate rule) in response to permanent monetary and fiscal disturbances in the rest of the world. Our major finding is that nominal income and inflation rate rules can be very effective in preventing any output and price deviations from their steady-state values, whereas pegging the real exchange rate is not a feasible policy when foreign disturbances affect the equilibrium real interest rate.

The second essay examines stabilization policy options for some of the European Economic Community countries by considering a two-country currency area model involving floating exchange rates with the rest of the world and real wage rigidity within the currency area. Recognition of the supply-side effects of the exchange rate in this model yields conclusions which differ markedly from the existing results concerning beggar-my-neighbour effects between currency area countries. Moreover, both currency area countries are adversely affected by a foreign interest rate hike.

The third essay preserves the three country (the two currency area countries and the rest of the world) setting of the second essay, but now the emphasis is on the short-run interdependence of monetary policies (i.e., open market operations) between the currency area's central banks. The portfolio balance model is used
to show how the conventional results encountered in the one and two country models need to be qualified once the responses of the one of the currency area's central banks both to policy actions by the other currency area central bank and to exogenous disturbances are taken into account.
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CHAPTER 1
INTRODUCTION

1.1 Overview

Fifteen years ago the conventional wisdom was that flexible exchange rates would allow countries to gear their monetary policies towards domestic objectives (the choice of the "optimal" point on their Phillips curve) -- free of balance of payments constraints. Furthermore, it was believed that if flexible exchange rates were in general use, the need for international coordination of economic policies would be reduced since the transfer of resources from surplus countries to deficit countries (inherent in a fixed exchange rate regime) would no longer be a necessary part of the policy landscape.

The early arguments supporting that insulation from foreign disturbances is greater under flexible exchange rates rested on the simplest Keynesian model of the determination of the levels of output and employment by aggregate demand and the assumption that the exchange rate would move to equate current account expenditures and receipts. (Friedman, (1953) based his
argument in favour of flexible exchange rates on the fact that it is easier — in disequilibrium situations — to change one variable, e.g., the exchange rate, than the whole wage-price structure, especially when wages and prices are slow to adjust). Therefore, foreign disturbances in the form of changes in the demand for a country's exports would not be transmitted through to the level of this country's economic activity. As late as 1969, Harry Johnson argued that "the fundamental argument for flexible exchange rates is that they would allow countries autonomy with respect to their monetary, fiscal and other policy instruments... The argument for flexible exchange rates can be put more strongly still: flexible exchange rates are essential to the preservation of national autonomy and independence consistent with efficient organization and development of the world economy" (Johnson, 1969).

Today, after more than ten years of experience with flexible exchange rates there is much less confidence that flexible rates and domestic policy autonomy are reconcilable. Quite on the contrary, exercise of policy autonomy becomes nearly impossible because many countries are too small and open to accept the exchange rate variations induced by policy. As Charles Kindleberger prophetically observed in 1969: "Along with one more variable there is one more
target -- the exchange rate" (Kindleberger, (1969)). Alternatively, in the case of large countries, the effects of policy are exported abroad and interfere with foreign internal stability.

The myth that flexible exchange rates would insulate economies from external disturbances started being dispelled in the early sixties, with the appearance of the Mundell-Fleming model. The original focus of the model was on the role of international capital mobility and the foreign exchange market, but later the model was used extensively to examine the implications for monetary and fiscal policy under alternative exchange rate regimes. The standard Mundell-Fleming model had prices fixed and ignored any dynamics arising from the servicing of foreign debt. (For a model paying attention to the repatriation of interest payments, see Rodriguez (1979)).

A fiscal expansion in the Mundell Fleming model increases domestic income and therefore the domestic interest rate (of course, in the limiting case of perfect capital mobility and asset substitutability and an exogenous foreign interest rate (small country) these effects vanish). The higher domestic interest rate results in a capital inflow which corresponds to a trade deficit. If capital mobility is sufficiently high,
then the domestic currency appreciates under floating exchange rates. The resultant foreign trade surplus increases foreign income -- a positive international transmission. The main ambiguity in the above story is whether capital mobility is high enough for the fiscal expansion to appreciate the currency. Some (but not all) of the large econometric models exhibit an asymmetry (see Frankel (1986)): fiscal expansion in the US appreciates its currency but (whether because of lower capital mobility, a flatter LM curve, monetary accommodation or other factors) fiscal expansion in Europe or Japan depreciates their currencies.

However, the effects of monetary expansion are more clear-cut. It reduces the domestic interest rate and therefore increases domestic income. The interest rate differential in favour of the foreign currency induces a capital outflow (which corresponds to a trade surplus) and the currency unambiguously depreciates. The corresponding worsening in the foreign trade balance reduces foreign income. This is the classic Mundell-Fleming result of inverse transmission: a contractionary monetary policy such as the United States adopted under the Reagan administration is (supposedly) expansionary for Europe. The open economy macroeconomics literature has
pointed out at least three ways by which the foregoing transmission results are modified. We consider each of them briefly.

First, if the consumer price index (CPI) is used to deflate the nominal money balances in the money demand function a depreciation of the domestic currency will lower the real money stock exerting a contractionary effect on foreign output. If the fall in the exchange rate originated from a domestic fiscal expansion, this effect can reverse the standard Mundell-Fleming result of positive transmission. In the case of domestic monetary expansion the domestic currency depreciates, the foreign CPI falls, the foreign real money stock rises and hence foreign income increases, e.g., the transmission is positive. Thus, both the positive transmission of fiscal policy and the negative transmission of monetary policy can be reversed. In the same vein, the negative effect of the exchange rate on the real stock of government bonds can be contractionary if real wealth enters the expenditure function (Ricardian equivalence is ignored).

Second, increases in intermediate input prices reduce aggregate supply for a given price of the final output. Thus to the extent that the price of oil is determined in US dollars, an appreciation of the US dollar
is contractionary for other countries. Therefore, fiscal expansions that appreciate the currency can be transmitted negatively and monetary expansions that depreciate the currency transmitted positively, rather than the other way around. This is another reason why some Europeans believe that the strong dollar reduced growth in Europe.

Third, when wages are indexed to the CPI (which includes import prices as well as the prices of domestically produced goods), a depreciation of the currency increases the real wage faced by producers (for a given price of domestically produced goods) and hence output supplied falls. In other words, domestic output is a positive function of the terms of trade (the ratio of the prices of domestic goods over the prices of foreign goods expressed in the home country's currency) while foreign output is a negative function of the (same) terms of trade. In the case of a domestic fiscal expansion that depreciates the foreign currency, there is a contractionary effect on foreign income -- which gives another reason why Europeans believe that the strong dollar has hurt Europe. One might expect that a domestic monetary expansion because it appreciates the foreign currency, would have the opposite effect from a fiscal expansion, e.g., it would increase foreign income. But this could only be true if domestic income had fallen (since there would be a fall in the terms of trade). However, this perverse
effect can be ruled out by realizing that domestic income (as determined from the demand side) cannot fall unless the domestic interest rate rises, or the domestic currency appreciates and reduces the trade balance, neither of which will follow from a monetary expansion. The only possible solution is that domestic prices rise by the same proportion as the value of the foreign currency (the terms of trade remain unchanged) and there is no change in either domestic or foreign income.

Given the multitude of channels through which stabilization policies are transmitted internationally even under flexible exchange rates (and sometimes because of them), it is not surprising to find that in the last five years a significant literature has emerged dealing with the coordination of economic policies among the major industrialized countries, and with proposals for international monetary reform (Buiter and Marston (1985), Hamada (1985), Oudiz and Sachs (1984), McKibbin and Sachs (1986)).

The purpose of the present study is to explore the implications of institutional (or other) asymmetries for the international transmission of stabilization policies. These asymmetries refer to the size of the economy(ies) in question, the degree of wage indexation and the nature of the exchange rate regime. Any of these
asymmetries has in the past been the reason for acute policy differences between Europe and the United States and raised the need for coordination of economic policies (and the difficulty of achieving it).

Until recently, it was differences in the degree of wage indexation (e.g., real versus nominal wage rigidity) that received the main attention of economists trying to explain the policy conflicts among OECD countries during the last decade. But as much as it is a fact that European countries "suffer" from real wage rigidity it is a fact as well that their currencies float more or less jointly against the US dollar. The European Monetary System imposes rather strict limitations on the movements of exchange rates among member countries. This is what we mean by the nature of the exchange rate regime. Members of the European Monetary System have to gear their monetary (and fiscal?) policies towards maintaining the agreed exchange rate parities at the same time their currencies freely fluctuate against those in the rest of the world. Finally, asymmetries arising out of large differences in the size of interacting economies were not explicitly modelled until 1979 (Flood (1979)). The traditional small country and two-country models did not address this issue at all. In the small open
economy literature there is no explicit modelling of the rest of the world, whereas in the two-country models the interaction is totally symmetric, e.g., each of the two countries is capable of influencing all the variables in the model.

At least one of these asymmetries is each time incorporated in three distinct models in order to investigate: (1) the importance of the main channel (e.g., terms of trade changes) through which the effects of economic policies are transmitted internationally (chapters 2 and 3); (2) ways by which economic authorities can ameliorate the effects of foreign stabilization policies on the domestic economy (chapter 2), and (3) the nature and interdependence of monetary policies (chapter 4).

1.2 The Content of the Succeeding Chapters

Changes in the terms of trade have been established as the main avenue for the transmission of economic disturbances among countries. As long as exchange rates fluctuate far more than inflation differentials, then there will be large swings in the terms of trade or the real exchange rate. One of the reasons for excessive fluctuations of nominal exchange
rates is either changes in expectations about the future course of fundamentals, or an actual change in them.

In chapter 2 we investigate the effects of actual changes in fundamentals -- a change in the rate of monetary growth and the level of government spending abroad -- under different monetary policy regimes of the home country. The proposition that under a floating exchange rate regime restrictive, say, monetary policy can lead to substantial overshooting of the nominal and real exchange rate (the terms of trade) is now accepted fairly widely (Dornbusch (1976), Wilson (1979), Buiter and Miller (1981). The main reason is the presence of nominal price stickiness in domestic factor and product markets combined with a freely flexible nominal exchange rate, which is set in a forward-looking efficient auction market. Nominal appreciation of the currency therefore amounts to real appreciation -- a change in the terms of trade.

Chapter 2 offers an analysis of the importance of this mechanism for transmitting economic disturbances among countries and examines ways by which monetary authorities can ameliorate the effects of foreign monetary and fiscal policies on the domestic economy. We start by tracing out the consequences for domestic output of permanent changes in the rate of monetary expansion and
the level of government expenditure abroad under a flexible exchange rate regime. Also, we examine four other monetary policy regimes: a fixed nominal exchange rate regime, a "modified" (Taylor, 1985) nominal income rule, an inflation rate rule, and a fixed real exchange rate regime. The model used to carry out these investigations is an "extended" small open economy model -- a hybrid of the Dornbusch-Buiter and Miller small open economy models. The model involves two countries: one of the countries is the Rest of the World (ROW) -- which is effectively treated as a closed economy -- whereas the other country is a small open economy which takes as exogenous the ROW income, prices and interest rate, though these variables are determined endogenously in the ROW. The first to use such a model was Flood (1979) who studied the relative advantages of fixed over flexible rates. Later, Turnovsky (1983), used a similar model to examine optimal foreign exchange market intervention policies in response to various stochastic shocks. Both the Flood and Turnovsky models have the disadvantage of assuming purchasing power parity throughout the analysis. Harkness (1982) used the same structure in order to examine the effects of the small country's policies on the large economy. These effects arise due to the existence of the signal extraction problem. Any time the
large country's agents misinterpret changes in the exchange rate as reflecting disturbances originating in the large country instead of policy disturbances originating in the small country, the large country is not insulated by the small country's policies. In our case, perfect foresight prevails and no such effects can arise. Moreover, we concentrate on the effects of the large country's policies on the small economy rather than the other way around.

The inflation rate and nominal income rules are found to dominate all other rules in terms of preventing output and price deviations from their full equilibrium levels whenever the steady-state rate of monetary growth represented the "core" rate of inflation in the Phillips curve. However, both rules involve non-uniqueness whenever the actual rate of monetary growth (as determined by the need to keep either the price level or the nominal income on a predetermined growth path) stands for the "core" rate of inflation. Regarding the choice between fixed and flexible nominal exchange rates, clear cut answers cannot be obtained. The choice depends both on the form of the foreign disturbance and on the specification of the Phillips curve. However, some clear cut answers can be obtained when one considers particular fiscal-monetary
policy mixes in the rest of the world. For example, consider the Mundellian (Mundell, 1971) fiscal-monetary policy mix as implemented in the United States five years ago (Sachs, 1985a). Under such a policy mix fixed nominal exchange rates are found to perform better than flexible nominal exchange rates. The same holds true for the opposite policy mix (expansionary monetary-restrictive fiscal). On the other hand, flexible rates perform better than fixed if the rest of the world policies are of the form expansionary-expansionary or contractionary-contractionary.

Another clear cut conclusion is that under any of the above policy mixes in the rest of the world, a strategy of maintaining the real exchange rate constant is not feasible, since a change in the level of government expenditure abroad requires a change in the real exchange rate if an equilibrium is to be attained. Changes in only the rate of monetary growth in the ROW do not require changes in the terms of trade, and hence policies that keep the real exchange rate constant are feasible.

The importance of wage rigidities for the international transmission of economic disturbances plays a dominant role in chapter 3 as well. However, since the focus of chapter 3 is on the comparative statics effects...
of stabilization policies within a currency area, exchange rate expectations are ignored and hence the short-run dynamics arising from terms of trade changes are not investigated.

The distinction between real and nominal wage rigidity and its meaning for the smooth operation of alternative exchange rate systems was evident to Meade thirty-five years ago "....for the gold-standard mechanism to work effectively there must be 'sufficient' flexibility of money wage rates; and for the variable exchange rate mechanism to work effectively there must be 'sufficient' divorce between movements in the cost of living and movements in money wage rates...." (Meade (1951)).

However, it was only recently -- in the light of acute macroeconomic policy differences between Europe and the United States -- that the distinction between nominal and real wage rigidity was incorporated into the standard Mundell-Fleming model. Casas (1975 was probably the first to examine the implications of wage indexation for the effectiveness of fiscal and monetary policies. He showed that with full indexation (real wage rigidity) and perfect capital mobility the Mundell-Fleming results are reversed. Argy
and Salop (1979) reached the same conclusion within the context of a small open economy which allows for different degrees of money illusion in the labour market. With no money illusion (a situation that can be related to real wage rigidity), monetary policy is impotent in affecting output, but influences prices and the exchange rate. Fiscal expansion increases output and decreases the domestic price level. As the degree of money illusion increases the effects of monetary and fiscal policies become similar to the standard Mundell-Fleming results.

Many writers have attempted interpreting macroeconomic policy conflicts in the OECD since 1973 as occurring in the context of fixed real wages in Europe and Japan and nominal wage rigidity in the United States (Branson and Rotemberg (1980), Sachs (1980)). Argy and Salop (1983) extended their previous model to the two country case and allowed for differences in the degree of money illusion between the two countries. Fiscal policy is capable of producing beggar-my-neighbour effects if money illusion is present in the expanding country but not in the rest of the world. However, under similar conditions expansionary monetary policy will increase income in both countries. Finally, if there is no money illusion in the first country expansionary monetary policy will have no output effects on either country regardless of the degree of money illusion in the second country.
Allen and Kenen (1980) and Levin (1983) were the first to show the existence of beggar-my-neighbour effects of fiscal policy within a currency area. Allen and Kenen did it in a model with fixed wages and flexible prices, whereas Levin's model had both wages and prices fixed. Their models captured the institutional setting within which most of the EEC countries operate (i.e., fixed exchange rates with other EEC members and flexible exchange rates with the rest of the world).

Although many open economy investigations have involved real wage rigidity only five have modelled aggregate demand with a three-sector specification, i.e., the domestic country, the rest of the currency area and the rest of the world (Allen and Kenen, Levin, Miller and Salmon (1985) and Marston (1984, 1985)). None have considered both of these institutional realities in the way which is specified in chapter 3. In that chapter we first show that combining these two assumptions yields conclusions which differ markedly from the existing results concerning beggar-my-neighbour effects between currency area countries, and second we show that by endogenizing the rest of the world, fiscal expansion there entails beggar-my-neighbour effects on both currency area countries.

In chapter 4 the currency area setting of the previous chapter remains intact and exchange rate expectations are
again ignored. However, the emphasis is no longer on the price and output effects of monetary and fiscal policies, but on the short-run behaviour of the main financial variables (so short that prices and output remain constant).

The interdependence of central banks actions was first studied by Girton and Henderson (1976) within a two-country short-run portfolio balance model. However, if the notion of interdependence is to be fully captured one has to go beyond the conventional two-country model. In a world containing only two countries there would be no way to study the responses of the two countries to an external disturbance, all disturbances would be internal. To do that a technique used in customs union theory is employed. A model is built that contains two countries that interact with each other and with the rest of the world.

Allen and Kenen (1980), and Marston (1983) used this setting to study the short-run and long-run effects of monetary and fiscal policies and the desirability of an exchange rate union when one of the currencies of the union is subject to disturbances involving portfolio shifts between this currency (i.e., mark) and rest of the world (i.e., dollar) assets. Although Allen and Kenen used their model to study questions of interdependence they focussed their attention on the "longer run" price and
output effects of monetary and fiscal policies and to what extent the integration of goods and assets markets affects the distribution of the effects of these policies, between the countries of a monetary union.

In chapter 4 we analyze the effects of monetary policies and examine the ways in which central banks interact in response to both endogenous policy changes and to exogenous disturbances. The conventional result in open economy macroeconomics that an expansionary open market operation or an increase in the ROW interest rate will result in a depreciation of the country's currency is not always validated in one model once the responses of the other currency area country are considered. Furthermore, as expected, different methods of intervening either to peg the money supply or the interest rate in response to a policy change have different qualitative results on the variables of the system. What was not expected was the finding that under certain responses of the second currency area country, an expansionary open market operation by the first country can result in a decrease in the money supplies of both currency area countries (the offset coefficient is greater than |1|). Moreover, if both countries are net debtors then all results pertaining to the common external exchange rate can be reversed though none of the results pertaining to the other variables is affected.
Finally, in chapter 5 the study's main findings are summarized, some limitations of the analysis are noted, and suggestions for further research rectifying these limitations are offered.
FOOTNOTES

Chapter 1

1. Of course, the 'invention' of a vertical Phillips curve invalidates this argument.

2. The insulation question is not the same as the question: Under what circumstances will the variance in domestic variable x be smaller: fixed or flexible exchange rates. Indeed, it is likely that with a high degree of insulation under flexible exchange rates, internal disturbances will be transmitted abroad more readily under fixed rates. Therefore it is possible that a fixed exchange rate regime can provide a lower level of variance for the domestic economy if internal disturbances have a higher variation than external disturbances (Corden, 1972).

3. For a more extensive discussion of the themes in the next few paragraphs see Buiter and Branson (1983), Dornbusch and Fischer (1984) and Frankel (1986).

4. Some other channels that at times have appeared in the theoretical literature but are usually ignored in policy discussions (at least within the OECD) include the Laursen-Metzler effect (e.g., a deterioration of the terms of trade decreases the amount saved out of a given income and therefore reduces the external surplus), and wealth redistribution effects arising from world real interest rate changes (e.g., a rise in the real interest rate is an intertemporal terms of trade change that benefits lenders whose real income rises, and hurts borrowers). Both of these channels are likely to reverse the standard Mundell-Fleming transmission results.

5. In this study the term "currency area" refers to an arrangement in which member countries maintain fixed exchange rates between them, but each country retains its own central bank with control over its national monetary policy. This limited type of union, which Corden (1972) calls a "pseudo exchange rate union", is to be distinguished from a monetary union with a single central bank and a union wide currency.
CHAPTER 2
MONETARY POLICY RULES IN AN OPEN ECONOMY

2.1 Introduction

Whereas, there is a burgeoning literature dealing with the consequences for domestic output of a reduction in the rate of growth of the domestic money supply (Buiter and Miller, 1981, 1982, 1983) up to date¹ there has been no theoretical attempt to investigate the consequences for a small open economy of a change in either the rate of monetary growth or the level of government expenditure abroad. (For an investigation using time-series (innovation-accounting) techniques of the links between the U.S. and Canadian economies see Burbidge and Harrison (1985).)

The reason for substantial effects on domestic output as a result of say, a change in the rate of monetary expansion abroad is the same as the one causing output to be different from its full equilibrium level, following a change in the rate of domestic monetary growth. The proposition that under a floating exchange rate regime restrictive monetary policy can lead to substantial overshooting of the nominal and real exchange rates, relies in the presence of nominal stickiness or inertia
in domestic factor and product markets combined with a freely floating nominal exchange rate. Current and anticipated future monetary policy actions are reflected immediately in the nominal exchange rate, which is set in a forward-looking efficient asset market, while they are reflected only gradually and with a lag in domestic costs and good prices. Nominal appreciation of the currency therefore amounts to real appreciation -- a loss in competitiveness -- and this is the reason for the transitory (but potentially quite persistent) loss of output. Similarly, an increase in the rate of growth of the money stock or a decrease in the level of government expenditure abroad, causes an appreciation of the nominal and real exchange rate, therefore forcing aggregate demand for the domestic good to decrease, which brings about a decline in domestic output.

If one interprets an increase in the rate of growth of the money supply as expansionary monetary policy (for an interpretation as fiscal policy see Sargent and Wallace (1981) and Scarth (1986a)), then this finding is reminiscent of the well known Mundell result, that in a two-country model expansionary monetary policy by one country is a "beggar-my-neighbour" policy, whereas fiscal policy is not.
However, in this chapter we do not analyze a traditional two-country model. One of the countries is the Rest of the World (ROW) -- which is effectively treated as a closed economy -- whereas the other country is a small open economy which takes as exogenous variables the ROW income, prices and interest rate, though these variables are determined endogenously in the ROW. The first one to use such a model was Flood (1979), who studied the relative advantages of fixed over flexible exchange rates. Later Turnovsky (1983) used a similar model to examine optimal exchange rate market intervention policies in response to various stochastic shocks. Both the Flood and Turnovsky models have the disadvantage of assuming purchasing power parity\(^2\) throughout the analysis.

In this chapter we first find out that the deviations of domestic output from its full employment level following either a permanent increase in the rate of monetary growth in the ROW or in its level of government spending can be quite significant for what we consider to be a quite long "short-run". Subsequently we examine the properties of four rules that have been suggested quite often in the literature as alternatives to money supply targetting -- these rules being a nominal exchange rate rule, a nominal income rule, an inflation rate rule, and a real exchange rate rule.
The extensive literature on fixed versus flexible exchange rates is another way of looking at the issue of choosing between the money stock and the exchange rate as policy targets. Theoretical and empirical work bearing on the determination of the exchange rate has in recent years given prominence to the money supply and its growth as determinants, and consequently accounts of how monetary targets, yield control of inflation indicate that in an open economy the exchange rate is a significant part of the transmission mechanism. This consideration leads one then to ask why if the exchange rate is an important part of the transmission mechanism of monetary policy, and the link between monetary policy and the exchange rate is not precise, would not exchange rate targets produce at least as good a regime for output and inflation control as one of monetary targets. Analyses of such a question include papers by Parkin (1977), Sparks (1979), Minford (1981), Artis and Currie (1981) and Longworth and Poloz (1986). The general conclusion emerging from this literature is that there is not clear-cut evidence of superiority of one rule over another but it depends on the particular shock being examined. Our conclusion, in the case of a permanent foreign monetary shock, is that whereas the absolute
deviations of domestic output from its full employment level are about the same in both cases, under fixed exchange rates there is a boom which persists for a long time period, while under flexible exchange rates, initially there is a significant recession which is reversed into a slight boom only after some time has elapsed. In the case of foreign fiscal expansion, both under fixed and flexible exchange rates domestic output rises initially above its full employment level. However, the subsequent path of output is significantly different. More specifically, under the fixed exchange rate regime, domestic output keeps increasing beyond the initial increase for a significant time period. This is true both when the actual rate of monetary growth (as determined by the requirement for equilibrium in the foreign exchange market) stands for the "core" rate of inflation in the Phillips curve and when the steady-state rate of monetary growth stands for the "core" rate of inflation. Under flexible exchange rates, the initial increase in domestic output, following a fiscal expansion in the ROW, is higher than under fixed nominal exchange rates. Moreover, the initial increase in output subsides with the passage of time, with output returning to its steady-state level asymptotically.
Nominal income targets have become fashionable among economists during the last decade. The first to suggest nominal income targetting was Meade (1978) and has been followed promptly by Tobin (1980, 1983), Hall (1983), Vines et. al. (1983), McCallum (1984), Gordon (1983, 1985) and Taylor (1985). The main argument in favour of nominal income targets is that it would automatically call for an offsetting adjustment in money growth in response to velocity shocks and this should stabilize real income, and that it would automatically call for a reduction in real income growth in response to price shocks and this should stabilize inflation. In other words, "a nominal income rule provides the economy with a nominal anchor, and represents a reasonable compromise between real output targetting, which provides no anchor for the price level, and price level targetting, which can result in rather large movements in real output". (Longworth and Poloz, 1986.)

Nominal income targets in an open economy context have been examined by Masson (1983), Currie and Levine (1985), Aizenman and Frankel (1986a) and Longworth and Poloz. Aizenman and Frenkel's conclusion is similar to the one reached by Bean (1983) in the context of a closed economy, e.g., the more inelastic is labour supply
the smaller are the welfare losses\(^3\) associated with nominal income targets relative to money supply, price level and interest rate targets. Longworth and Poloz found that a nominal income rule dominates all other rules in terms of real output and price level variance for all shocks. Masson reached the same conclusion when the shock is a step reduction in the growth rate of the targeted variable.

In this chapter we find that a "modified" nominal income rule (Taylor, 1985) is desirable only when the steady-state rate of monetary growth stands as the "core" rate of inflation. When the actual rate of monetary growth enters the Phillips curve the rule was found involving non-uniqueness\(^4\). It should be noted that non-uniqueness appears when a conventional nominal income rule is adopted under both specifications of the Phillips curve.

Price level rules have been examined recently in the literature by Currie and Levine and Longworth and Poloz. Currie and Levine's conclusion was that a simple price level rule was the most desirable from a single economy perspective. However, a general application of this rule by all countries in the world economy would have disastrous results because of the excessive use of
monetary policy to manipulate the exchange rate in order to control inflation. Moreover, the rule was found to perform disastrously by Longworth and Poloz in a discrete-time adaptive expectations model. The difference in results was interpreted by Longworth and Poloz as reflecting the different expectations assumptions. In our case since the model involves steady-state inflation, the price level rule is translated into an inflation rate rule. Our results support the Longworth and Poloz intuition since in our continuous time, perfect foresight model, an inflation rule dominates all other rules (except the nominal income rule) in terms of deviations of output and prices from their desired levels. Again -- as was the case with the nominal income rule -- the same qualification applies. The inflation rule involves non-uniqueness when the actual rate of monetary growth enters the Phillips curve.

Real exchange rate rules have been widely adopted among developing countries as a way of isolating the foreign trade sector from macroeconomic disturbances. Recent studies of purchasing-power-parity oriented exchange rate policies include Basevi and de Grauwe (1977) and Dornbusch (1982). Dornbusch concluded that adhering to real exchange rate rules can cause increased instability of both output and prices. Basevi and
De Grauwe found policies geared towards stable exchange rates desirable, except in the case of real disturbances, that require changes in the terms of trade between countries. Our conclusion is that whereas fixed real exchange rates can be as desirable as fixed nominal exchange rates in the case of monetary disturbances they don't constitute a feasible strategy in the case of real disturbances -- the system becomes indeterminate.

The rest of the chapter examines in detail the above mentioned monetary policy rules with the main conclusion and suggestions for further research offered at the end. Two appendices are offered as well.

2.2 The Model

Consider an extended small country version of the Dornbusch-Buiter and Miller open macroeconomic model with a freely floating exchange rate and perfect capital mobility and asset substitutability. All variables other than interest rates are in natural logarithms and the coefficients are positive. Country 1 will be referred to as the home country and country 2 as the rest of the world (ROW). Dots over a variable denote time derivatives.
(2.1) \[ m_1 - p_1^C = k_1 y_1 - \lambda_1 i_1 \]

(2.2) \[ y_1 = \delta (c + p_2 - p_1) - \gamma_1 (i_1 - p_1) + \varepsilon y_2 + \phi_1 g_1 + \beta_1 \]

(2.3) \[ p_1 = \psi_1 y_1 + \dot{m}_1 \]

(2.4) \[ i_1 = i_2 + \varepsilon \]

(2.5) \[ m_2 - p_2 = k_2 y_2 - \lambda_2 i_2 \]

(2.6) \[ y_2 = -\gamma_2 (i_2 - p_2) + \phi_2 g_2 + \beta_2 \]

(2.7) \[ p_2 = \psi_2 y_2 + \dot{m}_2 \]

(2.8) \[ p_1^C = \alpha p_1 + (1 - \alpha) (c + p_2) \]

(2.8') \[ p_2^C = p_2 \]

where \( m_j \) is the nominal money stock of country \( j \), \( p_j^C \) its consumer price index, \( p_j \) its output price, \( y_j \) the deviation of its output from its capacity level (by choice of units the logarithm of capacity output is equal to zero; its level is equal to one) and \( i_j \) its nominal interest rate. \( e \) is the
nominal exchange rate, expressed as the number of units of country 1's currency per unit of country 2's currency, \( g_j \) is a measure of fiscal stance in country j, and \( \beta_j \) its autonomous component of aggregate demand.

The model embodies the assumption that exchange rate and price expectations are formed rationally, which is equivalent to perfect foresight in this nonstochastic model. The exchange rate is set in an efficient, forward-looking asset market. It can make discrete "jumps" at a point in time in response to any change in "fundamentals". Prices, \( p_j \), are predetermined (i.e., given at a point in time) but their rates of change correspond to excess demand or supply and "core" inflation.

In the short-run the model displays Keynesian features while in the long-run the usual classical properties prevail. Each country's demand for real balances varies positively with its own output, \( y_i \), and negatively with its own nominal interest rate, \( i_j \). The deflator used to convert nominal money balances to real money balances was the deflator defined by the country's consumption bundle. The careful reader will have noticed that the "real" balances are defined in terms of each country's consumer price index, while the demand for money is not a function of "real" income but of real output. We have verified that by letting money demand be
a function of real income \( = y_1 + P_1 - P_1^c \) instead of real output, \( y_1 \), would not change at all the qualitative nature of the results. (We briefly report some quantitative changes in results in Appendix 2.)

The demand for the first country's output depends on its real interest rate \( i_1 - P_1 \), on the real exchange rate, \( (e + p_2 - p_1) \), on the second country's level of real income, \( y_2 \), and on the domestic fiscal stance, \( g_1 \). Given the small country assumption for the first country, it is reasonable to treat the rest of the world effectively as a closed economy. Whereas the foreign variables are exogenous to the small country they are themselves endogenously determined in the ROW, reflecting the various policies adopted there. It is important to recognize this endogeneity when it comes to determining optimal response policies, for the home country. Anytime there is an exogenous (viewed from the home country's viewpoint) change in a foreign variable, this change is itself endogenously determined -- within the foreign country -- along with changes in other foreign variables which influence in quite different ways the home country variables. To give an example, it is not appropriate to try to trace the consequences of, say, an increase in the foreign interest rate on the domestic
economy by assuming that all other foreign variables remain unchanged; the assumed increase in the foreign interest rate will necessarily cause changes in other foreign variables, whose changes will affect the domestic economy. Moreover, any given increase in the ROW interest rate can be effected by different economic policies of the foreign government making a given change in the ROW interest rate have diverse effects on domestic variables.

The demand for the second country's output does not depend on the real exchange rate \( (e + p_2 - p_1) \) but only on its real interest rate, \( i - p_2 \), and its fiscal stance, \( g_2 \). Domestic costs (prices) are governed by an augmented Phillips curve. The augmentation term in the Phillips curve is taken to be the "core" rate of growth of the money stock. Equation (2.4) reflects the assumption of perfect capital mobility and asset substitutability between domestic and foreign bonds. Risk neutral speculators equate the uncovered interest differential in favour of the domestic country to the expected rate of depreciation of the domestic currency which -- given the assumption of perfect foresight -- is equal to the actual rate of depreciation.

The dynamics of the model can be conveniently summarized in terms of three state variables, \( \lambda_1, \lambda_2 \) and \( c \):
\[ \lambda_1 = m_1 - p_1 \]
\[ \lambda_2 = m_2 - p_2 \]
\[ c = e + p_2 - p_1 \]

Real balances or liquidity in the two countries (\( \lambda_1 \) and \( \lambda_2 \)) are backward-looking or predetermined variables. They only make discrete jumps when policy instruments change discontinuously. The real exchange rate or competitiveness, \( c \) is a forward-looking or jump variable. It jumps whenever \( e \) jumps.

Having completed the description of the model we are now ready to analyze the effects of both a change in the rate of monetary growth in the ROW and change in its fiscal stance on the home economy under different policy settings. We start with the case of freely floating exchange rates.

2.3 Flexible Exchange Rates

With complete exchange rate flexibility the rate of monetary growth is an exogenous or policy-determined variable. It is then possible to interpret the flexible exchange rate regime as a fixed rate of monetary growth regime.
2.3.1 Dynamics

The dynamics of the model can be captured by reducing the system of equations (2.1) to (2.8) to a system of three differential equations in \( \lambda_1, \lambda_2 \) and \( c \).

Combining equations (2.5), (2.6) and (2.8) we get

\[
Y_2 = \frac{\gamma_2 \dot{\lambda}_2 + \gamma_2 \lambda_2 \dot{m}_2 + \lambda_2 \phi_2 \dot{s}_2 + \lambda_2 \beta_2}{w_2}
\]  

where \( w_2 = \lambda_2 + \gamma_2 (k_2 - \gamma_2 \lambda_2) \)

Together with equations (2.1), (2.2) and (2.8), equation (2.9) gives:

\[
Y_1(w_1) = (\lambda_1 \delta - \gamma_1 (1-\alpha))c + \gamma_1 \dot{\lambda}_1 + \frac{\lambda_1 \epsilon \gamma_2}{w_2} \lambda_2 + \frac{\lambda_1 \epsilon \gamma_2}{w_2} \dot{m}_2 + \frac{\lambda_1 \epsilon \gamma_2}{w_2} \dot{s}_2 + \frac{\lambda_1 \epsilon \gamma_2}{w_2} \dot{g}_2 + \frac{\epsilon \lambda_1 \lambda_2 \psi_2}{w_2} + \lambda_1 \beta_1
\]

where \( w_1 = \lambda_1 + \lambda_1 (k_1 - \lambda_1 \psi_1) \)

\( w_1 \) and \( w_2 \) are both assumed to be positive. When \( w_1 \) and
where are greater than zero, an increase in aggregate demand raises output at a given level of competitiveness, c. This is a weak condition, which amounts to assuming that in a diagram with the nominal interest rate on the vertical axis and output on the horizontal axis, the IS curve (after using the Phillips curve to substitute out the rate of inflation) is either downward-sloping or upward-sloping and flatter than the LM curve.

Equations (2.2), (2.3), (2.4) and (2.10) give

\[
(2.11) \quad c = e + p_2 - p_1 = i_1 - i_2 + p_2 - p_1 = \left( \frac{k_1}{\lambda_2} - \gamma_2 \right) Y_1 \\
- \left( \frac{k_2}{\lambda_1} - \gamma_1 \right) Y_2 - \frac{\lambda_1}{\lambda_2} + \frac{\gamma_2}{\gamma_1} + \left( 1 - \frac{c}{\lambda_1} \right) c \\
+ \frac{m_2 - m_1}{\lambda_2}
\]

Substituting equation (2.10) to equations (2.11), (2.3) and (2.7) we get the following system of differential equations written in matrix form.

\[
(2.12) \quad \begin{pmatrix} \dot{c} \\ \dot{y} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & a_3 \\ 0 & a_4 & 0 \\ a_5 & a_6 & a_7 \end{pmatrix} \begin{pmatrix} c \\ y \\ r \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + \begin{pmatrix} -k_1 \gamma_1 \frac{1}{\lambda_1} \\ -k_2 \gamma_2 \frac{1}{\lambda_2} \\ -\frac{\lambda_1}{\lambda_2} + \frac{\gamma_2}{\gamma_1} + \left( 1 - \frac{c}{\lambda_1} \right) c \end{pmatrix} + \frac{m_2 - m_1}{\lambda_2}
\]
where \( a_1 = -\psi \frac{y_1}{w_1}, a_2 = -\psi \frac{\lambda_1 \epsilon y_2}{w_1 w_2}, a_3 = \frac{-\psi}{w_1} (\lambda_1 \delta - y_1 (1-\alpha)) \)

\[ a_5 = -\frac{\psi_2 y_2}{w_2}, \quad a_7 = \frac{k_1}{\lambda_1} - \psi_1 \frac{y_1}{w_1} - \frac{1}{\lambda_1} \]

\[ a_8 = -\frac{k_2}{\lambda_2} - \frac{\psi_2}{w_2} \frac{y_2}{w_2} + \frac{k_1}{\lambda_1} - \psi_1 \frac{\lambda_1 \epsilon y_2}{w_1 w_2} + \frac{1}{\lambda_2} \]

\[ a_9 = \frac{k_1}{\lambda_1} - \psi_1 \left( \frac{\lambda_1 \delta - y_1 (1-\alpha)}{w_1} \right) + \frac{1-\alpha}{\lambda_1} \]

The determinant of this system is

\[ \text{Det} = \frac{\psi_2 y_2 \psi_1 \delta}{w_2 w_1} > 0 \]

The trace of the system is

\[ \text{Trace} = \frac{-\psi y_1}{w_1} - \frac{\psi_2 y_2}{w_2} + \frac{k_1}{\lambda_1} - \psi_1 \left( \frac{\lambda_1 \delta - y_1 (1-\alpha)}{w_1} \right) + \frac{1-\alpha}{\lambda_1} > 0 \]

If the trace is negative the trace condition (i.e., the sum of the characteristic roots equals the value of the trace) implies that there are at most 2 positive roots. Given that the determinant condition (e.g., the product of the characteristic roots equals the
value of the determinant) implies that there are either one or three positive roots we know that there is only one positive root. In a dynamic linear model with \( n_1 \) backward looking variables and \( n_2 \) forward looking variables there exists a unique saddle path converging to the long-run equilibrium provided there are \( n_1 \) stable (negative) characteristic roots (the ones corresponding to the \( n_1 \) predetermined variables) and \( n_2 \) unstable (positive) roots (the ones corresponding to the \( n_2 \) jump variables) (Blanchard and Kahn (1980)). In our three-equation dynamic model with one jump variable (competitiveness, \( c \)) and two predetermined variables (\( \lambda_1 \) and \( \lambda_2 \)) the existence of a unique convergent saddle path requires the presence of two stable roots and one unstable root. Since, when the trace is negative we do have 1 positive and 2 negative roots, the system is a saddle path.

If the trace is positive then it is possible that there exist 3 positive roots -- the system is unstable. However, even when the trace is positive there can never be three positive roots. The reasons for this is that \( a_3 = \frac{-\lambda_2 \gamma_2}{\lambda_2} \) is negative and since one of the characteristics roots equals \( a_3 \), there will always be one negative root.
2.3.2 The Long-Run Equilibrium

The long run comparative statics in this model are completely classical or monetarist. Output in each country is at its exogenously given full employment level and changes in the growth rates of nominal money stocks are reflected into corresponding changes in the rates of growth of prices and the exchange rate. In other words, in long-run equilibrium,

\[ y_i = 0 \quad i = 1,2 \]
\[ p_i = m_i \quad i = 1,2 \]
\[ \epsilon = m_1 - m_2 \]
\[ i_1 - p_1 = i_2 - p_2. \]

The following long-run multipliers can be derived by setting the left hand side of equation (2.12) equal to zero (bars over variables denote steady-state values):

\[ \frac{\bar{\gamma}_1}{\bar{m}_1} = -\lambda_1, \quad \frac{\bar{\gamma}_2}{\bar{m}_2} = -\lambda_2, \quad \frac{\bar{\gamma}_1}{\bar{m}_2} = \frac{\bar{c}_1}{\bar{m}_1}, \quad \frac{\bar{c}_1}{\bar{m}_2} = \frac{\bar{c}_2}{\bar{m}_1} = 0 \]

\[ \frac{\bar{\gamma}_1}{\bar{g}_2} = -\phi_2 (\lambda_1 \delta - \gamma_1 (1 - \alpha)) / \gamma_2 \delta, \quad \frac{\bar{\gamma}_2}{\bar{g}_2} = \frac{-\lambda_2 \phi_2}{\gamma_2} \]

\[ \frac{\bar{c}_1}{\bar{g}_2} = \frac{\gamma_1 \phi_2}{\gamma_2 \delta}, \quad \frac{\bar{c}_1}{\bar{g}_1} = -\frac{\phi_1}{\delta}, \quad \frac{\bar{c}_2}{\bar{g}_1} = 0 \quad \frac{\bar{c}_1}{\bar{g}_1} = -\frac{\phi_1 (-\alpha)}{\delta} \]
In the long-run fiscal expansion at home worsens home competitiveness (there is crowding-out of net exports) while fiscal expansion in the ROW causes home competitiveness to improve. Neither changes in the levels nor in the rates of growth of the nominal money stocks affects competitiveness or real interest rates. ROW increases in the rate of monetary growth do not affect home country liquidity, since they affect neither home nominal interest rates nor the rate of increase of domestic prices as nominal exchange rates (the rate of change) adjust to fully insulate in the long-run any transmission of inflation from the ROW.

2.3.3 The Effects of an Increase in the Rate of Monetary Growth in the ROW

In this section we calculate the costs in terms of home output deviations from its long-run equilibrium value after an increase in the rate of monetary expansion in the ROW. We do so by using a particular set of plausible parameter values. (Their interpretation is given in Appendix I, and they are similar to the ones used by Buitel and Miller (1981, 1982, 1983). These parameter values are:
\[ \lambda_1 = \lambda_2 = 2, \quad k_1 = k_2 = 1, \quad \psi_1 = \delta = \psi_2 = \gamma_1 = \gamma_2 = .5, \]
\[ \epsilon = .4 = \phi_1 = \phi_2, \quad \beta_1 = .4925 = \beta_2, \quad \alpha = .7 \]

When these values are substituted into equation (2.12) the system becomes:
\[
\begin{bmatrix}
\dot{x}_1 \\ \dot{x}_2 \\ \dot{c}
\end{bmatrix} =
\begin{bmatrix}
-.125 & -.05 & -.2125 \\
0 & -.125 & 0 \\
-.5 & .5 & .15
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\ \lambda_2 \\ c
\end{bmatrix} + \ldots.
\]

(2.13)

The characteristic roots of the matrix in equation (2.13) are: .366, -.341 and -.125.

Dixit (1980) has shown that if the system is to travel to its new long run equilibrium \((\bar{x}_1, \bar{x}_2, \bar{c})\) along the stable manifold, it must be the case that at each point in time

\[
c(0) - \bar{c} = u_1(\bar{x}_1(0) - \bar{x}_1) + u_2(\bar{x}_2(0) - \bar{x}_2)
\]

(2.14)

where \(c(0), \bar{x}_1(0)\) and \(\bar{x}_2(0)\) are the values of these variables immediately after (still at period 0) an exogenous policy change has taken place, and \(u_1\) and \(u_2\) are the elements of the row eigenvector satisfying the following condition:
\[(2.14') \quad (u_1 u_2 -1)(-A + \rho I) = (0 \ 0 \ 0)\]

where \(\rho\) is the positive characteristic root of the matrix in equation (2.13).

By inserting the value of .366 for \(\rho\), we find the elements of the associated eigenvector to be \((u_1 u_2 -1) = (1.02, -1.12, -1)\) as may be readily confirmed.

The terms measuring initial disequilibrium following an unanticipated change in \(m_2\), denoted by \(\dot{m}_2\), can be evaluated as follows:

\[
\begin{align*}
\lambda_1(0) - \lambda_1 &= 0, \quad \text{since} \quad \frac{d\lambda_1}{dm_2} = 0 \quad \text{and} \quad \lambda_1 \text{ is a predetermined variable.} \\
\lambda_2(0) - \lambda_2 &= \lambda_2 \dot{m}_2, \quad \text{since} \quad \frac{d\lambda_2}{dm_2} = -\lambda_2 \quad \text{and} \quad \lambda_2 \text{ is a predetermined variable.} \\
c(0) - \bar{c} &= dc, \quad \text{the jump in competitiveness,} \\
\text{since} \quad dc/d\dot{m}_2 &= 0
\end{align*}
\]

With the help of equation (2.14) we then find that \(c(0) - \bar{c} = dc = u_2 \lambda_2 \dot{m}_2 = -.0224\) (assuming that there was a one-point increase in monetary growth at \(t = 0\)).

In order to study the dynamic behaviour of the variables in the system we use the "starting" values for
$\lambda_1$, $\lambda_2$ and $c$ discussed above. These values were chosen so as to place the system on the stable manifold. The behaviour of the system on the stable manifold will depend only on the stable (negative) characteristic roots. In general, the evolution of the variables over time can be described by the differential equation:

$$x(t) = A_1 e^{r_1 t} + A_2 e^{r_2 t}$$

where $r_1$ and $r_2$ correspond to the two stable roots of the system and $A_1$ and $A_2$ are arbitrary constants which can be determined from initial conditions.

Since we are mainly interested in the dynamic behaviour of home output, $y_1$, we accordingly determine the arbitrary constants $A_1$ and $A_2$ of the differential equation (2.16)

$$y_1(t) = A_1 e^{r_1 t} + A_2 e^{r_2 t}$$

Evaluating equation (16) at $t = 0$, gives

$$y_1(0) = A_1 + A_2$$

while differentiating equation (2.16) with respect to time and evaluating its first derivative at $t = 0$, gives

$$y'_1(0) = r_1 A_1 + r_2 A_2$$

Equation (2.10) is now used to find $y_1(0)$. 
When the starting values (measuring initial disequilibrium) for \( \ell_1, \ell_2 \) and \( c \) are used, we find that \( \ell_1(0) = -0.0075 \). For \( \ell_1(0) \) to be calculated, \( \ell_1(0), \ell_2(0) \) and \( c(0) \) must be determined first. With the help of equation (2.13) we find that

\[
\begin{align*}
\ell_1(0) &= 0.00376, \\
\ell_2(0) &= -0.0025, \\
c(0) &= 0.00664
\end{align*}
\]

Equation (10) is now used to find that \( \ell_1(0) = 0.0075 \).

Inserting these values for \( \ell_1(0) \) and \( \ell_1(0) \) into equations (2.17) and (2.18) we get \( A_1 = -0.01196 \) and \( A_2 = 0.00446 \). Therefore, the deviation of the home country's output from its full employment level after a one-point increase in the rate of monetary growth in the ROW can be described by

\[
(2.19) \quad \ell_1(t) = (-0.01196)e^{-0.341t} + 0.00446e^{-0.125t}
\]

Figure 1 depicts the path \( \ell_1 \) after a one-point increase in the rate of monetary growth in the ROW.

![Graph](image)
Initially, there is a drop in home output of about .75 per cent. An increase in $m_2$ raises $p_2$ and the rate of appreciation of the home country's currency by the same amount in the long-run. Hence, the real exchange rate (competitiveness) remains constant. But in the short-run the home country's exchange rate appreciates more than the change in the relative prices $p_2 - p_1$ (the real exchange rate overshoots its long-run equilibrium value) and the resulting fall in competitiveness causes a decline in home output. A year after a one-point increase in $m_2$ output in the home country is still .45 per cent lower than its full employment level. It will take 5 years for output to return to its capacity level, remaining slightly above it till it comes back to its full employment level. Looking at equation (2.3) it is obvious that domestic prices follow the same pattern as domestic output, e.g., an increase in the rate of monetary growth in the ROW results in a decrease in the rate of growth of prices at home for a prolonged time period (5 years).

Our choice of parameter values is not crucial in obtaining the above results. Experimenting with different sets of parameter values we found that only when $\delta$ (the real exchange rate elasticity of aggregate
demand) obtains very small values ($\delta \leq .1$), it is possible for the home country's output to increase slightly (.1 per cent). For $\delta$ to obtain such low values the Marshall-Lerner condition has to be barely satisfied -- the sum of the import and export elasticities has to be only slightly greater than one -- and this seems to be at variance with recent evidence (Stern, 1976).

Although the cumulative output cost from a one-point increase in $m_2$ is very small (in Figure (2.1) the loss in output in the first 5 years is about as much as the gain in output in the remaining years) the absolute deviations of output from equilibrium are quite significant and according to Buiter and Miller (1983) might be a more appropriate index of economic waste. Moreover, governments time horizons are usually not longer than 5 years, thus the early losses in output are going to be weighted heavily in their welfare calculations.

2.3.4 The Effects of Fiscal Expansion in the ROW

In this section we consider the effects of a five (5) per cent increase in the level of government spending in the ROW. Since (as can be seen in the Appendix) we assumed that the share of government expenditure in total output was 30 per cent, this 5 per cent
increase in government expenditure will raise the share of government expenditure in total output from 30 to 31.5 per cent. When these changes are expressed in logarithms, $g_2$ will increase by .05 (from -1.2 to -1.15).

The terms measuring initial disequilibrium following an unanticipated change in $g_2$, denoted by $d_g_2$, can be evaluated as follows:

$$k_1(0) - \tilde{k}_1 = \frac{\phi_2(\lambda_1 \delta - \gamma_1(1 - \alpha))}{\gamma_2 \delta} \quad d_g_2 = .068$$

$$k_2(0) - \tilde{k}_2 = \frac{\lambda_2 \phi_2}{\gamma_2} \quad d_g_2 = .08$$

$$c(0) - \bar{c} = dc(0) - \frac{\gamma_1 \phi_2}{\gamma_2 \delta} \quad d_g_2$$

With the help of equation (2.14) we then find that $c(0) - \bar{c} = dc(0) - \frac{\gamma_1 \phi_2}{\gamma_2 \delta} \quad d_g = -.02$. Decomposing this result we realize that the real exchange rate depreciates on impact (e.g., $dc(0) = .04 - .02 = .02$), but it does not overshoot its long-run equilibrium -- as was the case when the ROW monetary growth increased.

The starting values for $k_1$, $k_2$ and $c$ calculated above are now used to study the dynamic behaviour of the
system. In general the evolution of the variables over time can be described by the differential equation

\[ x(t) = A_1 e^{r_1 t} + A_2 e^{r_2 t} \]

where \( r_1 \) and \( r_2 \) correspond to the two stable roots of the system and \( A_1 \) and \( A_2 \) are arbitrary constants which can be determined from initial conditions. Performing the same tasks as in the previous section we end up with equation (2.20) describing the deviations of the home country’s output from its full employment level after a five per cent increase in the level of government spending in ROW.

(2.20) \[ y_1(t) = -0.00097e^{-0.341t} + 0.01747e^{-0.125t} \]

Figure 2.2 depicts the path for \( y_1 \) implied by equation (2.20)

![Graph of equation (2.20)](image)

**FIGURE 2.2: Flexible Exchange Rates-Fiscal Expansion in the ROW**
Initially, there is an increase in home output of about 1.65 per cent. The increase in $g_2$ raises the ROW interest rate causing an immediate depreciation of the home country's currency, which along with the increased rate of growth of foreign prices results in increased home country competitiveness. While home country competitiveness continues to improve, liquidity starts falling, causing domestic output to gradually return to its original level. It is noteworthy that even after 10 years (time is measured in years, since all the elasticities used to derive the parameter values refer to one-year periods) have passed, output is still about half a per cent greater than its full employment level.

We thus find that in the case of ROW fiscal expansion (the graph for fiscal contraction would, of course, be the mirror image of Figure 2.2) both the cumulative and absolute deviations of home output from its full employment level are significant to justify our interest in different monetary policy rules that could be employed in order to lessen the impact on domestic variables of economic policies in the ROW.
2.4 **Fixed Exchange Rates**

In order to peg the nominal exchange rate the monetary authorities must be prepared to let the money supply be endogenously determined by the requirement for equilibrium in the foreign exchange market. For the model to have a long-run equilibrium the inflation rates in the two countries have to be equal in the steady-state. This further implies that their rates of monetary expansion are equal, e.g., \( \bar{m}_1 = \bar{m}_2 \) (bars over variables denote steady state values). For the moment we assume that it is the steady-state rate of monetary expansion that enters the home country's Phillips curve (\( \bar{m}_1 = \bar{m}_2 \)) and not the actual rate of monetary expansion (during the approach to the long-run equilibrium) as determined by the requirement for equilibrium in the foreign exchange market.

The model of equations (2.1) to (2.8) remains intact, except for the change noted above. Equation (2.3) is now written as

\[
(2.3') \quad \dot{p}_1 = \gamma_1 y_1 + \bar{m}_1
\]

Along with another endogenous variable, \( \dot{m}_1 \), another equation is added:

\[
(2.21) \quad \dot{e} = 0
\]
Now the model is recursive. Equation (2.1) residually determines the domestic money supply, \( m_1 \), after the values for all other endogenous variables have been determined from equations (2.2) to (2.8) along with (2.21). The dynamics can be conveniently summarized in terms of the two state variables, \( \lambda_2 \) and \( c \). Real balances, \( \lambda_2 \), and competitiveness, \( c \), are now backward looking or predetermined variables since \( e \) can no longer make discrete jumps.

Equations (2.4), (2.6), (2.7) and (2.21) imply that

\begin{equation}
(2.22)
\frac{i_1}{\gamma_2} = -\frac{\psi_2}{\gamma_2} + \frac{\phi_2}{\gamma_2} g_2 + \psi_2 y_2 + \frac{m_2}{\gamma_2} + \frac{\beta_2}{\gamma_2}
\end{equation}

Combining equations (2.2), (2.3), (2.9) and (2.2) we get

\begin{equation}
(2.23)
(1-\gamma_1 \phi_1) y_1 = \delta c + (\psi - \gamma_1 (\psi - \frac{1}{\gamma_2})) (\frac{\psi_2}{\gamma_2} + \frac{\phi_2 m_2 + \phi_2 g_2 + \lambda_2 g_2 + \beta_2}{\gamma_2})
\end{equation}

\[- \frac{\gamma_1 \psi_2}{\gamma_2} g_2 + \phi_1 g_1 + \beta_1 - \frac{\gamma_1 \beta_2}{\gamma_2}\]

Upon substituting equation (2.23) to equation (2.7) and to the differential of the equation that defines \( c \), we can write the resulting system of differentials equations in matrix form as follows:
Since the determinant of this system is positive and the trace negative, both of its characteristic roots are negative. Given that now both $\lambda_2$ and $c$ are predetermined variables the system is stable.

In order to find the path for domestic output, $y_1$, after an increase either in the rate of monetary growth or in the level of government spending abroad we assume the following initial values for the variables of the model (more details in Appendix 1).

$$y_1(0) = y_2(0) = c(0) = 0$$

$$i_1(0) = i_2(0) = .075$$

$$p_1(0) = p_2(0) = .05$$
Substituting the assumed parameter values in the above system of equations we get

\[
(2.25) \quad \dot{\lambda}_2 + 0.125\dot{\lambda}_2 = -0.25\dot{m}_2 - 0.00625
\]

\[
(2.26) \quad \dot{c} + 0.067\dot{\lambda}_2 + 0.333c = -0.133\dot{m}_2 - 0.003
\]

The solution of the differential equation (2.25) is

\[
(2.27) \quad \lambda(t) = A_1 e^{0.125t} - 2\dot{m}_2 - 0.05
\]

Upon inserting the initial values for the variables into equation (2.27) we are able to determine the arbitrary constant \(A_1\), which is found to be

\[A_1 = 0.02\]

(We have assumed that \(\dot{m}_2\) initially was equal to 0.05 and that it became equal to 0.06 at time \(t = 0\)).

We now substitute into equation (2.26) the found solution for \(\dot{\lambda}_2\) and obtain

\[
(2.28) \quad \dot{c} + 0.333c = -0.001\dot{c}
\]
whose solution is equation (2.27)

\[
(2.29) \quad c(t) = A_2 e^{-\frac{333}{3}t} - (0.005)e^{-\frac{125}{3}t}
\]

Again using the initial values for the variables we find \( A_2 \) to be

\( A_2 = 0.005 \)

Therefore using equation (23) and the assumed parameter values, the path for \( y_1 \) is given by the following equation:

\[
(2.30) \quad y_1(t) = 0.667c(t) + 0.383k_2(t) + 0.0651
\]

Figure 2.3 depicts the path for \( y_1 \) after a one-point increase in the rate of monetary growth in the ROW. Comparing Figures (2.1) and (2.3) we observe that whereas under flexible exchange rates (a constant rate of monetary growth rule) there is our initial drop in domestic output of about .75 per cent, under fixed exchange rates there is an increase in domestic output of about .77 per cent following an increase in the rate of monetary expansion in the ROW. The main reason for this difference is, of course, the avoidance of real exchange rate overshooting to such a degree that could overturn the positive effects on domestic output caused by the initial drop in the real
interest rate and the increase in foreign output, $y_2$. If one accepts the absolute deviations

\[
0.0080
\]

![Graph showing $y_1$ over time](image)

FIGURE 2.3: Fixed Exchange Rates-Monetary Expansion in the ROW

of output from its full employment level as appropriate index of economic waste, then Figures (2.1) and (2.3) indicate that there is little to choose between fixed and flexible exchange rates. However, the revealed preference of governments around the world has been that more output is better than less.

We now examine the effects of fiscal expansion in the ROW. Equations (2.25) and (2.26) are then written as

\[
\begin{align*}
(2.31) \quad \dot{x}_2 + 0.125x_2 &= (-0.2)g_2 - 0.2587 \\
(2.32) \quad c + 0.067x_2 + 0.333c &= (0.16)g_2 + 0.1823
\end{align*}
\]
The solution of the differential equation (2.31) is

\[ 2(t) = A_1 e^{\cdot125t} - (1.6)g_2 - 2.07 \]

Upon inserting the initial values for the variables (\( g_2 \) changes from -1.2 to -1.15) we are able to determine the arbitrary constant \( A_1 \), which is found to be

\[ A_1 = .08 \]

We now substitute into equation (2.32) the found solution for \( 2 \) and obtain

\[ 333c + .333c = .005e^{\cdot125t} + .0133 \]

whose solution is

\[ c(t) = A_2 e^{\cdot333t} - .024e^{\cdot125t} + .04 \]

Again, using the initial values for the variables we find \( A_2 \) to be

\[ A_2 = -.016 \]

Using equation (2.23) we find the path for \( y_1 \) after a five per cent increase in the level of government spending in the ROW, to be given by equation (2.36),
Figure 2.4 depicts the path for $y_1$ after a five per cent increase in the level of government spending abroad.

Comparing Figures 2.2 and 2.4 we observe that in both cases output increases instantaneously, the increase under flexible rates is about four times the increase under fixed exchange rates. The reason for this is quite obvious. Under fixed exchange rates, competitiveness does not improve as much as under flexible rates. However, whereas under flexible rates output returns asymptotically to its full employment value, under fixed
exchange rates it keeps increasing for a period of about 5 years, and only then asymptotically approaches its steady-state value. Despite this, the absolute deviations of output from its full employment level are greater under flexible exchange rates than under fixed rates. On this account, fixed exchange rates insulate the domestic economy better from foreign aggregate demand disturbances.

2.4.1 The "Core" Rate of Inflation is the Actual Rate of Monetary Growth

In this section we let the actual rate of monetary growth, as determined by the requirement for equilibrium in the foreign exchange market be the relevant term in the Phillips curve of the home country. Then the model of equations (2.1) to (2.8) along with equation (2.21) is no longer recursive.

Equations (2.2), (2.3), (2.4), (2.5), (2.8), and (2.21) imply that

\[ y_1 = \delta c + \varepsilon y_2 + \gamma_1 \left( -\frac{k_2 y_2 - \lambda_2}{\lambda_2} - \psi_1 y_1 - \dot{m}_1 \right) + \phi_1 g_1 + \beta_1 \]

whereas equations (2.1), (2.4), and (2.5) give
We can then write combining equations (2.37), (2.38) and (2.7)

\[
\begin{align*}
\dot{c} & = \dot{p}_2 - \dot{p}_1 = \psi_2y_2 + \dot{m}_2 - \psi_1y_1 - \frac{(1 - \gamma_1\gamma_2)}{\gamma_1}y + \frac{\delta}{\gamma_1}c + \\
& + \frac{\gamma_1k_2}{\lambda_2} \frac{\lambda_2}{\gamma_1}y_2 + \frac{1}{\lambda_2}y_2 + \frac{\phi_1}{\gamma_1}g_1 + \frac{\beta_1}{\gamma_1}.
\end{align*}
\]

In the usual way we form the \( \dot{x}_1 = m_1 - \dot{p}_1 = -\psi_1y_1 \)

and \( \dot{x}_2 = m_2 - \dot{p}_2 = -\psi_2y_2 \) equations, which, along with

equations (2.38), (2.39) and (2.9) give us the following

differential equation system:

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{c}
\end{pmatrix}
= 
\begin{pmatrix}
a_1 & a_2 & a_3 & x_1 \\
0 & a_5 & 0 & x_2 \\
a_7 & a_8 & a_9 & c
\end{pmatrix}
+ 
\begin{pmatrix}
(a_2 + \frac{\lambda_1}{k_1\lambda_2})\lambda_2 \\
\frac{\lambda_1}{k_1\lambda_2} \frac{\lambda_2}{\gamma_1}y_2 \\
(a_8 - \frac{\lambda_1}{\lambda_2\gamma_1k_1})\lambda_2 \\
\frac{1}{\lambda_2} - \frac{\lambda_1}{\lambda_2\gamma_1k_1} \gamma_2
\end{pmatrix}
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{c}
\end{pmatrix}
+ 
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}.
\]
Where
\[ a_1 = -\frac{\psi_1}{k_1}, \quad a_2 = -\psi_1\left(\frac{\lambda_1 k_2}{k_1 \lambda_2 w_2} - \frac{\lambda_1}{k_1 \lambda_2}\right), \]
\[ a_3 = \frac{\psi_1 (1 - \alpha)}{k_1}, \quad a_5 = -\frac{\psi_2 \gamma_2}{w_2}, \quad a_7 = -\frac{1}{\gamma_1 k_1} \]
\[ a_8 = \left(\frac{\gamma_1 \psi_2 + \delta - \frac{\gamma_1 k_2}{\lambda_2}}{\gamma_1}\right) - \frac{\lambda_1 k_2}{\lambda_2 \gamma_1 k_1} \frac{\gamma_2}{w_2} + \frac{1}{\lambda_2} + \frac{\lambda_1}{\lambda_2 \gamma_1 k_1}, \]
\[ a_9 = \frac{1 - \alpha}{\gamma_1 k_1} + \frac{\delta}{\gamma_1} \]

The determinant of the system is
\[ \text{Det} = \frac{\psi_1 \psi_2 \gamma_2 \delta}{k_1 w_2 \gamma_1} > 0 \]

and its trace
\[ \text{trace} = -\frac{\psi_1}{k_1} - \frac{\psi_2 \gamma_2}{w_2} + \frac{1 - \alpha}{\gamma_1 k_1} + \frac{\delta}{\gamma_1} > 0 \]

If the trace is positive then the possibility of three positive characteristic roots arises, i.e., the system is outright unstable. However, this would not be the case under any set of parameter values, given that
a₅ is always negative. Therefore we conclude that there is only one positive root -- the one corresponding to the jump variable. \( l_2 = m_2 - p_2 \) and \( c = p_2 - p_1 \) are the predetermined variables, whereas \( l_1 = m_1 - p_1 \) is the jump variable. The domestic money supply \( m_1 \) makes discrete jumps in order to keep the exchange rate fixed.

The long-run multipliers in this model are:

\[
\frac{d\bar{x}_1}{dm_2} = \lambda_1', \quad \frac{d\bar{x}_2}{dm_2} = \lambda_2', \quad \frac{dc}{dm_2} = 0
\]

\[
\frac{d\bar{x}_1}{dg_2} = \frac{\gamma_1(1 - \alpha)\phi_2 - \delta\phi_2\lambda_1}{\gamma_2^\delta}, \quad \frac{d\bar{x}_2}{dg_2} = -\frac{\phi_2\lambda_2}{\gamma_2}, \quad \frac{dc}{dg_2} = \frac{\gamma_1\phi_2}{\gamma_2^\delta}
\]

In the steady-state the rates of growth of the money supplies in the two countries will be equal, and hence their inflation rates, the home country's long run inflation and interest rates adjusting to the ones prevailing in the ROW. It is for this reason that home country liquidity falls, whenever the rate of monetary growth rises in the ROW, under fixed exchange rates, but remains unchanged under flexible exchange rates. As expected, fiscal expansion in the ROW improves home country competitiveness, reduces liquidity in the ROW and (most probably) in the home country.
2.4.1A  The Effects of Monetary Expansion in the ROW

When the assumed parameter values are substituted into equation (2.40) the system becomes

\[
\begin{pmatrix}
\dot{\xi}_1 \\
\dot{\xi}_2 \\
\dot{c}
\end{pmatrix} =
\begin{pmatrix}
-0.5 & 0.375 & 0.15 \\
0 & -0.125 & 0 \\
-2 & 2.2 & 1.6
\end{pmatrix}
\begin{pmatrix}
\xi_1 \\
\xi_2 \\
c
\end{pmatrix} + \ldots
\]

(2.41)

The characteristic roots of this matrix are 1.445, -0.346 and -0.125.

Solving for the \( u_1 \) and \( u_2 \) from equations (2.14') and (2.41) we get

\[ u_1 = 1.029 \quad u_2 = -1.155 \]

The terms measuring initial disequilibrium following an unanticipated change in \( \dot{m}_2 \), denoted by \( \dot{d}m_2 \), can be evaluated as follows:

\[ \xi_1(0) - \xi_1 = \lambda_1 \dot{m}_2 + d\lambda_1(0) = 0.02 + d\lambda_1(0) \]
\[ \xi_2(0) - \xi_2 = \lambda_2 \dot{m}_2 = 2(0.01) = 0.02 \]
\[ d(c) - \bar{c} = 0, \text{ since } d\bar{c}/d\dot{m}_2 = 0, \text{ and } c \text{ is now a predetermined variable.} \]
With the help of equation (2.14) we then find that

\[ \hat{y}_1(0) - \bar{y}_1 = d\hat{y}_1(0) + .02 = .0224 \]

The starting values for \( \hat{y}_1, \hat{y}_2 \) and \( c \) calculated above are now used to study the dynamic behaviour of the system. The evolution of home output over time after a one-point increase in the rate of monetary growth in the ROW is described by the following equation:

\[ (2.42) \quad y_1(t) = .00525e^{.364t} + .00215e^{.125t} \]

whose graph is depicted in Figure 2.5.

![Graph](image-url)
Comparing Figures 2.5 and 2.3 we observe that they are very similar. Hence changing the specification of the Phillips curve does not lead to different qualitative conclusions.

2.4.1B The Effects of Fiscal Expansion in the ROW

The terms measuring initial disequilibrium following an unanticipated change in \( g_2' \), denoted by \( \Delta g_2 \) can be evaluated as follows:

\[
\lambda_1(0) - \bar{\lambda}_1 = d\lambda_1(0) - \left( \frac{\gamma_1 (1 - \alpha) \phi_2 - \delta \phi_2 \lambda_1}{\gamma_2 \delta} \right) \Delta g_2
\]

\[
c(0) - \bar{c} = \frac{-\gamma_1 \phi_2}{\gamma_2 \delta} \Delta g_2 = -(.8)(.05) = -.04; \quad \lambda_2(0) \bar{\lambda}_2 - \bar{\lambda}_2 = .08
\]

Using equation (2.14) we find that

\[
\lambda_1(0) - \bar{\lambda}_1 = d\lambda_1(0) - \left( \frac{\gamma_1 (1 - \alpha) \phi_2 - \delta \phi_2 \lambda_1}{\gamma_2 \delta} \right) \Delta g_2 = .0509
\]

The starting values for \( \lambda_1, \lambda_2 \) and \( c \) calculated above along with equations (2.38) and (2.9) are now used to derive equation (2.43) describing the evolution of home output over time after a five percent increase in the level of government spending in the ROW

\[
(2.43) \quad \gamma_1(t) = .01832e^{-1.25t} - .01542e^{-1.346t}
\]

whose graph is depicted in Figure 2.6.
Comparing Figures 2.4 and 2.6 we reach the same conclusion reached when Figures 2.3 and 2.5 were compared. The specification of the "core" rate of inflation in the Phillips curve is not important (at least qualitative results do not change) for the dynamics of home output when either foreign final or monetary disturbances are concerned. We observe that when the actual rate of monetary growth enters the Phillips curve the initial increase in domestic output brought about by fiscal expansion in the ROW is smaller (than what would
have been the case with the steady-state monetary growth rate standing for the core rate of inflation) because home country liquidity starts falling due to the restrictive monetary policy required to keep the domestic currency from depreciating.

2.5 **Nominal Income Targetting**

In this section we will be looking at the properties of what Taylor (1985) called a "Modified GNP rule". In a strict sense this is not a nominal income rule. The modified GNP rule has the level of real GNP reacting on a one-to-one basis to inflation, that is:

$$ y_l + \dot{p}_l = \text{constant} = \epsilon $$

The reasons we examine this modified rule are, first it has been found to display better dynamic properties than the conventional nominal income rule (e.g., $y_l + \dot{p}_l = \epsilon$) (Taylor, 1985) and second, in our model in at least one of the specifications of the Phillips curve we examine, provides us with a definite answer.

Since in the steady-state, $y_l = 0$, the above rule allows the model to have a long-run equilibrium
only when the constant \((=c)\) is equal to the steady-state rate of monetary growth, \(\bar{m}_1\), e.g.,

\[(2.44) \quad \ddot{p}_1 + \ddot{y}_1 = \bar{m}_1\]

2.5.1 **The Steady-State Rate of Monetary Growth Is The "Core" Rate of Inflation**

In this case it is evident from equations (2.44) and (2.3) that output is always at its full employment level. The paths of \(\ell_2\) and \(c\) are such that home output remains forever at its steady-state value. In our perfect-foresight model the modified nominal GNP rule clearly outperforms the other rules we have thus far examined. However, this would not be the case had we adopted the conventional nominal income rule. Under the conventional nominal income rule the system is characterized by non-uniqueness (there are fewer positive characteristic roots than "jump" variables). The same non-uniqueness "problem" obtains when the actual rate of monetary growth enters the Phillips curve even when the modified nominal GNP rule is adopted. We now look at this case.
2.5.2 The Actual Rate of Monetary Growth is the "Core" Rate of Inflation

Equations (2.44) and (2.3) now imply that

\[(2.45) \quad (1 + \psi_1)Y_1 = \bar{m}_1 - \bar{m}_1\]

Combining equations (2.1), (2.2), (2.8), and (4.5) we get

\[(2.46) \quad xy_1 = \delta c + \varepsilon y_2 + \frac{\gamma_1}{\lambda_1} \ell_1 - \frac{\gamma_1(1-a)}{\lambda_1} c + \phi_1 g_1 + \quad + \gamma_1 m_1 + \beta_1\]

where \(x = 1 + \frac{\gamma_1 k_1}{\lambda_1} + \gamma_1\)

Using the definition of c we obtain

\[(2.47) \quad \dot{c} = \dot{e} + \dot{p}_2 - \dot{p}_1 = \left(\frac{k}{\lambda_1} + 1\right) y_1 - \left(\frac{k_2}{\gamma_2} - \psi_2\right) y_2 - \frac{\ell_1}{\lambda_1} + \frac{\ell_2}{\lambda_2} + \bar{m}_2 - \bar{m}_1 + \frac{l-a}{\lambda_1} c\]

Substituting equation (2.46) to equations (2.47), (2.3) and (2.7) we end up with the following system of differential equations in \(\ell_1, \ell_2\) and \(c\),
\[
\begin{vmatrix}
\begin{array}{ccc}
\varepsilon_1 & a_1 & a_2 & a_3 \\
0 & a_5 & 0 & v_2 \\
a_7 & a_8 & a_9 & c
\end{array}
\end{vmatrix}
\]

\[ v_2 = (2.4S) \]

where \( a_1 = \frac{-\psi \gamma_1}{\lambda_1 x} \), \( a_2 = \frac{-\psi \varepsilon \gamma_2}{xw_2} \), \( a_3 = -\psi_1 \left( \frac{\delta}{x} - \frac{\gamma_1 (1 - \alpha)}{\lambda_1 x} \right) \), \( a_5 = \frac{-\psi \gamma_2}{w_2} \), \( a_7 = (\frac{k_1}{\lambda_1} + 1) \frac{\gamma_1}{\lambda_1 x} - \frac{1}{\lambda_1} \), \( a_8 = -\left( \frac{k_1}{\lambda_1} - 1 \right) \frac{\gamma_1}{\lambda_1 x} + \left( \frac{k_1}{\lambda_1} + 1 \right) \frac{\varepsilon \gamma_2}{xw_2} + \frac{1}{\lambda_2} \), \( a_9 = \left( \frac{k_1}{\lambda_1} + 1 \right) \left( \frac{\delta}{x} - \frac{\gamma_1 (1 - \alpha)}{\lambda_1 x} \right) + \frac{1 - \alpha}{\lambda_1} \).

The determinant of this system is

\[ \text{Det} = \frac{\psi_2 \gamma_2 \psi_1 \delta_1}{w_2 \lambda_1 x} > 0 \]

and its trace is

\[ \text{Trace} = a_1 + a_5 + a_9 \geq 0. \]
If the trace is negative then there exists only one positive root. Given that the system possesses two "jump" variables, \( l_1 \) and \( c \), the problem of non-uniqueness arises. If the trace is positive then it is possible that there exist 3 positive characteristic roots -- the system is outright unstable. However, we know that this cannot be true since one negative root will always exist as long as \( a_5 \) is negative. Therefore (the) non-uniqueness will always be present in this model. It should be noted that non-uniqueness in itself is not a necessarily unattractive feature of the nominal income rule. In practice, the economic authorities peg the nominal value of a variable, and this should be enough to remove the non-uniqueness "problem" (Patinkin, 1965).

2.6 Pegging the Inflation Rate

Pegging the inflation rate in an economy with steady-state inflation is equivalent to pegging the price level in an economy without steady-state inflation. In our perfect foresight, expectations augmented Phillips curve, natural rate model, it is evident that pegging the rate of growth of prices at a level equal to the steady-state rate of monetary growth results in
having the domestic output being forever at its full employment level. When the actual rate of monetary growth enters the Phillips curve, then, the inflation rate rule displays the same non-uniqueness "problems" that the nominal income rule displays.

The attractiveness of the inflation rate rule (when the steady-state \( \dot{m} \) enters the Phillips curve) should not be overstated however. Currie and Levine (1985) found a price level rule to dominate nominal income, money supply and exchange rate rules in the context of a small open economy rational expectations model. Nevertheless, a general application of a price level rule by all countries in the world economy produces disastrous results. This arises because of excessive use of monetary policy to manipulate the exchange rate to achieve domestic price stability, a "beggar-my-neighbour" policy which individual countries might pursue, but is infeasible in the aggregate.

2.7 Fixed Real Exchange Rates

Fixing the real exchange rate -- in other words following purchasing power parity (PPP) oriented exchange rate policies -- has been accepted as a way of isolating the foreign trade sector from macroeconomic disturbances.
By and large such policies have been accepted as sensible, implicitly placing a high cost on the variability of real exchange rates.

In this section we question the desirability of PPP oriented exchange rate policies. In particular we find that whereas in the case of foreign monetary disturbances, fixing the real exchange rate is a policy as good as fixing the nominal exchange rate (but worse than nominal income and price level rules under certain specification of the Phillips curve) in the case of foreign fiscal ("real") disturbances, fixing the real exchange rate renders the system indeterminate. The fundamental reason for this is that foreign fiscal disturbances require changes in the real exchange rate for a new equilibrium to be established while foreign monetary disturbances do not.

2.7.1 **Foreign Monetary Expansion**

Setting $c = e + p_2 - p_1 = 0$, results in real interest rate equalization between the two countries, e.g.,

$$i_1 - \dot{p}_1 = i_2 - \dot{p}_2$$

(2.49)

The model is now recursive. Equations (2.2), (2.5), (2.6) (2.7), (2.8), (2.8') and (2.49) can now determine the behaviour
of domestic output after a one-point increase in the rate of monetary growth in the rest of the world.

Putting together equations (2.6) and (2.49) we get

\[ i_1 - \dot{p}_1 = \frac{-Y_2}{\gamma_2} + \frac{\phi_2}{\gamma_2} g_2 + \frac{\beta_2}{\gamma_2} \tag{2.50} \]

Substituting equation (2.50) to (2.2) we end up with

\[ y_1 = \delta \bar{c} + (\varepsilon + \frac{\gamma_1}{\gamma_2}) Y_2 - \frac{\gamma_1 \phi_2}{\gamma_2} g_2 - \frac{\gamma_1}{\gamma_2} \beta_2 + \phi_1 g_1 + \beta_1 \tag{2.51} \]

or

\[ y_1 = \delta \bar{c} + (\varepsilon + \frac{\gamma_1}{\gamma_2}) (\frac{\gamma_2 \phi_2 + \gamma_2 \lambda_2 m_2 + \lambda_2 \phi_2 g_2 + \lambda_2 \beta_2}{w_2}) - \frac{\gamma_1 \phi_2}{\gamma_2} g_2 - \frac{\gamma_1}{\gamma_2} \beta_2 + \phi_1 g_1 + \beta_1 \tag{2.52} \]

In the usual way we form

\[ \dot{\lambda}_2 = m - \dot{p}_2 = -\psi_2 Y_2 = \psi_2 (\frac{\gamma_2 \phi_2 + \lambda_2 \phi_2 g_2 + \lambda_2 \beta_2}{w_2}) \tag{2.53} \]

or

\[ \dot{\lambda}_2 + .125 \lambda_2 = -.25 m_2 - .00625 \text{ whose solution is} \]

\[ \lambda_2(t) = A_1 e^{-0.125t} - 2m_2 + 0.5 \]
Therefore using equation (2.52) and (2.54) we find the initial change in home output often a one-point increase in $m_2$ to be $y_1(0) = .007$. The equation describing the path for domestic output is

$$y_1(t) = .35t + .0595,$$

whose graph is depicted in Figure 2.7.

![Graph of $y_1(t)$](image)

**FIGURE 2.7: Fixed Real Exchange Rate-Monetary Expansion in the ROW**

The similarity between Figures 2.3, 2.5, and 2.7 is attributable to the fact that the change in the real exchange rate is not substantial in both cases (of course, the change is equal to zero when the authorities
peg the real exchange rate). Under variable real exchange rate, the loss in competitiveness (in the short-run) is offset by lower real interest rates, than would have been the case under a fixed real exchange regime.

2.7.2 Foreign Fiscal Expansion

We have seen that fiscal expansion in the ROW requires a change in the real exchange rate for a new long-run equilibrium to be reached. This is obviously evident when we look at equation (2.51). Any change in \( g_2 \) will not allow output, \( y_1 \), to return to its full employment level unless there is a change in \( c \) (the real exchange rate). Therefore, the system becomes indeterminate. We can make this point more formally by using equations (2.3), (2.7), and (2.52) to form the following system of differential equations.

\[
\begin{bmatrix}
\dot{\lambda}_1 \\
\dot{\lambda}_2
\end{bmatrix}
= \begin{bmatrix}
0 & -(c + \gamma_1/\gamma_2)\gamma_2/w_2 \\
0 & -\varphi_2\gamma_2/w_2
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\
\lambda_2
\end{bmatrix}
+ \ldots
\]

(2.55)

The determinant of this system is equal to zero -- the system is indeterminate. This conclusion (pegging the real exchange rate is neither a desirable nor a feasible policy
in the face of "real" disturbances) has been reached before outside the confines of the present model by Basevi and de Grauwe (1977) and Bruce and Purvis (1984). In addition, Dornbusch (1982) did not find the PPP rule particularly desirable in a model with staggered wage contracts.

2.8 Conclusion

We have constructed an asymmetrical two-country model (one of the countries is small enough to take as given the other country's variables) of the Dornbusch-Buiter and Miller variety to study the properties of five monetary policy rules. The small country is the home country and the large country is the ROW whose monetary and fiscal policy effects the home country monetary authorities try to ameliorate, by instituting one of the following monetary policy regimes: fixed rates of monetary growth, fixed nominal exchange rates, fixed real exchange rates, nominal income targets and inflation rate targets.
The inflation rate and nominal income rules were found to dominate all other rules in terms of preventing output and price deviations from their full equilibrium levels whenever the steady-state rate of monetary growth represented the "core" rate of inflation in the Phillips curve. However, both rules involve non-uniqueness whenever the actual rate of monetary growth (as determined by the need to keep either the price level or the nominal income on a predetermined growth path) stands for the "core" rate of inflation.

Regarding the choice between fixed and flexible nominal exchange rates, clear-cut answers cannot be obtained. The choice depends both on the form of the foreign disturbance and on the specification of the Phillips curve. However, some clear-cut answers can be obtained when one considers particular fiscal-monetary policy mixes in the ROW. For example, consider the Mundellian (Mundell, 1971) fiscal-monetary policy mix as implemented in the United States five years ago (Sachs, 1985a). Under such a policy mix fixed nominal exchange rates were found to perform better than flexible nominal exchange rates. The same holds true for the opposite policy mix (expansionary monetary-restrictive fiscal). Not surprisingly, flexible rates perform better than fixed when the ROW policies are of the form expansionary-expansionary or contractionary-contractionary.
Another clear cut conclusion is that under any of the above policy mixes in the ROW a strategy of maintaining the real exchange rate constant is not feasible, since a change in the level of government expenditure abroad requires a change in the real exchange rate if an equilibrium is to be attained. Changes in the rate of monetary growth in the ROW do not require changes in the terms of trade, and hence policies that keep the real exchange rate constant are feasible but not particularly desirable -- the home output deviations from its full employment level are as large as under a nominal exchange rate rule.
APPENDIX 1

This appendix derives the structural equations for the home country from more traditional equations (in level form) and interprets the different parameters in terms of this more traditional model.

We begin with an expression relating output to total domestic expenditure, exports and imports:

\[(A1) \quad Y_1 = C(Y^1_1, r) + X_1(Y_2, T) - T.IM (Y_1, T) + G_1\]

All variables are expressed in level form. \(Y_1\) and \(C_1\) are domestic output and expenditure; while \(X\) is the demand for exports of country by country 2 and \(1M\) is the demand for imports from country 2 by country 1. \(T\) stands for the terms of trade between country 1 and country 2. We assume that trade is initially balanced so that initially \(X = T.1M\), and that \(T\) is initially equal to one.

We now define the following elasticities, where the partial derivative with respect to the first (second) argument of a function has the superscript 1(2):

\[h = \text{income elasticity of foreign demand for country 1's good,}\]
= \frac{x_1^{1}(y_2/x)}{\theta_1}

n_f = \text{price elasticity of foreign for country 1's good,}

= \frac{x_1^{2}(T/X)}{n}

n = \text{price elasticity of domestic demand for the foreign good,}

= -\frac{IM_1^{2}(T/IM_1)}{n}

Also, define $\theta_1$ equal to the sum of the marginal propensities to save and to import by country 1,

$\theta_1 = 1 - C_1^{1} + IM_1^{1} > 0$. Then equation (A1) can be written in terms of percentage changes as follows:

\begin{align*}
\frac{dY_1}{Y_1} &= \frac{1}{\theta_1} \left( h \frac{x_1}{Y_1} \frac{dY_2}{Y_2} + (n_f + n-1) \frac{x_1}{Y_1} \frac{dT}{T} \right. \\
&\quad \left. + \frac{G_1}{Y_1} \frac{dG_1}{G_1} + C_1 \frac{x_1}{Y_1} \frac{d_1}{d_1} \right) \frac{dY_1}{Y_1} \\
\end{align*}

Thus the coefficients of the aggregate demand equation in the text, which is expressed in logarithms, can be written as:

$\delta = (n_f + n-1) \frac{x_1}{Y_1} / \theta_1$

$\epsilon = h \frac{x_1}{Y_1} / \theta_1$
\[ \gamma = c_1^2 \frac{r}{y_1} \frac{1}{\theta_1}, \text{ the interest rate semi-elasticity of aggregate demand.} \]

\[ \phi = \frac{G_1}{y_1} / \theta_1 \]

The Marshall-Lerner condition for the trade balance between countries 1 and 2 is \[ n_f + n - 1 > 0, \] a condition which is necessary for \( \delta \) to be positive.

The equation describing equilibrium in the money market (equation (1)) can be derived from the following equation expressed in levels:

\[ \frac{M_1}{P_1} = k_1 \lambda_1 e^{-\lambda_1 y_1} \]

It is then evident that \( k_1 \) and \( \lambda_1 \) are the income elasticity of money demand and the interest rate semi-elasticity of money demand, respectively.

In order to get the parameter values stated earlier in this chapter we have assumed that

1) The marginal propensity to consume (= \( c_1 \)) is the product of the propensity to consume out of the disposable income (assumed to be .9) times one minus the overall tax rate (taken as .4). The overall tax rate stands for the rate at which taxes increase (and transfer payments decrease) with income, for all levels of
government combined (see Blinder and Solow (1973))

2) The marginal and the average propensity to import are the same and equal to the share of imports (= exports) in domestic output. This share is 30 per cent.

3) The share of government expenditure in total output is 30 per cent.

4) By choice of units the full employment level of home output is equal to one (so that its logarithm is equal to zero), the nominal interest rate is equal to .075 and the rate of monetary growth equals to .05.

5) All prices and exchange rates are initially set to unity (so that the logarithms are zero).

6) The sum of the import and export price elasticities is equal to 2.266 -- a value consistent with the findings of empirical studies (Stern, 1976).

7) The income elasticity of money demand is equal to one (for supporting evidence see Laidler, 1985).

8) The interest rate elasticity of money demand is -.15 (for studies using similar values for this elasticity, see Hall (1977) and Bhandari, Driskill
and Frenkel (1984), among others). Since the semi-
elasticity of money demand with respect to the interest
rate, is the ratio \( \lambda_1 \) of its elasticity and the equilibrium
interest rate \((-0.075)\), \( \lambda_1 = 2 \).

9) The interest rate elasticity of aggregate
demand is \(-0.0285\). That this value is a sensible one
can be seen from the fact that when inserted in a closed
economy multiplier formula given by

\[
\frac{dY}{dG} = \frac{1}{1 - MPC + n_{m,y} \frac{n_{y,r}/n_{m,r}}}
\]

along with the assumed values of MPC, the income elasti-
city of money demand \( n_{m,y} = 1 \) and the interest rate
elasticity of money demand \( n_{m,r} = -0.15 \) produces a
standard value for the government expenditure multiplier,
d.e., \( \frac{dY}{dG} \approx 1.5 \)

10) The income elasticity of foreign demand
for the home country's exports is unity.

11) Domestic goods prices fall, ceteris
paribus, by half a percentage point for each one per cent
reduction in domestic output, e.g., \( \psi_1 = .5 \) (see Buiter
and Miller, 1981). In other words, we have assumed that
it takes two years for complete adjustment of prices to
output changes. For a similar interpretation see Bhandari, Driskill and Frenkel.

For convenience, the parameter values pertaining to the second country have been set equal to the ones assigned to the first (home) country. We have verified that none of the qualitative results would be different had we assumed different parameter values for the second country, arising say, from a higher propensity to spend on domestic goods in the second country. For example, assuming that $\gamma_2 = .83$, $\phi_2 = .65$ and $\beta_2 = .8$ (these values are attained when $\theta_2 = 1 - C_1^2 + IM_1^2 = .46$) an increase in the rate of monetary growth in the ROW results in a drop in domestic output of about .55 per cent under flexible exchange rates in the first instant and an increase of about .83 per cent under fixed exchange rates. Moreover, the paths depicting the behaviour of domestic output in these two cases are similar to the ones depicted by Figures 2.1 and 2.3, respectively.
Equation (2.1) has the nominal money balances deflated by the consumer price index (CPI), while nominal output has been deflated by the producer's price index. In other words, money demand has been assumed to be a function of real output and not of real income. We now briefly explore the consequences of this assumption.

Money demand is a function of real income, \( y_1 + p_1 - p_1^C \) and the nominal interest rate, i.e.,

\[
m_1 - p_1^C = k_1 (y_1 + p_1 - p_1^C) - \lambda_1 i_1
\]

or

\[
m_1 - \alpha p_1 - (1-\alpha)(e + p_2) = k_1 (y_1 + p_1 - \alpha p_1 + \lambda_1 i_1
\]

or

\[
m_1 - p_1 = k_1 y_1 - \lambda_1 i_1 + (k_1 - 1)(\alpha - 1)(e+p_2-p_1)
\]

This last equation would be exactly the same as equation (2.1) if \( \alpha = 1 \) and \( k_1 = 1 \). Since \( k = 1 \) is hardly a controversial assumption (and anyway has been used in our numerical analysis) one can easily analyze the consequences of allowing real income to enter the money demand equation, by setting \( \alpha = 1 \) in our model in the text.
Doing so we find that at least for the flexible exchange rate (or constant rate of monetary growth) case the quantitative changes in the results is rather important, though the qualitative nature of the results remains intact. More specifically, in the case of a foreign increase in the rate of monetary growth, domestic output falls by 1.13 per cent (in the first instant) compared to the .75 percentage drop found in the main text (Figure 2.1). When there is fiscal expansion in the ROW domestic output increases by 2.24 per cent (in the first instant) when $\alpha = 1$, compared to a 1.65 per cent increase found in the main text (Figure 2.2). However, in both cases the approach of domestic output towards its steady-state value is very similar to the one depicted in Figures 2.1 and 2.2.

When the nominal exchange rate is kept fixed and the steady-state rate of monetary growth enters the Phillips curve the change in the specification of the LM curve does not affect at all the dynamic behaviour of domestic output, the reason being the recursiveness of the model. In all other policy regimes we have examined, only the quantitative nature of the results changes (at most).
FOOTNOTES

Chapter 2

1. This statement needs qualification. Prachowny (1984) has enquired about the long-run effects of ROW policies on small open economy variables by explicitly modelling the ROW. However, only scant attention is paid to comparative dynamics and the properties of alternative monetary policy rules are not examined in any detail.

2. One of the monetary policy rules we examine in this chapter involves keeping the real exchange rate constant, e.g., purchasing power parity prevails. However, in our model it is the actions of monetary authorities that keep the real exchange rate constant (unlike the Flood and Turnovsky models where purchasing power parity prevails as a result of uninhibited actions by private agents) and hence the domestic money supply is endogenously determined by the requirement for equilibrium in the foreign exchange market.

3. Although we conceive the present study as an exercise in positive economics, judgement is passed occasionally about the "goodness" of the particular monetary policy rules we examine. This judgement is based on the assumption that the objective of monetary authorities is to minimize the deviations of domestic output from its full employment level. Aizenman and Frenkel (1986b) provide underpinnings for such an objective function, and use it for evaluating the performance of different monetary policy rules.

4. This finding is true even without the presence of the "Flood" effect (e.g., the explicit modelling of the second (large) country).

5. In all the policy regimes we examine, the values of the parameters remain unchanged -- the Lucas (1976) critique is ignored.

6. For example, letting $\lambda_1 = \lambda_2 = 3$, $\gamma_1 = \gamma_2 = .5$, $\delta = 3$, $\psi_1 = \psi_2 = 1$, $k_1 = k_2 = .5$, $\varepsilon = .5$, $\alpha = .7$ will not practically change at all the path for domestic output as depicted by Figure 2.1.
7. Integrating equation (2.19) we can find that the early output losses are approximately equal to the later output gains. However, if a reasonable (i.e., .03) discount factor is used, then the early output losses outweigh the somehow distant gains to a large degree.

8. For different specifications of the Phillips curve see Buiter and Miller (1985).
CHAPTER 3
STABILIZATION POLICY WITHIN A CURRENCY AREA

3.1 Introduction

Few macro models that are suitable for analyzing the stabilization policy options for European countries exist. This paper attempts to cover some gaps in this limited literature. The reason for the inapplicability of most standard models is that the European monetary system involves both a fixed exchange rate with other EEC members (i.e., within the currency area), and a flexible exchange rate with the rest of the world. As a result, an appropriate model requires an aggregate demand sector with three components (the domestic country, the rest of the currency area, and the rest of the world), and an aggregate supply sector that is consistent with empirical studies of European wage behaviour. These studies (see for example, Sachs (1980) and Branson and Rotemberg (1980) indicate that real wage rigidity is appropriate for European countries (but not for the U.S.).

Many open economy investigations have involved real wage rigidity (e.g., Argy and Salop (1983) and
Kouri (1982)), but only five have modelled aggregate demand with the three-sector specification mentioned above (Allen and Kenen (1980), Levin (1983), Miller and Salmon (1985) and Marston (1984, 1985)). None have considered both of these institutional realities in the way which is specified below. We show that combining these two assumptions yields conclusions which differ markedly from the existing results concerning beggar-by-neighbour effects between currency area countries.

At least two strands of macroeconomic literature that have been applied to Europe predict some version of beggar-my-neighbour effects. In the Kouri and Argy/Salop models, an expansion of demand in the rest of the world causes a rise in the world interest rate. This contracts aggregate demand in the second country (Europe), and the resulting depreciation of the second country's (Europe's) currency shifts its aggregate supply curve to the left, since wages react to the higher cost of living. Thus, stagflation occurs in the second country following expansionary demand policy in the first, and this requires the assumptions of flexible prices and real wage rigidity.

In the other branch of literature that has been applied to Europe, the rest of the world is considered exogenous for the currency area countries so that the world
rate of interest is given for the remaining two sectors which represent Europe. Levin has essentially taken the (fixed wage and price) Mundell-Fleming model with perfect capital mobility and asset substitutability and a flexible exchange rate and split the small country into two countries with pegged rates between them. If the two countries have symmetrical responses (that is, similar income elasticities of money demand, similar marginal propensities to import, etc.), then the currency area as a whole can be analyzed as a single small country with a floating exchange rate (as in the standard Mundell-Fleming model). Thus an increase in government spending by either country within the currency area or an increase of world demand for one country's exports will lead to appreciation and no change in the area's income. Since either of these disturbances is asymmetrical within the currency area -- that is, it affects one country more strongly than the other -- that country's income must rise relative to the other country's income. Since total income is unchanged a rise in one country's income implies a fall in the other country's income.

By contrast a symmetrical disturbance in Levin's model -- such as a rise in the world interest rate or a rise in world income -- will affect the incomes of the
currency area's countries equally (assuming, again, that they are symmetrical). For example, a rise in world income raising demands for both countries' exports equally, will cause the area's currency to appreciate and prevent any rise in the area's total income. If the countries are symmetrical, neither country's income will change. Only asymmetries between the two countries will result in a rise in one country's income and a fall in the other.

It should be noted that Levin's general findings do not require his assumption of fixed prices only the assumption of nominal wage rigidity. Most of Levin's results had already been derived by Allen and Kenen in what they called the Keynesian variant of their model (flexible prices and fixed nominal wages).

The purpose of this chapter is to add supply-side effects of the exchange rate and real wage rigidity to the basic three-sector specification of the Allen and Kenen and Levin models, and to show (contrary to the earlier results) that beggar-my-neighbour effects from one member of the currency area to the other do not occur.

Marston's papers have extended this existing literature in other ways. In his 1984 paper, Marston considered the benefits that exist for a small country
joining a currency union. However, the country considered is too small to have any effect on events even in the rest of the currency area. Another feature is that both the rest of the currency area and the rest of the world are constrained to have the same wage behaviour (e.g., both involving real wage rigidity or both involving nominal wage rigidity). This is inconsistent with the empirical work discussed above. Finally, Marston's paper considers only the effects of unanticipated stochastic shocks, and to address the issue of perverse effects following from a deliberate and lasting fiscal policy we prefer to examine permanent shocks. One appealing feature of Marston's work which we have not allowed for is that short-run exchange rate overshooting is considered. However, to achieve this feature, Marston specifies sticky wages in the short-run, with the contract wage being adjusted according to the error made in forecasting prices, even though labour supply is entirely independent of prices (see Kenen's (1984), p. 438) criticism of Marston's wage adjustment rule). By considering a somewhat longer time interval, and assuming wage and price flexibility within the currency area, we avoid these issues, but must therefore neglect short-term exchange rate overshooting that can stem from differential speeds
of adjustment in exchange rates and the prices of goods and labour. Marston's 1985 paper extends his analysis to consider imperfect asset substitutability, but at the expense of having to impose purchasing power parity. Also the only shocks considered are portfolio shifts.

We wish to focus on lasting fiscal policies and the possibility of beggar-my-neighbour effects between members of a currency union that are both of significant size. In this respect, our model is appropriate for considering the relationships between countries like England, France or Germany, while Marston's analysis is more appropriate for countries like Greece and Portugal. Given our focus, and our wish to have less nominal wage flexibility in the rest of the world (the US) than in the rest of the currency area, we have chosen to extend Levin's and Allen and Kenen's basic model.

The remainder of this chapter is organized as follows. The structure of the basic model is explained in section 2, where for comparability with Levin, the marginal physical product of labour is held constant and the rest of the world variables are exogenously determined. We examine the effects of an increase in world interest rates and of one country's stabilization policies on both countries within the currency area. In section 3 we drop
the assumption of constant marginal labour product and we find that none of the conclusions reached in section 2 is altered. In section 4 we endogenize the rest of the world to see if the Kouri and Argy/Salop predictions are sensitive to there being disaggregated components within the "European" part of their "world" model. Thus, in this section of the chapter we combine several institutional features stressed in each of the two existing strands of literature: real wage rigidity and supply-side exchange rate effects, combined with an endogenous rate of interest and the three-sector specification for aggregate demand. Concluding remarks are contained in section 5.

3.2 The Basic Model

The aggregate demand sector of the basic model is given by the following equations:

\[(3.1)\quad Y_1 = C^1(Y_1, r_w) + G_1 + B(P_2^d/P_1^d, Y_1, Y_2) + T^1(EP_w/P_1^d, Y_1, Y_w)\]

\[(3.2)\quad Y_2 = C^2(Y_2, r_w) + G_2 - B(P_2^d/P_1^d, Y_1, Y_2) + T^2(EP_w/P_2^d, Y_2, Y_w)\]

\[(3.3)\quad M_1 + M_2 + R = P_1^d L^1(Y_1, r_w) + P_2^d L^2(Y_2, r_w)\]

\[(3.4)\quad P_1 = \gamma P_1^d + \delta P_2^d + (1 - \gamma - \delta)EP_w\]
(3.5) \[ P_2 = \epsilon P_1^d + \phi P_2^d + (1 - \epsilon - \phi)EP_w \]

where the notation is defined as follows: \( Y \) = real output, 
\( C \) = private expenditures on goods, \( G \) = government expenditures on goods, \( T \) = net exports with the rest of the world (i.e., with countries other than the currency area partner), \( M \) = central bank's holdings of domestic securities, 
\( L \) = real money demand, \( P^d \) = price of domestically produced good denominated in the currency area's currency, \( P \) = consumer price index. The 1, 2 or w subscripts (on these variables) or superscripts (on these functions) indicate the two countries within the currency area and the rest of the world, respectively. The other variables are: 
\( B \) = country 1's net exports with the rest of the currency area (i.e., with country 2), \( r \) = rate of interest, \( \Xi \) = exchange rate defined as the amount of currency area money required to buy one unit of foreign exchange, \( R \) = total foreign exchange reserves of the currency area (measured in the currency area's monetary units). Partial derivatives are indicated by subscript numbers corresponding to each argument in the functions (e.g., the marginal propensity to consume in country 1 is \( C_1^1 \)), and the behavioural assumptions are:

\[ 0 \cdot C_1^1, C_2^2, B_3, T_1^1, T_2^2, \gamma, \delta, \zeta < 1, L_1^1, L_2^2, \beta_1, \beta_2 \]

\[ T_1^1 > 0, -1 < B_2, T_1^1, T_2^2 : 0, C_1^1, C_2^2, L_1^2, L_2^2 : 0 \]
Equations (3.1) and (3.2) are standard IS relationships, and (3.4) and (3.5) are standard price indices, so they require no explanation. Equation (3.3) is the currency area's LM relationship, which is derived by combining the two separate portfolio equilibrium conditions:

\[ M_1 + Z = P_1 L^1(Y_1, r_w) \quad \text{and} \quad M_2 + R - Z = P_2 L^2(Y_2, r_w) \]

by substituting out \( Z \), country 1's holdings of foreign exchange reserves. Perfect capital mobility is involved and speculation in foreign exchange is ignored, so the currency area takes the world rate of interest as an exogenous variable. Actual interest payments are also ignored. All these assumptions are made for two reasons: simplicity and comparability with the Levin's analysis of stabilization policy options for EEC countries.

The aggregate supply sector of the model involves labour demand conditions: \( P^d_1 A_1 = W_1 \) and \( P^d_2 A_2 = W_2 \) and labour supply responses: \( (dW_1/W_1) = \phi (dP_1/P_1) \) and \( (dW_2/W_2) = \phi (dP_2/P_2) \), where \( 0 \leq \phi \leq 1 \), \( W \) is the nominal wage, and \( A \) is the marginal product of labour (which is assumed to be constant for comparability with Levin). Levin assumed no wage indexation; that is \( \phi = 0 \). With a constant nominal wage, the labour demand conditions make \( P^d_1 \) and \( P^d_2 \) constant. Furthermore, in terms of our notation,
Levin assumed \( \gamma + \delta = \epsilon + \theta = 1 \), so that neither the domestic product prices nor the overall price indices were affected by the exchange rate. (Levin's complete model is our equations (3.1)-(3.3) with all P's set to unity.) We consider the more realistic assumptions: that real wage rigidity exists \( (\phi = 1) \), and that imported goods from the rest of the world represent a non-zero component of the consumption bundle \( ((\gamma + \delta), (\epsilon + \theta) < 1) \). These changes in specification permit a significant aggregate supply-side effect for exchange rates, which fundamentally changes the model's predictions.

Differentiating the labour demand conditions, substituting in the wage-change rules (with \( \phi = 1 \)) and the differentials of the price indices (equations (3.4) and (3.5)) and simplifying by setting the initial values of all prices and the exchange rate at unity, we derive the following aggregate supply equations:

\[
\begin{align*}
(3.6) & \quad dP^d_1 = dP_1 = dE \\
(3.7) & \quad dP^d_2 = dP_2 = dE
\end{align*}
\]

To analyze the model, we take the total differential of equation (3.1)-(3.3) and use (3.6) and (3.7) to get

\[
(3.8) \quad H.(dY_1 \ dY_2 \ dE)' = J(dG_1 \ dG_2 \ dM_1 \ dM_2 \ dr_w \ dY_w)'
\]
where
\[
\begin{bmatrix}
 k & -B_3 & 0 \\
 B_2 & \lambda & 0 \\
 L_1 & L_2 & \mu
\end{bmatrix}
\] and
\[
\begin{bmatrix}
 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 \\
 0 & 0 & 1 & 1 - (L_2^1 + L_2^2) & 0
\end{bmatrix}
\]

\[k = 1 - C_1^1 - B_2 - T_2^1; \quad 0 < k < 1\]
\[\lambda = 1 - C_1^2 + B_3 - T_2^2; \quad 0 < \lambda < 1\]
\[\mu > 0\]

The reader can readily verify the following results:

(i) \(\frac{dY_1}{dG_1} > 0, \frac{dY_2}{dG_1} > 0\);

(ii) \(\frac{dY_1}{dM_1} = \frac{dY_2}{dM_1} = 0\), and

(iii) \(\frac{dY_1}{dr_w} < 0, \frac{dY_2}{dr_w} < 0\),

and we discuss each of these results in turn.

An increase in government expenditure in the first of the currency area countries causes an outward shift of the aggregate demand schedule for that country. The resulting increase in income makes the aggregate demand schedule for the second country within the currency area shift to the right due to the trade linkage effect. At
the same time there is a tendency for interest rates to increase within the currency area and hence for the exchange rate of the currency area to appreciate. This causes the rest of the world component in the price level of both countries to fall, and since real wages are rigid the aggregate supply schedules in both countries move downwards. This brings a further increase in income in both countries. It is important to note that this analysis does not support the beggar-my-neighbour prediction \( \frac{dY_2}{dG_1} < 0 \) found in Levin's model of the EEC.

We now consider the other two results mentioned above. Monetary expansion in one of the currency area countries again shifts both aggregate demand schedules to the right, and because currency area interest rates now have a tendency to fall, the resulting exchange rate depreciation shifts both aggregate supply schedules upwards fully offsetting the "initial" increase in income in both countries. Given the absence of these supply-side effects of exchange rate changes in Levin's model, it is not surprising that he finds output increases in both countries following a monetary expansion in one of the countries.

A rise in interest rates in the rest of the world causes the currency area's exchange rate to depreciate, so that the aggregate supply schedules for both countries
shift upwards. The resulting fall in income in both countries shifts the aggregate demand schedules to the left (since $Y_1$ and $Y_2$ are shift variables for the demand curves of country 2 and 1, respectively), partly offsetting the expansionary effect on aggregate demand of the exchange rate depreciation. As a result there is a decrease in both currency area countries' income. Again the constancy of nominal wages and prices in Levin's analysis makes him conclude that a rise in interest rates in the rest of the world has expansionary effects in at least one of the currency area countries. Recognition of real wage rigidity allows the model to be consistent with the stylized fact that all EEC countries suffered from recessionary pressures following the increase in US interest rates during the early 1980s.

3.3 The Basic Model With a Variable Marginal Labour Product

In this section we relax the assumption of a constant marginal labour product in order to test for robustness of the results in the previous section. Equations (3.1) to (3.5) of the basic model remain intact. Given that later in our analysis we will have to
resort to numerical assumptions about the size of certain elasticities in order to be able to sign some of the multipliers, in developing the supply side of the model we start with an explicit production function.

Output is assumed to be produced by a Cobb-Douglas production function using labour as the variable input. Hence

\[(3.9) \quad Y_1 = L_1^{\beta_1}, \quad 0 < \beta_1 < 1\]

where \(Y\) denotes output, \(L\) labour input and \(\beta\) is a fixed parameter. Profit maximization along with the assumption that producers satisfy their demand for labour yields:

\[(3.10) \quad Y_1 = \frac{1}{\beta_1} W_1^{\pi_1} \frac{1}{P_1^{\pi_1}} \frac{1}{d_1^{\pi_1}} \pi_1 = \frac{\beta_1}{\beta_1-1} < 0\]

where \(P^d\) and \(W\) denote the product price and the nominal wage rate, respectively. Substituting for \(W\), the wage indexation rule (i.e.,

\[
\frac{dW_1}{W_1} = \frac{d(\gamma P_1^d + \delta P_2^d + (1-\gamma-\delta)E_{P,W})}{\gamma P_1^d + \delta P_2^d + (1-\gamma-\delta)E_{P,W}}
\]

setting all prices and output equal to unity (initially) -- the implied initial value for \(W\) is then \(W_1 = \beta_1\) -- and totally differentiating equation (3.10) we get
To analyze the model we take the total differential of equations (3.1) to (3.5) and use equations (3.11) and (3.12) to get

\begin{equation}
\begin{array}{c}
\text{(3.11)}
\\quad dY_1 = (\gamma - 1)\pi_1 dP_1^d + \delta \pi_1 dP_2^d + (1-\gamma-\delta)\pi_1 dE \\
\text{and}
\\quad dY_2 = \varepsilon \pi_2 dP_2^d + (\theta-1)\pi_2 dP_2^d + (1-\varepsilon-\theta)\pi_2 dE
\end{array}
\end{equation}

\begin{equation}
\begin{array}{c}
\text{(3.13)}
\quad u(dY_1\ dY_2\ dP_1^d\ dP_2^d\ dE)' = Z(dG_1\ dG_2\ dM_1\ dM_2\ dr_w\ dY_w')
\end{array}
\end{equation}
where \( x = (1-\gamma-\delta)L^1 + (1-\varepsilon-\theta)L^2; \quad x > 0 \)

It can be readily verified that the determinant of the system, \( \det u \), is positive. Nevertheless, in deriving some of the results, stated below, we had to make certain assumptions (given the empirical evidence at hand, we regard them as quite weak assumptions). We now explain using as an example, the \( \frac{dY_2}{dG_1} \) multiplier, what the nature of these assumption is.

After some simple manipulations the expression for the \( \frac{dY_2}{dG_1} \) multiplier becomes

\[
\frac{dY_2}{dG_1} = (-B_2(\pi_1\pi_2((1-\gamma)(1-\varepsilon)-\delta\varepsilon)) - \varepsilon(1-\varepsilon-\theta)\pi_2(n_{x_{21}} + n_{M_{21}} - n_{x_{2w}} - n_{M_{2w}}))(L^1 + L^2)/\det U
\]

where \( n_{x_{2i}} \) and \( n_{M_{2i}} \) \((i = 1, w)\) are the export and import price elasticities of the second currency area country with the first currency area country and the rest of the world, respectively. (These elasticities appear in the above expression, since by taking the total differential of the trade balance functions, assume that trade is balanced initially and setting all prices and output to unity (initially) we are able to express the \( B_1 \) and \( T_1^2 \) slope.
coefficients as
\[ B_1 = \varepsilon (n_{x_{21}} + n_{1M_{21}} - 1), \quad T_1^2 = (1 - \varepsilon - \delta) (n_{x_{2w}} + n_{1M_{2w}} - 1) \]

It is now evident that if
\[ n_{x_{21}} + n_{1M_{21}} > n_{x_{2w}} + n_{1M_{2w}} \]

\[ \frac{dY_2}{dG_1} > 0 \]

However, even if the opposite was true, e.g., the sum of the second (currency area) country's import and export price elasticities with respect to the first country was smaller than the respective sum of the second country with the rest of the world, it would take a very large difference, e.g.,
\[ (n_{x_{2w}} + n_{1M_{2w}}) - (n_{x_{21}} + n_{1M_{21}}) > 1.35 \]

for the sign of \( \frac{dY_2}{dG_1} \) to be reversed. (This can be seen by setting \( \gamma = \theta = .7, \delta = \varepsilon = .15, \delta_1 = \delta_2 = .75 \) and \( B_2 = -.15 \). In setting these values we assumed that:

1) the share of labour in national income is 75 per cent (a standard value for the exponent of labour in a constant-returns to scale Cobb-Douglas production function, and 2) the marginal propensity to import of the second currency area country from the first is equal to the average one, e.g., \( \varepsilon = .15 = -B_2 \)

The empirical evidence at hand suggests that 1.35 is indeed a very large difference. \( n_{x_{2w}} \) and \( n_{1M_{2w}} \) can
be interpreted as the import and export price elasticities of the rest of the world, respectively. Taking Japan and the United States to be the rest of world from the point of view of members of the European Monetary System (EMS) means that the standard values for $n_{x_{2w}}$ and $n_{1M_{2w}}$ are 1.75 and 1.75, respectively. (These values are for finished manufactures, refer to a three-year period and are approximately the same for both the United States and Japan.) That means that their sum is 3.5. The respective sums for the members of the EMS are: Belgium-Luxembourg 2.75, Denmark 2.75, France 3.25, Germany 3.0, Italy 2.75, and Netherlands 2.75. The above numbers suggest that the biggest difference is .75 for finished manufactures. The biggest difference for semi-finished manufactures is .75 again, for crude materials .15, and for fuels 0.0. (The above elasticities are based on various empirical studies made by the International Monetary Fund staff and presented by Stern (1976) in his overall review of existing empirical estimates of price elasticities). We therefore conclude that 1.35 is indeed a very large difference, and for this reason $dy_2/dG_1 > 0$. Given that the same assumptions about elasticities are sufficient for signing some of the other multipliers (for the $dy_1/dG_1$, $dy_1/dM_1$, $dy_2/dM_2$ multipliers no assumptions were needed), the reader can readily now verify the following results.
3.4 The Extended Model

In the previous section of this paper, we showed that beggar-my-neighbour effects from one member of the currency area to the other do not exist when prices are flexible. We now test whether this result is sensitive to a generalization which allows the currency area to be a significant fraction of the rest of the world. This extension requires that we endogenize the rest of the world, but this allows us to investigate whether there are beggar-my-neighbour effects on both currency area countries, when fiscal expansion occurs in the rest of the world.

The extended model involves equations (3.1), (3.2), (3.3), (3.6), and (3.7) from before, along with the following new relationships (the assumption that the marginal product of labour is again used in this section):

- \( \frac{dY_1}{dG_1} > 0 \quad \frac{dY_2}{dG_1} > 0; \)
- \( \frac{dY_1}{dM_1} = \frac{dY_2}{dM_1} = 0, \) and
- \( \frac{dY_1}{dr_w} < 0, \quad \frac{dY_2}{dr_w} < 0 \)

Since these results are exactly the ones we reached in the previous section we do not discuss them further here.
\[ (3.14) \quad Y_w = C^W(Y_w, r_w) + G_w - T^1(EP_w/P_{1d}, Y_1, Y_w) \]
\[ - T^2(EP_w/P_{2d}, Y_2, Y_w) \]

\[ (3.15) \quad M_w = I, L^W(Y_w, r_w) \]

\[ (3.16) \quad I = hP_{1d}/E + jP_{2d}/E + (1 - h - j)P_w \]

which are the IS, LM and price index relationships for the rest of the world. The only new variables are \( M_w \) and \( I \), the money supply and the price index in the rest of the world.

Since we apply this model to Europe, with the rest of the world interpreted roughly as America, we follow convention (e.g., Branson and Rotemberg (1980) and Sachs (1980)) and assume real wage rigidity within the currency area, and money wage rigidity in the rest of the world. The latter assumption, along with the specification that the marginal product of labour is fixed as before implies that \( P_w \) is constant. When this result and equations (3.6) and (3.7) are substituted into the differential of (3.16), we have \( dI = 0 \). Thus to analyze the model, we set \( P_w = I = 1 \); take the total differential of equations (3.1), (3.2), (3.3), (3.14) and (3.15), (3.6), use (3.7) and the differential of (3.15) to eliminate \( dP_{1d} = dP_1, \quad dP_{2d} = dP_2 \) and \( dr_w \) by substitution; and arrange the remaining relationships as
\[ (3.17) \quad F(dY_1 \, dY_2 \, dY_w \, dE) = K(dG_1 \, dG_2 \, dG_w) \]

where

\[
F = \begin{bmatrix}
  k & -B_3 & -(cC_2^1 + T_3^1) & 0 \\
  B_2 & \lambda & -(cC_2^2 + T_3^2) & 0 \\
  T_2^1 & T_2^2 & (\psi - cC_2^w) & 0 \\
  L_1^1 & L_1^2 & c(L_2^1 + L_2^2) & (L_1^1 + L_2^1)
\end{bmatrix}
\]

\[
K = \begin{bmatrix}
  1 & 0 & 0 \\
  0 & 1 & 0 \\
  0 & 0 & 1 \\
  0 & 0 & 0
\end{bmatrix}
\]

\[ \psi = 1 - C_1^w + T_3^1 + T_3^2; \ (0 < \psi < 1) \]

\[ c = -L_1^w/L_2^w > 0 \]

Since we wish to concentrate on the possibility of beggar-my-neighbour fiscal policies, we have omitted the money supply change terms from the right hand side of equation (3.14).²

The fiscal policy multipliers are

\[ (3.18) \quad dY_w/dG_w = (L_1^1 + L_2^1)(k\lambda + B_3B_2)/\det F \]

\[ (3.19) \quad dY_1/dG_w = (L_1^1 + L_2^1)(B_3(cC_2^2 + T_3^2) + \lambda(cC_2^1 + T_3^1))/\det F \]

\[ (3.20) \quad dY_1/dG_1 = (L_1^1 + L_2^1)(\lambda(\psi - cC_2^w) + T_2^2(cC_2^2 + T_3^2))/\det F \]
The reader can readily verify that \( \frac{dY}{dG} > 0 \) if \( \det F > 0 \). Also, sufficient (though not necessary) conditions for \( \det F > 0 \) are: \( (cC_2^1 + T_3^1) > 0 \) and \( (cC_2^2 + T_3^2) > 0 \).

Using the symbols \( n_y, r \), \( n_{m,y} \) and \( n_{m,r} \) to stand for the interest elasticity of aggregate demand, income elasticity of money demand and interest elasticity of money demand (with superscripts again denoting countries), these conditions can be rewritten as:

\[
(3.23) \quad n_{m,y} (n_{y,r}/n_{m,r}) > T_3^1
\]

with an analogous condition for country two.

Some feel for whether condition (3.23) is met can be had by realizing that a standard value for the government expenditure multiplier in a closed economy model of aggregate demand is 1.5. The closed economy multiplier formula is given by expression (3.24)

\[
(3.24) \quad \frac{1}{1 - MPC + n_{m,y} n_{y,r}/n_{m,r}}
\]
Since MPC is the product of the propensity to consume out of disposable income (say .9) times one minus the overall tax rate (say .4), the fact that expression (3.24) equals 1.5 implies that \( n_{m,y} n_{y,r} / n_{m,r} = .21. \) Surely this is greater than the rest of the world's marginal propensity to import from one of the currency area's countries. If so, condition (3.18) holds, so that both \( \delta t F \) and \( dY_w / dG_w \) are positive.

The reader can readily verify that condition (3.18) is also sufficient to make \( dY_1 / dG_1 > 0, dY_2 / dG_1 > 0 \) and \( dY_1 / dG_w, dY_2 / dG_w < 0. \) These results verify that there are no beggar-my-neighbour effects within the currency area (as we discovered in the section 3.2) but there are beggar-my-neighbour effects imposed on the entire currency area from the rest of the world. Thus, we have verified that the existing prediction regarding external fiscal policy is insensitive to the disaggregation which we have allowed for within the European part of their world model.
3.5 Conclusions

We have investigated the transmission of foreign disturbances and the effects of one country's stabilization policies on both countries within a two-country currency area. Our purpose has been to combine the assumption of real wage rigidity that is popular within one branch of literature that has been applied to European countries, with Levin's suggestion that a three sector model is required to discuss policy within the EEC. With this combination of assumptions, our model is consistent with all these institutional arrangements, and the fact that all currency area countries are adversely affected by a foreign interest rate hike. Furthermore, this is not the only difference in results from Levin. Perhaps most significant is that we do not obtain the beggar-my-neighbour effect of fiscal expansion in one currency-area country, that is stressed by Levin.

We feel that these results justify our emphasis (given in the introduction) on the need for combining several features of recent work within one model. To provide a satisfactory analysis of European stabilization policy options, the model requires both institutional
reality on the aggregate supply side (real wage rigidity, supply-side exchange rate effects and variable prices), and the three-sector specification for aggregate demand (which allows for fixed exchange rates within the currency area and flexible exchange rates with the rest of the world).

As noted in the introduction, a limitation of our analysis is that agents are assumed to have no forward-looking expectations. Miller and Salmon (1985) have stressed that models involving rational exchange rate expectations have the property that current effects in one economy depend on current and expected future policies in other economies. This means that each policy maker is in a strategic relationship with other policy makers. Miller and Salmon analyze optimal policy within a two-country currency area that is essentially the same as Levin's model, except that forward-looking exchange rates are added. They assume that there are no beggar-my-neighbour effects, in a model with fixed prices, which is opposite to what Levin derived. We hope that our analysis encourages those who follow up Miller and Salmon's numerical exploration of time inconsistency and policy coordination to allow for a realistic aggregate supply specification, and not just a disaggregated aggregate demand sector.
FOOTNOTES
Chapter 3

1. We have followed convention (Buiter and Miller 1981, 1982, 1983) and deflated money balances with the consumer price index, while nominal income has been deflated by the producer's price index. Writing real income in the money demand function as $P_1^{dY_1/Y_1}$ does not change the qualitative nature of our results at all.

2. It is easily verified that $dY_1/dM_1 = dY_2/dM_1 = dY_w/dM_1 = 0$, so that monetary policy undertaken within one currency area country has no real effects (as we discovered in section 2). This is also consistent with the results of both Argy and Salop and Kouri. Furthermore, we find $dY_w/dM_w > 0$, but $dY_1/dM_w$, $dY_2/dM_w < 0$, so there can be beggar-my-neighbour effects on currency area countries from monetary policy undertaken in the rest of the world. Argy and Salop did not find this ambiguity, so the three-sector specification for aggregate demand is important for establishing beggar-my-neighbour effects within the currency area from external monetary policies.

3. See, for example, Dornbusch (1980), p. 51.

4. This line of reasoning is borrowed from Blinder and Solow (1973). They note that the overall tax rate stands for the rate at which taxes increase (and transfer payments decrease) with income, for all levels of government combined.
CHAPTER 4

A PORTFOLIO BALANCE MODEL FOR A CURRENCY AREA

4.1 Introduction

During the past decade, theoretical and empirical research in open economy macroeconomics has made extensive use of the notion of interdependence\(^1\). Clearly the notion of interdependence has to do with economic size. The small country assumption, exploited heavily in the literature precludes interdependence. Those who design policies for a small country must respond to changes in other countries' policies, as a consequence of the openness. But they don't have to consider if changes in their policies will cause changes in other countries' policies. Their policy decisions must take account of decisions made elsewhere but are not interdependent with them. The relevance of policy interdependence arises from the fact that a country with the formal autonomy to conduct an independent monetary policy may not have the opportunity to do so ex-post because of other countries' responses. A country with a pegged exchange rate may not give up any real autonomy by joining a monetary union. If its asset markets are closely integrated with those of other members
of the union, an attempt at formal autonomy would not preserve much real autonomy.

In order to examine the notion of interdependence we have to go beyond the conventional two-country model. In a world containing only two countries there would be no way to study the responses of the two countries to an external disturbance; all disturbances would be internal. To do that, a technique used in customs-union theory has been employed. A model is built that contains two countries that interact with each other and with the rest of the world. Earlier analyses of a similar problem (Girton and Henderson, 1976), (Marston, 1980) used the conventional two-country setting.

Later analyses (Allen and Kenen, 1980, Marston, 1983) examined different aspects of exchange rate unions within the context of a three-country model, the three countries representing two members of the union as well as a non-member country. Allen and Kenen studied the short-run and long-run effects of monetary and fiscal policies within this setting, whereas Marston analyzed the desirability of an exchange rate union when one of the currencies of the union is subject to disturbances involving portfolio shifts between this currency (i.e., mark) and rest of the world (i.e., dollar) assets.
Although Allen and Kenen used their model to study questions of interdependence they focused their attention on the longer run, price and output effects of monetary and fiscal policies, and by how much the integration of goods and assets markets affects the distribution of the effects of these policies between the countries of a monetary union. However, they never enquired about the short-run interdependence of monetary (= open market operations) policies arising from unwanted changes in one country's financial variables, due to either exogenous disturbances (e.g., a rise in the rest of the world interest rate) or to conscious efforts by one of the currency area countries to alter the existing configuration of exchange rates, money supplies and interest rates. In other words, Allen and Kenen implicitly assumed (for short-run purposes) that the currency area's central banks are passive in accepting changes in the financial variables which are -- on principle -- under their control.

In this chapter we analyze the effects of monetary policies and examine the ways in which central banks interact in response to both endogenous policy changes and to exogenous disturbances, by constructing a model of the financial sector that captures the institutional setting in which countries of the European Monetary System operate (e.g., fixed exchange rates between them
and flexible rates with the rest of the world). The conventional result in open economy macroeconomics that an expansionary open market operation will result in a depreciation of the country's currency is not always validated in our model once the responses of the other currency area country are considered. Furthermore, as expected, different methods of intervening either to peg the money supply or the interest rate by the second country, have different qualitative results on the variables of the system. What was not expected was the finding that under certain responses of the second currency area country, an expansionary open market operation by the first country can result in a decrease in the money supplies of both currency areas countries (the offset coefficient is greater than |1|). If both countries are net debtors, then all results pertaining to the common external exchange rate can be reversed though none of the results pertaining to the other variables is affected. As with the other papers these results were derived employing the portfolio balance approach.\footnote{According to this approach, equilibrium in financial markets occurs when the available stocks of national monies and other financial assets are willingly held, by the public. The desired holdings of each financial asset by a group of wealth holders depend on}
the levels of the interest rates on the asset in question and on alternative assets, the size of the portfolio and other variables. Exchange rates and interest rates are jointly determined with desired asset holdings. When the history of the economy begins we can imagine the holdings of bonds and money of each individual to be given. We can think of an auctioneer calling out a vector of prices (interest rates and exchange rates) and the individuals valuing their initial holdings of assets to calculate their wealth and then announcing their demands. As the auctioneer varies the prices, the value of initial holdings will also change until the system comes to equilibrium. The equilibrium reached will in general, depend on the initial distribution of assets, and it is assumed that there will be a unique distribution of assets in the new equilibrium.

The rest of the chapter is organized as follows. The structural model is developed in Part 2 and Part 3. Part 4 explains the results, while Part 5 offers the main conclusions.
4.2 The Model

The model contains two countries and the rest of the world. The two countries, country 1 and country 2 are small, in that they can buy any quantity of bonds from the rest of the world at a constant world interest rate. For the sake of exposition, we term country 1 as France; country 2 as Germany, and ROW as the rest of the world.

The French currency is the franc, the German currency is the mark, and the ROW currency is the dollar. We term the franc-mark exchange rate as the bilateral exchange rate. The bilateral exchange rate is assumed to be pegged by one of the two central banks. Neither of the two central banks intervenes to monitor the value of its currency against the dollar. The price of dollars in terms of francs and marks is market determined. In other words, the external exchange rate is freely floating. The model features five financial assets: French, German, and ROW bonds, and French and German currencies. Each country's bonds are denominated in terms of its national currency. The nominal interest rate paid on both currencies is fixed at zero. Each bond is a bill and is a promise to pay the domestic rate of interest times one unit of the currency in which it is issued. When bonds are bills, as they are here, rather than
long-term securities, changes in bond prices caused by changes in interest rates have no effects on wealth. The results would not change substantially by assuming that bonds are long-term securities\(^3\). Only the algebra would be more complicated.

Wealth holders in each country hold domestic money and all three types of securities. They regard the four assets they hold as imperfect substitutes for two sets of reasons. First, the securities are issued in different countries so wealth holders might believe that the returns on the three securities are uncertain, and that these returns are not perfectly correlated across countries due say, to differences in business and political risks. Second, the securities are denominated in different currencies, so potential exchange rate movements add exchange risk to the other risks associated with holding "foreign" assets. Either set of reasons is sufficient to ensure that wealth holders in both countries would in general, want to hold all securities.

It is assumed that each of the three securities is traded in an integrated world market, so that the interest rate on a given security is the same everywhere, but that the differential between the interest rates on the three securities depends on the relative supplies of the three securities.
In other words there is perfect capital mobility, but imperfect substitutability of the three securities.

Wealth holders in France base their nominal demands in terms of francs for French securities ($B_1$), for German securities ($B_2$), for ROW securities ($B_0$), and for French money ($M_1$) on their existing franc denominated nominal wealth ($W$). The fraction $\frac{4}{5}$ of their nominal wealth which they wish to hold in each of these four assets depends on the interest rates on the three securities. Income does not appear in the asset demand functions because it is assumed constant due to the very short-run nature of the model so that,

\begin{align}
(4.1) & \quad B_1 = b_1(r_1, r_2, r_0)W \\
(4.2) & \quad \frac{\pi}{\pi'} B_2 = b_2(r_1, r_2, r_0)W \\
(4.3) & \quad \pi B_0 = b_0(r_1, r_2, r_0)W \\
(4.4) & \quad M_1 = m_1(r_1, r_2, r_0)W
\end{align}

where $r_1$, $r_2$, $r_0$ are the interest rates on French, German and ROW bonds, respectively. $\pi$ and $\pi'$ are the prices of dollars in units of the French and German currencies, respectively.
The bonds are gross substitutes. The demand for any bond increases with its interest rate and decreases with each of the other interest rates thus \( b_{ii} > 0, \ b_{ij} < 0 \ \forall \ i \neq j \ \text{and} \ i = 1, 2, 0 \). The demand for money varies inversely with each interest rate \( m_{li} < 0 \ \forall \ i = 1, 2, 0 \). All asset demand functions are homogeneous in wealth and this specification eliminates the price level from the asset equilibrium sector. This is a useful gain from a seemingly unobjectionable assumption (Branson (1977)).

The French household's (wealth holder's) balance sheet identity is

\[(4.5) \quad W = M_1 + B_1 + \frac{\Pi}{\pi} B_2 + \Pi B_0\]

The balance sheet constraint for French wealth holders requires that the sum of their nominal demands for all assets be equal to their wealth, which is defined to be the sum of the nominal values of the securities and money they currently hold. This implies that

\[b_1 + b_2 + b_0 + m_1 = 1\]

and

\[b_{11} + b_{21} + b_{01} + m_{11} = 0\]

where \( b_{ii} \) is the partial derivative of \( b_1(r_1, r_2, r_0) \) with respect to \( r_i \ (i = 1, 2, 0) \).
Similarly, German households base their mark denominated nominal demands for French bonds ($B_1$), for German bonds ($B_2$), for ROW bonds ($B_0$), and for German money ($M_2$), on their existing mark denominated nominal wealth ($W$), so that

\begin{align}
(4.6) \quad & \Pi B_1 = b_1(r_1, r_2, r_0)W \\
(4.7) \quad & B_2 = b_2(r_1, r_2, r_0)W \\
(4.8) \quad & \Pi B_0 = b_0(r_1, r_2, r_0)W \\
(4.9) \quad & M_2 = m_2(r_1, r_2, r_0)W
\end{align}

where

\begin{align}
(4.10) \quad & W' = \frac{\Pi}{\Pi} B_1 + B_2 + \Pi B_0 + M_2
\end{align}

Similar restrictions apply as in the French case, e.g.,

\[ b_1 + b_2 + b_0 + m_2 = 1 \]

and

\[ b_{11} + b_{21} + b_{01} + m_{21} = 0. \]

The liabilities of the Bank of France are the stock of money held by the French public, while its assets are its holdings of French bonds, German bonds, ROW bonds and foreign exchange reserves ($R$) denominated in dollars. Thus, the balance sheet of the Bank of France is
where the superscript C denotes holdings of bonds by a central bank. There are no commercial banks in this model. Accordingly, the balance sheet of the German central bank (Bundesbank) is

\[(4.12) \quad M^S_2 = B^C_1 + \frac{\pi}{\pi'} B^C_2 + \Pi B^C_0 + \Pi R'\]

The holdings of each central bank's bonds are determined according to its policy objectives. The bank's demand for foreign exchange reserves will necessarily reflect their exchange-rate policies. The exchange rate regime we study in this chapter is one of joint float. The German central bank refrains from intervention in the foreign exchange market. But the French bank intervenes to peg the bilateral exchange rate, \(\theta = \frac{\pi}{\pi'}\), by purchasing or selling marks against francs. Under this arrangement \(\pi\) and \(\pi'\) are endogenous and \(\theta\) exogenous. Moreover, changes in \(R\) and \(R'\) offset each other and \(\bar{R} = R + R'\) cannot change. This regime of joint float is analytically similar to a currency union with a flexible exchange rate vis-a-vis the outside world. Here it is assumed that reserves are held in world currency (dollars) not merely denominated in that currency. Then the
central bank's balance sheet means that neither bank holds the other country's currency. The problem then, is that it is not easy to model the effects of intervention in the foreign exchange market by one of the two central banks to influence the bilateral exchange rate. We can do so by assuming that the two central banks have reached an agreement concerning conversion into world currency all of the other's currency one central bank acquires by intervening in the foreign exchange market to influence the bilateral exchange rate. In other words, central banks wishing to avoid reciprocal currency accumulation must agree to "mandatory asset settlement". Under this scheme, a French purchase of German currency in the foreign exchange market causes an increase in French reserves and an equal decrease in German reserves. As a result, the two countries' money supplies will change by equal but opposite amounts.

The world market for French securities is in equilibrium when the supply of French securities available to the public equals the demand for these securities by wealth holders in the three countries:

\[
\begin{align*}
\text{b}_1(r_1, r_2, r_0)W + \text{b}_1'(r_1, r_2, r_0)W' + \\
\text{b}_0^0(r_1, r_2, r_0)W^0 &= B_1^S - B_1^C - B_1' C
\end{align*}
\]
Similarly, the total demand (in francs) for German securities by wealth holders in the three countries, must be equal to the supply of German securities available to the public (in francs):

\[
(4.14) \quad b_2(r_1, r_2, r_0)W + \theta b_2'(r_1, r_2, r_0)W' + \theta \pi' b_2(r_1, r_2, r_0)W^D = \theta (B^S_2 - B^C_2 - B^C_2)
\]

Making use of the central bank balance sheet identities we can express the money market equilibrium conditions for the two countries as follows:

\[
(4.15) \quad m_1(r_1, r_2, r_0)W - (B^C_1 + \theta B^C_2 + \theta \Pi' B^C_0 + \Pi R) = 0
\]

and

\[
(4.16) \quad \theta m_2(r_1, r_2, r_0)W' - (B^C_1 + \theta B^C_2 + \theta \Pi' B^C_0
\]

\[+ \theta \pi'(R - R) = 0\]

The endogenous variables to be determined from this system of four equations (4.13) through (4.16) are the French and German interest rates \(r_1, r_2\) the common external exchange rate (we assume that \(\Pi = \Pi'\) so that \(\theta = 1\)) and the stock of reserves held by either one of the two countries.
At this stage it should be noted that as a result of open market and intervention operations made in order to peg the bilateral exchange rate, θ, wealth remains constant unless there is a change in the external exchange rate. Open market and intervention operations as such do not change the wealth of private agents, since agents increase their holdings of one asset by the same amount they decrease their holdings of another asset. The algebraic implication of this statement is that \( \Delta W = B'_0 (\Delta \pi) \), \( \Delta W' = B'_0 (\Delta \Pi) \). This can be verified by substituting the "trading constraint" faced by private agents, e.g., \( dM_1 + dB_1 + db_2 + \Pi dB_0 = 0 \), into equations (3.5) and (3.11). Furthermore, any changes in the domestic value of the central bank's holdings of ROW securities and foreign exchange reserves due to changes in the external exchange rate are not allowed to affect the country's money supply. The central bank is assumed to pass over to the government any "gains" or "losses" incurred from exchange rate changes, and since Ricardian equivalence is not assumed, the wealth of private agents is unaffected.

To analyze the model, we take the total differential of equations (4.13) to (4.16) and get

\[
(b_{11}dr_1 + b_{12}dr_2 + b_{13}dr_0)W + b_1B_0(\Delta \pi) +
\]
\[ + (b_{11}dr_1 + b_{12}dr_2 + b_{13}dr_0)W + \]
\[ + b_1^0B_0(d\Pi) + b_{11}^0dr_1 + b_{12}^0dr_2 + b_{13}^0dr_0)W^0 + \]
\[ + b_1^0W^0(d\Pi) = dB^S_1 - dB^C_1 - dB'^C_1 \]

\[ (b_{21}dr_1 + b_{22}dr_2 + b_{23}dr_0)W + b_2B_0(d\Pi) + \]
\[ + (b_{21}^0dr_1 + b_{22}^0dr_2 + b_{23}^0dr_0)W' + b_2^0B_0(d\Pi) + \]
\[ + (b_{21}^0dr_1 + b_{22}^0dr_2 + b_{23}^0dr_0)W^{0'} + b_2^0W^0d\pi \]
\[ = dB^S_2 - dB^C_2 - dB'^C_2 \]

\[ (m_{11}dr_0 + m_{12}dr_2 + m_{13}dr_0)W + m_1B_0(d\pi) - dB^C_1 - \]
\[ - dB^C_2 - dB^C_0 - dR = 0 \]

\[ (m_{21}dr_1 + m_{22}dr_2 + m_{23}dr_0)W' + m_2B_0'(d\Pi) - \]
\[ - dB'^C_1 - dB'^C_2 - m dB'^C_0 + m dR = 0 \]

or
\[(4.18)\quad H[dr_1, dr_2, \xi, dr] = J[dB^C_1, dB^C_2, dB^C_1, dB^C_2, dB^C_0, dB^C_0, dr_0]^\prime + \phi[dB^S_1, dB^S_2]^\prime\]

where

\[
H = \begin{pmatrix}
al & \gamma & k & 0 \\
\gamma & a & k & 0 \\
\varepsilon & \eta & \lambda & \Pi \\
\eta & \varepsilon & \lambda & \Pi \\
\end{pmatrix}, \quad J = \begin{pmatrix}-1 & 0 & -1 & 0 & 0 & 0 & x \\
0 & -1 & 0 & -1 & 0 & 0 & x \\
1 & 1 & 0 & 0 & v & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & v & 0 \\
\end{pmatrix}, \quad \phi = \begin{pmatrix}1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0 \\
\end{pmatrix}
\]

\[
a = b_{11}^W + b_{11}^\prime W^\prime + b_{11}^0 \Pi W^0 > 0
\]

\[
\gamma = b_{21}^W + b_{21}^\prime W^\prime + b_{21}^0 \Pi W^0 < 0
\]

\[
\varepsilon = m_{11}^W = m_{22}^W < 0
\]

\[
\eta = m_{21}^W = m_{12}^W < 0
\]

\[
k = b_{1}B_0 + b_{1}^\prime B_0^\prime + b_{1}^0 W^0 = b_{2}B_0 + b_{2}^\prime B_0^\prime + b_{2}^0 W^0 > 0
\]

\[
\lambda = m_{1}B_0 = m_{2}B_0
\]

\[
x = b_{13}^W + b_{13}^\prime W^\prime + b_{13}^0 \Pi W^0 = b_{23}^W + b_{23}^\prime W^\prime + b_{23}^0 \Pi W^0 > 0
\]
In order to reduce the number of parameters and resolve some ambiguities we have assumed, as is evident from the above definitions and inequalities, that there exist symmetries in the demand for assets in the two currency area countries. (Our assumptions are similar to those made in Allen and Kenen (1980), and Branson and Henderson (1984).) In order to get the required symmetry in asset demands we have imposed the following restrictions: (1) the bilateral exchange rate, $\theta$, is equal to one, and at this value of $\theta$, the stocks of wealth in the two currency area countries are equal (i.e., $W = W'$). In addition, interest rates are initially equal in France and Germany (i.e., $r_1 = r_2$) but not necessarily equal to the ROW interest rate; (2) the roles of $r_1$ and $r_2$ are reversed in the two money demand functions (i.e., an increase in $r_1$ decreases the demand for $M_1$, by the same amount an increase in $r_2$ decreases the demand for $M_2$). The roles of ROW, interest rate, $r_0'$, and wealth, $W$, are identical. The same restrictions are imposed on the demands for foreign bonds, (i.e., an increase in $r_1$ decreases the demand for ROW securities by French wealth holders, by the same amount an increase in $r_2$ decreases the demand for ROW securities by German wealth holders); (3) Again the roles of $r_1$ and $r_2$ are reversed with respect to the two demands for French bonds and those
for German bonds. The French wealth holders demand for French bonds is assumed to be identical in every other way to the German's demand for German bonds, and French demand for German bonds is assumed to be identical in every other way to the German demand for French bonds; (4) French hold more French bonds and add more to their holdings with an increase in their wealth, so that $b_1 > b_2$, and $b_1 > b_0$ for all sets of interest rates. (There is local asset preference); (5) French money is a closer substitute for French bonds than German money for French bonds; (6) The demand for either French or German money is unresponsive to the interest rate on ROW bonds, $r_0$. (This is the only assumption not made by Allen and Kenen and Branson and Henderson. It is made here in order to clarify some ambiguities arising when there is an increase in $r_0$.)

All these restrictions produce: (1) pairwise equalities in money and bond holdings so that initially

$$m_1 = m_2, \quad b_1 = b_2', \quad b_2 = b_1', \quad b_0 = b_0';$$

(2) pairwise equalities in responses in the desire to hold money and bonds due to changes in interest rates.

$$m_{11} = m_{22}, \quad b_{11} = b_{22}', \quad b_{22} = b_{11}', \quad b_{01} = b_{02}';$$

$$(4.19) \quad m_{12} = m_{21}, \quad b_{12} = b_{21}', \quad b_{21} = b_{12}', \quad b_{02} = b_{01}'$$
\[
\begin{align*}
   m_{13} &= m_{23} & b_{13} &= b_{23} & b_{23} &= b_{13} & b_{03} &= b_{03} \\
\end{align*}
\]

and
\[
   b_{13}^0 = b_{23}^0
\]

(3) inequalities: \( a > -(y + \varepsilon + n), \varepsilon > n, \ x > 1 \).

We are now ready to begin analyzing the model.

We do so by first establishing that the effects of open market operations on the currency area interest rates and the common external exchange rate do not depend on which central bank undertakes the operation but on the security used for the operation. We then present the most important results using both diagrams and algebra. Given the symmetry built into the model, the results display the same sort of symmetry (a result which we establish below), and for this reason it is sufficient to examine only the effects of open market operations undertaken by the first country's central bank.

4.3 Analysis

In Figure (4.1) four schedules which show the pairs of \( \pi \) and \( \pi_1 \) which are compatible with equilibrium in each of the four asset markets are depicted. It should be noted at the outset that changes in the other two endogenous variables, \( \pi_2 \) and \( x \), make all four schedules shift (of course
changes in the exogenous variables move the schedules as well). The way the position of these schedules is affected by changes in $r_2$ and $R$ is explained below. $B_1B_1$ is the locus of $r_1$ and $\Pi$ for which the market for French securities is in equilibrium. The algebraic expression for its slope is given by

$$\frac{d\pi}{dr_1} = \frac{- (b'_{11}W + b'_{11}W + b_{11}W^0)}{b_{11}B_0 + b'_{11}B_0 + b_{11}W^0} = \frac{-a}{K} < 0$$

FIGURE 4.1: The Initial Equilibrium
The $B_B_1$ curve slopes downwards because an increase in $r_1$ causes excess demand for French securities, so that $\pi$, the common external exchange rate, must decline in order to cut back demand (by decreasing wealth in both countries, assuming that both countries are net creditors; the implications of this assumption are considered later) until it matches the given supply of French securities available to wealth holders in all countries. It should now be evident that, for example, an increase in $r_2$ will shift $B_B_1$ to the right, since for a given value of $\pi$, an increase in $r_1$ is required to offset the excess supply of French securities caused by the increase in $r_2$.

$B_B_2$ represents the combinations of $r_2$ and $\pi$ for which the market for German securities is in equilibrium. Its slope is given by the expression:

$$\frac{d\pi}{dr_1} = \frac{-(b_21w + b_21w' + b_21w_0)}{b_2b_0 + b_2b_0' + b_2w_0} = \frac{-\gamma}{K} > 0$$

The $B_B_2$ is positively sloped since an increase in $r_1$ causes an excess supply of German securities, so that $\pi$ must rise in order to increase demand until it equals the supply of German securities available to the public. In this case an increase in $r_2$ will shift $B_B_2$.
to the right, since an increase in $r_1$ is required to offset the excess demand for German securities generated by the rise in $r_2$.

Combinations of $r_1$ and $\Pi$ that equate the demand for and supply of French money (francs) given a fixed supply of francs for the public to hold, are plotted as the $M_1M_1$ curve. For this curve

$$\frac{ds}{dr_1} = \frac{-m_{11}W}{m_1B_0} = \frac{-\alpha}{\lambda} > 0$$

This curve is positively sloped because an increase in $r_1$ reduces the demand for francs while an increase in $\Pi$ increases the demand for francs. An increase in $r_2$ will shift $M_1M_1$ to the left since a decrease in $r_1$ is required to offset the excess supply of francs generated by the rise in $r_2$. Similarly, an increase in $R$ generates excess supply of francs which can be eliminated by a decrease in $r_1$, hence, $M_1M_1$ shifts to the left.

Similar reasoning explains why combinations of $r_1$ and $\Pi$ that preserve the market for German money (marks) in equilibrium, plotted as the $M_2M_2$ curve is upward sloping as can be seen from the expression for the slope of $M_2M_2$:

$$\frac{dc}{dr_1} = \frac{-m_{21}W}{m_2B_0} = \frac{-\mu}{\lambda'} > 0$$
In Figure (4.1) we have drawn the $B_2B_2$ schedule steeper than the $M_1M_1$ and $M_2M_2$ schedules, and $M_1M_1$ steeper than $M_2M_2$. From the above slope expressions for $M_1M_1$ and $M_2M_2$ it is easily seen that the assumption that $|\epsilon| > |\eta|$ makes $M_1M_1$ steeper than $M_2M_2$. In order for $B_2B_2$ to be steeper than $M_1M_1$, the term $|\gamma\lambda|$ has to be greater than $|K\varepsilon|$. All the results relating to $\Pi$ movements carry through even if $|\gamma\lambda| < |K\varepsilon|$. However, there are two cases in which the movement in one of the interest rates and the level of reserves depends on the $(\gamma\lambda - K\varepsilon)$ terms. We discuss these two cases later. Furthermore, it can be noticed that at a point like $P$ in Figure 1, we have excess demand for all four assets. While this at first sight seems disturbing (violation of Walras' Law), it is not, since all four market equilibrium conditions are independent. At a point like $P$, there is "excess supply" of ROW bonds by the currency area's wealth holders, which is eliminated by instantaneous adjustment in the quantity supplied, given that the ROW interest rate $r_0$ is assumed constant.
4.3.1 Open Market Operations

We are now ready to examine the effects of open market operations. Consider an expansionary open market operation. One can easily verify from equation (4.18) and the implied restrictions as shown in equation (4.19) that

\[
\begin{align*}
\frac{dr_1}{dB_1^C} &= \frac{dr_1}{dB_1^C} = \frac{dr_2}{dB_2^C} = \frac{dr_2}{dB_2^C} = \\
&= \frac{-2a\lambda + k\varepsilon + \gamma k - ak + uk}{\Delta_1} < 0
\end{align*}
\]

\[
\begin{align*}
\frac{dr_2}{dB_1^C} &= \frac{dr_2}{dB_1^C} = \frac{dr_1}{dB_2^C} = \frac{dr_1}{dB_2^C} = \\
&= \frac{k\gamma + \gamma \lambda + uk - k\varepsilon - ka + \gamma \lambda}{\Delta_1} < 0
\end{align*}
\]

\[
\begin{align*}
\frac{d\Pi}{dB_1^C} &= \frac{d\Pi}{dB_1^C} = \frac{d\Pi}{dB_2^C} = \frac{d\Pi}{dB_2^C} = \\
&= \frac{a^2 - \gamma \varepsilon + na - \gamma n + \varepsilon a - \gamma^2}{\Delta_1} > 0
\end{align*}
\]
In order to gain some understanding of the above mentioned results consider an expansionary open market operation in the first country (France). The initial equilibrium position is given by the intersection of the solidly drawn lines at point a in Figure 4.2. If the French authorities increase their holdings of French securities by a given amount and then allow no further changes in these holdings, the $B_1B_1'$ curve must shift to the left, say from $B_1B_1$ to $B_1'B_1'$, since, for a given value of $\Pi$, the public will hold a reduced
supply of French securities only if \( r_1 \) is lower. The \( M_1M_1 \) schedule shifts to the left as well, say from \( M_1M_1 \) to \( M_1'M_1' \), since for a given value of \( \Pi \), the public will hold an increased supply of francs only if \( r_1 \) is lower. In order to increase its holdings of French securities by a given amount, the French central bank offers to supply franc deposits in exchange for French securities. At the initial values of interest rates and exchange rates, there is excess demand for French

\[ \text{FIGURE 4.2: Effects of Open Market Operations} \]
securities and excess supply of French money. Bank of France (BOF) bidding for French securities leads to a downward movement in $r_1$, an increase in private demand in France, Germany and the rest of the world (ROW) for German and ROW securities, and a downward movement in $r_2$. As the interest rate on German securities is bid down, an excess demand for francs, marks and French securities and an excess supply of German securities appears. These developments mean that the $B_1B_1'$ schedule shifts to the left, say to $B_1''B_1''$ since for a given value of $\pi$ a fall in $r_1$ is required to offset the excess demand for French securities that has been created due to the fall in $r_2$. Similarly, $B_2B_2'$ has to shift to the left, say to $B_2''B_2''$, since a decrease in $r_1$ is required to offset the excess supply for German securities that has been created due to the fall in $r_2$. The $M_1M_1'$ shifts to the right, say, to $M_1''M_1''$, since a fall in $r_2$ creates excess demand for francs that requires a rise in $r_1$ in order to keep the market for French money in equilibrium. For similar reasons, the $M_2M_2'$ schedule has to shift to the right, say to $M_2''M_2''$.

At the same time the attempts by one of the currency area's central banks to stabilize the bilateral exchange rate increases (through an increase in reserves
R') Germany's money supply by the same amount it reduces (through a decrease in reserves, R) France's money supply, hence shifting \( M'_2 M'_2 \) to the left and \( M''_1 M''_1 \) to the right. The point of intersection of all four schedules \( S \) (changes in \( r_2 \) and \( R \) make all schedules pass through a common point) is the new equilibrium. At the new equilibrium, \( \Pi \), the common external exchange rate is higher (there has been a depreciation of the external exchange rate) both \( r_1 \) and \( r_2 \) are lower, and \( R \) (B.O.F. foreign exchange reserves) is lower. (There has been partial offsetting \((0 < |dR/dB^C_1| < 1)\) of the initial increase in the French money supply, German's money supply being higher as a result). These results are in line with both the existing theory of exchange rate determination (in both small country and two country models) and the Allen and Kenen finding that there will be an exchange rate depreciation (Branson (1977), Branson and Henderson (1984), Allen and Kenen (1980)).

Looking at equations (4.20), (4.21), and (4.22), we observe that the effects of open market operations on the currency area's interest rates, \( r_1 \) and \( r_2 \), and the external exchange rate, \( \Pi \), do not depend on the central bank undertaking the open market operation, but only on the security involved in the open market operation. That this is the case, can be understood from the fact that
whether the BOF buys French securities or the Bundesbank buys French securities, the stock of each kind of bonds available to the public remains the same in both cases, hence, there can be only one configuration of interest rates \( r_1 \) and \( r_2 \) -- given that the ROW interest rate \( r_0 \) is constant) and exchange rates, for which both bonds markets remain in equilibrium. This further implies that movements of reserves have to be such that the money supply of each country remains the same in both situations.

\[
(dM_1 = \frac{dB_1^C}{dB_l} + \frac{\Pi dR}{dB_l} = \frac{\Pi dR}{dB_l} \quad \text{and} \quad dM_2 = \frac{dB_1^C}{dB_l} - \frac{\Pi dR}{dB_l} = -\frac{\Pi dR}{dB_l})
\]

as can be easily deduced from equations 4.23 and 4.24.

4.3.1.A Pegging the Interest Rate

Until now we have assumed that the second country (Germany) does not respond to prevent changes in either "its" interest rate, \( r_2 \), or its money supply, \( M_2 \), brought about by France's monetary policies. We think that the purpose of building disaggregated models is precisely to ask questions about the effects of differences in the ways a currency area's countries respond either to changes in exogenous variables (a change in \( r_0 \)) or to changes in the other currency area country's policies.
Although Allen and Kenen used their model to study the question of interdependence, they focussed their attention on the "longer-run" price and output effects of monetary and fiscal policies.) To do so we examine the effects on interest rates and exchange rates of expansionary open market operations in France when Germany: (1) pegs the interest rate on its own currency denominated securities \( r_2 \), and (2), pegs its money supply.

It is evident that there are three ways by which the German Central Bank (Bundesbank) can peg \( r_2 \). One way to do so is by being prepared to buy or sell French securities when open market operations, by the BOF change \( r_2 \). In this case \( B_C' \) (the Bundesbank's holdings of French securities) becomes endogenous in the place of \( r_2 \). One then easily verifies from equation (18) that

\[
\frac{d\Pi}{dB_C} = \frac{dr_1}{dB_C} = 0, \quad \frac{dR}{dB_C} = -1, \quad \frac{dB_C'}{dB_C} = -1
\]

Figure 4.3 can be of some help in understanding why we obtain the above results. The initial equilibrium is at point a. If the BOF increases its holdings of French securities and allows no further changes in these holdings, then we know that \( r_2 \) (the interest rate paid
on German securities) will be lower in the new equilibrium. To prevent this from happening the Bundesbank has to decrease its holdings of French securities by the same amount the BOF increased its holdings of French securities, since only then the stocks of both kinds of bonds (French and German bonds) available for the public to hold remains the same, and hence there can only be one configuration (the initial one) of interest rates and exchange rates for which both bonds markets remain in equilibrium. This further implies that the movements of reserves are such that the money supplies in both countries remain what they were in the initial equilibrium. In other words France loses through a decrease in its reserves, \( R \), all the initial increase in its money supply and Germany gains back through an increase in its reserves, \( R' \), all the initial decrease in its money supply brought about by the selling of French bonds. In terms of Figure 4.3 \( B_1B_1 \) and \( M_1M_1 \) after the initial movement to \( B_1B_1' \) and \( M_1M_1' \) they shift back to point \( a \).

Another way in which Bundesbank can respond to a declining \( r_2 \), is by reducing its holdings of ROW bonds. Since \( r_2 \) is to remain unchanged and the stocks of German bonds available to the public remains the same, the new equilibrium has to be on the \( B_2B_2 \) curve of Figure 4.3.
When the BOF buys French securities, the $B_1'B_1'$ schedule shifts to the left and the same is true for the $M_1'M_1$ schedule. Since no further changes in any central bank's holdings of French bonds take place, that means that the final equilibrium has to be at point $\beta$ where $B_1'B_1'$ and $B_2'B_2'$ cross. This is accomplished by reserve shifts between France and Germany (as well as the initial decline in the German money supply due to the selling of ROW bonds), so that the $M_1'M_1$ and $M_2'M_2$ schedules finally pass from

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig4.3}
\caption{Pegging the Interest Rate}
\end{figure}
point $\beta$. But in order for the $M_1M_1$ and $M_2M_2$ schedules to pass from point $\beta$ there has to be a shift to the right of both schedules which can only happen if there is a decrease in both countries' money supplies (given the constancy of $r_2$). An heuristic explanation for this result goes as follows: Since the supply of German bonds available to the public is constant and $r_1$ will be lower in the new equilibrium, an excess demand for German securities will develop (given the constancy of $r_2$) unless there is a decline in wealth caused by an appreciation of the common external exchange rate. It is then possible that the fall in wealth decreases money demand in both countries by more than the drop in $r_1$ increases it. Therefore money supplies have to be lower in the new equilibrium.

That this can indeed be the case can be verified from equation (4.18) after making $B_0^C$ (the holdings of ROW bonds by the Bundesbank) endogenous, in the place of $r_2$. Then,

$$\frac{dr_1}{dB_1} = \frac{1}{-a + \gamma} < 0$$

(4.27)

$$\frac{d\pi}{dB_1} = \frac{-\gamma}{-ak + \gamma k} < 0$$

$$\frac{dR}{dB_1} = \frac{ak - \gamma \lambda + \varepsilon k - \gamma k}{-ak\Pi + \Pi \gamma k} < 0,$$

$$\frac{dB_0^C}{dB_1} = \frac{-2\gamma \lambda + kn + kn + k\varepsilon - k\gamma + ka}{-ak\Pi + \Pi \gamma k} < 0$$
The change in France's money supply is given by:

\[ dM_1 = \frac{dB^C_1}{dB^C} + \frac{\Pi dR}{dB^C_1} \leq 0 \text{ if } |\gamma| \geq |\kappa|, \text{ and the change in } \]

Germany's money supply is given by

\[ dM_2 = \frac{dB^C_0}{dB^C} - \frac{\Pi dR}{dB^C_1} \leq 0 \]

if \( |\gamma| \geq |\kappa| \). Since it is the assumptions \( |\gamma| > |\kappa| \)
and \( |\gamma| > |\kappa| \) that make the \( B_2B_2 \) schedule steeper than the \( M_1M_1 \) and \( M_2M_2 \) schedules, one can easily see now that

if the opposite was true (\( B_2B_2 \) flatter than \( M_1M_1 \) and \( M_2M_2 \)), then the money supplies of both countries would not be lower in the final equilibrium (the \( M_1M_1 \) and \( M_2M_2 \) schedules would have to shift to the left to attain the new equilibrium) though there would still be an appreciation of the external exchange rate, \( \Pi \), and drop in \( r_1 \).

The above suggests the possibility of an offset coefficient greater (in absolute value) than one. The offset coefficient measures the degree to which an increase in the domestic component of the money supply \( (B^C_1) \) is offset by a reduction in the stock of international reserves. For the small country and two country (fixed exchange rate) portfolio balance models it is well known
(Branson (1977), Girton and Henderson (1976)) that the highest value of this coefficient (in absolute terms) obtains when there is perfect asset substitutability. In this case the value of the offset coefficient becomes -1. The closer this coefficient is to -1 the less effective monetary policy is in influencing the equilibrium stock of the money supply and domestic interest rate. If the offset coefficient is equal to -1, the monetary authorities cannot influence the equilibrium stock of domestic money supply, and this happens in the case of perfect asset substitutability. From the above results one can see that even without perfect asset substitutability this coefficient can be bigger than | -1 |. Furthermore, one can easily find from Figure 4.3 and equations (4.20) and (4.26) that the drop in the French interest rate, \( r_1 \), is higher when the Bundesbank does not intervene to peg its own interest rate, \( r_2 \). The reason for this is that with \( r_2 \) constant the \( B_2 B_2 \) and \( B_1 B_1 \) schedules do not shift to the left as they do in the case of a falling \( r_2 \), and hence the final equilibrium is on the initial \( B_2 B_2 \) line on which \( r_1 \) is higher (given the leftward shift of the \( B_1 B_1 \) schedule caused by a falling \( r_2 \) when \( r_2 \) is held constant.

A third way by which Bundesbank can prevent \( r_2 \)
from falling after a purchase of French securities
by the BOF is by reducing its holdings of German bonds, $B_2'$. Since in this case the same qualitative results
obtain as in the case Bundesbank was passive in accepting
changes in $r_2$ and $M_2$ we will not discuss it further than
giving the algebraic expressions for the changes in $r_1$, $\Pi$, $R$ and $B_2'\!$ ($B_2'$ is now the endogenous variable in the
place of $r_2$):

\[
\frac{dr_1'}{dB_1'} = \frac{-2\lambda - 2k}{\Delta_2} < 0, \quad \frac{d\Pi}{dB_1'} = \frac{a + \varepsilon + \gamma + n}{\Delta_2} > 0
\]

\[
\frac{dR}{dB_1'} = \frac{\lambda(n + \gamma - \varepsilon - a) + k(\gamma - \varepsilon + n - a)}{\Pi(\Delta_2)} < 0
\]

\[
\frac{dB_2'}{dB_1'} = \frac{2\gamma\lambda - \varepsilon k - kn + k\gamma - ak}{\Delta_2} > 0
\]

where $\Delta = 2a\lambda - \varepsilon k - \gamma k + ak - nk$

Thus far, we have seen that the German central bank can
peg $r_2$ if it wants, but only a particular method will
ensure full insulation of other key variables (and that
is method 1). But this method emaciates the actions of
the French Central Bank.
4.3.1B Pegging The Money Supply

We can now start looking at the ways by which Bundesbank can insulate the German money supply from the effects of open market operations in France. A way of doing so is by reducing its holdings of French securities $B_1^C$ by the same amount the inflow of reserves tends to increase the German money supply ($dB_1^C = \Pi dR = \Pi dR'$). By now it should be evident (as can be seen from equations (4.23) and (4.24) that due to the symmetry of the model, the amount by which the Bundesbank has to reduce its holdings of French securities so as to keep the German money supply constant is equal to the increase in the BOF's holdings of French securities, and hence in the new equilibrium both interest rates and the external exchange rate remain the same as can be confirmed by the following expressions:

\[
\frac{d\Pi}{dB_1^C} = \frac{dr_1}{dB_1^C} = \frac{dr_2}{dB_2^C} = 0, \quad \frac{dB_1^C}{dB_1} = -1
\]

derived after substituting the constraint $dB_1^C = \Pi dR$ in equation (4.18). We therefore find that when the Bundesbank uses its holdings of French bonds to prevent changes in $r_2$ or $M_2$, the results for the currency area
interest rates \((r_1 \text{ and } r_2)\) and the common external exchange rate are independent of whether Bundesbank sells French bonds in order to peg the interest rate on German securities \((r_2)\) or to peg the German money supply. Furthermore, the offset coefficient faced by BOF is \(-1\), in both cases.

In the case Bundesbank tries to peg \(M_2\) by selling ROW bonds to the public \((dB_0^C = dR)\), then as in the case Bundesbank pegged \(r_2\) by selling ROW bonds, there will be an appreciation of the common external exchange rate, \(\Pi\), and a drop in \(r_1\). Unlike the case of pegging \(r_2\), however, the offset coefficient will always be less than \(|1|\). It will be the movement of \(r_2\) that will depend on the \(|k_\Pi - \lambda Y|\) sign. The algebraic expressions for these results derived from equation (4.18) are

\[
\frac{dr_1}{dB_1^C} = \frac{\epsilon k - a\lambda}{\Delta 3} < 0, \quad \frac{dr_2}{dB_1^C} = \frac{\lambda Y - k_\Pi}{\Delta 3} > 0
\]

\[
\frac{d\Pi}{dB_1^C} = \frac{na - \gamma \varepsilon}{\Delta 3} < 0,
\]
\[-1 < \frac{\mathrm{d}R}{\mathrm{d}B_1^C} = \frac{- (a^2 \lambda + \gamma k \varepsilon + \gamma kn - nka - \varepsilon ka - \lambda \gamma^2)}{\Pi \Delta_3} + \gamma n \lambda + a \lambda n + k \varepsilon^2 - n^2 k - \varepsilon \lambda \gamma - \varepsilon \lambda a / \Pi \Delta < 0\]

where

\[\Delta_3 = (a^2 \lambda + \gamma k \varepsilon + \gamma kn - nka - \varepsilon ka - \lambda \gamma^2)\]

Still another way by which the German Central Bank can keep \(M_2\) constant is by selling German bonds to the public (\(\mathrm{d}B_2^C = \Pi \mathrm{d}R\)). In this case \(r_2\) will be higher in the new equilibrium, partly because the stock of German bonds available for the public to hold will be higher after the Bundesbank has reduced its holdings of German bonds. This can be confirmed from the following expressions which can be derived from equation (4.18) as well:

\[
\frac{\mathrm{d}r_1}{\mathrm{d}B_1^C} = \frac{2k \varepsilon - \lambda n - \lambda \gamma + \lambda \gamma - a \lambda}{\Delta_4} < 0,
\]

\[
\frac{\mathrm{d}r_2}{\mathrm{d}B_1^C} = \frac{-2k n - n \lambda + a \lambda + \varepsilon \lambda + \lambda \gamma}{\Delta_4} > 0
\]

\[
\frac{\mathrm{d}\Pi}{\mathrm{d}B_1^C} = \frac{\gamma (n - \varepsilon) + a (n - \varepsilon) + n^2 - \varepsilon^2}{\Delta_4} > 0
\]

\[
\frac{\mathrm{d}R}{\mathrm{d}B_1^C} = \frac{\lambda n (\gamma + a) + (\varepsilon^2 - n^2) - \varepsilon \lambda (\gamma + a) - \lambda (a^2 - \gamma^2) + k \gamma (n + \varepsilon) - ka (\varepsilon + n)) / \Pi \Delta_4}{\Delta_4} < 0
\]
where
\[ \Delta_4 = \lambda n (\gamma + a) - k (\varepsilon^2 - n^2) - \varepsilon \lambda (\gamma + a) + \lambda (a^2 - \gamma^2) + k \gamma (n + \varepsilon) - k a (\varepsilon + n) > 0. \]

We have thus seen so far that from the six (6) different ways Bundesbank can utilize to peg either \( r_2 \) or \( M_2 \) (three ways to peg \( r_2 \) plus three ways to peg \( M_2 \)) in only one case, the qualitative results for the currency area interest rates and the common external exchange rate are similar to those obtained under complete passiveness of the German Central Bank in the face of a falling \( r_2 \) and an increasing \( M_2 \) brought about by expansionary open market operations in France.

4.3.2 Intervention Operations

What we mean by intervention operations are exchanges by a central bank of one type of securities for another type of securities, so that initially there is no change in the money supply (sterilized intervention). It is evident that there can be three types of intervention operations. Intervention of type I is an exchange of home currency securities for ROW securities \( (dB^C_1 = -dB^C_0) \); of type II is an exchange of home currency securities for the other currency area's country securities \( (dB^C_1 = -dB^C_2) \); and of type III an exchange of the other currency area's country securi-
ties for ROW securities \( dB_2^C = -\pi dB_0^C \). Type III operations do not provide us with a significantly different policy to be analyzed. For this reason we restrict our discussion to the first two types of intervention policy. We start by looking at the type I operation.

4.3.2.A Type I Operations

We demonstrate the effects of this type of operations with the help of Figure 4.4. The initial position is given by the intersection of solidly drawn lines at point a. If the Bank of France (BOF) increases its holdings of French securities by the same amount it reduces its holdings of ROW securities, the \( B_1B_1 \) curve shifts to the left, say, from \( B_1B_1 \) to \( B_1'B_1' \), since for a given value of \( \pi \), the public will be willing to hold a reduced supply of French securities only if \( r_1 \) is lower. At the initial values of interest rates and exchange rates there is excess demand for French securities and excess supply of ROW securities. BOF bidding for French securities leads to downward movement in \( r_1 \), an increase in private demand in France,
Germany and ROW for German and ROW securities (hence offsetting to some extent the initial excess supply of ROW securities), and a downward movement in $r_2$. As the interest rate on German securities is bid down, an excess demand for francs, marks, French securities and ROW securities (thus offsetting further the initial excess supply), and an excess supply of German securities appears. These developments mean that the $B_1B_1'$ schedule shifts to the left, say to $B_1'B_1$, $B_2B_2$ shifts to $B_2'B_2$, $M_1M_1$ to $M_1'M_1$, and $M_2M_2$ to $M_2'M_2$. At the same time the commitment by one of the currency area's central banks to stabilize the bilateral exchange rate increases France's money supply (through an increase in reserves $R$) by the same amount it reduces Germany's money supply, hence shifting $M_2'M_2$ to the right and $M_1'M_1$ to the left. The point of intersection of all four schedules $S$, is the new equilibrium. At the new equilibrium, $\Pi$, the common external exchange rate is lower (there has been an appreciation) and $r_1$ is lower as well. The final effect on $r_2$ is ambiguous. What is worth noting is that a "sterilized" open market operation does affect the money supplies of both countries through movements of reserves. The above
are confirmed by the expressions (derived from equation (18) after substituting the "constraint" $dB_1^C = -\Pi B_0^C$:

FIGURE 4.4: Type I Operations
\[
\frac{dr_1}{dB_{1c}} = \frac{-2a\lambda - 2k\varepsilon}{\Delta 5} < 0, \quad \frac{dr_2}{dB_{1c}} = \frac{2\gamma\lambda - k\varepsilon}{\Delta 5} > 0
\]

\[
\frac{d\Pi}{dB_{1c}} = \frac{an - \gamma n - \varepsilon\gamma + \varepsilon a}{\Pi\Delta 5} < 0,
\]

\[
\frac{dR}{dB_{1c}} = \frac{n(\lambda + a) + k(\varepsilon^2 - n^2) - \varepsilon\lambda(\gamma + a)}{\Pi\Delta 5} > 0
\]

where \( \Delta 5 = a^2 + \gamma kn + \gamma k\varepsilon - nka - \varepsilon ka - \lambda \gamma^2 > 0 \)

Again, if the Bundesbank wants to prevent \( r_2 \) from changing it can do so in three ways. But since the qualitative results for \( r_1 \) and \( \Pi \) do not depend on either the security used for pegging \( r_2 \) or whether Bundesbank is passive or active, we will not discuss these cases further here. It should be noted though, that both the method used and the decision to be passive or not, have implications for the movements of reserves between the two currency area countries. The qualitative nature of the results does not change when the German Central Bank pegs \( M_2 \).
4.3.2.B **Type II Operations**

Intervention of type II is an exchange of home currency securities for the other currency area's country securities \((\text{dB}^C_1 = -\text{dB}^C_2)\). This type of intervention operation can be used when a currency area country wants to alter the structure of interest rates in the currency area without affecting the external exchange rate. Given the symmetry restrictions imposed earlier it is easy to understand that this type II intervention will have no effect on the common external exchange rate. In order to demonstrate the effects of this type of intervention on both currency area interest rates it is convenient to work in \((r_1, r_2)\) space instead of \((r_1, \pi)\) space. (Now \(\pi\) instead of \(r_2\) becomes a shift variable.) In Figure 4.5 four schedules which show the pairs of \(r_1\) and \(r_2\) which are compatible with equilibrium in each of the four asset schedules markets are depicted. The logic behind the slopes of these schedules is similar to the logic behind the slopes of Figure 4.1. A rise in one of the interest rates has to be accompanied by a rise in the other interest rate in order to keep the markets for the two bonds in equilibrium \((B_1B_1\) and \(B_2B_2\) schedules) and by a decline in the other interest rate in order to keep the markets for the two currencies in equilibrium \((M_1M_1\) and \(M_2M_2\) schedules).
The algebraic expressions for the slopes of the schedules are:

\[ \frac{dr_2}{dr_1} = \frac{- (b_{11}W + b_{11}'W' + b_{11}''W'')}{b_{12}W + b_{12}'W' + b_{12}''W''} = \frac{-a}{\gamma} > 0 \]

\[ \frac{dr_2}{dr_1} = \frac{- (b_{21}W + b_{21}'W' + b_{21}''W'')}{b_{22}W + b_{22}'W' + b_{22}''W''} = \frac{-\gamma}{a} > 0 \]

\[ \frac{dr_2}{dr_1} = \frac{-M_{11}}{M_{12}} = \frac{-\varepsilon}{n} < 0 \]

\[ \frac{dr_2}{dr_1} = \frac{-M_{21}}{M_{21}} = \frac{-n}{\varepsilon} < 0 \]
The relative slopes of the schedules are always as drawn in Figure 4.5. The initial equilibrium is given by the intersection of the four schedules at point a. If the BOF increases its holdings of French securities, by the same amount it reduces its holdings of German securities, the $B_1B_1$ curve shifts to the left, since for a given $r_2$ the public will be willing to hold a reduced supply of French securities only if $r_1$ is lower. The $B_2B_2$ schedule shifts to the left as well, since for a given $r_1$, a higher $r_2$ is required to absorb the increased supply of German securities. Attempts by one of the currency area's central banks to peg the bilateral exchange rate, increases France's money supply by the same amount it reduces Germany's money supply, so that $M_1M_1$ and $M_2M_2$ shift to pass through the new equilibrium point B, where $r_1$ is lower and $r_2$ higher.

The exact amount of change in the four variables is:

\[
\frac{\text{dr}_1}{\text{dB}_1^C} = \frac{4k_\varepsilon - 2a\lambda - 2\gamma \lambda}{\Delta_6} < 0
\]

\[
\frac{\text{dr}_2}{\text{dB}_1^C} = \frac{-2k_\varepsilon - 2k \gamma + 2a \lambda + 2\lambda \gamma}{\Delta_6} > 0
\]

\[
\frac{\text{dp}}{\text{dB}_1^C} = 0, \quad \frac{\text{dR}}{\text{dB}_1^C} = \frac{2\gamma (n\lambda - \varepsilon \lambda) + 2a(n\lambda - \varepsilon \lambda) + 2k(\varepsilon^2 - n^2)}{\pi \Delta_6} > 0
\]
where \( \Delta_6 = 2(a^2 \lambda + \gamma k \varepsilon + \gamma k n - \varepsilon k a - n k a - \lambda \gamma^2) > 0 \)

We observe that as a result of this intervention operation there has been a decrease in Germany's money supply and an increase in \( r_2 \). To counter these effects Bundesbank can itself be involved in open market operations. We demonstrate here only the effects of Bundesbank's pegging the interest rate on German securities, \( r_2 \), through changes in its holdings of ROW securities, \( B_0^C \), in the face of type II interventions operations by the BOF. To do so we still make use of Figure 4.5. The initial shifts due to the increase in BOF's holdings of French securities, by the same amount it reduces its holdings of German securities are the ones that shift the \( B_1B_1 \) and \( B_2B_2 \) schedules to \( B_1'B_1 \) and \( B_2'B_2 \), respectively. In order to prevent \( r_2 \) from rising the Bundesbank buys ROW securities. The final equilibrium is at point \( \gamma \), at which there has been a depreciation of the external exchange rate, \( \Pi \), causing \( B_1'B_1 \) and \( B_2'B_2 \) schedules to shift to \( B_1''B_1'' \) and \( B_2''B_2'' \), respectively, and an increase in Germany's money supply witnessed by the leftward shift of the \( M_2'M_2 \) schedule to \( M_2''M_2'' \). These are confirmed by
The other ways by which Budnesbank can peg either $r_2$ or $M_2$ do not provide us with significantly different results for discussion, and for this reason they are omitted.

4.3.3 A Rise in the ROW Interest Rate, $r_0$

The initial equilibrium position is given by the interaction of the solidly drawn lines at point in a in Figure 4.6. The increase in $r_0$ means that the $B_1 B_1'$ schedule has to shift to the right, say to $B_1 B_1'$, since given $\Pi$ an increase in $r_1$ is required to offset the excess supply of French securities that has been
created due to the increase in \( r_0 \). Similarly, the \( B_2B_2 \) schedule has to shift to the left, say \( B_2'B_2 \) since a drop in \( r_1 \) is required now to restore equilibrium in the market for German securities after a rise in \( r_0 \). At the initial values of interest rates and exchange rates an increase in \( r_0 \) results in a decreased demand for both French and German securities and as a result in an upward movement of both \( r_1 \) and \( r_2 \). The increase in \( r_2 \) means that the \( B_2'B_2 \) shifts to the right, say to

FIGURE 4.6: A Rise in the ROW Interest Rate
since an increase in \( r_1 \) is required for a given value of \( \Pi \), to offset the excess demand for German securities generated by the increase in \( r_2 \). The \( B^i B_1 \) schedule has to shift to the right as well, say to \( B_1 B_2 \), since an increase in \( r_1 \) is required for a given value of \( \Pi \) to offset the excess supply for French securities generated by the increase in \( r_2 \). For similar reasons the \( M_1 M_1 \) and \( M_2 M_2 \) schedules shift to the left, to \( M_1 M_1 \) and \( M_2 M_2 \), respectively. Because of the symmetry conditions imposed there is no change in \( R \) due to the change in \( r_0 \), and the new equilibrium is the common intersection of all schedules at point \( \beta \), with the values of \( r_1, r_2 \) and \( \Pi \) are

\[
\frac{dr_1}{dr_0} = \frac{(x\lambda - \lambda xy)}{\Delta_8} = \frac{dr_2}{dr_0} > 0
\]

\[
\frac{d\pi}{dr_0} = \frac{\gamma x(\epsilon + n) - 2ax(\epsilon + n)}{\Delta_8} > 0
\]

where \( \Delta_8 = (a^2 \lambda + \gamma \kappa \epsilon + \gamma kn - \epsilon ak - nka - \lambda \gamma^2) > 0 \)

If the rise of the currency area interest rates is deemed to be undesirable, then again there are three ways which can be used by the central banks to peg them. We start by first assuming that the Bundesbank
pegs $r_2$ by allowing its stock of ROW bonds to be endogenously determined. We use Figure 4.7 to illustrate the case. The assumed rise in the ROW interest rate will shift both the $B_1B_1$ and $B_2B_2$ schedules. $B_1B_1$ will shift to the right, $B_2B_2$ to the left, but both will shift by the same vertical distance, the reason being that the excess supply of French and German bonds generated by the rise in $r_0$ is the same and hence takes the same increase in wealth to be eliminated ($\Pi$ has to rise by the same amount for a given $r_1$, in order for the markets for French and German bonds to be again in equilibrium). Once this is realized it is easy to see that the final equilibrium will be at point $\beta$, since there will be no more shifts of the $B_1'B_1'$ and $B_2'B_2'$ schedules, given that $r_2$ is kept constant.

FIGURE 4.7: Pegging $r_2$ in the Face of Changes in the ROW Interest Rate
Increases in the money supplies of both currency area countries (by the same amount) shift the $M_1M_1$ and $M_2M_2$ schedules till they pass from point $\beta$. Using equation (18) we indeed get algebraically

$$\frac{dr_1}{dR_0} = 0, \quad \frac{d\Pi}{dr_0} = \frac{x}{k} > 0, \quad \frac{dR}{dR_0} = \frac{x\lambda}{k\Pi^2} > 0$$

$$\frac{dB_0'}{dr_0} = \frac{2x\lambda}{k\Pi} > 0, \quad dM_1 = dM_2 = \frac{x\lambda}{k\Pi} > 0$$

What if France's Central Bank wants to keep $M_1$ constant when Bundesbank intervenes to peg $r_2$ in the face of an exogenous increase in $r_0$? Let's start by looking at the case where BOF pegs $M_1$ by setting $dB_1^C = -\Pi dR$. Then, since both $M_1$ and $r_2$ are kept constant, there will be no shift at all of the $M_1M_1$ schedule, in Figure 4.7. The new equilibrium then will be on the $M_1M_1$ schedule, and since $B_2'B_2'$ does not shift at all ($r_2$ pegged, no changes in central banks holdings of German securities), the new point of equilibrium has to be where $M_1M_1$ and $B_2'B_2'$ cross (point $\gamma$). Changes in the holdings of French securities by the BOF and changes in reserves make the $B_1'B_1'$ and $M_2'M_2'$ schedules pass from this point too. At this point we observe that there has been an appreciation
of the external exchange rate (a fall in $\pi$), a decline in $r_1$, and a decline in Germany's money supply witnessed by the rightward shift of the $M_2M_2$ curve. As should be evident by now, the results are reversed if the relative slopes of the $B_1B_1$ and $M_1M_1$ schedules are reversed ($B_1B_1$ is steeper than $M_1M_1$ if $|k\varepsilon| < |\gamma\lambda|$). The following expressions (derived from equation (4.18)) confirm these findings:

$$\frac{dr_1}{dr_0} = \frac{-v\lambda}{\Delta\pi} < 0, \quad \frac{d\pi}{dr_0} = \frac{\varepsilon v}{\Delta\pi} < 0$$

$$\frac{dR}{dr_0} = \frac{x\lambda(\gamma - a)}{\Pi\Delta\pi} < 0, \quad \frac{dB^C_0}{dr_0} = \frac{-x\lambda(a + n - \varepsilon - \gamma)}{\Delta\pi} < 0$$

where $\Delta\pi = \Pi(\varepsilon k - \gamma\lambda) > 0$.

It should be noted that if the BOF tries to peg $M_1$ by setting $dB^C_0 = -dR$ the result is indeterminate when the Bundesbank uses its holdings of ROW bonds to peg $r_2$ in the face of an exogenous rise in $r_0$. The system can then be interpreted to be in a state of perpetual disequilibrium, since the actions of the two central banks push the system into different directions, given that the pegging of $r_2$ requires that the Bundesbank increases its holdings of ROW securities (the amount of
ROW securities available to be held by the public is reduced), whereas the pegging of $M_1$ requires that the BOF reduce its holdings of ROW securities (the amount of ROW securities available to be held by the public is increased). The same indeterminancy would arise if for example, $M_2$ was pegged by setting $\Pi dR = dB_1^C$, and $r_1$ was pegged, by using $B_1^C$, when there was a rise in $r_0$.

Another way by which Bundesbank can peg $r_2$ is by allowing its stock of French bonds to be endogenously determined. In Figure 4.8 the initial shifts due to the increase in $r_0$ take place. $B_1B_1'$ shifts to $B_1'B_1$ and $B_2B_2'$ to $B_2'B_2$. In order to prevent $r_2$ from rising, Bundesbank increases its holdings of French securities, $B_1'C$, thus shifting the $B_1'B_1$ schedule to the left (a lower $r_1$ is required to absorb the reduced quantity of French bonds available to the public). Since $r_2$ is kept constant, and no open market operations involving German bonds are undertaken the $B_2'B_2'$ schedule does not shift further. The new equilibrium is then at point $B$, the cross of $B_1'B_1'$ and $B_2'B_2'$, with changes in reserves making the $M_1M_1$ and $M_2M_2$ schedules ($M_2'M_2$ has already shifted to the left due to the increase in $M_2$ brought about by the increase in $B_1'C$) pass from $\delta$. The exact changes in the variables are:
FIGURE 4.8: Pegging r₂ -- Using the Other Country's Bonds

\[
\frac{dr_1}{dr_0} = -\frac{2\lambda x}{\Delta_{10}} < 0, \quad \frac{d\Pi}{dr_0} = \frac{x(a + n + \varepsilon - \gamma)}{\Delta_{10}} > 0
\]

\[
\frac{dR}{dr_0} = \frac{x(\lambda n - \lambda \gamma + \lambda a - \lambda \varepsilon)}{\Pi \Delta_{10}} > 0
\]

\[
\frac{dB'_C}{\Delta_{10}} = \frac{2x\lambda(a - \gamma)}{\Delta_{10}} > 0
\]

where \(\Delta_{10} = 2\lambda \gamma + k(n - \gamma + a + \varepsilon) > 0\)

If the Bundesbank was pegging \(r₂\) by allowing its stock of German bonds \((B'_2)\) to be endogenously determined then there would still be a depreciation of the external exchange rate, but now the interest rate
on French securities $r_I$ will rise instead of falling as in the previous case. Thus we again find that the way in which one of the currency area countries reacts to exogenous events has important qualitative implications for the other country's variables, as was the case when one country chose different reactions to a policy taken by the other currency area country.

4.4 Net Debtors

Up to this point we have assumed that both countries were net creditors -- an increase in $\Pi$ would increase their wealth. We now briefly look at the consequences of assuming that both countries are net debtors -- an increase in $\Pi$ would reduce their wealth. If the net debtor positions of the two currency area countries are large enough ($|b_1 B_0 + b_1' B'_0| > b_1^0 w^0$, $B_0, B'_0 < 0$), then one can see from the definitions of $k$ and $\lambda$ that their signs are reversed ($k, \lambda < 0$).

With the signs of the coefficients $k$ and $\lambda$ reversed, the slopes of the schedules in Figure 1 are reversed as well. $B_1 B_1$ is now upward sloping and $B_2 B'_2$, $M_1 M_1$ and $M_2 M_2$ are now downward sloping. Figure 4.9 depicts them and the point of their intersection is the initial equilibrium (point a). We will consider an expansionary
market operation by the BOF. If the French authorities increase their holdings of French securities by a given amount and then allow no further changes in these holdings the $B_1B_1$ curve shifts to the left, say from $B_1B_1$ to $B_1'B_1'$, since for a given value of $\Pi$, the public will be willing to hold a reduced supply of French securities only if $r_1$ is lower. The $M_1M_1$ shifts to the left as well, say from $M_1M_1$ to $M_1'M_1'$, since for a given value of $\Pi$, the public will hold an increased supply of francs only if $r_1$ is lower. In order to increase its holdings of French securities by a given amount, the BOF offers to supply francs deposits in exchange for French securities. At the
initial values of interest rates and exchange rates, there is excess demand for French securities and excess supply of French money. BOF bidding for French securities leads to downward movement in \( r_1 \), an increase in private demand in France, Germany and the ROW for German and ROW securities, and a downward movement in \( r_2 \). As the interest rate on German securities is bid down, an excess demand for francs, marks and French securities and an excess supply of German securities appears. These developments mean that the \( B_1B_1 \) shifts to the left, say to \( B_1'B_1' \), \( B_2B_2 \) shifts to the left as well, say to \( B_2'B_2' \), \( M_1M_1 \) shifts to the right to \( M_1'M_1' \), and \( M_2M_2 \) shifts to the right, to \( M_2'M_2' \).

At the same time stabilization of the bilateral exchange rate results in an increase in Germany's money supply by the same amount France's money supply is reduced, hence shifting \( M_2'M_2 \) to the left and \( M_1'M_1 \) to the right. The point of intersection of all four schedules \( \beta \) (changes in \( r_2 \) and \( R \) make all schedules pass through a common point) is the new equilibrium. At the new equilibrium, \( \Pi \), the common external exchange rate is lower (there has been an appreciation of the external exchange rate). This result of appreciation when the currency area countries are net debtors stands in contrast with our previous finding of
depreciation when both currency area countries are net creditors. The qualitative results for all other variables remain the same. One can easily see that even the quantitative results would not change, since in equation (4.18) the only thing that changes is the sign of all the coefficients of the third column of the matrix H. This then, further implies that only the results regarding the external exchange rate are reversed in all other situations we have discussed.

4.5 CONCLUSION

We have constructed a portfolio balance model to analyze monetary policy options and responses in a two country currency area interacting with the rest of the world. Three sets of conclusions emerge from this exercise. First, the conventional result in open-economy macroeconomics that an expansionary open market operation will result in a depreciation of the country's currency is not always validated in our model once the responses of the other currency area country are considered. Furthermore, as expected, different methods of intervening either to peg the money supply or the interest rate by the second country, have different qualitative results on the variables of the system. What was not expected
was the finding that under certain responses of the second currency area country, an expansionary open market operation by the first country can result in a decrease in the money supplies of both currency area countries (the offset coefficient is greater than $|1|$). If both countries are net debtors, then all results pertaining to the external exchange rate can be reversed, though none of the results pertaining to the other variables is affected.

Second, the qualitative results of open market or intervention operations do not depend on the central bank undertaking the operation, but only on the security(ies) involved. Clearly this is a result of an integrated world market in securities, so that the interest rate on a given security is the same everywhere, and the differential between the interest rates on the two securities depends on the relative supplies of the two securities. Note that this is a model with perfect capital mobility but not perfect asset substitutability, and for this reason even sterilized open market operations have effects on the common external exchange rate, unlike the monetarist model of exchange rate determination where asset supplies (other than money) do not matter -- only money supplies matter. However, due to the structure of our model, even
a "sterilized" intervention by one of the currency area countries, affects the money supplies of both currency area countries, the reason being reserves shifts needed for the perseverance of a common external exchange rate.

Third, following a disturbance in the ROW (a change in the ROW interest rate) a new equilibrium is not always attained whatever the behaviour of the monetary authorities. If the same security is used by both central banks to peg either their money supply and/or their interest rate, the system becomes indeterminate. For example, if, in the face of a rise in the ROW interest rate, the Bank of France pegs either the interest rate on the French securities $r_1$, or the French money supply using its holdings of French bonds, and the German monetary authority pegs either the German money supply or the German interest rate, using its holdings of French bonds, then an equilibrium cannot be attained. The question of whether or not there is an automatic adjustment mechanism under which balance of payments surpluses or deficits set in motion forces which lead to their elimination and whether or not policy making authorities can block such an automatic mechanism has long been a concern of international economics. Mundell (1968), using a model of the IS-LM variety argued that if the monetary authorities allow surpluses (deficits) to expand (contract) the money
supply, these surpluses (deficits) will set in motion expansionary (contractionary) forces which result in their elimination. However, if the monetary authorities sterilize the effects of imbalances, these imbalances can persist indefinitely, so that an equilibrium cannot be reached. Although our short-run portfolio balance model is not ideally suited for a complete analysis of questions relating to balance of payments adjustment mechanism, it yields the insight that the sterilization of financial capital movements effects on money supplies does not in itself lead to an "international disequilibrium system". As we explained above, it is only when certain methods for either stabilizing money supplies or interest rates are employed that the "inherent" adjustment mechanisms are not allowed to work.
FOOTNOTES
Chapter 4


2. It appears that several lines of thought independently originated the portfolio balance approach. McKinnon and Oates (1966) early pointed out the deficiencies of treating capital flows as a function of interest rate differentials. Branson (1975) modelled the portfolio approach at a fairly early state, basing his model on Tobin's monetary models. Kouri and Porter (1974), Boyer (1977), Branson (1977), Dornbusch (1975), and Obstfeld (1980) used the portfolio balance approach to study a host of problems.

3. All securities are assumed to be short-term fixed-price assets to avoid the complications introduced by capital gains resulting from changes in interest rates. These generally do not change the qualitative results (see Allen and Kenen (1976), and Allen and Kennen (1980)).

4. This is a portfolio balance model with postulated asset demand functions. In order to establish their plausibility, the builders of portfolio balance models have appealed to microeconomic theory -- in the case of non-monetary asset, to the theory of portfolio selection and in the case of monetary assets to the theory of money demand. There is widespread agreement on the importance of exploring the implications of macroeconomic asset demand functions derived from explicit utility maximizing behaviour. The argument that in general utility maximizing behaviour leads to modifications in asset demands when the policy regime changes (the Lucas critique) has increased the urgency for firmer microeconomic underpinnings.

5. When bonds are bills, however, it is not easy to explain why households invest simultaneously in all four financial assets, even when these households are risk averse. Uncertainty about the exchange rate explains why they hold ROW bonds, but not why they hold both bonds and money in the absence of transactions costs. One must attach some other source of uncertainty to the return on bonds, and this function can be played by the possibility of default.
6. The inclusion of fractional-reserve commercial banks would have no significant impact on the behaviour of the model (see Kenen, 1976).

7. There are four markets, not five, to be cleared by French and German variables, because the market for the ROW bond is cleared by instantaneous adjustments in the quantity supplied.

8. Negative net foreign asset positions have been associated with instability in flexible exchange rate regimes (Boyer (1977), Enders (1977), Branson, Haltunen and Masson (1979), Masson (1980), Henderson and Rogoff (1982)). When home country residents have a negative net foreign asset position the conventional valuation effect is reversed; home currency depreciation lowers the home currency value of home wealth rather than raising it, thereby reducing demand for home currency assets. This "perverse" valuation effect may cause instability. If a disturbance creates excess demand for home currency assets, restoration of short-run asset market equilibrium requires home currency depreciation. As home residents reduce their absorption and acquire financial assets, demand for home currency assets increases leading to further depreciation.

9. Recently this proposition has been challenged: it has been argued that securities being imperfect substitutes is neither a necessary nor a sufficient condition for intervention policy to have effects. It is not a necessary condition, since even if securities are perfect substitutes, a policy that changes expectations about the future values of other variables will affect the exchange rate. It is not a sufficient condition, if Ricardian equivalence is assumed, e.g., that private agents can "see through" government transactions. The basic insight is that if private agents regard the authorities holdings of domestic and foreign securities as their own, then when the authorities decide to alter their holdings through intervention operations, private agents will simply alter their direct holdings in an offsetting way, leaving the exchange rate unchanged. For more on this issue see Obstfeld (1982b) and Henderson (1984).
5.1 Summary

In this chapter we present the main findings of the previous chapters, note some limitations of one analysis and provide suggestions for further research.

What our analysis in chapter 2 suggests is that inflation rate and nominal income rules can be very effective in preventing any short-run home output deviations from its long-run equilibrium level for a small open economy in response to foreign disturbances, when the steady-state rate of monetary growth stands for the "core" rate of inflation in the Phillips curve. Moreover, fixing the real exchange rate does not represent a viable strategy whenever foreign shocks effect the equilibrium real rate of interest.

In chapter 3, the main conclusion was that beggar-my-neighbour effects of fiscal policies do not occur between members of a currency area "suffering" from real wage rigidity. However, it is still true that fiscal expansion in the rest of the world inflicts beggar-my-
neighbour effects on both currency area countries.

Finally, in chapter 4 we concluded that many of the results reached within the traditional one and two country portfolio balance models do not repeat themselves once the interdependence of monetary policies between the currency area countries is realized. For example, given certain responses of the central banks it is possible that an expansionary open market operation by one of the currency area countries can reduce both countries money supplies (i.e., the offset coefficient is greater than |1|), or appreciate the common external exchange rate.

5.2 Suggestions for Further Research

In this section we discuss some limitations of our analysis in the previous chapters and offer suggestions for future research.

An obvious suggestion for further work would be to incorporate all the asymmetries mentioned in the introduction into a model that incorporates exchange rate and price expectations as well. However, even the large scale econometric models typically embody some but not all, of these asymmetries. This accounts to a large extent for the rather large quantitative (and sometimes
qualitative) differences in the predictions of twelve leading large international econometric models (see Frankel, 1986). For this reason what we consider as a more realistic extension is to investigate the existence of beggar-my-neighbour effects of fiscal policies in a two-country model that incorporates exchange rate and price expectations and allows for different degrees of wage indexation in the two countries.

Chapter 2 is silent about what has come to be known as the Lucas (1976) critique of econometric policy evaluation. According to Lucas, because the structure of a model emerges from an optimal decision-making process that takes account of policy rules in place any change in these rules will be accompanied by a change in model structure. More specifically, the parameter values we have assumed should not be expected to remain the same when the underlying monetary policy regime changes. Usually (see Scarth (1986b)) the Lucas critique is used to point out that the slope parameter of the short-run Phillips curve is not a free parameter-like taste and technology coefficients that are presumably independent of government policy -- but depends on a host of taste and technology parameters as well as the policy rule(s) being in operation. However, if one wants to pay full service to the Lucas critique, formal rationale for the consumption, investment and money
demand functions should be provided as well. Doing so would be beyond the scope of the present study.

What we consider more important is the lack of explicit modelling of the supply side effects of exchange rate changes. Disturbances that require a change in the terms of trade for an equilibrium to be re-established affect the full employment level of output. Firms demand labour according to the real wage in terms of the domestically produced good, while workers supply labour according to the real wage in terms of a bundle of consumption goods, which includes the imported as well as the home good. A change in the terms of trade induces a change in employment through its differential impact on the real wage rates relevant to the decisions of firms and workers. However, since our interest was about the short-run dynamic properties of alternative monetary policy rules, we went along with the main thrust of the literature and ignored permanent aggregate supply effects of exchange rate changes.

These effects are the main focus of the analysis in chapter 3. Our interest in that chapter was on the long-run output effects of monetary and fiscal policies within a currency area "suffering" from real wage rigidity. An unattractive feature of our analysis was that agents were assumed to have no forward-looking expectations. Miller and Salmon (1985) have stressed that models
involving rational exchange rate expectations have the property that current effects in one economy depend on current and expected future policies in other economies. This means that each policy maker is in a strategic relationship with other policy makers. Miller and Salmon analyzed optimal policy within a two-country policy area that is essentially the same as Levin's (1983) model, except that forward-looking exchange rates are added. They assume that there are no beggar-my-neighbour effects in a model with fixed prices, which is opposite to what Levin derived. An obvious extension of our analysis would be to perform Miller and Salmon's numerical explorations of time inconsistency and policy coordination within a model that allows for a realistic aggregate supply specification as well, and not just a disaggregated aggregate demand sector.

Of course, the same strategic considerations should be relevant in chapter 4. The knowledge by one of the central banks that its actions can be emaciated by the policies of the other central bank, will probably force it to avoid policies that would be deemed to have undesirable effects on variables for whose the other central bank is considered to be responsible. But this would not necessarily be the case, especially when there are
asymmetries either in the distribution of power among central banks (e.g., the liabilities of one central bank constitute the reserve currency of the other) and/or in the relationships between the financial variables under the control of the central banks and the "real" side of the economy (e.g., different interest rate elasticities of aggregate demand, different term structures of interest rates, etc.).

The term structure of interest rates and its pivotal role in both connecting the real and financial sides of the economy and making subtle the interdependence of short run and long run actions of economic agents has been examined very recently by Aoki (1986) and Turnovsky (1986b). We think that incorporating permanent supply side effects of exchange rate changes and/or accounts of the determination of the term structure of interest rates, will prove to be a useful extension of the work done in all the chapters.
FOOTNOTE

Chapter 5

1. It should be noted that in chapter 2's asymmetrical two country-model, it is monetary and not fiscal expansion in the large country that inflicts beggar-my-neighbor effects on the small country for an extended time period -- fiscal expansion in the large country is positively transmitted to the small one.
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