DECISION THEORY MODELS

OF

SEQUENTIAL MULTIPLE OBSERVATIONS

STATISTICAL DECISION THEORY MODELS
OF SEQUENTIAL MULTIPLE OBSERVATIONS
BY HUMAN OBSERVERS

By

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SCOPE AND CONTENTS:

Kinchla (1969) has proposed a general model for multiple observation tasks which assumes that human observers respond in such tasks on the basis of the sum of the sensory information available to them. In this thesis Kinchla's simple sum model is compared to a model which assumes that observers respond on the basis of the weighted sum of the sensory information available to them.

Three experiments were carried out. The major findings were that, in a two interval sequential multiple observation task:

1) Stimulus Frequency within an interval does not induce weighting.

2) Instructions to concentrate on one interval induce weighting.

3) False information feedback about one interval induces weighting.

The relevance of the results to theories of "attention" is discussed.

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CONTENTS

FIGURES

INTRODUCTION

Consider the following practical decision problem. A doctor is examining a patient for signs of heart disease: at time A he obtains one sample of the patient's heart beats through his stethoscope and at time B he obtains a second sample of heart beats. The doctor's task is to decide if he has heard *at least one* indication of heart disease in the two samples, he must make a diagnostic decision at this point. How does the doctor combine the information from the two samples to arrive at a diagnostic decision, particularly when he suspects that one of the two observations is more likely to contain the indication (signal) he is listening for or that the information from one observation may be more reliable than the other? In this paper we shall attempt to test some mathematical models of the decision process in situations such as this.

We may formally define the decision problem outlined above as a *multiple-observation task*; the observer is presented with a series of n well defined consecutive observation intervals and is required to decide whether a signal was presented during *at least one* of these intervals. Note that the observer must defer his response until all n intervals have elapsed and that he need *not* specify in which interval or intervals he believes the signal to have occurred.

A particular sequence of signal and no signal intervals will be referred to as a *stimulus pattern*. We shall denote a stimulus pattern of two observation intervals as S_{ij} (for i and j equal to 1 or 0, where 1 indicates the presence of a signal and 0 the absence of a signal). Thus in the two observation cases there are four possible stimulus patterns: A signal in both observation intervals (S_{11}), a signal in the first interval and no signal in the second (S_{10}), no signal in the first interval and a signal in the second (S_{01}), and no signal in either interval (S_{00}). An observer's decision that no signals occurred in either interval will be denoted by A_0, and his decision that at least one signal occurred in the 2 observation intervals by A_1. Thus the observer's performance can be summarized by estimates of the four conditional probabilities, $P(A_1|S_{ij})$. Note that these correspond to the probability of a "false alarm" when i and j both equal 0 and to the probabilities of a "hit" when either i or j or both equal 1.

Two special cases of this general multiple-observation problem have been defined. The first special case occurs when signals occur either in all of the n observation intervals or in none of the n observation intervals, this has been termed the multiple-look task (Swets, et al., 1959; Swets and Green, 1961; Green and Swets, 1965; Swets and Birdsall, 1967; and Kinchla, 1969). The second special case occurs when only t signals ($t>n$) occur in n observation intervals (Kinchla, 1969). The theoretical treatments of the authors cited has been to assume that the observer sums the sensory information available in each observation interval and then bases his decision on the value of this sum. The alternative model that has been examined (Green and Swets, 1966) has been one in which the observer keeps track of decisions made after each observation interval. In general the decision theoretic model has been quite clearly supported.

All of the above work is predicated on the hypothesis that if a model of the detection process adequately specifies the information contained in a single observation it should then be possible to deduce the additional information contained in each successive observation interval. An explicit assumption of Kinchla's (1969) temporal uncertainty model was that each of the t signals be equally likely to occur in any of the n observation intervals. On the basis of the strategy and the assumption described above Kinchla arrived at a model which describes the behavior of observers in multiple observation tasks by assuming that they base their decisions on the sum of the sensory information available from each observation interval. The present paper presents a model of the behavior of observers in multiple-observation tasks which assumes that observers base their decisions on the weighted sum of the sensory information available in each of the n consecutive observation intervals. The present paper then seeks to experimentally identify some of the variables which lead to unequal weighting of the separate observation intervals.

In this paper we shall consider three variables which might induce an observer to treat the two observation intervals differentially. These variables are:

1) Signal probability in a given interval
2) Instructions
3) Feedback reliability.

Intuitively it seems reasonable that if a signal can occur in either one or both of two intervals but is very much more probable in one of the two intervals the observer will give more weight to observations of the more probable interval. This hypothesis is tested in Experiment 1.

In Experiment 2 we test the hypothesis that if an observer is told that it is very important to detect almost all of the signals in a given interval he will treat the observations of that interval differentially.

Finally in Experiment 3 we test the hypothesis that unreliable feedback about the events in one interval will lead the subject to give observations in that interval less weight than observations from an interval where stimulus events always predict feedback reliably.

THEORETICAL MODELS

The Theory of Signal Detection (TSD) (cf. Green and Swets, 1966) represents the observable relations between stimuli and responses in elementary discriminations as the product of two processes: an activation process and a decision process. The activation process relates the external stimulus event to a set of internal, hypothetical, sensory states of the observer. The decision process specifies the relations between the hypothetical sensory states and the observable responses. Basically, the physical stimulus is converted by the activation process into sensory information which is then employed as the basis for response. In general the activation process is thought to be solely determined by the physical properties of the stimulus and the sensory limitations of the observer, i.e., his "sensitivity". The decision process is generally considered to be the product of non-sensory variables such as instructions, signal probability or the costs and values of the various decision outcomes. The TSD model yields the prediction that, while an observer may modify his performance by altering his decision process, for each level of sensitivity there exists a unique set of possible performances, his <u>receiver operating characteristic</u> (ROC). Thus to measure an observer's sensitivity is to specify his ROC for that particular signal.

For the single observation case the decision theory model represents the set of possible sensory states as a continuous variable, X, that ranges from 0 to plus infinity. The activation process is represented as two over-lapping normal distributions of this subjective sensory state variable. The distribution with the higher mean value is associated with presentations of the signal, S_1, while the other distribution is associated with the sensory events evoked when no signal is present, S_o. These distributions are probability density functions specifying the relative probability of each value of the variable occurring. The observer's decision process can be summarized by a criterion value, β, of the sensory state variable such that if and only if the value of X on any particular observation is greater than β will the observer make an affirmative response, A_1. The ROC is generated by allowing β to assume all possible values, and noting the pair of hit and false-alarm probabilities for each value of β. The two parameters of this ROC function are d' and k, with

$$d' = \frac{\mu_1 - \mu_0}{\sigma_0} ,$$

and

$$k = \frac{\sigma_1}{\sigma_0} ,$$

where μ_1 is the expected value of the signal distribution of sensory state values, μ_0 is the expected value of the no signal distribution; σ_0 is the standard deviation of the no signal distribution and σ_1 is the standard deviation of the signal distribution. These two parameters, d' and k, specify the sensory information available to the observer in the one observation task.

The way in which this process has been generalized to multiple observation tasks in the past has been to simply assume that the observer sums the sensory information available in each observation and then sets a criterion value of this sum, which when exceeded will lead to an affirmative response. Green and Swets, (1966) give an expression for the ROC produced by such summing for the multiple-look case. Kinchla (1969) gives a more general expression for the multiple observation ROC which subsumes the multiple-look form as a special case. We shall now derive a set of expressions for two observation ROC curves which do not make the assumption that in arriving at the summed value of the sensory state variable that each observation interval is treated identically by the observer.

If we represent the value of the sensory variable evoked during an observation interval by x, then the two probability density functions representing the activation process can be represented as $g_0(x)$ and $g_1(x)$. Transforming x to x′, where

$$x' = \frac{x - \mu_0}{\sigma_0}$$

the following is true:

$$\mu_0' = E(x'|S_0) = 0, \tag{1}$$

$$\mu_1' = E(x'|S_1) = d', \tag{2}$$

$$\sigma_0' = \sqrt{Var(x'|S_0)} = 1, \tag{3}$$

$$\sigma_1' = \sqrt{Var(x'|S_1)} = k. \tag{4}$$

Consider then the distributions of the weighted sums of sensory values produced by the four possible stimulus patterns S_{00}, S_{11}, S_{10}, S_{01}. We shall denote this weighted sum as the random variable Y where

$$Y = \omega x_i' + (1-\omega)x_j' \tag{5}$$

where ω is a weighting factor such that $0 \leq \omega \leq 1$.*

*It should be noted that any system of two weights may be reduced to a scheme of ω and $(1-\omega)$.

It follows if x' is a Gaussian random variable that Y is a Gaussian random variable with

$$E(Y|S_{ij}) = \omega E(x_i') + (1-\omega)E(x_j') \quad (6)$$

and

$$Var(Y|S_{ij}) = \omega^2 Var(x_i') + (1-\omega)^2 Var(x_j') \quad (7)$$

From Equation 6 it follows that:

$$E(Y|S_{00}) = 0 \quad (8)$$

$$E(Y|S_{11}) = \omega d' + (1-\omega)d' = d' \quad (9)$$

$$E(Y|S_{10}) = \omega d' \quad (10)$$

$$E(Y|S_{01}) = (1-\omega)d' \quad (11)$$

and from Equation 7 that:

$$Var(Y|S_{00}) = \omega^2 + (1-\omega)^2 \quad (12)$$

$$Var(Y|S_{11}) = k^2[\omega^2+(1-\omega)^2] \quad (13)$$

$$Var(Y|S_{10}) = \omega^2 k^2+(1-\omega)^2 \quad (14)$$

$$Var(Y|S_{01}) = \omega^2+(1-\omega)^2 k^2 \quad (15)$$

From the above we may derive the parameters d_{ij}' and k_{ij} for the ROC functions for the detectability of any given stimulus pattern in relation to any other of the four possible stimulus patterns. Thus we obtain:

$$d_{11}' = \frac{d'}{\sqrt{\omega^2+(1-\omega)^2}}, \quad (16)$$

and

$$k_{11} = k; \quad (17)$$

$$d_{10}' = \frac{\omega d'}{\sqrt{\omega^2+(1-\omega)^2}}, \quad (18)$$

and

$$k_{10} = \frac{\sqrt{\omega^2 k^2+(1-\omega)^2}}{\sqrt{\omega^2+(1-\omega)^2}}; \quad (19)$$

$$d_{01}' = \frac{(1-\omega)d'}{\sqrt{\omega^2+(1-\omega)^2}}, \quad (20)$$

and

$$k_{01} = \frac{\sqrt{\omega^2+(1-\omega)^2k^2}}{\sqrt{\omega^2+(1-\omega)^2}}. \tag{21}$$

Thus we now have a set of predictions for ROC functions which do not make the assumption that observers always treat each interval in a two interval multiple observation task identically. It should be noted that given the above model if $\omega = 0.5$ the predictions are identical to Kinchla's (1969) unweighted sum model, against which the weighted sum model will be evaluated.

EXPERIMENT I

In the first experiment three observers performed a visual form of the multiple-observation task, specifically, they attempted to discriminate whether or not a light was briefly extinguished (a signal occurred) in at least one of two observation intervals defined by the offsets of a warning tone. The ratio of the probability of a signal occurring in interval 1, (γ_1) to the probability of a signal occurring in interval 2, (γ_2), ($r = \frac{\gamma_1}{\gamma_2}$), was varied from session to session while the proportion of trials on which no signal occurred in either interval, $(1-\gamma_1)(1-\gamma_2)$, was held constant at .40. ROC functions for the single-observation case were obtained by the use of instructions intended to manipulate the observer's criterion.

Procedure

Each of three paid ($2.00 per hour) observers was tested for five blocks of 100 trials each during each of two daily sessions for 15 days. Each session took approximately 45 min. The two daily sessions were separated from each other (end of the first to beginning of second) by a period of 2 hours. All observers had 20/20 vision with correction if needed.

The observers sat alone in a dark room. A circular area was rear illuminated by a glow modulator lamp situated at eye-level 3.5m from the observer. The circular area had an angular subtense of 0.041° and a luminance of 0.36 ft-L. A signal consisted of a 14 msec darkening of this light. Feedback was provided by briefly flashing either or both of two small dim red lights, one at either side of the signal lamp. All control was carried out by a PDP 8/S computer.

The single and double observation trials were defined as follows. Each trial in the _single observation_ case began with a 1 sec _warning tone_, the offset of which indicated the occurrence of a 14 msec _observation interval_. This was followed by a 2 sec _response interval_ during which the observer indicated whether or not a signal had occurred by pressing one of two buttons. Finally the trial concluded with a 0.5 sec _feedback_ interval during which the two red lights came on if a signal had occurred on that trial.

Each trial in the _double observation_ case consisted of a 1 sec _warning tone_, the offset of which indicated the _first_ 14 msec _observation interval_. This was followed by a 0.5 sec _delay_ which was followed by a second 1 sec _warning tone_ the offset of which indicated the _second_ 14 msec _observation interval_. This was followed by a 2 sec _response interval_ during which the observer indicated whether or not at least one signal had occurred by pressing one of two buttons. Finally the trial concluded with a .5 sec _feedback interval_ during which: a) the red light to the left of the signal lamp was illuminated if the signal had occurred during the first observation interval; b) the red light to the right of the signal lamp was illuminated if the signal had occurred during the second observation interval; c) both red lights were illuminated if the signal had occurred in both observation intervals; or, d) neither red light came on when no signals had occurred.

There were 3 single observation _instructional conditions_ designed to manipulate the observer's response bias. The "_Liberal_" instructions encouraged the observer to report a signal if he was in any doubt as to whether a signal had occurred. The "_Conservative_" instructions encouraged the observer to report no signal if he was in any doubt about whether a signal occurred. The "Neutral" instruction encouraged the observer to report a signal equally as often as he reported no signal if he had any doubt as to whether a signal had occurred. The probability of a signal, P(S), was 0.5 for all of the single observation conditions.

There were 3 double observation experimental conditions which may be designated as the "Equal", "First Interval" and "Second Interval" conditions. In all three conditions the probability of no signal occurring in either interval, $(1-\gamma_1)$ $(1-\gamma_2)$, was set at 0.4. The ratio, $\gamma_1/\gamma_2=r$, took the value 1.0 for the "Equal" condition; 2.5 for the "First Interval" condition; and 0.4 for the "Second Interval" condition. Thus in the "Equal" condition a signal was equally as likely in the first observation interval as in the second observation interval. In the "First Interval" condition a signal was more likely to occur in the first observation interval than in the second. The signal was more likely to occur in the second observation interval than the first in the "Second Interval" condition. The observers were informed of the probability of each pattern of stimulus occurrence before the start of each session; e.g., "Today there will be no signal on 40 per cent of the trials. There will be a signal in the first interval only on 40 per cent of the trials. There will be a signal in the first interval only on 40 per cent of the trials, in the second interval only on 10 per cent of the trials and in both intervals on 10 per cent of the trials".

During the first 6 sessions the observers were exposed only to the single observation task under the "Neutral" instruction, these were unrecorded practice trials. Each succeeding group of 6 sessions constituted a complete replication of the experiment, each observer was tested under each of the 6 conditions in random order. Four replications were carried out.

Results

The results of Experiment 1 are shown in Tables 1 and 2. Table 1 presents the data for each observer, pooled over the appropriate sessions, for each of the single observation conditions. Note that as the instructions change from Conservative to Moderate to Liberal that both $\hat{P}(A_1|S_1)$ and $\hat{P}(A_1|S_0)$ increase and that in all cases $\hat{P}(A_1|S_1)$ is greater than $\hat{P}(A_1|S_0)$. Both of these effects are significant ($p<.01$) by a chi-square analysis (Anderson and Goodman, 1957). The increase in the hit and false alarm rates over conditions indicates that the instructions had their desired effect. The fact that the hit rate is always greater than the False alarm rate indicates that the observers were capable of discriminating the presence of the signal from its absence.

Insert Table 1 about here

Table 2 presents the data for each observer, pooled over the appropriate sessions, for each of the double observation conditions. Chi-square analysis (Anderson and Goodman, 1957) indicates that the independent variable, r, had a significant effect on the performance of two of the three observers in so far as $\hat{P}(A_1|S_{00})$ was higher under the Equal condition then in the other two conditions for Observers 1 and 3. In addition, for Observer 1, $\hat{P}(A_1|S_{01})$ is lower under the Equal condition than under the other two conditions. The hypothesis that $\hat{P}(A_1|S_{10})$ and $\hat{P}(A_1|S_{01})$ would vary as a function of r has not been supported.

Insert Table 2 about here

TABLE 1

Estimates, pooled over sessions, of $P(A_1|S_i)$ for each observer under each single observation condition in Experiment 1.

Condition	Observer	$\hat{P}(A_1\|S_0)$	$\hat{P}(A_1\|S)$
Conservative	1	.10	.78
	2	.05	.60
	3	.06	.78
Moderate	1	.15	.88
	2	.14	.79
	3	.12	.86
Liberal	1	.19	.89
	2	.43	.92
	3	.22	.89

TABLE 2

Estimates, pooled over sessions, of $P(A_1|S_{ij})$ for each observer under each double observation condition in Experiment 1.

Condition	Observer	$\hat{P}(A_1\|S_{00})$	$\hat{P}(A_1\|S_{11})$	$\hat{P}(A_1\|S_{10})$	$\hat{P}(A_1\|S_{01})$
Equal	1	.22	.96	.74	.69
	2	.24	.95	.73	.73
	3	.20	.97	.79	.77
First Interval	1	.18	.97	.78	.81
	2	.23	.93	.72	.74
	3	.15	.97	.81	.70
Second Interval	1	.15	.97	.83	.80
	2	.23	.97	.66	.79
	3	.14	.96	.73	.77

Theoretical Analysis and Discussion.

Table 3 presents estimates of d′ and k for each observer obtained by visually fitting a straight line through the observed data points from the single observation conditions on normal deviate co-ordinates. As may be seen in Figure 1 these straight lines provide a fair account of these data.

Insert Table 3 and Figure 1 about here

Since the data failed to reveal a significant effect of r the data for the three double observation conditions were pooled for each observer as shown in Table 4.

Insert Table 4 about here

Table 5 presents estimates of d'_{ij} for each observer obtained by assuming the unweighted sum model to be true insofar as its predictions about k_{ij} are concerned, as well as predicted values of d'_{ij} under this model. This procedure was adopted since no empirical estimates of k_{ij} were obtained. It is obvious that this procedure will aid the fit of the model. It is quite clear that the model is quite well supported by the observed data.

Insert Table 5 about here

TABLE 3

Estimates of d′ and k for each observer from the single observation data of Experiment 1.

Observer	$\hat{d}'$	$\hat{k}$
1	1.90	0.869
2	2.03	1.266
3	3.05	1.818

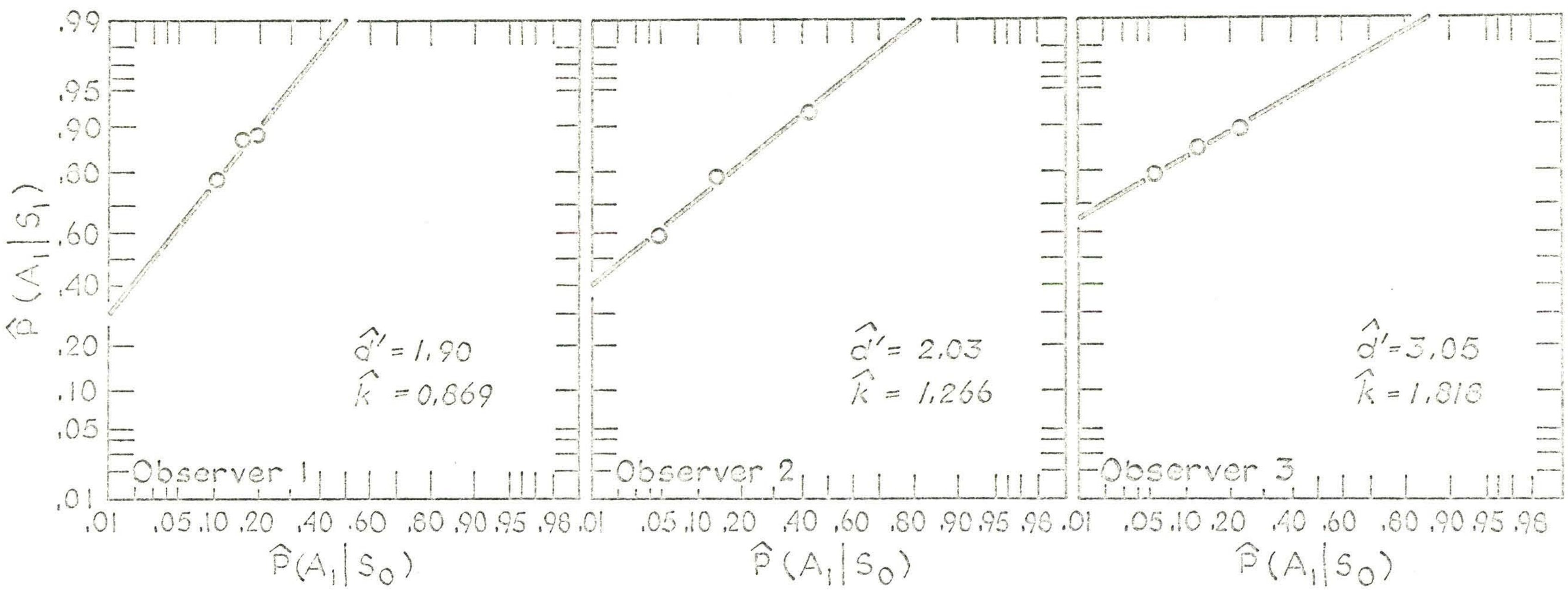

Figure 1. $\hat{P}(A_1|S_0)$ as a function of $\hat{P}(A_1|S_1)$ for each observer in Experiment 1.

TABLE 4

Pooled estimates of $\hat{P}(A_1|S_{ij})$ for each observer in Experiment 1.

Observer	$\hat{P}(A_1\|S_{00})$	$\hat{P}(A_1\|S_{11})$	$\hat{P}(A_1\|S_{10})$	$\hat{P}(A_1\|S_{01})$
1	.182	.965	.762	.765
2	.231	.949	.714	.763
3	.197	.967	.789	.727

TABLE 5

Observed and predicted values of d'_{ij} for each observer based on the pooled data from all conditions in Experiment 1. The observed values assume the predictions of k_{ij} of the unweighted sum model.

Observer		Observed	Predicted
1	d'_{11}	2.49	2.69
	d'_{10}	1.58	1.34
	d'_{01}	1.59	1.34
2	d'_{11}	2.82	2.87
	d'_{10}	1.39	1.44
	d'_{01}	1.56	1.44
3	d'_{11}	4.22	4.31
	d'_{10}	2.04	2.16
	d'_{01}	1.74	2.16

It is worth noting here that in Kinchla's (1969) work on this model the predictions concerning an improvement under multiple-look conditions and a decrement under temporal uncertainty conditions were tested in separate experiments. We see in Table 5 that the model also holds for what may be termed a "complete" multiple observation experiment. Thus Experiment 1, while it failed to bear out the hypothesis under investigation, has replicated Kinchla's (1969) findings and extended them to what we have termed a complete multiple observation experiment.

EXPERIMENT II

In the second experiment the same three observers performed the same multiple observation task as in Experiment I with $r = 1$, and $(1-\gamma_1)(1-\gamma_2)=.40$. The procedure was identical in all respects to the Equal condition of Experiment I with the following exceptions:

1. Before each of the first four sessions each observer was instructed to "concentrate more on the first interval; if you think no signal occurred in the first interval be very sure you think a signal occurred in the second interval before you make a "yes response".
2. Before each of the last four sessions each of the observers read our instruction which was the opposite of the instruction in 1) above, i.e., it urged the observer to "concentrate more on the second interval".

Results

The data for each observer, pooled over the four sessions under each condition in Experiment 2 are shown in Table 6. Chi-square analysis (Anderson and Goodman, 1957) reveals that the effect of the instructions approached statistical significance with Observer 1 ($p<.10$) and reached statistical significance with Observers 2 and 3 ($p<.05$). In general the effect was similar in all observers, $\hat{P}(A_1|S_{01})$ was smaller under instructions to concentrate more on the first interval than under instructions to concentrate more on the second interval.

Insert Table 6 about here

Theoretical Analysis and Discussion

The estimates of d′ and k from Experiment 1 (cf Table 3) were employed to obtain estimates of d'_{ij} both predicted and observed, and predicted values of k_{ij} for each observer under each condition of this experiment.

Table 7 presents the observed and predicted values of d'_{ij} for the data of this experiment obtained by employing Kinchla's unweighted sum model. As can be seen this model does not give too bad an account of these data.

Insert Table 7 about here

Figure 2 presents, plotted on normal deviate coordinates, $\hat{P}(A_1|S_{10})$ versus $\hat{P}(A_1|S_{01})$ for each observer under each condition in Experiment 2. If the assumptions of the unweighted sum model were met all of these points would lie on the major diagonal of these plots, that is $\hat{P}(A_1|S_{10})$ would equal $\hat{P}(A_1|S_{01})$. As is plain from Figure 2 this is not the case. Instead, in all cases, the point for the First Interval condition lies above the major diagonal and that for the Second Interval condition lies below the major diagonal, as would be predicted by a weighted sum model. Thus the weighted sum model will now be evaluated.

Insert Figure 2 about here

TABLE 6

Estimates, pooled over sessions, for each observer under each condition in Experiment 2.

Condition	Observer	$\hat{P}(A_1 \mid S_{00})$	$\hat{P}(A_1 \mid S_{11})$	$\hat{P}(A_1 \mid S_{10})$	$\hat{P}(A_1 \mid S_{01})$
Concentrate	1	.22	.99	.86	.81
on First Interval	2	.29	.98	.83	.80
	3	.17	.96	.78	.68
Concentrate	1	.23	.99	.86	.89
on Second Interval	2	.28	.99	.86	.94
	3	.18	.96	.72	.78

TABLE 7

Observed and predicted values of d_{ij} for each observer in Experiment 2 under Kinchla's (1969) unweighted sum model.

		Observer 1		Observer 2		Observer 3	
Condition		obs.	pred.	obs.	pred.	obs.	pred.
First Interval	d'_{11}	2.79	2.69	3.15	2.87	4.13	4.31
	d'_{10}	1.78	1.34	1.64	1.44	2.09	2.16
	d'_{01}	1.59	1.34	1.51	1.44	1.64	2.16
Second Interval	d'_{11}	2.76	2.69	3.53	2.87	4.10	4.31
	d'_{10}	1.75	1.34	1.82	1.44	1.77	2.16
	d'_{01}	1.89	1.34	2.36	1.44	2.05	2.16

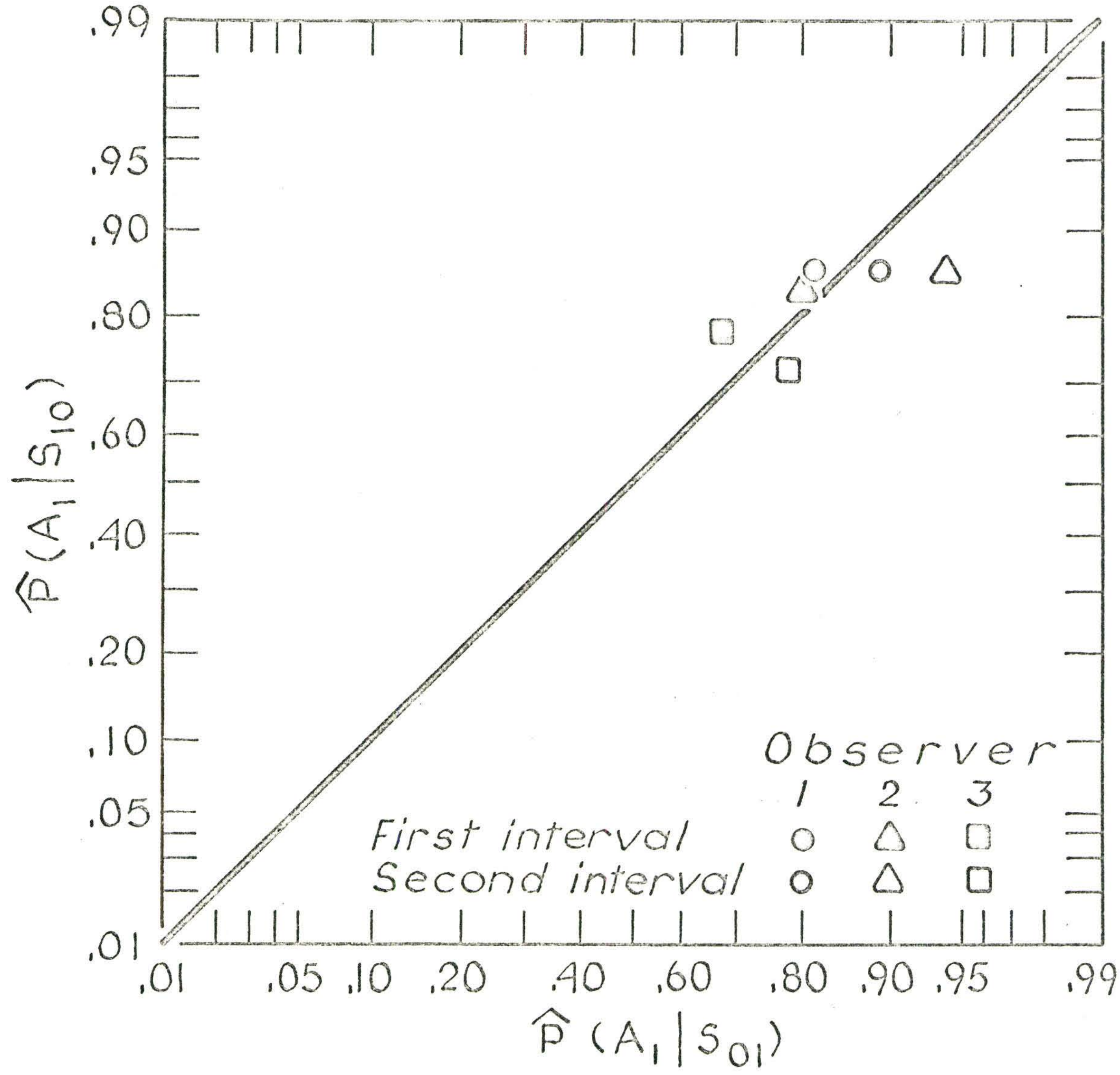

Figure 2. $\hat{P}(A_1|S_{10})$ as a function of $\hat{P}(A_1|S_{01})$ for each observer in Experiment 2.

Table 8 presents the observed and predicted values of d'_{ij} for each observer under each condition of Experiment 2 as well as the value of $\hat{\omega}$ employed in calculating the predictions. As before the values of d'_{ij} shown under the observed columns assume the predicted value of k_{ij} under the model in their calculation. $\hat{\omega}$ was chosen to minimize the sum of squared deviations between the observed and predicted values of d'_{ij}. Comparing Tables 7 and 8 we see that the addition of a weighting parameter, ω, improves slightly our predictive ability. If we take the ratio of the sums of squared deviations between the observed and predicted values of d'_{ij} for each observer under the two models we find that for Observers 1 and 2 that the sum of squared deviations is about 1.04 times greater under the unweighted sum model than under the weighted sum model. For Observer 3 the ratio is somewhat better, 1.88.

Insert Table 8 about here

In general one must conclude from this experiment that the cost (in terms of parsimony) of adding an additional weighting parameter, ω, to Kinchla's unweighted sum model is too great in relation to the gain in predictive ability obtained, although the improvement in prediction obtained for Observer 3 suggests that this may not be the case under all conditions.

TABLE 8

Observed and predicted values of d'_{ij} and estimated value of ω for each observer in Experiment 2.

Condition		Observer 1 obs.	Observer 1 pred.	Observer 2 obs.	Observer 2 pred.	Observer 3 obs.	Observer 3 pred.
		$\hat{\omega}$ = .52		$\hat{\omega}$ = .52		$\hat{\omega}$ = .59	
First Interval	d'_{11}	2.79	2.68	3.15	2.87	4.14	4.25
	d'_{10}	1.78	1.40	1.65	1.49	2.19	2.50
	d'_{01}	1.60	1.29	1.50	1.38	1.57	1.74
		$\hat{\omega}$ = .48		$\hat{\omega}$ = .47		$\hat{\omega}$ = .43	
Second Interval	d'_{11}	2.76	2.68	3.53	2.87	4.10	4.27
	d'_{10}	1.76	1.29	1.80	1.35	1.71	1.84
	d'_{01}	1.88	1.40	2.38	1.52	2.13	2.43

EXPERIMENT III

This experiment was carried out to test the hypothesis that if the feedback about one of the two intervals in a two interval multiple observation task was unreliable, i.e., non-veridical, the observer would be likely to treat the sensory information contained in that interval differentially.

Three initially naive observers took part in this experiment; each had 20/20 or better corrected vision and each was paid at the rate of \$2.00 per hour for each of two daily, one hour sessions separated by two hours from each other.

There were three single observation conditions that correspond perfectly with three single observation conditions of Experiment I. There were also four double observation conditions. For the first two of these conditions, 1 and 2, $r = 1.0$ and $\gamma_1 = \gamma_2 = 0.5$. For conditions 3 and 4, $r = 1.0$ and $\gamma_1 = \gamma_2 = 0.7$. We may designate the probability of being correctly informed of the presence of a signal in a given interval as Π_j, where j designates the particular interval. For conditions 1 and 4, $\Pi_1 = 1.0$ and $\Pi_2 = 0.1$, while for conditions 2 and 3, $\Pi_1 = 0.1$ and $\Pi_2 = 1.0$. Similarly let θ_j equal the probability of being correctly informed of the absence of a signal, where j designates the particular interval. For all conditions $\theta_j = 1.0$.

Each observer was run for four 480 trial sessions under the Neutral instructions of Experiment I. This was followed by six further single observation sessions, three under the Conservative instructions and three under the Liberal instructions of Experiment I. Each observer then ran for eight consecutive sessions under each of the four experimental double observation conditions. The first block of trials each day was an unrecorded "practice" block. Under conditions 1 and 2 each block of trials consisted of 80 trials; under conditions 3 and 4 each block consisted of 100 trials. Thus under conditions 1 and 2 there were six blocks per session with a one minute rest between blocks, while for conditions 3 and 4 there were five blocks per session with a one minute rest between blocks. The trials, with the exception of the false feedback, were identical to those of Experiments I and II. The final eight sessions were run as single observation sessions, four under the Conservative instructions and four under the Liberal instructions of Experiment I.

At the end of each of the 32 double observation sessions a "score" for the session was posted, this score consisted of the per cent "correct" on trials having a signal where correct was defined by the feedback. This was instituted to encourage the observers to attend to the feedback.

Results

The results of Experiment III are summarized in Tables 9 and 10. Table 9 presents the single observation data, pooled over each instructional condition, for each observer in the final phase of the experiment. Note that both $\hat{P}(A_1|S_1)$ and $\hat{P}(A_1|S_0)$ are higher for all observers under the Liberal instruction than under the Conservative instruction. Chi-square analysis (Anderson and Goodman, 1957) reveals this difference to be significant ($p<.01$) for all observers. Note as well that in all cases $\hat{P}(A_1|S_1)$ is greater than $\hat{P}(A_1|S_0)$. This difference is also significant and shows only that the observers were capable of discriminating the occurrence of signals from their non-occurrence.

Insert Table 9 about here

Table 10 presents the double-observation data for each observer, pooled over sessions within each condition, under each of the four experimental conditions. For all observers the effects of both the false feedback variable, Π, and the signal probability variable, γ, are significant ($p<.01$) as shown by chi-square analysis (Anderson and Goodman, 1957). The effect of the false feedback variable may be seen in the $\hat{P}(A_1|S_{10})$ and $\hat{P}(A_1|S_{01})$ vary directly with the value of Π_i, when Π_1 is high $\hat{P}(A_1|S_{10})$ is high, when Π_1 is low $\hat{P}(A_1|S_{10})$ is low; when Π_2 is high $\hat{P}(A_1|S_{01})$ is high, when Π_2 is low $\hat{P}(A_1|S_{01})$ is low. The effect of the signal probability variable manifests itself in that the values of $\hat{P}(A_1|S_{ij})$ are higher when γ is larger.

Insert Table 10 about here

Theoretical Analysis and Discussion

As was the case in the preceding experiments, the estimates of $\hat{P}(A_1|S_1)$ and $\hat{P}(A_1|S_0)$ from the single observation data (from the final phase of the experiment) were employed to graphically obtain estimates of both d' and k for each observer which are shown in Table 11.

Insert Table 11 about here

These single observation estimates were then employed in calculating all further *predicted* values. In calculating the *observed* values of d'_{ij},

TABLE 9

Estimates, pooled over sessions, of $P(A_1|S_i)$ for each observer during the last eight sessions of Experiment 3.

Instruction	Observer	$\hat{P}(A_1\|S_0)$	$\hat{P}(A_1\|S_1)$
Conservative	1	.09	.58
	2	.18	.66
	3	.03	.65
Liberal	1	.22	.77
	2	.50	.83
	3	.49	.92

TABLE 10

Estimates, pooled over sessions, of $P(A_1|S_{ij})$ for each observer under each double observation condition in Experiment 3.

Condition			Observer	$\hat{P}(A_1\|S_{00})$	$\hat{P}(A_1\|S_{11})$	$\hat{P}(A_1\|S_{10})$	$\hat{P}(A_1\|S_{01})$
π_1	π_2	γ					
1.0	0.1	0.5	1	.08	.63	.51	.27
			2	.26	.71	.58	.43
			3	.12	.86	.84	.17
0.1	1.0	0.5	1	.11	.71	.19	.56
			2	.33	.76	.36	.64
			3	.15	.91	.11	.84
0.1	1.0	0.7	1	.21	.88	.21	.79
			2	.53	.86	.48	.75
			3	.25	.96	.26	.90
1.0	0.1	0.7	1	.28	.86	.87	.34
			2	.46	.87	.86	.45
			3	.13	.96	.96	.17

TABLE 11

Estimates of d′ and k for each observer from the single observation data of the last eight sessions of Experiment 3.

Observer	$\hat{d}'$	$\hat{k}$
1	1.55	1.059
2	1.61	1.690
3	2.58	1.820

the theoretically appropriate value of k_{ij} was employed, that is, the model under consideration was assumed to be correct in predicting changes in the ratio of standard deviations parameter.

Table 12 presents the observed and predicted values of d'_{ij} for each observer under each condition as calculated by assuming Kinchla's (1969) unweighted sum of sensory information model of multiple observation tasks. It is readily apparent that this model gives a very poor account of the data, insofar as it predicts that d'_{10} will equal d'_{01} under all conditions while the observed values of d'_{10} are invariably greater than the observed values of d'_{01} under the first and fourth conditions while d'_{01} is invariably greater than d'_{10} under the second and third conditions. This may be seen graphically in Figure 3, which present plots of $\hat{P}(A_1|S_{01})$ against $\hat{P}(A_1|S_{01})$ (on normal deviate co-ordinates) for each observer under each of the four double observation conditions of Experiment III. Note that the unweighted sum model predicts that all of these points should fall on the major diagonal, while none of them do.

Insert Table 12 and Figure 3 about here

Table 13 presents estimated values of ω and both predicted and observed value of d'_{ij} for each observer under each condition in Experiment III. The values of $\hat{\omega}$ shown were chosen to minimize the sum of squared deviations between the observed and predicted values of d'_{ij}. It is apparent from Table 13 that the assumption that the observers make use of a <u>weighted</u> sum of sensory information rather than a simple sum leads to a much greater agreement between the observed and predicted values. The weighted sum assumption leads to a reduction in the sums of squared deviations between

TABLE 12

Predicted and observed values of d'_{ij} for each observer under each condition of Experiment 3 under the unweighted sum assumption.

Condition	Measure	Observer 1		Observer 2		Observer 3	
		Obs.	Pred.	Obs.	Pred.	Obs.	Pred.
1	d'_{11}	1.76	2.20	1.58	2.28	3.14	3.64
	d'_{10}	1.43	1.10	0.92	1.14	2.64	1.82
	d'_{01}	0.77	1.10	0.40	1.14	-0.23	1.82
2	d'_{11}	1.81	2.20	1.63	2.28	3.48	3.64
	d'_{10}	0.32	1.10	-0.06	1.14	-0.76	1.82
	d'_{01}	1.28	1.10	0.94	1.14	2.50	1.82
3	d'_{11}	2.05	2.20	1.75	2.28	3.86	3.64
	d'_{10}	-0.02	1.10	-0.15	1.14	-0.27	1.82
	d'_{01}	1.63	1.10	0.86	1.14	2.56	1.82
4	d'_{11}	1.73	2.20	2.00	2.28	4.31	3.64
	d'_{10}	1.74	1.10	1.60	1.14	3.70	1.82
	d'_{01}	0.13	1.10	-0.11	1.14	-0.27	1.82

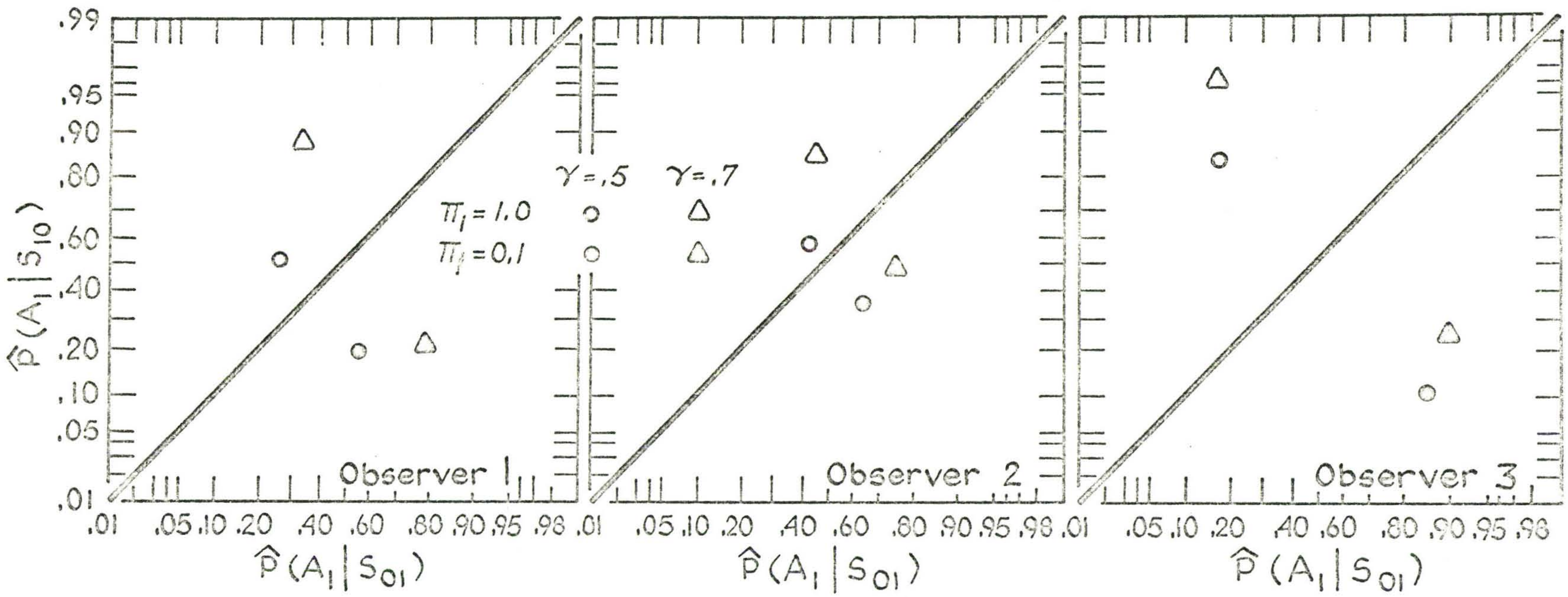

Figure 3. $\hat{P}(A_1|S_{10})$ as a function of $\hat{P}(A_1|S_{01})$ for each observer in Experiment 3.

the predicted and observed d'_{ij}'s by factors ranging from approximately four for Observer 3 to thirteen for Observer 1. Figure 4 presents a plot of observed versus predicted values of d'_{ij} for all observers under all four double observation conditions. It is clear that the fit of the model is quite good.

Insert Table 13 and Figure 4 about here

The primary reason for varying signal probability, γ, was to generate ROC curves under the false feedback conditions. It had been hoped that ω would remain constant over the signal probability manipulation thus allowing an experimental evaluation of the predictions of the weighted sum model with regard to k_{ij}. Unfortunately ω seems to interact with either γ or time (it is impossible to determine which from the present experiment) in such a way that the planned analysis could not be carried out. It seems likely that the best way to test the predictions of the model with regard to k_{ij} will require the use of category rating procedures (Green and Swets, 1966) in the future.

It thus seems quite clear that the primary hypothesis of this experiment was confirmed, namely, that observers will give less weight to the sensory information contained in an observation interval about which unreliable feedback information has been given.

An important difference between the present study and a previous study (Kinchla and Atkinson, 1964) employing false feedback should be noted. Kinchla and Atkinson report that the effect of false feedback in a single observation, auditory detection task was on response bias and not on sensitivity. In the present study, which is very different, the effect of

TABLE 13

Estimated values of ω and observed and predicted values of d'_{ij} for each observer under each double observation condition in Experiment 3 under the weighted sum assumption.

Condition	Measure	Observer 1		Observer 2		Observer 3	
		Observed	Predicted	Observed	Predicted	Observed	Predicted
1		$\hat{\omega} = 0.70$		$\hat{\omega} = 0.80$		$\hat{\omega} = 0.89$	
	d'_{11}	1.76	2.03	1.58	1.95	3.14	2.88
	d'_{10}	1.43	1.42	0.98	1.56	2.98	2.56
	d'_{01}	0.79	0.61	0.46	0.39	0.21	0.32
2		$\hat{\omega} = 0.18$		$\hat{\omega} = 0.08$		$\hat{\omega} = 0.09$	
	d'_{11}	1.81	1.85	1.63	1.74	3.48	2.82
	d'_{10}	0.35	0.33	0.08	0.14	-0.20	0.25
	d'_{01}	1.39	1.51	1.04	1.60	2.84	2.57
3		$\hat{\omega} = 0.15$		$\hat{\omega} = 0.00$		$\hat{\omega} = 0.17$	
	d'_{11}	2.05	1.80	1.75	1.61	3.86	3.04
	d'_{10}	0.00	0.27	-0.13	0.00	0.00	0.52
	d'_{01}	1.66	1.53	1.06	1.61	2.97	2.53
4		$\hat{\omega} = 0.91$		$\hat{\omega} = 0.90$		$\hat{\omega} = 0.79$	
	d'_{11}	1.73	1.70	2.00	1.78	4.31	3.16
	d'_{10}	1.78	1.54	1.92	1.60	4.29	2.49
	d'_{01}	0.14	0.15	-0.05	0.18	0.10	0.66

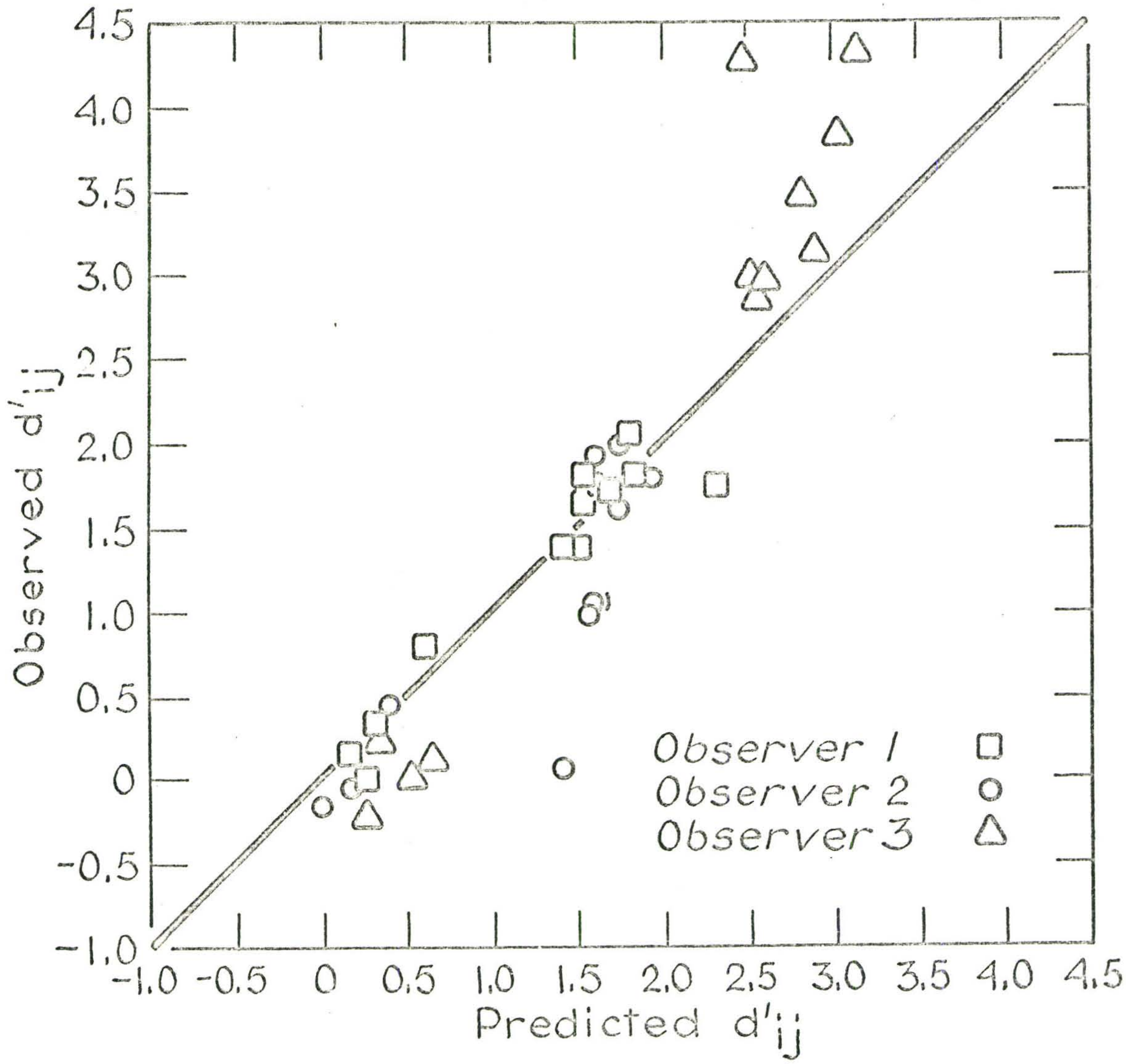

Figure 4. Observed versus predicted values of d'_{ij} for each observer in Experiment 3 under the weighted sum assumption.

false feedback in sequential multiple observation tasks seems to be a depression in a sensitivity measure. It seems reasonable to assume, that sensitivity is a relatively stable property of the organism. Given this assumption we would argue that the effects of false feedback are seen in the present experiment not on the activation or sensory process and not on the decision (response bias) process but rather on a third, post-sensory and pre-decision cognitive process which is responsible for the combination of information and is responsive to reinforcement variation.

DISCUSSION

The results of the preceding three experiments raise a number of points which must be discussed. The first of these has to do with the necessity of adding a third parameter to signal detection theory and the interpretation of that parameter, and the second point has to do with the differences between simultaneous and sequential multiple observation tasks. We turn now to these points.

It is always a major step to add yet another parameter to any model or theory and not something one does lightly. Indeed on the basis of Experiments 1 and 2 the addition of a third parameter to current signal detection theory models of multiple observation tasks would be unjustified. The results of Experiment 3, on the other hand, confirm our initial bias that under some conditions it is probably unlikely that in arriving at a decision observers will treat the sensory information contained in each observation interval identically. Under these conditions it becomes necessary to assume an information combination process that is more complex than the simple addition of sensory information. The information combination process that has been proposed here is a weighted summing of the sensory information. By adding this third, weighting parameter, ω, it is possible to greatly improve the predictive power of the simple summing model since it is easily shown that that model is the special case of the weighted sum model when $\omega = .5$.

It was argued above that ω may be interpreted as representing the quantitative action of a pre-decision, post-sensory cognitive process, that is responsive to reinforcement variables. Future research will have to be directed at determining the precise relation of ω to the parameters of the reinforcement (feedback) schedules employed.

Finally the results of the experiments reported here have some bearing on the problem of "attention" in psychology. Carruthers (1968) and Kinchla (1969b) have reported data from a simultaneous multiple observation task which except for being simultaneous was identical to that employed here. They instructed their subjects to "pay more attention" to one or the other of two spatially separated signal sources. Given the separation they employed (5°) they found that on multiple look trials that observers could do no better than their single observation performance. Swets and Kristofferson (1969) and Kinchla (1969b) interpret this finding as support for an all or none attention hypothesis or what Treisman (1969) has termed a "selection of inputs" view of attention.

An input selective model of sequential multiple observation tasks might argue that on some trials the observer "attends" to interval 1, on some trials to interval 2, on some trials to both intervals and on some trials to neither interval. To be sure such a model can account for the results of the experiments reported here by assuming that in Experiments 1 and 2 on all, or nearly all, of the trials the observers attended to both inputs and that on trials where both inputs are attended to, summation occurs, while in Experiment 3 the observers attending strategies varied as a function of Π and Υ. It is unclear, however, how one would measure the various probabilities of attending to the different observation intervals.

The major point to be made here is that an alternative view of what is causing the effects found in experiment 3 can account for the results of that experiment in a more parsimonious way. If we assume, as does the weighted sum model, that all inputs are attended on every trial, i.e., get into the system, then by postulating a post-sensory, pre-decision weighting process we may account for the results of all those experiments with the addition of only one parameter, ω. Note that the action of this process is to act upon the sensory information available to the observer in such a manner as to change his decision axis, and thereby to affect his response selection. This view is akin to what Treisman (1969) has termed the response selective view of attention (Deutsch and Deutsch, 1963, 1967).

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Appendix A

Instructions employed in Experiment 1.

This experiment is designed to study your ability to discriminate brief changes in the illumination of a light. During each experimental session you will be seated in a dark room where you can see a point of light a few feet in front of you. Your task will be to detect very brief off-periods of the light. We shall refer to these brief "flicks" of the light as "signals."

One hour's work in the lab will be called a "session." Each session will consist of a series of "trials." Each trial will begin with a brief (1 second) warning tone. At the offset of the tone there may or may not be a signal on the light. Your task will be to report whether or not you perceived a signal by pushing the appropriate button on the arm of your chair. After the response period two red lights, one on either side of the signal light, will come on for ½ second if a signal occurred during the trial. If no signal occurred, the red lights will not come on. We shall refer to this type of session as the "Single Observation Case."

In the "Double Observation Case" the session will be comprised of trials which require 2 observations each. There will be one observation to be made at the offset of the first tone. A short time later there will be a second tone. At the offset of the second tone, there will be another observation to be made. In all sessions where there are two observations to be made, your task will be to report whether or not you observed at least one signal by pressing the appropriate push button. After you have responded, the left red light will come on if only the first observation interval contained a signal, the right red light will come on if only the second observation interval contained a signal, both red lights will come on if both observation intervals contained signals. If no signal occurred during either observation interval neither light will come on.

In both the single and double observation cases you will have about 2 seconds to indicate your response. (This may seem a very short time, but with practice you will find you have plenty of time to make your decision.) After this response period and the ½ second "feedback" period the next trial will begin with the sound of another warning tone.

Each session will consist of 100 "warm-up" or "practice" trials, followed by 4 "blocks" of trials. A block will consist of 100 trials. There will be a short (1 minute) rest period between blocks of trials.

There will be three variations of the single observation case, and three of the double observation case, making a total of 6 different kinds of sessions.

Condition 1

Single observation   Liberal

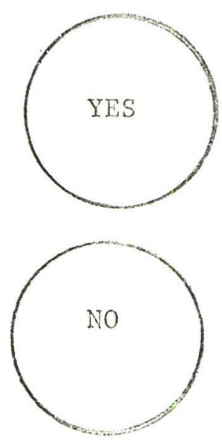
For this condition there will be just <u>1 observation</u> to be made per trial. You should report a signal on about 80 per cent of the trials. Since you are to report a signal unless you are quite sure none occurred, we will call this the "<u>liberal</u>" case.

If you are quite sure no signal occurred, press the bottom "NO" button, otherwise press the top "YES" button.

Condition 2

Single observation Neutral

For this condition there will be just 1 observation to be made per trial. You should report a signal on about 50 per cent of the trials. Since you should respond "YES" and "NO" about equally often, we will call this the "neutral" case.

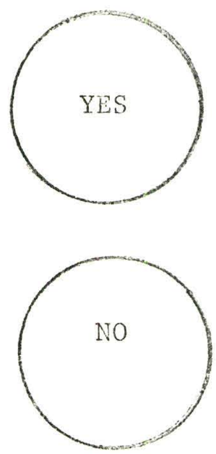


If you perceived a signal press the top "YES" button. If you did not perceive a signal press the bottom "NO" button.

Condition 3

Single observation — Conservative

For this condition there will be just 1 observation to be made per trial. You should report a signal on only about 20 per cent of the trials. Since you should not report a signal unless you are quite sure you saw one, we will call this the "conservative" case.



If you are quite sure a signal occurred, press the top "YES" button, otherwise press the bottom "NO" button.

Condition 4

Double Observation — Equal

For this condition there will be 2 observations to be made per trial.

At the offset of the first warning tone a signal may, or may not, occur on the light. After ½ second there will be a second warning tone and again a signal may or may not occur. After watching for the signal a second time you will have 2 seconds to indicate your response

Today there will be no signal on 40 per cent of the trials. There will be a signal in the first interval only on 23 per cent of the trials, in the second interval only on 23 per cent of the trials and in both intervals on 14 per cent of the trials.

If you think that a signal occurred after the first tone or after the second tone or after both tones press the top "YES" button. If you perceived no signal either time press the bottom "NO" button. You should respond "YES" about 60 per cent of the time.

Condition 5

Double Observation First Interval

For this condition there will be <u>2 observations</u> to be made per trial.

At the offset of the first warning tone a signal may, or may not, occur on the light. After ½ second there will be a second warning tone and again a signal may or may not occur. After watching for the signal a second time you will have 2 seconds to indicate your response.

Today there will be no signal on 40 per cent of the trials. There will be a signal in the first interval only on 40 per cent of the trials, in the second interval only on 10 per cent of the trials, and in both intervals on 10 per cent of the trials.

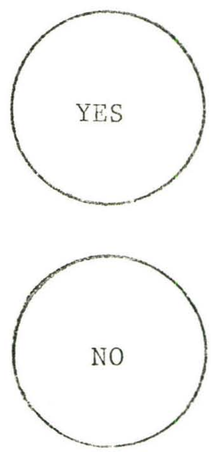
If you think that a signal occurred after the first tone or after the second tone or after both tones press the top "YES" button. If you perceived no signal either time press the bottom "NO" button. You should respond "YES" about 60 per cent of the time.

Condition 6

Double Observation | Second Interval

For this condition there will be 2 observations to be made per trial.

At the offset of the first warning tone a signal may, or may not, occur on the light. After ½ second there will be a second warning tone and again a signal may or may not occur. After watching for the signal a second time, you will have 2 seconds to indicate response.



Today there will be no signal on 40 per cent of the trials. There will be a signal in the first interval only on 10 per cent of the trials, in the second interval only 40 per cent of the trials and in both intervals on 10 per cent of the trials.

If you think that a signal occurred after the first tone or after the second tone or after both tones, press the top "YES" button. If you perceived no signal either time, press the bottom "NO" button. You should respond "YES" about 60 per cent of the time.

APPENDIX B

Daily frequencies of A, responses for each observer under each condition of Experiment 1.

TABLE B. 1

Single Observation Data

	Observer 1		Observer 2		Observer 3	
Condition	$F(A_1 \mid S_1)$	$F(A_1 \mid S_0)$	$F(A_1 \mid S_1)$	$F(A_1 \mid S_0)$	$F(A_1 \mid S_1)$	$F(A_1 \mid S_0)$
Liberal	183	34	176	31	183	45
	176	46	191	100	183	37
	174	38	187	108	173	50
	176	37	188	101	173	46
Moderate	173	18	152	29	169	17
	177	34	167	22	165	29
	178	33	156	26	169	34
	172	35	155	34	181	18
Conservative	142	12	167	32	157	5
	160	27	120	3	158	9
	169	22	104	1	155	14
	151	22	85	3	157	20

TABLE B. 2

Double observation data

Cond'n	Observer 1 $F(A_1\|S_{00})$	$F(A_1\|S_{11})$	$F(A_1\|S_{10})$	$F(A_1\|S_{01})$	Observer 2 $F(A_1\|S_{00})$	$F(A_1\|S_{11})$	$F(A_1\|S_{10})$	$F(A_1\|S_{01})$	Observer 3 $F(A_1\|S_{00})$	$F(A_1\|S_{11})$	$F(A_1\|$
	30	49	65	64	30	54	68	63	30	56	73
r = 1.0	33	55	67	57	25	53	71	66	32	53	76
	43	56	62	67	47	51	62	69	31	55	73
	32	56	77	67	49	55	69	69	33	54	67
	41	36	107	27	29	39	108	25	15	37	128
r = 2.5	23	40	125	32	33	39	113	31	27	40	134
	29	40	138	37	34	33	118	33	25	39	130
	21	39	130	34	49	37	119	30	32	39	125
	24	38	30	128	27	38	26	118	21	38	28
r = 0.4	28	38	33	131	35	40	21	134	25	38	28
	15	40	36	128	44	38	33	123	20	39	33
	31	39	33	125	41	39	26	130	25	38	27

APPENDIX C

Summary of χ^2 Statistics

for Experiment 1.

TABLE C.1

Single Observation Data

Effect		Obs 1	Obs 2	Obs 3
Instructions	χ^2	69.1	618.3	127.4
	df	4	4	4
	$p \leq$	.01	.01	.01
Stimuli	χ^2	2332.5	1522.3	2406.7
	df	1	1	1
	$p \leq$	.01	.01	.01

TABLE C.2

Double Observation Data

Observer	χ^2	df	$p \leq$
1	28.93	6	.05
2	11.67	6	.10
3	18.87	6	.05

APPENDIX D

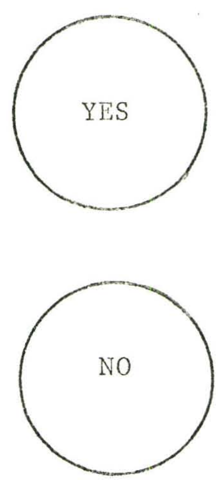
Instructions employed in Experiment 2

Condition 1

Double Observation — First

For this condition there will be <u>2 observations</u> to be made per trial.

At the offset of the first warning tone a signal may, or may not, occur on the light. After $\frac{1}{2}$ second there will be a second warning tone and again a signal may or may not occur. After watching for the signal a second time you will have 2 seconds to indicate your response.



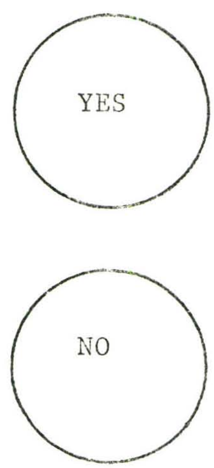
Today there will be no signal on 40% of the trials. You should concentrate on reporting signals in the first interval. It is important that you say "YES" on all of the trials that have a signal in the first interval. If you saw "NO" signal in the first interval you should be very sure there was a signal during the second interval before you say "YES".

Condition 2

Double Observation | Second

For this condition there will be 2 observations to be made per trial.

At the offset of the first warning tone a signal may, or may not, occur on the light. After ½ second there will be a second warning tone and again a signal may or may not occur. After watching for the signal a second time you will have 2 seconds to indicate your response.



Today there will be no signal on 40% of the trials. You should concentrate on reporting signals in the second interval. It is important that you say "YES" on all of the trials that have a signal in the second interval. If you saw "NO" signal in the second interval you should be very sure there was a signal during the first interval before you say "YES".

APPENDIX E

Daily Frequencies of A_1 responses

to each stimulus pattern

for each observer under

each condition of Experiment 2

TABLE E.1

Double Observation Data

Obs.	Condition	$F(A_1\|S_{00})$	$F(A_1\|S_{11})$	$F(A_1\|S_{10})$	$F(A_1\|S_{01})$
1	First Interval	35	55	74	72
		33	56	79	75
		39	56	80	72
		35	56	83	80
	Second Interval	45	56	74	75
		42	55	80	83
		27	56	83	86
		34	56	80	85
2	First Interval	34	55	74	67
		40	55	72	72
		45	54	80	75
		64	56	79	80
	Second Interval	77	54	75	84
		54	56	75	83
		80	56	82	87
		106	56	84	91
3	First Interval	28	54	71	60
		19	56	76	62
		36	52	72	63
		28	52	68	64
	Second Interval	30	53	68	68
		21	54	63	75
		31	54	75	77
		33	54	60	68

APPENDIX F

Summary of χ^2 statistics

for Experiment 2

TABLE F.1

Observer	χ^2	df	$p \leq$
1	9.9	4	.10
2	91.2	4	.05
3	13.8	4	.05

APPENDIX G

Instructions employed in

Experiment 3

This experiment is designed to study your ability to discriminate brief changes in the illumination of a light. During each experimental session you will be seated in a dark room where you can see a point of light a few feet in front of you. Your task will be to detect very brief off-periods of the light. We shall refer to these brief "flicks" of the light as "signals".

One hour's work in the lab will be called a "session". Each session will consist of a series of "trials". Each trial begins with a brief (1 second) warning tone. At the offset of the tone there may, or may not, be a signal on the light. Your task will be to report whether or not you perceived a signal by pushing the appropriate button on the arm of your chair. After the response period a red light, on the left side of the signal light, will come on for $\frac{1}{2}$ second if a signal occurred during the trial. If no signal occurred the red light will not come on. We shall refer to this type of session as the "Single Observation Case".

In the "Double Observation Case" the session will be comprised of trials which require 2 observations each. There will be one observation to be made at the offset of the first tone. A short time later there will be a second tone. At the offset of the second tone there will be another observation to be made. Signals can occur in either the first observation interval, the second observation interval or both observation intervals. Your task is to report whether or not you observed at least one signal during the trial. If you observed at least one signal press the "yes" button, if you observed no signal press the "no" button. If at the end of the trial either or both red lights have come on a "yes" response was correct. If, on the other hand, no red lights have come on, a "no" response was correct. You should always try your best to be correct as often as you can. The _left_ red light is related to the occurrence of signals in the _first_ interval and the _right_ red light is related to the occurrence of signals in the _second_ interval.

In both the single and double observation cases you will have about 2 seconds to indicate your response. (This may seem a very short time, but with practice you will find you have plenty of time to make your decision.) After this response period and the $\frac{1}{2}$ second "feed-back" period the next trial will begin with the sound of another warning tone.

Each session will consist of 80 "warm-up" or "practice" trials, followed by 5 "_blocks_" of trials. A block will consist of 80 trials. There will be a short (1 minute) rest period between blocks of trials.

APPENDIX H

Daily frequencies of A_1 responses to each stimulus pattern for each observer under each condition in Experiment 3.

TABLE H.1

Single observation data, all observers.

Condition	Observer 1 $F(A_1 \mid S_1)$	Observer 1 $F(A_1 \mid S_0)$	Observer 2 $F(A_1 \mid S_1)$	Observer 2 $F(A_1 \mid S_0)$	Observer 3 $F(A_1 \mid S_1)$	Observer 3 $F(A_1 \mid S_0)$
Conservative	113	20	152	54	123	3
	117	18	118	30	141	5
	127	16	145	33	131	4
	108	20	115	43	123	8
Liberal	145	87	167	91	190	111
	157	46	167	102	182	104
	153	27	158	114	179	99
	157	19	173	95	184	77

TABLE H.2

Observer 1, Double observation data.

Condition	$F(A_1 \mid S_{00})$	$F(A_1 \mid S_{11})$	$F(A_1 \mid S_{10})$	$F(A_1 \mid S_{01})$
$\gamma = .5$	10	64	48	33
$\pi_1 = 1.0$	10	48	31	30
$\pi_2 = 0.1$	6	64	47	26
	10	80	62	33
	6	63	54	21
	10	60	57	27
	10	67	48	30
	5	62	60	18
	11	60	26	54
$\gamma = .5$	9	71	16	57
$\pi_1 = 0.1$	9	77	13	61
$\pi_2 = 1.0$	23	62	26	52
	7	75	13	65
	10	64	13	57
	10	76	12	64
	10	81	4	63
$\gamma = .7$	5	165	12	55
$\pi_1 = 0.1$	5	173	10	65
$\pi_2 = 1.0$	6	173	16	61
	9	185	16	59
	10	180	16	63
	11	182	20	71
	13	176	22	65
	7	175	20	65
$\gamma = .7$	8	162	67	30
$\pi_1 = 1.0$	11	167	70	22
$\pi_2 = 0.1$	8	170	73	23
	9	174	72	13
	18	166	66	31
	12	180	71	29
	13	181	72	41
	11	173	65	30

Table H. 3

Observer 2, double observation data.

Condition	$F(A_1 \mid S_{00})$	$F(A_1 \mid S_{11})$	$F(A_1 \mid S_{10})$	$F(A_1 \mid S_{01})$
$\gamma = .5$	33	69	62	41
$\pi_1 = 1.0$	23	78	50	51
$\pi_2 = 0.1$	34	63	60	45
	15	69	55	38
	29	62	47	38
	22	78	70	47
	40	68	59	50
	10	83	62	37
$\gamma = .5$	20	74	33	55
$\pi_1 = 0.1$	26	72	36	63
$\pi_2 = 1.0$	39	67	40	64
	38	77	38	62
	37	85	37	75
	43	82	40	65
	34	65	34	64
	31	83	29	67
$\gamma = .7$	17	170	36	57
$\pi_1 = 0.1$	26	168	41	53
$\pi_2 = 1.0$	25	175	44	62
	14	174	29*	56
	17	176	45	66
	22	168	35	62
	26	170	38	66
	24	181	39	61
$\gamma = .7$	22	161	63	55
$\pi_1 = 1.0$	23	181	72	33
$\pi_2 = 0.1$	11	174	68	26
	13	182	67	31
	19	171	67	35
	27	178	73	46
	15	175	70	29
	18	176	66	34

* $n = 79$.

TABLE II.4

Observer 3, double observation data

Condition	$F(A_1 \mid S_{00})$	$F(A_1 \mid S_{11})$	$F(A_1 \mid S_{10})$	$F(A_1 \mid S_{01})$
γ = .5	14	86	76	37
π_1 = 1.0	12	77	84	16
π_2 = 0.1	12	92	79	11
	6	89	86	17
	17	85	80	13
	4	83	90	15
	10	92	91	13
	13	81	87	10
γ = .5	12	93	12	79
π_1 = 1.0	8	93	9	81
π_2 = 0.1	11	95	5	83
	18	93	6	90
	8	91	6	86
	26	85	20	72
	21	89	16	84
	16	89	11	89
γ = .7	13	191	24	75
π_1 = 0.1	8	188	20	72
π_2 = 1.0	19	179	43	70
	12	194	18	70
	7	194	12	72
	4	196	19	73
	7	195	14	76
	10	192	14	70
γ = .7	6	193	75	6
π_1 = 1.0	2	188	79	14
π_2 = 0.1	4	194	77	13
	6	189	77	16
	6	193	78	20
	4	192	76	14
	5	191	78	14
	9	193	74	12

APPENDIX I

Summary of χ^2 Analyses of Experiment 3

TABLE I.1

Single Observation Data

Effect		Obs. 1	Obs. 2	Obs. 3
Instructions	χ^2	113.1	220.9	623.9
	df	2	2	2
	$p \leqslant$	.01	.01	.01
Stimuli	χ^2	860.2	505.7	887.6
	df	1	1	1
	$p \leqslant$	.01	.01	.01

TABLE I.2

Double Observation Data

Effect		Obs. 1	Obs. 2	Obs. 3
Π	χ^2	796.4	450.3	2930.1
	df	4	4	4
	$p \leq$	.01	.01	.01
Υ	χ^2	3972.0	956.1	158.5
	df	4	4	4
	$p \leq$	.01	.01	.01