# THEORY OF MONETARY POLICY

# FOUR ESSAYS ON THE THEORY OF MONETARY POLICY

By

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To "Maan Ji"

#### Abstract

This dissertation focuses on the construction of macroeconomic models that can be used to evaluate monetary policy. The theoretical models developed for this purpose emphasize the importance of intertemporal optimizing behaviour, the role of expectations and nominal price rigidities.

The first essay (chapter 2) compares the implications of the interest rate channel and the cost channel (when firms' marginal cost depends directly on the nominal interest rate) of the monetary policy transmission mechanism under inflation and price-level targeting regime for determining the optimal trade-off between stabilizing inflation and the output-gap in a forward-looking closed-economy model. In contrast to the traditional literature, which focuses only on the interest rate channel, research presented in chapter 2 demonstrates that the demand shocks cannot be completely stabilized with inflation targeting when the cost channel is operative: the central bank faces an inflation outputgap stabilization trade-off. Moreover, it is shown that there are gains (in terms of reduced expected losses for the central bank) to be made from conducting monetary policy with commitment rather than in a discretionary fashion even if there is no inflation bias. Also, these gains are larger in the presence of cost channel; so there is a stronger case for commitment. Furthermore, building on the notion that it is difficult to conduct monetary policy with full commitment due to practical considerations, chapter 2 seeks an answer to the question: Is it possible to replicate the results of inflation targeting under commitment with price-level targeting under discretion? The results in chapter 2 confirm the earlier results that it is possible. However, it also points out that the results need to be interpreted carefully in the presence of the cost channel. In particular, the relative weight on outputgap stabilization versus inflation stabilization needs to be appropriately adjusted in comparing the expected value of the losses incurred in the two regimes: otherwise the results could be different.

Chapter 2 also proposes an alternative method of calculating the implicit interest rate rule that implements the optimal policy. Unlike the existing literature, this method correctly depicts the inertial behaviour of optimal policy with commitment and derives conditions that can avoid indeterminacies of output and inflation. Moreover, in the presence of the cost channel, this method is the only correct method in the commitment case for the sake of internal consistency of results.

The second essay (chapter 3) uses continuous-time modeling approach instead of the more conventional discrete-time approach and compares the performance of two rulebased targeting regimes --- price-level targeting and nominal income targeting with and without the cost channel of monetary policy for a closed economy. Chapter 3 considers a series of macroeconomic models with different specifications for both the aggregate demand side and the aggregate supply side of the economy to check for robustness of results. It is assumed that the two targeting regimes generate the same outcome regarding long-term inflation. Thus, the criterion for evaluating the performance of a monetary regime is its ability to minimize the volatility in real output in response to aggregate demand shocks.

The main result of chapter 3 is that the cost channel matters in the sense that the volatility of real output increases under both price-level and nominal income targeting when the cost channel is included in the models. However, the inclusion of the cost channel does not say much on the choice between the two regimes. It appears that nominal income targeting performs better than price-level targeting in bringing down the volatility of real output in almost all the specifications of the macro models used in the analysis regardless of the cost channel.

Using the type of models analyzed in chapter 3, chapter 4 introduces open economy considerations and looks at the performance of monetary policy (in terms of reducing volatility in real output) under three alternative targeting regimes: exchange rate targeting (fixed exchange rates), price-level targeting, and nominal income targeting (both flexible exchange rate options). Although the supply-side effects of the interest rate (the cost channel) are ignored, the supply-side effects of exchange rate changes (due to the existence of intermediate imported inputs) are highlighted. Under these settings, chapter 4 explores the impact of an increased degree of price flexibility on the volatility of real output. Chapter 4 finds support for Keynes' concern that under a flexible exchange rate regime a higher degree of price flexibility can raise output volatility. This result is consistent with central banks that argue that their low inflation-flexible exchange rate policy has increased contract length and so decreased the degree of nominal price flexibility. Chapter 4 also finds support for central banks that favour fixed exchange rates. A fixed exchange rate regime implicitly implies that the economy is willing to accept structural reforms of the sort that promote flexibility in prices to absorb the effects of shocks. Chapter 4 shows that with fixed exchange rates output volatility indeed goes down as a result of increased price flexibility. Thus, there is internal consistency within both views about exchange rate policy.

Chapter 5 develops a discrete-time dynamic stochastic general equilibrium model with incomplete asset markets, nominal price rigidities and monopolistic competition to shed light on the role of the exchange rate and its relation with current account dynamics and domestic inflation in the formulation of monetary policy. Chapter 5 argues that the assumption of complete asset markets is not realistic in a model with imperfections and rigidities in goods market because, with nominal rigidities, monetary policy will affect real variables including the current account. With incomplete asset markets the dynamics of current account does matter for monetary policy because then, besides dealing with the distortions created by monopolistic competition, the central bank needs to address the inefficiencies caused by incomplete asset markets.

The main result of chapter 5 is the breakdown of a widely accepted result that optimal monetary policy for an open economy calls for adjusting the interest rate to completely offset demand shocks, which implies no trade-off between output volatility and inflation volatility. Due to the direct effect of real exchange rate on domestic inflation, the optimal monetary policy calls for trading-off some output volatility for less inflation volatility. Moreover, due to the dynamics of current account, it entails a response to variations in the net foreign asset position, since this mechanism makes the impact of the demand shock last well beyond the time interval for which prices are rigid.

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#### **Chapter 1**

#### Introduction

Academic thinking about monetary economics has changed drastically over the past decade and so has the practice of monetary policy. Almost simultaneously, big advances were made in the management of monetary policy on the one hand and in theoretical and empirical research on monetary policy on the other hand. For example, a consensus has grown in many countries that the central banks should primarily focus on policies that promote long-term price stability in the economy, and that a rule-based monetary policy is superior to discretion-based monetary policy. Many developments in the macroeconomic theory along with actual events have led the economics profession, academicians and policy makers alike, to embrace this ideology. The theoretical developments have culminated in a simple framework that is based on intertemporal optimizing behaviour, rational expectations, and temporary price rigidities and in which monetary policy have both short-term or temporary output effects and long-term or permanent price effects. This new framework is generally referred to as the 'New Neoclassical Synthesis model' (Goodfriend and King (1997)) or the 'New Keynesian' model (Clarida, Gali and Gertler (1999)) and is widely used to assess the desirability of alternative monetary policies.<sup>1</sup>

<sup>&</sup>lt;sup>1</sup> For an excellent survey of recent developments and results in this area, see Gali (2003).

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This dissertation utilizes this framework to analyze different transmission mechanisms through which monetary policy affects aggregate output and inflation. For example, chapter 2 and 3 explore the implications of the cost channel, that is, when monetary policy directly affects the cost of production of firms. Chapter 2 focuses on deriving the optimal monetary policy under discretion and commitment and compares the performance of two targeting regimes --- inflation targeting and price-level targeting. The metric used for this purpose is the expected losses of the central bank. Chapter 3, instead of deriving the optimal monetary policy, uses interest rate based monetary policy rule and studies price-level and nominal income targeting regimes. In addition, a series of sensitivity tests regarding the specification of the model are also considered.

Chapter 4 and 5 close the cost channel of monetary policy and extend the analysis to an open economy. Apart from comparing the performance of fixed versus flexible exchange rates in terms of reduced volatility in real output, chapter 4 also studies the implications of an increased degree of price flexibility on the volatility of real output. Chapter 5 incorporates the dynamic interaction of exchange rate, domestic inflation and current accounts in deriving the optimal monetary policy response under discretion. Thus, the dissertation explores a host of issues that are of interest to central banks --- the institute responsible for the conduct of monetary policy.

### 1.1- Why a New Framework? --- Some Historical Background

In order to fully appreciate the importance of the new framework that is applied to

various issues related to monetary policy in this dissertation, it is useful to understand the historical developments<sup>2</sup> that have led to its inception.

The dominant paradigm in macroeconomics in the 60s and early 70s that was used to explain short-run fluctuations in aggregate output, and the role of monetary policy in this context, was the traditional IS-LM model augmented with a relationship that depicted the gradual adjustment of prices and wages --- the Phillips curve (PC). This reference framework contained key elements of the Keynesian approach as it assigned a central role to demand shocks, and the gradual adjustment of wages and prices, as factors accounting for short-term fluctuations. The model also incorporated some elements of the Classical approach, such as neutrality of money in the long run and thus was referred to as the "neoclassical synthesis" by Samuelson.

The rational expectations revolution of the 70's disturbed the consensus around the 'neoclassical synthesis' model. It criticized the traditional IS-LM-PC framework for lacking in microfoundations especially regarding the Phillips curve and demonstrated its weaknesses in terms of evaluating alternative policy regimes that is popularly known as the Lucas critique.<sup>2</sup> Also, empirical observations such as the stagflation of the 70's and the inability of demand management policies to achieve full employment, further called into question the relevance of the traditional framework.

<sup>&</sup>lt;sup>2</sup> For a detailed review, see Blanchard (2000) and Woodford (1999).

 $<sup>^3</sup>$  For a detailed discussion on the criticism of the Keynesian paradigm, see Lucas and Sargent (1979).

Based on the Classical assumption of perfect competition and price flexibility, an alternative framework was developed. This model, known as Lucas' "island model", relied on the assumption of imperfect information, that is, the inability of economic agents to distinguish between changes in relative prices and changes in the general price level. However, this model did not succeed due to its empirical shortcomings; it failed to generate fluctuations of significant magnitude and persistence.

The early 80's saw the emergence of a new paradigm --- Real Business Cycle (RBC) theory.<sup>4</sup> In retrospect, it can be concluded that the RBC theory has had an unquestionable influence on the macroeconomic theory and policy. It proposed a number of revolutionary ideas. First, it developed a dynamic stochastic general equilibrium model with optimizing consumers and firms thus providing firm microfoundations for aggregate macro relationships and a systematic way of answering Lucas critique. Second, it emphasised on quantitative analysis of calibrated models, and it compared implied statistical properties of the models with those observed in data. Third, in contrast to the traditional approach, its focus on technological variations as a central source of aggregate economic fluctuations demonstrated the possibility of explaining such fluctuations without reference to any monetary variable. Although the RBC school may have had permanent effects, the initial enthusiasm around its conception of fluctuations has gradually diminished mainly due to two reasons: excessive reliance on technology shocks

<sup>&</sup>lt;sup>4</sup> For an early contribution in this literature, see Kydland and Prescott (1982).

as a source of business cycles, and (contrary to empirical evidence) its predictions of monetary neutrality.

In the late 90's a convergence of Keynesian assumptions and RBC methodology took place. Thus, a new framework ---- "new neoclassical synthesis" or "new Keynesian model" has emerged. This new framework has many advantages. First, it integrates Keynesian elements ---- imperfect competition and nominal rigidities into a dynamic general equilibrium framework that until recently was largely associated with the Real Business Cycle (RBC) or "new Classical" paradigm. So, the mechanism that Keynesians regard as essential for generating short-run real effects from demand shocks is an integral part of the structure. Second, it is firmly grounded in inter-temporal optimization, so the desire on the part of "new Classicals" for well-articulated micro-foundations is respected. Third, with each equation being structural, the Lucas critique can be respected as the model is applied to policy questions. Fourth, the model is conveniently analyzed at the same level of aggregation as was common with earlier generations of policy-oriented discussions. Fifth, it permits an explicit utility-based welfare analysis of the consequences of alternative monetary policies.

#### **1.2-** An Outline of the New Framework

In recent years, there has been an explosion in research that applies the type of framework referred to as the "new Keynesian" model to analyze various aspects of monetary policy.<sup>5</sup> Most of this research takes as given that the central bank has an explicit or implicit inflation target. This assumption makes the research useful to most central bankers. The purpose of this research is then to find policy rules to guide central banks in setting the policy instruments so as to bring the inflation rate close to the target and keep it there in order to achieve the ultimate target of price stability, while taking account of the short-run trade-offs that impinge on output and exchange rate variability.

A typical closed economy 'new Keynesian' model used for monetary policy analysis, and that serves as a benchmark in this dissertation, is composed of three relationships:

First, the "new IS Curve" relates the output gap negatively to the real interest rate, and positively to the expected future output gap.

$$x_{i} = E_{i} x_{i+1} - \sigma (u_{i} - E_{i} \pi_{i+1}) + u_{i} \qquad \sigma > 0$$

where,  $x_i$  is the gap between actual output level and the level of output under flexible prices,  $i_i$  is the nominal interest rate --- the monetary policy instrument,  $E_i \pi_{i+1}$  is the future expected inflation based on information available up-until and including period t and  $u_i$  is the disturbance term representing demand shocks.

<sup>&</sup>lt;sup>5</sup> Two excellent advanced-level textbook treatments on the applications of this framework are Walsh (2003a) and Woodford (2003).

Chapter 2 derives this relationship from a dynamic general equilibrium discretetime model. Chapter 3 derives it in continuous-time settings. Based on the works of Fuhrer (2000) and Amato and Laubach (2003), chapter 3 also introduces a lagged outputgap term in the new IS function. Both these papers have pointed out that the standard Euler equation for consumption (that gives rise to the new IS relationship) fails to capture the dynamics of the aggregate output. Fuhrer (2000) allows for habit persistence in preferences maintaining the assumption of optimal consumption choice on the part of consumers, while Amato and Laubach (2003) introduce 'rule of thumb' behaviour on the part of a fraction of the households (assuming that it is costly to reoptimize every period) with the remaining fraction of the household optimizing consumption in a usual fashion. Chapter 4 derives the continuous-time version of the standard "new IS" relationship and also uses the alternative specifications mentioned above for an open economy. Chapter 5 involves the assumption that all imports are consumption goods, it derive the new IS relationship for an open economy in a general equilibrium setting, and it emphasizes the importance of current account dynamics in affecting the aggregate output-gap.

The second component of the "new Neo-Classical synthesis model" is the "new Keynesian Phillips Curve" that relates inflation positively to both the output gap and expectations of future inflation.

$$\pi_i = \beta E_i \pi_{i+1} + \lambda x_i + v_i \qquad \beta > 0, \lambda > 0$$

where,  $\pi_i$  is the actual inflation in period t and v, is the disturbance term representing supply-side or cost-push shocks.

Chapter 2 takes this specification as a benchmark (nonetheless it formally derives it) and introduces the direct effect of nominal interest rate on inflation by assuming that firms need to borrow cash from banks in order to finance their wage-bills before the receipt of sales revenues. This introduces an independent channel of monetary policy transmission mechanism, known as the cost channel. Apart from deriving the "new Keynesian Phillips curve" in a continuous-time framework, chapter 3 augments this basic forward-looking inflation adjustment equation with a lagged inflation term in addition to the cost channel. Fuhrer and Moore (1995) introduce the lagged inflation term in an adhoc fashion while Mankiw and Reis (2002) suggest an alternative approach. They depart from the assumption of sticky prices and replace it with that of sticky information. According to their model of price adjustment firms gather and process the information about the state of the economy slowly over time, thus, although prices are always changing but firms are slow to update their pricing strategies in response to new information. The motivation for doing this is the criticism of many researchers that the basic 'new Keynesian Phillips curve', which is based on Calvo's staggered price model, generates inertia in the price level and not the inflation rate and that this is inconsistent with the stylized facts on inflation dynamics. Chapter 4, assuming that a fraction of imports are used as raw material in domestic production, incorporates the lagged inflation term in an open economy continuous-time inflation adjustment equation. Chapter 5 models imports only as consumption goods and derives the baseline "new Keynesian Phillips curve" for an open economy. As will be discussed in detail in chapter 5, the

channel through which exchange rate movements affects domestic inflation are quite different from the existing specifications.

The third equation in the "new Neo-Classical synthesis model" is the description of monetary policy behaviour. Broadly speaking, there are two approaches to modeling the monetary policy. First is the specification of an objective function (also termed as the loss function) that explicitly translates the behaviour of the target variables into a welfare measure. A widely used specification of the loss function takes the output gap  $x_i$  and inflation  $\pi_i$  as the target variables and takes the following general form<sup>6</sup>:

$$L_{i} = \frac{1}{2} E_{i} \left[ \sum_{i=0}^{\infty} \beta' \left( \alpha x_{i+i}^{2} + \pi_{i+i}^{2} \right) \right]$$

where,  $\alpha$  is the weight the central bank places on output deviations relative to inflation fluctuations. This approach involves deriving the optimal monetary policy by minimizing the loss function subject to the constraints imposed by the structure of the economy as captured by the "new IS" and the "new Keynesian Phillips curve" relationships. Chapter 2 and chapter 5 model the behaviour of the central bank using this approach. Svensson (1999a) has labelled this approach as a "flexible" inflation targeting regime. It is flexible in the sense that the central bank is not only concerned with the deviation of inflation from its target but also with fluctuations in the output-gap because movements in the output-gap have a direct effect on inflation. Chapter 2 also considers an alternative

<sup>&</sup>lt;sup>6</sup> Woodford (2002) derives this loss function for a typical 'new Keynesian model'

targeting regime, namely, price-level targeting and chapter 5 also reports results for an exchange rate targeting regime.

An alternative approach<sup>7</sup> to incorporate the role of monetary policy is to specify an interest rate based monetary policy rule.<sup>8</sup> An example of one such rule, that has gained significant popularity in recent years, is the Taylor rule<sup>9</sup>:

$$i_t^* = (\overline{r} + \overline{\pi}) + \alpha_\pi (\pi_t - \overline{\pi}) + \alpha_x x_t \qquad \qquad \alpha_\pi > 0, \alpha_x > 0$$

where,  $i_t^*$  is the desired nominal interest rate set by the central bank and  $\bar{r}$  and  $\bar{\pi}$  represent the long-run equilibrium values of the real interest rate and inflation respectively.  $\alpha_{\pi}$  is the central bank's reaction coefficient of deviation of inflation from its target and is usually taken to be bigger than one.  $\alpha_x$  is the coefficient on fluctuations in the output gap. Chapter 3 and 4 use variants of such interest rate based monetary policy rules. In particular, they employ interest rate rules that entail a response to movements in the price-level and nominal income. In this sense these chapters study price-level and nominal income targeting regimes. The chapters assume that the two targeting regimes generate the same outcome regarding long-term inflation. Thus, the criterion for evaluating the performance of a monetary regime is its ability to minimize the volatility in real output in response to ongoing aggregate demand shocks.

<sup>&</sup>lt;sup>7</sup> For a critical survey of the two approaches, see Pill and Rostagno (2001)

<sup>&</sup>lt;sup>8</sup> Taylor (1999) is an excellent collection of papers that apply these rules in various settings.

<sup>&</sup>lt;sup>9</sup> For critical assessment of Taylor-type interest-rate based rules, see Kozicki, (1999), Hetzel, (2000) and Svensson, (2003).

#### 1.3- Monetary Policy Transmission Mechanisms --- An Overview<sup>10</sup>

The monetary policy instrument, for instance the short-term nominal interest rate. affects the economy through various channels influencing various markets and variables. Identifying these transmission mechanisms is useful and important since they determine the main restrictions that central banks face in implementing monetary policy. Figure 1 below provides an overview of the relationship between monetary policy actions and the various transmission mechanisms.



#### **Figure 1: Monetary Policy Transmission Mechanisms**

<sup>&</sup>lt;sup>10</sup> For a detailed discussion and overview, see Bank of England (1999) and Taylor (2002).

The first channel is the traditional interest rate-aggregate demand channel of monetary policy transmission. It relies on at least two key relationships that elaborate the link between changes in the short-term nominal interest rate that is regarded as the monetary policy instrument and the long-term real interest rate that ultimately affect consumption and investment activity in an economy. First is the relationship between the nominal interest rate and the real interest rate and is explained by theories of price and/or wage rigidities. The second is the relationship between short-term and long-term real interest rates and is captured by various theories of the term structure of interest rates. It must be noted that it is the changes in the long-term real interest rates that affect aggregate consumption, business and residential investment and other components of aggregate demand. However, in most macroeconomic models, including the models employed in this dissertation, an explicit distinction between short-term and long-term interest rates is not made; they only refer to the interest rate. The distinction between nominal and real interest rates, however, remains crucial in explaining the effects of monetary policy and accordingly the issue of nominal rigidities underlying this distinction. All four chapters in this dissertation treat the interest rate-aggregate demand channel as the main channel through which monetary policy action affects aggregate output and prices.

The second channel of monetary policy transmission focuses on monetary and credit aggregates. In a limited and a typical classical monetarist's sense this channel relies on a direct impact of the monetary aggregates on aggregate demand and prices. For determining long-run inflation this is still regarded as a relevant theory. However, a broad and more recent view emphasizes credit aggregates to determine the short to medium run effects of monetary policy. This important transmission mechanism rests on the direct and indirect impact of interest rate changes on the balance sheet of borrowers (and in some instances, balance sheet of lenders such as banks). The balance sheet plays an important role for a firm's ability to obtain external finance (e.g., access to bank credit), especially when there are information asymmetries in the financial markets that lead to adverse selection and moral hazard problems. The more leveraged the balance sheet of firms (that is, a higher debt to equity ratio), the more important this channel becomes. A change in monetary policy (often reflected in changes in short-term interest rates) directly affects the 'cost of debt' or 'cost of external funds' of firms, altering firms' balance sheet positions and thus their net worth. Moreover, a change in the interest rate indirectly leads to a change in the prices of assets held by firms affecting its collateral. Thus, changes in monetary policy affect the investment activity (by affecting the net worth and/or collateral of firms) in an economy leading to a change in the aggregate demand and thus aggregate output and prices. This channel is usually termed the financial accelerator mechanism.

A closely related channel --- the bank lending channel --- focuses exclusively on the balance sheet of banks and argues that if banks' balance sheets are disturbed by the monetary policy action affecting their ability to lend, then firms' cost of production (and thus inflation) can be directly affected if they are taking loans from banks. It is these lending and cost channels that are the focus of chapter 2 and 3 of the dissertation. Chapter 2 incorporates this cost channel in a dynamic general equilibrium setting by assuming that firms need to borrow cash from banks to finance the wage bill before receiving the revenues form sales. The chapter then studies its implications for optimal monetary policy under inflation targeting and price-level targeting. Chapter 3, using the framework of chapter 2, analyzes the cost channel in comparing the performance of a rule-based monetary policy that implements price-level targeting regime or nominal-income targeting regime.

The third channel of monetary policy transmission focuses on asset prices other than the interest rate such as market value of securities (bonds and equities) and prices of real estate. According to this channel, a policy-induced change in the nominal interest rate affects the price of bonds and stocks that may make investment more or less attractive. This relationship is often referred to as Tobin's q and is defined as the ratio of market value of an enterprise to the replacement value of the enterprise. Moreover, a change in the prices of securities entails a change in wealth which can affect the consumption of households. It is worth noting that in the baseline 'new Keynesian' model investment (a change in the capital stock over time) is not modeled, thus making this channel non-operative.<sup>11</sup>

<sup>&</sup>lt;sup>11</sup> For a general discussion on monetary policy and asset prices, see Bernanke and Gertler (1999).

Since exchange rate is the relative price of domestic and foreign money, the exchange rate channel depends on both domestic and foreign monetary policy conditions. However, other things equal, a policy-induced change in the domestic interest rate affects the exchange rate (through the interest parity condition) that in turn affects net exports and thus aggregate demand. Moreover, exchange rate changes lead to changes in the domestic price of imported consumption goods and imported production inputs affecting inflation directly. The implications of this channel are explored in chapter 4 and 5. Chapter 4 models imports as intermediate inputs and compares the performance of fixed exchange rate regime with price-level and nominal income targeting under flexible exchange rate regime. Chapter 5 models imports as consumption goods and sheds light on the dynamic interaction of exchange rate and current accounts in the context of an optimal discretionary inflation targeting regime.

So far I have described how changes in the *current* monetary policy actions work their way through the economy via four channels. However, the literature sometimes identifies a separate fifth channel that is based on the economic agents' *expectations* of the future stance of monetary policy. According to this 'expectations' channel, all variables that are determined in a forward-looking fashion (such as aggregate demand and inflation) are affected by future expected monetary policy actions. The direction in which such expectational effects work is hard to predict and can vary from time to time. But, the very possibility of such effects contributes to the uncertainty of the impact of any

policy change, and increases the importance of having a credible and transparent monetary policy regime.

#### 1.4- An Intuitive Discussion of Results Derived in the Dissertation

To understand the mechanics of the monetary policy in a baseline 'new Keynesian' model, and thus the dissertation, suppose that the central bank is conducting monetary policy in a discretionary fashion and that there is a negative demand shock that lowers the aggregate real output and as a result also inflation. Assuming that the central bank is able to observe these shocks, a typical response would be to decrease the nominal interest rate. This policy response increases the level of output through the traditional interest rate-aggregate demand channel bringing it back at the target level of output under flexible prices. Since inflation is positively related to output, it also increases reaching the original level. Thus, the central bank is able to perfectly offset the demand shock and faces no trade-off between stabilizing the output-gap and inflation. In the case of a cost-push shock, the central bank does face an output-gap-inflation variability trade-off. An adverse cost-push shock that raises inflation will be countered by raising the nominal interest rate thus bringing inflation back at its targeted level. However, the output level decreases, so the cost-push shock cannot be completely stabilized.

Chapter 2 analyzes the optimal monetary policy response to both demand and cost-push shocks by incorporating the cost channel of monetary policy transmission mechanism in the new Keynesian model summarized above. As discussed, the cost PhD Thesis - Hamza Ali Malik, Department of Economics, McMaster University

channel is based on the assumption that firms need to borrow cash from financial intermediaries or simply banks before the receipt of sales revenues to finance the wage bill. This makes firms' marginal cost (and thus inflation) depend directly on the interest rate generating supply side effects of changes in the monetary policy instrument --- the nominal interest rate. Several researchers (e.g., Christiano and Eichenbaum (1997, 2001) have emphasized on the cost channel as a powerful collaborator of the traditional interest rate channel in the transmission of short-run effects of monetary policy. The contribution of this chapter is its ability to analyze these two alternative and independent channels of monetary policy transmission, in one unified, widely accepted 'new Keynesian' model.

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An important result is that, with the cost channel operating, the central bank faces a trade-off in stabilizing output-gap and inflation in the presence of both demand and cost-push shocks. Building from the previous example, suppose there is a negative demand shock that lowers output and inflation. The central bank, using its discretion, responds by lowering the nominal interest rate. As before, this policy increases output that also indirectly increases inflation. However, due to the cost channel there is a direct negative effect on inflation as well. Thus the central bank cannot completely stabilize both output-gap and inflation; it will be better-off in trading some volatility in the outputgap for reduced volatility in inflation.

Chapter 2 also evaluates the performance of monetary policy under commitment, that is, when the central bank commits itself to following a certain rule. The debate over
rules versus discretion and the role of credibility in monetary policy has a long tradition starting from the influential works of Kydland and Prescott (1977) and Barro and Gordon (1983). The basic insight of this literature is that if the central bank has a desire to push the economy's output level beyond the natural rate then discretion would only lead to higher average inflation (generally termed as inflation bias) with no effect on output. Committing to a rule would eliminate the inflation bias. An interesting new insight provided by the forward-looking 'new Keynesian' model is that there are gains from commitment even in the absence of an over-ambitious output target and an inflation bias. Clarida, Gali and Gertler (1999) refer to this mefficiency due to discretion as a stabilization bias. It arises simply because of a lack of history dependence or mertia in the policy actions of the central bank. Since optimal monetary policy under commitment exhibits a considerable degree of mertia, it is superior to discretionary optimal policy. Chapter 2 takes this result one step further and shows that the gains from commitment are even larger in the presence of a cost channel. The intuition for this result is simple. With monetary policy conducted under commitment, the future expected values of output-gap and inflation affect central bank's current behaviour as well. When the central bank lowers the nominal interest rate to counter the effects of a negative demand shock it not only increases current output but also increases the future expected output-gap and thus expected future inflation. The latter two outcomes have a direct effect on current inflation. Chapter 2 shows that these direct effects of expected future output-gap and its indirect effect via future expected inflation on current inflation are only operational in the presence of the cost channel and are precisely the reason why the gains from commitment increase.

It is standard practice in the literature on optimal monetary policy to derive the implicit interest rate rule that implements the optimal discretionary and commitment policy. Chapter 2 proposes an alternative method of calculating this implicit interest rate rule. Unlike the existing literature, this method correctly depicts the inertial behaviour of optimal policy with commitment and derives conditions that can avoid indeterminacies of output and inflation. Moreover in the presence of cost channel, this method is the only correct method in the commitment case for the sake of internal consistency of results.

Although a pre-committed monetary policy yields better outcomes, it is timeinconsistent. That is, the central bank has an incentive to deviate from its announced policy. Moreover, in practice no central bank can commit to a policy rule for all time periods.<sup>12</sup> Keeping in view these considerations, chapter 2 then explores the possibility of replicating the commitment solution with inflation targeting while monetary policy is conducted in a discretionary fashion under an alternative price-level targeting regime. As expected, price-level targeting regime performs better in terms of lower expected losses for the central bank even in the presence of the cost channel. The intuitive reason for this is that it depicts the inertial behaviour of the central bank which is the feature of the commitment solution. Thus, monetary policy actions entail a more gradual response to

<sup>&</sup>lt;sup>12</sup> "Rarely does society solve a time-inconsistency problem by rigid pre-commit.....Enlightened discretion is the rule." Blinder (1998).

shocks that allow the central bank to appropriately affect private sector expectations. However, it is worth noting that this result needs to be interpreted carefully in the presence of the cost channel. In particular, the relative weight on output-gap stabilization versus inflation stabilization needs to be appropriately adjusted when comparing the expected value of the losses incurred in the two regimes; otherwise the results can be different.

Chapter 3 is similar in its approach employed in chapter 2. It compares the performance of two targeting regimes --- price-level targeting and nominal income targeting with and without the cost channel of monetary policy for a closed economy. However, there are some important differences. First, it uses continuous-time modeling approach instead of the more conventional discrete-time approach. Apart form the advantage in terms of analytical simplicity, continuous-time models avoid the unappealing problems regarding the model properties being dependent on small changes in assumptions concerning information availability. Second, rather than deriving the optimal policy chapter 3 makes use of the Taylor-type interest rate based rules which have become quite popular in policy circles in recent years. Third, chapter 3 considers a series of macroeconomic models with different specifications for both the aggregate demand side and the aggregate supply side of the economy to check for robustness of results. Fourth, instead of the nominal interest rate, the real interest rate appears in the Phillips curve relationships to represent the cost channel. Fifth, the aggregate demand shock is modeled as an ongoing cycle to autonomous spending depicted by a sine-curve.

This is in contrast to the more traditional approach of treating shocks as a one time (serially correlated) change. Sixth, it is assumed that the two targeting regimes generate the same outcome regarding long-term inflation. Thus, the criterion for evaluating the performance of a monetary regime is its ability to minimize the volatility in real output in response to ongoing aggregate demand shocks.

The main result of chapter 3 is that the cost channel matters in the sense that the volatility of real output increases under both price-level and nominal income targeting. The intuition for this is simple. In response to an autonomous ongoing demand cycle, the central bank adjusts the nominal interest rate to manipulate aggregate demand (and thus the aggregate price level) in order to keep the resulting output volatility at a minimum. In the presence of the cost channel, however, adjustments in the nominal interest rate directly affect the aggregate supply side of the model as well; thereby increasing output volatility. As a result, with price-level targeting, the central bank would have to manipulate the aggregate demand by a large magnitude that would ensure the achievement of the original level of prices at the cost of an increased volatility in output. On the other hand, with nominal income targeting it would adjust aggregate demand just enough to reach a targeted level of nominal income with slightly high level of prices but a lower volatility in output. Since the metric used to evaluate the performance of a targeting regime is the minimization of real output volatility and only demand shocks are considered, nominal income targeting is preferred to price-level targeting. A somewhat surprising result is that the inclusion of the cost channel does not say much on the choice

between the two regimes. It appears that nominal income targeting performs better than price-level targeting in bringing down the volatility of real output in almost all the specifications of the macro models used in the analysis regardless of the cost channel.

Using the type of models analyzed in chapter 3, chapter 4 introduces open economy considerations and looks at the performance of monetary policy (in terms of reducing volatility in real output) under three alternative targeting regimes: exchange rate targeting (fixed exchange rates), price-level targeting, and nominal income targeting (both flexible exchange rate options). Although the supply-side effects of the interest rate (the cost channel) are ignored, the supply-side effects of exchange rate changes (due to the existence of intermediate imported inputs) are highlighted.

Under these settings, chapter 4 explores the impact of an increased degree of price flexibility on the volatility of real output. Previous analysis of this question offered mixed conclusions especially in an open economy context. Chapter 4 finds support for Keynes' prediction that under a flexible exchange rate regime a higher degree of price flexibility can raise output volatility. This result is consistent with the claims of central bankers that argue that their low inflation-flexible exchange rate policy has increased contract length and so decreased the degree of nominal price flexibility. However, it should be noted that this result holds only with nominal income targeting under flexible exchange rates; with price-level targeting the volatility of real output remains unchanged. Chapter 4 also finds support for central banks that favour fixed exchange rates. By adopting a fixed exchange rate regime, the authorities are embracing the challenge of accepting structural reforms of the sort that promote flexibility in prices to absorb the effects of shocks. Chapter 4 shows that with fixed exchange rates output volatility indeed goes down as a result of increased price flexibility. Thus, there is internal consistency within both views about exchange rate policy. However, this result does not hold when an alternative specification of the demand and supply side of the economy are considered

Chapter 5 develops a discrete-time dynamic stochastic general equilibrium model with incomplete asset markets, nominal price rigidities and monopolistic competition to shed light on the role of exchange rate and its relation with current account dynamics in the formulation of monetary policy. In some ways it is similar to the approach employed in chapter 2 as it also derives the optimal monetary policy under discretion. The important difference (apart from ignoring the cost channel and price-level targeting regime) is its focus on open economy issues, in particular the dynamic relationship between exchange rates, domestic inflation and current accounts. Two basic questions are explored in the chapter. First, how does the overall design of monetary policy change when we move from closed economy models to open economy models? Second, what is the role of exchange rate and its dynamic relation with the current account in formulating monetary policy in an open economy setting?

In the recent literature, labelled as new open economy macroeconomics (NOEM)<sup>13</sup>, the dynamics of the current account do not matter for monetary policy due to the assumption of complete asset markets in most of the research. However, chapter 5 argues that the assumption of complete asset markets is not realistic in a model with imperfections and rigidities in goods market because with nominal rigidities monetary policy will affect real variables including the current account. With incomplete asset markets the dynamics of current account do matter for monetary policy because then, besides dealing with the distortions created by monopolistic competition, the central bank needs to address the inefficiencies caused by incomplete asset markets.

Another issue, which is far from being settled but is explored in chapter 5, is the role of exchange rate in the formulation of monetary policy in NOEM models. In particular, it provides new insights on the relationship between domestic inflation and movements in the real exchange rate. In contrast to the traditional literature, chapter 5 shows that real exchange rate depreciation can lower domestic inflation. The key to understand this seemingly counter-intuitive result is to note that imports are introduced only as final consumption goods and that the nominal wage is completely flexible. Consider a real depreciation. It would lead to a decrease in the overall consumption bundle because imports are now relatively expensive. Since labour supply is a positive function of consumption, it also decreases nominal wage claims. With the assumption of

<sup>&</sup>lt;sup>13</sup> For a wealth of articles related to monetary policy in open economies a very useful website is maintained by Brian Doyle,<u>http://www.geocities.com/monetaryrules/mpoe.htm</u>

sticky goods prices, real wages also go down. Falling real wages imply a lower cost of production for domestic firms and thus results in lower domestic inflation.

The main result of the chapter is the breakdown of a widely accepted result that optimal monetary policy for an open economy calls for adjusting the interest rate to completely offset demand shocks, which implies no trade-off between output volatility and inflation volatility. Chapter 5 shows that when exchange rate affects both aggregate demand and inflation, adjusting the interest rate to stabilize output in the face of a demand shock will cause fluctuations in inflation. For example, consider a negative demand shock that lower output-gap and thus domestic inflation. The central bank operating under a discretionary domestic inflation targeting regime responds by lowering the nominal interest rates which, via the interest parity condition, causes real depreciation. Both these effects help increase the output-gap and thus inflation. However, when real depreciation also has a direct negative effect on domestic inflation, the central bank faces a dilemma; it cannot stabilize both output-gap and inflation. Thus, the optimal monetary policy calls for trading-off some output volatility for less inflation volatility.

Chapter 5 also demonstrates that allowing the exchange rate to float in order to stabilize the output from demand shocks may not be a good idea. In other words, it paysoff (in terms of reduced volatility in the key variables and reduced expected losses for the central bank) to assign a positive weight to stabilize exchange rate in the objective function of the central bank. The reason for this is the dynamic interaction between real exchange rate and current account and the effect of current account movements on the aggregate demand. Thus, the dynamic relationship between current account and the exchange rate plays a crucial role in obtaining the optimal monetary policy and in propagating the effect of shocks. The optimal monetary policy entails a response to variations in the net foreign asset positions and the impact of the demand shock last well beyond the time interval for which prices are rigid.

The remainder of the dissertation is organized as follows. Chapter 2 compares the performance of inflation targeting and price-level targeting and derives the optimal monetary policy in the presence of the cost channel. Chapter 3 evaluates the performance of price-level targeting and nominal income targeting in a series of closed economy models with the cost channel. Chapter 4 extends the framework of chapter 3 for an open economy and sheds light on the issue of price flexibility. Chapter 5 derives the optimal discretionary monetary policy while focusing on the dynamic relationship between domestic inflation, exchange rate and current account. Chapter 6 concludes and chapter 7 briefly discusses the possible extensions of the dissertation.

## **Chapter 2**

# The Cost Channel in the New Keynesian Model --- Comparing Inflation Targeting and Price-level Targeting

## **<u>1- Introduction</u>**

The 'New Neoclassical Synthesis model' or the 'New Keynesian' model that is based on intertemporal optimizing behaviour, rational expectations, and temporary nominal price rigidities and that is widely used to assess the desirability of alternative monetary policies has come a long way in integrating the ideas of New Keynesians with those of New Classicals.<sup>14</sup> However, it still lacks in its treatment of the monetary policy transmission mechanism. The objective of this chapter is to fill this gap in the literature, at least partially, by developing a model in the tradition of new Keynesian models that incorporate two alternative and independent channels of monetary policy transmission mechanism. The model is then used to analyze two popular monetary policy targeting regimes --- inflation targeting and price-level targeting.

In a typical new Keynesian model, the traditional interest rate-aggregate demand channel is used to explain the effects of monetary policy. According to this channel, a change in the interest rate affects the spending decisions of households and firms and thus

<sup>&</sup>lt;sup>14</sup> Typically, New Keynesians focus on market imperfections and nominal rigidities (such as price stickiness) to generate real effects of monetary policy while new Classical emphasize on intertemporal optimization and micro-foundations for the aggregate macroeconomic behavior based on a flexible price dynamic general equilibrium model.

operates only through the aggregate demand side of the model. An alternative channel, that has received considerable attention in recent years, emphasizes the existence of frictions (such as limited participation) and information asymmetries (that lead to the problem of adverse selection and moral hazard) in the financial markets as an important source of monetary non-neutralities. This channel is usually termed the credit channel and has the potential to affect the spending decision of firms by affecting their net worth --- the financial accelerator mechanism, and their production decisions by affecting their cost of production --- the bank lending channel. Thus, the broad credit channel can have an effect on the demand side and the supply side of the model. Until recently, the credit channel was mostly studied in flexible price models. For example, Christiano, Eichanbaum and Evans, in a number of important papers have introduced the bank lending channel in a flexible-price dynamic general equilibrium model<sup>15</sup>. Bernanke, Gertler and Gilchrist (1999) is an excellent example where the financial accelerator mechanism is incorporated in a sticky-price general equilibrium model. However, the bank lending channel, that affects the supply side of the model, has received very little attention in a simple new Keynesian model.<sup>16</sup>

This chapter incorporates the bank lending channel or simply the cost channel of monetary policy (because the bank lending channel affects the cost of production of firms) in an otherwise standard new Keynesian model. By analyzing both the traditional

<sup>&</sup>lt;sup>15</sup> See Christiano and Eichenbaum (1992) and Christiano, Eichanbaum and Evans (1997).

<sup>&</sup>lt;sup>16</sup> Two exceptions are two-yet to be published- working papers: Christiano, Eichenbaum and Evans (2001) and Walsh and Ravenna (2003).

and the cost channel of monetary policy in one unified framework, I attempt to bridge the gap between these two strands of literature. The model is then used to provide new insights on two well-known questions in the literature on monetary policy. First, which variable(s) should the central bank target to achieve the long-term goal of price stability? Second, should the central bank commit to rules or follow discretion in achieving those targets? In recent years, a consensus has emerged that central banks should primarily focus on policies that promote long-term price stability in the economy, and that there are gains from credibly committing to a rule-based monetary policy regime. However, more research needs to be done on the properties of specific intermediate targeting regimes and the feasibility and implications of commitment. As Clarida, Gali and Gertler (1999, page 1683) conclude, "though substantial progress has been made, our understanding of the full practical implications of commitment for policy-making is still at a relatively primitive stage, with plenty of territory that is worth exploring". This chapter is one step forward in that direction.

Distinguishing the relative importance of the traditional and the cost channel is useful for various reasons.<sup>17</sup> First, it improves our understanding of the link between the financial and real sectors of the economy. Second, it provides alternative indicators to help gauge the stance of monetary policy and thus increases its ability to offset particular types of adverse shocks. Third, a clear understanding of the transmission mechanism has the potential to give more information regarding the choice of intermediate targets.

<sup>&</sup>lt;sup>17</sup> For a detailed discussion see Kashyap and Stein (1994)

Moreover, several researchers like Christiano and Eichenbaum (1992), Christiano, Eichenbaum and Evans (1997, 2001) and Barth and Ramey (2001) have emphasized on the cost channel as a powerful collaborator of the traditional channel in the transmission of short run effects of monetary policy.

Building on these observations the chapter demonstrates two results for an inflation targeting regime.<sup>18</sup> First, the central bank faces a trade-off between stabilizing inflation and the output-gap in the presence of both demand and cost-push shocks. This result is different from the one reported by Clarida. Gali and Gertler (1999) in a standard new Keynesian model. They argue that the central bank is able to perfectly off-set demand shocks and faces an inflation and output-gap volatility tradeoff only in the presence of cost-push shock. However, when the cost channel is operating, an increase in the nominal interest rate to counter the effects of a positive demand shock not only reduces the output-gap thus reducing inflation but also increases inflation directly. Thus, the central bank is better off in trading some volatility in the output-gap for reduced volatility in inflation. Second, the presence of cost channel increases the gains from commitment. Moreover, the outcome of optimal commitment monetary policy is superior compared to optimal discretionary policy even in the absence of a classic inflation bias<sup>14</sup> and even in the absence of persistence in the shock processes. Emphasizing the

<sup>&</sup>lt;sup>1</sup><sup>N</sup> By inflation targeting. I mean when the central bank attempts to stabilize both output-gap and inflation. Svensson (1999a) calls this 'flexible' inflation targeting.

<sup>&</sup>lt;sup>19</sup> The classic literature, in the tradition of Barro and Gordon (1983), on rules versus discretion and the role of credibility in monetary policy maintains that if the central bank has a desire to push the economy's output level beyond the natural rate then discretion would only lead to higher average inflation (thus the term inflation bias) with no effect on output; credibly committing to a rule would eliminate the inflation bias

importance of forward-looking behaviour, the standard new Keynesian model predicts that the central bank only gains from commitment when the cost-push shock exhibits some persistence (even if there is no inflation bias).

This chapter extends the analysis further by seeking an answer to the question: If (for practical reasons) it is difficult to credibly commit to a monetary policy rule, is it possible to delegate to the central bank an alternative discretionary targeting regime (a different loss function) that would replicate the commitment solution? Woodford (1999b) argues that this is possible if the central bank is assigned a loss function with the interest rate as an explicit argument. Jensen (2002) has derived a similar answer in which the central bank targets nominal income growth rate. Vestin (2000) has also reached a similar conclusion by considering a price-level targeting regime. The intuitive explanation for this conclusion is the same in all these papers: they imply inertial behaviour in the discretionary monetary policy that is a feature of the commitment solution. It is the pricelevel targeting regime that I consider as an example to highlight the importance of cost channel of monetary policy to verify the above mentioned claims.

### 1.1- Comparison of Inflation Targeting and Price-level Targeting

In this sub-section, I first briefly highlight the differences between inflation targeting and price-level targeting and what the literature says on their relative merits and then report the results derived in this chapter.

Inflation targeting has become extremely popular in recent years in many developed countries. It appeals to policy makers as a way of directly achieving goals of price stability and is easily understood by the public which promotes low inflation expectations. Moreover, inflation targeting avoids several problems arising from money-growth targeting and exchange rate targeting --- the two targeting regimes used by several countries in the past. In particular, it avoids the problem of velocity shocks and allows a country to maintain an independent monetary policy. Perhaps the biggest advantage of inflation targeting is that it is not necessarily an ironclad rule as it allows for "constrained discretion" (Bernanke and Mishkin (1997)).

Moreover, for all practical purposes inflation targeting is forward-looking, with monetary policy being aimed at keeping future inflation within the defined target zone. Under inflation targeting, a central bank does not seek to compensate for past breaches of the inflation target. For example, if the inflation over-shoots the inflation target in one period, the central bank does not seek to compensate for that by reducing inflation below the target; it merely seeks to bring inflation back to the target. Therefore, under an inflation targeting regime "bygones are bygones" and the central bank worries only about the future path of inflation. The alternative direct method of seeking to achieve price stability is to target the general level of prices.<sup>20</sup> Although price-level targeting is quite similar to inflation targeting and it shares many of its benefits, the two regimes have a fundamental difference. Unlike inflation targeting which is forward-looking in nature, price-level targeting does not allow "bygones to be bygones". More specifically, if there is an unexpected increase in prices then according to price level targeting the monetary authority will attempt to tighten monetary policy so as to restore the price level back to the target in order to prevent the base drift in the price level. Under inflation targeting no action will be taken and the new level of prices would be maintained. Gavin and Stockman (1988) show that this base drift problem under inflation targeting leads to a higher level of uncertainty about the future price level. The central bank may miss its inflation target by a very small percentage in some years, but if these misses are not offset, they will accumulate and may become quite large over a long time horizon.

Price-level targeting offers the potential benefit of delivering greater certainty of the level of prices through time and may provide greater prospects for maintaining price stability in the longer run than under an inflation targeting regime. However, short-term price volatility (and thus output volatility) may be higher under price-level targeting

<sup>&</sup>lt;sup>20</sup> The only country that has adopted formalized price-level targeting is Sweden between 1931 and 1937. Berg and Jonung (1998) argue that the price-level targeting allowed Sweden to have less deflation, and one of the least severe depressions in that period. They suggest that price-level targeting can be used to raise inflation expectations in the face of deflation. This is one of the reasons why price-level targeting is suggested in Svensson (2001) as a solution for the deflation problem in Japan

because unexpected rises in the price level will be followed by attempted reductions in the price level.

The conventional literature (e.g., Fischer (1994) and Haldane and Salmon (1995)) focus on this alleged increased output-gap volatility under price-level targeting to argue against it. Kiley (1998) has also reached a similar conclusion using a new Keynesian Phillips curve. The reason is that price-level targeting raises output variability because inflationary mistakes must be reversed in order to prevent base drift in the price level, and the reversal of inflationary mistakes leads to extra output variability. However, Dittmar, Gavin and Kydland (1999) and Svensson (1999b) have challenged this conventional wisdom and, employing a neo-classical Phillips curve, shown the price-level targeting to be preferred over inflation targeting. Svensson (1999b) analyzes price-level targeting and inflation targeting by endogenously deriving the price-level and inflation rules when central banks acts under discretion and faces output persistence. He finds that price-level targeting results not only in lower variability in the price-level but also delivers lower inflation variability in the presence of output persistence.

More recently, Dittmar and Gavin (2000) and Vestin (2000) have confirmed this result using the new Keynesian Phillips curve by demonstrating that price-level targeting provides a better inflation-output-gap variability trade-off compared to inflation targeting with discretionary policy making regardless of the degree of importance of past levels of output for current output. Vestin (2000) takes this line of reasoning a step further and

proposes that the price-level target replicates the commitment solution of inflation targeting when there is no persistence in the cost-push shock.

Thus, the debate over the relative benefits of inflation targeting and price-level targeting is far from being settled.<sup>21</sup> As Mishkin (2001) has correctly pointed out, the results in favour of or against a price level target are very model specific, especially regarding the specification of the Phillips curve. In particular, the assumptions about private sector's inflation expectations entering the Phillips curve, amount of persistence in the output gap and whether policy is conducted under a commitment rule or in a discretionary fashion play important roles in determining the desirability of price level targeting. In this chapter, I add one more consideration: namely, the cost channel of monetary transmission and demonstrate that the price-level targeting is preferable to inflation targeting as it lowers the expected value of the loss incurred by the central bank. The framework used for this purpose is a variant of the standard "new Keynesian" model. However, this comparison is conducted after appropriately adjusting the relative weight the central bank places on output-gap stabilization and inflation stabilization. If this weight is not adjusted then the results are different.

<sup>&</sup>lt;sup>21</sup> For an in-depth analysis of the conditions under which price level targeting would be preferred over inflation targeting see Barnett and Engineer (2001).

## **<u>2- The Model</u>**

The model developed here introduces supply-side effects of interest rates or the cost channel of monetary policy in an otherwise standard New Keynesian model widely used for analysis of monetary policy. The basic framework employed is a variant of a cash-in-advance model with sticky prices. There are four types of economic agents in the economy: households, firms, the monetary authority and financial intermediaries or simply banks. Given their preferences, households decide how much to consume the differentiated composite consumption good, how to allocate time between leisure and work --- the labour supply decision, and how much of their money holdings and wage earnings should they deposit with the bank. The firms operate in a monopolistically competitive environment and thus command some monopoly power. They also take three decisions: how much of the differentiated good should they produce using the labour services of the households, how much loan to take from the bank to pay the wages of the hired workers and how to set the price for their output. Each firm sets the price of the good it produces, but not all firms reset their price in each period. The monetary authority issues money and employs nominal interest rates as an instrument of monetary policy to achieve certain well-specified goals. The role of the banks is quite trivial. They receive deposits from the households and a cash injection from the monetary authority and lend this amount to the firms at an interest rate set by the monetary authority.

#### 2.1- Households

The economy consists of a continuum of identical households. The model 1s described in terms of a representative household making decisions in the presence of uncertainties about the future. A typical household seeks to maximize the expected present discounted value of utility:

$$U(C_t, N_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-1\sigma}}{1-1\sigma} - \psi \frac{N_t^{1+\sigma}}{1+\phi} \right]$$
(1)

where  $0 < \beta < 1$  is the discounted rate of time preference, ' $\sigma$ ' represents elasticity of intertemporal substitution, ' $\phi$ ' is the elasticity of labour supply and  $E_0$  denotes the expectation based on the information set available at time zero.  $C_i$  is the composite consumption good,  $N_i$  denotes labour supply, and  $P_i$  is the price index for  $C_i$ .

The composite consumption index,  $C_i$ , consists of differentiated goods produced by firms operating in a monopolistically competitive environment. I assume that consumption is differentiated at the individual goods level. Thus, the goods consumption index can be written as a CES aggregator of the quantities consumed of each type of good.

$$C_{i} = \left(\int_{0}^{1} C_{i}(j)^{\frac{\ell-1}{c}} dj\right)^{\frac{\ell}{c-1}}$$
(2)

where the parameter ' $\epsilon$ ' is the elasticity of substitution within each category and is assumed to be greater than one, i.e.,  $\epsilon > 1$ .

The demand functions for goods within each category can be determined by maximizing equation (2) with respect to the total expenditure on that good, given as  $Z_{i} = \int_{0}^{i} P_{i}(j)C_{i}(j)dj, \text{ where } P_{i}(j) \text{ is the price of the consumption good } C_{i}(j). \text{ The}$ 

demand function that emerges from this maximization exercise is given as:

$$C_i(j) = \left(\frac{P_i(j)}{P_i}\right)^{-i} C_i$$
(3)

where  $P_i$  is the aggregate price index for composite good  $C_i$  that satisfy the expenditure equation expressed as  $Z_i = P_i C_i$ . The expression for  $P_i$  is given as:

$$P_{i} = \left(\int_{0}^{1} P_{i}(j)^{1-i} dj\right)^{\frac{1}{1-i}}$$
(4)

In maximizing utility given by (1), the representative household faces two types of constraints --- a cash-in-advance constraint and the intertemporal budget constraint. She enters period t with some initial cash holding,  $M_i$  and receives her wage income.  $W_iN_i$  as cash. From this total cash amount  $(M_i + W_iN_i)$  she deposits amount  $D_i$  in a bank and uses the remaining amount to purchase consumption goods  $P_iC_i$ . Thus, her cash-inadvance constraint can be written as:

$$M_i + W_i N_i - D_i \ge P C_i$$
<sup>(5)</sup>

At the end of the period t, she receives from the bank the principal amount she deposited  $D_r$ , interest accrued on this amount  $i_rD_r$ , and her share of bank profits  $\Pi^{tr}$ . Moreover, she also receives the share of profits  $\Pi^{tr}$  from the firms. The remaining amount is carried as cash over to the next period, t+1. Thus, the intertemporal budget constraint faced by the representative households can be written as:

$$M_{i} + W_{i}N_{i} - D_{i} - P_{i}C_{i} + (1 + i_{i})D_{i} + \Pi^{b} + \Pi^{f} = M_{i+1}$$
(6)

By maximizing (1) subject to the constraints (5) and (6), I can derive the following set of first order optimality conditions:

$$\frac{C_{i}^{-1\,\sigma}}{P_{i}} = \beta(1+i_{i})E_{i}\frac{C_{i+1}^{-1\,\sigma}}{P_{i+1}}$$
(7)

$$\psi C_t^{\dagger \sigma} N_t^{\phi} = \frac{W_t}{P_t}$$
(8)

Equation (7) is the standard Euler equation for the optimal intertemporal allocation of consumption. It has the usual interpretation that at a utility maximum, the household cannot gain from feasible shifts of consumption between period t and t+1. Equation (8) is the intratemporal optimality condition representing the labour supply decision.

Moreover, the cash-in-advance constraint will be binding in equilibrium:

$$M_t + W_t N_t - D_t = P_t C_t \tag{9}$$

#### 2.2)- Firms

This section outlines the mechanics of monopolistic competition in a dynamic general equilibrium setting. Like every firm operating in a monopolistically competitive market, each firm in this model has to take two types of decisions --- how much output to produce and at what prices to sell this output that would maximize profits. In doing so, a representative firm 'j' is subject to a number of constraints. First is the specification of the production function. Following McCallum and Nelson (1999a), I assume that there is no capital in the economy and so the firm only employs the labour input supplied by households to produce the differentiated consumption good consumed by the household.

$$C_i(j) = A_i N_i(j) \tag{10}$$

where  $A_i = \exp(z_i)$  and  $z_i$  represents aggregate technology shock specified as:

$$z_i = \rho_z z_{i-1} + \varepsilon_i^z$$

The second constraint is the demand function for these differentiated goods, which is given by equation (3). The third constraint introduces price stickiness by assuming that each period some firms are unable to adjust their price. This staggered price adjustment behaviour is based on Calvo (1983). Firms are assumed to face a constant probability  $1-\rho$  in every period to alter their price in an optimal fashion. This probability is independent of how long their prices have been fixed and the expected duration of price stickiness is  $1/\rho$ . It is easy to verify that with a large number of firms in the economy, the fraction of firms adjusting price optimally in a period is equal to the

probability of price adjustment  $1 - \rho$ ; the remaining fraction of firms ' $\rho$ ' do not adjust their price. Thus, the parameter ' $\rho$ ' captures the degree of nominal price rigidity.

The fourth constraint is used to incorporate the cost channel of monetary policy. I assume that the representative differentiated-good-producing firm needs to pay the hired workers before receiving the revenues from the sale of output produced. For this purpose, it will borrow an amount  $W_i N_i(j)$  from the bank at the interest rate  $i_i$  to finance the wage bill.

For analytical simplicity, I assume initially that all firms are able to adjust their prices every period, that is, the third constraint is not binding yet. Then, the profit function for a representative firm 'j' can be written as:

$$\pi_{i}(j) = P_{i}(j)C_{i}(j) - W_{i}N_{i}(j) - i_{i}W_{i}N_{i}(j)$$
(11)

The differentiated-good-producing firm chooses  $P_i(j)$  and  $N_i(j)$  to maximize these profits subject to the conditional demand for their variety of output given by equation (3) and the production function given by equation (10). The expressions for  $P_i(j)$  and  $N_i(j)$  are given respectively as:

$$P_{i}(j) = \frac{\varepsilon}{\varepsilon - 1} MC_{i}$$
(12)

$$\frac{W_i(1+i_i)}{P_i(j)} = \frac{\varepsilon - 1}{\varepsilon} F_{\Lambda}$$
(13)

and

where  $\left(\frac{\varepsilon}{\varepsilon-1}\right) = \upsilon$  is the constant mark-up and  $MC_i$  is the minimized nominal marginal cost.  $F_N$  is the marginal product of labour, which, given the production function is simply  $A_i$ .

Note that equation (12) just depicts the relationship between the 'flexible' price chosen by all firms and the minimized marginal cost of production under monopolistic competition; it does not say anything about prices being sticky. Combining equation (12) and (13) I can write:

$$MC_i = \frac{W_i(1+i_i)}{A_i} \tag{14}$$

or in real terms as:

$$mc_i = \frac{W_i(1+i_i)}{P_i A_i} \tag{15}$$

Note that, combining equation (13) --- equilibrium labour demand with equation (8) --- equilibrium labour supply, using equation (10) --- the production function, and the goods market equilibrium condition  $Y_t = C_t$ . I can derive the equilibrium level of output produced in the economy. This equilibrium level of output represents the flexible-price equilibrium of the economy since I have not introduced sticky prices yet in the model and is given as:

$$Y_{i}^{\prime} = \left[\frac{1}{\psi'} \frac{\varepsilon - 1}{\varepsilon} \frac{A_{i}^{1+\varphi}}{(1+i_{i}^{\prime})}\right]^{\frac{\varphi}{1+\varphi_{\theta}}}$$
(16)

In addition to technology, the flexible-price output level depends on the mark-up (due to the presence of monopolistic competition) and the nominal interest rate (which represents monetary policy). Thus, even if the distortion created by the monopolistic competition is eliminated.  $Y^{T}$  will not be efficient as long as  $i_{t}^{T} > 0$ .

Now, I introduce price stickiness by assuming that price adjustment does not take place simultaneously for all firms. Following Rotemberg (1987), suppose that a representative firm '*j*' that is allowed to change its price, set its price to minimize the expected present discounted value of deviations between the price it sets and the minimized nominal marginal cost.

$$\sum_{k=0}^{\infty} \rho^{k} \beta^{k} E_{i} (P_{i}(j) - MC_{i+k})^{2}$$
(17)

where  $MC_t$  is the minimized nominal marginal cost. Note that there are two parts to discounting. The first,  $\beta$  represents a conventional discount factor, and the second.  $\rho$  reflects the fact that the firm that has not adjusted its price after 'k' periods, still has the same price in period t + k that she set in period t. The first order condition with respect

to  $P_t(j)$  gives the following optimal value denoted by  $P_t^*(j)$ .<sup>22</sup>

$$P_i^* = (1 - \rho\beta)MC_i + \rho\beta E_i P_{i-1}^*$$
(18)

Thus, the optimally chosen price in period t is a weighted average of nominal marginal cost and expected value of optimal price in the future. However, following Calvo (1983) I assume that in period t only a fraction  $1 - \rho$  of firms set their price. The remaining firms are stuck with the prices set in previous periods. Thus, the parameter  $\rho$  is a measure of the degree of nominal rigidity. The average price of the previous period is the price of the fraction of firms that are unable to adjust their price this period. Therefore, the overall aggregate price level in period t is a weighted average of current optimally chosen and past prices. This can be written as:

$$P_{t} = \left[\rho^{1-c}P_{t-1} + (1-\rho)^{1-c}P_{t}^{*}\right]^{\frac{1}{1-c}}$$
(19)

where  $P_i^*$  is the price chosen by all adjusting domestic firms in period t.

<sup>&</sup>lt;sup>22</sup> It is reasonable to set  $P_i^*(j) = P_i^*$  because all firms are identical except for the timing of their price adjustment.

#### 2.3)- Banks

Banks operate costlessly in a competitive environment and play a trivial role in this model. They receive deposits,  $D_i$  from the households and lump sum cash injection,  $X_i$  from the monetary authority. This amount is supplied/lent to firms at the nominal interest rate,  $i_i$ . The demand for these loans comes from the firms who need to finance their wage bill,  $W_i N_i$ . Thus, equilibrium in the loan market requires that:

$$W_t N_t = D_t + X_t \tag{20}$$

The bank pays  $(1+i_t)D_t$  to households in return for their deposits and distributes  $(1+i_t)X_t$  to households in the form of profits.

## **3- Log-linearized Model**

In this section, the model is log-linearized around the steady state. A variable in lower case represent the log deviation with respect to the steady state. In equilibrium firms are assumed to be symmetric and taking identical decisions. This implies that prices are equal for each variety of good and is equal to the price index given by equation (4). That is,  $P_i(j) = P_i$ . Also,  $N_i(j) = N_i$  and  $C_i(j) = C_i$ .

#### 3.1 - Goods Market Equilibrium --- the new IS-curve

The log-linearized version of the resource constraint of this economy.  $Y_r = C_r$  can be written as:

$$y_t = c_t \tag{21}$$

In order to derive an IS-type relationship that relates output level to the real interest rate, I need to make use of the Euler equation for consumption. The loglinearized version of this relationship is:

$$c_{i} = E_{i}c_{i+1} - \sigma(l_{i} - E_{i}\pi_{i+1}) + s_{i}$$
(22)

Note that in deriving equation (22), I omitted constant terms, ignored Jensen's inequality and used the fact that  $\ln(1 + x) \approx x$  for small x. Also, I have included an additive disturbance term  $s_i$  that represents uncertainty (see, McCallum and Nelson (1999a)). It can be justified on the grounds that some non-linear terms are ignored while linearizing the Euler equation for consumption. The disturbance term  $s_i$  could also include the taste shocks had I introduced them in the specification of the utility function or it could represent the change in the government spending if it was not assumed to be zero. My objective is not to pin-down the source of this shock rather to have some disturbance term that represents the demand shocks.

After substituting equations (22) in equation (21), 1 get a relationship that represents equilibrium in the goods market --- the new IS equation:

$$y_{i} = E_{i} y_{i+1} - \sigma(i_{i} - E_{i} \pi_{i+1}) + s_{i}$$
(23)

Following the tradition in the recent literature on monetary policy, let  $x_t = y_t - y_t^T$ be defined as the output gap, where  $y_t^T$  is defined as the level of output that arises with perfectly flexible prices as shown in equation (16). Similarly, let  $r_t^T$  denote the real interest rate that arise in the frictionless equilibrium. Then, I can write equation (23) as:

$$x_{t} = E_{t} x_{t+1} - \sigma(i_{t} - E_{t} \pi_{t+1} - r_{t}^{f}) + s_{t}$$
(24)

where  $r_i^{T}$  is defined as follows:

$$r_i^{\prime\prime} = \left(\frac{1}{\sigma}\right) E_i \left(y_{i+1}^{\prime} - y_i^{\prime\prime}\right) + \left(\frac{1}{\sigma}\right) s_i$$
(25)

 $y'_t$  can be calculated by log-linearizing equation (16):

$$y_i^{\ t} = \frac{\sigma}{1+\sigma\phi} \Big[ (1+\phi)z_i - i_i^{\ t} \Big]$$
(26)

## 3.2 - Inflation Adjustment Behaviour --- the new Phillips curve

The log-linearized version of equation (18) and equation (19) can be combined to produces the following Phillips curve type relationship (see appendix I for a detailed derivation):

$$\pi_{t} = \beta E_{t} \pi_{t+1} + \gamma m c. \tag{27}$$

where  $\gamma = \frac{(1-\rho)(1-\rho\beta)}{\rho}$ 

The expression for  $mc_i$  (real marginal cost) can be had by log-linearizing equation (15).

$$mc_t = w_t - p_t + i_t - z_t \tag{28}$$

Using the log-linearized version of the labour supply equation (equation 8) to eliminate  $w_t - p_t$ , and using equation (21) to replace  $c_t$  with  $y_t$  and the production function to eliminate  $n_t$ . I can rewrite the equation for  $mc_t$  as:

$$mc_{i} = \left(\frac{1+\sigma\phi}{\sigma}\right) y_{i} - (1+\phi)z_{i} + i_{i}$$
<sup>(29)</sup>

Note that by setting,  $mc_i = 0$  1 get the same expression for  $y_i^{t}$  as given in equation (26). Thus, by subtracting equation (26) from equation (29),  $mc_i$  can be expressed in output gap form:

$$mc_{t} = \left(\frac{1+\sigma\phi}{\sigma}\right)x_{t} + (i_{t} - i_{t}^{\prime})$$
(30)

Thus, the inflation adjustment behaviour defined in equation (27) can be written as:

$$\pi_{i} = \beta E_{i} \pi_{i+1} + \gamma \left( \frac{1 + \sigma \phi}{\sigma} \right) x_{i} + \gamma \left( i_{i} - i_{i}^{\prime} \right)$$
(31)

Consider three ad-hoc modifications to this optimally derived Phillips curve. First, consistent with the current literature on monetary policy, I will introduce an ad-hoc costpush shock,  $v_i$  to study the trade-off between stabilizing output-gap variability and inflation variability faced by the monetary authority. In the standard new Keynesian model (without the cost channel), this supply-side shock is necessary to generate a meaningful monetary policy problem because the demand-side shocks are completely stabilized. However, in this paper it is not necessary to introduce this cost-push shock because demand shocks are not completely stabilized. However, the inclusion of the cost-push shock would be useful for comparing the results with the baseline case. Second, a new parameter  $\kappa$  will be introduced. By setting its value equal to 0 or 1.1 can study the properties of the model when the cost channel is closed and when it is operational. Third, the nominal interest rate under flexible-price equilibrium.  $i'_t$  will be set equal to zero. This is done for the sake of simplicity. With flexible prices, the goods market will clear automatically and the monetary authority does not need to alter its policy instrument ---the nominal interest rate ---- to stabilize output. In this sense, the nominal interest rate can be treated as a constant and normalized to zero. Thus, a change in monetary policy instrument can be thought of as a deviation from the level of interest rate that prevailed under flexible prices. The resulting Phillips curve is:

$$\pi_i = \beta E_i \pi_{i+1} + \lambda x_i + \gamma \kappa i_i + v_i$$
(32)

where  $\lambda = \gamma \left( \frac{1 + \sigma \phi}{\sigma} \right)$ .

The complete model is summarized by the following four equations:

$$x_{i} = E_{i} x_{i+1} - \sigma(i_{i} - E_{i} \pi_{i+1}) + u_{i}^{2/5}$$
(33)

$$\pi_{i} = \beta E_{i} \pi_{i+1} + \lambda x_{i} + \gamma \kappa i_{i} + v_{i}$$
(34)

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \qquad \qquad 0 < \rho_u < 1 \tag{35}$$

$$\mathbf{v}_i = \rho_i \mathbf{v}_{i-1} + \varepsilon_i^{\gamma} \qquad \qquad 0 < \rho_\gamma < 1 \tag{36}$$

The key difference between this model and the standard new Keynesian model is the presence of nominal interest rate in equation (34) --- the Phillips curve or the inflation

<sup>&</sup>lt;sup>23</sup> Equation (33) is obtained after eliminating  $r_t^{f}$  from equation (24) using equation (25) and (26). To be precise  $u_t = \frac{1+\phi}{1+\sigma\phi} (E_t(z_{t+1}-z_t) + \frac{1}{\sigma}s_t)$ , and as mentioned above it could concervable include taste and government spending shocks.

adjustment equation. The reason for its inclusion is the assumption that firms borrow cash from banks to finance their wage bills. Thus, a change in the interest rate directly affects the costs faced by firms and thus their price adjustment behaviour. Since the interest rate is the instrument of monetary policy, its inclusion can be termed the cost channel of monetary policy. Therefore, in addition to the traditional demand-side channel of monetary policy, the model also captures the supply-side effects of monetary policy. With  $\kappa = 0$ , the cost channel can be closed and the model then becomes the standard new Keynesian model. The next section analyzes the implications of the cost channel for optimal monetary policy under commitment and discretion. It also highlights the significance of the cost channel while comparing the performance of two alternative monetary policy regimes --- inflation targeting and price-level targeting.

## 4)- Optimal Monetary Policy<sup>24</sup>

This section describes the behaviour of the monetary authority --- the central bank. In simple words, the central bank uses its policy instrument (the nominal interest rate) together with the knowledge of the economy as represented by the IS curve (depicting the behaviour of aggregate output) and the Phillips curve (describing the behaviour of inflation (and prices)) to achieve certain well-specified goals represented by an objective function (also termed as the loss function) that explicitly translates the behaviour of the target/goal variables into a welfare measure. The objective function thus

<sup>&</sup>lt;sup>24</sup> By optimal monetary policy I mean that, given the dynamic general equilibrium structure, the effects of all sources of sub-optimality like nominal price rigidities are fully neutralized and the efficient flexible price equilibrium allocation is restored.

serves as a guide for the central banks to formulate monetary policy. For a new Keynesian model, Woodford (2002) has derived this monetary policy objective function by taking a second-order linear approximation to the utility function of the representative agent.<sup>25</sup> According to Woodford's specification of the loss function, the central bank seeks to minimize the discounted sum of squared deviations of output-gap and inflation from their target paths.

$$L_{i} = \frac{1}{2} E_{i} \left[ \sum_{k=0}^{\infty} \beta^{k} \left( \alpha x_{i+k}^{2} + \tau_{i+k}^{2} \right) \right] \qquad \qquad \alpha > 0 \qquad (37)$$

where ' $\alpha$ ' is the weight that the central bank places on stabilizing output gap relative to inflation. As Woodford has shown ' $\alpha$ ' depends on the deep parameters of the model

Although equation (37) is very similar to the standard quadratic loss function used extensively in the classic time-consistency literature (pioneered by Kydland and Presscott (1977) and Barro and Gordon (1983)) and in most recent papers (for example, Clarida, Gali and Gertler (1999)) analyzing the properties of basic new Keynesian model, two differences are worth noting. First, the output gap is defined as the difference in the actual aggregate output level and the equilibrium output under flexible prices rather as output level relative to the natural rate of output. The natural rate output typically depends only on technology shocks and the production function parameters. However, as evident from expression (26), the output under flexible prices depends on the utility function parameters as well. Thus, for example, households labour supply decision will

<sup>&</sup>lt;sup>25</sup> Rather than deriving the objective function for the model in this paper. I will intuitively explain the differences that may arise due to the presence of cost channel and how I assume away from those differences.

have an impact on the level of output under flexible prices. In addition, in the present model it also depends on the instrument of monetary policy, the nominal interest rate, due to the presence of the cost channel. This can have important implications for policy. It may not be optimal anymore to minimize the gap between actual output level and the output level under flexible prices. This leads to the next point.

Second, it is assumed that the central bank's target for output is equal to the economy's equilibrium level of output under flexible prices, that is, a target of zero output-gap  $x_i = 0$ . Unlike the traditional analysis, this assumption essentially ensures that the central bank has no incentive to increase the economy's output beyond this level. This seems a little odd in the presence of monopolistic competitive firms. However, consistent with the literature, 1 have assumed that government fully offsets this market power distortion by subsidizing employment, which is financed through lump sum taxes on households. Accordingly, it would be optimal for the central bank to eliminate the effects of nominal price rigidities and attain the flexible price equilibrium. This reasoning would hold in a model without the cost channel. However, with the cost channel present, setting  $x_i = 0$  may not be consistent with keeping inflation around zero because targeting  $x_i$  at zero requires a change in the interest rate which will cause inflation to move as well. Another way to understand this point is to consider equation (30) ---  $mc_i = \left(\frac{1+\sigma\phi}{\sigma}\right)x_i + (i_i - i_i^{-1})$ . In the absence of the cost channel, setting  $mc_i = 0$  would

guarantee  $x_i = 0$ . With the cost channel operating, achieving  $x_i = 0$  would require setting
$i_i = t_i^{\prime}$ . But, i' is not independent of the output level under flexible prices ---  $y_i^{\prime}$ . In order to get around this problem, I need to define the concept of output-gap with care. I define it as the gap between actual output level and the output level that would prevail under flexible prices and constant nominal interest rate. As explained above, this constant nominal interest rate with flexible prices is normalized to be equal to zero.

A well-known result in the classic literature on optimal discretionary and commitment monetary policy is that if the central bank has no incentive to increase the economy's level of output beyond the equilibrium flexible-price level of output then there would be no inflation bias problem and the gains from commitment (in terms of eliminating this bias) would essentially be zero. However, Clarida, Gali and Gertler (1999) have argued that in a forward-looking model, such as the basic new Keynesian model, there still may be gains from credible commitment. The reason is that in a forward-looking model, expectations play a crucial role and discretion leads to what is known as a stabilization bias. Clarida, Gali and Gertler (1999) then use this result to propose the appointment of a conservative central banker (originally proposed by Rogoff (1985) to reduce the average inflation bias of discretionary policy) that puts more weight on inflation stabilization, that is, a smaller value of  $\alpha$  to reduce this stabilization bias. The general idea here is that the outcomes of policy can be improved by assigning the central bank an objective function that differs from the optimally derived objective function (Walsh (1995)). A large body of literature has emerged that analyzes the impact of alternative central bank objective functions, termed as targeting regimes, on the outcome of policy. In what follows, I focus on two of such targeting regimes, namely, inflation targeting and price-level targeting. The loss functions corresponding to these targeting regimes can be expressed respectively as:

$$L_{t} = \frac{1}{2} E_{t} \left[ \sum_{k=0}^{\infty} \beta^{k} \left( \alpha_{t} x_{t+k}^{2} + \pi_{t+k}^{2} \right) \right] \qquad \text{(Inflation Targeting)} \tag{38}$$

and 
$$L_{t} = \frac{1}{2} E_{t} \left[ \sum_{k=0}^{\infty} \beta^{k} \left( \alpha_{PT} x_{t+k}^{2} + p_{t+k}^{2} \right) \right] \qquad \text{(Price-level Targeting)} \tag{39}$$

where,  $\alpha_{II}$  is the weight on output stabilization relative to inflation stabilization and  $\alpha_{PI}$  is the weight on output stabilization relative to price-level stabilization.

### 4.1)- Inflation Targeting

This is the most popular and widely studied targeting regime. Accordingly, I will treat the policy outcomes under inflation targeting as a benchmark and then compare them with the policy outcomes under price-level targeting. A similar approach was employed by Vestin (2000). The key questions I seek to answer are: First, what is the implication of the cost channel in comparing the outcomes of discretionary optimal monetary policy and commitment optimal policy? Second, whether optimal discretionary monetary policy under price-level targeting captures the outcome of optimal policy with commitment under an inflation targeting regime. The main difference between Vestin's analysis and the analysis conducted in this paper is the presence of the cost channel. As will be evident below, with the presence of cost channel, the IS curve will not be irrelevant in deriving the optimal policy and the gains from commitment will be larger.

#### **4.1.1)- Discretionary** Case

In deriving the optimal discretionary monetary policy, it is assumed that the central bank uses nominal interest rate  $i_i$  as its instrument variable. It chooses the time path of this instrument to influence the time paths of the target variables  $x_i$  and  $\pi_i$  in such a way that it minimizes equation (38) subject to the constraints on their behaviour implied by the system of equations, (33) – (36). The solution to the constrained minimization exercise yields the following first order conditions:

$$\frac{\partial L}{\partial \pi_i} = \pi_i - \psi_i = 0 \tag{40}$$

$$\frac{\partial L}{\partial x_i} = \alpha_{iT} x_i + \lambda \psi_i - \phi_i = 0$$
(41)

$$\frac{\partial L}{\partial i_i} = \psi_i \gamma \kappa - \sigma \phi_i = 0 \tag{42}$$

where, ' $\psi_i$ ' and ' $\phi_i$ ' are the Langrangian multipliers associated with the new Phillips curve and the new IS curve respectively.

The main difference between this model and the basic new Keynesian model is evident from equation (42). In the basic new Keynesian model ( $\kappa = 0$ ),  $\phi_i$  would be zero. This explains the reason why the new IS curve is usually ignored in deriving the optimal discretionary policy. However, with the cost channel present, that is, with  $\kappa = 1$ ,  $\phi_i$  is different from zero and the IS curve will be relevant. Eliminating  $\psi_i$  and  $\phi_i$  from the first order conditions, yields the following optimality condition:

$$x_{i} = -\left(\frac{\sigma\lambda - \gamma\kappa}{\sigma\alpha_{iT}}\right)\pi_{i}$$
(43)

Equation (43) has the usual interpretation that the central bank should contract output by raising the interest rate whenever inflation is above target and vice versa (as long as  $(\sigma\lambda > \gamma)$ ). With  $\kappa = 0$  the equation is exactly the same as derived in Clarida, Gali and Gertler (1999) for optimal discretionary policy. However, the important difference emerges when the parameter ' $\kappa$ ', that depicts the effect of the cost channel of monetary policy, is set equal to one. It is straight forward to verify that the response of the central bank in trading-off fluctuations in the output for stabilization of inflation is less aggressive when  $\kappa = 1$  compared to when  $\kappa = 0$ . Thus, the presence of cost channel makes inflation stabilization more costly. The reason for this is that as the central bank increases the interest rate to bring inflation down, output goes down that reduces inflation (captured by  $\sigma\lambda$ ) but at the same time this policy action increases inflation directly due to the cost channel (captured by  $\gamma$ ). The net effect, however, would be that the inflation comes down because for all plausible parameter values the condition  $\sigma\lambda > \gamma$  holds.

To obtain a solution for the model that describes the equilibrium behaviour of  $\pi_i$ and  $x_i$ , first eliminate *i*, from equation (37) using equation (36) to get: PhD Thesis - Hamza Ali Malik, Department of Economics, McMaster University

$$\pi_{t} = (\beta + \gamma \kappa) E_{t} \pi_{t+1} + (\gamma \kappa / \sigma) E_{t} x_{t+1} + ((\sigma \lambda - \gamma \kappa) / \sigma) x_{t} + (\gamma \kappa / \sigma) u_{t} + v_{t}$$
(44)

then combine this equation with the optimality condition (43) to find the reduced form expressions for  $x_t$  and  $\pi_t$  using the method of undetermined coefficients. The only relevant state variables are ' $u_i$ ' and ' $v_i$ '. Thus, the trial solution takes the form:

$$\pi_i = b_{IT}^d u_i + c_{IT}^d v_i \tag{45}$$

$$x_t = e_{TT}^d u_t + f_{TT}^d v_t \tag{46}$$

The solution expressions for  $b_{IT}^d$ ,  $c_{IT}^d$ ,  $e_{IT}^d$  and  $f_{IT}^d$  are given as follows:

$$b_{IT}^{d} = \frac{(\gamma \kappa \cdot \sigma)}{q_{1}}$$
(47)

$$c_{II}^{J} = \frac{1}{q_2} \tag{48}$$

$$c_{II}^{d} = -\Omega_{II} b_{II}^{\prime} \tag{49}$$

$$f_{II}^{D} = -\Omega_{II} c_{II}^{D} \tag{50}$$

where.

$$\Omega_{II} = \left(\frac{\sigma\lambda - \gamma\kappa}{\sigma\alpha_{IT}}\right)$$

$$q_1 = 1 - \rho_u(\beta + \gamma\kappa) + \lambda\Omega_{IT} - (\gamma\kappa/\sigma)(1 - \rho_v)\Omega_{IT}$$

$$q_n = 1 - \rho_n(\beta + \gamma\kappa) + \lambda\Omega_{IT} - (\gamma\kappa/\sigma)(1 - \rho_v)\Omega_{IT}$$

and

$$q_2 = 1 - \rho_v (\beta + \gamma \kappa) + \lambda \Omega_{II} - (\gamma \kappa / \sigma) (1 - \rho_v) \Omega_{II}$$

Note that, in the absence of cost channel ( $\kappa = 0$ ), both  $b_{lT}^d$  and  $e_{lT}^d$  are equal to zero. This is a standard result in a basic new Keynesian model meaning that demand shock  $u_i$  has no effect on output-gap and inflation and thus the central bank faces no trade-off in stabilizing the output-gap and inflation in the face of demand shocks. However, when the cost channel is operating ( $\kappa = 1$ ), the demand shock  $u_i$  will lead to a trade-off between output-gap fluctuation and inflation fluctuation. Consider a positive realization of  $u_i$ . As a result, both the output-gap  $x_i$  and inflation  $\pi_i$  increase. The central bank responds by increasing the interest rate. This lowers the output-gap and thus inflation (through the demand channel) but it also increases inflation directly (through the cost channel). In order to stabilize both output-gap and inflation in inflation.

In order to draw comparisons across alternative monetary policy regimes, it would prove convenient to evaluate the performance of a policy by calculating the unconditional expected value of the loss function expressed in terms of variances of inflation and output-gap. Thus, the unconditional expected value of the loss function given by equation (38) can be approximately written as:

$$E(L_t^a) = \operatorname{var}(\pi_t) + \alpha_{tT} \operatorname{var}(x_t)$$
(51)

Using expressions (47) - (50), I can calculate the variance expressions for inflation and output-gap as follows:

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$$\operatorname{var}(\pi_{i}) = b_{i1}^{d^{2}} \operatorname{var}(u_{i}) + c_{iT}^{d^{2}} \operatorname{var}(v_{i})$$
(52)

$$\operatorname{var}(x_t) = e_{IT}^{d^2} \operatorname{var}(u_t) + f_{II}^{d^2} \operatorname{var}(v_t)$$
(53)

where the variances for the demand and supply shocks are given as;

$$\operatorname{var}(u_{t}) = \frac{1}{1 - \rho_{u}^{2}} \operatorname{var}(\varepsilon_{t}^{u})$$
(54)

$$\operatorname{var}(v_{i}) = \frac{1}{1 - \rho_{i}^{2}} \operatorname{var}(\varepsilon_{i}^{\nu})$$
(55)

In order to learn more about the outcome of policy and its sensitivity to various parameter values, I use some specific parameter values to evaluate the discretionary case. The same parameter values will then be used to evaluate the commitment case under inflation targeting and the discretionary case under price-level targeting. Calibrating the model in this fashion makes it easy to compare the properties of the model with and without the cost channel and highlight the significance of the cost channel of monetary policy.<sup>20</sup> The following baseline parameter values are used:

 $\beta = 0.99$   $\sigma = 0.60$   $\phi = 1$   $\lambda = 0.3$   $\gamma = 0.1$   $\rho_u = \rho_v = .3$  $\operatorname{var}(\varepsilon_v^u) = \operatorname{var}(\varepsilon_v^v) = .000225$ 

<sup>&</sup>lt;sup>26</sup> The same methodology is used in McCallum and Nelson (2000a) in a model with no cost channel. They compare the expected value of the losses under discretionary policy with timeless perspective of monetary policy.

The value for  $\beta$  is standard implying that the time interval should be interpreted as one quarter.  $\sigma$  is the elasticity of intertemporal substitution and is generally quite small. Although the value for the elasticity of labour supply  $\phi$  is slightly high, however, it is considered quite standard in the recent literature on monetary policy (e.g., see Walsh (2003a)).  $\lambda$  is the slope parameter for the Phillips curve and is given as:  $\frac{(1-\rho)(1-\rho\beta)(1+\sigma\phi)}{\rho\sigma}$ , where  $\rho$  depicts the degree of price rigidity. Various estimates of  $\rho$  put its value somewhere between 0.66 and 0.80. I have picked a value of 0.72 for

 $\rho$  that, combined with the values of  $\beta$ ,  $\sigma$  and  $\phi$ , imply a value of approximately 0.3 for  $\lambda$ . The effects of the cost channel (that is, the sensitivity of inflation to nominal interest rate changes) are captured by  $\gamma$  that is given as:  $\frac{(1-\rho)(1-\rho\beta)}{\rho}$ . Given  $\rho = 0.72$ , the

value for  $\gamma$  turns out to be approximately 0.1. The results of the paper are not sensitive to alternative parameter values

The variances for the white noise demand and cost-push shocks are taken to be 0.000225 with a persistence parameter of 0.3. Taken together these numbers imply an annualized standard deviation of a 6% for the model economy.<sup>27</sup> The values chosen for the variances of the shock have a direct effect on the absolute magnitude of expected losses, but do not influence the relative magnitudes of the losses with and without the

<sup>&</sup>lt;sup>27</sup> For example, standard deviation of the demand shock

 $<sup>=\</sup>sqrt{\operatorname{var}(u_t)} = \sqrt{\left(1/(1-(0.3)^2)\right)^* 0.000225} = 0.015$ . Multiplying this number by 4 gives the annualized standard deviation of 6° o

cost channel of monetary policy. As discussed below, it is the relative losses that are used for comparison. Similarly, it is the relative losses that I use in comparing the losses under discretionary and commitment policies.

Another parameter of interest is  $\alpha_{IT}$  in the objective (loss) function of the central bank. It depicts the relative weight the central bank places on stabilizing output versus stabilizing inflation. Walsh and Ravenna (2003) derives this loss function using the method proposed by Woodford (2002) for a model with the cost channel and pins down the parameter  $\alpha_{II}$  to be a function of 'deep parameters' of the model and reports its value to be approximately 0.02. I consider three values for this parameter that incorporate not only Walsh and Ravenna's (2003) value but also values reported by other papers. The parameter  $\kappa$  is set to zero if the cost channel is closed and 1 if the cost channel is operating. The results are reported in table 1 and 2 for both demand and cost-push shocks respectively.

It is obvious from table 1 that a demand shock implies an inflation-output-gap stabilization tradeoff in the presence of the cost channel. Thus, the central bank incurs positive losses in the presence of cost channel and these losses increases as the central bank puts more weight on stabilizing output-gap. This has a simple intuitive explanation. For example, in case of a positive demand shock that increases the output-gap and thus inflation, the central bank responds by increasing the nominal interest rate. This lowers the output-gap and indirectly inflation. However, in the presence of cost channel this policy response also directly increases inflation. If the central bank puts a higher weight

 Table 1: Inflation Targeting with Discretion --- Demand shock<sup>28</sup>

Cost channel closed				Cost channel operating			
( <i>k</i> = 0)				( <i>K</i> = 1)			
	$\alpha_{17} = .01$ $\alpha_{17} = .25$ $\alpha_{11} = 1.0$				$\alpha_{11} = .25$	$\alpha_{II} = 1.0$	
$\operatorname{var}(\pi_{t})$	0	0	0	0.055	1.145	1.451	
var(x_)	0	0	0	1.241	0.416	0.033	
$Loss(L_i^d)$	0	0	0	0.178	1.249	1.484	

on stabilizing output-gap then she would (relatively) ignore this extra volatility in inflation thus incurring some extra losses.

Figure 1 captures this monetary policy trade-off between stabilizing inflation and output-gap. Note that this efficiency policy frontier is only possible in case of demand shocks when the cost channel is operating. Thus, the claim made by Clarida, Gali and Gertler (1999) that the central bank will face no trade-off in case of demand shocks is refuted by introducing the cost channel of monetary policy in a new Keynesian model.

 $<sup>^{28}</sup>$  Reported values are multiplied by  $10^5\,$  in all the tables.



Figure 1: Efficient Policy Frontier

Cost channel closed				Cost channel operating			
$(\kappa = 0)$				$(\kappa = 1)$			
$\alpha_{II} = .01  \alpha_{II} = .25  \alpha_{II} = 1.0$				$\alpha_{ll} = .01$	$\alpha_{rr} = .25$	$\alpha_{i_1} = 1.0$	
$\operatorname{var}(\pi_{i})$	0.341	28.446	51.113	2.452	51.386	65.136	
$\operatorname{var}(x_{t})$	307.26	40.962	4.600	557.26	18.684	1.480	
$Loss(L_t^3)$	3.414	38.686	55.714	8.025	56.057	66.617	

 Table 2: Inflation Targeting with Discretion --- Cost-push shock

The importance of the cost channel of monetary policy is further highlighted in the presence of cost-push shocks --- the losses incurred by the central bank increases for all values of alpha. (See, table 2). With the exception of very small values of  $\alpha_{ir}$ , (approximately less than 0.05, as is evident from figure 3 below), the cost channel lowers the volatility in output-gap at the cost of increased volatility in inflation. For example, consider a positive cost-push shock that increases inflation. A typical response by the central bank is to increase the nominal interest rate. This policy response creates a negative output-gap and thus helps in lowering inflation. However, with the cost channel present, an increase in the interest rate pushes up the inflation even further. Thus, the central bank would be less aggressive in increasing the interest rate. This leads to a smaller negative output-gap at the cost of higher inflation. Figures 2 and 3 compare the volatility in inflation and output-gap with and without the cost channel for cost-push shocks for all possible values of  $\alpha_{ii}$  between 0 and 1. The important point is that the cost of discretion is higher in the presence of the cost channel of monetary policy and this strengthens the case for optimal commitment policy even more.

 $var(\pi)$ 



Figure 2: Inflation volatility for cost-push shocks



Figure 3: Output-gap volatility for cost-push shocks

Figures 4 and 5 compare the efficient policy frontier for cost-push shocks with and without the cost channel. It is evident that the policy frontier with the cost channel (figure 5) is higher than the policy frontier without the cost channel (figure 4) for all values of alpha. Thus, it is more 'costly' to stabilize inflation and output-gap.



Figure 4: Efficient Policy Frontier without cost channel



Figure 5: Efficient Policy Frontier with cost channel

## The Implicit Interest rate rule

The interest rate rule that implements the discretionary monetary policy can be derived by using the optimality condition (equation 43), the solution expressions for  $\pi_i$ 

and  $x_i$  as given by equations (45) and (46), and the IS relationship given by equation (33). It is given as follows:

$$i_{t} = \left(1 + \frac{\Omega_{tt}(1 - \rho_{t})}{\sigma\rho_{t}}\right) E_{t} \pi_{t+1} + \left(\frac{1}{\sigma} + \frac{b_{tt}^{d} \Omega_{tt}(1 - \rho_{u})}{\sigma} + \frac{b_{tt}^{d} \rho_{u} \Omega_{tt}(1 - \rho_{t})}{\sigma\rho_{t}}\right) u_{t}$$
(56)

With no cost channel, equation (56) reduces to exactly the same expression as derived in Clarida, Gali and Gertler (1999) since  $b_{II}^d = 0$  and  $\Omega_{IT}$  reduces to  $\lambda/\alpha$ implying that the demand shocks is perfectly offset and the central bank faces a tradeoff between stabilizing inflation and output-gap only in the presence of a cost-push shock. Moreover, in response to a one percent change in the expected inflation rate, the central bank should change the nominal interest rate by more than one percent (as evident from the coefficient in front of the expected inflation term) so that the real interest rate is affected in the 'right' direction. This principal ensures determinacy and is called the Taylor principle.

In the presence of the cost channel, however, the demand shock is not completely offset as  $b_{II}^d \neq 0$ . Also, the response of the central bank to expected movements in the inflation rate is less aggressive. That is, the central bank changes the nominal interest rate by a smaller amount (still greater than one) in response to expected inflation. The reason for this, as explained above, is that now a change in the interest rate directly affects inflation in addition to its indirect effect through output-gap. An alternative method of deriving the interest rate rule is to solve the IS relationship (equation (33)), the Phillips curve (equation (34)) and the central bank's optimality condition (equation 43)) simultaneously for  $i_t, \pi_t$  and  $x_t$ . Although in the discretionary case there is no difference in the two methods, in case of commitment it makes a difference as will be demonstrated shortly. Using this alternative approach, the solution expressions for inflation and output-gap remain the same as described by equation (45) and (46) and the expression for the implicit interest rate rule is determined as follows:

$$i_{t} = \left(\frac{1}{\sigma} + \frac{b_{II}^{d} \Omega_{II} (1 - \rho_{u})}{\sigma} + b_{II}^{d} \rho_{u}\right) u_{t} + \left(\frac{c_{II}^{d} \Omega_{II} (1 - \rho_{v})}{\sigma} + c_{II}^{d} \rho_{v}\right) v_{t}$$
(57)

#### 4.1.2)- Commitment Case

This section demonstrates that there are gains from commitment even in the absence of inflation bias and that these gains are larger with the cost channel present. Under the commitment policy, central bank seeks to minimize the loss function given in (38) by choosing the current and future values of inflation, output-gap, and the nominal interest rate subject to the constraints, equation (33) - (36). The first order conditions for this minimization exercise are:

$$\frac{\partial L_t}{\partial \pi_t} = \pi_t - \psi_t = 0 \qquad \text{for } t = 0 \tag{58}$$

$$\frac{\partial L_{i}}{\partial \pi_{i}} = \beta \pi_{i} + \beta \psi_{i-1} - \beta \psi_{i} + \sigma \phi_{i-1} = 0 \qquad \text{for } t \ge 1 \qquad (59)$$

$$\frac{\partial L_i}{\partial x_i} = \alpha_{IF} x_i + \lambda \psi_i - \phi_i = 0 \qquad \text{for } t = 0 \tag{60}$$

$$\frac{\partial L_{t}}{\partial x_{t}} = \alpha_{tT} \beta x_{t} + \beta \lambda \psi_{t} + \phi_{t-1} - \beta \phi_{t} = 0 \qquad \text{for } t \ge 1 \qquad (61)$$

$$\frac{\partial L_t}{\partial i_t} = \gamma \kappa \psi_t - \sigma \phi_t = 0 \qquad \text{for } t = 0 \qquad (62)$$

$$\frac{\partial L_i}{\partial i_i} = \beta \gamma \kappa \psi_i - \beta \sigma \phi_i = 0 \qquad \text{for } i \ge 1 \qquad (63)$$

Eliminating the Langrangian multipliers,  $\psi_i$  and  $\phi_i$ , I can simplify the first order conditions to get:

$$x_{t} = -\left(\frac{\sigma\dot{\lambda} - \gamma\kappa}{\sigma\alpha_{tT}}\right)\pi_{t} \qquad \text{for } t = 0 \qquad (64)$$

$$x_{t} = x_{t-1} - \left(\frac{\sigma\lambda - \gamma\kappa}{\sigma\alpha_{tt}}\right)\pi_{t} \qquad \text{for } t \ge 1 \qquad (65)$$

Thus, as reported by Clarida, Gali and Gertler (1999), Woodford (1999b) and McCallum and Nelson (2000a), the optimal commitment policy entails inertial behaviour; rather than adjusting the level of output-gap in response to fluctuations in inflation, the commitment policy requires the adjustment in the change in the output gap to changes in inflation. However, in the initial period, t=0, the central bank behaves as if it were operating in a discretionary fashion. Thus, the commitment solution is not time-consistent. To get around this problem Woodford (1999c) has suggested that the central

bank should commit to implement in each period the policy that it would have been optimal to commit to if the same problem had been considered at a date far in the past. This procedure avoids treating the current period (t = 0) as the initial one by setting inflation in that period as if it were one of many future periods when policy was considered in the distant past. Woodford (1999c) labels this approach the timeless perspective of monetary policy.<sup>29</sup> In the context of the present model, the timeless perspective policy amount to implementing equation (65) for all time periods including the initial period. The basic idea behind this timeless perspective pre-commitment policy is that the policymaker commits to a policy that disregards the conditions that happen to prevail at the time in which the policy begins.

The gains from commitment with the cost channel present are easily understood by looking at equation (44):

$$\pi_{i} = (\beta + \gamma \kappa) E_{i} \pi_{i+1} + (\gamma \kappa / \sigma) E_{i} x_{i+1} + ((\sigma \lambda - \gamma \kappa) / \sigma) x_{i} + (\gamma \kappa / \sigma) u_{i} + v_{i}$$
(44)

With the cost channel operating, the future expected output gap also affects the current inflation in addition to current output-gap and future expected inflation. The reason the expected output-gap  $E_t x_{t+1}$  directly affects current inflation  $\pi_t$  is because a lower  $E_t x_{t+1}$  reduces the nominal interest rate associated with any given current output-

<sup>&</sup>lt;sup>29</sup> It is important to note that a number of authors such as Dennis (2001a), Blake (2001) and Jensen and McCallum (2002) have questioned the timelessness of Woodford's approach and have demonstrated that timeless perspective policy is not the time-invariant rule and does not in general minimizes the loss function. This issue, though important, is beyond the scope of the present chapter.

gap. And since the nominal interest rate directly effects  $\pi_i$  because of the cost channel, current inflation goes down. Also,  $E_i x_{i+1}$  will affect  $E_i \pi_{i+1}$  that in turn affects  $\pi_i$  in addition to the usual effect. Now consider a positive demand shock that pushes up the current output-gap and inflation. The central bank increases the nominal interest rate. As a result current output-gap goes down also reducing inflation. With the cost channel, this increased interest rate also increases inflation directly. Up until now the analysis is the same as in the discretionary case. However, with commitment policy, the central bank does not take the expectations as given, so they also affect current behaviour of inflation and output-gap. More precisely, the expected output-gap also decreases when the central bank increases the interest rate as long as the shock is serially correlated. This fall in the expected output-gap will lower current inflation as explained above (using equation (44)). Moreover, a fall in the expected output-gap will also lower future expected inflation that also directly affects inflation. With the cost channel operating the effect of future expected inflation on current inflation is bigger. Thus, inflation goes down by a bigger amount --- due to a fall in the current output-gap and the expectations of a lower future expected output-gap and inflation --- than it goes up due to an increase in the interest rate. The direct effect of expected future output gap and its indirect effects via future expected inflation on current inflation are only operational in the presence of the cost channel and are precisely the reason why the gains from commitment increase.

In order to obtain the solution with optimal commitment policy (which is the timeless perspective policy of Woodford), I combine equation (44) (the inflation

adjustment equation after substituting for  $i_i$  using equation (33)) with equation (65). The trial solution used for this purpose is given as:

$$\pi_{t} = a_{IT}^{c} x_{t-1} + b_{IT}^{c} u_{t} + c_{IT}^{c} v_{t}$$
(66)

$$x_{t} = d_{II}^{\epsilon} x_{t-1} + e_{II}^{\epsilon} u_{t} + f_{II}^{\epsilon} v_{t}$$
(67)

The solution expressions for  $a'_{II}$ ,  $b'_{II}$ ,  $c'_{II}$ ,  $d'_{II}$ ,  $e'_{II}$  and  $f'_{II}$  are given as.

$$a_{IT}^{\epsilon} = (\beta + \gamma \kappa) a_{II}^{\epsilon} d_{II}^{\epsilon} + (\gamma \kappa / \sigma) d_{IT}^{\epsilon} + ((\sigma \lambda - \gamma \kappa) / \sigma) d_{II}^{\epsilon}$$
(68)

$$b_{II}^{c} = (\beta + \gamma \kappa)(a_{II}^{c} e_{II}^{c} + \rho_{u}b_{II}^{c}) + (\gamma \kappa/\sigma)(d_{II}^{c} e_{II}^{c} + \rho_{u}e_{II}^{c}) + ((\sigma\lambda - \gamma\kappa)/\sigma)e_{II}^{c} + (\gamma\kappa/\sigma)$$
(69)

$$c_{II}^{c} = (\beta + \gamma \kappa)(a_{II}^{c} f_{II}^{c} + \rho_{c} c_{II}^{c}) + (\gamma \kappa / \sigma)(d_{II}^{c} f_{II}^{c} + \rho_{c} f_{II}^{c}) + ((\sigma \lambda - \gamma \kappa) / \sigma)f_{II}^{c} + 1$$
(70)

$$d_{II}^{c} = 1 - \Omega_{II} d_{II}^{c} \tag{71}$$

$$e_{IT}^{\iota} = -\Omega_{II} b_{IT}^{\iota} \tag{72}$$

$$f_{IT}^{\epsilon} = -\Omega_{IT} c_{IT}^{\epsilon}$$
(73)

It is obvious from the above expressions that their solution would involve multiple values for  $a_{TT}^{c}$  and thus for all other undetermined coefficients. The solution for  $a_{TT}^{c}$  can be obtained by solving the following quadratic equation:

$$q_{3}a_{11}^{c^{*}} + q_{4}a_{11}^{c} - \lambda = 0$$
(74)

where

and

$$q_{3} = (\beta + \gamma \kappa)\Omega_{II} - (\gamma \kappa/\sigma)\Omega_{II}^{2}$$
$$q_{4} = 1 - (\beta + \gamma \kappa) + \lambda\Omega_{II} - (\gamma \kappa/\sigma)\Omega_{II}$$

73

The solution for equation (74) that satisfies  $0 < a_{IT}^c < 1$  (and thus  $0 < d_{IT}^c < 1$ ) is given as:

$$a_{IT}^{c} = \frac{-q_4 + \sqrt{q_4^2 + 4\lambda q_3}}{2q_3} \tag{75}$$

Accordingly,  $b_{IT}^c$  and  $c_{IT}^c$  are determined as follows.  $d_{IT}^c$ ,  $e_{IT}^c$  and  $f_{IT}^c$  can then be determined from equations (71) – (73):

$$b_{IT}^{c} = \frac{(\gamma \kappa / \sigma)}{q_{5}}$$
(76)

$$c_{IT}^c = 1/q_6 \tag{77}$$

where,

$$q_{5}=1+(\beta+\gamma\kappa)(a_{TT}^{c}\Omega_{TT}-\rho_{u})+\Omega_{TT}\left(((\sigma\lambda-\gamma\kappa)/\sigma)+(\gamma\kappa/\sigma)(d_{TT}^{c}+\rho_{u})\right)$$
$$q_{6}=1+(\beta+\gamma\kappa)(a_{TT}^{c}\Omega_{TT}-\rho_{v})+\Omega_{TT}\left(((\sigma\lambda-\gamma\kappa)/\sigma)+(\gamma\kappa/\sigma)(d_{TT}^{c}+\rho_{v})\right)$$

The variance of inflation and output-gap can now be calculated using equation (66) and (67) (see appendix II for detailed derivations):

$$\operatorname{var}(\pi_{t}) = \left(\frac{a_{IT}^{c^{2}} e_{IT}^{c^{2}} (1 + \rho_{u} d_{IT}^{c})}{(1 - d_{IT}^{c^{2}})(1 - \rho_{u} d_{IT}^{c})} + b_{IT}^{c^{2}} + \frac{2a_{IT}^{c} b_{IT}^{c} e_{IT}^{c} \rho_{u}}{1 - \rho_{u} d_{IT}^{c}}\right) \operatorname{var}(u_{t}) + \left(\frac{a_{IT}^{c^{2}} f_{IT}^{c^{2}} (1 + \rho_{v} d_{IT}^{c})}{(1 - d_{IT}^{c^{2}})(1 - \rho_{v} d_{IT}^{c})} + c_{IT}^{c^{2}} + \frac{2a_{IT}^{c} c_{IT}^{c} f_{IT}^{c} \rho_{v}}{1 - \rho_{v} d_{IT}^{c}}\right) \operatorname{var}(v_{t})$$
(78)

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$$\operatorname{var}(x_{t}) = \frac{1}{(1 - d_{H}^{c})} \left( \frac{e_{H}^{c^{2}}(1 + d_{H}^{c}\rho_{u})}{1 - \rho_{u}d_{H}^{c}} \operatorname{var}(u_{t}) + \frac{f_{H}^{c^{2}}(1 + \rho_{v}d_{H}^{c})}{1 - \rho_{v}d_{H}^{c}} \operatorname{var}(v_{t}) \right)$$
(79)

Thus, the unconditional expected value of the loss function in the commitment case, given by equation (38), can be calculated as:

$$E(L_{t}^{c})^{T} = \operatorname{var}(\pi_{t}) + \alpha_{T} \operatorname{var}(x_{t})$$
(80)

Using the same parameter values mentioned above, the calibrated results of the model for demand and cost-push shock are reported respectively in table 3 and table 4:

Table 3: Inflation Targeting with Commitment --- Demand shock

	Cost	channel clo	Cost channel operating			
		( <i>k</i> <sup>-</sup> = 0)	$(\kappa = 1)$			
	$\alpha_{i_1} = .01$ $\alpha_{i_1} = .25$ $\alpha_{i_2} = 1.0$				$\alpha_{II} = .25$	$\alpha_{II} = 1.0$
$\operatorname{var}(\pi_{\tau})$	0	0	0	0.036	0.502	0.804
$\operatorname{var}(x_i)$	0	0	0	7.90	0.453	0.074
$Loss(L_i)$	0	0	0	0.115	0.615	0.878

Although the cost channel leads to an inflation-output-gap volatility tradeoff, quantitatively the effects are small. To appreciate the importance of cost channel and gains from commitment compare the losses reported in table 1 and table 3. For example, when  $\alpha_{IT} = 0.25$ , the expected loss decreases by almost 50% due to commitment. Thus, these results confirm the proposition that there are gains from commitment even in the absence of inflation bias and that these gains are larger in the presence of cost channel.

Cost channel closed				Cost channel operating			
( <i>κ</i> = 0)				$(\kappa = 1)$			
	$\alpha_{IT} = .01$	$\alpha_{IT} = .25$	$\alpha_{TT} = 1.0$	$\alpha_{IT} = .01$	$\alpha_{\rm IT} = .25$	$\alpha_{IT} = 1.0$	
$\operatorname{var}(\pi_{t})$	0.377	15.367	30.375	1.607	22.537	36.082	
$\operatorname{var}(x_t)$	274.235	41.491	9.361	354.606	20.339	3.341	
$Loss(L_t^c)$	3.119	25.739	39.736	5.153	27.621	39.423	

 Table 4: Inflation Targeting with Commitment --- Cost-push shock

Again, the impact of the cost channel on the losses of the central bank is quite small and insignificant in the commitment case. However, this does not undermine the result that the presence of cost channel strengthens the case for commitment. Comparing the losses reported in table 2 and 4 reveals that the losses are lower with commitment. As in the case of demand shock, when  $\alpha_{11} = 0.25$ . the expected loss decreases by almost  $50^{\circ}$  due to commitment in case of cost-push shocks as well.

Figures 6 and 7 compare the volatility in inflation and output-gap with and without the cost channel for cost-push shocks for all possible values of  $\alpha_{IT}$  between 0 and 1 when monetary policy is conducted with commitment.



Figure 6: Inflation volatility for cost-push shocks

 $var(x_t)$ 



Figure 7: Output-gap volatility for cost-push shocks

Figures 8 and 9 compare the efficient policy frontier for cost-push shocks with and without the cost channel in case of commitment. As in the discretionary case, the policy frontier with the cost channel (figure 9) is higher than the policy frontier without the cost channel (figure 8) for all values of alpha. Thus, it is more 'costly' to stabilize inflation and output-gap in the presence of the cost channel.



Figure 8: Efficient Policy Frontier without cost channel



Figure 9: Efficient Policy Frontier with cost channel

## The Implicit Interest rate rule

Following the method of Clarida. Gali and Gertler (1999), I can derive the interest rate rule that implements the optimal pre-commitment monetary policy by writing the central bank's optimality condition (equation (65)) one period forward, taking expectations and sunbstituting the result in the IS relationship (equation (33)).

$$i_{t} = \left(1 - \frac{\Omega_{II}}{\sigma}\right) E_{t} \pi_{t-1} + \frac{1}{\sigma} u_{t}$$
(81)

Two points are worth noting about equation (81). First, the coefficient associated with expected inflation is less than one. Accordingly, an increase in expected inflation leads to a small (less than one) increase in the nominal interest rate implying that the real interest rate moves in the 'wrong' direction, that is, it actually decreases. Clarida, Gali and Gertler (1999) argue that a rule of this type violate the Taylor principle and may lead to indeterminacies of output and inflation. Second, the rule in equation (81) implies that the demand shocks will be completely stabilized whether the cost channel is present or not. This implication is inconsistent with the result derived and discussed above that the demand shock cannot be completely stabilized in the presence of cost channel. The reason for this inconsistency is the method of deriving equation (81). To be specific, the following version of the optimality condition (equation (65)) is used to get equation (81):

$$E_{t}x_{t+1} = x_{t} - \left(\frac{\sigma\lambda - \gamma\kappa}{\sigma\alpha_{IT}}\right)E_{t}\pi_{t+1}$$
(65a)

But, it is important to note that while equation (65) implies equation (65a), the opposite is not true. Equation (65) may not hold even if equation (65a) does. Thus, the interest rate rule implied by equation (81) does not implement optimality condition (65). For this reason and to confirm the possibility of indeterminacy, I use an alternative method (described in the discretionary case) to derive the interest rate rule that implements the optimality condition given by equation (65).

The solution method involves solving the IS relationship (equation (33)), the Phillips curve (equation (34)) and the optimality condition --- equation (65) simultaneously for  $i_i, \pi_i$  and  $x_i$  using the following trial solutions:

$$\pi_{t} = a_{IT}^{c} x_{t-1} + b_{IT}^{c} u_{t} + c_{IT}^{c} v_{t}$$
(66)

$$x_{t} = d_{IT}^{c} x_{t-1} + e_{IT}^{c} u_{t} + f_{IT}^{c} v_{t}$$
(67)

$$i_{l} = g_{lT}^{c} x_{l-1} + h_{lT}^{c} u_{l} + j_{lT}^{c} v_{l}$$
(82)

Not surprisingly, the expressions for  $a_{IT}^c$ ,  $b_{IT}^c$ ,  $c_{IT}^c$ ,  $d_{IT}^c$ ,  $e_{IT}^c$  and  $f_{II}^c$  turns out to be the same as defined by equations (75), (76), (77), (71), (72) and (73) respectively. The implied interest rate rule is given as follows:

$$i_{t} = \left(1 - a_{IT}^{c} \Omega_{II} \left(\frac{a_{II}^{c} (\sigma - \Omega_{II})}{\sigma}\right) x_{t-1} + \left(\frac{1}{\sigma} + \frac{\gamma \kappa}{\sigma} \left(\rho_{u} - a_{IT}^{c} \Omega_{II} \left(\frac{\sigma - \Omega_{II}}{\sigma q_{5}}\right)\right) u_{t} + \left(\rho_{v} - a_{IT}^{c} \Omega_{II} \left(\frac{1 + \Omega_{II}}{q_{6}}\right) v_{t}\right)$$

$$(83)$$

Equation (83) offers lot of insight in the implementation of optimal precommitment monetary policy. The first important point to note is that, unlike equation (81), it captures the inertial behaviour of the central bank implied by the optimality condition (65) as it entails a response to previous period's output-gap. The second point is that the coefficient in front of the lagged output-gap term must be positive to avoid self-fulfilling fluctuations in output and inflation; current interest rate must increase if in the previous period output-gap goes up. This condition is satisfied if  $(\sigma - \Omega_{IT}) > 0$ . It is easy to verify that this condition may be violated for small values of  $\alpha_{IT}$ . Similarly,  $(\rho_i - a_{IT}^i \Omega_{IT})$  must be positive if the interest rate were to increase in response to a positive cost-push shock. Lastly, equation (83) confirms the previous result that the central bank cannot completely stabilize both output-gap and inflation in response to demand shocks because the coefficient of  $u_t$  is different from  $1/\sigma$  in the presence of cost channel.

## **4.2)-** Price-level Targeting

Assuming that in practice no central bank can commit to a policy rule for all time periods and acknowledging at the same time that there are gains from commitment (as demonstrated in the above section), it is a reasonable question to ask: can we delegate to the central bank a targeting regime that would come close to replicating the commitment solution while operating in a discretionary fashion? Vestin (2000) answers this question in the affirmative, provided the cost-push shock is serially uncorrelated. In this section I reconsider the question in a more general model in which demand shocks are also relevant due to the presence of cost channel of monetary policy.

The central bank seeks to minimize the following loss function subject to the constraints implied by equation (33)- (36):

$$L_{t} = \frac{1}{2} E_{t} \left[ \sum_{k=0}^{\infty} \beta^{k} \left( \alpha_{PT} x_{t+k}^{2} + p_{t+k}^{2} \right) \right]$$
(39)

Since the loss function is in terms of the price-level rather than the inflation rate, it would be useful to express the model in terms of price-level as well. Thus, the new IS curve and the new inflation adjustment equation are written as follows:

$$x_{t} = E_{t} x_{t+1} - \sigma (i_{t} - (E_{t} p_{t+1} - p_{t})) + u_{t}$$
(84)

$$p_{t} - p_{t-1} = \beta(E_{t}p_{t+1} - p_{t}) + \lambda x_{t} + \theta \kappa i_{t} + v_{t}$$
(85)

The only new first order condition will be with respect to the price-level; the other two (with respect to  $x_i$  and  $i_j$ ) will be the same as in the inflation targeting case with discretion:

$$\frac{\partial L_i}{\partial p_i} = p_i - (1+\beta)\psi_i - \sigma\phi_i + E_i\psi_{i+1} = 0$$
(86)

$$\frac{\partial L}{\partial x_i} = \alpha_{PT} x_i + \lambda \psi_i - \phi_i = 0$$
(87)

$$\frac{\partial L}{\partial i_i} = \psi_i \gamma \kappa - \sigma \phi_i = 0 \tag{88}$$

Combining these equations will give the following optimality condition describing the behaviour of the central bank:

$$x_{t} = \left(\frac{1}{1+\beta+\gamma\kappa}\right) E_{t} x_{t+1} - \left(\frac{\sigma\lambda-\gamma\kappa}{\sigma\alpha_{pq}\left(1+\beta+\gamma\kappa\right)}\right) p_{t}$$
(89)

An interesting point to note from equation (89) is that it depicts the inertial behaviour of the central bank, which is a feature of the commitment solution implying the possibility that if a relative weight of  $\alpha_{pT}$  is assigned to the central bank instead of  $\alpha_{T}$ , the commitment solution can be replicated. However, that would be a premature conclusion. To be concrete, I solve the model using the method of undetermined coefficients with the following trial solutions:

$$p_{i} = a_{PT} p_{t-1} + b_{PT} u_{t} + c_{PT} v_{t}$$
(90)

$$x_{t} = d_{PT} p_{t-1} + e_{PT} u_{t} + f_{PT} v_{t}$$
(91)

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Combining equation (85) (after eliminating  $i_i$  using equation (84)) and equation (89) with the trial solutions yields the following identifying restrictions:

$$a_{PT} = \frac{1}{1 + \beta + \gamma \kappa} \left( 1 + (\beta + \gamma \kappa) a_{PT}^2 + (\sigma \lambda - \gamma \kappa) / \sigma \right) d_{PT} + (\gamma \kappa / \sigma) a_{PT} d_{PT} \right)$$
(92)

$$b_{p_{T}} = \frac{1}{1 + \beta + \gamma \kappa} ((\beta + \gamma \kappa)(a_{p_{T}}b_{p_{T}} + \rho_{\kappa}b_{p_{T}}) + (\sigma\lambda - \gamma\kappa)/\sigma)e_{p_{T}} + (\gamma\kappa/\sigma)(b_{p_{T}}d_{p_{T}} + \rho_{u}e_{p_{T}}) + \gamma\kappa/\sigma)$$

$$c_{p_{T}} = \frac{1}{1 + \beta + \gamma\kappa} ((\beta + \gamma\kappa)(a_{p_{T}}c_{p_{T}} + \rho_{\kappa}c_{p_{T}}) + (\sigma\lambda - \gamma\kappa)/\sigma)f_{p_{T}} + (\gamma\kappa/\sigma)(c_{p_{T}}d_{p_{T}} + \rho_{\kappa}f_{p_{T}}) + 1)$$

$$d_{PT} = \frac{-\Omega_{PI} a_{PI}}{(1 + \beta + \gamma \kappa)(1 - a_{PT})}$$
(95)

$$e_{FT} = \frac{b_{PT} \left( d_{PT} - \Omega_{PT} \right)}{\left( 1 + \beta + \gamma \kappa \right) \left( 1 - \rho_u \right)} \tag{96}$$

$$f_{PT} = \frac{c_{PT} (d_{PT} - \Omega_{PT})}{(1 + \beta + \gamma \kappa)(1 - \rho_{\gamma})}$$
(97)

where,

$$\Omega_{PT} = \frac{(\sigma \lambda - \gamma \kappa)}{\sigma \alpha_{PT}}$$

It is difficult to derive an analytical expression for unique values of these coefficients. Thus, I resort to the calibrated version of the model and pick a solution that satisfies  $0 < a_{p_1} < 1$ .

The variance of the price-level and output-gap can now be calculated using equation (90) and (91) (see appendix II for detailed derivations):

$$\operatorname{var}(x_{r}) = \left(\frac{b_{PI}^{2} d_{PT}^{2} (1 + \rho_{u} a_{PT})}{(1 - a_{PT}^{2})(1 - \rho_{u} a_{PT})} + c_{PT}^{2} + \frac{2b_{PI} d_{PI} e_{PI} \rho_{u}}{1 - \rho_{u} a_{PT}}\right) \operatorname{var}(u_{t}) \\ + \left(\frac{c_{PI}^{2} d_{PI}^{2} (1 + \rho_{v} a_{PT})}{(1 - a_{PT}^{2})(1 - \rho_{v} a_{PT})} + f_{PT}^{2} + \frac{2c_{PT} d_{PI} f_{PT} \rho_{v}}{1 - \rho_{v} a_{PT}}\right) \operatorname{var}(v_{t})$$
(98)

$$\operatorname{var}(p_{t}) = \frac{1}{(1 - a_{PT}^{2})} \left( \frac{b_{PI}^{2} (1 + a_{PT} \rho_{u})}{1 - \rho_{u} a_{PT}} \operatorname{var}(u_{t}) + \frac{c_{PT}^{2} (1 + \rho_{v} a_{PT})}{1 - \rho_{v} a_{PT}} \operatorname{var}(v_{t}) \right)$$
(99)

The implied variance of inflation under price-level targeting is given as follows:

$$\operatorname{var}(\pi_{t}) = \left(\frac{2b_{PT}^{2}(1-\rho_{u})}{(1+a_{PT})(1-\rho_{u}a_{PT})}\right) \operatorname{var}(u_{t}) + \left(\frac{2c_{PT}^{2}(1-\rho_{v})}{(1+a_{PT})(1-\rho_{v}a_{PT})}\right) \operatorname{var}(v_{t})$$
(100)

The unconditional expected value of the loss function with price-level targeting can be calculated as:

$$E(L_t)^{PT} = \operatorname{var}(p_t) + \alpha_{PT} \operatorname{var}(x_t)$$
(101)

It would be incorrect to compare the absolute value of this expected loss under price-level targeting with expected loss under inflation targeting with pre-commitment (given by equation (80)) because the relative weights in the two expressions ---  $\alpha_{II}$  and  $\alpha_{PT}$  ---represent different benchmarks.  $\alpha_{IT}$  is the relative weight assigned to the variability of output-gap compared to the variability of inflation, while  $\alpha_{PT}$  is the relative weight placed on the variability of output-gap compared to the variability of the pricelevel. As noted by Vestin (2000), failing to appreciate this difference may have created a bias in favour of the free-lunch result of Svensson (1999b). However, using the expression for variance of inflation (equation (100)) and variance of the price-level (equation (99)), I can establish a link between  $\alpha_{IT}$  and  $\alpha_{PT}$ . Taking the ratio of the two variance expressions, I get:

$$\frac{\operatorname{var}(\pi_{t})}{\operatorname{var}(p_{t})} = \frac{(1 - a_{pT}) \left[ 2b_{pT}^{2} (1 - \rho_{u})(1 - a_{pT}\rho_{v}) \operatorname{var}(u_{t}) + 2c_{pT}^{2} (1 - \rho_{v})(1 - a_{pT}\rho_{u}) \operatorname{var}(v_{t}) \right]}{b_{pT}^{2} (1 + a_{pT}\rho_{u})(1 - a_{pT}\rho_{v}) \operatorname{var}(u_{t}) + c_{pT}^{2} (1 + a_{pT}\rho_{v})(1 - a_{pT}\rho_{u})}$$
(102)

Using equation (102) I can calculate the following loss function:

$$E(L_t^*)^{PT} = \operatorname{var}(\pi_t) + \alpha_{PT}^* \operatorname{var}(x_t)$$
(103)

where

$$\alpha_{PT}^{*} = \frac{(1 - a_{PT}) \left[ 2b_{PT}^{2} (1 - \rho_{u})(1 - a_{PT}\rho_{v}) \operatorname{var}(u_{t}) + 2c_{Pt}^{2} (1 - \rho_{v})(1 - a_{PT}\rho_{u}) \operatorname{var}(v_{t}) \right]}{b_{PT}^{2} (1 + a_{PT}\rho_{u})(1 - a_{PT}\rho_{v}) \operatorname{var}(u_{t}) + c_{PT}^{2} (1 + a_{PT}\rho_{v})(1 - a_{PT}\rho_{u})} \alpha_{PT}$$

Now, the two expected losses, given by equation (80) and equation (103), can be compared.

The calibrated values for variance of price-level, inflation and output-gap and for the expected losses in case of demand and cost-push shocks are reported in table 5 and 6 respectively. In addition to the result that demand shocks matter even with price-level targeting when the cost channel is operating, an important result is that the price-level targeting is preferred over inflation targeting (except for small values of  $\alpha_{IT}$ ). For example, comparing the expected losses in table 5 (the last row) with those reported in table 3 reveals that the losses decreases by almost 50% when the central bank targets price-level in a discretionary manner as compared to a pre-commitment inflation target. The only exception to this result is when  $\alpha_{II} = 0.01$ . The difference, however, is negligible.

Cost channel closed				Cost channel operating			
( <sub>K</sub> = 0)				$(\kappa = 1)$			
$\alpha_{PT} = .01  \alpha_{PT} = .25  \alpha_{PT} = 1.0$			$\alpha_{II} = .01$	$\alpha_{pp} = .25$	$\alpha_{r_I} = 1.0$		
$\operatorname{var}(p_i)$	0	0	0	0.081	0.997	2.108	
$\operatorname{var}(x_i)$	0	0	0	0.074	0.183	0.029	
$\operatorname{var}(\pi_i)$	0	0	0	5.69	0.356	0.474	
$Loss(L_i)^{PT}$	0	0	0	0.138	1 043	2.137	
$Loss(L_t^*)^{PT}$	0	0	0	0.125	0.367	0.474	

Table 5: Price-level Targeting with Discretion --- Demand shock

In general, these results confirm the previous results obtained by Vestin (2000) and Dittmar and Gavin (2000). However, it is worth noting that these authors derived this result for a cost-push shock only; in case of demand shocks central bank is indifferent in choosing between the two regimes. In this sense the result that price-level targeting regime is preferred over an inflation targeting regime in the presence of a demand shock can be considered a new result. Note that if the relative weight on inflation stability versus output-gap stability is not appropriately adjusted, the result could be opposite ---- an inflation targeting regime would be preferred. (compare the second last row in table 5 with the expected losses in table 3).

Cost channel closed				Cost channel operating			
$(\kappa = 0)$				$(\kappa = 1)$			
	$\alpha_{IT} = .01$	$\alpha_{IT} = .25$	$\alpha_{IT} = 1.0$	$\alpha_{IT} = .01$	$\alpha_{\rm IT} = .25$	$\alpha_{IT} = 1.0$	
$\operatorname{var}(p_t)$	0.798	26.945	73.382	3.630	45.012	93.944	
$\operatorname{var}(x_t)$	210.532	20.069	4.362	254.939	8.291	1.267	
$\operatorname{var}(\pi_t)$	0.910	12.576	19.459	3.297	16.072	21.102	
$Loss(L_t)^{PT}$	2.903	31.962	77.744	6.179	47.084	95.21	
$Loss(L_t)^{PT}$	3.111	14.918	20.616	5.605	16.813	21.386	

Table 6: Price-level Targeting with Discretion --- Cost-push shock

As obvious from the comparison between the expected values of losses in table 6 and table 4, price-level targeting is preferred over inflation targeting except for small values of  $\alpha_{II}$ . However, it is important to recognize that this result would not hold if the relative weight in the loss function is not appropriately adjusted. Put differently, most of the earlier analyses (for example, Dittmar and Gavin (2000) and Svensson (1999b)) while comparing the benefits of inflation targeting and price-level targeting, ignore this point. Had I followed the same strategy I would have ended up concluding that inflation targeting is preferable to price-level targeting when the cost channel is introduced.

# 5- Concluding Remarks

Relying on the ample empirical evidence and a number of previous theoretical flexible-price models this chapter developed a model, in the tradition of the "new Keynesian" literature on monetary policy, which involved the bank lending channel or the cost channel of monetary policy. The "new Keynesian" model has emerged as the standard framework, because this compact structure combines three very desirable features. First, it is firmly grounded in inter-temporal optimization, so the desire on the part of "new Classicals" for well-articulated micro-foundations is respected. With each equation being structural, the Lucas critique can be respected as the model is applied to policy questions. Second, a degree of nominal rigidity is allowed for, so the mechanism that Keynesians regard as essential for generating short-run real effects from demand shocks is an integral part of the structure. Third, the model is conveniently analyzed at
the same level of aggregation as was common with earlier generations of policy-oriented discussions. One of the contributions of the present chapter is that it analyzes both the traditional and the cost channel of monetary policy in one unified framework, therefore, embracing the strand of literature that studies only the cost channel in the folds of the new Keynesian framework.

The model is applied to two basic questions commonly analyzed in the context of optimal monetary policy: performance of discretionary policy versus pre-commitment policy, and relative benefits of inflation targeting versus price-level targeting. A number of important results emerged.

First, with the introduction of cost channel the demand shock leads to a tradeoff between stabilizing inflation and output-gap in addition to cost-push shocks. This result is in stark contrast compared to a widely accepted result reported by Clarida, Gali and Gertler (1999) that only cost-push shocks generate meaningful monetary policy problems.

Second, the gains from pre-commitment exist even in the absence of an inflation bias and these gains are larger due to the presence of the cost channel of monetary policy thus strengthening the case for commitment.

Third, the chapter proposes an alternative method of calculating the implicit interest rate rule that implements the optimal monetary policy. It is shown that this alternative method is the correct method in the commitment case for the sake of internal consistency of results. For example, the interest rate rule in the commitment case, when derived using the existing approach, suggests that the demand shock is completely stabilized even in the presence of the cost channel. However, the calibrated results demonstrate the opposite thus implying an internal inconsistency of results. On the other hand when the alternative method proposed in the paper is used, this problem is resolved.

Fourth, the chapter finds some support for earlier results that price-level targeting with discretion is preferable over inflation targeting with commitment even in the presence of the cost channel except for very small values of the preference parameter of the central bank. However, unlike the previous work, the chapter is careful in comparing the expected values of the losses incurred in the two regimes by appropriately adjusting the relative weight on output-gap stabilization as compared to inflation stabilization. Using the commonly employed approach gives the opposite result. In this sense the chapter provides new insights on the issue of comparing inflation targeting with price-level targeting.

### **Appendix I**

The purpose of this appendix is to show in detail the derivation of the inflation adjustment equation --- the 'new Keynesian' Phillips curve (equation 27).

A representative firm 'j' that is allowed to change its price, set its price to minimize the expected present discounted value of deviations between the price it sets and the minimized nominal marginal cost.

$$\sum_{k=0}^{\infty} \rho^k \beta^k E_t (P_t(j) - MC_{t+k})^2$$
(A1-1)

where  $MC_t$  is the minimized nominal marginal cost.

Can write equation (A1-1) as:

$$\sum_{k=0}^{\infty} \rho^{k} \beta^{k} E_{t} (P_{t}^{2}(j) - MC_{t+k}^{2} - 2P_{t}(j)MC_{t+k})^{2}$$

First order condition with respect to  $P_t(j)$  is:

$$2\sum_{k=0}^{\infty} \rho^{k} \beta^{k} E_{t} P_{t}^{*}(j) - 2\sum_{k=0}^{\infty} \rho^{k} \beta^{k} E_{t} M C_{t+k} = 0$$

since  $E_t P_t^*(j) = P_t^*(j)$ 

$$\Rightarrow P_t^*(j) \sum_{k=0}^{\infty} \rho^k \beta^k = \sum_{k=0}^{\infty} \rho^k \beta^k E_t M C_{t+k}$$
  
or 
$$P_t^*(j) \left(\frac{1}{1-\rho\beta}\right) = \sum_{k=0}^{\infty} \rho^k \beta^k E_t M C_{t+k}$$

or 
$$P_i^*(j) = (1 - \rho\beta) \sum_{k=0}^{\infty} \rho^k \beta^k E_i M C_{i+k}$$
 (AI-2)

Writing out the terms in equation (AI-2) and assuming that  $P_t^*(j) = P_t^*$ , I get:

$$P_{t}^{*} = (1 - \rho\beta)MC_{t} + (1 - \rho\beta)\rho\beta E_{t}MC_{t+1} + (1 - \rho\beta)\rho^{2}\beta^{2}E_{t}MC_{t+2} + \dots$$
  
or 
$$P_{t}^{*} = (1 - \rho\beta)MC_{t} + \rho\beta(1 - \rho\beta)[E_{t}MC_{t+1} + \rho\beta E_{t}MC_{t+2} + \rho^{2}\beta^{2}E_{t}MC_{t+3}\dots]$$

or 
$$P_t^* = (1 - \rho\beta)MC_t + \rho\beta(1 - \rho\beta)\sum_{k=0}^{\infty} \rho^k \beta^k MC_{t+k+1}$$

But, 
$$(1 - \rho\beta) \sum_{k=0}^{\infty} \rho^k \beta^k M C_{i+k+1} = E_i P_{i+1}^*$$
 (from equation (AI-2).  

$$\Rightarrow P_i^* = (1 - \rho\beta) M C_i + \rho\beta E_i P_{i+1}^*$$
(AI-3)

This is equation (18) in the chapter.

Note that the final expression for the inflation adjustment equation is in terms of real marginal cost, which is given as  $\frac{MC_t}{P_t}$  or in log-linear version as:  $mc_t = MC_t - p_t$ , where  $mc_t$  is the real marginal cost. Therefore, I replace nominal marginal cost ( $MC_t$ ) term in equation (AI-3) with real marginal cost. Accordingly, the log-linearlized version of equation (AI-3) equation becomes:

$$p_{t}^{*} = (1 - \rho\beta)(mc_{t} + p_{t}) + \rho\beta E_{t}p_{t+1}^{*}$$
(AI-4)

The log-linearized version of equation (19) of the chapter can be written as:

$$p_{t} = \rho p_{t-1} + (1 - \rho) p_{t}^{*}$$
(AI-5)

Writing equation (AI-5) one period forward and taking expectation of both sides, I get:

$$E_{t}p_{t+1} = \rho p_{t} + (1-\rho)E_{t}p_{t+1}^{*}$$

Subtract  $p_t$  from both sides

$$\Rightarrow \quad E_{t}p_{t+1} - p_{t} = -(1-\rho)p_{t} + (1-\rho)E_{t}p_{t+1}^{*}$$

But,  $E_t p_{t+1} - p_t = E_t \pi_{t+1}$ 

$$\Rightarrow \quad E_{t}\pi_{t+1} = -(1-\rho)p_{t} + (1-\rho)E_{t}p_{t+1}^{*}$$

or  $E_t p_{t+1}^* = \left(\frac{1}{1-\rho}\right) E_t \pi_{t+1}^* + p_t$ 

Use this to eliminate  $E_t P_{t+1}^*$  from equation (AI-4)

$$\Rightarrow \qquad p_i^* = (1 - \rho\beta)(mc_i + p_i) + \rho\beta \left\{ \left(\frac{1}{1 - \rho}\right) E_i \pi_{i+1}^* + p_i \right\}$$
(AI-6)

Substitute equation (AI-6) in equation (A-5):

$$p_{t} = \rho p_{t-1} + (1-\rho) \left[ (1-\rho\beta)(mc_{t}+p_{t}) + \rho\beta \left\{ \left(\frac{1}{1-\rho}\right) E_{t} \pi_{t+1}^{*} + p_{t} \right\} \right]$$

Simplifying:

$$[1 - (1 - \rho)(1 - \rho\beta) - \rho\beta(1 - \rho)]p_{t} - \rho p_{t-1} = (1 - \rho)(1 - \rho\beta)mc_{t} + \rho\beta E_{t}\pi_{t+1}$$
$$\rho(p_{t} - p_{t-1}) = (1 - \rho)(1 - \rho\beta)mc_{t} + \rho\beta E_{t}\pi_{t+1}$$

or

or 
$$(p_t - p_{t-1}) = \left(\frac{(1-\rho)(1-\rho\beta)}{\rho}\right)mc_t + \beta E_t \pi_{t+1}$$

But,  $p_t - p_{t-1} = \pi_t$ 

$$\Rightarrow \quad \pi_{i} = \beta E_{i} \pi_{i+1} + \left(\frac{(1-\rho)(1-\rho\beta)}{\rho}\right) mc_{i}$$
  
or 
$$\quad \pi_{i} = \beta E_{i} \pi_{i+1} + \gamma mc_{i}$$
(AI-7)  
where, 
$$\gamma = \frac{(1-\rho)(1-\rho\beta)}{\rho}$$

Equation (AI-7) is equation (27) of the chapter.

### **Appendix II**

#### **Calculation of Variances**

This appendix calculates the variances of inflation and the output-gap under inflation targeting with commitment and under price-level targeting with discretion.

#### A- Inflation targeting with Commitment

The model described in the chapter gives the following evolution of inflation and the output-gap under inflation targeting with commitment:

$$\pi_{t} = a_{IT}^{t} x_{t-1} + b_{IT}^{t} u_{t} + c_{IT}^{t} v_{t}$$
(AII-1)

$$x_{i} = d_{II}^{c} x_{i+1} + e_{II}^{c} u_{i} + f_{II}^{c} v_{i}$$
(AII-2)

Applying the variance formula to equation (AII-2) gives:

$$\operatorname{var}(x_{t}) = d_{II}^{c^{2}} \operatorname{var}(x_{t-1}) + e_{II}^{c^{2}} \operatorname{var}(u_{t}) + f_{II}^{c^{2}} \operatorname{var}(v_{t}) + 2d_{II}^{c} e_{II}^{c} \operatorname{cov}(x_{t-1}, u_{t}) + 2d_{II}^{c} f_{II}^{c} \operatorname{cov}(x_{t-1}, v_{t}) + 2e_{II}^{c} f_{II}^{c} \operatorname{cov}(u_{t}, v_{t})$$

where,

$$cov(u_{t}, v_{t}) = 0$$

$$cov(u_{t}, v_{t}) = cov(d_{1T}^{*} x_{t-2} + e_{1T}^{*} u_{t-1} + f_{1T}^{*} v_{t-1}, \rho_{u} u_{t-1} + \varepsilon_{t}^{u})$$

$$= d_{1T}^{*} \rho_{v} cov(x_{t-2}, u_{t-1}) + e_{1T}^{*} \rho_{u} var(u_{t-1}) + f_{1T}^{*} \rho_{u} cov(u_{t-1}, v_{t-1})$$

$$e_{u-t} \rho_{u}$$
(AII-3)

or  $\operatorname{cov}(x_{-1}, u_{i}) = \frac{e_{ii} \rho_{ii}}{1 - d_{ii} \rho_{ii}} \operatorname{var}(u_{i})$  (AII-4)

Similarly,

$$\operatorname{cov}(x_{i-1}, v_i) = \frac{f_{II}^{c} \rho_i}{1 - d_{IT}^{c} \rho_i} \operatorname{var}(v_i)$$
(All-5)

Therefore,

$$\operatorname{var}(x_{t}) = \frac{1}{1 - d_{tt}^{c^{2}}} \left[ e_{tT}^{c^{2}} \operatorname{var}(u_{t}) + f_{TT}^{c^{2}} \operatorname{var}(v_{t}) + \frac{2d_{tT}^{c} e_{tL}^{c} (e_{tT}^{c} \rho_{u})}{1 - d_{tT}^{c} \rho_{u}} \operatorname{var}(u_{t}) + \frac{2d_{tT}^{c} f_{tL}^{c} (f_{tT}^{c} \rho_{u})}{1 - d_{tT}^{c} \rho_{u}} \operatorname{var}(v_{t}) \right]$$

$$= \frac{1}{1 - d_{II}^{v^{2}}} \left[ e_{II}^{v} \left( 1 + \frac{2d_{II}^{v} \rho_{u}}{1 - d_{II}^{v} \rho_{u}} \right) \operatorname{var}(u_{t}) + f_{II}^{v^{2}} \left( 1 + \frac{2d_{II}^{v} \rho_{t}}{1 - d_{II}^{v} \rho_{v}} \right) \operatorname{var}(v_{t}) \right]$$

$$= \frac{1}{1 - d_{II}^{v^{2}}} \left[ e_{II}^{v^{2}} \left( \frac{1 - d_{II}^{v} \rho_{u} + 2d_{II}^{v} \rho_{u}}{1 - d_{II}^{v} \rho_{u}} \right) \operatorname{var}(u_{t}) + f_{II}^{v^{2}} \left( \frac{1 - d_{II}^{v} \rho_{v} + 2d_{II}^{v} \rho_{v}}{1 - d_{II}^{v} \rho_{v}} \right) \operatorname{var}(v_{t}) \right]$$

$$\operatorname{var}(v_{t}) = \frac{1}{1 - d_{II}^{v^{2}}} \left[ \frac{e_{II}^{v^{2}} (1 + d_{II}^{v} \rho_{u})}{1 - d_{II}^{v} \rho_{u}} \operatorname{var}(u_{t}) + \frac{f_{II}^{v^{2}} (1 + d_{II}^{v} \rho_{v})}{1 - d_{II}^{v} \rho_{v}} \operatorname{var}(v_{t}) \right]$$
(AII-6)

Equation (AII-6) is equation (79) of the chapter.

To find the variance of inflation, apply the variance formula to equation (AII-1):  $\operatorname{var}(\pi_{t}) = a_{TT}^{c} \operatorname{var}(x_{t+1}) + b_{TT}^{c} \operatorname{var}(u_{t}) + c_{TT}^{c} \operatorname{var}(v_{t}) + 2a_{TT}^{c} b_{TT}^{c} \operatorname{cov}(x_{t+1}, v_{t}) + 2b_{TT}^{c} c_{TT}^{c} \operatorname{cov}(u_{t}, v_{t})$ 

Substituting the expressions for the covariances using equation (AII-3), (AII-4) and (AII-5), and the expression for the variance of the output-gap using equation (AII-6), I get:

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$$\operatorname{var}(\pi_{t}) = \frac{a_{IT}^{c^{2}}}{1 - d_{IT}^{c^{2}}} \left[ \frac{e_{IT}^{c^{2}}(1 + d_{IT}^{c}\rho_{u})}{1 - d_{IT}^{c}\rho_{u}} \operatorname{var}(u_{t}) + \frac{f_{IT}^{c^{2}}(1 + d_{IT}^{c}\rho_{v})}{1 - d_{IT}^{c}\rho_{v}} \operatorname{var}(v_{t}) \right] \\ + b_{IT}^{c^{2}} \operatorname{var}(u_{t}) + c_{IT}^{c^{2}} \operatorname{var}(v_{t}) + \frac{2a_{IT}^{c}b_{IT}^{c}e_{IT}^{c}\rho_{u}}{1 - d_{IT}^{c}\rho_{u}} \operatorname{var}(u_{t}) + \frac{2a_{IT}^{c}c_{IT}^{c}f_{IT}^{c}\rho_{v}}{1 - d_{IT}^{c}\rho_{v}} \operatorname{var}(v_{t})$$

Simplifying,

$$\operatorname{var}(\pi_{t}) = \left[ \frac{a_{TT}^{c^{2}} e_{TT}^{c^{2}} (1 + d_{TT}^{c} \rho_{u})}{(1 - d_{TT}^{c^{2}})(1 - d_{TT}^{c} \rho_{u})} + b_{TT}^{c^{2}} + \frac{2a_{TT}^{c} b_{TT}^{c} e_{TT}^{c} \rho_{u}}{1 - d_{TT}^{c} \rho_{u}} \right] \operatorname{var}(u_{t}) \\ + \left[ \frac{a_{TT}^{c^{2}} f_{TT}^{c^{2}} (1 + d_{TT}^{c} \rho_{v})}{(1 - d_{TT}^{c^{2}})(1 - d_{TT}^{c} \rho_{v})} + c_{TT}^{c^{2}} + \frac{2a_{TT}^{c} c_{TT}^{c} f_{TT}^{c} \rho_{v}}{1 - d_{TT}^{c} \rho_{v}} \right] \operatorname{var}(v_{t})$$
(AII-7)

Equation (AII-7) is equation (78) of the chapter.

#### **B-** Price-level Targeting with Discretion

The model described in the chapter gives the following evolution of price-level and the output-gap under price-level targeting with discretion:

$$p_{t} = a_{PT} p_{t-1} + b_{PT} u_{t} + c_{PT} v_{t}$$
(AII-8)

$$x_{t} = d_{PT} p_{t-1} + e_{PT} u_{t} + f_{PT} v_{t}$$
(AII-9)

Applying the variance formula to equation (AII-8) gives:

$$\operatorname{var}(p_{t}) = a_{PT}^{2} \operatorname{var}(p_{t-1}) + b_{PT}^{2} \operatorname{var}(u_{t}) + c_{PT}^{2} \operatorname{var}(v_{t})$$
$$+ 2a_{PT}b_{PT} \operatorname{cov}(p_{t-1}, u_{t}) + 2a_{PT}c_{PT} \operatorname{cov}(p_{t-1}, v_{t}) + 2b_{PT}c_{PT} \operatorname{cov}(u_{t}, v_{t})$$

where,

$$\operatorname{cov}(p_{t-1}, u_{t}) = \operatorname{cov}(a_{p_{1}} p_{t-2} + b_{p_{1}} u_{t-1} + c_{p_{1}} v_{t-1}, \rho_{u} u_{t-1} + \varepsilon_{t}^{u})$$

$$= a_{PT} \rho_u \operatorname{cov}(p_{t-2}, u_{t-1}) + b_{PT} \rho_u \operatorname{var}(u_{t-1}) + c_{PT} \rho_u \operatorname{cov}(u_{t-1}, v_{t-1})$$

or 
$$\operatorname{cov}(p_{i-1}, u_i) = \frac{b_{p_I} \rho_u}{1 - a_{p_T} \rho_u} \operatorname{var}(u_i)$$
 (AII-10)

Similarly,

$$\operatorname{cov}(p_{t,1}, v_t) = \frac{c_{PI} \rho_t}{1 - a_{PI} \rho_v} \operatorname{var}(v_t)$$
(AII-11)

Therefore,

$$\operatorname{var}(p_{t}) = \frac{1}{1 - a_{p_{t}}^{2}} \left[ b_{p_{t}}^{2} \operatorname{var}(u_{t}) + c_{p_{T}}^{2} \operatorname{var}(v_{t}) + \frac{2a_{p_{t}}b_{p_{t}}(b_{p_{t}}\rho_{u})}{1 - a_{p_{t}}\rho_{u}} \operatorname{var}(u_{t}) + \frac{2a_{p_{t}}c_{p_{t}}(c_{p_{t}}\rho_{u})}{1 - a_{p_{T}}\rho_{u}} \operatorname{var}(v_{t}) \right]$$

$$= \frac{1}{1 - a_{p_{t}}^{2}} \left[ b_{p_{t}}^{2} \left( 1 + \frac{2a_{p_{T}}\rho_{u}}{1 - a_{p_{T}}\rho_{u}} \right) \operatorname{var}(u_{t}) + c_{p_{t}}^{2} \left( 1 + \frac{2a_{p_{t}}\rho_{u}}{1 - a_{p_{T}}\rho_{v}} \right) \operatorname{var}(v_{t}) \right]$$

$$= \frac{1}{1 - a_{p_{t}}^{2}} \left[ b_{p_{t}}^{2} \left( \frac{1 - a_{p_{t}}\rho_{u} + 2a_{p_{t}}\rho_{u}}{1 - a_{p_{T}}\rho_{u}} \right) \operatorname{var}(u_{t}) + c_{p_{t}}^{2} \left( \frac{1 - a_{p_{t}}\rho_{v} + 2a_{p_{t}}\rho_{v}}{1 - a_{p_{T}}\rho_{v}} \right) \operatorname{var}(v_{t}) \right]$$

$$\operatorname{var}(p_{t}) = \frac{1}{1 - a_{p_{t}}^{2}} \left[ \frac{b_{p_{t}}^{2} (1 + a_{p_{t}}\rho_{u})}{1 - a_{p_{T}}\rho_{v}} \operatorname{var}(u_{t}) + \frac{c_{p_{t}}^{2} (1 + a_{p_{t}}\rho_{v})}{1 - a_{p_{T}}\rho_{v}} \operatorname{var}(v_{t}) \right]$$

$$\operatorname{var}(p_{t}) = \frac{1}{1 - a_{p_{t}}^{2}} \left[ \frac{b_{p_{t}}^{2} (1 + a_{p_{t}}\rho_{u})}{1 - a_{p_{T}}\rho_{v}} \operatorname{var}(u_{t}) + \frac{c_{p_{t}}^{2} (1 + a_{p_{t}}\rho_{v})}{1 - a_{p_{T}}\rho_{v}} \operatorname{var}(v_{t}) \right]$$

$$\operatorname{var}(p_{t}) = \frac{1}{1 - a_{p_{t}}^{2}} \left[ \frac{b_{p_{t}}^{2} (1 + a_{p_{t}}\rho_{u})}{1 - a_{p_{T}}\rho_{v}} \operatorname{var}(u_{t}) + \frac{c_{p_{t}}^{2} (1 + a_{p_{t}}\rho_{v})}{1 - a_{p_{T}}\rho_{v}} \operatorname{var}(v_{t}) \right]$$

$$\operatorname{var}(p_{t}) = \frac{1}{1 - a_{p_{t}}^{2}} \left[ \frac{b_{p_{t}}^{2} (1 + a_{p_{t}}\rho_{u})}{1 - a_{p_{T}}\rho_{v}} \operatorname{var}(u_{t}) + \frac{c_{p_{t}}^{2} (1 + a_{p_{t}}\rho_{v})}{1 - a_{p_{T}}\rho_{v}} \operatorname{var}(v_{t}) \right]$$

$$\operatorname{var}(p_{t}) = \frac{1}{1 - a_{p_{t}}^{2}} \left[ \frac{b_{p_{t}}^{2} (1 + a_{p_{t}}\rho_{u})}{1 - a_{p_{T}}\rho_{v}} \operatorname{var}(u_{t}) + \frac{c_{p_{t}}^{2} (1 + a_{p_{t}}\rho_{v})}{1 - a_{p_{T}}\rho_{v}} \operatorname{var}(v_{t}) \right]$$

Equation (AII-12) is equation (99) of the chapter.

To find the variance of inflation, I need to re-express the evolution of the pricelevel in terms of inflation. To do this, subtract  $p_{t-1}$  from both sides of equation (AII-8): PhD Thesis - Hamza Ali Malik, Department of Economics, McMaster University

$$\Rightarrow \qquad \pi_t = -(1 - a_{PT})p_{t-1} + bu_t + cv_t \qquad (AII-13)$$

Applying the variance formula to equation (AII-13) gives:

$$\operatorname{var}(\pi_{t}) = (1 - a_{PT})^{2} \operatorname{var}(p_{t-1}) + b_{PT}^{2} \operatorname{var}(u_{t}) + c_{PT}^{2} \operatorname{var}(v_{t}) - 2(1 - a_{PT})b_{PT} \operatorname{cov}(p_{t-1}, u_{t})$$
$$- 2(1 - a_{PT})c_{PT} \operatorname{cov}(p_{t-1}, v_{t}) + 2b_{PT}c_{PT} \operatorname{cov}(u_{t}, v_{t})$$

Substituting the expressions for the covariances using equation (AII-3), (AII-10) and (AII-11), and the expression for the variance of price-level using equation (AII-12), I get:

$$\operatorname{var}(\pi_{t}) = \frac{(1-a_{PT})^{2}}{1-a_{PT}^{2}} \left[ \frac{b_{PT}^{2}(1+a_{PT}\rho_{u})}{1-a_{PT}\rho_{u}} \operatorname{var}(u_{t}) + \frac{c_{PT}^{2}(1+a_{PT}\rho_{v})}{1-a_{PT}\rho_{v}} \operatorname{var}(v_{t}) \right] + b_{PT}^{2} \operatorname{var}(u_{t}) \\ + c_{PT}^{2} \operatorname{var}(v_{t}) - \frac{2b_{PT}^{2}(1-a_{PT})\rho_{u}}{1-a_{PT}\rho_{u}} \operatorname{var}(u_{t}) - \frac{2c_{PT}^{2}(1-a_{PT})\rho_{v}}{1-a_{PT}\rho_{v}} \operatorname{var}(v_{t}) \\ \operatorname{var}(\pi_{t}) = \left[ \frac{(1-a_{PT})b_{PT}^{2}(1+a_{PT}\rho_{u})}{(1+a_{PT})(1-a_{PT}\rho_{u})} + b_{PT}^{2} - \frac{2b_{PT}^{2}(1-a_{PT})\rho_{u}}{(1-a_{PT}\rho_{u})} \right] \operatorname{var}(u_{t}) \\ + \left[ \frac{(1-a_{PT})c_{PT}^{2}(1+a_{PT}\rho_{v})}{(1+a_{PT})(1-a_{PT}\rho_{v})} + c_{PT}^{2} - \frac{2c_{PT}^{2}(1-a_{PT})\rho_{v}}{(1-a_{PT}\rho_{v})} \right] \operatorname{var}(v_{t}) \\ \operatorname{var}(\pi_{t}) = \left[ \frac{2b_{PT}^{2}(1-\rho_{u})}{(1+a_{PT})(1-a_{PT}\rho_{u})} \right] \operatorname{var}(u_{t}) + \left[ \frac{2c_{PT}^{2}(1-\rho_{v})}{(1+a_{PT})(1-a_{PT}\rho_{v})} \right] \operatorname{var}(v_{t}) \quad (AII-14)$$

Equation (AII-14) is equation (100) of the chapter.

To find the variance of the output-gap, apply the variance formula to equation (AII-9):

$$\operatorname{var}(x_{t}) = d_{PT}^{2} \operatorname{var}(p_{t-1}) + c_{PT}^{2} \operatorname{var}(u_{t}) + f_{PT}^{2} \operatorname{var}(v_{t}) + 2d_{PT}c_{PT} \operatorname{cov}(p_{t-1}, u_{t}) + 2d_{PT}f_{PT} \operatorname{cov}(p_{t-1}, v_{t}) + 2e_{PT}f_{PT} \operatorname{cov}(u_{t}, v_{t})$$

Substituting the expressions for the covariances using equation (AII-3), (AII-10) and (AII-11), and the expression for the variance of the price-level using equation (AII-12), l get:

$$\operatorname{var}(x_{t}) = \frac{d_{p_{t}}^{2}}{1 - a_{p_{t}}^{2}} \left[ \frac{b_{p_{t}}^{2} \left(1 + a_{p_{t}} \rho_{u}\right)}{1 - a_{p_{T}} \rho_{u}} \operatorname{var}(u_{t}) + \frac{c_{p_{t}}^{2} \left(1 + a_{p_{t}} \rho_{u}\right)}{1 - a_{p_{T}} \rho_{u}} \operatorname{var}(v_{t}) \right] \\ + e_{p_{T}}^{2} \operatorname{var}(u_{t}) + f_{p_{T}}^{2} \operatorname{var}(v_{t}) + \frac{2d_{p_{t}} e_{p_{t}} b_{p_{t}} \rho_{u}}{1 - a_{p_{T}} \rho_{u}} \operatorname{var}(u_{t}) + \frac{2d_{p_{t}} f_{p_{t}} c_{p_{t}} \rho_{u}}{1 - a_{p_{T}} \rho_{u}} \operatorname{var}(u_{t}) + \frac{2d_{p_{t}} f_{p_{t}} c_{p_{t}} \rho_{u}}{1 - a_{p_{T}} \rho_{u}} \operatorname{var}(v_{t}) + \frac{2d_{p_{t}} f_{p_{t}} c_{p_{t}} \rho_{u}}{1 - a_{p_{T}} \rho_{u}} \operatorname{var}(v_{t}) + \frac{2d_{p_{t}} f_{p_{t}} c_{p_{t}} \rho_{u}}{1 - a_{p_{T}} \rho_{u}} \operatorname{var}(v_{t}) + \frac{2d_{p_{t}} f_{p_{t}} c_{p_{t}} \rho_{u}}{1 - a_{p_{T}} \rho_{u}} \operatorname{var}(v_{t}) + \frac{2d_{p_{t}} f_{p_{t}} c_{p_{t}} \rho_{u}}{1 - a_{p_{t}} \rho_{u}} \operatorname{var}(v_{t}) + \frac{2d_{p_{t}} f_{p_{t}} c_{p_{t}} \rho_{u}}{1 - a_{p_{t}} \rho_{u}}} \operatorname{var}(v_{t})$$

Simplifying,

$$\operatorname{var}(x_{t}) = \left[\frac{b_{PI}^{2} d_{PI}^{2} (1 + a_{PI} \rho_{u})}{(1 - a_{PT}^{2})(1 - a_{PT} \rho_{u})} + e_{PI}^{2} + \frac{2b_{PI} d_{PT} e_{PI} \rho_{u}}{(1 - a_{PI} \rho_{u})}\right] \operatorname{var}(u_{t}) + \left[\frac{c_{PI}^{2} d_{PI}^{2} (1 + a_{PI} \rho_{u})}{(1 - a_{PT}^{2})(1 - a_{PT} \rho_{u})} + f_{PI}^{2} + \frac{2c_{PI} d_{PI} f_{PI} \rho_{u}}{(1 - a_{PT} \rho_{u})}\right] \operatorname{var}(v_{t})$$
(AII-15)

Equation (AII-15) is equation (98) of the chapter.

#### **Chapter 3**

## Price-level vs. Nominal Income Targeting and the Cost Channel of Monetary Policy Transmission

#### **1- Introduction**

As discussed in the previous chapters, a consensus has grown in many countries in recent years that central banks should primarily focus on policies that promote price stability in the economy, and that a rule-based monetary policy is superior to discretionbased monetary policy actions. Price stability eliminates the costs associated with inflation (the rate of change of prices) and makes the economy more efficient. Moreover, a rule-based monetary policy disciplines the central banks by "tying their hands" and ensures systematic action. This kind of commitment leads to more transparency in the policymaking process and increases credibility and accountability. A number of variables that can be (and have been) used as targets in the conduct of rule based monetary policy to achieve the ultimate target of long-term price stability have appeared in the literature. Some notable examples are: money-growth, exchange rate, nominal income, inflation rate and the price level.

While there is broad understanding regarding the overall monetary policy strategy, the mechanisms through which monetary policy affects the real economy are not completely understood. For example, in chapter 2, we learned that in the presence of the

cost channel of monetary policy, in addition to the traditional interest rate-aggregate demand channel, demand shocks cannot be completely stabilized and there is a trade-off between stabilizing inflation and output-gap. The traditional interest rate channel operates by affecting the spending decisions of households and firms and thus works through the aggregate demand side of the model, while the cost channel operates by affecting the cost of production and thus the aggregate supply side of the model. Most of the literature has, so far, concentrated mainly on the traditional channel of monetary policy while assessing alternative targeting regimes. However, several researchers such as Christiano and Eichenbaum (1992), Christiano, Eichenbaum and Evans (1997) and Barth and Ramey (2001) have emphasized the cost channel as a powerful collaborator in the transmission of short run effects of monetary policy. By analyzing both the traditional and the cost channel of monetary policy in one unified framework, this chapter, like the previous one, is an attempt to bridge the gap between these two strands of literature.

In the previous chapter, I have already mentioned the reasons why distinguishing the relative importance of the traditional and the cost channel is useful.<sup>30</sup> I briefly mention them again. First, it improves our understanding of the link between the financial and real sectors of the economy. Second, it provides alternative indicators to help gauge the stance of monetary policy and thus increases its ability to offset particular types of adverse shocks. Third, a clear understanding of the transmission mechanism has the potential to give more information regarding the choice of intermediate targets. Informed

<sup>&</sup>lt;sup>30</sup> For a detailed discussion see Kashyap and Stein (1994)

by these observations, especially the last one, the objective of this chapter is to assess the robustness of policy recommendations for a closed economy in the presence of the cost channel of monetary policy.

Thus, in spirit, this chapter is similar to the previous chapter. However, there are some important differences. First, it uses a continuous-time modeling approach instead of the more conventional discrete-time approach. Apart from the advantage in terms of analytical simplicity, continuous-time models avoid the unappealing problems regarding the model properties being dependent on small changes in assumptions concerning information availability. I explain this point further while explaining the structure of the model below. Second, rather than deriving the optimal policy as in chapter 2, this chapter makes use of the Taylor-type interest rate based rules which have become quite popular in policy circles in recent years. In particular, I study two interest rate based monetary policy rules --- price-level targeting and nominal income targeting in a continuous-time version of the 'new Keynesian model' that incorporates both the traditional interest rate channel and the cost channel of monetary policy transmission mechanism. Third, the chapter also studies alternative specifications for the aggregate demand side of the economy --- the IS-type relationship and for the aggregate supply side of the economy --a Phillips curve type relationship that has been proposed recently in the literature in light of empirical considerations. Also, as explained in the last chapter, the results in favour of or against a price level target are very model specific, especially regarding the specification of the Phillips curve. This consideration adds robustness in assessing the

role of the cost channel. Fourth, instead of the nominal interest rate, the real interest rate appears in the Phillips curve relationships to represent the cost channel. In chapter 2, I assumed that firms needed to pay the hired workers before the receipt of the sales revenues. For this purpose, they borrowed from banks at the nominal interest rate t. Thus, there was a payment lag involved and the relevant 'cost of borrowing' was represented by the nominal interest rate. On the other hand, in this chapter I assume that firms borrow from banks to pay for the wage-bill before the production process begins. Thus, there is a production lag involved here and the relevant 'cost of borrowing' is represented by the real interest rate r. Fifth, it is assumed that the two targeting regimes generate the same outcome regarding long-term inflation. Thus, the criterion for evaluating the performance of a monetary regime is its ability to minimize the volatility in real output in response to aggregate demand shocks

The main results of this chapter are that the cost channel matters in the sense that the volatility of real output increases under both price-level and nominal income targeting when the cost channel is included in the models. However, the inclusion of the cost channel does not say much on the choice between the two regimes. It appears that nominal income targeting performs better than price-level targeting in bringing down the volatility of real output in almost all the specifications of the macro models used in the analysis regardless of the cost channel.

## 1.1- Comparison of Price-level Targeting and Nominal Income Targeting

The relative benefits of price-level targeting and the related literature have already been discussed in chapter 2. We learned that the long-term price level is more certain under price level targeting while it may wander around randomly over long periods under inflation targeting. However, short-term price volatility (and thus output volatility) may be higher under price-level targeting because unexpected rises in the price level will be followed by attempted reductions in the price level. Moreover, the literature on the relative benefits of price-level targeting is divided with a central role being played by the specification of the Phillips curve. Since, unlike chapter 2, this chapter also studies alternative specifications for the Phillips curve (and the IS relationship for that matter) in addition to the presence of the cost channel, it has the potential to shed more light on the issue of an appropriate targeting regime including price-level targeting.

Nominal income targeting is another desirable strategy for monetary policy as it shares many positive features of inflation targeting. But, the most attractive feature of nominal income targeting is that it is closely related to both real output and prices --- the two variables that central bank seem to care about most. In addition, nominal income targeting allows the monetary policy to adjust to offset disturbances to both aggregate demand and aggregate supply. For example, in case of an adverse demand shock (that would cause both real output and prices to go below target), policymakers would ease monetary policy that would return nominal income (the product of real income and prices) to target. Similarly, an adverse supply shock results in falling real output and rising price levels. This could pose a dilemma if central bank is pursuing price level targeting. Stabilizing the price level would mean further decline in real output. Nominal income targeting would help policy makers resolve the dilemma as it places equal emphasis on stability of both real output and price level.<sup>31</sup>

Frankel and Chinn (1995) have shown nominal income targeting to be superior to money-growth. exchange rate and price level targeting in a simple time-consistent model of monetary policy. More recently, several contributions in the literature have been made that study the stability properties of the nominal income-targeting regime. Two key papers in this regard are Ball (1999a) and McCallum (1997). Using a backward looking macro model, Ball (1999a) has forcefully argued that nominal income targets are not merely inefficient, but also disastrous: they imply that output and inflation have infinite variances. Svensson (1999a) replicates Ball's instability result and suggests that it is the stylized fact that policy affects real output before inflation which Ball (1999a) builds into his model that lies at the heart of the instability result. Challenging the negative assessment of nominal income targeting, McCallum (1997) has shown that Ball's instability result is not robust; it critically depends on the specification of the Phillips curve relationship.<sup>32</sup> Using a forward-looking model McCallum demonstrates that nominal income targeting does not generate instability. Using a Philips curve with mixed

<sup>&</sup>lt;sup>31</sup> The case in favour of nominal income targeting has been well documented in Hall and Mankiw (1994).

<sup>&</sup>lt;sup>32</sup> The issue of the importance of Phillips curve or the supply side of the economy for the performance of nominal income targeting is not new; it has been previously highlighted by Bean (1983) and West (1986).

expectations. Dennis (2001b) has shown that nominal income targeting will not generate instability as long as inflation expectations contain some forward-looking component. More recently Rudebusch (2002), however, has shown that nominal income targeting performs poorly after taking into account of the range of model and data uncertainty that policy makers face.

It is evident from the above discussion that the case for or against price level targeting and nominal income targeting relies critically on how inflation expectations are formed in the Phillips curve or more generally on the specification of the model. For this reason, I evaluate the performance of price-level targeting and nominal income targeting in a series of macroeconomic models with different specifications for the 1S relationship and Phillips curve relationship. In particular, based on the works of Fuhrer (2000) and Amato and Laubach (2003), I introduce a lagged output-gap term in the new IS relationship. Both these papers have pointed out that the standard Euler equation for consumption (that gives rise to the new IS relationship) fails to capture the dynamics of the aggregate output. Similarly, many researchers have criticized the basic 'new Keynesian Phillips curve', which is based on Calvo's staggered price model, on the ground that it generates inertia in the price level and not the inflation rate and that this is inconsistent with the stylized facts on inflation dynamics. Thus, I introduce a lagged inflation term to the baseline Phillips curve based on the work of Fuhrer and Moore (1995) and study the Mankiw and Reis's (2002) Phillips curve relationship that is based not on sticky prices but on sticky information. In addition. I also explore the implications

of adding the supply side effects --- the cost channel --- of interest rates to each specification. It has been argued in the literature that such effects can be significant in evaluating the performance of monetary policy (Myatt and Scarth (2003)). These considerations provide an additional and comprehensive contribution to the ongoing debate between choosing an appropriate targeting regime. Thus, the analysis not only allows for a direct comparison between price-level and nominal income targeting in a range of macroeconomic models, but also highlights the importance of the transmission mechanism of monetary policy.

#### 2- The Baseline Continuous-time 'new Keynesian' Model

The model is defined by equations (1) through (4). These equations define (respectively) the "new" *IS* relationship (aggregate demand), the "new" Phillips curve (aggregate supply), monetary policy, and the exogenous cycle in autonomous spending. The definition of variables and a more detailed description of the structure are given following the equations.

$$\dot{v} = \alpha(r - \bar{r}) + \beta \dot{a} \tag{1}$$

$$\ddot{p} = -\lambda(y - \bar{y}) + \psi(a - \bar{a}) - \kappa\gamma(r - \bar{r})$$
<sup>(2)</sup>

$$p + \mu y = 0 \tag{3}$$

$$a = \overline{a} + \delta \sin(t) \tag{4}$$

All variables except the interest rate (r) and the time index (t) are the natural logarithms of the associated variable. Dots and bars above a variable denote (respectively) the time derivative, and the full-equilibrium value of that variable. All coefficients (the Greek letters) are positive. The variables are: a – autonomous spending, p – the general price level, r – the real interest rate, and y – the level of real output.

Before discussing each equation in turn, I discuss the continuous-time specification. Discrete-time specifications are more common, but following this practice can involve model properties being dramatically dependent on small changes in assumptions concerning information availability. For example, consider the original "policy relevance" paper by Sargent and Wallace (1976). The central conclusion in this study does not emerge if it is assumed that the information available to agents when deciding how much to spend is the same as what is now usually assumed (that is, when the assumption involved in McCallum and Nelson (1999a) is invoked). Also, if the McCallum and Nelson analysis (p. 309) is reworked with the information-availability assumption used by Sargent and Wallace, the entire undetermined coefficients solution procedure breaks down (with restrictions on structural, not reduced form, coefficients being called for).<sup>33</sup> A continuous-time specification precludes such unappealing problems from developing.

<sup>&</sup>lt;sup>33</sup> See, Lam and Scarth (2002).

Equation (1) is the "new" IS relationship which states that the rate of change of real output depends positively on the real interest rate and on the rate of change of autonomous spending. The motivation for such a relationship can be appreciated by referring to a dynamic general equilibrium macro model with optimizing economic agents. I start with a log-linear approximation of the economy's resource constraint:  $y = \alpha c + \beta a$ , where 'c' is the log of consumption expenditure, 'a' is the log of the autonomous spending. The parameters ' $\alpha$ ' and ' $\beta$ ' are the steady-state ratios of household spending and autonomous spending to total real output respectively. The Ramsey model is used to model forward-looking domestic households. If the instantaneous utility function involves separable terms, log consumption and the square of labour supply, the first-order conditions are  $\dot{c} = r - \bar{r}$ , and (ignoring constants) n = w - p - c. 'n' and 'w' denote the log of employment and the nominal wage. Equation (1) follows by taking the time derivative of the resource constraint and substituting in the Euler equation for consumption. (See appendix II for details). The labour supply function is used below. For detailed derivation and discussion, see Clarida, Gali and Gertler (1999), McCallum and Nelson (1999), Kerr and King (1996) and Walsh (2003a). Almost all the models being referred to are set in discrete- time with rational expectations. I use a continuous-time deterministic setting so that rational expectations and perfect foresight mean the same thing. The focus of this chapter is to exploit this tractable model of aggregate demand with a variety of aggregate supply specifications to analyze various monetary policies in the presence of the cost channel of monetary policy.

Equation (2) is the "new" Phillips curve that relates the rate of change of inflation to the output gap, autonomous-spending gap and the real rate of interest gap. This relationship essentially captures the supply side of the economy and can be derived by incorporating nominal price rigidities using Calvo's (1983) model of sluggish price adjustment and imperfect competition a la Dixit and Stiglitz (1977) in a dynamic general equilibrium macro model. Only proportion  $(1-\tau)$  of firms can change prices at each point in time. Firms minimize the undiscounted present value of the squared deviations between the log of marginal cost (mc) and price (p). Many authors have shown that optimal behaviour at the individual firm level leads to  $\ddot{p} = -[(1-\tau)^2/\tau](mc-p)$  at the aggregate level. To represent this price-adjustment process in a format that resembles the traditional Phillips curve, I follow King (2000) and replace real marginal cost with the output gap (and any other term that emerges as relevant given that I have autonomous spending and supply-side effects of interest rate in the model). In order to incorporate the cost channel I assume that firms borrow from banks to pay for the wage-bill before the production process begins. Thus, there is a production lag involved here and the relevant 'cost of borrowing' is represented by the real interest rate r. This assumption allows me to explicitly analyze the supply-side effects (the cost channel) of monetary policy. The cost channel makes firms' marginal costs depend directly on the rate of interest. I assume a standard Cobb-Douglas production function of the form  $Y = N^{\theta}$ . Thus, in log terms,  $y = \theta n$  and the marginal product of labour, MPL, equals  $\theta Y/N$ . Now, the marginal cost is defined as MC = W(1+r)/MPL; we can (ignoring constants) approximate the log of real marginal cost by mc - p = w - p + r - y + n. Equation (2) is then derived in three more

steps. Use the labour supply function, the production function and the resource constraint to eliminate (w - p), *n* and *c* by substitution; define units so that, in full equilibrium, all prices are unity (so that  $\overline{mc} - \overline{p} = 0$ ); and substitute out the deviation of real marginal cost from its full-equilibrium value (see appendix I for details). The coefficients in (2) have the following interpretations:  $\lambda = (1 - \tau)^2 ((2/\theta) + (1/\alpha) - 1)/\tau$ ,  $\psi = (1 - \tau)^2 \beta / \alpha \tau$ , and  $\gamma = (1 - \tau)^2 / \tau$  Thus parameters ' $\lambda$ ', ' $\gamma$ 'and ' $\psi$ ' are functions of "deep" parameters like the fraction of firms adjusting their prices, labour's exponent in the production function and ' $\alpha$ ' and ' $\beta$ '. The parameter ' $\kappa$ ' is introduced to capture the cost channel of monetary transmission. By setting  $\kappa = 0$ , I can close this channel.

Equation (3) defines monetary policy and encompasses both price-level targeting,  $\mu = 0$  and nominal income targeting,  $\mu = 1$ . Equation (4) depicts the anticipated ongoing cycles in exogenous spending defined by the sine curve. Since the focus of the paper is on the role of the cost channel in affecting the volatility of output under alternative monetary policy regimes, the simplest way to introduce fluctuations in output is to assume that these are caused by exogenous variations in the autonomous spending.

Before analysing the model and discussing the results I briefly talk about the parameter values that are used in calibrating the model(s) below. Consumption is 80% of the total output, that is,  $\alpha = 0.8$ . This implies that  $\beta = 0.2$ . The other summary coefficients for the baseline Phillips curve relationship can be calculated by referring to the corresponding values of the 'deep' parameters. For example, if labour's exponent in

the Cobb-Douglas production function is two-thirds ( $\theta = 0.67$ ) and the fraction of firms that are able to adjust their prices once a year is approximately one-fourth ( $1 - \tau = 0.27$ ), then  $\lambda = 0.33$ ,  $\psi = 0.03$  and  $\gamma = 0.10$ .<sup>34</sup> The parameter  $\delta$  in equation (4) is taken as 1.

#### <u>3- Analysis</u>

In this section I derive the reduced form for real output to see how the cost channel affects the amplitude of the cycle in y, and to see the relative performance of price-level and nominal income targeting in this regard. I explain this derivation in the baseline case only. The reader can use similar steps to verify the results that I report for other cases in the following sections.

First, take second time derivative of equation (3) and use the result to eliminate  $\ddot{p}$ in equation 2. Also, use equation (1) to eliminate  $r - \bar{r}$  from equation 2. The result is:

$$-\mu \ddot{y} = -\lambda (y - \bar{y}) + \psi (a - \bar{a}) - (\kappa \gamma / \alpha) \dot{y} + (\kappa \gamma \beta / \alpha) \dot{a}$$
(5)

Using the undetermined coefficient solution procedure as described in Chiang (1984), the solution for output can be written as:

$$y = \overline{y} + B[\cos(t)] + C[\sin(t)]$$
(6)

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where B and C are arbitrary constants that must be related to the underlying parameters

<sup>&</sup>lt;sup>34</sup> In order to ensure that my results are not dependent on particular values of these parameters, 1 have considered a range of other parameter values as well. For example, if we assume that the fraction of firms with sticky prices is two-thirds 0.67 rather than 0.73 than the values of all summary parameters change accordingly. In particular, they are:  $\lambda = 0.55$ ,  $\psi = 0.05$  and  $\gamma = 0.18$  However, the results are not sensitive to these alternative values for various parameters.

of the model. To solve for B and C, first take the time derivatives of (6),  $y = -B\sin(t) + C\cos(t)$  and  $\ddot{y} = -B\cos(t) - C\sin(t)$  along with the time derivative of (4),  $\ddot{a} = \delta\cos(t)$  and then substitute these results and equation (4) and (6) in equation (5). The resulting coefficient-identifying restrictions are:

$$B = \frac{\kappa \gamma (\beta \delta - C)}{\alpha (\lambda + \mu)} \tag{7}$$

$$C = \frac{\alpha^2 \psi(\lambda + \mu) + \beta \delta(\kappa \gamma)^2}{(\kappa \gamma)^2 + \alpha^2 (\lambda + \mu)^2}$$
(8)

The amplitude of the cycles in real output that correspond to the ongoing cycles in autonomous spending can be examined by substituting the calibrated expressions for B and C in equation (6)

Note that if  $\kappa = 0$ , that is, if the cost channel is not operating, B = 0. Thus, only the reduced-form parameter C represents the amplitude of the cycle in real output. It is straightforward to verify that the amplitude of the cycle in real output decreases as  $\mu$ increases, that is, changes from zero to one. This is the first main result of this chapter.

**Result 1**: In a baseline 'new Keynesian' model, nominal income targeting performs better as compared to price-level targeting in terms of reducing the volatility of real output in the face of demand shocks.

To investigate the role of the cost channel, I set  $\kappa = 1$ . The results are reported in table 1.

Amplitude of ongoing cycle in real output		
	Cost channel closed ( $\kappa = 0$ )	Cost channel operating ( $\kappa = 1$ )
Price-level targeting		
$(\mu = 0)$	0.061	0.123
Nominal Income		
targeting $(\mu = 1)$	0.016	0.036

#### Table 1: Output Effects --- Baseline 'new Keynesian' Model

Looking at table 1, it is clear that the volatility of real output goes up in the presence of cost channel irrespective of the monetary policy regime. This result is consistent with the claims of many empirical papers like Barth and Ramey (2001) that there are important supply side effects of monetary policy. Moreover, nominal income targeting performs better than price-level targeting with or without the cost channel.

**Result 2**: In a baseline 'new Keynesian' model with the cost channel the volatility of real output is larger than in a baseline 'new Keynesian'' model without the cost channel. However, nominal income targeting still performs better in terms of reducing the volatility of real output.

The intuition for this is simple. In response to an autonomous ongoing demand cycle, the central bank adjusts the nominal interest rate to manipulate the aggregate demand (and thus the aggregate price level) in order to keep resulting output volatility at a minimum. In the presence of the cost channel, however, adjustments in the real interest rate directly affect the aggregate supply side of the model as well: thereby increasing output volatility. As a result, with price-level targeting, the central bank would have to manipulate the aggregate demand by a large magnitude that would ensure the achievement of the original level of prices at the cost of an increased volatility in output. On the other hand, with nominal income targeting it would adjust aggregate demand just enough to reach a targeted level of nominal income with slightly higher level of prices but a lower volatility in output. Since the metric used to evaluate the performance of a targeting regime is the minimization of real output volatility, nominal income targeting is preferred to price-level targeting.

#### 4- Extension I: Alternative Specifications of the Phillips Curve

It has been pointed out by many researchers that 'the new Keynesian Phillips curve' based on Calvo's (1983) sticky price model generates inertia in the price level and not the inflation rate and that this is inconsistent with stylized facts on inflation dynamics. The empirical evidence (for example, Nelson (1998)) indicates that inflation responds sluggishly to economic shocks. The 'new Keynesian Phillips curve' implies that inflation is determined by the current output gap and current expectations of future inflation. Inflation is, therefore, very flexible and responds immediately to monetary policy shocks and hence does not accord with stylized facts. In order to capture the inflation persistence found in the data, it is common to augment the basic forward-looking inflation adjustment equation with the addition of lagged inflation. Fuhrer and Moore (1995) is one such example. Mankiw and Reis (2002) suggest an alternative approach, which departs from the assumption of sticky prices and replaces it with that of sticky information. According to their model of price adjustment firms gather and process the information about the state of the economy slowly over time. Unlike the sticky price model, prices are always changing but firms are slow to update their pricing strategies in response to new information. Empirical research of Gali and Gertler (1999) and Fuhrer (1997) have generally found that when lagged inflation is added to the basic 'new Keynesian Phillips curve', its coefficient is statistically and economically significant. Since the debate over the relative benefits of price-level and nominal income targeting rests critically on the specification of the Phillips curve, it is a worthwhile exercise to redo the analysis with these more general specifications for the Phillips curve.

If the weight on the lagged inflation term is assumed to be 0.5, then the Fuhrer and Moore (1995) type Phillips curve can be written as follows (see appendix I for a detailed derivation):

$$\ddot{p} = -2\lambda(v - \bar{y}) + 2\psi(a - \bar{a}) - 2\kappa\gamma(r - \bar{r})$$
(2a)

# Table 3: Output Effects --- 'new' IS relationship with Mankiw - Reis'Phillips curve

	Amplitude of ongoing cycle in real output		
	Cost channel closed ( $\kappa = 0$ )	Cost channel operating ( $\kappa = 1$ )	
Price-level targeting			
$(\mu = 0)$	0.080	0.352	
Nominal Income			
targeting $(\mu = 1)$	0.072	0.081	

Once again the volatility of real output increases under both price-level and nominal income targeting when the cost channel is operating. This time the difference between the performance of nominal income targeting regime and price-level targeting regime is more pronounced when the cost channel is operating; nominal income targeting performs far better in keeping the volatility of output low. Comparing the results of table 2 and 3, a point can be made (with some caution) that it is the structure of the model regarding how the backward looking behaviour is introduced that is important and not just the backward looking behaviour itself.

#### 5- Extension II: Alternative Specification of the IS Relationship

In all the three models discussed above, I have studied various specifications for the Phillips curve relationship combined with the 'standard' Ramsey type specification of the aggregate demand relationship or the IS curve. In this section I consider the change in the specification of the aggregate demand relationship and then combine it with the three different specifications of the Phillips curve. This exercise is useful in shedding more light on the robustness of the results derived above. In particular, I introduce a lag output term in the IS function. The motivation for doing this modification is taken from the works of Fuhrer (2000) and Amato and Laubach (2003).

Fuhrer (2000) and Amato and Laubach (2003) have pointed out that the standard Ramsey type Euler Equation for consumption (which gives rise to an IS-type relationship) fails to capture the dynamics of the aggregate output. Fuhrer (2000) allow for habit formation in preferences while maintaining the assumption of optimal consumption choice on the part of consumers. Amato and Laubach (2003), on the other hand, introduce the 'rule of thumb' behaviour on the part of a fraction of the household: the remaining fraction of the household is able to optimize their consumption in a usual fashion. Their modification to the standard consumer problem is justified on the grounds that it is costly to reoptimize every period. Both these modifications, introducing habit persistence and incorporating 'rule of thumb behaviour'. leads to a lagged output gap term with some positive weight in the IS equation. Thus, it can be considered as the 'hybrid' version used by Ball (1999a) and McCallum (1997). If the weight on the lagged output term is taken as 0.5, then the 'hybrid' IS relationship can be written in continuous time as (see appendix II for a detailed derivation):

$$\ddot{v} = 2\alpha(r - \bar{r}) + 2\beta \dot{a} \tag{1a}$$

The quantitative results for the calibrated version of the models that combine this hybrid IS relationship with the three specifications of the Phillips curve discussed above are reported in table 4, 5 and 6.

# Table 4: Output Effects --- 'hybrid' IS with 'new Keynesian' Phillips curve

	Amplitude of ongoing cycle in real output		
	Cost channel closed ( $\kappa = 0$ )	Cost channel operating ( $\kappa = 1$ )	
Price-level targeting			
$(\mu = 0)$	0.061	0.226	
Nominal Income			
targeting $(\mu = 1)$	0.015	0.039	

# Table 5: Output Effects --- 'hybrid' IS with Fuhrer-Moore's Phillips curve

Amplitude of ongoing cycle in real output		
	Cost channel closed ( $\kappa = 0$ )	Cost channel operating ( $\kappa = 1$ )
Price-level targeting		
$(\mu = 0)$	infinity	infinity
Nominal Income		
targeting ( $\mu = 1$ )	0.62	0.606

### Table 6: Output Effects --- 'hybrid' IS with Mankiw and Reis' Phillips

#### curve

	Amplitude of ongoing cycle in real output		
	Cost channel closed ( $\kappa = 0$ )	Cost channel operating ( $\kappa = 1$ )	
Price-level targeting			
( $\mu = 0$ )	0.080	0.111	
Nominal Income			
targeting ( $\mu = 1$ )	0.070	0.101	

A stark difference between the results reported in table 2 and table 5 is that now the volatility of output is infinite under price level targeting and much higher under nominal income targeting in the presence of the cost channel. Moreover, the performance of nominal income targeting regime is not much different with or without the cost channel. Thus, the initial results do not hold completely when an alternative specification of the IS relationship is considered. This contradicts the claim made by McCallum (1997, Page 1) that it is only the specification of the Phillips curve that is important. "…replacement of the Ball-Svensson Phillips curve with the mentioned alternative (new Keynesian Phillips curve) results in a model in which both output and inflation are dynamically stable under nominal income targeting whether or not the IS relationship is re-specified". This point is strengthened even more when results of table 6 are compared with the results of table 3; the amplitude of output is quite different under price-level targeting in the presence of the cost channel.

#### 6- Extension III: A More General Monetary Policy Rule

So far the analysis implicitly assumed that in response to the ongoing cycles in autonomous spending the central bank adjusted its policy instrument in an infinitely aggressive fashion. Depending on the value of parameter ' $\mu$ ', the central bank used its one degree of freedom to focus entirely on achieving either the price-level target or the nominal income target. This behaviour could lead to excessive volatility in the policy instrument. Thus, while this scenario was treated as a baseline case, this section provides a more general specification of the monetary policy rule that incorporates the aggressive response only as a special case. In particular, I replace equation (3) of the baseline case with the following monetary policy reaction function

$$i = i + \Omega(p + \mu y - 0) \tag{3a}$$

According to this rule, the central bank adjusts the nominal interest rate above its steady-state value whenever either the price level is above its target (assumed to be zero, with  $\mu = 0$ ), or the nominal income is above its target (also assumed to be zero, with  $\mu = 1$ ). Thus, as before, price-level targeting is involved when  $\mu = 0$  and nominal income targeting is considered when  $\mu = 1$ . However, now I can consider various degrees of 'leaning against the wind' in both cases. For example,  $\Omega = 1$  depicts the case when the central bank is conducting monetary policy in a 'modest' manner. On the other hand,  $\Omega$  approaching infinity would give me the baseline case when central bank responds in an 'aggressive' manner.

Here, I explain the solution procedure only in the baseline 'new Keynesian' model with the general monetary policy rule. For other cases, only the quantitative results for the calibrated version of the model with  $\Omega = 1$  are reported in table 7 and 8.

First, I substitute equation (3a) and the relationship r = i - p in equation (1) to get:

$$\dot{v} = \alpha \Omega p + \alpha \Omega \mu v - \alpha \dot{p} + \beta \dot{a} \tag{9}$$

Also, I substitute equation (3a) and r = i - p in equation (2) and eliminate p using equation (9) to yield:

$$\ddot{p} = -\lambda(y - \bar{y}) + \psi(a - \bar{a}) - (\kappa \gamma / \alpha)\dot{y} + (\kappa \gamma \beta / \alpha)\dot{a}$$
(10)

Now, by taking the appropriate number of time derivatives of equation (9) and using equation (10) to eliminate  $\ddot{p}$ , I get the following third order differential equation:

$$\ddot{y} = -\alpha \lambda \Omega (y - \bar{y}) + \alpha \psi \Omega (a - \bar{a}) + (\alpha \lambda - \kappa \gamma \Omega) \dot{y} + (\alpha \mu \Omega + \kappa \gamma) \dot{y} + (\kappa \gamma \beta \Omega - \alpha \psi) \dot{a}$$

$$-\kappa \gamma \beta \ddot{a} + \beta \ddot{a}$$
(11)

As before the solution for output is given by equation (6). To solve for B and C, I first take the time derivatives of (6),  $\dot{v} = -B\sin(t) + C\cos(t)$  and  $\ddot{y} = -B\cos(t) - C\sin(t)$ and  $\ddot{y} = B\sin(t) - C\cos(t)$  along with the time derivatives of (4),  $\dot{a} = \delta\cos(t)$ .  $\ddot{a} = -\delta\sin(t)$  and  $\ddot{a} = -\delta\cos(t)$  and then substitute these results and equation (4) and (6) in equation (11). The resulting coefficient-identifying restrictions are:

$$B = \frac{\delta(\alpha \eta \Omega + \kappa \gamma \beta)}{A_1} - \frac{A_2}{A_1}C$$
(12)

$$C = \delta A_1 \frac{(\beta + \alpha \psi - \kappa \gamma \beta \Omega)}{A_1^2 + A_2^2} + \delta A_2 \frac{(\alpha \psi \Omega + \kappa \gamma \beta)}{A_1^2 + A_2^2}$$
(13)

where,  $A_1 = 1 + \alpha \lambda - \kappa \gamma \Omega$  and  $A_2 = \alpha \Omega (\lambda + \mu) + \kappa \gamma$ 

A significantly different result emerges when the central bank conducts monetary policy in a modest fashion (see table 7 and 8). The cost channel turns out to be
insignificant in affecting the behaviour of output in all the six models. However, nominal income targeting still performs relatively better in comparison with the price-level targeting case. The only exception is model # 2 but the difference is negligible.

Table 7: Output Effects with a General Monetary Policy Rule --- 'new Keynesian' IS relationship with the Three Specifications of the Phillips curve

	Amplitude of ongoing cycle in real output		
	Cost channel closed ( $\kappa = 0$ )	Cost channel operating ( $\kappa = 1$ )	
Model # 1			
Price-level targeting	0 196	0.195	
Nominal Income targeting	0.152	0.140	
Model # 2			
Price-level targeting	0.084	0.068	
Nominal Income targeting	0.095	0.079	
Model # 3			
Price-level targeting	0.192	0.197	
Nominal Income targeting	0.167	0.172	

Note to Table 7: Model 1 combines the baseline 'new Keynesian' IS relationship with the baseline 'new Keynesian' Phillips curve while model 2 and 3 combine the base line IS relationship with Fuhrer and Moore (1995) Phillips curve and Mankiw and Reis (2002) Phillips curve respectively.

# Table 8: Output Effects with a General Monetary Policy Rule ---'hybrid' IS relationship with the Three Specifications of the Phillips curve

Amplitude of ongoing cycle in real output			
	Cost channel closed ( $\kappa = 0$ )	Cost channel operating ( $\kappa = 1$ )	
Model # 4			
Price-level targeting	0.251	0.240	
Nominal Income targeting	0.085	0.077	
Model # 5			
Price-level targeting	0.241	0.247	
Nominal Income targeting	0.195	0.175	
Model # 6			
Price-level targeting	0.341	0.352	
Nominal Income targeting	0.186	0.195	

Note to Table 8: Model 4 combines the hybrid IS relationship with the baseline 'new Keynesian' Phillips curve while model 5 and 6 combine the hybrid IS relationship with Fuhrer and Moore (1995) Phillips curve and Mankiw and Reis (2002) Phillips curve respectively.

### 7- Concluding Remarks

The chapter studied the main developments in the macroeconomic theory regarding the specifications for the aggregate demand and aggregate supply side of the model in the context of alternative rule-based monetary policy regimes such as pricelevel targeting and nominal income targeting. In addition, it also incorporated the cost channel of monetary policy transmission to various models of the economy. Thus, the chapter assessed the robustness of policy recommendations for a closed economy by comparing results in a series of macroeconomic models. The results indicate that analysing both the traditional and the cost channel of monetary policy in one unified framework has been worthwhile. They confirm the results of earlier theoretical and empirical research on the potency of supply side effects of monetary policy (the cost channel) in effecting the real economy. Moreover, the chapter also finds strong support for a case in favour of nominal income targeting when compared with price-level targeting as it keeps the volatility of real output low. There is a growing literature that studies and compares the performance of these targeting regimes and a consensus has not been reached yet. Thus, the results of this chapter can be considered as an addition to this debate. An important point in this regard is that the specification of both the demand side and the supply side of the model are crucial while analysing various monetary policy targeting regimes.

However, I agree with McCallum when he concluded while comparing the performance of inflation targeting and nominal income targeting, "This demonstration

does not establish that nominal income targeting is preferable to inflation targeting or to other rules for monetary policy. To reach such a conclusion would require an extensive combination of theoretical and empirical analyses, conducted in a manner that gives due emphasis to the principle of robustness to model specification, plus attention to concerns involving policy transparency and communication with the public". (McCallum (1997, page 9). The point of this chapter was not to attempt any such ambitious undertaking. However, the results can be considered as a small step in that direction.

## **Appendix I**

The purpose of this appendix is to help the reader understand the derivations involved in specifying the various Phillips curve relationships in continuous-time.

# A- Derivation of the Baseline 'new Keynesian' Continuous-time Phillips Curve with the Cost Channel

As derived in chapter 2. the baseline 'new Keynesian' discrete-time Phillips curve can be written as:

$$\pi_{\tau} = E_{\tau} \pi_{\tau+1} + \left(\frac{(1-\tau)^2}{\tau}\right) m c_{\tau}$$
(AI-1)

Note that I have set the discount factor  $\beta$  equal to 1 for simplicity.  $(1 - \tau)$  represent the proportion of firms that can change prices at each point in time and  $mc_{\tau}$  is the real marginal cost.

Given the labour supply function  $-- w_i - p_i = n_i + c_i$ , the production function  $-- y_i = \theta n_i$ , the real marginal cost  $-- mc_i = w_i - p_i + r_i - y_i + n_i$  and the economy's resource constraint  $-- y_i = \alpha c_i + \beta a_i$ , I can write the Phillips curve (equation (AI-1) as (see chapter 3, page 112-113 for details):

$$\pi_t = E_t \pi_{+1} + \lambda(y_t - \overline{y}) - \psi(a_t - \overline{a}) + \kappa \gamma(r_t - \overline{r})$$
(AI-2)

where parameters  $\lambda, \psi$  and  $\gamma$  are function of deep parameters and are defined in the chapter. The parameter  $\kappa$  is introduced to capture the cost channel of monetary policy transmission. By setting  $\kappa = 0$ , I can close this channel.

To express equation (AI-2) in continuous-time, define  $E_i \pi_{i+1} - \pi_i = \pi$  and drop the time-subscript t.

$$\Rightarrow \quad \vec{\pi} = -\lambda(y - \overline{y}) + \psi(a - \overline{a}) - \kappa \gamma(r - \overline{r})$$

Since  $\pi_i = p_i - p_{i-1}$ , can define  $\pi = p$ .

$$\Rightarrow \qquad \ddot{p} = -\lambda(y - \bar{y}) + \psi(a - \bar{a}) - \kappa\gamma(r - \bar{r})$$
(AI-3)

Equation (AI-3) is equation (2) of the chapter

# B- Derivation of the Fuhrer-Moore's Continuous-time Phillips Curve with the Cost Channel

In the baseline 'new Keynesian' Phillips curve inflation is entirely forwardlooking, while in the traditional literature it is entirely backward-looking. The idea behind a hybrid theory of a Phillips curve, such as Fuhrer-Moore's Phillips curve, is that inflation is a weighted average of both forward-looking and backward-looking behaviour. Using equation (AI-2), a simple discrete-time hybrid version of the Phillips curve with the cost channel can be written as:

$$\pi_{t} = \omega E_{\tau} \pi_{t+1} + (1-\omega)\pi_{t-1} + \lambda(y_{\tau} - \overline{y}) - \psi(a_{\tau} - \overline{a}) + \kappa \gamma(r_{\tau} - \overline{r})$$
(AI-4)

where,  $\omega$  is the weight assigned to the forward-looking component of inflation. For simplicity, 1 set  $\omega = 0.5$ .

Subtracting  $\pi_i$  from both sides, equation (AI-4) can be written after some re-arranging as:

$$0.5(E_{i}\pi_{i+1} - \pi_{i}) - 0.5(\pi_{i} - \pi_{i-1}) = -\lambda(y_{i} - \overline{y}) + \psi(a_{i} - \overline{a}) - \kappa\gamma(r_{i} - \overline{r})$$

Define,  $E_{i}\pi_{i+1} - \pi_{i} = k_{i+1}$  and  $\pi_{i} - \pi_{i-1} = k_{i}$ 

$$\Rightarrow \quad 0.5(k_{i+1}-k_i) = -\lambda(y_i-\overline{y}) + \psi(a_i-\overline{a}) - \kappa\gamma(r_i-\overline{r})$$

or 
$$(k_{i+1} - k_i) = -2\lambda(y_i - \overline{y}) + 2\psi(a_i - \overline{a}) - 2\kappa\gamma(r_i - \overline{r})$$
 (AI-5)

To express equation (AI-5) in continuous-time, define  $k_{i+1} - k_{i} = k$  and drop the time-subscript t.

$$\dot{k} = -2\lambda(y - \bar{y}) + 2\psi(a - \bar{a}) - 2\kappa\gamma(r - \bar{r})$$

Since  $k_i = \pi_i - \pi_{i-1}$ , can define  $k = \dot{\pi}$ . Also,  $\pi_i = p_i - p_{i-1}$ , can define  $\pi = p$ .

$$\Rightarrow \qquad \ddot{p} = -2\lambda(v - \bar{v}) + 2\psi(a - \bar{a}) - 2\kappa\gamma(r - \bar{r})$$
(AI-6)

Equation (Al-6) is equation (2a) of the chapter.

# C- Derivation of the Mankiw-Reis's Continuous-time Phillips Curve with the Cost Channel

Mankiw and Reis (2002) suggested an alternative model of price dynamics in which every firm sets its price every period, but firms gather information and recompute optimal prices slowly over time. In each period, a fraction  $(1 - \tau)$  of firms obtain new information about the state of the economy and computes a new path of optimal prices. The remaining fraction  $(\tau)$  continue to set prices based on outdated information. Following Mankiw and Reis (2002), 1 assume that a firms that last updated its price k periods ago sets the price

$$p_{i}^{*} = E_{i-k}(mc_{i} + p_{i})$$
 (Al-7)

where,  $mc_t + p_t$  is the minimized nominal marginal cost.

The aggregate price-level is the weighted average of the prices of all firms in the economy:

$$p_{t} = (1 - \tau) \sum_{k=0}^{\infty} \tau^{k} p_{t}^{*}$$
(AI-8)

Combining equation (AI-7) and (AI-8) yield the following equation for the price level:

$$p_{t} = (1 - \tau) \sum_{k=0}^{\infty} \tau^{k} E_{t-k} (mc_{t} + p_{t})$$
(AI-9)

Writing out the terms in equation (AI-9), I get:

$$p_{t} = (1 - \tau) \Big[ E_{t} (mc_{t} + p_{t}) + \tau E_{t-1} (mc_{t} + p_{t}) + \tau^{2} E_{t-2} (mc_{t} + p_{t}) + \dots \Big]$$
$$p_{t} = (1 - \tau) (mc_{t} + p_{t}) + (1 - \tau) \Big[ \tau E_{t-1} (mc_{t} + p_{t}) + \tau^{2} E_{t-2} (mc_{t} + p_{t}) + \dots \Big]$$

or

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or 
$$p_t = (1 - \tau)(mc_t + p_t) + (1 - \tau) \sum_{k=0}^{\infty} \tau^{k+1} E_{t-1-k}(mc_t + p_t)$$
 (A1-10)

Using equation (AI-9), the previous period's price level can be written as:

$$p_{t-1} = (1 - \tau) \sum_{k=0}^{\infty} \tau^{k} E_{t-1-k} \left( m c_{t-1} + p_{t-1} \right)$$
(Al-11)

Subtracting equation (AI-11) from equation (AI-10) and rearranging yields the following equation for the inflation rate:

$$\pi_{t} = (1 - \tau)(mc_{t} + p_{t}) + (1 - \tau)\sum_{k=0}^{\infty} \tau^{k} E_{t-1-k} \{(mc_{t} - mc_{t-1}) + \pi_{t}\}$$
$$- (1 - \tau)^{2} \sum_{k=0}^{\infty} \tau^{k} E_{t-1-k} (mc_{t} + p_{t})$$
(AI-12)

Re-writing equation (AI-10), it can be shown that:

$$(1-\tau)\sum_{k=0}^{\infty}\tau^{k}E_{i-1-\kappa}(mc_{i}+p_{i}) = \frac{p_{i}-(1-\tau)(mc_{i}+p_{i})}{\tau}$$
(AI-13)

Use equation (AI-13) to eliminate the last term in equation (AI-12) to get:

$$\pi_{t} = (1 - \tau)(mc_{t} + p_{t}) + (1 - \tau)\sum_{k=0}^{\infty} \tau^{k} E_{t-1-k} \{(mc_{t} - mc_{t-1}) + \pi_{t}\}$$
$$-(1 - \tau) \left(\frac{p_{t} - (1 - \tau)(mc_{t} + p_{t})}{\tau}\right)$$
or
$$\pi_{t} = \left(\frac{1 - \tau}{\tau}\right) mc_{t} + (1 - \tau)\sum_{k=0}^{\infty} \tau^{k} E_{t-1-k} \{(mc_{t} - mc_{t-1}) + \pi_{t}\}$$
(AI-14)

But,  $mc_i = w_i - p_i + r_i - y_i + n_i$  which after using the labour supply function  $w_i - p_i = n_i + c_i$ , production function  $y_i = \theta n_i$ , and economy's resource constraint  $v_i = \alpha c_i + \beta a_i$  can be written as (see chapter 3, page 112-113 for more details):

$$mc_{i} = \left(\frac{1}{\alpha} + \frac{2}{\theta} - 1\right)(y_{i} - \overline{y}) - \left(\frac{\beta}{\alpha}\right)(a_{i} - \overline{a}) + \kappa(r_{i} - \overline{r})$$
(AI-15)

Use equation (AI-15) and its lagged version to substitute out real marginal cost terms from equation (AI-14):

$$\pi_{i} = \lambda'(y_{i} - \bar{y}) - \psi'(a_{i} - \bar{a}) + \kappa\gamma'(r_{i} - \bar{r}) + (1 - \tau)\sum_{k=0}^{\infty} \tau^{k} E_{i-k} \left\{ \eta'(y_{i} - y_{i+1}) - \psi'(a_{i} - a_{i-1}) + \kappa(r_{i} - r_{i-1}) + \pi_{i} \right\}$$
(AI-16)

where,

$$\eta' = \left(\frac{1}{\alpha} + \frac{2}{\theta} - 1\right), \upsilon' = \frac{\beta}{\alpha}, \quad \gamma' = \left(\frac{1 - \tau}{\tau}\right), \lambda' = \gamma' \eta, \psi' = \gamma' \upsilon$$

Equation (AI-16) is the sticky-information Phillips curve with the cost channel.

To express equation (AI-16) in continuous-time, define  $\pi = p$ ,  $y_i - v_{i-1} = \dot{y}^*$ ,  $a_i - a_{i-1} = \dot{a}^*$ , and  $r_i - r_{i-1} = r^*$ . Also, drop the time-subscript t.

$$\Rightarrow \dot{p} = \lambda'(y - y) - \psi'(a - a) + \kappa\gamma(r - r) + \dot{p}' + \eta\dot{y}' - v\dot{a}' + \kappa\dot{r}'$$
(AI-17)  
$$\ddot{p}^* = \phi(p - \dot{p}^*)$$
  
$$\ddot{v}^* = \phi(\dot{y} - \dot{y}^*)$$
  
$$\ddot{a}^* = \phi(\dot{a} - \dot{a}^*)$$
  
$$\ddot{r}^* = \phi(\dot{r} - \dot{r}^*)$$

The system of equations defined by (AI-17) is equation (2b) of the chapter.

## **Appendix II**

This appendix explains the derivation of the two IS-type relationships in continuous-time used in the chapter.

# A- Derivation of the Baseline 'new Keynesian' Continuous-time IS Relationship

Assuming that the elasticity of intertemporal substitution ' $\sigma$ ' is equal to one, the discrete-time Euler equation for consumption can be written as:

$$c_{i} = E_{i}c_{i+1} - (r_{i} - r)$$
 (AII-1)

Using the economy's resource constraint,  $y_i = \alpha c + \beta u_i$ , and writing equation (AII-1) one period forward, I get:

$$y_{t} = E_{t}y_{t+1} - \alpha(r_{t} - r) - \beta(E_{t}a_{t+1} - a_{t})$$
(AII-2)

Define,  $E_i y_{i+1} - y_i = \dot{y}$  and  $E_i a_{i+1} - a_i = \dot{a}$ . Also, drop the time subscript t.

$$\Rightarrow \quad \dot{y} = \alpha(r - \bar{r}) + \beta i \tag{AII-3}$$

Equation (AII-3) is equation (1) of the chapter.

#### **B-** Derivation of the Hybrid Continuous-time IS Relationship

Just as in the case of Fuhrer-Moore's Phillips curve, the idea behind a hybrid theory of an IS relationship is that consumption (and accordingly output) is a weighted average of both forward-looking and backward-looking behaviour. Using equation (AII-2), a simple discrete-time hybrid version of an IS relationship can be written as:

$$y_{t} = \omega E_{t} y_{t+1} + (1 - \omega) y_{t-1} - \alpha (r_{t} - r) - \beta (E_{t} a_{t} - a_{t})$$
(AII-4)

where,  $\omega$  is the weight assigned to the forward-looking component of output. For simplicity, I set  $\omega = 0.5$ .

Subtracting  $y_i$  from both sides, equation (AII-4) can be written after some re-arranging as:

-----

$$0.5(E_t y_{t+1} - y_t) - 0.5(y_t - y_{t-1}) = \alpha(r_t - r) + \beta(E_t a_{t+1} - a_t)$$

Define,  $E_{i} y_{i+1} - y_{i} = m_{i+1}$  and  $y_{i} - y_{i+1} = m_{i}$ 

$$\implies 0.5(m_{i+1} - m_i) = \alpha(r_i - r) + \beta(E_i a_{i+1} - a_i)$$

or 
$$(m_{t+1} - m_t) = 2\alpha(r_t - r) + 2\beta(E_t a_{t+1} - a_t)$$
 (AII-5)

To express equation (AI-5) in continuous-time, define  $m_{t+1} - m_t = \dot{m}$ ,  $E_t a_{t+1} - a_t = \dot{a}$  and drop the time-subscript t.

$$\dot{m}=2\alpha(r-\bar{r})+2\beta\dot{a}$$

Since  $m_i = y_i - y_{i-1}$ , can define  $m = \dot{y}$ 

$$\Rightarrow \qquad \ddot{v} = 2\alpha(r - r) + 2\beta \dot{a} \qquad (AII-6)$$

Equation (AII-6) is equation (1a) of the chapter.

## **Chapter 4**

## Is Price Flexibility De-Stabilizing? A Reconsideration

### **<u>1-Introduction</u>**

At least since Keynes (1936), macroeconomists have recognized that a one-time reduction in the price *level* involves higher aggregate demand, while an ongoing expected *rate* of deflation involves lower aggregate demand. Stressing the second of these two propositions, Keynes argued that an increased degree of price flexibility leads to increased output volatility. Perhaps one of the reasons that he favoured the Bretton Woods arrangements was that he expected (1936, p. 276) the destabilizing feature of increased price flexibility to be more powerful under flexible exchange rates.

Formal evaluation of Keynes' concern did not take place until 50 years after he wrote on the subject. Howitt (1986). Chadha (1988) and Fleming (1987) offered analytical explorations of this issue, based on the first generation of rational expectations models (that involved descriptive structural equations). DeLong and Summers (1986), Driskill and Sheffrin (1986), King (1988) and Myers and Scarth (1990) used numerical versions of similar models to evaluate Keynes' concern. Mixed conclusions emerged. For example, using a closed-economy analysis, DeLong and Summers (1986, p. 1031) concluded that "simulations based on realistic parameter values suggest that increases in price flexibility might well increase the cyclical variability of output in the United

States." In contrast, in a small open-economy setting, Myers and Scarth reported only very limited support for the proposition that an increased degree of price flexibility leads to increased output volatility. Further, they found more support for Keynes' concern if the exchange rate is fixed – a result that is opposite to what Keynes expected.

Since this spate of interest in evaluating the output volatility effects of changes in the degree of price flexibility, there has been a fundamental change in the focus of policyoriented macro theory. The "new neoclassical synthesis" has emerged as the standard framework, because this compact structure combines a number of very desirable features<sup>35</sup>. The purpose of this chapter is to re-examine the question of de-stabilizing price flexibility in the context of this new neoclassical framework

We find it surprising that this reconsideration has yet to be explored. For one thing, this question is a central one, and with a change in the core framework of mainstream macroeconomics, one would expect a sensitivity check on all previously derived aspects of conventional wisdom. But the lack of this checking is particularly surprising given that, recently, policy makers have taken positions on this very question. For example, central bankers have noted that average contract length has varied rather dramatically over the last 30 years -- falling as the core inflation rate rose, and rising again as the underlying inflation rate came back close to zero. It appears to be presumed that longer contract length (a lower degree of price flexibility) is one of benefits of low

<sup>&</sup>lt;sup>25</sup> See chapter 1 for a detailed discussion

equilibrium inflation. That is, it appears that central bankers agree with Keynes on this question. On the other hand, when answering the concern that their adoption of a common currency may involve a loss on macroeconomic built-in stability grounds, policy makers in Europe argue that this very fact may induce agents to embrace structural reforms in a more thorough-going fashion. One dimension of more flexible labour markets may be an increased degree of wage flexibility. <sup>36</sup> If so, these analysts appear to be arguing that such increased flexibility, if it were to develop, would help stabilize real output. In other words, they disagree with Keynes on this question. It seems that a reconsideration of this question – within the now-accepted framework for policy analysis – is of relevance for both these debates.

The remainder of the chapter is organized as follows. In the next section, a small open economy version of the new neoclassical synthesis model is outlined. We assume that the main reason for business cycles in this model is that there is an ongoing cycle in export demand from the rest of the world. In section 3, the implications of this ongoing variation in demand for real output volatility is derived, and the effect of a change in average contract length on the amplitude of this real output cycle is calculated. Section 4 provides a sensitivity test on the baseline model by developing a 'hybrid' version of the model. Finally, concluding remarks are offered in section 5.

<sup>&</sup>lt;sup>36</sup> I am not arguing that the common currency was adopted with a view to increasing the appeal of (or pressure for) more flexible wages. What I mean is that given the common currency has now been embraced, it makes sense to argue in favour of more flexible wages.

#### 2- A Continuous-time Open-Economy 'new Keynesian' Model

The model is defined by equations (1) through (5). These equations define (respectively) the "new" *IS* relationship (aggregate demand), the "new" Phillips curve (aggregate supply), interest parity, monetary policy (assuming flexible exchange rates), and the exogenous cycle in autonomous export demand. The definition of variables, and a more detailed description of the structure are given following the equations:

$$\dot{y} = \alpha (r - \bar{r}) + \Omega (\dot{f} + \dot{c} - \dot{p}) + \beta \dot{u}$$
<sup>(1)</sup>

$$\ddot{p} = -\lambda(y - \bar{y}) + \gamma((f - \bar{f}) + (c - \bar{e}) - (p - \bar{p})) + \psi(a - \bar{a})$$
<sup>(2)</sup>

$$r = \overline{r} + \dot{f} + \dot{e} - p \tag{3}$$

$$p + \mu v = 0 \tag{4}$$

$$a = \overline{a} + \delta \sin(t) \tag{5}$$

All variables except the real interest rate (r) and the time index (t) are the natural logarithms of the associated variable. Dots and bars above a variable denote (respectively) the time derivative, and the full-equilibrium value of that variable. All coefficients (the Greek letters) are positive. The variables are: a – autonomous export demand, e – nominal exchange rate (the domestic currency price of foreign exchange), f – the price of foreign goods, p – the price of domestically produced goods, r – the domestic real interest rate, and y – the level of real output.

In chapter 3, I discussed in detail the reasons for using the continuous-time specification. The main reason is that although discrete-time specifications are more common, following this practice can involve model properties being dramatically dependent on small changes in assumptions concerning information availability. A continuous-time specification precludes such unappealing problems from developing.

Equation (1) is the expectational *IS* relationship. We start with a log-linear approximation of the economy's resource constraint:  $y = \alpha c + \beta u + (1 - \alpha - \beta)x$ , where *c* is the log of domestic consumption expenditure, *a* is the log of the autonomous part of exports and *x* is the log of the part of exports that is sensitive to the real exchange rate. The Ramsey model is used to model forward-looking domestic households. If the instantaneous utility function involves separable terms, log consumption and the square of labour supply, the first-order conditions are  $\dot{c} = r - \bar{r}$ , and (ignoring constants) n = w - p - c, where  $\bar{r}$  is now also interpreted as the rate of time preference, and *n* and *w* denote the log of employment and the nominal wage. We follow McCallum and Nelson (2000b) and do not formally model the rest of the world; we simply assume  $x = \xi(f + e - p)$ . Equation (1) follows by taking the time derivative of the resource constraint, substituting in the domestic consumption and the export functions, and interpreting summary parameter  $\Omega$  as  $\xi(1 - \alpha - \beta)$  (See appendix II for details). The labour supply function is used below.

Equation (2) summarizes Calvo's (1983) model of sticky prices. Only proportion  $(1 - \tau)$  of firms can change prices at each point in time. Firms minimize the undiscounted present value of the squared deviations between the log of marginal cost (mc) and price (p). Many authors have shown that optimal behaviour at the individual firm level leads to  $\ddot{p} = -[(1 - \tau)^2 / \tau](mc - p)$  at the aggregate level. To represent this price-adjustment process in a format that resembles the traditional Phillips curve, we must follow King (2000) and replace real marginal cost with the output gap (and any other terms that emerge as relevant given that we assume firms use intermediate imports). We assume a Leontieff production relationship between intermediate imports and domestic value added.  $\phi$  is the unit requirement coefficient for intermediate imports. and  $\theta$  is labour's exponent in the Cobb-Douglas domestic value added process (so that Y = N'', or in log terms,  $y = \theta n$  and the marginal product of labour, MPL, equals  $\theta Y/N$ ). Since marginal cost is defined as  $MC = W / [MPL(1 - \phi(FE/P))]$ , we can (ignoring constants) approximate the log of real marginal cost by:

$$mc - p = w - p - y + n + (\phi/(1 - \phi))(f + e - p).$$

Equation (2) is derived in three more steps. We use the labour supply function, the production function and the resource constraint to eliminate (w - p), n and c by substitution; we define units so that, in full equilibrium, all prices are unity (so that  $\overline{mc} - \overline{p} = 0$ ); and we substitute out the deviation of real marginal cost from its full-

equilibrium value (See appendix I for details). The coefficients in (2) then have the following interpretations:

$$\lambda = (1-\tau)^2 ((2/\theta) + (1/\alpha) - 1) \tau, \qquad \gamma = (1-\tau)^2 ((\Omega/\alpha) - (\phi/(1-\phi))) \tau, \qquad \text{and}$$
$$\psi = (1-\tau)^2 \beta / \alpha \tau.$$

For a closed economy with no autonomous spending term,  $\beta = \Omega = \phi = 0$ , so only the output gap appears in the "new" Phillips curve. But in this open-economy setting, there are direct supply-side effects of both the real exchange rate and the exogenous variation in exports.

There is much discussion of the importance of exchange-rate "pass-through" in the literature that compares the efficacy of fixed and flexible exchange rates<sup>37</sup>. One advantage of specifying imports as intermediate products is that no independent assumption concerning exchange-rate pass-through needs to be made. Indeed, the Calvo nominal flexibility parameter,  $(1 - \tau)$ , also stands for the proportion of firms that pass changes in the exchange rate through to customers at each point in time.

The focus of this chapter is on alternative degrees of price flexibility. As just noted, increased price flexibility is modeled as a reduction in parameter  $\tau$ . As is evident in the previous paragraph, this development increases the size of the three "new" Phillips

<sup>&</sup>lt;sup>27</sup> see, for example, Devereux (2001) and Devereux and Engel (2002)

curve parameters ( $\lambda$ ,  $\gamma$  and  $\psi$ ) by the same proportion. We rely on this fact in the next section of the chapter. We also make reference to a representative calibration of the model below. For illustration, the following parameter values are considered. Domestic consumption is 60% of output ( $\alpha = 0.6$ ). Exports and imports are 40% of output and the real exchange rate elasticity of exports is unity ( $\beta = 0.2$ ,  $\xi = 1.0$ ,  $\Omega = 0.2$ ,  $\phi = 0.4$ ). The annual effect of a one percent change in the output gap on inflation is one third ( $\lambda = 0.33$ ). Labour's exponent in the Cobb-Douglas production function is two thirds ( $\theta = 0.67$ ). The other supply side parameters ( $\tau$ ,  $\gamma$  and  $\psi$ ) are then determined by the model. Two things are noteworthy with this calibration: the direct "cost-of-living" effect of domestic currency depreciation is inflationary ( $\gamma < 0$ ), and the coefficients on the real exchange rate and the autonomous spending terms in the new Phillips curve are very small ( $\psi = -\gamma = 0.03$  while  $\lambda = 0.33$ ).

Equation (3) defines interest arbitrage. With perfect foresight, the domestic nominal interest rate,  $r + \dot{p}$ , must exceed the foreign nominal interest rate,  $\bar{r} + \dot{f}$ , by the expected depreciation of the domestic currency,  $\dot{e}$ . Price stability exists in the rest of the world ( $f = \bar{f} = \dot{f} = 0$ ), so the domestic central bank can achieve domestic price stability in two ways. One option is to fix the exchange rate (imposed in the model by assuming  $e = \bar{e} = \dot{e} = 0$  and ignoring equation (4)). The second option is to peg a linear combination of the domestic price level and domestic real output (imposed in the model by assuming equation (4)). Equation (4) encompasses two interesting cases: targeting the price level ( $\mu = 0$ ), and targeting nominal *GDP* ( $\mu = 1$ ).

Given our focus on a Keynesian question, it is natural to assume that business cycles in this small open economy are caused by exogenous variations in export demand, as defined by the sine curve in equation (5). We now proceed to derive the reduced form for real output, to see how the amplitude of the resulting cycle in y is affected by changes in the degree of price flexibility, and to see how the answer to this question depends on monetary/exchange-rate policy.

#### 3- Analysis

We explain the derivation of the reduced form for real output in the flexible exchange rate case. The reader can use similar steps to verify the result that we simply report for fixed exchange rates.

First, we simplify by setting  $f = \tilde{f} = \tilde{f} = 0$ . Next, we take time derivatives of (4) and use the results to eliminate the first and second time derivatives of p. Then, we substitute (3) into (1) to eliminate the interest rate, and use the result to eliminate the term involving the time derivative of the exchange rate in the time derivative of equation (2). The result is:

$$-\mu \ddot{y} = A_1 y + A_2 a \tag{6}$$

where

$$\begin{split} A_{1} &= \gamma \mu - \lambda + (\gamma / (\alpha + \Omega))(1 - \mu (\alpha + \Omega)), \\ A_{2} &= \psi - (\gamma \beta / (\alpha + \Omega)). \end{split}$$

We use the undetermined coefficient solution procedure. Following Chiang (1984, p. 472) the solution for output can be written as

$$y = \overline{v} + B[\cos(t)] + C[\sin(t)]. \tag{7}$$

The time derivatives of (7),  $\dot{y} = -B\sin(t) + C\cos(t)$  and  $\ddot{y} = B\sin(t) - C\cos(t)$ , along with the time derivative of (5),  $a = \delta \cos(t)$ , are substituted into (6). The resulting coefficient-identifying restrictions are B = 0, and

$$C_{\mu\alpha} = \delta A_3 / (A_4 + \mu(\alpha + \Omega)), \tag{8}$$

where

$$\begin{split} A_3 &= \psi(\alpha + \Omega) - \gamma \beta \,, \\ A_4 &= \lambda(\alpha + \Omega) - \gamma . \end{split}$$

The similar expression for the fixed exchange rate version of the model is

$$C_{tr} = \delta(A_3 + \beta)/(A_4 + 1).$$
 (9)

Since reduced-form parameter C represents the amplitude of the cycle in real output, it is our summary measure of output volatility. Before investigating how both C expressions are affected by a change in the degree of price flexibility, we compare expressions (8) and (9). Subtracting the former from the latter, we have

$$C_{\mu\nu} - C_{\mu\nu} = \frac{\delta \left[ (\alpha + \Omega) \left\{ \psi \mu (\alpha + \Omega) + \beta \mu (1 - \gamma) + (\beta \lambda - \psi) \right\} \right]}{(A_4 + 1)(A_4 + \mu (\alpha + \Omega))}$$
(10)

The reader can verify that a sufficient, though not necessary, condition for  $A_4$  to be positive is that intermediate imports be less than half of *GDP*. Assuming this to be true, expression (10) is positive as long as  $(\beta \lambda - \psi) > 0$ . The illustrative parameter values cited in the previous section indicate that this condition is certainly satisfied. We conclude that – as long as the degree of price flexibility is not affected by exchange rate policy – the model supports a flexible exchange rate. In addition, it is worthwhile to note that under a flexible exchange rate regime nominal income targeting outperforms price level targeting. Table 1 reports the quantitative results when the degree of price stickiness ( $\tau$ ) is 0.75.

**Table 1: Baseline Model: Output Effects ---- Baseline Parameters** 

Amplitude of Ongoing Output cycles			
Flexible exchange rate			Fix exchange rate
	Price level targeting	Nominal income targeting	
Baseline			
Model	0.102	0.027	0.178

We now investigate the effect of changing price flexibility directly. We do so by re-interpreting the slope coefficients of the "new" Phillips curve as  $\lambda = \sigma \lambda^*$ ,  $\gamma = \sigma \gamma^*$ 

and  $\psi = \sigma \psi^*$ , where  $\sigma = (1 - \tau)^2 / \tau$ ,  $\lambda^* = (2/\theta) + (1/\alpha) - 1$ ,  $\gamma^* = (\Omega/\alpha) - (\phi/(1 - \phi))$ and  $\psi^* = \beta / \alpha$ . We differentiate both the *C* expressions with respect to  $\sigma$ . Since a higher  $\sigma$  corresponds to an increased degree of price flexibility, we interpret a finding of  $\partial C / \partial \sigma > 0$  as support for Keynes' concern that more price flexibility increases the amplitude of the ongoing cycle in real output.

For the case of flexible exchange rates (expression (8)),  $\partial C / \partial \sigma$  is given as:

$$\frac{\partial C}{\partial \sigma} = \frac{\delta \left[ \mu(\alpha + \Omega)(\psi^* - \beta \gamma^*) \right]}{\left[ \sigma \lambda^*(\alpha + \Omega) - \sigma \gamma^*) + \mu(\alpha + \Omega) \right]^2}$$
(11)

For the parameter values noted above, this expression must be positive, and the model unambiguously supports Keynes' concern; increased price flexibility must accentuate output volatility. Two comments on this result are warranted. First, it is appealing on intuitive grounds that there is more support for Keynes' concern in the new neoclassical synthesis model, compared to what emerged from the corresponding "old" small open economy analysis. This is because Keynes' concern is based on the presumption that the *de*-stabilizing effect of the expected inflation/deflation *rate* dominates the stabilizing effect of one-time changes in the price *level*. Since the "new" approach involves agents that are more forward-looking, it makes sense that the destabilizing effect matters more. The second point worth noting is that the analysis supports central banks that have taken pride in the fact that their low-inflation, flexible exchange rate policy has decreased the degree of nominal price flexibility. The analysis

says that both dimensions of this policy (the flexible exchange rate and the lower degree of price flexibility) help to lower output volatility.

With a fixed exchange rate (expression (9)),  $\partial C / \partial \sigma$  is given as:

$$\frac{\partial C}{\partial \sigma} = \frac{\delta \left[ (\alpha + \Omega)(\psi^* - \beta \lambda^*) \right]}{\left[ \sigma \lambda^* (\alpha + \Omega) - \sigma \gamma^* + 1 \right]^2}$$
(12)

This expression must be negative if  $(\beta\lambda' - \psi') > 0$ . This is the same condition that is important for determining a preference for one exchange rate regime or the other – if a direct effect of exchange rate policy on the degree of price flexibility is ignored. At first glance, the analysis appears not to support European countries that have opted for currency union, since the almost certainly satisfied  $(\beta\lambda - \psi) > 0$  condition suggests that a flexible exchange rate involves lower output volatility. But policy makers in these countries have argued that, without the stability offered by a flexible rate, there will be increased incentive for their citizens to accept structural reforms. If this prediction turns out to be true, and if a decreased degree of price rigidity is one of the outcomes of this general move toward flexibility, then this same condition,  $(\beta\lambda - \psi) > 0$ , is sufficient for the model to support this view. Thus, there is internal consistency within both views about exchange rate policy. European policy makers appear to believe that Keynes' concern is *not* applicable in a fixed exchange rate setting, and they are right: Others, such as the Canadian authorities, appear to believe that Keynes' conjecture *is* applicable in a

,

flexible exchange rate setting, and they are correct as well. As an illustration, Table 2 reports the results when the degree of price stickiness decreases from 0.75 to 0.50.

# Table 2: Baseline Model: Output Effects --- Parameters with Increased Price Flexibility

Amplitude of Ongoing Output cycles			
Flexible exchange rate			Fix exchange rate
	Price level targeting	Nominal income targeting	
Baseline			
Model	0.102	0.066	0.142

### 4- Sensitivity Test --- Endogenous Persistence

In this section we explore "hybrid" *IS* and Phillips curve relationships. Fuhrer (2000) and Amato and Laubach (2003) have pointed out that the standard Ramsey type Euler Equation for consumption (which gives rise to an IS-type relationship) fails to capture the dynamics of the aggregate output. Fuhrer (2000) allows for habit formation in preferences while maintaining the assumption of optimal consumption choice on the part of consumers. Amato and Laubach (2003), on the other hand, introduce the 'rule of thumb' behaviour on the part of a fraction of the household; the remaining fraction of the household is able to optimize their consumption in a usual fashion. Their modification to

the standard consumer problem is justified on the grounds that it is costly to reoptimize every period. Both these modifications, introducing habit persistence and incorporating ' rule of thumb behaviour', leads to a lagged output gap term with some positive weight in the IS equation.

Similarly, it has been pointed out by many researchers that 'the new Keynesian Phillips curve' based on Calvo's (1983) sticky price model generates inertia in the price level and not the inflation rate and that this is inconsistent with stylized facts on inflation dynamics. The empirical evidence (for example, Nelson (1998)) indicates that inflation responds sluggishly to economic shocks. The 'new Keynesian Phillips curve' implies that inflation is determined by the current output gap and current expectations of future inflation. Inflation is, therefore, very flexible and responds immediately to monetary policy shocks and hence does not accord with stylized facts. In order to capture the inflation persistence found in the data, it is common to augment the basic forward-looking inflation adjustment equation with the addition of lagged inflation. Fuhrer and Moore (1995) is one such example. Mankiw and Reis (2002) suggest an alternative approach, which departs from the assumption of sticky prices and replaces it with that of sticky information. Empirical research of Gali and Gertler (1999) and Fuhrer (1997) have generally found that when lagged inflation is added to the basic 'new Keynesian Phillips curve', its coefficient is statistically and economically significant.

It appears that it is a worthwhile exercise to pose the Keynesian question once again, in this more complete setting. The 'hybrid' version of the IS and Phillips curve relationship, with a weight of 0.5 on lagged output gap term and lagged inflation term, can be written as follows (See appendix I and II for detailed derivations). The rest of the model remains the same.

$$\ddot{y} = (1-\alpha)y + \alpha(r-\dot{r}) + \Omega(\dot{f} + \dot{e} - \dot{p}) - \Omega(f + e - p) + \beta \dot{a} - \beta a$$
(1a)

$$\ddot{p} = -2\lambda(y-\bar{y}) + 2\gamma((f-\bar{f}) + (e-\bar{e}) - (p-\bar{p})) + 2\gamma((a-\bar{a})$$
(2a)

A solution procedure identical to the one explained in section 3 of the paper is used to derive the coefficient identifying restrictions:

$$B_{flex} = \frac{2\delta A_5(\psi(\alpha + \Omega) - \gamma\beta)}{A_5^2 + A_6^2} + \frac{2\delta A_6(\psi\Omega - \gamma\beta)}{A_5^2 + A_6^2}$$
(13)

$$C_{dex} = \frac{2\delta(\psi\Omega - \gamma\beta)}{A_5} - \frac{A_6}{A_7}B$$
(14)

where,

$$A_{5} = 2(\gamma(2-\alpha) - \lambda\Omega) - \mu(\alpha + \Omega)$$
$$A_{0} = \mu\Omega - 2\lambda(\alpha + \Omega)$$

The similar expressions for the fixed exchange rate case are quite messy so we do not report them here. Instead, we just provide in table 3 the quantitative results of the calibrated version of the model. Also, note that unlike the previous analysis the reducedform parameter B is not equal to zero. This means that the parameter C does not independently represent the amplitude of the cycle in real output and thus cannot be treated as a summary measure of output volatility.

# Table 3: Model with Endogenous Persistence: Output Effects -- Baseline Parameters

Amplitude of Ongoing Output cycles			
Flexible exchange rate		Fix exchange rate	
	Price level targeting	Nominal income targeting	
Hybrid			
Model	0.113	0.061	0.389

Table 4 provides the results with increased price flexibility, i.e., when  $\tau = 0.5$ . While the hybrid version of the model supports the initial conclusions in the flexible exchange rate case, an interesting result emerges in the fiedx exchange rate case. The output volatility does not go down with increased price flexibility, as was the case in the base line model.

# Table 4: Model with Endogenous Persistence: Output Effects --

	А	mplitude of Ongoing Output c	ycles
	Flexible exchange rate		Fix exchange rate
	Price level targeting	Nominal income targeting	
Baseline			
Model	0.113	0.105	0.421

#### Parameters with Increased Price Flexibility

#### 5- Concluding Remarks

This chapter has used the new neoclassical synthesis model to reconsider Keynes' concern that an increased degree of price flexibility may increase output volatility. Earlier open-economy analyses of this question involved models with less complete micro-foundations and less forward-looking agents. Our results indicate that the reconsideration has been worthwhile. The earlier research found only limited support for Keynes' proposition overall, with somewhat higher support under fixed exchange rates. With the new analysis, which gives additional emphasis to forward-looking expectations, we find much stronger support for Keynes' proposition. Indeed, it *must* apply under flexible exchange rates – the very policy regime that Keynes highlighted when drawing attention to his concern.

We find two things reassuring. First, since we are uncomfortable with the proposition that real-world policy makers are completely irrational, we find it appealing that the new neoclassical synthesis model can identify the internal consistency that was noted in the previous section. According to the model, those who favour fixed exchange rates are correct when they expect that increased price flexibility will lower output volatility. Also according to the model, those who favour flexible exchange rates and a domestic monetary policy that focuses on price stability are correct when they argue that decreased price flexibility will lower output volatility. Second, since we are uncomfortable with the proposition that Keynes had bad intuition, we find it appealing that the macro framework that has more reliable underpinings supports his conjecture that the de-stabilizing effect of higher price flexibility is only likely to emerge under flexible exchange rates. As noted in the introduction, traditional rational-expectations macro policy analysis could provide neither of these reassurances.

While we find the results encouraging, we stress the value of further investigation. For one thing, it would be worth modeling the rest of the world. In this chapter, we have assumed a cycle in demand for the domestic economy's exports (presumed to reflect an unexplained business cycle in the rest of the world). But if that cycle were modeled, it is likely that there would be a corresponding cycle in both the foreign interest rate and the foreign price level. To proceed along this line, the domestic economy would have to involve an extension (such as overlapping generations) so the rate of time preference of individual domestic agents (a constant) could differ from the world interest rate (a variable following a cyclical time path). We hope that the present note stimulates others to join us in pursuing these further analyses.

## **Appendix I**

This appendix shows the derivations of the two open economy Phillips curves used in the chapter. In spirit, the approach is similar to the one used in deriving the Phillips curves for a closed economy.

# A- Derivation of the Baseline 'new Keynesian' Continuous-time Open Economy Phillips Curve

As before, the baseline 'new Keynesian' discrete-time Phillips curve can be written as:

$$\pi_{\tau} = E_{\tau}\pi_{\tau+\tau} + \left(\frac{(1-\tau)^2}{\tau}\right)mc_{\tau}$$
(AI-1)

Given the labour supply function ---  $w_t - p_t = n_t + c_t$ , the domestic value added process ---  $y_t = \theta n_t$ , the real marginal cost -- $mc_t = w_t - p_t + r_t - y_t + n_t + (\phi/(1 - \phi))(f_1 + e_t - p_t)$  and the economy's resource constraint ---  $y_t = \alpha c_t + \beta a_t + \Omega(f_t + e_t - p_t)$ . I can write the open economy Phillips curve (equation (AI-1) as (see chapter 4, page 145-146 for details):

$$\pi_i = E_i \pi_{i+1} + \lambda(y_i - \overline{y}) - \gamma \left( (f_i - \overline{f}) + (e_i - \overline{e}) - (p_i - \overline{p}) \right) - \psi(a_i - \overline{a})$$
(AI-2)

where parameters  $\lambda, \psi$  and  $\gamma$  are function of deep parameters and are defined in the chapter.

To express equation (AI-2) in continuous-time, define  $E_i \pi_{i+1} - \pi_i = \dot{\pi}$  and drop the time-subscript t.

$$\Rightarrow \quad \dot{\pi} = -\lambda(y-\overline{y}) + \gamma((f-\overline{f}) + (e-\overline{c}) - (p-\overline{p})) + \psi(a-\overline{a})$$

Since  $\pi_i = p_i - p_{i-1}$ , can define  $\pi = \dot{p}$ .

$$\Rightarrow \quad \ddot{p} = -\lambda(y - \overline{y}) + \gamma \left( (f - \overline{f}) + (e - \overline{e}) - (p - \overline{p}) \right) + \gamma (a - \overline{a}) \tag{AI-3}$$

Equation (AI-3) is equation (2) of the chapter.

# **B-** Derivation of the Hybrid Continuous-time Open Economy Phillips Curve

Just as in the closed economy case, a simple discrete-time hybrid version of the open economy Phillips curve can be written, using equation (AI-2), as:

 $\pi_t = \omega E_t \pi_{t+1} + (1 - \omega) \pi_{t-1} + \lambda (y_t - \overline{y}) - \gamma ((f_t - \overline{f}) + (e_t - \overline{e}) - (p_t - \overline{p})) - \psi (a_t - \overline{a})$  (AI-4) where,  $\omega$  is the weight assigned to the forward-looking component of inflation. For simplicity, I set  $\omega = 0.5$ . Subtracting  $\pi_t$  from both sides, equation (AI-4) can be written after some re-arranging as:

$$0.5(E_t\pi_{i+1} - \pi_i) - 0.5(\pi_t - \pi_{i-1}) = -\lambda(y_t - \overline{y}) + \gamma((f - \overline{f}) + (e_t - \overline{e}) - (p_t - \overline{p})) + \psi(a_t - \overline{a})$$

Define,  $E_{i}\pi_{i+1} - \pi_{i} = k_{i+1}$  and  $\pi_{i} - \pi_{i-1} = k_{i}$ 

$$\Rightarrow \quad 0.5(k_{t+1} - k_t) = -\lambda(y_t - \overline{y}) + \gamma((f_t - \overline{f}) + (e_t - \overline{e}) - (p_t - \overline{p})) + \psi(a_t - \overline{a})$$

or 
$$(k_{t+1} - k_t) = -2\lambda(y_t - y) + 2\gamma((f_t - f') + (e_t - c) - (p_t - p)) + \psi(a_t - a)$$
 (AI-5)

To express equation (AI-5) in continuous-time, define  $k_{t+1} - k_t = \dot{k}$  and drop the time-subscript t.

$$\dot{k} = -2\lambda(y-\overline{y}) + 2\gamma \left( (f-\overline{f}) + (e-\overline{e}) - (p-\overline{p}) \right) + 2\psi(a-\overline{a})$$

Since  $k_i = \pi_i - \pi_{i-1}$ , can define  $k = \dot{\pi}$ . Also,  $\pi_i = p_i - p_{i-1}$ , can define  $\pi = \dot{p}$ .

$$\Rightarrow \qquad \ddot{p} = -2\lambda(y-\overline{y}) + 2\gamma\left((f-\overline{f}) + (e-\overline{e}) - (p-\overline{p})\right) + 2\psi(a-\overline{a}) \qquad (AI-6)$$

Equation (AI-6) is equation (2a) of the chapter.

## **Appendix II**

This appendix explains the derivation of the two open economy IS-type relationships in continuous-time used in the chapter.

# A- Derivation of the Baseline 'new Keynesian' Continuous-time Open Economy IS Relationship

Similar to the closed economy case, assuming that the elasticity of intertemporal substitution  $\sigma$  is equal to one, the discrete-time Euler equation for consumption can be written as:

$$c_t = E_t c_{t+1} - (r_t - \bar{r})$$
 (All-1)

Using the economy's resource constraint,  $v_i = \alpha c_i + \beta a_i + \Omega(f_i + e_i - p_i)$ , and writing equation (AII-1) one period forward, I get:

$$y_{t} = E_{t} v_{t+1} - \alpha(r_{t} - \bar{r}) - \beta(E_{t} a_{t+1} - a_{t}) - \Omega(f_{t+1} - f_{t}) + (e_{t+1} - e_{t}) - (p_{t+1} - p_{t}))$$
(AII-2)

Define,  $E_{i}y_{i+1} - y_{i} = y$ ,  $E_{i+1}f_{i} - f_{i} = \dot{f}$ ,  $E_{i+1}e_{i+1} - e_{i} = \dot{e}$ ,  $E_{i+1}p_{i+1} - p_{i} = \dot{p}$  and

 $E_t a_{t+1} - a_t = \dot{a}$ , Also, drop the time subscript t.

$$\Rightarrow \qquad \dot{y} = \alpha(r - \dot{r}) + \Omega(\dot{f} + \dot{e} - \dot{p}) + \beta \dot{a} \qquad (AII-3)$$

Equation (AII-3) is equation (1) of the chapter.
# B- Derivation of the Hybrid Continuous-time Open Economy IS Relationship

The approach adopted here is slightly different from the one used in deriving the closed economy hybrid continuous-time IS relationship. The basic idea behind a hybrid theory of an IS relationship is still the same --- it combines the two theories of consumption behaviour. The first theory of consumption behaviour is the one used in deriving the baseline 'new Keynesian' consumption function given as equation (AII-1). The second theory simple assumes that consumption in period t is a function of previous period's output and can be written as  $c_i = y_{i-1}$ . Given the economy's resource constraint  $y_i = \alpha c_i + \beta a_i + \Omega(f_i + e_i - p_i)$ , these two consumption theories yields the following two IS relationships respectively.

$$y_{t}^{1} = E_{t}y_{t+1} - \alpha(r_{t} - r) - \beta(E_{t}a_{t+1} - a_{t}) - \Omega(f_{t+1} - f_{t}) + (e_{t+1} - e_{t}) - (p_{t+1} - p_{t}))$$
  
$$y_{t}^{2} = \alpha y_{t-1} + \Omega(f_{t} + e_{t} - p_{t}) + \beta a_{t}$$

The hybrid IS relationship is simply a weighted average of these two IS relationships:

$$y_{i} = \omega y_{i}^{4} + (1 - \omega) y_{i}^{2}$$
(AII-4)

where,  $\omega$  is the weight assigned to the baseline consumption theory. For simplicity, I set  $\omega = 0.5$ .

$$\Rightarrow \qquad y_{i} = 0.5 \Big[ E_{i} y_{i+1} - \alpha (r - \overline{r}) - \beta (E_{i} a_{i+1} - a_{i}) - \Omega (f_{i+1} - f_{i}) + (e_{i+1} - e_{i}) - (p_{i+1} - p_{i})) \Big] \\ + 0.5 \Big[ \alpha v_{i+1} + \Omega (f_{i} + e_{i} - p_{i}) + \beta a_{i} \Big]$$

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or 
$$0.5(E_{t}y_{t+1} - y_{t}) = 0.5(y_{t} - y_{t-1}) + 0.5y_{t-1} - 0.5\alpha v_{t-1} + 0.5\alpha (r_{t} - r) + 0.5\Omega(f_{t+1} - f_{t}) + (e_{t+1} - e_{t}) - (p_{t+1} - p_{t})) - 0.5\Omega(f_{t} + e_{t} - p_{t}) + 0.5\beta (E_{t}a_{t+1} - a_{t}) - 0.5\beta a_{t}$$
(AII-5)

Define,  $E_i y_{i+1} - y_i = m_{i+1}$  and  $y_i - v_{i+1} = m_i$ 

$$\Rightarrow \quad 0.5(m_{t+1} - m_t) = 0.5(1 - \alpha)y_{t-1} + 0.5\alpha(r_t - r) + 0.5\beta(E_t(a_{t+1} - a_t) - 0.5\beta a_t) + 0.5\Omega(f_{t+1} - f_t) + (e_{t+1} - e_t) - (p_{t+1} - p_t)) - 0.5\Omega(f_t + e_t - p_t)$$
  
or 
$$(m_{t+1} - m_t) = (1 - \alpha)y_{t-1} + \alpha(r_t - r) + \beta(E_t(a_{t+1} - a_t) - \beta a_t)$$

+ 
$$\Omega(f_{t+1} - f_t) + (c_{t+1} - e_t) - (p_{t+1} - p_t)) - \Omega(f_t + e_t - p_t)$$
 (AII-6)

To express equation (AII-6) in continuous-time, define,  $m_{t+1} - m_t = \dot{m}$ ,  $E_{t+1}f_t - f_t = f$ ,  $E_{t+1}e_{t+1} - e_t = \dot{e}$ ,  $E_{t+1}p_{t+1} - p_t = \dot{p}$  and  $E_ta_{t+1} - a_t = \dot{a}$ , Also, drop the time subscript t.

$$\Rightarrow \quad \dot{m} = (1-\alpha)y + \alpha(r-\bar{r}) + \beta \dot{a} - \beta a + \Omega (f+\dot{c}-p) - \Omega (f+c-p)$$

Since  $m_i = y_i - y_{i-1}$ , can define  $m = \dot{v}$ .

$$\Rightarrow \qquad \ddot{y} = (1 - \alpha)y + \alpha(r - r) + \beta \dot{a} - \beta a + \Omega (\dot{f} + \dot{c} - \dot{p}) - \Omega (f + e - p) \qquad (AII-7)$$

Equation (AII-7) is equation (1a) of the chapter.

### **Chapter 5**

# Monetary Policy Analysis in an Open Economy with Incomplete Asset Markets

#### **<u>1-Introduction</u>**

Traditionally, the effects of monetary policy on aggregate demand in an open economy setting have been largely studied within the framework developed by Mundell (1963), Fleming (1962) and Dornbusch (1976). Although the Mundell-Fleming-Dornbusch (MFD) framework has played a dominant role in shaping the literature on open economy macroeconomics (largely due to its empirical success and popularity among policy makers), it has certain important methodological drawbacks. It does not provide any microfoundations for the aggregate macroeconomic relationships and thus is unable to provide any well-defined welfare criteria by which to evaluate the effectiveness of alternative macroeconomic policies. The MFD-type models also disregard the role of the intertemporal budget constraints, which is central in the analysis of the current account and exchange rate dynamics. Moreover, they fail to provide an explicit account of how monetary policy affects firm's production and price-setting decisions.

A tremendous amount of research has been done in recent years to overcome these drawbacks by developing dynamic general equilibrium models with an explicit focus on the presence of nominal rigidities and monopolistically competitive firms. This approach essentially forms a new paradigm in international macroeconomic theory and has been labelled as the 'new open economy macroeconomics' (NOEM)<sup>38</sup>. Some notable examples that have particularly inspired and influenced this chapter are Clarida. Gali and Gertler (2001), Gali and Monacelli (2004). Walsh (1999), Svensson (2000), McCallum and Nelson (1999a, 2000) and Ghironi (2003). The contribution of the present paper to this literature is that it provides new insights to the questions pertaining to optimal monetary policy in an open economy by clarifying the role of the exchange rate and its dynamic relation with current accounts.

The debate over the role of the exchange rate in the formulation of monetary policy in a NOEM model is far from being settled.<sup>30</sup> Taylor (2001) provides a very basic discussion of this issue. He suggests that although the monetary policy rules, which respond to exchange rate movements produce only negligible improvements. the issue needs to be researched further. Moreover, what is the implication of dynamic relationship between exchange rate and current accounts for the formulation of monetary policy is very much an open question. In most recent papers on monetary policy, (e.g., Clarida, Gali and Gertler (2001)), the dynamics of current accounts do not matter due to the assumption of complete asset markets. However, as pointed out by Obstfeld and Rogoff (1995a), the assumption of complete asset markets is not realistic in a model with

<sup>&</sup>lt;sup>38</sup> Philip Lane (2001) is a good reference for a detailed survey. For monetary policy analysis in an open economy context, two useful websites are <u>www geocities.com/monetaryrules/mpoe.htm</u> and <u>http://www2.bc.edu/~ghironu/monesyll.html</u>

<sup>&</sup>lt;sup>39</sup> By a role, I mean responding to exchange rate movements and not targeting exchange rate.

imperfections and rigidities in goods market. The reason is that with nominal rigidities monetary policy will affect real variables including the current account.

To address these issues, I develop a dynamic general equilibrium monetary model of an open economy with monopolistic competition (and thus nominal rigidities) and incomplete asset markets. With incomplete asset markets the dynamics of the current account does matter for monetary policy because then, besides dealing with the distortions created by monopolistic competition, the central bank needs to address the inefficiencies caused by incomplete asset markets. Monetary policy is conducted in an endogenous fashion, that is, the short-term nominal interest rate is the instrument of policy rather than an exogenous stochastic process for some monetary aggregate.

The main result of the chapter is the breakdown of a widely accepted result reported by Clarida, Gali and Gertler (2001) in a similar model of an open economy. They report that under certain standard conditions, the optimal monetary policy design problem for the small open economy is isomorphic to the policy problem for the closed economy. More specifically, they argue that optimal monetary policy calls for adjusting the interest rate to completely offset demand shocks, which implies no trade-off between output volatility and inflation volatility. There is no additional need (or very little additional need (Batini, Harrison and Millard (2003)) for the interest rate to respond to exchange rate movements as long as the central bank implements a rule in which the nominal interest rate responds to expected future inflation. It is assumed that the effect of exchange rate movements is incorporated in its effect on expected future inflation.<sup>40</sup> However, I have shown in this chapter that optimal monetary policy<sup>41</sup> calls for a less than full offset of the output effects of an aggregate demand shock and thus central bank faces an inflation-output gap volatility trade-off. The basic insight is that when the exchange rate affects both aggregate demand and aggregate supply, adjusting the interest rate to stabilize output in the face of a demand shock will cause fluctuations in inflation. Thus, the optimal monetary policy calls for trading-off some output volatility for less inflation volatility. The exchange rate thus provides an additional piece of information to be considered when adjusting interest rates. Walsh (1999), Ball (1999b) and Svensson (2000) have also reached similar conclusions in their ad-hoc extensions of closed economy models. Moreover, due to the dynamic interaction between current accounts and the real exchange rate. the optimal monetary policy also entails a response to net foreign asset positions and the impact of the demand shock last well beyond the time interval in which prices are rigid.

#### **1.1- Structure of Asset Markets**

Since the structure of international asset markets play a crucial role in this chapter, this sub-section explains, at considerable length, the difference between

<sup>&</sup>lt;sup>40</sup> Leitemo and Soderstorm (2001) and Guender (2001) have argued that because of the uncertainty about how exchange rates are determined and about the true economic environment, central banks may respond to exchange rate movements inappropriately: tightening when they should be loosening and vice versa.

<sup>&</sup>lt;sup>41</sup> By optimal monetary policy I mean that, given the dynamic general equilibrium structure, the effects of all sources of sub-optimality (like nominal price rigidities and imperfect asset markets) are fully neutralized and the efficient flexible price/complete asset market equilibrium allocation is restored. (see, Gali and Monacelli (2004), Benigno, P. (2001)).

complete and incomplete asset markets and their implications for the current account dynamics.

In an open economy, economic agents can insulate themselves from countryspecific shocks by running current account deficits or surpluses. To what extent this international risk sharing would take place and how monetary policy would affect this behaviour depends on the structure of international asset markets. Moreover, it is a well known observation that the predictions of a dynamic general equilibrium model of a small open economy regarding the current account behaviour depend critically on the structure of the asset markets. Broadly speaking, there are two types of asset market structures that have been studied in the literature --- complete versus incomplete asset markets.

Complete asset markets mean that economic agents are able to trade as many state-contingent assets as there are future states of nature thus insuring themselves against any type of risk or shock that may hit the economy. This paradigm of complete contingent claims markets was first developed by Arrow (1964) and Debreu (1959). The implication of having access to a complete set of Arrow-Debreu securities is that there are no current account imbalances and no asset accumulation takes place.<sup>42</sup> The reason as explained by Obstfeld and Rogoff (1995b, 1996) is that with complete asset markets "individuals everywhere in the world equalize their marginal rates of substitution of

<sup>&</sup>lt;sup>42</sup> Note that this would not be true in a model with capital accumulation.

present for future state-contingent consumption on the same Arrow-Debreu prices, so for all countries 'i' and 'j', and all dates 't',  $\frac{u'(C'_{t+1})}{u'(C'_{t})} = \frac{u'(C'_{t+1})}{u'(C'_{t+1})}$  given similar rates of time preference. '<sup>43</sup> (Obstfeld and Rogoff, 1995b, page 1765). According to this optimality condition, complete risk pooling takes place among countries against any (expected and unexpected) future country specific shocks. So, on any future date or in any possible state, there will be no gains from intertemporal trade. That is, the current account will remain zero. Thus, the assumption of complete asset markets simplifies the analysis to a great extent by closing the current account channel as a dynamic propagation mechanism.

Incomplete asset markets, on the other hand, mean that domestic economic agents do not have access to a complete set of Arrow-Debreu securities. In its simplest form, the agents in a model with incomplete asset markets have access to only a one-period riskless (non-contingent) bond. Thus, all country specific shocks/risks cannot be pooled, so there is a possibility that current account imbalances may occur. For example, after a positive (negative) shock, the domestic agents smooth their consumption by lending to (borrowing from) the foreign country and thus run a current account surplus (deficit). In Obstfeld and Rogoff's (1995b, 1996) language, the above equation only holds in expectation:

$$E_t \frac{u'(C_{t+1})}{u'(C_t)} = E_t \frac{u'(C_{t+1})}{u'(C_{t+1})}$$
. In this case, unlike the previous one, differences between the

<sup>&</sup>lt;sup>43</sup> More generally, in monetary models this condition should be appropriately adjusted for the presence of exchange rate terms.

marginal rates of intertemporal substitution of the two countries can arise in the face of an unexpected shock.

It is important to note, however, that in all future periods after the shock, the consumption in the domestic country follows a random walk meaning that there is no well defined endogenously determined steady state. This is the well-known stationarity problem of open economy models with infinitely-lived economic agents. Essentially what happens is that under incomplete asset markets (thus imperfect risk sharing) and a given initial choice for the international distribution of asset holdings, shocks cause the distribution to change permanently by generating differences in the time path of consumption. Since there is a possibility of an infinite number of steady state equilibria, log-linearization of the model is also problematic and can be very inaccurate (Kim and Kose, 2003) because one would be approximating the dynamics of the model around a moving steady state.

The reason why most recent papers have developed frameworks that deemphasize the role of current account imbalances is to avoid such non-stationarity problems. For example, Corsetti and Pesenti (2001a, 2004), Obstfeld and Rogoff (2000) assume that the intratemporal elasticity of substitution between domestic and foreign goods is one and in the initial steady state net foreign assets are zero. The advantage of their approach is that they were able to solve the model without resorting to loglinearization. However, the problem is that the choice of initial zero steady state is arbitrary. Any other non-zero initial steady state will lead to non-stationarity (See, Ghironi (2000)).

Assuming complete asset markets is another alternative to avoid the nonstationarity problems. As explained above in detail, complete asset markets imply perfect risk sharing and thus current accounts do not respond to shocks. Important examples of recent work in this vein are Clarida, Gali and Gertler (2001) and Gali and Monacelli (2004). Although these models are tractable, the concern here, as pointed out by Obstfeld and Rogoff (1995a, 1996), is that the assumption (and thus the implication) of complete asset markets and the presence of imperfections and nominal rigidities in the goods market are just not compatible because in the presence of sticky prices, unexpected monetary shocks can have real effects that include the effect on current accounts.

There are some papers that do not close the current account channel and still try to address the (in)determinacy of the steady state and the non-stationarity issue. One approach is to assume an endogenous rate of time preference, more specifically, assuming that the rate of time preference is a function of consumption. This was originally proposed by Uzawa (1968) and recently adopted by Mendoza (1991) among others. Nuemeyer and Perri (2001a) and Benigno (2001) assume that the domestic agents face a positive cost when acquiring the foreign bonds. They call such costs "portfolio adjustment costs" and use the existence of intermediaries in the foreign asset markets as a rationale for such costs. Yet another approach to bring about stationarity is to adopt the overlapping generation framework rather than the more popular infinitely lived representative agent framework. Cardia (1991) and Smets and Wouters (2002) use Blanchard's (1985) model in which domestic agents face a non-zero probability of death at each point in time. Ghironi (2003), on the other hand, uses Weil's (1989) specification, where agents are born on different dates with no assets, to attain determinacy in his model.

In order to explore the current account channel and to attain stationarity in my open economy model, I have assumed that the interest rate faced by the domestic agents is equal to the world interest rate plus a risk premium that is increasing in the domestic country's foreign debt. This assumption ensures that the rate of consumption growth depends on asset holdings; so setting consumption to be constant pins down a steady state distribution of net foreign assets. Senhadji (1997) and Schmitt-Grohe and Uribe (2001) have used this debt-elastic risk premium approach to achieve stationarity. The choice among different stationarity-generating approaches is quite adhoc and difficult to distinguish quantitatively as shown by Schmitt-Grohe and Uribe (2003).

The rest of the chapter is organized as follows: Section 2 discusses in detail the basic structure of the model. Section 3 deals with finding the equilibrium and the derivation of the aggregate demand (the new IS equation) and the aggregate supply (new Phillips curve) relationship. Section 4 talks about the behaviour of the monetary authority. In section 5, I discuss the results and section 6 includes concluding remarks.

### 2 - The Model

There are three types of economic agents in the economy: households, firms, and the monetary authority. Given their preferences, households decide how much to consume (both domestically produced goods and imported goods) and how to allocate time between leisure and work. The firms have some monopoly power and take two decisions: how much to produce using the labour services of the households and how to set the price for their output. The monetary authority issues money and employs nominal interest rates as an instrument of monetary policy to achieve certain well-specified goals.

#### 2.1 - Households

The economy consists of a continuum of identical households. The model is described in terms of a representative household making decisions in the presence of uncertainties about the future. A typical household seeks to maximize the expected present discounted value of utility<sup>44</sup>

$$E_0 \sum_{i=0}^{r} \beta' U(C_i, N_i, \frac{M_i}{P_i})$$
(1)

where  $0 < \beta < 1$  and  $E_0$  denotes the expectation based on the information set available at time zero.  $C_i$  is a composite consumption index,  $N_i$  denotes labour supply, and  $\frac{M_i}{P_i}$ 

<sup>&</sup>lt;sup>44</sup> Although I have adopted a money-in-the-utility-function approach to introduce money, a cashin-advance version of the model produces similar results. Feenstra (1986) has discussed the equivalence between the two approaches. It should be noted, however, that this equivalence is obtained by redefining consumption as being inclusive of transaction costs and is therefore not independent of money holdings on the part of the consumer. This implies that leaving out a money demand equation from the model is technically incorrect. However, McCallum (2001) has shown that making transaction costs explicit does not affect the results.

represents real money balances, with  $M_{e}$  being nominal quantity of money, and  $P_{e}$  being the overall consumer price index (CPI).

The composite consumption index is a function of domestic goods and foreign goods, and is defined as

$$C_{t} = \left[ (1-a)^{\frac{1}{\eta}} C_{H_{x}} \frac{\eta-1}{\eta} + a^{\frac{1}{\eta}} C_{L_{x}} \frac{\eta-1}{\eta} \right]^{\frac{\eta}{\eta-1}}$$
(2)

where  $C_{H,t}$  denotes the index of domestic consumption goods and  $C_{F,t}$  the index of foreign consumption goods (imports). ' $\eta$ ' is a measure of the elasticity of substitution between domestic and foreign goods and is assumed to be positive, i.e.,  $\eta > 0$ . 'a' represents the share of foreign (imported) goods in the consumption index. I assume that consumption is differentiated at the individual goods level. Thus, the domestic and foreign goods consumption indices can be written as CES aggregators of the quantities consumed of each type of good.

$$C_{H,j} = \left(\int_{0}^{1} C_{H,j}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon-1}{\varepsilon}}$$
(3)

$$C_{I,i} = \left(\int_{0}^{1} C_{I,i}(j)^{\frac{i+1}{i}} dj\right)^{\frac{i}{i-1}}$$
(4)

where ' $\epsilon$ ' is the elasticity of substitution within each category and is assumed to be greater than one, i.e.,  $\epsilon > 1$ .

The demand functions for goods within each category can be determined by maximizing equation (3) and (4), individually, with respect to the total expenditure on the

respective category of good, given as 
$$Z_{H,t} = \int_{0}^{1} P_{H,t}(j) C_{H,t}(j) dj$$

and  $Z_{F,i} = \int_{0}^{1} P_{F,i}(j) C_{F,i}(j) dj$ .  $P_{H,i}(j)$  and  $P_{F,i}(j)$  are the prices of the consumption goods

 $C_{H_I}(j)$  and  $C_{F_I}(j)$  respectively. The demand functions that emerge from this maximization exercise, are given as:

$$C_{H,i}(j) = \left(\frac{P_{H,i}(j)}{P_{H,i}}\right)^{-\varepsilon} C_{H,i}$$
(5)

$$C_{FJ}(j) = \left(\frac{P_{FJ}(j)}{P_{FJ}}\right)^{-\varepsilon} C_{FJ}$$
(6)

where  $P_{H,t}$  and  $P_{F,t}$  are the price indices for domestic and foreign (imported) goods respectively that satisfy the expenditure equations that can be expressed as  $Z_{H,t} = P_{H,t}C_{H,t}$  and  $Z_{F,t} = P_{F,t}C_{F,t}$ . The expressions for  $P_{H,t}$  and  $P_{F,t}$  are given as:

$$P_{H,t} = \left(\int_{0}^{1} P_{H,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$$
(7)

$$P_{I,i} = \left(\int_{0}^{1} P_{I,i}(j)^{1-\varepsilon} dj\right)^{1-\varepsilon}$$
(8)

By maximizing equation (2) subject to the total expenditure on home and foreign goods, I can derive the demand functions for home and foreign consumption goods. The total expenditure equation can be written as  $Z_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$ . The optimality condition yields the following equations:

$$C_{H_{t}} = (1-a) \left(\frac{P_{H_{t}}}{P_{t}}\right)^{-\eta} C_{t}$$
(9)

$$C_{F_{T}} = a \left( \frac{P_{F_{T}}}{P_{r}} \right)^{-\eta} C_{r}$$
(10)

where  $P_{i}$  is the overall price index and is given as

$$P_{i} = \left[ (1-a)P_{H,i}^{-1-\eta} + aP_{\Gamma,i}^{-1-\kappa} \right]^{\frac{1}{1-\eta}}$$
(11)

In the rest of the world a representative household faces a problem identical to the domestic household's problem. It is assumed that a foreign individual's utility function is analogous to that of a domestic household. Thus, relationships similar to equation (10) and (11) can be written as:

$$C_{j_{i_{l_{l_{l}}}}}^{*} = (1 - d) \left( \frac{P_{j_{l_{l}}}^{*}}{P_{j_{l}}^{*}} \right)^{-\eta} C_{j_{l_{l}}}^{*}$$
(12)

$$C_{H,r}^{*} = d \left( \frac{P_{H,r}^{*}}{P_{r}^{*}} \right)^{-7} C_{r}^{*}$$
(13)

where 'd' is the share of foreign goods (domestic economy's exports) in the overall consumption of the foreign country.  $P_i^*$  is the overall price index in the foreign country and is given as:

$$P_{t}^{*} = \left[ dP_{H_{t}}^{*}^{-1-\eta} + (1-d)P_{F_{t}}^{*}^{-1-\eta} \right]_{t=\eta}^{1}$$
(14)

Moreover, it is also assumed that the law of one price holds for each good which implies after aggregation that  $P_t = c_r P_t^{*45}$ , where 'e<sub>t</sub>' is the nominal exchange rate (the price of foreign currency in terms of domestic currency). The real exchange rate can be defined as

$$q_{i} = \frac{e_{i}P^{*}}{P_{i}}$$
(15)

In the model, I have made a clear distinction between domestic price level  $P_{H,i}$ and the overall price level (CPI),  $P_i$ . Thus, it is natural to discuss the relationship between the terms of trade and the real exchange rate. The terms of trade is defined as the price of imported goods relative to the domestic goods:

$$s_{t} = \frac{P_{F,t}}{P_{H,t}} = \frac{e_{t}P_{F,t}}{P_{H,t}}$$
(16)

<sup>&</sup>lt;sup>45</sup> This is assumed for the sake of analytical simplicity. It implies that the law of one price holds and the real exchange rate is going to be constant. But, the empirical evidence supporting the law of one price is fairly weak. (Rogoff (1996)). A number of authors have introduced international market segmentation that allows firms to charge different prices for the same good in home and foreign market This practice is commonly referred to as pricing-to-market (PTM). See, Betts and Devereux (2000).

Assume that the foreign country is very large relative to the domestic economy One way to think of it is to consider the foreign country as the rest of the world. This assumption implies that the share of domestic goods consumed by the rest of the world is negligible; so  $P_{t,t}^* = P_t^*$ . Thus, we can write the relationship linking the terms of trade and the real exchange rate as:

$$q_{i} = s_{i} \frac{P_{H_{i}}}{P_{i}}$$
(17)

Moreover, using the log-linearized version of the definition of the terms of trade (equation 16) and the definition of CPI (equation 11), I can derive a simple relationship between domestic price level,  $P_{H,t}$  and  $P_t$  (see Gali and Monacelli (2004) for details):

$$p_{H,i} - p_i = -as_i \tag{18}$$

or in percentage terms:

$$\pi_t = \pi_H \, , + a\Delta s_t \tag{19}$$

Thus, the gap between domestic inflation and CPI inflation is proportional to the percentage change in the terms of trade. Note that the appreciation of the terms of trade will lower the domestic inflation relative to the overall CPI inflation.

In order to complete the specification of the household problem, I need to describe the representative agents' intertemporal budget constraint. In nominal terms this constraint can be written as:

$$W_{t}N_{t} + P_{t}T_{t} + P_{t}\Pi_{t} + M_{t-1} + B_{t-1}(1+i_{t-1}) + e_{t}B_{t-1}^{*}(1+i_{t-1}^{*}) = P_{t}C_{t} + M_{t} + B_{t} + e_{t}B_{t}^{*}$$
(20)

The left hand side of this equation represents the resources the consumer has at his disposal at the beginning of period t. These consist of wage earnings  $W_t N_t$ , obtained by supplying his labour services to the firm, transfers  $P_t T_t$  from the monetary authority, share of profits  $P_t \Pi_t$  from firms', amount of money  $M_{t-1}$  held, the amount of one-period domestic bonds  $B_{t-1}$  and the amount of foreign currency denominated bonds  $B_{t-1}^*$ purchased.  $i_{t-1}$ ' denote the nominal interest rate earned on domestic bonds between period t-1 and t. The right hand side corresponds to the uses of these resources. The household can use these to consume goods, acquire new money balances or purchase new bonds.

Let the functional form of the utility function be given by<sup>40</sup>

$$U(C_{i}, N_{i}, \frac{M_{i}}{P_{i}}) = \frac{C_{i}^{1-\sigma}}{1-\sigma} - \psi \frac{N_{i}^{1+\phi}}{1+\phi} + \frac{\chi}{1-\xi} \left(\frac{M_{i}}{P_{i}}\right)^{1-d}$$
(21)

where ' $\sigma$ ' represents the intertemporal elasticity of substitution, ' $\phi$ ' is the elasticity of substitution between consumption and leisure, that is, it measures the elasticity of labour supply and ' $\xi$ ' is the interest rate elasticity of money demand. All three parameters are

 $<sup>^{46}</sup>$  The functional form of the utility function is very similar to the one in Devereux and Lane (2000).

assumed to be positive. By maximizing (1) subject to (20) we can derive the optimality conditions for the household<sup>47</sup>

$$\beta E_{i} \left(\frac{C_{i+1}}{C_{i}}\right)^{-\sigma} \left(\frac{P_{i}}{P_{i+1}}\right) = (1+i_{i})^{-1}$$
(22)

$$\beta E_{i} \left(\frac{C_{i+1}}{C_{i}}\right)^{-\sigma} \left(\frac{P_{i}}{P_{i+1}}\right) \left(\frac{e_{i+1}}{e_{i}}\right) = (1+i_{i}^{*})^{-1}$$
(23)

$$\frac{W_i}{P_i} = \gamma C_i^{\sigma} N_i^{\phi}$$
(24)

$$\frac{M_{i}}{P_{i}} = \left(\frac{1}{\chi}C_{i}^{-\sigma}\frac{i_{i}}{1+i_{i}}\right)^{\frac{1}{2}}$$
(25)

Equation (22) is the standard Euler equation for the holding of domestic bond. It has the usual interpretation that at a utility maximum, the household cannot gain from feasible shifts of consumption between periods. Similarly, equation (23) is the efficiency condition for the holding of foreign bonds. Equation (24) represents the labour supply decision and equation (25) gives the money demand function.

By combining equations (22) and (23), I can derive the familiar uncovered interest parity condition depicting the optimal portfolio choices of the economic agent:

$$E_{i}\left(\frac{C_{i+1}}{C_{i}}\right)^{-\sigma}\left(\frac{P_{i}}{P_{i+1}}\right)(1+i_{i}) = E_{i}\left(\frac{C_{i+1}}{C_{i}}\right)^{-\sigma}\left(\frac{P_{i}}{P_{i+1}}\right)\left(\frac{e_{i+1}}{e_{i}}\right)(1+i_{i}^{*})$$
(26)

 $<sup>4^{\</sup>circ}$  In addition to the optimality conditions (22) – (25) and the consumer's budget constraint (20) the equilibrium also requires the satisfaction of the standard transversality conditions

After some approximations the above relationship can be written as:

$$(1+i_{t}) = (1+i_{t}^{*})E_{t}\frac{e_{t+1}}{e_{t}}$$
(27)

Using the definition of real exchange rate, equation (15) and equation (16), the interest parity condition can be written as:

$$(1+i_{r}) = (1+i_{r}^{*})E_{r}\frac{s_{r+1}}{s_{r}}\frac{P_{H,r+1}}{P_{H,r}}\frac{P_{r}^{*}}{P_{r+1}^{*}}$$
(28)

The approximation mentioned above ignores the non-linear terms involving marginal utilities and prices. As noted by Bergin (2004), these omitted terms represent risk premium and Jensen's inequality. It is also a well-known fact that the form of the uncovered interest parity condition described in equation (27) is strongly rejected by the data. Moreover, for reasons explained earlier in the context of stationarity problem in open economy models with incomplete markets, it is justifiable to generalize the interest parity expression by adding a risk premium term.<sup>48</sup> Following Schmidt-Grohe and Uribe (2001a), the risk premium is assumed to be a decreasing function of the economy's stock of net foreign assets and is given by:

$$RP(nfa_t) = \kappa(e^{\frac{n}{n}a - nta_t} - 1)$$
(29)

where,  $\kappa$  is some constant and  $\overline{nfa}$  is the steady state level of real net foreign assets. This equation combined with the steady sate version of (28) and the steady state

<sup>&</sup>lt;sup>48</sup> In the traditional small open economy, the interest rate is exogenous. So, tacking on an arbitrary term is not problematic. But, in my 'semi-small' open economy model, adding an arbitrary term to an endogenously derived equation can be problematic. However, the foreign interest rate can be interpreted as exogenous (using equation 55 (below)) with foreign output modelled as an exogenous process.

relationships  $\beta(1+i) = 1$  and  $\beta(1+i^*) = 1$  (from equations 22 and 23) pins down the steady state value of net foreign assets.

Analogous to the domestic household's optimization problem. I can derive the similar optimality conditions for the representative household in the foreign country. For example, the counterpart of equation (22) can be written as:

$$\beta E_{t} \left( \frac{C_{t+1}^{*}}{C_{t}^{*}} \right)^{-\sigma} \left( \frac{P_{t}^{*}}{P_{t+1}^{*}} \right) = (1 + i_{t}^{*})^{-1}$$
(30)

Combining equation (30) with (23) and using the definition of real exchange rate and terms of trade yields the following relationship:

$$E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = E_t \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \left(\frac{P_t}{P_{t-1}}\right) \left(\frac{q_t}{q_{t+1}}\right)$$
(31)

This equilibrium condition reflects how the representative households in each country share consumption risk.

#### 2.2 - Net Foreign Assets and Current Account

Assuming no spending by the government implies that for the government budget constraint to hold, all the seignorage revenue associated with money creation must be returned to the households in the form of lump-sum transfers in each period<sup>49</sup>:

$$M_{t} - M_{t-1} = P_{t}T_{t}$$
(32)

<sup>&</sup>lt;sup>49</sup> I have assumed that in equilibrium the net supply of domestic bonds is zero

Also, in equilibrium, we have:

$$W_i N_i + P_i \Pi_i = P_i Y_i \tag{33}$$

Combining these two relationships, I can write the consumer budget constraint as:

$$B_{t-1}(1+i_{t-1}) + e_t B_{t-1}^{*}(1+i_{t-1}^{*}) + P_t Y_t = P_t C_t + B_t + e_t B_t^{*}$$
(34)

Define the net foreign assets as:

$$NFA_t = B_t + e_t B_t^* \tag{35}$$

or in real terms as:

$$nfa_{i} = \frac{B_{i} + e_{i}B_{i}^{*}}{P_{i}}$$
(36)

Thus, equation (34) can be written as:

$$nfa_{i} = (1 + r_{i-1})nfa_{i-1} + Y_{i} - C_{i}$$
(37)

where  $r_{t-1}$  is the real interest rate and  $Y_t - C_t = NX_t$ , where  $NX_t$  is net exports. This equation represents the dynamics of domestic economy's current accounts.

#### 2.3- Firms

This section outlines the mechanics of monopolistic competition in a dynamic general equilibrium setting. To make the presentation clear, I define two types of firms. One type of firms operates in a monopolistically competitive environment and are called

intermediate-good-producing firms, and the other operates in competitive markets and are called final-good-producing firms.

Like every firm operating in a monopolistically competitive market, each intermediate-good-producing firm takes two types of decisions --- how much output to produce and at what prices to sell this output that would maximize profits. In doing so, a representative firm 'j' is subject to a number of constraints. First is the specification of the production function. Following McCallum and Nelson (1999), I assume that there is no capital in the economy and so the firm only employs the labour input supplied by households to produce the differentiated good:

$$Y_{i}(j) = A_{i}N_{i}(j) \tag{38}$$

where  $Y_i(j)$  is the intermediate-good produced by firm 'j' and is used by the final-good producer.  $A_i = \exp(z_i)$  and 'z,' represents aggregate technology shock given as:

$$z_t = \rho_z z_{t-1} + \varepsilon_t^2$$

The representative firm j then supplies this good to the final good-producers. If the output of the final good, which is produced by using the inputs supplied by a continuum of intermediate-goods-producing firms indexed by  $j \in [0,1]$ , is denoted by  $Y_i$ , then the production function for the final output can be written as:

$$Y_{t} = \left[\int_{0}^{1} Y_{t}(j)^{\frac{k-1}{k}} dj\right]^{\frac{k}{k-1}}$$
(39)

Profit maximization (max  $P_{H_i}Y_i - \int_0^1 P_{H_i}(j)Y_i(j)dj$ ) by final-goods-producers

yields the following input demand function:

$$Y_{i}(j) = \left[\frac{P_{H,i}(j)}{P_{H,i}}\right]^{-z} Y_{i}$$
(40)

This input demand function describes the second constraint faced by intermediategoods-producing firm j'. Note that these goods are consumed both domestically and abroad. The demand function for the domestically produced differentiated good is given by equation (5). A similar function (the export-demand function) exists for the demand of these goods in the foreign country.

The third constraint introduces price stickiness by assuming that each period some firms are unable to adjust their price. This staggered price adjustment behaviour is based on Calvo (1983). Firms are assumed to face a constant probability  $1-\rho$  in every period to alter their price in an optimal fashion. This probability is independent of how long their prices have been fixed and the expected duration of price stickiness is  $1/\rho$ . It is easy to verify that with a large number of firms in the economy, the fraction of firms adjusting price optimally in a period is equal to the probability of price adjustment  $1-\rho$ ; the remaining fraction of firms ' $\rho$ ' do not adjust their price. Thus, the parameter ' $\rho$ ' captures the degree of nominal price rigidity. To facilitate the tractability of the model, I assume initially that all firms are able to adjust their prices every period, that is, the third constraint is not binding yet.<sup>50</sup> Then, the profit function for a representative firm 'j' can be written as:

$$\pi_{B_{i}}(j) = P_{B_{i}}(j)Y_{i}(j) - W_{i}N_{i}(j)$$
(41)

The differentiated-good-producing firm chooses  $P_{H_{t}}(j)$  and  $N_{t}(j)$  to maximize these profits subject to the conditional demand for their variety of output given by equation (40) and the production function given by equation (38). The expressions for  $P_{H_{t}}(j)$  and  $N_{t}(j)$  are given respectively as:

$$P_{H,\varepsilon}(j) = \frac{\varepsilon}{\varepsilon - 1} MC_{\varepsilon}$$
(42)

$$\frac{W_{\ell}}{P_{H_{\ell}}(I)} = \frac{\varepsilon - 1}{\varepsilon} F_{\chi}$$
(43)

where  $\left(\frac{\varepsilon}{\varepsilon-1}\right) = \upsilon$  is the constant mark-up and  $MC_{\iota}$  is the minimized nominal marginal

cost.  $F_{\rm v}$  is the marginal product of labour, which, given the production function is simply  $A_{\rm v}$ .

Equation (42) just depicts the relationship between the 'flexible' price chosen by all firms and the minimized marginal cost of production under monopolistic competition;

and

<sup>&</sup>lt;sup>50</sup> Put differently, I assume that firm's price-setting decisions are completely independent from their factor demand decisions. One way to interpret this separation of decisions is to think of firms as having two departments, one that decides what price to set each period, and the other that decides how much output to produce taking the prices of the inputs as given.

it does not say anything about prices being sticky. Combining equation (42) and (43) I can write:

$$MC_{i} = \frac{W_{i}}{A_{i}}$$
(44)

or in real terms as:

$$mc_{t} = \frac{W_{t}}{P_{H,t}A_{t}}$$
(44a)

Now, I introduce price stickiness by assuming that price adjustment does not take place simultaneously for all firms. Following Rotemberg (1987), suppose that a representative firm 'j' that is allowed to change its price, set its price to minimize the expected present discounted value of deviations between the price it sets and the minimized nominal marginal cost.

$$\sum_{k=0}^{\infty} \rho^{k} \beta^{k} E_{t} (P_{H,t}(j) - MC_{t+k})^{2}$$
(45)

where  $MC_t$  is the minimized nominal marginal cost. Note that there are two parts to discounting. The first,  $\beta$  represents a conventional discount factor, and the second,  $\rho$  reflects the fact that the firm that has not adjusted its price after 'k' periods, still has the same price in period t + k that she set in period t. The first order condition with respect

to  $P_{H_{i}}(j)$  gives the following optimal value denoted by  $D_{H_{i}}(j)$ .<sup>51</sup>

$$D_{h_{t,t}} = (1 - \rho\beta)MC_t + \rho\beta E_t D_{H_{t+1}}$$
(46)

Thus, the optimally chosen price in period t is a weighted average of nominal marginal cost and expected value of optimal price in the future. However, in period t, only a fraction  $1 - \rho$  of firms set their price according to equation (46). The remaining firms are stuck with the prices set in previous periods. Since the fraction of firms that are able to optimally set their price is randomly chosen, the average price of the previous period will be the price of the fraction of firms that are unable to adjust their price this period. Therefore, the overall aggregate price level in period t is a weighted average of current optimally chosen and past prices. This can be written as:

$$P_{H,t} = \left[\rho^{1-\epsilon} P_{H,t-1} + (1-\rho)^{1-\epsilon} D_{H,t}\right]^{\frac{1}{1-\epsilon}}$$
(47)

where  $D_{H_{c}}$  is the price chosen by all adjusting domestic firms in period t.

## <u>3- Log-linearized Model<sup>52</sup></u>

In this section, the model is log-linearized around the steady state. A variable in lower case represent the log deviation with respect to the steady state. In equilibrium

<sup>&</sup>lt;sup>51</sup> It is reasonable to set  $D_{H,i}(i) = D_{H,i}$  because all firms are identical except for the timing of their price adjustment

 $<sup>^{\</sup>rm C2}$  In getting most of the relationships in this section, I have closely followed Clarida, Gali and Gretler (2001)

firms are assumed to be symmetric and taking identical decisions. This implies that prices are equal for each variety of good and is equal to the price indices given by equation (7) and (8). That is,  $P_{H,i}(j) = P_{H,i}$ ,  $P_{F_i}(j) = P_{F_i}$ .

#### 3.1 - Deriving the new IS-curve

The resource constraint for this small economy can be written as:

$$Y_t = C_{H,t} + C_{H,t}^*$$
 (48)

This constraint simply says that the economy's output is either consumed domestically or exported to the rest of the world.

Log-linearizing around the steady state. I can write equation (48) as:

$$y_{i} = (1 - d)c_{H_{i}} + dc_{H_{i}}^{*}$$
(49)

where, 'd' is the share of the exports. Similarly, the log-linearized version of equation (2) can be written as:

$$c_{i} = (1 - a)c_{H,i} - ac_{F,i}$$
(50)

Now, consider equation (9) and (10) --- the demand curves for the domestic and foreign goods. Combining the log-linearized version of the two equations and using (18) I can derive a relationship that relates the two demand curves:

$$c_{H,i} - c_{F,i} = \eta s_i \tag{51}$$

Relationships similar to (9) and (10) hold for the foreign country. Thus, an expression analogous to equation (51) is:

$$c_{H_t}^* - y_t^* = \eta s_t \tag{52}$$

Combining equations (49) - (52), I can write:

$$y_{t} = (1-d)c_{t} + dy_{t}^{2} + \eta(a-ad+d)s_{t}$$
(53)

Thus, domestic output is a weighted average of domestic and foreign expenditures, plus an 'expenditure-switching factor' which is proportional to the terms of trade.

In order to derive an IS-type relationship that relates output of the small open economy to the real interest rate. I need to make use of the Euler equations for domestic consumption, foreign consumption and the uncovered interest parity condition. The loglinearized versions of these relationships are:

$$c_{i} = E_{i}c_{i+1} - \frac{1}{\sigma} \left[ i_{i} - (E_{i}\pi_{H_{i}+1} + aE_{i}\Delta s_{i+1}) \right] + u_{i}$$
(54)

$$y_{i}^{*} = E_{i} y_{i+1}^{*} - \frac{1}{\sigma} \Big[ i_{i}^{*} - E_{i} \pi_{i+1}^{*} \Big]$$
(55)

$$E_{i}\Delta s_{i+1} + i_{i}^{*} - E_{i}\pi_{i+1}^{*} - \kappa\eta fa_{i} = i_{i} - E_{i}\pi_{H_{i}+1}$$
(56)

Note that in deriving equation (54), I have included an additive disturbance term that represents uncertainty. (See, McCallum and Nelson (1999b)). It can also be justified

on the grounds that some non-linear terms are ignored while linearizing the Euler equation for consumption.

Thus, after substituting equations (54) - (56) in equation (53), we get a relationship that resembles an IS equation:

$$y_{i} = E_{i} y_{i+1} - \left(\frac{1+w}{\sigma}\right) (i_{i} - E_{i} \pi_{H_{i+1}}) + \left(\frac{w}{\sigma}\right) (i_{i}^{*} - E_{i} \pi_{i+1}^{*}) - \kappa \left(\frac{w+d}{\sigma}\right) n f a_{i} + u_{i}$$
(57)

where,

$$w = ad - (a + d) + \sigma\eta(a + d - ad)$$

Following Clarida, Gali and Gertler (2001), let  $x_t = y_t - y_t^0$  be defined as the output gap, where  $y_t^0$  is the level of output that arises with perfectly flexible prices. Similarly, let  $r_t^0$  and  $r_t^*$  be the real interest rates for the domestic and foreign economy respectively that arise in the frictionless equilibrium. Also,  $nfa_t^0$  (which equals zero) corresponds to the net asset holdings in the complete asset market case. Then, I can write equation (57) as:

$$x_{i} = E_{i} x_{i+1} - \left(\frac{1+w}{\sigma}\right) (i_{i} - E_{i} \pi_{Hi+1} - r_{i}^{0}) - \kappa \left(\frac{w+d}{\sigma}\right) n f a_{i} + u_{i}$$
(58)

where,

$$r_{i}^{0} = \left(\frac{\sigma}{1+w}\right) E_{i} \left(y_{i+1}^{0} - y_{i}^{0}\right) + \left(\frac{w}{1+w}\right) r_{i}^{*}$$
(59)

Since  $y_t^0$  is defined as the level of output that arises with perfectly flexible prices, it can be calculated by setting  $mc_t = 0$ .

#### 3.2 - Deriving the new Phillips curve

The log-linearized version of equation (45) and equation (47) can be combined to produces the following Phillips curve type relationship.

$$\pi_{H_{i}} = \beta E_{i} \pi_{H_{i+1}} + \theta m c_{i} \tag{60}$$

where  $\theta = \frac{(1-\rho)(1-\rho\beta)}{\rho}$ 

The expression for  $mc_r$  (real marginal cost) can be had by log-linearizing equation (44).

$$mc_{1} = w_{1} - p_{H,1} - z_{1}$$
 (61)

Using the log-linearized version of the labour supply equation (equation 24) and the relationship,  $p_{H_i} - p_i = -as_i$  from equation (18) I can rewrite the equation for  $mc_i$ as:

$$mc_{t} = \sigma c_{t} + \phi n_{t} + as_{t} - z_{t}$$
(62)

After substituting for  $c_i$  using equation (53) and  $n_i$  using the production function, 1 get the following relationship for  $mc_i$ :

$$mc_{t} = \left(\frac{\sigma}{1-d} + \phi\right) v_{t} - \left(\frac{d\sigma}{1-d}\right) v_{t}^{*} - (1+\phi)z_{t} - \left(\frac{w+d}{1-d}\right) s_{t}$$
(63)

Note that in the above equation an appreciation (depreciation) of the terms of trade would increase (decrease) the marginal cost. It seems a little puzzling and counterintuitive. But, rather than substituting out  $c_i$  from equation (62), if we eliminate  $y_i$  using the same equation (53), I get:

$$mc_{t} = (\sigma + \phi(1 - d))c_{t} + (a + \phi(a\eta(1 - d) + \eta d)s_{t} - (1 + \phi)z_{t} + \phi dy_{t}^{*}$$
(63a)

Since my purpose is to derive an expression that is analogous to the new Phillips curve (that involves  $v_i$  rather than  $c_i$ ) I continue to use (63). However, this does raise an issue, which should be ironed out. Some other papers (for example Walsh (1999)), in trying to get the new Phillips curve from similar steps simply assume real marginal cost to be positively proportional to the output gap term. King (2000) calls it a "heroic" step.

Setting,  $mc_t = 0$  I get the expression for  $y_t^0$ :

$$y_{t}^{0} = \frac{(1+\phi)z_{t} + \frac{d\sigma}{1-d}y_{t}^{*} + \left(\frac{w+d}{1-d}\right)s_{t}^{0}}{\left(\frac{\sigma}{1-d} + \phi\right)}$$
(64)

where  $s_t^0$  is defined as the value of the terms of trade that arise with balanced trade.

Subtracting equation (63) from (64). I can write:

$$mc_{t} = \left(\frac{\sigma}{1-d} + \phi\right) x_{t} - \theta \left(\frac{w+d}{1-d}\right) (s_{t} - s_{t}^{0})$$
(65)

Thus, equation (55) can be written as:

$$\pi_{H,i} = \beta E_i \pi_{H,i+1} + \theta \left( \frac{\sigma}{1-d} + \phi \right) x_i - \theta \left( \frac{w+d}{1-d} \right) (s_i - s_i^0)$$
(66)

#### 3.3 - Net Foreign Assets and Current Account Dynamics

In order to find  $s_1^0$ , the value of the terms of trade that arise with balanced trade, I need to find an expression for the net exports. Log-linearizing and combining equation (10) and (13) gives this expression:

$$mx_{i} = (2\eta - \eta(a+d) - 1)s_{i} - (c_{i} - c_{i}^{*})$$
(67)

Eliminating  $c_i$  using equation (53) and  $c_i^*$  using  $y_i^* = c_i^*$ . I can write:

$$nx_{i} = \frac{\left((2\eta - 1)(1 - d) - ad\eta + d\eta(a + d)\right)}{1 - d}s_{i} - \frac{1}{1 - d}(y_{i} - y_{i}^{*})$$
(68)

or

$$nx_{t} = \left(\frac{(2\eta - 1)(1 - d) - ad\eta + d\eta(a + d)}{1 - d}\right)(s_{t} - s_{t}^{0}) - \frac{1}{1 - d}x_{t}$$
(69)

where,

$$s_{i}^{0} = \frac{1}{(2\eta - 1)(1 - d) - ad\eta + d\eta(a + d)} (y_{i}^{0} - y_{i}^{*})$$
(70)

The equation describing the dynamic behaviour of the real exchange rate/terms of trade can be derived by combining the log-linearized version of equation (17) and (31) with equation (53). The log-linearized version of equation (31) and (17) are given as:

$$E_{t}(c_{t+1} - c_{t}) = E_{t}(c_{t+1}^{*} - c_{t}^{*}) + \frac{1}{\sigma}E_{t}(q_{t+1} - q_{t}) + u_{t}$$
(71)

and

$$q_t = (1-a)s_t$$
 (using the relationship  $p_{H_t} - p_t = -as_t$ ) (72)

Thus, the dynamic equation for the real exchange rate can be written after some simplification as:

$$E_{t}s_{t+1} = s_{t} + \frac{\sigma}{1+w} \left( E_{t}(x_{t+1} - x_{t}) \right) + E_{t}(s_{t+1}^{0} - s_{t}^{0}) + u_{t}$$
(73)

Finally, the dynamic equation for net foreign asset can be had after some approximations by log-linearizing equation (37):

$$nfa_{i} = \frac{1}{\beta} nfa_{i-1} + \frac{1}{\beta} (i_{i-1} - \pi_{i}) + \frac{nx}{nfa} nx_{i}$$
(74)

# 3.4 - Summary of the model

$$x_{t} = E_{t} x_{t+1} - \left(\frac{1+w}{\sigma}\right) \left(i - E_{t} \pi_{H_{t+1}} - r_{t}^{0}\right) - \kappa \left(\frac{w+d}{\sigma}\right) n f a_{t} + u_{t}$$
(58)

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \theta \left( \frac{\sigma}{1-d} + \phi \right) x_t - \theta \left( \frac{w+d}{1-d} \right) (s_t - s_t^{t_t})$$
(66)

$$s_{t} = E_{t}s_{t+1} - \left(\frac{\sigma}{1+w}\right)(E_{t}x_{t+1} - x_{t}) - E_{t}\left(s_{t+1}^{0} - s_{t}^{0}\right) - u_{t}$$
(73)

$$nfa_{t} = \frac{1}{\beta}nfa_{t-1} + \frac{1}{\beta}(i_{t-1} - \pi_{t}) + \frac{\overline{nx}}{\overline{nfa}}nx_{t}$$
(74)

$$nx_{t} = \frac{h}{1-d}(s_{t} - s_{t}^{0}) - \frac{1}{1-d}x_{t}$$
(69)

$$r_{t}^{0} = \left(\frac{\sigma}{1+w}\right) E_{t} \left(y_{t+1}^{0} - y_{t}^{0}\right) + \left(\frac{w}{1+w}\right) r_{t}^{*}$$
(59)

$$y_{i}^{0} = \frac{(1+\phi)z_{i} + \frac{d\sigma}{1-d}y_{i}^{*} + \left(\frac{w+d}{1-d}\right)s_{i}^{0}}{\left(\frac{\sigma}{1-d} + \phi\right)}$$
(64)

$$s_{i}^{0} = \frac{1}{h} \left( y_{i}^{0} - y_{i}^{*} \right)$$
(70)

$$r_{i}^{*} = \sigma E_{i} \left( y_{i+1}^{*} - y_{i}^{*} \right)$$
(55)

where .

$$\theta = \left(\frac{(1-\rho)(1-\rho\beta)}{\rho}\right)$$
$$w = ad - (a+d) + \sigma\eta(a+d-ad)$$
$$h = (2\eta - 1)(1-d) - ad\eta + \eta d(a+d)$$

Exogenous processes:

$$u_{t} = \gamma_{u}u_{t-1} + \varepsilon_{t}^{u} \qquad 0 < \gamma_{u} < 1$$

$$z_{t} = \gamma_{z}z_{t-1} + \varepsilon_{t}^{z} \qquad 0 < \gamma_{z} < 1$$

$$y_{t}^{*} = \gamma_{y}, y_{t-1}^{*} + \varepsilon_{t}^{y} \qquad 0 < \gamma_{z} < 1$$

### **<u>4 - The Conduct of Monetary Policy</u>**

In this section, I describe the behaviour of the monetary authority. One approach to incorporate the role of monetary authority and study the issues relating to the conduct of monetary policy is to specify an objective function (also termed as the loss function) for the central bank that explicitly translates the behaviour of the target variables into a welfare measure. The objective function thus serves as a guide for the monetary authority to formulate monetary policy. A widely used specification of the loss function takes the output gap  $x_i$  and inflation  $\pi_{H_i}$  as the target variables. This is particularly true for closed economies. Clarida, Gali and Gertler (2001) have shown that under certain conditions this would also be true for open economies because in their model, terms of trade gap is proportional to the output gap only hold in expectations as seen in equation (73). Moreover, terms of trade movements affect the net foreign asset holdings, which in turn affect the aggregate demand. Nonetheless, to ease comparison with Clarida, Gali and Gertler (2001), I have used the standard approach in the literature to assume that the monetary authority aims to minimize the expected value of a loss function that depends on output
gap and domestic inflation only. In order to provide some sensitivity test, however, I have also reported the results with a loss function that also incorporates the terms of trade gap in it. The objective function of the central bank can be written as:

$$\max -\frac{1}{2} E_{t} \left[ \sum_{k=0}^{\infty} \beta^{k} \left( \alpha x_{i+k}^{2} + \pi_{H^{-}i+k}^{2} \right) \right]$$
(75)

where ' $\alpha$ ' is the relative weight that the policy authority places on output deviations.

In the above specification, it is assumed that the central bank's target for output is equal to the economy's natural rate of output. This assumption essentially ensures that the central bank has no incentive to increase the economy's output beyond the natural rate level corresponding to flexible prices. This seems a little odd in the presence of monopolistic competitive firms. Following Gali and Monacelli (2004), I have assumed that government fully offsets this market power distortion by subsidizing employment, which is financed through lump sum tax on households. Accordingly, it would be optimal for the central bank to eliminate the effects of nominal rigidities and attain the efficient flexible price equilibrium. The other source of sub-optimality in the model is the absence of complete asset markets (see, Benigno, P (2001)). This explains the reason for including the term  $s - s_i^0$  in the loss function of the central bank rather than just having  $s_i$  or completely ignoring the terms of trade term. The central bank tries to stabilize the terms of trade around the 'efficient' complete asset market (with zero current account) level. As mentioned above, I have reported the results both with and without the inclusion of this term of trade gap term in the loss function.

In deriving the optimal monetary policy, it is assumed that the central bank uses the nominal interest rate *i*, as its instrument variable. It chooses the time path of this instrument to influence the time paths of the target variables  $x_i$  and  $\pi_{H_i}$  in such a way that it maximizes equation (75) subject to the constraints on behaviour implied by the system of equations described on the previous pages. The optimal outcome --- the optimal monetary policy --- that emerges is a feedback policy that links the policy instrument to the current state of the economy in a very specific way. The model is calibrated and simulated by using the technique provided by Soderlind (1999). The software used for this purpose is MATLAB.

#### **4.1- Optimal Monetary Policy under Discretion**

Following the method described in Clarida, Gali and Gertler (1999), the central bank first chooses  $x_i$  and  $\pi_{H,i}$  to maximize the objective function (75), given the inflation equation (66) (after substituting out terms of trade,  $s_i$  using equation (73)) and then conditional on the optimal values of  $x_i$ ,  $s_i$  and  $\pi_i$ , it determines the value of  $i_i$  implied by the IS curve. Combining equations (66) and (73). I can write the inflation equation as:

$$\pi_{H,i} = \lambda x_{i} + \left(\frac{\theta \sigma(w+d)}{(1+w)(1-d)}\right) E_{i} x_{i+1} + \beta E_{i} \pi_{H(i+1)} - \left(\frac{\theta(w+d)}{1-d}\right) \left(E_{i} s_{i+1} - E_{i} s_{i+1}^{0}\right) + \left(\frac{\theta(w+d)}{1-d}\right) u_{i}$$
(76)

where

$$\lambda = \theta \left( \frac{\sigma}{1-d} + \phi \right) - \left( \frac{\theta \sigma(w+d)}{(1-d)(1+w)} \right)$$

Note that  $\lambda$  depicts both the direct effect of output gap on inflation and the indirect effect on inflation operating through the impact of output gap on the terms of trade.

The solution to the constrained maximization exercise yields the following first order condition for an optimal monetary policy that requires that:

$$x_{i} = \frac{-\lambda}{\alpha} \pi_{H_{i}}$$
(77)

This is a very similar expression reported in Clarida. Gali and Gertler (1999) for optimal discretionary monetary policy in the closed economy. The slight difference is that the parameter  $\lambda$  incorporates the open economy features. Equation (77) has the usual interpretation that the central bank should contract demand by raising the interest rate whenever inflation is above target and vice versa.

To obtain a solution for the model (that is, the reduced form expressions for  $x_i$ ,  $\pi_i$  and  $s_i$ ), combine the first order condition (77) with (66) and (73) (after eliminating  $s_i^0$ )

and  $E_i s_{i+1}^0$  using equations (70) and (64)) to get two expectational difference equations involving inflation and the terms of trade:

$$\pi_{H,i} = \frac{\beta}{\Delta} E_i \pi_{H,i+1} - \frac{\theta(w+d)}{\Lambda(1-d)} s_i + \frac{\theta(w+d)}{\Delta\Omega(1-d)} v_i$$
(78)

$$s_{t} = E_{t}s_{t+1} + \left(\frac{\sigma\lambda}{\alpha(1+w)}\right)E_{t}\pi_{Ht+1} - \left(\frac{\sigma\lambda}{\alpha(1+w)}\right)\pi_{Ht} - v_{t} - u_{t}$$
(79)

where.

$$\Delta = 1 + \frac{\lambda^{2}}{\alpha} + \frac{\lambda \sigma \theta(w+d)}{\alpha(1-d)(1+w)}$$

$$\Omega = 1 + \frac{a\kappa(w+d)}{\beta(1+w)} - \frac{h\kappa nx(w+d)}{nfa(1-d)(1+w)}$$

$$v_{t} = \left(\frac{(1-d)(1+\phi)(\gamma_{z}-1)}{h(\sigma+\phi(1-d)) - (w+d)}\right) z_{t} + \left(\frac{(1-d)(\sigma+\phi)(\gamma_{z},-1)}{h(\sigma+\phi(1-d)) - (w+d)}\right) y_{t}^{*}$$

 $v_i$  is the combined error term and follows the usual AR(1) process:

$$v_r = \gamma_1 v_{r+1} + \varepsilon_r^3$$

The trial solution takes the form:

$$\pi_{H_{\ell}} = c_1 v_{\ell} + c_2 u_{\ell} \tag{80}$$

$$s_r = d_1 v_r + d_2 u_r \tag{81}$$

The solution equations for  $\pi_{H,i}$ ,  $s_i$  and  $x_i \left( = -\frac{\lambda}{\alpha} \pi_{H,i} \right)$  can be derived and written

as follows:

$$\pi_{H,i} = \left(\frac{\partial \Delta(w+d)}{(1-d)(\Delta - \beta\gamma_{y})} + \frac{\partial^{2}(w+d)^{2}\sigma\lambda\Delta}{(1-d)(\Delta - \beta\gamma_{y})q_{1}} + \frac{\alpha\theta(1+w)(w+d)}{(1-\gamma_{y})q_{1}}\right)v,$$

$$+ \left(\frac{\alpha\theta(w+d)(1+w)}{(1-\gamma_{u})q_{1}}\right)u_{i}$$

$$s_{i} = -\left(\frac{\sigma\lambda\theta(w+d)\Delta}{q_{1}} + \frac{\alpha(1+w)(1-d)(\Delta - \beta\gamma_{y})}{(1-\gamma_{y})q_{1}}\right)v,$$

$$- \left(\frac{\alpha(1+w)(1-d)(\Delta - \beta\gamma_{u})}{(1-\gamma_{y})q_{1}}\right)u_{i}$$
(82)

where,

$$q_1 = \alpha (1+w)(1-d)(\Delta - \beta \gamma_v) - \sigma \lambda \theta (w+d)$$

It is clear from the above expressions that domestic inflation and terms of trade are affected by demand shocks under the optimal monetary policy. Thus, some inflation and output volatility must be traded-off in the presence of demand shock. The reason is that when the central bank adjusts the nominal interest rate in response to a demand shock, it also effects the terms of trade, which has a direct effect on domestic inflation.

Finally, substituting the expressions for  $x_i$  (from 77) in the IS equation we can derive the interest rate feedback rule that implements the optimal monetary policy:

$$i_{t} = r_{t}^{0} + \left(1 + \frac{\sigma\lambda c_{1}(1 - \gamma_{1})}{\alpha(1 + w)c_{1}\gamma_{1}}\right)E_{t}\pi_{H_{t}+1} - \left(\frac{\kappa(w + d)}{1 + w}\right)nfa_{t}$$
$$+ \left(\frac{\sigma}{1 + w} + \frac{\sigma\lambda c_{2}(1 - \gamma_{u})}{\alpha(1 + w)} - \frac{\sigma\lambda c_{1}c_{2}\gamma_{u}(1 - \gamma_{v})}{\alpha(1 + w)c_{1}\gamma_{1}}\right)u_{t}$$
(85)

As long as there is some positive risk premium, the optimal monetary policy entails a response to net foreign assets; adjusting nominal interest rate in response to changes in net foreign assets in addition to the changes in the expected inflation rate.

# 5 – Simulation Results

In this section I discuss the simulated dynamic effects of a domestic demand shock given the structure of the model and the behaviour of the monetary authority described above. The following parameter values are used in order to obtain the results.

- $\sigma$  --- Intertemporal elasticity of substitution = 1.5
- $\phi$  --- elasticity of labour supply = 1
- $\eta$  --- elasticity of substitution between domestic and foreign goods = 1.5
- a --- share of imports in domestic output = 0.4
- d --- share of exports in domestic output = 0.35
- $\beta$  --- rate of time preference = 0.99
- $\rho$  --- fraction of firms with sticky prices = 0.75

The parameter ' $\kappa$ ' is taken to be equal to 0.007. Two other parameter values required are  $\overline{nx}$  and  $\overline{nfa}$ . From equation (74) it is obvious that they appear only in ratio form. Assuming zero steady-state inflation, the nominal (and real) interest rate must be equal to 0.01 ( $\beta(1+i) = 1$ ). The steady-state version of equation (37) gives: rnfa = -nx. implying a ratio  $\frac{\overline{nx}}{\overline{nta}} = -0.01$ .

$$\underset{nfa}{\text{mprying a ratio}} =$$

The impulse responses of key variables to a one percent of demand shock are shown in figure 1. Some sensitivity tests are also reported. For example, figure 2 reports the results of a case when central bank's objective function also includes a terms of trade gap term. It is obvious from the figures that under optimal policy, the monetary authority is unable to completely stabilize the demand shock. This result is in direct contrast with the one reported in Clarida. Gali and Gertler (2001). In their paper they have argued that, "under certain conditions, the monetary policy design problem for the small open economy is isomorphic to the problem of the closed economy". In particular, they have reported in their closed economy (CGG (1999)) paper that optimal policy calls for adjusting the interest rate to completely offset demand shocks. (See figure 3). Simply put, they have claimed that for open economies the exchange rate does not independently affect both aggregate demand and inflation and only act as a demand shock. Thus, the optimal monetary policy completely stabilizes the domestic economy from terms of trade.' exchange rate disturbances. The monetary authority increases the nominal interest rate in response to a positive demand shock, which dampens demand through the

traditional interest rate channel and via appreciation of the exchange rate. Since, domestic inflation is not independently affected by the appreciation of the exchange rate; the optimal monetary policy is able to hit its inflation and output targets simultaneously while allowing the exchange rate to fluctuate. The situation changes drastically when the fluctuations in exchange rate have a direct impact on the domestic inflation, as in my model. (Also see, Walsh 1999). In this case (figure 1), a positive demand shock, which leads to an increase in the real interest rate, also causes appreciation that affects domestic inflation. Thus, stabilizing aggregate demand in the face of an exchange rate disturbance leads to fluctuations in inflation, which require a less-than-full offset of the output effects of the demand shock. Thus, the optimal monetary policy trade-off some output variability for reduced inflation variability.

An additional channel that is ignored by CGG (2001) and Walsh (1999) is the dynamic relationship between the exchange rate and the current account. This is the channel explored explicitly in this chapter and generates interesting impulse responses. Two points need to be noted in understanding this channel after the central bank increases the interest rate due to a positive demand shock. First, an increase in the interest rate directly affects the net foreign asset holding which in turn affects the aggregate demand. Second, an increase in the interest rate that causes appreciation in the exchange rate also affect the net foreign asset holding (and thus aggregate demand) via its effect on the net exports. Thus, due to the presence of such considerations the central bank will not be able to fully offset the demand shock. The reason I am able to generate this additional channel

is because of the assumptions of incomplete asset markets and the presence of a risk premium term that is a decreasing function of net asset holdings. These two assumptions combined means that the Ricardian Equivalence hypothesis breaks down making bond holdings matter for aggregate demand. In short, the chapter has shown that greater inflation stability can only be achieved at the cost of greater output volatility. That is, the central bank does face a trade-off between output volatility and inflation volatility not only in the presence of supply (cost-push) shocks, but also in the presence of demand shocks.

Another interesting feature to note from figure 1 is the impulse response of output; it is a very nice hump-shape response. The impact of the shock does not die out immediately; rather, it first has a growing impact and then it gradually fades away. Note that I have picked the price adjustment parameter to be 0.75, which implies an expected period of one year (4 quarters) between price adjustments. This means that the effect of the shock should go away after this period. But, from the figure it is clear that the impact of the shock last well beyond this time (around 15<sup>th</sup> quarter). The reason is the short run wealth accumulation via current account. Thus, the model is capable of producing some realistic persistence with a value of only 0.2 for the persistence parameter in the demand shock.

Some of the other results reported in CGG (1999, 2001) do hold in this chapter. For example, the nominal interest rate rises more than the domestic expected inflation causing real interest rate to go up. (Result # 3, CGG (1999)). Moreover, since domestic inflation gradually converges to its target over time, optimal monetary policy can also be interpreted as incorporating inflation targeting. However, In CGG (1999) this was only true for cost-push shocks; in my case this is also true with demand shocks.

Figure 2 reports the results of a case when central bank's objective function also includes a terms of trade gap term. Benigno (2001), in a similar model, has explicitly derived the objective function of the central bank (based on the utility function of the representative economic agent) with incomplete asset markets and has shown that the terms of trade term does appear to play a role. Rather than re-deriving the objective function, I have only reported the results when the central bank places a positive weight on the terms of trade gap term. Some very interesting results can be observed in this case. The volatility of all the key variables goes down. Initially inflation goes up quite drastically but approaches the steady state far more quickly than in the case when the central bank places no weight on the terms of trade fluctuations. This can be interpreted that it is optimal for the central bank to gradually stabilize the exchange rate. The popular notion that allowing the exchange rate to float completely stabilizes output from demand shocks does not hold in the model presented in this chapter.

# 6 - Concluding Remarks

The usefulness of the model developed in this chapter is that it allows us to combine the salient features of the two leading schools of thought, namely, New Keynesians (the adherents of MFD-type models) and New Classicals (the proponents of real business cycle (RBC) models) for an open economy. Keynesian analysis emphasizes on the presence of sticky nominal prices and the absence of perfect competition in the various markets of the economy, while Classicals put more emphasis on explicit microfoundations and dynamic intertemporal optimizing behaviour of the economic agents. The demand side of the model is represented by a "New IS curve" and the supply side is depicted by, what has been labelled as, "New Keynesian Phillips curve". In addition, the relationships capturing the dynamics of exchange rate and current accounts are explicitly derived. The chapter closely follows the model developed in Clarida. Gali and Gertler (2001) (which in turn is quite similar to the model developed in Gali and Monacelli (2004)). The main difference, which turns out to be quite significant, is the incorporation of incomplete assets markets.

The framework developed in the chapter can be used to answer a number of monetary policy related questions; I focused on two basic questions. First, how would the overall design of monetary policy change when we move from closed economy models to open economy models? More specifically, would the central bank face an inflation-output volatility trade-off in responding to a demand shock? Second, what is the role of exchange rate and its dynamic relation with the current account in formulating monetary policy in an open economy setting? As shown in the chapter, when the exchange rate effects both aggregate demand and inflation, optimal monetary policy --- using nominal interest rate as the policy instrument --- calls for trading-off some output volatility for less inflation volatility. Thus, unlike Clarida. Gali and Gertler (2001), the optimal

monetary policy design problem is not isomorphic to the problem of closed economy. Moreover, movements in exchange rate not only lead to expenditure switching effects on aggregate demand thus reinforcing the impact of monetary policy, but with incomplete asset markets they also effect current account and net foreign asset accumulation. It is shown in the chapter that these changes in net foreign assets affect aggregate demand. Thus, the dynamic relationship between movements in exchange rate and current account plays an important role in deriving the optimal monetary policy response. The highlighted results of the models are:

- The central bank faces a trade-off between output volatility and inflation volatility not only in the presence of supply (cost-push) shocks, but also in the presence of demand shocks.
- Allowing the exchange rate to float in order to stabilize the output from demand shocks may not be a good idea. In other words, it pays-off (in terms of reduced volatility in the key variables) to assign a positive weight to stabilize exchange rate.
- The dynamic relationship between current account and the exchange rate plays a crucial role in obtaining the optimal monetary policy and in propagating the effect of shocks.
- The impact of the shock last well beyond the time interval in which prices are rigid.
- The impulse response of output has a hump-shape.

# **Figure 1:** Impulse Responses --- No Weight on the Exchange Rate Term in the Objective Function of the Central Bank.



Loss: 3.5568

#### **Standard Deviations**

Output Gap	0.0698

Inflation 0.0427

Exchange Rate 0.0720

# **Figure 2**: Impulse Responses --- Positive Weight on the Exchange Rate Term in the Objective Function of the Central Bank.



Loss: 1.1010

#### **Standard Deviations**

Output Gap	0.0012
Inflation	0.0104
Exchange Rate	0.0039

# Figure 3: Benchmark Case: Clarida, Gali and Gertler (2001)



# **Chapter 6**

#### Conclusion

Many central banks have adopted long-term price stability, either explicitly or implicitly, as a primary goal for monetary policy. Although there is no precise definition of price stability, in practice price stability has often been interpreted as low and stable average rate of inflation. This ultimate target of long-run price stability can be attained by announcing a particular nominal anchor, such as the growth rate of money, the exchange rate, the price-level, nominal GDP and even the short-term inflation rate, that can serve as an intermediate target. Thus, the objective of a tremendous amount of research in recent years has been to find policy rules that guide central banks in setting policy instruments so as to bring the inflation rate close to the target and keep it there, while taking account of the short-run trade-offs that impinge on output and exchange rate variability. An important consideration in this regard is the focus on the monetary policy transmission mechanisms --- the channel(s) through which monetary policy actions affects the economy. Identifying these transmission mechanisms is useful and important since they determine the main restrictions that central banks face in implementing monetary policy. This dissertation has dealt with a host of such issues --- intermediate targets and transmission mechanisms --- and provided several new insights. The reference framework used for this purpose was the widely accepted "new-Keynesian" model.

In much of the recent literature on monetary policy, cost-push shocks have occupied the centre stage in discussions on monetary policy and the role of demand shocks has been downplayed. Chapters 2 and 3 address these issues by introducing an alternative and independent channel of monetary policy transmission --- the cost channel --- and demonstrate the significance of demand shocks in affecting the monetary policy objectives for a closed economy. To introduce the cost channel it was assumed that firms need to borrow cash to finance their wage-bills before receiving the proceeds of sales revenues. Thus the short-term interest rate, which is also taken as the monetary policy instrument, directly affected the dynamics of inflation.

Although both chapters 2 and 3 study the implication of the cost channel and uses the "new-Keynesian" framework, the methodology employed for this purpose was quite different across the two chapters. For example, the first key difference is the use of more common and popular discrete-time framework in chapter 2 and a relatively less conventional continuous-time framework in chapter 3. Second, chapter 2 derives the optimal monetary policy and thus the optimal interest rate rule, while chapter 3 uses Taylor-type interest rate rule that may not be optimal. Third, chapter 2 compares the performance of inflation targeting versus price-level targeting regime and chapter 3 compares the performance of price-level targeting and nominal-income targeting regime. Fourth, the significance of demand shocks in the presence of the cost channel is further highlighted in chapter 2 by comparing the monetary policy outcomes when policy is conducted with discretion and with commitment. Fourth, to check for robustness of results, chapter 3 also considers a series of specifications for the demand side of the model --- the IS relationship and for the inflation adjustment relationship. Fifth, chapter 2 specifies the cost channel as involving the nominal interest rate, while chapter 3 considers a direct effect of real interest rates in the 'new Keynesian' Phillips curve. Thus, taken together these two chapters provide a lot of new insights to various monetary policy questions by considering alternative transmission mechanisms and alternative targeting regimes.

In chapter 2 we learned that the central bank operating under a flexible inflation targeting regime faces a trade-off between stabilizing inflation and output-gap in the presence of both demand and cost-push shocks. The reasons for this trade-off in response to cost-push shocks are straightforward and have been discussed before in the literature. In response to a positive cost-push shock that puts inflation above its target, a central bank that acts under discretion raises the nominal interest rate. This increase in the nominal interest rate affects the real interest rate (because prices adjust gradually in the short-run) which in turn affects the spending decisions of households thus lowering the output-gap, which in turn brings inflation. the central bank raises the nominal interest rate which brings both the output-gap and inflation back to the target. However, if the nominal interest rate also directly affects the inflation, as would be the case in the stabilizing inflation and output-gap. This is the basic insight used throughout chapter 2 and 3.

Chapter 2 also evaluated the performance of monetary policy under commitment and derived a new result that the gains from commitment are larger in the presence of the cost channel. The basic insight of the traditional literature, in the context of this debate over commitment versus discretion, is that if the central bank has a desire to push the economy's output level beyond the natural rate then discretion would only lead to higher average inflation (generally termed as inflation bias) with no effect on output; conducting the policy with commitment would eliminate the inflation bias. In the recent literature that uses the forward-looking "new Keynesian" model, the inefficiency due to inflation bias has received little attention because it is assumed that the central bank targets a zero output-gap. Nevertheless, even in the absence of an over-ambitious output target and an inflation bias, discretionary policy remains inefficient since it leads to a stabilization bias. It arises simply because of a lack of history dependence or inertia in the policy actions of the central bank. Since optimal monetary policy under commitment exhibits a considerable degree of inertia, it is superior to discretionary optimal policy.

What is the role of the cost channel in determining the optimal commitment solution? With monetary policy conducted under commitment, the future expected values of output-gap and inflation affect their current behaviour as well. When the central bank lowers the nominal interest rate to counter the effects of a negative demand shock it not only increases current output but also increases the future expected output-gap and thus expected future inflation. In the presence of the cost channel, the direct effect of this policy on the expected future output-gap and its indirect effect via future expected inflation affect current inflation. Thus, in order to bring inflation back to its target level, the central bank needs to affect the current output-gap by a small amount which improves the trade-off between stabilizing inflation and output-gap and thus reduces expected losses faced by the central bank leading to increased gains from commitment.

Yet another new insight provided in chapter 2 deals with the derivation of the implied interest rate rule that implements the optimal discretion and commitment monetary policy. The standard method involves substituting the central bank's optimality condition (in the commitment case it is written one period forward) along with the solution expressions for inflation and the output-gap in the IS relationship and then solving for the nominal interest rate  $i_i$ . The problems with this method are best illustrated in the commitment case and when the cost channel is operative. The derived interest rate rule violates the Taylor principal (that the coefficient in front of the expected inflation term should be greater than one), fails to capture the inertial behaviour of the optimal commitment policy, and predicts that demand shocks will be completely stabilized even in the presence of the cost channel (which is not the case as discussed above). The alternative method of deriving the optimal interest rate rule, proposed in the chapter, solves the IS relationship, the inflation adjustment relationship and the central bank's optimality condition simultaneously for nominal interest rate, inflation and output-

gap. This method correctly depicts the inertial behaviour of optimal policy with commitment and derives conditions that can avoid indeterminacies of output and inflation. Moreover in the presence of cost channel, this method is the only correct method in the commitment case for the sake of internal consistency of results.

As discussed in the beginning of this chapter, the central bank can achieve its goal of long-run price stability by targeting a host of nominal anchors. One example, that has received considerable attention in recent years, is adopting a price-level targeting regime. The motivation for price-level targeting is twofold. First is the widely reported result that although a pre-commitment monetary policy under an inflation targeting regime yields a better outcome, it is time-inconsistent. That is, the central bank has an incentive to deviate from its announced policy. Second, in practice, no central bank can commit to a policy rule for all time periods. Thus, an important question is: How can a central bank avoid the time-inconsistency problem related to pre-commitment and at the same time take advantage of the superior results provided by commitment? Put differently, can a central bank replicate the commitment solution by adopting an alternative targeting regime conducted in a discretionary fashion? Assigning a specific discretionary pricelevel targeting regime to the central bank is one example of answering these questions and was used in chapter 2. The simple reason as to why this strategy works in terms of lowering expected losses for the central bank and converging to the commitment solution is that it depicts the inertial behaviour of the central bank which is the feature of the

commitment solution. Thus, monetary policy actions entail a more gradual response to shocks that allow the central bank to appropriately affect private sector expectations.

Since the debate over the relative benefits of price-level targeting as compared to inflation targeting is an ongoing one, it makes sense to check for the robustness of the result just mentioned. Chapter 2, utilizing the presence of the cost channel in a standard "new Keynesian" model, did just that. Due to the presence of the cost channel monetary policy actions cause movements in the aggregate supply side of the model in addition to affecting the demand side. Therefore, the conventional wisdom had argued that targeting the price-level under this scenario could lead to increased volatility in economic activity. However, in contrast to the traditional literature and in line with the recent work (that employs the forward-looking "new Keynesian" model) on this issue, chapter 2 tinds support for the price-level targeting regime. Two new insights are worth noting. First, the price-level targeting is preferred over inflation targeting even in the presence of the demand shock. Recent literature derived this result for a cost-push shock only; in case of demand shocks central bank was indifferent in choosing between the two regimes. Second, this result needs to be interpreted carefully in the presence of the cost channel. In particular, the relative weight on output-gap stabilization versus inflation stabilization needs to be appropriately adjusted in comparing the expected value of the losses incurred in the two regimes; otherwise the results could be different.

Chapter 3 modeled the demand shocks as an ongoing cycle rather than a one-time change and compared the performance of two rule-based targeting regimes --- price-level targeting and nominal income targeting --- with and without the cost channel of monetary policy for a closed economy. It assumed that the two targeting regimes generate the same outcome regarding long-term inflation. Thus, the criterion for evaluating the performance of a monetary regime was its ability to minimize the volatility in real output.

An important feature of chapter 3 is the use of the continuous-time framework as opposed to discrete-time specifications. The main reason for employing this approach is that the properties of a discrete-time model are very sensitive to small changes in assumptions concerning information availability. In some cases, the entire undetermined coefficients solution procedure breaks down (with restrictions on structural, not reduced form, coefficients being called for). A continuous-time specification precludes such unappealing problems from developing. Moreover, as discussed extensively in chapters 2 and 3, the choice between alternative targeting regimes rests critically on the specification of the model. For this purpose, chapter 3 considered various alternative specifications for both the new IS and inflation adjustment relationships. Due to its analytical simplicity, the continuous-time framework proved extremely helpful in this regard.

As argued in the chapter, the standard Euler equation for consumption that gives rise to the new IS relationship fails to capture the dynamics of the aggregate output. One way to avoid this problem, and to make the model come close to capturing the empirical regularities, is to allow for habit persistence in preferences. Another approach is to introduce a 'rule of thumb' behaviour on the part of a fraction of the households (assuming that it is costly to reoptimize every period) with the remaining fraction of the household optimizing consumption in a usual fashion. Both these approaches essentially makes sure that a lagged output-gap term is included in the new IS relationship. Chapter 3 makes use of a continuous-time version of this so-called 'hybrid' IS relationship.

Similarly, the basic forward-looking inflation adjustment equation --- the 'new Keynesian Phillips curve' --- is accused of generating inertia in the price level and not the inflation rate and that is considered as being inconsistent with the stylized facts on inflation dynamics. A simple method of getting around this problem is to add a lagged inflation term to the baseline inflation adjustment relationship. Another very recent approach has suggested an alternative set of assumptions to capture the inflation dynamics. It replaces the assumption of sticky prices with that of sticky information. In particular, it assumes that firms gather and process the information about the state of the economy slowly over time. Thus, although prices are always changing, firms are slow to update their pricing strategies in response to new information. Chapter 3 employs a continuous-time version of both these alternative specifications of the inflation adjustment relationship.

Thus, building on the new insights provided by chapter 2, chapter 3 provides further sensitivity tests on the role of the cost channel of monetary policy transmission. Not surprisingly, the cost channel matters in the sense that the volatility of real output increases under both price-level and nominal income targeting. The intuition for this is simple. In response to an autonomous ongoing demand cycle, the central bank adjusts the nominal interest rate to manipulate the aggregate demand (and thus the aggregate price level) in order to keep the resulting output volatility at a minimum. In the presence of the cost channel, however, adjustments in the nominal interest rate directly affect the aggregate supply side of the model as well: thereby increasing output volatility. As a result, with price-level targeting, the central bank would have to manipulate the aggregate demand by a large magnitude that would ensure the achievement of the original level of prices at the cost of an increased volatility in output. On the other hand, with nominal income targeting it would adjust aggregate demand just enough to reach a targeted level of nominal income with slightly high level of prices but a lower volatility in output. Since the metric used to evaluate the performance of a targeting regime is the minimization of real output volatility, nominal income targeting is preferred to price-level targeting. A somewhat surprising result is that the inclusion of the cost channel does not say much on the choice between the two regimes. It appears that nominal income targeting performs better than price-level targeting in bringing down the volatility of real output in almost all the specifications of the macro models used in the analysis regardless of the cost channel.

After thoroughly analyzing the implications of the cost channel for the choice among alternative targeting regimes in chapters 2 and 3, the dissertation focused on open economy models in chapters 4 and 5. Theoretically modeling the open economy is not an easy task as a host of new issues need to be addressed. For instance, the structure of international asset markets in which economic agents trade domestic and foreign assets. As discussed in chapter 5 in detail, the structure of asset markets have important implications for the risk sharing behaviour of domestic and foreign residents and can alter the specification of the uncovered interest parity relationship that is the cornerstone of open economy models. The international risk sharing in turn affects the current account dynamics of an economy The "new-Keynesian" modeling framework has tended to ignore the effects of current account dynamics on aggregate demand. Chapter 5 addressed this issue and provided important new insights.

Another key variable in the open economy is the exchange rate. The debate over the role of exchange rate in affecting the domestic inflation (the exchange rate passthrough) and thus monetary policy objectives is far from being settled. Moreover, the dynamic relationship between exchange rate movements and current account dynamics has received little attention in the recent literature. Chapter 5 is an attempt to fill this gap. The way to model imports has also been controversial. Some researchers tend to model imports as intermediate-inputs used in the domestic production process. An immediate implication is that the domestic inflation is directly affected by movements in the exchange rate as it alters the cost of production. Other researchers model imports as final consumption goods. Thus, depending on the assumption regarding the structure of international asset markets or the nature of international risk sharing, movements in exchange rate may or may not affect domestic inflation directly. This has led to the conclusion that central bank should focus only on domestic inflation while allowing the exchange rate to float. Thus, the classic debate over fixed versus flexible exchange rate regimes is still very much alive.

In light of these alternative methodologies, chapter 4 models imports as intermediate-inputs while chapter 5 models them as final consumption goods. However, this is not the only difference in the two chapters. Chapter 4 extends the continuous-time models used in chapter 3 for an open economy and compares the performance of monetary policy (in terms of reducing volatility in real output) under three alternative targeting regimes: exchange rate targeting (fixed exchange rates), price-level targeting and nominal income targeting (flexible exchange rates). Moreover, it also sheds light on a relatively old question: the effects of an increased degree of price flexibility on the volatility of real output. Chapter 5 extends the discrete-time general equilibrium model developed in chapter 2 for an open economy and focuses on the implications of the dynamic relationship between domestic inflation, exchange rate and current accounts on the performance of a discretionary inflation targeting regime.

The first result in chapter 4 confirms the result derived in chapter 3 that nominal income targeting outperforms price-level targeting, in terms of reducing volatility in real

output, in an open economy with flexible exchange rates for all the alternative specifications of the IS and inflation adjustment relationships considered. But the main focus of the chapter was to explore the impact of an increased degree of price flexibility on the volatility of real output. Previous analysis of this question offered mixed conclusions especially in an open economy context. Keynes' had argued back in the 1930s that under a flexible exchange rate regime, a lower degree of price flexibility helps lower output volatility. The intuition was simple. A lower degree of price flexibility essentially means that the aggregate price level in the economy is stickier; therefore a flexible exchange rate would allow the economy to absorb the effects of demand shocks leading to a lower volatility of output. Chapter 4 supports Keynes' intuition. Moreover, this result is consistent with the behaviour of central banks that argue that their low inflation-flexible exchange rate policy has decreased the degree of nominal price flexibility (by increasing average contract length).

Chapter 4 also finds support for central banks that favour fixed exchange rates. A fixed exchange rate regime implicitly implies that the economy is willing to accept structural reforms of the sort that promote flexibility in prices to absorb the effects of shocks. The chapter shows that with fixed exchange rates output volatility indeed goes down as a result of increased price flexibility. Thus, there is internal consistency within both views about exchange rate policy. However, this result does not hold when an alternative specification of the demand and supply side of the economy are considered.

Chapter 5 studied the properties of an optimal discretionary domestic inflation targeting regime under a flexible exchange rate. It also reported results for the case when the central bank explicitly incorporates minimization of the exchange rate fluctuations in its objective function. In a sense, this can be considered as targeting CPI inflation as opposed to domestic inflation. The main contribution of the chapter was the incorporation of current account dynamics and their relationship with the exchange rate and domestic inflation. The broad question explored in the essay was: how would the overall design of monetary policy change when we move from closed economy models to open economy models?

The reason current account dynamics have been missing from the 'new Neo-Classical synthesis' literature is the assumption of complete international asset market. As discussed in chapter 5, the implication is that the dynamics of current account become irrelevant for monetary policy. However, chapter 5 argued that the assumption of complete asset markets is not realistic in a model with imperfections and rigidities in goods market because with nominal rigidities monetary policy affects real variables that include current account. This was the main reason for assuming incomplete asset markets since it made the dynamics of current account matter for monetary policy.

Another important contribution of chapter 5 was that it provided new insights on the relationship between domestic inflation and movements in the real exchange rate. In contrast to the traditional approach, it derives a domestic inflation adjustment relationship in which real exchange rate depreciation lowers domestic inflation. The key to understanding this seemingly counter-intuitive result is to note that imports are introduced only as final consumption goods and that the nominal wage is completely flexible. Consider a real depreciation. It leads to a decrease in the overall consumption bundle because imports are now relatively expensive. Since labour supply is a positive function of consumption, it also decreases nominal wages, and with the assumption of sticky goods prices, real wages also go down. Falling real wages imply a lower cost of production for domestic firms and thus results in lower domestic inflation.

Building on these new theoretical insights, chapter 5 was able to question a widely accepted result that optimal monetary policy for an open economy calls for adjusting the interest rate to completely offset demand shocks, which implies no trade-off between output volatility and inflation volatility. According to the open economy model of chapter 5, the exchange rate affects both aggregate demand and aggregate supply (inflation): adjusting the interest rate to stabilize output in the face of a demand shock causes fluctuations in inflation. For example, consider a negative demand shock that lower output-gap and thus domestic inflation. The central bank, operating under a discretionary domestic inflation targeting regime, responds by lowering the nominal interest rates which, via the interest parity condition, causes real depreciation. Both these effects help increase the output-gap and thus inflation. However, when real depreciation also has a direct negative effect on domestic inflation, the central bank faces a dilemma,

it cannot stabilize both output-gap and inflation. Thus, the optimal monetary policy calls for trading-off some output volatility for less inflation volatility.

Exchange rate movements not only lead to a trade-off in policy objectives, but also interact with current account dynamics that in turn cause fluctuations in the outputgap. Thus, it makes sense for the central bank to try to stabilize the exchange rate directly, indeed, the volatility of key variables and the expected losses decrease when the central bank includes exchange rate stabilization in its objective function. Thus, an important lesson learned in chapter 5 is that the dynamic relationship between the current account, the exchange rate, and domestic inflation play a crucial role in obtaining the optimal monetary policy. Moreover, it also influences the mechanism through which the effects of the shock are propagated through the economy. Thus, it was no surprise that the optimal monetary policy entailed a response to net foreign asset positions and the impact of the demand shock lasted well beyond the time interval for which prices were rigid.

To summarize, what we learned from this dissertation is that the "new Keynesian" framework, as applied to different monetary policy issues in various chapters, has a lot of potential to increase our understanding regarding the formulation of monetary policy. Thus, efforts to improve this framework even further could be very rewarding for both academic economists and central bankers.

### Chapter 7

#### **Directions for Future Research**

This chapter discusses some of the possible extensions and applications of the "new-Keynesian" framework that has occupied the centre stage in research on monetary policy related issues. The four core chapters in this dissertation themselves constituted an attempt to build upon and contribute to the existing models in this area. However, due to time considerations, not all of the issues that have intrigued me over the course of writing this dissertation could be included in those four chapters. Thus, the following discussion can be considered as a part of my future research agenda.

One result --- reported in chapter 2 --- was that the outcome of monetary policy with commitment is superior to the results attained with discretion. But the commitment solution has its own problem --- it is time-inconsistent; in the initial period the central bank behaves as if it were operating in a discretionary fashion. Chapter 2 ducked this issue by adopting the "timeless perspective" of monetary policy suggested by Woodford (1999c). He suggested that the central bank should commit to implement in each period the policy that would have been optimal to commit to if the same problem had been considered at a date far in the past. This procedure, thus, avoids treating the current period as the initial one and disregards the conditions that happen to prevail at the time in which policy begins. It is worth noting, however, that a number of authors such as Dennis (2001), Blake (2001), Blake and Kirsanova (2004), Jensen and McCallum (2002) and Jensen (2003) have questioned the timelessness of Woodford's approach and have demonstrated that "timeless perspective" policy is not the time-invariant rule and does not in general minimize the loss function of the central bank. In my opinion, this is an important issue and further research would be extremely fruitful in shedding light on the true nature and implication of commitment for achieving the goal of price stability.

Recently, though, some economists have argued that an unwavering commitment to price stability by an independent central bank may not be a sufficient requirement: price stability requires not only an appropriate monetary policy but also an appropriate fiscal policy. Most of the arguments on these lines have built upon the insight provided by Sargent and Wallace (1981). They contended that an economy's monetary authority cannot prevent inflation by its own control of base money creation if an uncooperative or irresponsible fiscal authority behaves so as to generate a continuing stream of primary deficits. Since then, a growing literature has raised questions concerning the ability of monetary policy to control inflation and proposed that it can only do so with the cooperation of the fiscal authority. Some important recent literature in this area, which has been labeled as "the fiscal theory of the price level", include Leeper (1991), Sims (1994, 1997), Woodford (1995, 1998). Cochrane (2001) and Canzoneri. Cumby and Diba (2001). A critical assessment of this literature has been provided by Buiter (2002), Kocherlakota and Phelan (1999), Carlstorm and Fuerst (2000) and McCallum (1998).<sup>53</sup>

<sup>&</sup>lt;sup>53</sup> For a general discussion, see Christiano and Fitzgerald (2000) and McCallum (1999).

While a number of papers in this literature have analyzed price level determination in models with flexible prices (the exceptions are Canzoneri and Diba (2000) and Woodford (1998)), few have considered the role of monetary policy in models with nominal rigidities. Thus an explicit analysis of the relationship between monetary and fiscal policy in a "new-Keynesian model", which endogenously incorporates nominal rigidities, is an important research project.

Chapter 2 and 5 used the expected loss function of the central bank as welfare metric to evaluate the performance of alternative monetary policy regimes. For example, chapter 2 specifies two loss functions for the central bank; one depicting the inflation targeting regime and one focusing on the price-level targeting regime. Similarly, chapter 5 reports results for two loss functions; one that only includes domestic inflation gap and output gap and one that also includes a terms-of-trade-gap term. But, none of the specifications of the loss functions used in these two chapters were formally derived. They relied on the works of Walsh and Ravenna (2003) and Benigno (2001) respectively that derived similar loss functions using a general method proposed by Woodford (2002). However, it is important to derive formally how the appropriate loss function changes with several changes in the model. For example, in a closed economy model of chapter 2 a consideration that whether the cost channel stems from a production lag or a payment lag can have an important bearing on the value of the parameter describing the relative weight central bank places on key variables in its loss function. This is important since the verdict concerning which policy regime delivers the smallest variances depend on that

parameter. Similarly, in the open economy model of chapter 5 with incomplete asset markets it was argued that there is a potential role for terms of trade of exchange rate gap in the loss function alongside the output gap and domestic inflation. The intuition was simple. In the presence of incomplete asset markets and thus incomplete risk sharing output gap and consumption gap may not move together. Welfare depends on both gaps. If the central bank does not monitor the consumption gap directly, then they need to consider the exchange rate gap alongside the output gap in setting the policy. A formal derivation of this intuition can have important bearing on alternative monetary policy targeting regimes for an open economy.

All the chapters of the thesis focused on only two types of shocks: goods demand and supply (cost-push) shocks. Implications of other types of shocks, such as financial sector shocks, exchange rate shocks, technology shocks, productivity shocks, for monetary policy will be considered in future work.

In practice, central banks make the distinction between core and headline inflation rates, but most of the chapters did not focus on this issue. The only distinction made in the thesis regarding the definition of inflation highlighted the difference between domestic inflation and CPI inflation for an open economy. However, carefully defining inflation even for a closed economy can be important. For example, the implications of the role of the cost channel in chapter 2 may depend on whether the inflation measure used incorporates these interest costs or not. Based on empirical evidence, chapter 3 and 4 considered "hybrid" versions (a mix of forward and backward-looking elements) of the new IS and Phillips curve relationships. Increasing number of recent papers are paying attention to this issue, However, as noted by Walsh (2003b) and Amato and Laubach (2003), there is not complete agreement on how this should be done. Further research on this issue will be most desirable.

A significant drawback with the new Keynesian closed economy model that was applied to many monetary policy questions in chapters 2 and 3 is perhaps its assumption regarding the capital accumulation process. Based on the work of McCallum and Nelson (1999a), recent research has tended to ignore endogenous variations in the capital stock by arguing that little is lost for the purpose of short-run fluctuations in the economy if an exogenous process for the capital stock is assumed. However, something is lost. Without capital, the models cannot capture the broad credit channel of monetary policy and thus the implications of asymmetric information paradigm for monetary policy cannot be analyzed. Casares and McCallum (2000) can be considered as one of the first attempts to incorporate endogenous investment into a typical discrete-time "new Keynesian" model. Dupor (2001) adds investment to a standard continuous-time sticky price model and reverses the well-established results on stabilization property of interest rate policy rules (the Taylor principle): only increases in the nominal interest rate *less* than one-for-one with increases in inflation will ensure determinacy. Li (2002) develops a discrete-time version of Dupor's model and questions Dupor's claims. He further argues that before
using continuous-time approach it is important to verify that continuous time limit does not distort the dynamic property of a discrete-time model as periods are made short. Since investment is both an important and highly volatile component of aggregate output, it is crucial to understand whether the introduction of investment to a model has indeed a major reverse impact on stabilization property of monetary policies. This investigation has practical importance for central bank policies and thus further research on this issue is in order.

Just as in closed economy models, the open economy "new-Keynesian" model also ignored the role of endogenous capital accumulation and thus the role of the credit channel of monetary policy in explaining, for instance, the recent currency crises episodes that have rocked the international financial system during the late 1990s. The discussion of credit markets with asymmetric information (the financial accelerator mechanism) and the role of balance sheet problems of both the firms and the banks have occupied the centre stage in the recent theoretical research on currency crises. The seminal contribution of this approach (for a closed economy) is Bernanke, Gertler and Gilchrist (1999). Very few papers have attempted to extend this line of work to the open economy. Some notable exceptions are Gertler, Gilchrist, and Natalucci (2003), Faia, and Monacelli (2001). Devereux and Lane (2001). Caspedes, Chang and Velasco, (2004), Krugman (2000a) and Aghion. Bacchetta, Banerjee, (2000). Even fewer papers have explored the dynamic interaction between a firm's balance sheet (demand for credit) and a bank's balance sheet (supply of credit). A possible exception is perhaps Gersbach (2000). Moreover, links between a private sector's balance sheet and dynamics of current account with an endogenous monetary policy are also unexplored so far. Thus, incorporating 'financial accelerator' mechanism with a non-trivial role of banks in an open economy model, such as the one developed in chapter 5, can prove extremely useful in modeling the currency crises and in learning about the optimal monetary policy responses.

There are at least two observations, theoretical and empirical, that provide the necessary motivation to pursue this project and to shed more light on monetary policy issues. On the theoretical front the challenge is to break the famous Modigliani-Miller Theorem (1958). They derived the formal proposition that real aggregate economic behavior is independent of the financial structure, thus making the financial conditions useless and redundant. More specifically, this theorem states that if households and firms have unrestricted access to complete/perfect financial markets, then at the competitive equilibrium the size and composition of financial balance sheets have no impact on real economic decisions. However, financial markets may not be complete/perfect due to imperfect and asymmetric information structure. According to the asymmetric information paradigm, different economic agents possess different pieces of information on relevant economic variables, and that agents use this information for their own profit. It has been widely recognized in the literature that these informational asymmetries lead to problems such as adverse selection, moral hazard and costly state verification in the

credit markets and provide the justification for an independent credit channel of monetary policy.

On the empirical front, it is the observation that the last decade of the twentieth century witnessed a drastic increase in the number of currency, financial and economic crises, for example, the Latin America in 1994-95 and South East Asia in 1997-98. These crises episodes have increased awareness, both in academic and policy circles, and have heightened interest in investigating their origins, persistence and propagation mechanism. Leo Tolstoy noted once, "*Every happy family is the same. Every unhappy family is miserable in its own way*". Similarly, every financial crisis is different and involves its own distinctive elements; yet, there are some issues that are common to many of the recent emerging market crises.

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