EMPIRICAL HOUSEHOLD BEHAVIOUR

THREE ESSAYS ON EMPIRICAL HOUSEHOLD BEHAVIOUR

By

XIAODI XIE, B. Eng, M.A.

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AUTHOR:	Xiaodi Xie,	B.Eng.	(Lanzhou University, P.R. China)
		M.A.	(University of New Brunswick)

SUPERVISOR: Professor M. Browning

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ABSTRACT

This thesis consists of three essays on empirical household behaviour: in particular, on demand and female labour supply decisions. The first essay examines empirically whether the condition for aggregation across goods and the condition for aggregation across individuals are accepted using Canadian family expenditure surveys. A new demand system is developed to test both conditions. Both conditions are rejected by the data. This places some doubt on the use of single good, representative agent models in macroeconomics.

The second essay is concerned with the impact of children on labour supply decisions of married women using the 1975 labor supply data from Panel Study of Income Dynamics. Some previous research treated the number of children as a continuous variable and has found that the number of children is exogenous in the hours of work equation of married women, suggesting that the number of children is determined independently of hours worked. This essay finds that treating the number of children as a discrete integer may be important when testing for their individual exogeneity in the hours equation. It also finds that children and labour force participation are a joint decision for married women. The findings in this essay emphasize the importance of

considering both the potential endogeneity and the discrete count data nature of the number of children when estimating the policy-related parameters in labour supply equations, especially in the participation equation.

The third essay investigates the cost of children through equivalence scales using Canadian family expenditure surveys. It generalises Blackorby and Donaldson's (BD) condition concerning the structure of preferences. Both the BD condition and its generalization determine the equivalence scale uniquely from the demand data. It is found that the restriction implied by the BD condition regarding the budget share for the 'children only' good is rejected, whereas that of its generalization is not. A new rank three demand system is developed to examine the testable implications of the generalized BD condition, which are then rejected. Nevertheless, equivalence scales are estimated under the generalized BD condition. It is found that the cost of a child increases with both the age of a child and the labour force involvement of the female adult, and decreases with the income level of the household.

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CHAPTER ONE

INTRODUCTION

The structural modelling of individual choices using survey data and the analysis of microeconomic policy issues have interested many economists. For public policy makers, the consideration of both the positive and normative implications of government policies is an important part of policy analysis. If one wants to carry out a proper public policy analysis, one needs to know the structural parameters of the model employed. Therefore, it is important to be clear about assumptions of the model and investigate their validity. It is important to take into consideration the endogeneity of variables. It is important to compute empirical estimates of policy-related parameters from a model which has a sound theoretical basis.

From a strict microeconomic perspective, the only true behavioral constants are taste and technology parameters. The structural equations in a model are the first order conditions that follow from a constrained maximization problem involving tastes, technology, and market constraints. Structural parameters are estimated from the utility function which represents the tastes of individuals and from the production function which represents the technology of firms. Structural parameters are independent of policy changes. Public policy works by changing the market constraints facing individuals. The impact of alternative policies can then be investigated by changing the values of policy parameters.

Many public policies involve issues concerning household behaviour such as labour supply. In the recent discussion paper released by the Ministry of Human Resources Development, proposed reforms on the unemployment insurance system and the Child Tax Benefit program focus on removing the disincentive to work and providing better income support for low-income families with children. The Child Tax Benefit program recognizes the additional costs borne by families that raise children, especially by poor families. A study of the impact of these reforms (if implemented) can be best carried out through microeconometric analysis of the data at the individual level because these reforms intend to affect households' decisions on labour supply and fertility.

This thesis is composed of three essays. Each is contained separately in the three chapters that follow. All are concerned with empirical household behaviour, in particular, demand and female labour supply decisions. All the empirical results are obtained through the programs written in Gauss programming language (version 2.1) by the author. The connecting thread of this thesis is the interest in structural modelling of household behavior. In the following paragraphs, a brief introduction to each essay is provided.

The first essay examines empirically whether the condition for aggregation across goods and the condition for aggregation across individuals are accepted using data from the Canadian family expenditure surveys. There are two kinds of aggregation problems in the study of systems of consumer demand equations: aggregation across commodities and aggregation across individuals. Conditions are known under which both types of aggregation are possible in the economic literature (Gorman (1959), Muellbauer, (1975)).

The problem of aggregation across individuals concerns the connection between the microeconomics of consumer behaviour and the analysis of aggregate market demand. There is no obvious reason why macroeconomic relations should replicate their microeconomic foundations. The conditions under which aggregation across commodities is possible are interesting because of the need to reduce the number of parameters necessary to characterize a system of demand equations.

The primary goal of consumer demand modelling is to determine empirically the price and income elasticities of demand for specific commodities. These elasticities play a critical role in projecting demand and evaluating the impact of economic policy on consumer welfare. To test both aggregation conditions, a new demand system is developed, which nests both a demand system where the condition for aggregation across individuals is satisfied and a demand system where the condition for aggregation across commodities is satisfied. Evidence is found that both conditions are rejected by the data. This places some doubt on the use of single good, representative agent models in macroeconomics.

The second essay is concerned with the impact of children on female labour supply decisions. The labour supply behaviour of women is interesting because it has important implications for many policy related topics, including the distribution of income and poverty problems, wage differentials, welfare payments and social security. It is realized that many aspects of household decisions are interconnected. For example, the presence of children in the household is closely associated with female labour supply decisions. Inclusion of possibly endogenous children variables in the labour supply equations will likely lead to biased estimates of labour supply parameters. Hence, in order to estimate labour supply parameters more precisely, careful modelling of children variables is essential. Some studies on female labour supply decisions (e.g., Mincer (1962), Heckman (1974), Heckman and MaCurdy (1980)) treat children as exogenously imposed constraints on the household decision-making process. Other studies (e.g., Schultz (1978, 1980), Mroz (1987)) have argued that there are mutual dependencies between the labour supply of married women and fertility (usually proxied by the number of children in specified age groups in the household) and have allowed for this endogeneity of fertility in models of female labour supply. None of these studies have explicitly taken into consideration the count nature of the children variables when testing the exogeneity of the children variables in equations for female labour supply decisions (i.e., the labour force participation decision and the hours of work decision if participating). Hence, in the second essay, the count nature of the children variables is modeled explicitly when the exogeneity assumption of the children variables in equation for the labor supply decisions of married women is investigated using the 1975 labour supply data from the Panel Study of Income Dynamics. This essay finds that treating children variables as count data may be important when testing for their individual exogeneity in the hours equation. Furthermore, there is evidence that children and labour force participation are a joint decision for married women. The findings emphasize the importance of considering both the potential endogeneity and the discrete count data nature of the children variables when estimating the policy-related parameters in labour supply equations, especially in the participation equation.

The third essay investigates the cost of children through equivalence scales. Households differ in their characteristics, and the needs of their individual members vary with their ages, work status and other characteristics. Therefore expenditure behaviour across households is expected to be different. In terms of designing or implementing government welfare policies, it is essential to know how well off the members of one household are relative to those of another. In the literature, the notion of equivalence scales is developed to bridge demand behaviour and welfare comparisons across households of differing composition. Equivalence scales can be defined as the ratio of the cost of a household obtaining a given level of utility to the cost of a different household with a different set of characteristics obtaining the same level of utility. There are several ways of estimating equivalence scales, most of them are based on empirical analyses of household demand behaviour. However, it is argued in the literature that without any restrictions on consumers' preferences, not all the information required to make welfare comparisons is identifiable from demand analysis alone. Blackorby and Donaldson (BD, 1988) propose a condition under which the equivalence scale can be uniquely determined from demand behaviour alone. Blundell and Lewbel (1991), and Dickens, Fry and Pashardes (1992) empirically investigated the BD condition. Their empirical results rejected the BD condition. These empirical rejections put some doubt on the validity of the identified equivalence scales. Therefore, in the third essay, the BD condition is generalized. It is found on a sample of Canadian data that the restriction implied by the BD condition regarding the budget share for the 'children only' good is rejected, whereas that of its generalization is not. A new rank three demand system is developed to examine the testable implications of the generalized BD condition, which are then rejected. Nevertheless, equivalence scales are estimated under the generalized BD condition. It is found that the cost of a child increases with both the age of a child and the labour force involvement of the female adult, and decreases with the income level of the household.

CHAPTER TWO

TESTING THE CONDITIONS FOR AGGREGATION ACROSS COMMODITIES AND INDIVIDUALS

1. Introduction

There are two kinds of aggregation problem in the study of systems of consumer demand equations: aggregation across commodities and aggregation across individuals. Conditions are known under which both types of aggregation are possible (see, for example, Deaton and Muellbauer, 1980b).

The conditions under which aggregation across commodities is possible are interesting because of the need to reduce the number of parameters necessary to characterize a system of demand equations. The primary goal of consumer demand modelling is to determine empirically the price and income elasticities of demand for specific commodities. These elasticities play a critical role in projecting demand and evaluating the impact of economic policy on consumer welfare. However, because the number of commodities consumed by households is typically large, and the number of own-price and cross-price elasticities of demand increases with the square of the number of commodities, the estimation of a complete demand system is econometrically intractable. One approach used to overcome this problem is to impose restrictions on the structure of the utility function, which lead to Strotz's (1957) two stage budgeting[1]. Using

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[1] See section 3.3 for the explanation of two stage budgeting.

this approach, the computational problems of estimating a system of demand equations for a large number of commodities can be circumvented by estimating group demand systems separately. Gorman (1959) examined the relationship between two stage budgeting behaviour and the form of a consumer's utility function. In particular, he showed that if the utility function is additively separable over groups of commodities and each group utility function is of the generalized Gorman polar form (GGPF) then goods can be aggregated.

The problem of aggregation across individuals concerns the connection between the microeconomics of consumer behaviour and the analysis of aggregate market demand. There is no obvious reason why macroeconomic relations should replicate their microeconomic foundations. The demand theory of an individual household cannot be directly applied to aggregate market data, though often data are available only for aggregates of households. Thus it is important to find conditions under which macro relations are consistent with micro relations, i.e., when aggregation across individuals is possible. Muellbauer (1975) derived one such necessary and sufficient condition for aggregation across individuals, which he calls "Generalized Linearity" (GL). GL is the same as Freixas and Mas-Colell's (1987) notion of no torsion.

Lau (1982) explored the restrictions of aggregation across individuals under which an aggregate demand function relies only on prices of individual commodities, symmetric functions of household income and demographic characteristics. The restrictions require that individual demand functions be linear in functions of income and demographic characteristics, with the addition of a function dependent only upon prices of commodities. An aggregation market demand function satisfying Lau's (1982) conditions of exact aggregation is not necessarily consistent with a utility-maximizing or cost-minimizing process, whereas an aggregation market demand function satisfying Muellbauer's (1975) GL has a corresponding cost function.

Households differ in two respects: their demographic characteristics and their incomes. Some previous researchers have used aggregate time series data either to estimate price and income elasticities or to test the propositions of consumer theory (Stone, 1954; Theil, 1965, Deaton and Muellbauer, 1980a). This kind of research treats aggregate data as if they come from a single consumer and imposes the restrictions for aggregation across individuals. Other empirical work, for example the paper by Nicol (1989), focuses on the effects of heterogeneity in households' demographic characteristics on aggregate market demand by testing Lau's theory of exact aggregation using cross-section Canadian micro-data. The empirical model of aggregate consumer behaviour developed by Jorgenson et al (1982), which allows the simultaneous estimation of price and demographic effects in a demand system using pooled aggregate time series data and cross-sectional micro-data, is based on Lau's conditions of exact aggregation.

This paper focuses on the effects of household income and tests the conditions for aggregation across individuals and aggregation across commodities implied by the functional forms derived by Gorman (1959) and Muellbauer (1975); it uses cross-section micro data from four Canadian family expenditure surveys (FAMEX) from 1978, 1982, 1984 and 1986. To test the restrictions implied by the aggregation conditions, a new demand system is developed. This system nests both a demand system in which the condition for aggregation across individuals is satisfied (Muellbauer, 1975) and a demand system in which the condition for aggregation across commodities is satisfied (Gorman, 1959).

Muellbauer's (1975) GL condition for aggregation across individuals is described in section 2 and Gorman's (1959) condition for aggregation over commodities in section 3. In addition, it is argued that one linearly homogeneous price index may not be sensitive enough to capture changes in consumers' behaviour due to exogenous price changes of commodities, and hence may not be able to describe the first stage allocation accurately in the two stage budgeting framework. Therefore, two different linearly homogeneous price indices from each period may be needed to allocate total expenditure in the first stage.

A more general utility function, called a generalized GGPF (GGGPF) is proposed in section 4. It employs two linearly homogeneous price indices in the first stage allocation of the two stage budgeting framework. The demand system corresponding to the proposed GGGPF is presented in section 4, which allows the aggregation conditions to be tested. The restrictions implied by GL and GGPF are also described.

Section 5 describes the empirical results using FAMEX data. The systems (unrestricted and restricted) derived in section 4 are estimated for the several data sets for two-adult households, stratified on the basis of the employment status of the female adult and presence of children. We stratify the data because the labour force status of wife and the presence of children in a household affect household demands (see Browning and Meghir, 1991). Section 6 presents a summary of the main conclusions.

2. A Brief Review of the Condition for Aggregation across Individuals

In the individual utility maximization framework, an individual demand for each commodity is a function of the total expenditure and a vector of prices. Consumers have different levels of total expenditure; thus in order for aggregation across individuals to be valid (i.e., for income responses of aggregate demand at the macro level to consistently reflect the income responses of individual demand at the micro level), the income responses at different levels of individual income at the micro level must take a particular form.

The aggregate budget share takes the form:

$$\bar{w}_i = \sum_h p_i q_{ih} / \sum_h y_h \equiv \sum_h y_h w_{ih} / \sum_h y_h$$

where w_{ih} (q_{ih}) is the consumer h's budget share (demand) for commodity i. In general, $\bar{w}_i$ is a function of a vector of commodity prices **p** and the expenditure distribution vector ($y_1, y_2, \ldots, y_H$). If the aggregate budget share $\bar{w}_i$ can be written as a function of prices and the representative budget share level y_o, which itself is a function of prices and the expenditure distribution vector, that is,

$$\bar{w}_i = g_i(\mathbf{p}, y_0), \quad y^o = y^o(y_1, y_2, \ldots, y_H, \mathbf{p}) \qquad (2.1.1)$$

then aggregation across individuals is possible.

Muellbauer (1975) showed that for (2.1.1) to hold, the individual budget share equation must satisfy "Generalized Linearity" (GL), which specifies the form of income responses at the micro level. There are four equivalent ways to express GL in the context of identical preferences:

(1) Differential restrictions on budget shares:

$$\frac{\partial}{\partial y}\left(\frac{\partial w_i / \partial y}{\partial w_j / \partial y}\right) = 0$$

for all i and j, where w_i is the budget share for commodity i of a consumer and y is the consumer's expenditure level.

(2) Budget shares:

$w_i(y, \mathbf{p}) = V(y, \mathbf{p})A_i(\mathbf{p}) + B_i(\mathbf{p})$

for all i with $\Sigma A_i = 0$, $\Sigma B_i = 1$.

(3) Cost function:

$C(u, \mathbf{p}) = G(u, H(\mathbf{p}))B(\mathbf{p})$

where u is the level of utility, $B(\mathbf{p})$ is linearly homogeneous, and $H(\mathbf{p})$ is homogeneous of degree zero.

(4) Relationship between budget shares:

$w_i(y,\mathbf{p}) = k_{ij}(\mathbf{p})w_j(y, \mathbf{p}) + l_{ij}(\mathbf{p})$

where $k_{ij}(\mathbf{p}) = A_i(\mathbf{p}) / A_j(\mathbf{p})$, $l_{ij}(\mathbf{p}) = B_i(\mathbf{p}) - B_j(\mathbf{p})[A_i(\mathbf{p}) / A_j(\mathbf{p})]$.

There are two important special cases of GL. One is called price independent generalized linearity (PIGL)[2], whose cost function is defined by

$C(u, \mathbf{p}) = (a(\mathbf{p})^{\varepsilon}(1-u) + b(\mathbf{p})^{\varepsilon}u)^{1/\varepsilon}$

where ε is a scalar, and both $a(\mathbf{p})$ and $b(\mathbf{p})$ are linearly homogeneous. The other is price independent generalized linearity logarithmic form (PIGLOG), whose cost function is expressed by

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[2] Under GL, the representative expenditure level depends on both prices and the distribution of expenditures, whereas under PIGL the representative expenditure level is independent of prices and dependent only on the distribution of expenditures. PIGL satisfies the conditions of no torsion (NT) and uniform curvature (UC). The NT condition indicates that in the relevant range of income, the Engel curve is contained in a plane through the origin. The UC condition requires that in addition, the Engel curve be either convex or concave to the origin. See Freixas and Mas-Colell (1987) for more details.

$C(u, \mathbf{p}) = H(\mathbf{p})^u B(\mathbf{p})$

The utility function corresponding to the almost ideal demand system (AIDS) (Deaton and Muellbauer, 1980a) belongs to the PIGLOG class.

3. Two Stage Budgeting and Aggregation across Commodities

3.1. Introduction

Since the work of Gorman (1959) and Strotz (1957, 1959), the assumption of separability of preferences has become a popular practice in applications of consumer theory. The ability to group commodities by type or time period stands as one of the most valuable forms of restrictions on consumer preferences. The basic idea of separability originates from the ordinary properties of goods. It is assumed that commodities may be broadly grouped such that goods which interact closely in the yielding of utility are grouped together while goods which are in different groups interact only in a general way. For example, service and recreation are usually classified as two different groups. If a relationship between one type of service and one type of recreation exists, then that relationship will be much the same for all pairs of commodities chosen from these two groups.

The separability concept in terms of functional structures was first introduced by

Sono (1961)[3], and Leontief (1947a, 1947b). A group of variables is said to be separable from the remaining variables in a utility function if the marginal rates of substitution between variables in that group are independent of the values of variables outside that group. About a decade later, Strotz (1957, 1959) and Gorman (1959) made use of the concept of separability in the context of demand analysis.

In this section, the concept of separability of preferences and the corresponding specifications of utility functions, two stage budgeting and the implications of assuming the separability of preferences for the empirical analysis of consumer expenditure data are presented.

3.2. Concepts of Separability and Corresponding Functional Structures

Leontief and Sono realized that separability is equivalent to the functional structure of utility functions. Assume that U is twice differentiable, and let U_i be the partial derivative of U with respect to the ith variable. Following Leontief and Sono, the variables i and j are separable from the variable k if and only if

[3] Sono's paper, originally published in Japanese in 1945, was later translated into English in 1961.

$$\frac{\partial}{\partial x_k} \left(\frac{U_i}{U_j}\right) = 0$$

That is, the pair (i, j) is separable from k if the marginal rate of substitution between commodities i and j is independent of the quantity of commodity k. Several types of separability are defined according to the respective groups to which commodities i, j and k belong.

(a) Weak Separability

A utility function U is said to be weakly separable with respect to a partition (n_1, ..., n_R)[4] if the marginal rate of substitution between two commodities i and j belonging to the same subset is not affected by the quantity of commodity k consumed in any other subsets.

$$\frac{\partial}{\partial x_k} \left(\frac{U_i}{U_j}\right) = 0, \quad for\ i,\ \ j \in n_r\ ,\ k \notin n_r\ ,\ 1 \leq r \leq R$$

or equivalently, if and only if the utility function U is of the form:

$U = f[v_1(\mathbf{x}_1), \ldots, v_R(\mathbf{x}_R)]$

where f(.) is increasing in all of its arguments, and for each r, $v_r(\mathbf{x}_r)$ is a function of sub-vector $\mathbf{x}_r$.

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[4] (n_1, ..., n_R) is a partition of the set of n commodities, $n_1 \cup n_2 \cup \ldots \cup n_R = n$. A small letter denotes a single variable, a bold one represents a vector.

(b) Strong Separability

A utility function U is said to be strongly separable to a partition (n_1, ..., n_R) if the marginal rate of substitution between goods i and j from different subsets n_i and n_j is not related to the quantity consumed of good k from subset n_k which does not contain goods i and j, that is,

$$\frac{\partial}{\partial x_k}\left(\frac{U_i}{U_j}\right) = 0, \quad \textit{for all } i \in n_i\ , j \in n_j\ , k \notin n_i \cup n_j\ , n_i \neq n_j$$

or equivalently, if and only if the functional form of U is

$U = f[v_1(\mathbf{x}_1) + ... + v_R(\mathbf{x}_R)]$

where f(.) is a monotonic increasing function. For each r, $v_r(\mathbf{x}_r)$ is a function of sub-vector $\mathbf{x}_r$. In the case where there is only on commodity in each group, preferences are said to be additively separable. Additive separability is a special case of strong separability, which in turn is a special case of weak separability.

3.3. Two Stage Budgeting

Following Leontief (1947a, 1947b), interest in functional structure and separability was refreshed by Strotz (1957, 1959) in the area of consumer budgeting and price aggregation. Two stage budgeting was first proposed by Strotz (1957). In the first stage, group expenditures are obtained by allocating the total expenditure among commodity groups with reference only to price indices from individual groups. This process involves aggregation across goods, and all commodities consumed in the same group are treated as a single unit. In the second stage, group expenditures are allocated to individual commodities based on commodity prices within that group.

Commodities can be grouped either by type or by time period. The time period is used to classify commodities here. It is assumed that there are T periods, n commodities in each period.

In the first stage, if the expenditure in period t, Y_t, can be expressed as:

$$Y_t = f_t(y, \Pi_1, \Pi_2, \ldots, \Pi_T) \text{ and } \Pi_t = \Pi_t(\mathbf{p}_t), \quad r = 1, 2, \ldots, T \qquad (3.3.1)$$

where $\mathbf{y}$ is the total expenditure over T periods, $\mathbf{p}_t$ is the commodity price vector $(p_{t1}, \ldots, p_{tn})$ in period t and Π_t is a linearly homogeneous function of $\mathbf{p}_t$, then aggregation across commodities is possible. As long as price indices are known from all periods, one can carry out the allocation of total expenditure across each period without needing to know individual commodity prices. Nevertheless, this is only true when the representation of preferences takes particular forms. Anderson (1979) calls Π_t the perfect price indices (PPI)[5].

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[5] Frisch (1936) classified index numbers into two categories: 'statistical' index numbers and 'functional' index numbers. The former are purely descriptive statistics, measuring the variations in a set of comparable phenomena (prices, quantities, etc.) along an axis of observation (eg., time, space, etc.), without any direct theoretical underpinning. The latter are based on a formal theory, and have a precise theoretical interpretation. PPI belongs to the latter.

Gorman (1959) showed that for (3.3.1) to hold, the utility function must take particular forms: either (i) the utility function is weakly separable across each period and each period utility function (hereafter called a sub-utility function) is homothetic, or (ii) the utility function is additively separable over time periods and each sub-utility function takes the generalized Gorman polar form (GGPF), or (iii) a mixture of both (Gorman, 1959 or Deaton and Muellbauer, 1980b). That is, the utility function U over T periods takes the form:

$$U = v_1 + \ldots + v_t + g(v_{t+1}, \ldots, v_T) \qquad T \geq 2 \,,\ 1 \leq t \leq T \qquad (3.3.2)$$

where g(.) is increasing in all of its arguments,

(i) $v_r = G_r(Y_r/\Pi_r(\mathbf{p}_r))$ $(r = t+1, \ldots, T)$, where $G_r(.)$ is an increasing function, and $\Pi_r(\cdot)$ is linearly homogeneous, v_r represents homothetic preferences and

(ii) $v_s = H_s[Y_s/\Pi_s(\mathbf{p}_s)] + \varphi_s(\mathbf{p}_s)$ $(s = 1, \ldots, t)$, is called the generalized Gorman polar form (GGPF), where $H_s(\cdot)$ is monotonically increasing, $\varphi_s(\cdot)$ is homogeneous of degree zero, and $\Pi_s(\cdot)$ is linearly homogeneous. If H_s is the identity function, v_s, which represents quasi-homothetic preferences, is called the Gorman polar form. If φ_s equals zero, GGPF reduces to a representation of homothetic preferences.

If the expenditure in a period is divided by a price index, a single aggregate is obtained. This aggregate is called 'consumption' in conventional macroeconomics. In the literature, concerns about what kind of price index should be used in order to define a consumption good were first discussed by Gorman (1959) and explicitly was raised two decades later by Anderson (1979). He suggested that the perfect price index is the appropriate price index that should be used to construct a 'consumption' good. He utilized one PPI from each commodity group to allocate broad group expenditures in the first stage, and showed that the application of the PPI leads to more accurate estimates of group expenditure functions than those obtained using some ordinary price indices which leave the aggregation across commodities implicit.

Both quasi-homothetic (i.e., Gorman polar form) and homothetic preferences are generally considered too restrictive for demand analysis since both imply linear Engel curves. However, Engel curves from a GGPF utility function are nonlinear. Thus, a GGPF utility function is considered empirically much more interesting and flexible, and it is chosen in studies by Browning (1989), Yen and Roe (1989), Heien and Wessells (1990) and Browning and Meghir (1991) as an indirect utility function.

In general, two stage budgeting takes the following structure: in stage one, the total expenditure y is allocated over T periods in a way such as

Max U in (3.3.2) with respect to Y_t subject to $\sum_{t=1}^{T} Y_t = y$

The first order condition gives us the optimal period expenditure Y_t ($1 \leq t \leq T$), which is a function of T period PPIs (one from each period) and the total expenditure. Each period PPI is a function of individual prices of that period.

$Y_t = \theta_t(y, \Pi_1, ..., \Pi_T)$ where $\Pi_t = \Pi_t(\mathbf{p}_t)$ for $1 \leq t \leq T$.

In stage two, demand for each commodity in a period is obtained from

Max $U_t(\mathbf{x}_t)$ subject to $\mathbf{x}_t\mathbf{p}_t = Y_t$

where $\mathbf{x}_t = (x_{t1}, ..., x_{tn})$ and x_{ti} is the demand for the ith commodity in period t. The solution for this problem is

$$x_{ti} = \psi_{ti}(Y_t, p_{t1}, ..., p_{tn}) \text{ for } t = 1, ..., T \text{ and } i = 1, ..., n \qquad (3.3.3)$$

That is, in the second stage, the consumer only needs to know the individual prices in period t with no need to take into account the allocation of any other group. Individual commodity demands are a function of the period expenditure and the individual commodity prices of the same period.

As a result, demand changes in one period due to detailed price changes in any other period are taken into account merely through the corresponding period expenditure, which acts as the only medium for all intertemporal substitutions. Perfect price indices capture the changes in individual prices in any periods. However, it is possible for price changes in different commodities within the same period to balance out, leading to no change in the perfect price index of that period, in turn causing no change in each period expenditure in the two stage budgeting framework. This would imply that even if some prices change, consumers will not alter their intertemporal allocation decision unless a period PPI changes. Relaxing this assumption leads to the introduction of a second PPI for each period. It is expected that the utilization of two PPIs from each period in the first stage may allow the modelling of changes in consumers' behaviour which cannot be picked up by using only one PPI from each period. Therefore, instead of employing one PPI, two different PPIs for each period are used to describe consumers' behaviour in the first stage. That is, the expenditure for each period is

$Y_t = \theta_t\,(y, \Pi_1, \Lambda_1, \ldots\ldots\Pi_1, \Lambda_T)$

where $\Pi_t = \Pi_t(\mathbf{p}_t)$, $\Lambda_t = \Lambda_t(\mathbf{p}_t)$, $\Pi_t \neq \Lambda_t$ for all t, both of them are linearly homogeneous. In section 4, a more general utility function will be specified, which allows the validity of the aggregation conditions to be investigated.

4. Model Specification

4.1. The Relationships between GL and GGPF

The general representations of PIGL, GGPF and GL are as follows:

$C(u, \mathbf{p}) = (a(\mathbf{p})^{\epsilon}(1\text{-}u) + b(\mathbf{p})^{\epsilon}u\ )^{1/\epsilon}$ (PIGL)

$v(Y, \mathbf{p}) = F(Y/D(\mathbf{p})) + A(\mathbf{p})$ (GGPF)

$C(u, \mathbf{p}) = G(u, H(p))B(\mathbf{p})$ (GL)

where $F(\cdot)$ and $G(.)$ are a monotonically increasing function, $\epsilon \neq 0$, $a(\mathbf{p})$, $b(\mathbf{p})$, $B(\mathbf{p})$, and $D(\mathbf{p})$ are linearly homogeneous functions of prices, and $A(\mathbf{p})$ and $H(\mathbf{p})$ are homogeneous of degree zero functions of prices. When $\in$ approaches 0, PIGL becomes PIGLOG. Inverting the PIGL cost function yields the PIGL indirect utility function $v(Y, \mathbf{p}) = Y^{\in} / [b(\mathbf{p})^{\in} - a(\mathbf{p})^{\in}] + a(\mathbf{p})^{\in} / [b(\mathbf{p})^{\in} - a(\mathbf{p})^{\in}]$. To prove that a canonical form of PIGL utility is a GGPF subutility, i.e., PIGL $\subset$ GGPF, let $D(\mathbf{p}) = [b(\mathbf{p})^{\in} - a(\mathbf{p})^{\in}]^{1/\in}$, $A(\mathbf{p}) = a(\mathbf{p})^{\in} / (a(\mathbf{p})^{\in} - b(\mathbf{p})^{\in})$. Then, in terms of $v(.)$, $D(\mathbf{p})$ and $A(\mathbf{p})$, PIGL utility is $v(.) = (Y/D(\mathbf{p}))^{\in} + A(\mathbf{p})$, which is the form of GGPF having $F(Y/D(\mathbf{p})) = (Y/D(\mathbf{p}))^{\in}$. Similarly, rearranging the general form of GGPF with $u = v(Y, p)$ and $Y = C(u, p)$ yields the GGPF cost function $C(u, \mathbf{p}) = F^{-1}(u - A(\mathbf{p}))D(\mathbf{p})$. To prove GGPF $\subset$ GL, let $B(\mathbf{p}) = D(\mathbf{p})$, and $H(\mathbf{p}) = A(p)$. Then, in terms of $C(.)$, $H(\mathbf{p})$ and $B(\mathbf{p})$, GGPF cost function is $C(.) = F^{-1}(u - H(\mathbf{p}))B(\mathbf{p})$, which is the form of GL having $G(u, H(\mathbf{p})) = F^{-1}(u - H(\mathbf{p}))$. In summary, PIGL $\subset$ GGPF $\subset$ GL. Since the objective of the paper is to test whether the aggregation conditions are satisfied, a utility function, which nests GL as a special case, is needed. Section 4.2 will deal with this problem.

4.2. A Generalized GGPF Utility Function and Demand System

To test the aggregation conditions and to have a consistent micro foundation for the use of two different PPIs in the first stage, a corresponding utility function must be specified. It is known that the GGPF utility function used in a number of studies results in employing one PPI in the first stage. It is natural to think that if a GGPF utility function can be generalised, then two different PPIs may be introduced in the first stage. It is also required that the generalization of this particular GGPF includes GL as a special case.

A particular GGPF utility function is chosen such as

$$v_t = \varphi_t(\mathbf{p}_t)^{-1}\ln(Y_t / \Pi_t(\mathbf{p}_t)) \qquad (4.2.1)$$

This function represents the preferences associated with the almost ideal demand system (Deaton and Muellbauer, 1980a). We generalize (4.2.1) into the form

$$v_t = \varphi_t(\mathbf{p}_t)^{-1} [\ \ln (Y_t / \Pi_t (\mathbf{p}_t) + b\ (Y_t / \Lambda_t(\mathbf{p}_t))] \quad \text{for } t = 1, 2, ..., T \qquad (4.2.2)$$

which we name as "a generalized GGPF" (GGGPF). It is a linear combination of the utility function corresponding to the AIDS and the utility function representing homothetic preferences. It provides a flexible functional form which allows the budget shares to vary with the expenditure level more freely than (4.2.1) does. If $b = 0$, (4.2.2) represents the same preferences as (4.2.1). b is called a functional flexibility parameter. It is assumed that preferences are additively separable across T periods. The utility function for each period is specified in (4.2.2).

The demand system in period t can be obtained from v_t for all t. However, single cross section data set does not typically reveal much price variation in a single period. In order to identify the effects of price changes, we use four cross section data and assume that budget share systems are the same in all periods, and hence subscript t (representing time) is dropped.

We specify functional forms for $\varphi(\mathbf{p})$, $\Pi(\mathbf{p})$ and $\Lambda(\mathbf{p})$ as follows:

$$Ln(\Pi_t(\boldsymbol{p})) = \sum_{i=1}^{n} \alpha_i Ln(p_i) + \frac{1}{2} \sum_{k=1}^{n} \sum_{l=1}^{n} \gamma_{kl}^{*} \; Ln(p_k) \; Ln(p_l)$$

$$Ln(\varphi(\boldsymbol{p})) = \sum_{i=1}^{n} \delta_i Ln(p_i) \textit{ with the restriction } \sum_{i=1}^{n} \delta_i = 0$$

with the constraints:

$$\sum_{k=1}^{n} \alpha_k = 1, \quad \sum_{k=1}^{n} \gamma_{kl}^{*} = \sum_{l=1}^{n} \gamma_{kl}^{*} = 0 \qquad \text{and}$$

$$Ln(\Lambda(\boldsymbol{p})) = Ln(\Pi(\boldsymbol{p})) + \sum_{k=1}^{n} \beta_k \; Ln(p_k), \quad \textit{with} \sum_{k=1}^{n} \beta_k = 0$$

Since demographic variables affect consumers' preferences, we let the α_i parameters be linear functions of household demographic characteristics[6].

$$\alpha_k = \alpha_{k0} + \sum_{j=1}^{m} \alpha_{kj} \, d_j \;\; , \quad k = 1, 2, ..., n$$

with the constrains:

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[6] This method of accommodating demographic variables is known as demographic translation. It preserves the linearity of the corresponding parts of the system. See Pollak and Wales (1981) and Browning (1991) for a full discussion of the ways of incorporating the effects of demographic variables in demand analysis.

$$\sum_{k=1}^{n} \alpha_{k0} = 1, \ \sum_{k=1}^{n} \alpha_{kj} = 0 \ \ for\ j = 1, 2, ..., m.$$

where α_{**} are parameters to be estimated and d_i are the demographic variables.

Using a logarithmic form of the Roy's identity, the budget share for good i associated with these specifications from (4.2.2) is derived as:

$$w_i^* = \alpha_i + \sum_{k=1}^{n} \gamma_{ik} Ln(p_k) + \frac{\delta_i Ln(y_1) + b(\delta_i + \beta_i) y_2}{1 + by_2} \qquad (4.2.3)$$

where $y_1 = Y/\Gamma(\mathbf{p})$ and $y_2 = Y/\Lambda(\mathbf{p})$, $\gamma_{ik} = 0.5(\gamma_{ik}^* + \gamma_{ki}^*)$ for all i.

We regard the demand system from the GGGPF as a maintained structure which characterizes the application of the two different PPIs in the first stage. The alternative structures are laid out below. As mentioned above, GL is equivalent to

$$\frac{\partial}{\partial Y}\left(\frac{\partial w_i / \partial Y}{\partial w_j / \partial Y}\right) = 0 \qquad (4.2.4)$$

for all i, j. It can be shown that this, in our case, is equivalent to $\beta_i\delta_j - \beta_j\delta_i = 0$, so we can set $\beta_i/\delta_i = \zeta$ or $\beta_i = \zeta\delta_i$ for all i. When estimating we only work with n - 1 commodities to allow for adding-up (though we have n goods in our system) and ζ is a new parameter to be estimated. Therefore, the number of restrictions imposed by GL on the proposed functional form GGGPF is n-2:

(GL) $\beta_i = \zeta\delta_i$ for all i

The restrictions from GGPF and AIDS on the GGGPF are:

(GGPF) (GL) and $\zeta = 0$ for all i

and

(AIDS) $b = 0$

To estimate (4.2.3), a random component, e_i, is added to each equation in the system. This captures any random differences in individual decision-making. The unrestricted system (4.2.3) becomes:

$$w_i = \alpha_i + \sum_{k=1}^{n} \gamma_{ik} Ln(p_k) + \frac{\delta_i Ln(y_1) + b(\delta_i + \beta_i)y_2}{1 + by_2} + e_i \tag{4.2.5}$$

We estimate this system using the method of non-linear three stage least squares. Since total expenditure is often considered endogenous, we use the instrumental variable method (see Gallant, 1987) to estimate (4.2.5)[7]. The instrument variables for the total expenditure are net income, log net income and reciprocal of net income so that the system is overidentified.

By substituting the restrictions from GL, GGPF and AIDS, we can get the corresponding restricted systems. The restricted systems are estimated in a similar way.

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[7] We also make corrections for heteroscedasticity in the error term.

For each set of estimates, we calculate a Sargan statistic[8]. The difference in Sargan statistics between the restricted and unrestricted systems is the "likelihood ratio" test statistic, which is distributed as $\chi^2(q)$, where q is the number of the restrictions.

5. Empirical Evidence

5.1. Likelihood Ratio Test Results

To test the restrictions presented in the last section, we used the Canadian family expenditure (FAMEX) survey data of 1978, 1982, 1984 and 1986. A brief description of data and the grouping of certain variables are provided in Appendix 1. The whole data set used in this paper consists of 5542 households of married couples with and without children, with husband in full time employment, and no other adult living in the household. Households with zero expenditures on any goods[9] and households in which the gross income of the head of the household is zero are dropped. This deletion leaves 5268 households which are stratified into five sub-data sets based upon the employment status of wife and presence of children in the household. We stratify the data because the

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[8] Sargan statistic examines the covariances between the estimated residuals from the instrument variables method and a set of instruments that need not have been used in the estimation of the model under the null hypothesis (see Godfrey, 1988).

[9] The FAMEX gives annual expenditures, so there are very few zeros.

labour force status of wife and the presence of children in a household affect household demands (see Browning and Meghir, 1991). By doing so, demand behaviour is modelled conditional on data stratification. The first stratum (NC-WFT) consists of households with no children and wife working full time, the second (NC-WNFT) includes households with no children and wife working part time or not working; the third (C-WFT) represents households with children and wife working full time; the fourth (C-WPT) contains households with children and wife working part time; and the fifth (C-WNW) selects households with children and wife not working. The number of observations in each stratum is 995, 1153, 815, 1068 and 1237, respectively. Descriptive statistics for the variables in each data set are given in Table B1 through Table B5 in Appendix 2. We choose to work with five non-durable commodities: food, clothing, services, recreation[10] and vices based on the implicit assumption that these five commodities are weakly separable from any other non-durable goods and durable goods.

We estimate (4.2.5) with and without restrictions for each stratum using the instrumental variable estimation method. The test results for a variety of restrictions for the five strata are summarized in Table 1, as are the estimation results in Table C1 to Table C5 in Appendix 3.

[10] Note that recreation also includes some durable goods, for example, recreation vehicles.

The test results in Table 1 indicate that GL, GGPF and AIDS are rejected for the data sets NC-WFT and C-WNW, however, are accepted for the data sets NC-WNFT, C-WFT and C-WPT. These results suggest that the demand structure from GGGPF fits the data sets NC-WFT and C-WNW better than alternative restricted structures, whereas AIDS fits the rest of data sets relatively better than any other structures under consideration. We find that once the condition for aggregation across individuals is accepted, the condition for aggregation over commodities is also accepted. The test results tell us that we can aggregate across individuals and commodities for some data sets, but not for others. Nevertheless, if we started with GL as a maintained hypothesis, then the restrictions from GGPF and AIDS are accepted for the data sets under investigation. This is consistent with the finding of Lewbel (1991) that most household demands can be reasonably modelled as PIGLOG. In general, if we started with a more general structure than that implied by GL, the conditions for aggregation across individuals and commodities are rejected.

5.2. Further Analysis on Test Results

The test results reported in Table 1 indicate that the data set C-WNW exhibits the strongest rejection to the restricted structure from GL, whereas the data set C-WFT shows the strongest acceptance to the restrictions from GL. To investigate further, some graphical analyses are conducted for the data sets C-WFT and C-WNW.

As mentioned in section 2, GL implies that the relationship between any two budget shares is linear: $w_i(y, \mathbf{p}) = k_{ij}(\mathbf{p})w_j(y, \mathbf{p}) + l_{ij}(\mathbf{p})$. Note that the total expenditure variable does not appear in either k_{ij} and l_{ij}. In order to eliminate the effects of commodity prices and demographic attributes, each budget share is regressed on commodity prices and household demographic characteristics. Based on this feature of GL, the residuals from regressions are graphed against each other. In particular, the graphs from plotting recreation budget share residual against food budget share residual for the data sets C-WFT and C-WNW are presented in Figures 1 and 2.

Figure 1 looks approximately linear, which is consistent with the implication of GL and the statistical test results about the data set C-WFT. Nevertheless, Figure 2 looks nonlinear, which is also consistent with the test results about the data set C-WNW. Notably, in Figure 2, the curve is stretched by two points, which correspond to two households spending more than 70% of their expenditure on recreation (expenditure on which contains spending on recreation vehicles, a durable good). In order to check the influences of these particular points, they are dropped. The restrictions from GL are re-tested, and budget share residuals re-graphed. The χ^2 test statistic is equal to 15.75, and the corresponding probability value is 0.13%. The restrictions from GL are still rejected for the data set C-WNW when these two points are dropped. The graph for the data set

C-WNW with the two observations removed is shown in Figure 3 and the curve still appears nonlinear. Hence statistical and graphical analyses are consistent with each other. Dropping the two observations does not change the nature of the test result from the statistical analysis, it only reduces the χ^2 test statistic for the restricted structure from GL by 0.44. Therefore, the strong rejection from the data set C-WNW to the restrictions from GL is not due to those seemingly peculiar observations.

6. Summary and Conclusions

From Muellbauer (1975) it is learned that aggregation across consumers is possible if and only if 'Generalized Linearity' (GL) is satisfied, and from Gorman (1959) that aggregation across commodities is possible if the utility function is additive across the sub-utility functions, and if and only if the sub-utility functions are of the 'generalized Gorman polar form' (GGPF). The demand system developed in this paper nests a demand system which satisfies GL and a demand system which satisfies GGPF. The aggregation conditions are investigated empirically using this system as a maintained hypothesis. Test results indicate that the aggregation conditions across individuals and commodities are accepted for some data sets, but not for others. This has implications for the macroeconomic treatment of 'consumption' as obtained by dividing nominal expenditure by one price index. It also has implications for 'representative agent' models. Given the result that the aggregation conditions are rejected, the concepts of a single 'consumption' good and 'representative agent' cannot be fully supported by the empirical investigations here. Further research will be needed: perhaps an alternative unrestricted demand system which includes a demand system from the GGPF and GL utility functions as special cases should be investigated.

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Table 1. Tests of Restrictions

		DATA			SET	
		NC-WFT	NC-WNFT	C-WFT	C-WPT	C-WNW
Restriction	DF	LR			Statistics	
GL	3	15.21 (0.16%)	1.22 (74.8%)	0.80 (85.2%)	6.993 (7.21%)	16.19 (0.10%)
GGPF	4	18.36 (0.11%)	1.23 (87.4%)	2.04 (72.9%)	6.994 (13.6%)	16.21 (0.27%)
AIDS	5	20.65 (0.09%)	1.59 (90.2%)	4.26 (51.2%)	7.93 (16.0%)	16.26 (0.61%)

Note: The percentage values in parentheses are probability levels.

Figure 1

Recreation Budget Share Residual vs. Food Budget Share

Residual for Data Set C-WFT

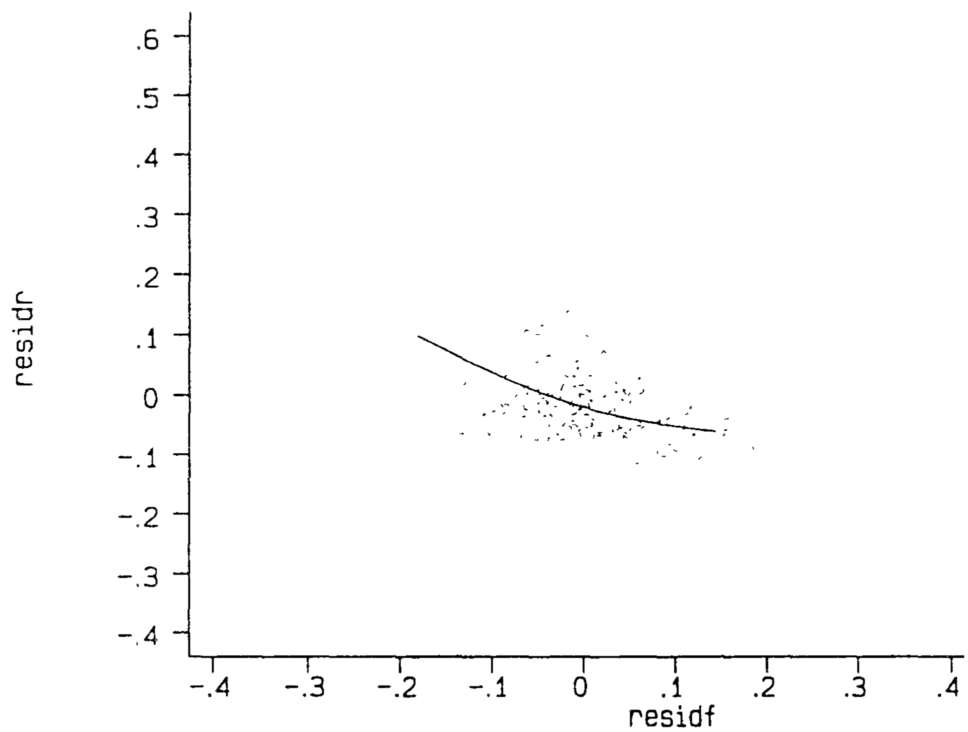


Figure 2

Recreation Budget Share Residual vs. Food Budget Share

Residual for Data Set C-WNW

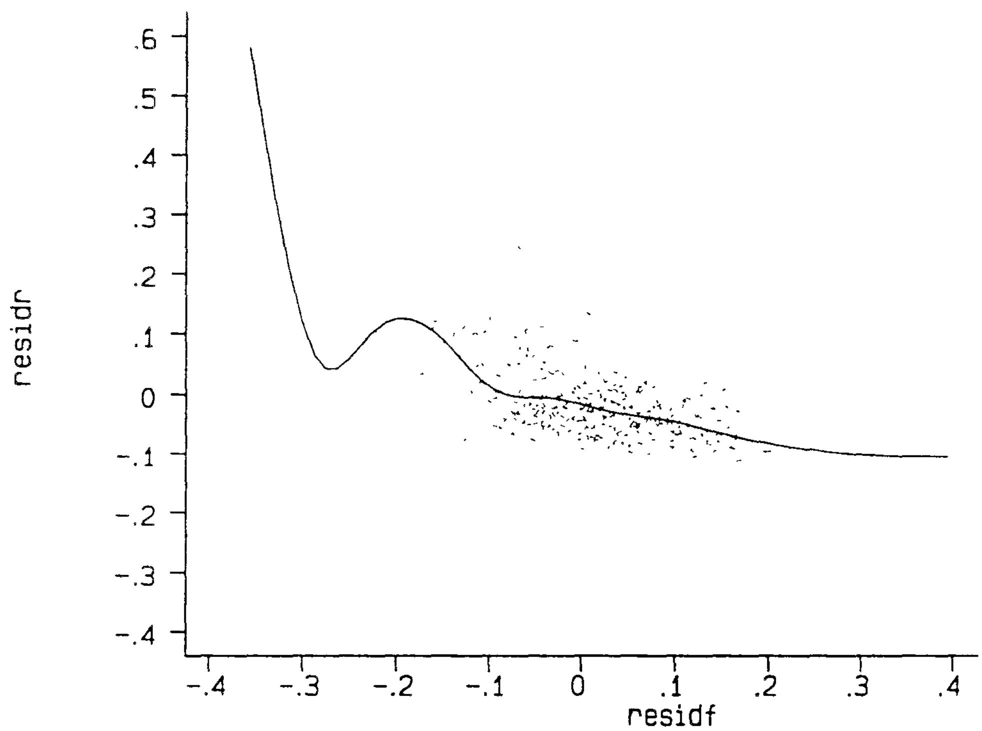
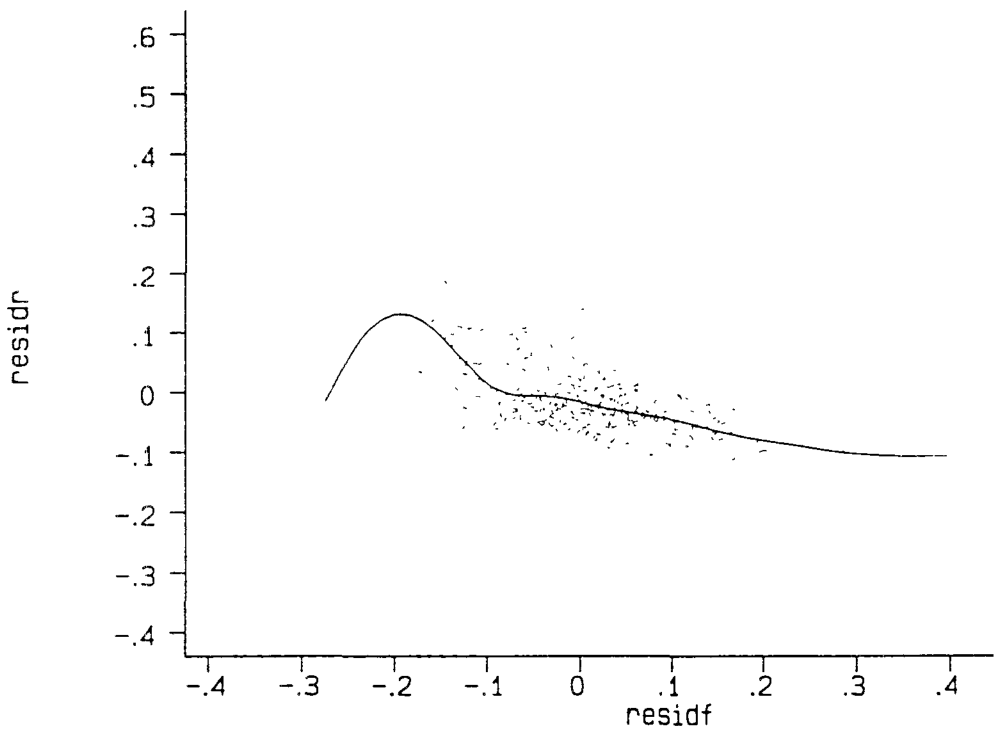


Figure 3

Recreation Budget Share Residual vs. Food Budget Share

Residual for Data Set C-WNW



Appendix 1. Data

The data used in this paper is from Canadian family expenditure surveys of 1978, 1982, 1984 and 1986, which were conducted by the Family Expenditure Survey Section, Household Surveys Division of Statistics Canada. For details, see its publications on Survey of Family Expenditure ---- Public Use Micro Data File.

The sample extracted contains 30 variables. The definition of each variable is:

year: 1978, 1982, 1984, 1986

province: 1 = Atlantic province, 2 = Quebec, 3 = Ontario, 4 = Prairie, 5 = BC

Tenure: 1 = owner without mortgage, 2 = owner with mortgage, 3 = renter

adult: the number of adults in a household

Mardum (dummy variable for married): 1 for married couple, 0 for single

ych: the number of children aged under 5 (6 for 1978)

och: the number of children aged between 5 and 18 (6 and 18 for 1978)

nety: after tax household income

hgy: gross income of head of household

sgy: gross income of spouse

hage: age of head of household

hsex: sex of head of household, 1 for male, 2 for female

heduc: education of head of household = 1 for less than 9 years, 2 for some secondary, 3 for some post-secondary, 4 for post-secondary certificate, 5 for university degree

hoccup: occupation = 1 for managerial, 2 for professional, 3 for clerical, 4 for sales, 5 for services, 6 for farming, 7 for other, 8 for not working

hempl: employment status of head of household = 1 for full time, 2 for part-time, 3 for not in the labor force

sage: age of spouse

ssex: sex of spouse, 1 for male, 2 for female

seduc: education of spouse, the classification same as in heduc

soccup: occupation of spouse, the classification same as in hoccp

sempl: employment status of spouse, same as in hempl

recr: expenditure on recreation, reading and education

food: expenditure on food at home, in restaurants and on tips

cloth: expenditure on clothing for all members of household

vices: expenditure on alcohol and tobacco

services: expenditure on household operations, personal care and medical care

pfood: price of food

precr: price of recreation

pcl: price of clothing

pvices: price of vices

pserve: price of services

The followings are the dummy and age variables used in the estimation:

dprov = 1, if province = quebec, 0 otherwise;

down = 1, if tenure < 3, 0 otherwise;

dheduc (dseduc) = 1, if heduc (seduc) > 3, 0 otherwise;

dhoccup (dsoccup) = 1, if hoccp (soccp) < 6, year = 1978 or if < 7, year = 1978,

0 otherwise;

Hage (Sage) = 0.1*Hage (Sage)

Appendix 2. Descriptive Statistics of the Data

(Note: In Tables B1-B5, all the expenditure and income variables are divided by 1000, and all the price variables are divided by 100)

Table B1
Data set NC_WFT
N = 995

Variable	Mean	Std. Dev.	Min	Max
year	82.384	2.907	78.000	86.000
province	3.000	1.165	1.000	5.000
tenure	2.219	0.716	1.000	3.000
adult	2.000	0.000	2.000	2.000
mardum	1.000	0.000	1.000	1.000
hage	37.412	12.602	20.000	76.000
hsex	1.000	0.000	1.000	1.000
heduc	2.999	1.243	1.000	5.000
hoccup	3.927	2.235	1.000	7.000
hempl	1.000	0.000	1.000	1.000
sage	34.887	12.363	18.000	69.000
ssex	2.000	0.000	2.000	2.000
seduc	2.917	1.189	1.000	5.000
soccup	3.230	1.489	1.000	7.000
sempl	1.000	0.000	1.000	1.000
ych	0.000	0.000	0.000	0.000
och	0.000	0.000	0.000	0.000
nety	36.699	14.697	10.711	124.230
hgy	27.354	14.749	0.600	161.000
sgy	19.224	9.172	1.300	101.100
cloth	2.610	1.934	0.131	15.938
recr	2.227	2.213	0.009	24.982
food	4.704	1.912	0.704	17.460
services	2.591	1.692	0.388	28.039
vices	1.214	1.173	0.002	8.372
pcl	1.372	0.231	0.968	1.666
pfood	1.468	0.275	0.980	1.821
pserv	1.484	0.293	0.925	1.881
precr	1.412	0.268	0.938	1.775
pvices	1.683	0.485	0.939	2.862

Table B2
Data set: NC_WNFT
N = 1153

Variable	Mean	Std. Dev.	Min	Max
year	82.458	2.913	78.000	86.000
prov	3.056	1.239	1.000	5.000
tenure	2.062	0.835	1.000	3.000
adult	2.000	0.000	2.000	2.000
mardum	1.000	0.000	1.000	1.000
hage	47.075	14.892	19.000	80.000
hsex	1.000	0.000	1.000	1.000
heduc	2.654	1.289	1.000	5.000
hoccup	4.177	2.236	1.000	7.000
hempl	1.000	0.000	1.000	1.000
sage	44.820	15.149	18.000	77.000
ssex	2.000	0.000	2.000	2.000
seduc	2.520	1.155	1.000	5.000
soccup	5.962	2.430	1.000	8.000
sempl	2.526	0.500	2.000	3.000
ych	0.000	0.000	0.000	0.000
och	0.000	0.000	0.000	0.000
nety	27.515	13.546	6.089	126.678
hgy	27.253	17.559	0.112	162.000
sgy	7.082	8.714	0.000	67.100
cloth	1.782	1.512	0.008	12.030
recr	1.634	2.115	0.010	25.793
food	4.226	1.733	0.795	13.705
services	2.194	1.112	0.262	8.397
vices	1.060	1.030	0.002	6.320
pcl	1.378	0.229	0.968	1.666
pfood	1.476	0.271	0.980	1.821
pserv	1.500	0.295	0.921	1.885
precr	1.417	0.269	0.938	1.775
pvices	1.707	0.514	0.912	2.936

Table B3
Data set: C_WFT
N = 815

Variable	Mean	Std. Dev.	Min	Max
year	82.746	2.879	78.000	86.000
prov	2.874	1.135	1.000	5.000
tenure	2.070	0.583	1.000	3.000
adult	2.000	0.000	2.000	2.000
mardum	1.000	0.000	1.000	1.000
hage	37.384	7.280	21.000	68.000
hsex	1.000	0.000	1.000	1.000
heduc	2.942	1.269	1.000	5.000
hoccup	3.989	2.295	1.000	7.000
hempl	1.000	0.000	1.000	1.000
sage	34.787	6.733	20.000	63.000
ssex	2.000	0.000	2.000	2.000
seduc	2.795	1.185	1.000	5.000
soccup	3.256	1.650	1.000	7.000
sempl	1.000	0.000	1.000	1.000
ych	0.339	0.532	0.000	2.000
och	1.231	0.729	0.000	2.000
nety	39.120	15.478	9.564	128.008
hgy	29.351	15.130	3.000	151.687
sgy	20.050	10.148	0.000	71.534
cloth	3.112	2.179	0.167	18.357
recr	2.636	3.486	0.080	75.691
food	5.995	2.270	1.104	18.013
services	4.288	2.469	0.507	17.972
vices	1.148	1.116	0.002	13.130
pcl	1.390	0.220	0.968	1.666
pfood	1.502	0.267	0.980	1.821
pserv	1.540	0.294	0.929	1.887
precr	1.446	0.261	0.938	1.775
pvices	1.766	0.518	0.972	2.936

Table B4
Data set: C_WPT
N = 1068

Variable	Mean	Std. Dev.	Min	Max
year	82.732	2.916	78.000	86.000
prov	2.964	1.220	1.000	5.000
tenure	2.067	0.593	1.000	3.000
adult	2.000	0.000	2.000	2.000
mardum	1.000	0.000	1.000	1.000
hage	36.128	7.244	20.000	66.000
hsex	1.000	0.000	1.000	1.000
heduc	3.078	1.322	1.000	5.000
hoccup	3.871	2.305	1.000	7.000
hempl	1.000	0.000	1.000	1.000
sage	33.713	6.693	18.000	60.000
ssex	2.000	0.000	2.000	2.000
seduc	2.890	1.173	1.000	5.000
soccup	3.382	1.517	1.000	7.000
sempl	2.000	0.000	2.000	2.000
ych	0.487	0.619	0.000	2.000
och	1.213	0.802	0.000	2.000
nety	32.137	12.009	8.480	91.373
hgy	30.840	14.457	1.638	118.958
sgy	9.173	7.476	0.030	87.920
cloth	2.472	1.652	0.190	16.706
recr	2.241	2.097	0.071	27.545
food	5.412	1.916	1.542	18.120
services	3.308	1.848	0.565	18.864
vices	0.961	0.898	0.005	9.064
pcl	1.392	0.224	0.968	1.666
pfood	1.497	0.266	0.980	1.821
pserv	1.521	0.293	0.933	1.882
precr	1.439	0.267	0.938	1.775
pvices	1.753	0.516	0.912	2.936

Table B5
Data set: C_WNW
N = 1237

Variable	Mean	Std. Dev.	Min	Max
year	81.675	3.082	78.000	86.000
prov	2.749	1.209	1.000	5.000
tenure	2.074	0.620	1.000	3.000
adult	2.000	0.000	2.000	2.000
mardum	1.000	0.000	1.000	1.000
hage	36.948	7.898	20.000	64.000
hsex	1.000	0.000	1.000	1.000
heduc	2.968	1.310	1.000	5.000
hoccup	4.146	2.381	1.000	7.000
hempl	1.000	0.000	1.000	1.000
sage	34.205	7.648	18.000	65.000
ssex	2.000	0.000	2.000	2.000
seduc	2.520	1.062	1.000	5.000
soccup	8.000	0.000	8.000	8.000
sempl	3.000	0.000	3.000	3.000
ych	0.605	0.687	0.000	2.000
och	1.216	0.783	0.000	2.000
nety	25.664	11.208	6.443	95.036
hgy	31.359	16.208	0.193	157.300
sgy	1.010	3.478	0.000	51.001
cloth	1.882	1.401	0.005	14.180
recr	1.707	2.212	0.039	56.582
food	4.826	1.794	1.280	13.410
services	2.312	1.348	0.333	20.781
vices	0.724	0.723	0.001	5.900
pcl	1.304	0.247	0.968	1.666
pfood	1.398	0.301	0.980	1.821
pserv	1.416	0.324	0.932	1.881
precr	1.348	0.286	0.938	1.775
pvices	1.590	0.522	0.932	2.936

CHAPTER THREE

RE-EXAMINING THE ASSUMPTION THAT

CHILDREN ARE EXOGENOUS TO FEMALE LABOR SUPPLY

1. Introduction

The labor force participation rate of women, particularly married women, has increased substantially in most advanced economies since the 1960s. The labor supply behavior of women is interesting because it has important implications for many policy related topics, including the distribution of income and poverty problems, wage differentials, welfare payments and social security. It is realized that many aspects of household decisions are interconnected. Especially the presence of children in the household is closely associated with female labor supply decisions. Inclusion of possibly endogenous children variables in the labor supply equations will likely lead to biased estimates of labor supply parameters. In order to estimate labor supply parameters more precisely, careful modelling of children variables is essential.

Many previous studies of female labor supply decisions treat children as exogenously imposed constraints on the household decision-making process (see, for example, Mincer (1962), Heckman (1974), Heckman and MaCurdy (1980)). However, some studies have argued that there are mutual dependencies between the labor supply of married women and fertility, and have allowed for this endogeneity of fertility, either in a life cycle model or in a static model of female labor supply. Moffit (1984) builds a life cycle model of labor supply, fertility and wages and estimates his model with a full-information maximum likelihood technique. Hotz and Miller (1988) embody a contraceptive choice index function into a life cycle model of female labor supply. None of these test the endogeneity of fertility variables in the life cycle models.

Using static models, studies by Cain and Dooley (1976), Fleisher and Rhodes (1979), Schultz (1978, 1980), Link and Settle (1981), and Dooley (1982) acknowledge the interrelationship between fertility and female labor supply. They all employ some variant of a simultaneous equation model of female labor supply, fertility and wages and/or earnings, using individual data or grouped data aggregated across geographical locations. With the exception of the papers by Schultz (1978, 1980), the labor force participation rate of married women is used as the only measure of labor supply behavior. Fertility is directly introduced to the set of explanatory variables in the labor force participation equation except in Dooley (1982), where fertility[1] and labor force participation equations are specified as "seemingly unrelated". That is, fertility does not appear as an explanatory variable in the labor force participation equation, neither does labor force participation rate in the fertility equation, yet the two equations are estimated jointly to allow for possible correlation between disturbances from these two equations.

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[1] Fertility is measured by the number of children ever born if individual data are used, or the average number of children ever born per 1,000 married women if grouped data are used.

Schultz[2] (1978, 1980) analyzes female labor supply decisions through both an hours of work equation and a labor force participation equation, which are estimated separately. He estimates both equations with and without the inclusion of fertility variables. This allows him to examine the quantitative importance of alternative plausible assumptions regarding the determination of fertility for labor supply projections. In order to avoid biased and inconsistent estimates of the underlying structural parameters due to the possible endogeneity of fertility variables, he replaces them in the labor supply relationships with an imputed linear combination of exogenous instruments.

An approach to dealing with the endogeneity of fertility used by Nakamura and Nakamura (1985a, 1992) and Lehrer (1992) is to introduce measures of past labor supply to the analysis. Consequently, they emphasize the role of unobservable variables such as individual preferences[3] regarding children and work, which affect both fertility and female labor supply decisions.

In all studies mentioned above except for Lehrer[4] (1992), fertility is proxied by

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[2] There are four variables used to measure fertility in his paper, three of which are defined as the number of children in three age bands. The fourth one is the number of additional children the wife expects to have, which is only defined for wives between the ages of 18 and 29.

[3] It is believed that the measure of past labor supply contains information about individuals' preferences for work and children.

[4] She uses dummy variables defined according to age ranges.

the number of children in specified age groups in the household. None of the studies mentioned above have explicitly tested the exogeneity assumption of fertility in female labor supply equations in the static framework. The exception is a recent paper by Mroz (1987), which examines the sensitivity of estimates in an empirical model of married women's hours of work to economic and statistical assumptions. Using 1975 labor supply data from the Panel Study of Income Dynamics (PSID), Mroz does not reject the exogeneity of fertility in a married women's hours of work equation. Mroz measures fertility with two children variables, defined as (1) the number of children under age 6 and (2) the number of children aged from 6 to 18, in the household. Both Mroz and Schultz (1978, 1980) instrument the children variables via a simple OLS method, without considering the count nature[5] of these variables.

The objectives of this paper can be perceived from the above brief review[6]. First, taking into consideration the count nature of the children variables and using the same framework as Mroz, we re-investigate the exogeneity assumption of children in a female hours of work equation. Second, the case where the children variables do not directly appear in an hours of work equation is considered. By doing so, the question of whether

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[5] See Cameron and Trivedi (1986), Hausman and Griliches (1984), King (1988, 1989), Maddala (1983), Mullahy (1986), Winkelmann and Zimmermann (1991a, 1991b) for analysis and examples of the treatment of count data

[6] For a much more comprehensive surveys of literature on female labor supply, see for example Killingsworth and Heckman (1986), and particularly the effects of children on female labor supply, see Browning (1992) and Nakamura and Nakamura (1992).

the children equations and hours of work equation could be "seemingly unrelated" is investigated. This allows us to study whether children could be endogenous to the decision on hours of work of married women due to omitted factors (usually unobservable) or due to their direct correlation with the error terms in the labor supply relations. Third, we examine the exogeneity assumption of children in the participation equation, an issue not addressed by Mroz. The data used in this paper are essentially the same as in Mroz's study[7] (and the comparable results are very similar). The hours of work and participation equations are estimated separately to account for the fixed cost of employment (see for example Cogan, 1981).

The paper is organized as follows. Since only Mroz explicitly tested the null hypothesis of exogeneity of the children variables in an hours of work equation of married women, we want to see whether the results of testing for exogeneity are robust to the way the children variables are treated. Hence, section 2 describes the specification of the hours of work equation used by Mroz and the main conclusions in Mroz's paper. Section 3 describes two methods to perform a limited information test of exogeneity. Section 4 reports the empirical results. Section 5 summarizes the main conclusions.

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[7] The data set used is from the diskette in Berndt's (1991) book. It contains the data used by Mroz, except for two variables: observations on the number of years of schooling of the husband's mother and father. I am grateful to Professor Mroz for discussion of this point. However, in Mroz these variables were used only as instruments and the comparable results using a slightly smaller instrument set are very similar to those of Mroz. For details of the data, see Appendix 1, Berndt (1991) and Mroz (1987).

2. Brief Summary of Mroz's Study

In Mroz (1987), the married women's hours of work equation is given by

$$h_i = \alpha_0 + \alpha_1 \ln(ww_i) + \alpha_2 nwm_i + \alpha_3' Z_i + e_i \quad (2.1)$$

where h_i is the wife's annual hours of work (the product of the number of weeks the wife worked for money and the average number of hours of work per week during the weeks she worked), and $\ln(ww_i)$ is the natural logarithm of wife's average hourly earnings (the wife's total labor income divided by her annual hours of work). nwm_i is non-wife income (the household's total income minus the wife's labor income). Z_i is a set of variables, which includes $k16_i$ (the number of children under age 6), $k618_i$ (the number of children from age 6 to 18), wa_i (the wife's age), and we_i (the wife's education measured in number of years). e_i is a stochastic error term, and α_0, α_1, α_2, and the vector α_3 are the parameters in the hours of work equation.

The data used by Mroz are from the University of Michigan Panel Study of Income Dynamics (PSID) for the year 1975. It contains 753 married white women between the ages of 30 and 60 in 1975, of whom 428 worked some time during the year 1975. The definition of variables and some descriptive statistics are provided in Appendix 1.

Mroz tested the exogeneity assumption of wife's wage rate, non-wife income, children in the household, and the wife's labor market experience as well as the importance of statistical control for self-selection into the labor force. He concluded that in the hours of work equation: (1) wife's wage rate is endogenous, (2) wife's labor market experience is endogenous, (3) non-wife income is exogenous, (4) when labor market experience is treated as endogenous, control for self-selection into the labor force is not important, and (5) children are exogenous.

Let us state the hypotheses about the exogeneity of children in Mroz's framework. The joint null hypothesis H_0 is: k16 and k618 both are exogenous in the labor supply equations. The alternative hypothesis H_a is: k16 or k618 is not exogenous in the labor supply equations. The variables in the instrument set under the null hypothesis are those involving location, family background, education, age, non-wife income and the children variables (i.e., B, F3, H3[9], nwm, k16 and k618), and the variables in the instrument set under the alternative hypothesis are the same except for the children variables (namely, B, F3, H3 and nwm).

Based on Mroz's conclusions (2) and (4), we can focus on the sample for those working without considering the sample selection bias (as the wife's labor market experience is not included in the set of instrument variables). For further simplicity, we

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[9] See Appendix 1 for the definitions of B, F3, H3.

shall also accept conclusions (1) and (3) and hence concentrate on conclusion (5), the exogeneity of children. In studying this issue, Mroz did not take into account the integer nature of the children variables. In section 4, their count nature will be considered. In summary, we accept Mroz's conclusions (1) to (4). We use the sample of working women to test the hypothesis mentioned in this section.

3. Test of Exogeneity with Limited Information Methods

Consider the following regression equation

$$y_i = X_i\alpha + Y_i\beta + W_i\gamma + v_i, \quad i = 1, 2, \ldots, n, \tag{3.1}$$

where n is the sample size, X_i is a $1xk_1$ exogenous vector, Y_i is a $1xk_2$ endogenous vector, W_i is 1xr vector, and $\theta = \{\alpha', \beta', \gamma'\}$, which is a $(k_1+k_2+r)x1$ vector of structural parameters. W_i is a set of variables suspected to be correlated with the error term v_i. The null hypothesis is that W_i is not correlated with the error term v_i, or equivalently, the W_i variables are exogenous. The alternative hypothesis is that the W_i variables are endogenous. Define Z^*_i as the set of exogenously determined variables having no overlap with the set of X_i. Under the null hypothesis of exogeneity, the auxiliary equations for Y are specified as functions of a set of exogenous variables $\{Z^*, X, W\}$. Under the alternative hypothesis, the auxiliary equations for both Y and W are specified as functions of a set of exogenous variables $\{Z^*, X\}$. Two different methods can be used to test for the exogeneity of W depending on how one makes use of the auxiliary equations. Methods for testing the null hypothesis are presented in the following sub-sections.

3.1. Durbin-Wu-Hausman-White Test (DWHW Test or Substitution Method)

For the test of exogeneity of explanatory variables W_1 in the linear regression model (3.1), we first substitute the predicted values of Y_1 and W_1 from the auxiliary equations[10] for the actual values of Y_1 and W_1 in equation (3.1), and obtain one set of estimates of the parameter vector θ by estimating (3.1) under the alternative hypothesis. Since Y_1 is known to be endogenous, the predicted values of Y_1 from the auxiliary equations are used to replace the actual values of Y_1 in equation (3.1) under the null hypothesis as well, but the actual values of W_1 are retained. We then obtain the second set of estimates of the parameter vector θ. Then we construct the covariance matrix of these two sets of estimates, and test whether the difference between the estimates of the coefficients in front of W_1 are significantly different from zero using a χ^2 test statistic with degrees of freedom equal to r (For more detail, see the specification tests proposed by Durbin, 1954; Wu, 1973; Hausman, 1978; White, 1982; Spencer and Berk, 1981 and 1982; and Ruud, 1984).

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[10] See Appendix 3 for the description of the auxiliary equations and the steps needed for obtaining the estimates of the parameters θ.

Two ways of constructing the covariance matrix for the two sets of estimates of the parameter vector θ are used: one is the simple formula as given in Case 1 of Mroz's Appendix 2, the other is derived from Duncan (1987). The formula of the covariance matrix from Duncan is described in Appendix 2 of this paper. The simple formula can be used only in the case of the usual linear two-stage least squares estimator, where the asymptotic distribution of the second-stage estimator is independent of the distribution of the first-stage estimator. The formula from Duncan is more general and can be used when the asymptotic distribution of the first-stage estimator affects the distribution of the second-stage estimator[11]. These two formulae are asymptotically equivalent if the variables that are being tested for exogeneity have linear auxiliary equations. To help show this, a simulation experiment is conducted. The simulation model and results are described in Tables 4a and 4b in Appendix 4.

3.2. Generalized Residuals Method

For the test of exogeneity of explanatory variables W_i in the linear regression model (3.1), we add the generalized residuals[12] obtained from the auxiliary equations for

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[11] Note that both the simple formula and Duncan formula of computing the covariance matrix are heteroscedasticity-corrected.

[12] The ith generalized residual is defined as the first derivative with respect to the constant term of the natural logarithm of the portion of the likelihood function corresponding to the observation i (see, Gourieroux, C., et al, 1987). For example, if we have a linear regression, and the error term is assumed to

W_i and Y_i under the alternative hypothesis to the right hand side of equation (3.1) as extra variables, and estimate the parameter vector θ and the coefficients of the appended generalized residual terms. Second, we calculate the covariance matrix for all the estimates. Third, the t-value on the coefficient of the generalized residual term of a specific W_i is used to determine whether to accept or reject the null hypothesis of the exogeneity of this variable. Alternatively, a χ^2 statistic on the coefficients of a set of these terms is employed to judge whether to accept or reject the null hypothesis of the joint exogeneity of a set of the corresponding variables. Unlike the first method described in section 3.1, we need to estimate the model under the alternative hypothesis only.

Two different approaches can be utilized to compute the covariance matrix of estimates of the parameter vector θ and the estimated coefficients of the added generalized residual terms. The simple approach treats the generalized residuals as if they were ordinary variables like X_i in equation (3.1). This is valid only if the null hypothesis is true and if the first-stage estimators are from linear regressions. The other method based on Duncan takes into consideration the effects of the asymptotic distribution of the estimator used to compute the generalized residuals[13] on the subsequent stage estimation.

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have the standard normal distribution, then the generalized residuals are the ordinary residuals. The generalized residuals are useful for various types of hypothesis testing (see Gourieroux et al, 1987; Blundell and Smith, 1989).

[13] See pagan (1984) for the econometric issues in the analysis of regressions with generated regressors.

Duncan's method is always valid when using the generalized residuals method for exogeneity testing. If the null hypothesis is not true and/or the first-stage regressions are not linear, the results from these two methods can be substantially different. The simulation results in Tables 4c and 4d of Appendix 4 to demonstrate this.

4. Empirical Results

4.1. Introduction

There are two aspects to labor supply[14]: participation and the hours of work if participating. The choice between working and not working is referred to as the participation decision. The number of hours supplied in the labor market conditional upon participation is called the hours of work decision. Some researchers believe that these two aspects of female labor supply are governed by the same underlying sources and suggest that they should be estimated jointly. For example, Heckman and MaCurdy (1980) present a life cycle female labor supply model, which integrates annual hours of work and annual participation. Others (for example, Cogan, 1981) argue that there may be a discontinuity in labor supply decisions due to fixed cost of employment. The econometric issues of discontinuity in labor supply are discussed by Killingsworth (1983).

[14] For more details on the measurement of labor supply and the estimation of labor supply equations, see Boskin (1973), Cain and Watts (1973), Heckman (1976, 1981), Nelson and Olson (1978), Hanoch (1981) and Lee(1982).

In this paper, hours of participation equations are estimated separately to account for the possible discontinuity in labor supply.

The participation and hours of work decisions for married women may be correlated with the presence and ages of children in the household. Two possible reasons for endogeneity of children variables in female labor supply relationships may be: (Case 1) households treat the choices on female labor supply and children as simultaneous aspects of a joint decision, and (Case 2) there are unobservable factors that affect both decisions on how many children to have and female labor supply.

Case 1 suggests a simultaneous system for labor supply decisions and children variables, where children variables are directly introduced along with other explanatory variables in equations of labor supply decisions and similarly a labor supply variable also appears as an explanatory variable in the children equations. Case 2 implies a "seemingly unrelated" equation system for labor supply variables and children variables, where children variables do not appear in the equations of labor supply decisions and neither does a labor supply variable appear in the children equations. However, the error terms from each children equation and the labor supply equations are correlated due to unobservables[15]. Section 4.2 reports the test results for the exogeneity of children

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[15] Each individual has her own preferences for children and work, which are unobservable and can be regarded as fixed effects. If panel data were available, the fixed effects could be dealt with by the approach implemented in Browning et al (1985). If using a single cross section, they can only be left in the regression residuals. For example, the regression residuals from both a labor supply equation, say, an hours

variables in the labor supply equations in Case 1. Section 4.3 presents the test results for exogeneity of children variables in the labor supply equations in Case 2.

4.2. Testing for Exogeneity of Fertility in Case 1

In this section, the exogeneity of the children variables is re-examined in the hours of work equation and is also investigated in the participation equation using Mroz's data set[16]. Unlike Mroz, in each situation we take into consideration the count nature of the children variables by assuming that they have Poisson distributions[17] conditional on exogenous variables. The Poisson distribution imposes the equality of mean and variance of the variate. However, if this restriction is violated, consistency of estimates is still obtained as long as the relationship between the conditional expectation of the number of children and the explanatory variables is correct (since violation of equidispersion has effects similar to those of heteroscedasticity in the linear regression model, for example,

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of work equation, and the children variable equations contains some of the same information about individuals' preferences towards children and work. This may induce correlation between the regression residuals. Including residual terms in the labor supply equations may therefore help allow for the heterogeneous tastes of individuals.

[16] The analysis of this paper has to be confined to a static framework since a single cross section data is used.

[17] In the sample, some women older than, say, 40 have finished childbearing, but some younger than 40 have not. Thus, the analysis here does not model the whole process of fertility. The Poisson distribution is used to model the children variables in order to improve the quality of instrumentation of the children variables.

see Winkelmann and Zimmermann, 1991a and 1991b). Therefore, violation of equidispersion does not violate the assumptions required for the tests for exogeneity.

In Case 1, the female labor supply and children are chosen simultaneously, and the children variables are introduced along with the other explanatory variables in both an hours of work equation and a participation equation, and a labor supply measure appears in the children equations. We are interested in testing the exogeneity of the children variables in the labor supply equations, and the model is estimated by the limited information method. Thus, in the following specification, labor supply equations are written in structural form, and the children equations are in reduced form:

$$h = X^*\beta + k16^*\beta_y + k618^*\beta_o + e_h \qquad (4.2.1)$$

$$k16 = f\,(Z_y^*\gamma,\ e_y\ ) \qquad (4.2.2)$$

$$k618 = f\,(Z_o^*\delta,\ e_o) \qquad (4.2.3)$$

$$lfp = Z^*\lambda + k16^*\lambda_y + k618^*\lambda_o + e_p \qquad (4.2.4)$$

where f is a linear function if the count nature of k16 and k618 is not considered, i.e., young children $k16 = Z_y{}^*\gamma + e_y$ and old children $k618 = Z_o{}^*\delta + e_o$; otherwise, $k16 = \exp(Z_y{}^*\gamma) + e_y$ and $k618 = \exp(Z_o{}^*\delta) + e_o$ if k16 and k618 are modelled using the Poisson distribution. X is a matrix containing all the variables in (2.1) except the children variables, and Z_y and Z_o contain variables concerning location, family background, education, age, non-wife income (B, F3, H3, nwm) and a column vector of ones, i. Z is a matrix including all the right hand side variables in (2.1) except for the wife's log wage rate and the children variables, e_h, e_y, e_o and e_p are disturbance terms[18], uncorrelated across individuals but potentially related, that is, $cov(e_h, e_y) \neq 0$, $cov(e_h, e_o) \neq 0$, $cov(e_p, e_y) \neq 0$ and $cov(e_p, e_o) \neq 0$. Hence, k16 and k618 may be correlated with the error terms in the labor supply equations.

The results of the tests of exogeneity of the children variables in the hours of work equation in Case 1 are reported in Tables 1, 2 and 3. In Table 1, we first compare Mroz's estimates (listed in columns (5), (6) and (5)-(6)) with our own (listed in columns (1), (2) and (1)-(2)) obtained with slightly different data (see footnote 7). For example, the coefficient of young (old) child variable k16 (k618) from Mroz in column (5) is -344 (-116) and the corresponding standard error is 130 (30), whereas our estimate of the coefficient of young (old) child variable k16 (k618) in column (1) is -343 (-115) and the corresponding standard error is 131 (30). In particular, the results of the tests regarding exogeneity of the children variables reported in columns (1)-(2) and (5)-(6) are more or less the same. Therefore, we conclude that the minor difference in the data set used in this paper and the one in Mroz's paper is not a problem for further testing for the exogeneity of the children variables.

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[18] Assume $cov(e_y, e_o) = 0$. In the linear reduced forms for k16 and k618, this is not necessary if the right hand side variables for them are the same. However, if their reduced forms are nonlinear, this assumption is necessary. The additional complications entailed in testing when relaxing this assumption are topics for future research. We use the limited information method to estimate the h and lfp equations separately if $cov(e_h, e_p) \neq 0$, then we lose some efficiency.

Tables 1 and 2 show that both the joint and the individual exogeneity null hypotheses of the children variables are not rejected in the hours of work equation no matter which method is used to calculate the standard errors, when no attention is paid to the count nature of the children variables. Individually, the exogeneity of the young child variable is not rejected, and that of the old child variable is rejected only when both children variables are included in the hours of work equation and each of them is modelled using the Poisson[19] distribution. Even though the χ^2 values become larger when the count nature of the children variables are considered, the joint exogeneity of the children variables is still not rejected. In Table 1, for example, when we use the formula from Duncan (1987) to calculate the covariance matrix, the χ^2 value from using the estimation method SM_(PSN+OLS) (or SM_PSN) is 5.5 (or 5.43), whereas it is 4.41 from using SM_OLS (see Appendix 3 for explanations of each method).

Tables 1 and 2 also indicate that when the children variables are treated as exogenous, their coefficients are significantly different from zero individually and jointly; however, when they are treated as endogenous, their coefficients are not significantly different from zero either individually or jointly regardless of whether their count nature is taken into consideration. Hence, we summarize that for the hours of work equation,

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[19] A goodness of fit test is used to examine the prediction performance of Poisson regression model for the young and old child variables. The results indicate that the simple Poisson distribution is rejected for the old child variable; it can not be rejected for the young child variable for the sample with working women, whereas it is rejected for the young child variable in the whole sample. The results are summarized in Appendix 5. A different count model could be considered for further research.

we cannot reject both the null hypothesis of joint exogeneity of the children variables, and the null hypothesis that their coefficients are zero. This finding leads us to examine the alternative structure where the children variables do not enter the hours of work equation in section 4.3.

Table 3 provides evidence that the exogeneity of the children variables is rejected jointly in the labor force participation equation. The presence of children, especially young children, has a pronounced negative influence on female labor force participation. This is consistent with findings reported in previous studies (for example, Schultz, 1978, 1980). The coefficients of the children variables are jointly significantly different from zero, no matter how they are treated. However, when they are treated as endogenous, their coefficients are much larger than when they are treated as exogenous. For example, the coefficient of k16 (k618) is about 6.6 (20)[20] times larger with no consideration of their integer nature, and is about 5.5 (15.3)[21] times larger when each is modelled using the Poisson distribution. This suggests that the children variables should be in the labor force participation equation and are endogenous, and thus it is important to take into consideration both their endogeneity and count nature when estimating the participation equation.

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[20] Compare the coefficients in column (2) with those in column (1) in Table 3

[21] Compare the coefficients in column (4) with those in column (1) in Table 3.

4.3. Testing for Exogeneity of Fertility in Case 2

There may be unobservable factors that affect decisions both on how many children to have and on female labor supply. This implies children variables might not appear directly in the hours of work and participation equations, and labor supply measures might not appear directly in the children equations. The specification of Case 2 can be expressed as

$$h = X^{*}\beta + u_h \qquad (4.3.1)$$

$$k16 = f(Z_y^{*}\gamma, u_y) \qquad (4.3.2)$$

$$k618 = f(Z_o^{*}\delta, u_o) \qquad (4.3.3)$$

$$lfp = Z^{*}\lambda + u_p \qquad (4.3.4)$$

where $cov(u_h, u_y) \neq 0$, $cov(u_h, u_o) \neq 0$, $cov(u_p, u_y) \neq 0$, $cov(u_p, u_o) \neq 0$. The error terms in the labor supply relationships are related to the error terms in the children equations and therefore the children variables are related to the errors terms in the labor supply relationships.

Tables 4 and 5 present the results for testing the null hypothesis of exogeneity of the children variables in the labor supply equations in Case 2. Table 4 provides evidence that the exogeneity of both children variables in the hours of work equation are rejected individually and jointly no matter what method is used to compute the standard errors of estimates and whether or not their count nature are taken care. This finding suggests that when the children variables do not appear directly as the explanatory variables in the hours of work equation, they are endogenous to the hours of work decision of married women. Table 5 shows that the null hypothesis that children are exogenous cannot be rejected if these variables are not included in the set of the explanatory variables in the labor force participation equation. This result holds for all approaches used.

Column (2) to (4) of Table 2 indicates a general structure where the children variables are treated as endogenous and they are included in the hours of work equation. This general structure allows us to test two hypotheses in the hours equation: (A) Children are exogenous when they are included in the hours equation; (B) Children are excluded from the hours equation when they are treated as endogenous. The χ^2 and p values for the hypotheses (A) and (B) are shown in the interaction cells of columns (2) to (4) and rows 12 to 13 in Table 2. They indicate the acceptance of the hypotheses (A) and (B). The acceptances of (A) and (B) lead us to two sub-structures: one is implied in column (1) of Table 2 where the children variables are treated as exogenous and they should be in the hours of work equation; the other is suggested in columns (2) to (4) of Table 4 where the children variables are excluded from the hours equation and they are endogenous. The acceptances of these two non-nested sub-structures indicate that the data set used does not allow us to distinguish between them.

Columns (2) to (4) of Table 3 indicate a general structure where children are treated as endogenous and they are included in the participation equation. This general structure allows us to test two hypotheses in the participation equation: (C) Children are exogenous when they are included in the participation equation; (D) Children are excluded from the participation equation when they are treated as endogenous. The χ^2 and p values for the hypotheses (C) and (D) are shown in the interaction cells of columns (2) to (4) and rows 11 to 12 of Table 3. They indicate the rejection of the hypotheses (C) and (D). Therefore, we conclude that the specification of the labor force participation suggested by Case 1 is more suitable to the data set under investigation. This suggests that the children variables are endogenous to the labor force participation decision of married women as a result of a joint decision on how many children to have and whether to work in the job market.

5. Summary and Conclusion

Some previous studies argued that there is an interrelationship between fertility and the labor supply decisions of married women. In particular, Mroz (1987) explicitly tested the exogeneity assumption of fertility in an hours of work equation of married women using the 1975 labor supply data from the Panel Study of Income Dynamics, and he did not rejected the exogeneity of fertility as a result of a joint decision on fertility and hours of work. Two children variables are used to measure fertility in Mroz: (1) the number of the children under age 6 and (2) the number of the children from age 6 to 18. He instrumented them via a simple OLS method without considering the integer nature of these variables. Using essentially the same data set as in Mroz, this paper takes into consideration the count nature of the children variables using the Poisson distribution and re-examines the exogeneity of the children variables in a female hours of work equation employed by Mroz. It also considers a case where fertility and labor supply decisions are both affected by omitted factors, which reflect individuals' heterogenous tastes for children and work. The exogeneity assumption of the children variables in the labor force participation equation, which is not addressed by Mroz, is also examined in this paper.

The empirical results suggest that for the data set analyzed in this paper, two interpretations can be made on the relationship between children and hours of work of married female: (1) the children variables are exogenous and appear in the hours of work equation, which is in accord with Mroz's conclusion about children variables; (2) the children variables are endogenous to the hours of work decision and do not appear in the hours of work equation. However, when testing for individual exogeneity of children variables in the hours equation and each is treated as count data, we reject the individual exogeneity of the old child variable and do not reject that of the young child variable. We find evidence that the children variables appear on the right hand side of the labor force participation equation and they are endogenous to the participation decision no matter how the children variables are treated, the effect of accounting for their endogeneity and integer nature on the estimates of parameters in the participation equation is substantial. For example, when each child variable is treated as endogenous and modelled using the Poisson distribution, the coefficient of the young (old) child variable is about 5.5 (15.3) times larger than those when children are treated as exogenous. The evidence suggests that fertility and participation are a joint decision of married women. Thus, it is important to model children variables properly in the labor supply equations of married women. For future research, more complicated count models and alternative data set may be used.

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Table 1. Endogeneity of Children in the Hours of Work Equation
Substitution Method, Children Variables Included
(Standard Errors in Parentheses)

Variable	Estimates under Ho (1)	Estimates under Ha (2)	(3)	(4)	Differences in Estimates under Ho & Ha (1)-(2)	(1)-(3)	(1)-(4)	Mroz's Estimates under Ho (5)	under Ha (6)	Differences (5)-(6)
int	2115 (352)a	1279 (772)a (937)b	880 (768)b	837 (792)b	836 (635)a (806)b	1235 (620)b	1278 (634)b			
lnww	-18 (175)a	-48 (195)a (243)b	-58 (241)b	-78 (247)b	30 (67)a (108)b	40 (132)b	60 (128)b	-30 (174)m	-52 (189)m	22 (57)m
nwm/10^3	-4.2 (3.3)a	-6.2 (3.7)a (3.7)b	-6.6 (3.9)b	-6.5 (4.1)b	2 (1.3)a (1.4)b	2.4 (1.6)b	2.3 (1.9)b	-4.2 (3.3)m	-5.4 (3.6)m	1.7 (1.2)m
wa	-7.7 (5.8)a	4.5 (13.9)a (17)b	12.2 (13.2)b	13.1 (13.7)b	-12.2 (11.6)a (14.7)b	-19.9 (10.9)b	-20.8 (11.1)b			
we	-14.4 (25.8)a	-0.01 (27.3)a (30.3)b	-0.28 (31.3)b	1.2 (32.3)b	-14.3 (9)a (13)b	-14.1 (16.5)b	-15.6 (16.8)b			
k16	-343 (131)a	-267 (415)a (523)b	-8.1 (374)b	30 (394)b	-76 (377)a (469)b	-334.9 (320)b	-373 (332)b	-344 (130)m	-298 (380)m	-45 (344)m
k618	-115 (30)a	36 (91)a (110)b	83 (93)b	85 (94)b	-151 (85)a (102)b	-198 (86)b	-200 (86)b	-116 (30)m	14 (85)m	-129 (79)m
*χ^2 value P value					4.66a, 4.41b 9.7%a, 11%b	5.5b 6.4%b	5.43b 6.6%b			
**χ^2 value P value	17.80sa 0.01%sa	1.60sa, 1.43sb 44.9%sa, 48.9%sb	1.28sb 52.7%sb	1.12sb 57.1%sb						

Note. Ho -- the null hypothesis of exogeneity of the children variables, k16 and k618. Ha -- the alternative hypothesis of Ho. a & b -- the standard errors calculated by Mroz's formula and Duncan's formula respectively. (2), (3) and (4) -- estimates by the method SM_OLS, SM_(PSN+OLS) and SM_PSN respectively (see Appendix 3 for the explanations of each method). P value -- the probability level. m -- Mroz's estimates. * is for the joint test of exogeneity of k16 and k618. ** is for the joint test of significance of k16 and k618.

Table 2. Endogeneity of Children in the Hours of Work Equation

Generalized Residuals Method. Children Variables Included

(Standard Errors in Parentheses)

Variable	(1)	(2)	(3)	(4)
int	2115 (352.2)a (351.9)b	1279 (771)a (937)b	880 (768)b	894 (803)b
lnww	-18 (175)a (231)b	-48 (193)a (243)b	-58 (241)b	-77 (245)b
nwm/10^3	-4.2 (3.27)a (3.32)b	-6.2 (3.4)a (3.7)b	-6.6 (3.9)b	-6.4 (4)b
wa	-7.7 (5.81)a (5.83)b	4.5 (13.9)a (17.0)b	12.2 (13.2)b	12.1 (13.9)b
we	-14.4 (25.8)a (29.1)b	-0.01 (27)a (30)b	-0.28 (31)b	0.74 (31.6)b
k16	-342.55 (130.86)a (130.89)b	-267 (417)a (522)b	-8 (374)b	11 (420)b
k618	-115 (29.7)a (30.7)b	36 (91)a (111)b	83 (93)b	76 (95)b
ulnww	0.64 (187.5)a (238.5)b	30.2 (204)a (252)b	37 (250)b	54.7 (255)b
uk16	----	-36 (405)a (501)b	-332 (353)b	-356 (395)b
uk618	----	-181 (95)a (113)b	-232 (98)b	-224 (99)b
χ^2 value P value (JE)		4.86a, 4.44b 9%a, 11%b	5.71b 5.7%b	5.30b 7%b
χ^2 value P value (JS)	17.80a, 18.1b (0.01%)a, (0.01%)b	1.70a, 1.43b (42.7%)a, (48.9%)b	1.28b (52.8%)b	0.99b (61%)b
R^2		0.23(y), 0.32(o)	0.25(y), 0.33(o)	
Adjusted R^2		0.19(y), 0.28(o)	0.20(y), 0.29(o)	

Note (1) -- estimates when treating k16 and k618 as exogenous. (2), (3) and (4) -- estimates using the estimation method GR_OLS, GR (PSN+OLS) and GR_PSN respectively (see Appendix 3 for the explanations of each method). a and b -- the standard errors calculated by the ordinary method and Duncan method. u* -- the generalized residual term corresponding to the variables *. R^2 is from the first stage estimation of k16 and k618. y and o refer to k16 and k618 respectively. JE denotes the joint test of exogeneity of k16 and k618. JS denotes the joint test of significance of k16 and k618.

Table 3. Endogeneity of Children in the Labor Force Participation Equation

Probit and Generalized Residuals Method. Children Variables Included

(Standard Errors in Parentheses)

Variable	(1)	(2)	(3)	(4)
int	0.42 (0.47)	9.77 (2.83)b	8.86 (2.97)b	7.74 (2.62)b
nwnw/10^3	-0.02 (0.005)	-0.013 (0.011)b	-0.014 (0.011)b	-0.016 (0.009)b
wa	-0.03 (0.008)	-0.22 (0.049)b	-0.20 (0.052)b	-0.18 (0.047)b
we	0.16 (0.024)	0.18 (0.05)b	0.19 (0 048)b	0.19 (0.042)b
k16	-0.89 (0.12)	-5.86 (1.316)b	-5.18 (1.26)b	-4.86 (1.369)b
k618	-0.038 (0.041)	-0.76 (0.368)b	-0.71 (0.378)b	-0.58 (0.30)b
uk16	----	5.79 (1.312)b	5.14 (1.257)b	4.70 (1.356)b
uk618	----	0.67 (0.368)b	0.62 (0.376)b	0.50 (0.296)b
VNLLF	454	319	331	342
X^2 value **P value (JE)**		19.27b 6.5E-03%	29.5b 0.00%	12 02 0.25%
X^2 value **P value (JS)**	59.84 0.00%	19.85b 5E-03%b	17.09b 1.9E-02%b	12 61b 0.18%b
R^2		0.40(y), 0 31(o)	0 40(y), 0.31(o)	
Adjusted R^2		0.36(y), 0.29(o)	0.38(y), 0.29(o)	

Note (1) -- estimates when treating k16 and k618 as exogenous. (2), (3) and (4) -- estimates using the estimation method GR_OLS, GR_(PSN+OLS) and GR_PSN, respectively (see Appendix 3 for the explanations of each method). b -- the standard errors calculated by the Duncan method. VNLLF -- value of negative log likelihood function. u* -- the generalised residual term corresponding to the variables *. R^2 is from the first stage estimation. y and o refer to k16 and k618 respectively. JE denotes the joint test of exogeneity of k16 and k618. JS denotes the joint test of significance of k16 and k618.

Table 4. Endogeneity of Children in the Hours of Work Equation

Generalized Residuals Method. Children Variables Excluded

(Standard Errors in Parentheses)

Variable	(1)	(2)	(3)	(4)
int	1445 (312)a (325)b	1288 (317)a, (326)b	1298 (322)b	1288 (326)b
lnww	249 (178)a (259)b	-82 (180)a, (235)b	-61 (232)b	-82 (235)b
nwm/10^3	-6.2 (3.7)a (3.6)b	-5.3 (3.2)a, (3.6)b	-5.4 (3.6)b	-5.3 (3.6)b
wa	4.5 (5.1)a (5.3)b	6.6 (5.0)a, (5.3)b	6.4 (5.27)b	6.6 (5.3)b
we	-40.1 (26.1)a (32.5)b	-5.0 (26.6)a, (30.6)b	-7.2 (30.1)b	-5.0 (30.6)b
ulnww	-266 (199)a (266)b	64.2 (191)a, (244)b	39 (239)b	60.5 (244)b
uk16	----	-303 (133)a, (126)b	-340 (123)b	-344 (114)b
uk618	----	-145 (32)a, (31)b	-149 (31)b	-149 (31)b
χ^2 Value **P value (JE)**		25.87a, 27.5b 2E-04%a, 1E-04%b	28.5b 0.0001%b	30.8b 0%b
R^2		0.23(y), 0.32(o)	0.25(y), 0.33(o)	
Adjusted R^2		0.19(y), 0.28(o)	0.20(y), 0.29(o)	

Note See notes on Tables 2 and 3. a and b -- the standard errors calculated by the ordinary method and Duncan method respectively. R^2 is from the first stage estimation of k16 and k618. y and o refer to k16 and k618 respectively. JE denotes the joint test of exogeneity of k16 and k618.

Table 5. Endogeneity of Children in the Labor Force Participation Equation
Probit and Generalized Residuals Method, Children Variable Excluded
(Standard Errors in Parentheses)

Variable	(1)	(2)	(3)	(4)
int	-0.815932 (0.384053)	-0.818167 (0 381391)b	-0.818080 (0.381422)b	- 0.817857 (0.381422)b
nwm/10^3	-0.020888 (0.004427)	-0.020978 (0.004570)b	-0.020978 (0.004571)b	-0.020970 (0.004570)b
wa	-0.006614 (0.005887)	-0.006620 (0.005892)b	-0.006621 (0.005893)b	-0.006618 (0.005893)b
we	0.138326 (0.022770)	0.138720 (0.022909)b	0.138716 (0 022911)b	0.138672 (0.022916)b
uk16		-0.002409 (0.007029)b	-0.004158 (0.007387)b	-0.006248 (0.007042)b
uk618		-0 056410 (0.043027)b	-0.057894 (0.032212)b	-0.055751 (0 043079)b
VNLLF	488 16	487.28	487.23	487.30
χ^2 Value **P value (JE)**		1.76b 41.6%b	3.27b 19.5%b	1 84b 39.8%b
R^2		0.40(y), 0.31(o)	0.40(y), 0.31(y)	
Adjusted R^2		0.36(y), 0.29(o)	0.38(y),0.29(o)	

Note: See notes in Table 2 and 3 b -- the standard errors calculated by the Duncan method VNLLF -- value of negative log likelihood function u* -- the generalised residual term corresponding to the variables * R^2 is from the first stage estimation y and o refer to k16 and k618 respectively JE denotes the joint test of exogeneity of k16 and k618

Appendix 1. Definition of Variables and Summary Statistics

(A) The Definition of the Variables:

lfp:	Labor force participation lfp = 1, if participating, lfp = 0, otherwise
w(h)hrs:	Wife's (Husband's) annual hours of work
k16:	Number of children under age six
k618:	Number of children aged from six to eighteen
w(h)a:	Wife's (Husband's) age
w(h)e:	Wife's (Husband's) education, in number of years
w(h)w:	Wife's (Husband's) hourly wage
faminc:	Family income (in US$1000 dollars)
wmed:	Number of years of schooling of the wife's mother
wfed:	Number of years of schooling of the wife's father
un:	County unemployment rate
cit:	Standard Metropolitan Statistical Areas (SMSA) dummy, SMSA = 1, if SMAS area, SMSA = 0, otherwise
B:	un, SMSA dummy, wmed, wfed
F3:	wa, wa^2, we, we^2, wa*we, wa^3, we^3, wa^2*we, we^2*wa
H3:	the husband's analogous terms to the variables in F3.

(B) Means of the Data (Standard Deviation in Parentheses)

Variable	Whole Sample	Working Sample
whrs	740.6 (873.1)	1302.9 (776.3)
k16	0.24 (0.52)	0.14 (0.39)
k618	1.35 (1.32)	1.35 (1.32)
wa	42.5 (8.1)	42 (7.72)
we	12.3 (2.3)	12.7 (2.29)
ww	---	4.18 (3.31)
hhrs	2267 (595.6)	2234 (582.9)
hw	7.48 (4.23)	7.23 (3.57)
ha	45.2 (8.06)	44.6 (7.95)
he	12.5 (3.02)	12.6 (3.04)
faminc	23081 (12190)	24130 (11671)
wmed	9.25 (3.37)	9.52 (3.31)
wfed	8.81 (3.57)	8.99 (3.52)
un	8.6 % (3.1%)	8.55% (3.03%)
cit	0.64 (0.48)	0.64 (0.48)
Number of observation	753	428

Appendix 2. Duncan Method of Computing Standard Errors

(1) Definition of M-estimator

If $\Sigma_1^n \psi(z_i; b)/n = 0$ is a system of equations depending upon an unknown parameter vector b and data vector z_i and if b solves the system uniquely, then b will be called an M-estimator. Let y_i, x_i, w_i and z_i be a sequence of independent, identically distributed row vectors and consider the following examples of M-estimators:

(i) the normal equations for the least squares estimate of b in $y_i = x_i b + \varepsilon_i$, where the errors ε_i may be conditionally heteroscedastic, define $\psi(y_i, x_i; b) = x'(y_i - x_i b)$, and let b solve $\Sigma_1^n \psi(y_i, x_i ; b)/n = \Sigma_1^n x'(y_i - x_i b) = 0$, which is the ordinary least squares estimator, so the OLS estimator may be considered an M-estimator.

(ii) the equations for an instrumental variable estimator of b in $y_i = x_i b + \varepsilon_i$, define $\psi(y_i, x_i, w_i; b) = w'(y_i - x_i b)$ where w_i is instrumental variable vector, and let b solve $\Sigma_1^n \psi(y_i, x_i, w_i; b)/n = \Sigma_1^n w_i'(y_i - x_i b) = 0$. So simple instrumantal variables estimators are M-estimators.

(2) Definition of Two Stage M-estimator (TSME)

Let $\hat{b}_1$ be an M-estimator satisfying

$$\sum_{i=1}^{n} \psi^1(z_i;\ \hat{b}_1)\ /\ n = 0$$

and let b_2 be an M-estimator for b_1, satisfying

$$\sum_{i=1}^{n} \psi^2(z_i;\ \hat{b}_1,\ \hat{b}_2)\ /\ n = 0$$

Then $b = (b_1^T, b_2^T)^T$ will be said to be TSME.

(3) The formula for calculating the covariance matrix

Let $\psi(z_i, b_1, b_2) = [\psi^1(z_i, b_1)^T, \psi^1(z_i, b_2)^T]^T$, satisfying Assumptions 1-7 given in Duncan (1987, page 377), then the TSME b is a strongly consistent estimator for $b_0 = (b_1^T, b_2^T)^T$ and $n^{1/2}(b - b_0) \rightarrow N(0, V)$, with

$$V = \begin{bmatrix} E\nabla_1\psi_0^1 & 0 \\ E\nabla_1\psi_0^2 & E\nabla_2\psi_0^2 \end{bmatrix}^{-1} \begin{bmatrix} E(\psi_0^1\psi_0^{1T}) & E(\psi_0^1\psi_0^{2T}) \\ E(\psi_0^2\psi_0^{1T}) & E(\psi_0^2\psi_0^{2T}) \end{bmatrix} \begin{bmatrix} E\nabla_1\psi_0^{1T} & E\nabla_1\psi_0^{2T} \\ 0 & E\nabla_2\psi_0^{2T} \end{bmatrix}^{-1}$$

where ∇_i represents the gradient of Ψ^i with respect to parameter vector i, and the expectations are taken over z.

Appendix 3. Estimation Methods

(A) Substitution Methods

SM_OLS (used by Mroz): The auxiliary equations of the variables under testing for endogeneity are specified in the usual way, as linear in the chosen instrumental variables. In the first stage, the predicted values for the variables under testing are estimated from the auxiliary equations. In the second stage, their predicted values are substituted into the labor supply relationships. The covariance matrix can be obtained through using the formula given in case 1 of Mroz's (1987) Appendix 2 and the formula of Duncan (1987) described in Appendix 1 of this paper.

SM_PSN: The auxiliary equations are estimated under the assumption of the Poisson distribution for k16 and k618 (k16 = $\exp(Z_y*\gamma) + e_y$ and k618 = $\exp(Z_o*\delta) + e_o$). Second, the predicted values of the mean of the Poisson distribution[22] (i.e., $\exp(Z_y*\gamma)$ and $\exp(Z_o*\delta)$) replace the original variables in the labor supply equations. In this case, the auxiliary equations are no longer linear. The formula used by Mroz (1987) cannot be employed to compute the variance matrix, since it requires the linearity in the first stage. The formula derived by Duncan (1987) can be utilized to calculate the covariance matrix

[22] For the expression for the Poisson distribution, see e.g., Maddala (1983), p51.

of the parameters.

SM_(PSN+OLS): First, the auxiliary equations are estimated under the assumption of the Poisson distribution for k16 and k618. Second, the predicted values from the auxiliary equation based on the Poisson distribution are added into the set of the existing instrumental variables in the first step and the similar steps as in SM_OLS are conducted.

(B) <u>Generalized Residuals Method</u>

GR_OLS: The same auxiliary equations are used as in SM_OLS, and the generalized residuals are the OLS residuals under the normal distribution of the dependent variable of the auxiliary equations.

GR_(PSN+OLS): The same auxiliary equations are specified as in SM_(PSN+OLS). The generalized residuals in this case are still normal residuals.

GR_PSN: The auxiliary equations are specified based on the Poisson distribution. The generalized residual from the Poisson distribution is the difference between the dependent variable and estimated mean.

For all the methods in (B), the generalised residual terms are appended to the set of explanatory variables and the model is estimated under the alternative hypothesis.

Appendix 4. Simulation: Comparison of Covariance Estimation Method

The simulation experiment demonstrates that the simple covariance formula and the formula from Duncan (1987) (see Appendix 2) tend to give the same results asymptotically for the substitution method (in the linear case), but not always for the generalised residuals methods. Consider the simple model (1) and (2) used in the simulation

$$y_1 = \beta_1 + \beta_2 y_2 + u \qquad (1)$$

$$y_2 = z\theta + v \qquad (2)$$

where u and v are normally distributed random variables, θ is a parameter vector of ones, and $\beta_1 = \beta_2 = 1$. There are two ways of computing variances of β_1 and β_2. Simulations are performed for two cases:

(a) Cov(y_2, u) (the population correlation between y_2 and u) = 0.98; (b) Cov(y_2, u) = 0.

For Cov(y_2, u) = 0.98 and the substitution method, Table 4a shows that the proportional difference between the mean results of the two methods (MXX/MXO) approaches zero as the number of observations grows larger. Table 4b gives a very similar result when Cov(y_2, u) = 0. For Cov(y_2, u) = 0.98 and the generalised residuals method, Table 4c shows that results of the methods do not converge asymptotically (namely, MXX/MXO does not approach zero). Table 4d shows that the results do converge when Cov(y_2, u) = 0.

The results of Table 4c suggest that the simple formula (namely, the one used in linear two stage least squares structures, which ignores the effects of the asymptotic distribution of the first stage estimator on the distribution of the second stage estimator) cannot be used to compute the variances of parameters in the model when some right hand side variables are highly correlated to the error term. Instead, Duncan's formula can be employed. The simple formula for the generalised method is valid only if the null hypothesis of exogeneity of the variables is statistically accepted and the model is of linear two stage structure.

Let XO = stardard error of β calculated from the simple formula.

XD = standard error of β calculated from the formula by Duncan (1987).

XX = XD - XO.

MXX = Mean of XX.

MXO = Mean of XO.

Std. XX = Standard Error of XX.

Table 4a

Simulation Results of the Variances in the Substitution Method, $Cov(y_2, u) = 0.98$

No. of Observation	No. of Replication	MXO	MXX	MXX/MXO	Std. XX
500	600	6.3E-02 4.4E-02	-2.7E-17 -5.8E-20	-4.4E-16 -1.3E-18	4.7E-16 4.3E-17
1000	600	4.4E-02 3.1E-02	-4.9E-17 2.1E-19	-1.1E-15 6.9E-18	7.0E-16 3.9E-17
10000	600	1.4E-02 9.8E-03	-2.5E-18 -1.2E-18	-1.8E-16 -1.2E-16	1.7E-15 3.7E-17

Note: In columns 3 - 6, the first row in each cell is the calculated stard error or standard error related measures of β_1, the second that of β_2.

Table 4b

Simulation Results of the Variances in the Substitution Method, $Cov(y_2, u) = 0$

No. of Observation	No. of Replication	MXO	MXX	MXX/MXO	Std. XX
500	600	6.3E-02 4.5E-02	5.4E-19 4.6E-19	8.6E-18 1.0E-17	4.0E-17 1.9E-17
1000	600	4.5E-02 3.2E-02	2.7E-18 1.8E-18	6.1E-17 5.8E-17	3.9E-17 1.7E-17
10000	600	1.4E-02 1.0E-02	-4.2E-19 -1.3E-19	-3.0E-17 -1.32E-17	2.5E-17 1.1E-17

Note: See the note of Table 4a for the meaning of numbers in each cell in columns 3 - 6.

Table 4c

Simulation Results of the Variances in the Generalised Residual Method, $Cov(y_2, u) = 0.98$

No. of Observation	No. of Replication	MXO	MXX	MXX/MXO	Std. XX
500	600	6.21E-02 4.4E-02 8.8E-02	6.2E-02 4.4E-02 2.9E-02	1.00 1.00 0.32	5.5E-03 4.7E-03 4.4E-03
1000	600	4.4E-02 3.1E-02 6.2E-02	4.4E-02 3.1E-02 2.0E-02	1.00 1.00 0.32	2.7E-03 2.3E-03 2.2E-03
10000	600	1.4E-02 9.8E-03 2.0E-02	1.4E-02 9.8E-03 6.3E-03	1.00 1.00 0.32	2.9E-04 2.3E-04 2.2E-04

Note: The first row in each cell in columns 3 - 6 is the computed standard error or standard error related measures of β_1, the second is that of β_2, and the third is that of estimated residual from y_2 equation.

Table 4d

Simulation Results of the Variances in the Generalised Residual Method, $Cov(y_2, u) = 0$

No. of Observation	No. of Replication	MXO	MXX	MXX/MXO	Std. XX
500	600	6.3E-02 4.4E-02 6.3E-02	3.9E-04 2.7E-04 1.9E-04	6.1E-03 6.1E-03 3.1E-03	5.7E-04 3.9E-04 3.0E-04
1000	600	4.5E-02 3.2E-02 4.5E-02	1.3E-04 8.9E-05 5.8E-05	2.8E-03 2.8E-03 1.3E-03	2.0E-04 1.4E-04 1.0E-04
10000	600	1.4E-02 1.0E-02 1.4E-02	4.3E-06 3.0E-06 2.1E-06	3.0E-04 3.0E-04 1.5E-04	7.2E-06 5.2E-06 4.1E-06

Note: See the note under Table 4c for the explanation of each number in columns 3 - 6.

Appendix 5. Examination of Prediction Performance of Poisson Regression Models for k16 and k618

This Appendix examines the prediction performance of Poisson regression models for k16 and k618[23]. We compare the actual frequencies, the predictions of the Poisson regression model. The χ^2 test rejects Poisson for k618. The χ^2 test does not reject Poisson for k16 in the sample for working women, whereas it accepts Poisson for k16 in the whole sample.

Poisson Distribution

$$f(y_i) = \frac{\lambda_i^{y_i} e^{-\lambda_i}}{y_i!} \qquad \lambda_i \in R^+, \quad y_i = 0,\ 1,\ 2,\ \ldots$$

$$E(y_i \mid \mathbf{z}_i) = \lambda_i = \exp(\mathbf{z}_i * \beta)$$

Goodness of the Fit Test :

$$x^2(m-2) = \sum_{i=1}^{m} \frac{(O_i - E_i)^2}{E_i}$$

where m is the number of the category, m-2 is the degree of freedom of χ^2, O_i is the observed frequency and E_i is the predicted frequency, which is obtained by summing over

[23] See Winkelmann and Zimmermann (1992)

the individual predicted probabilities for each categoty. For χ^2 test, categories are grouped together to obtained expected frequencies of 5 or greater.

Frequency distribution of the young child variable (k16) in the sample for working women

k16	O_i
0	375
1	46
2	7

H_0: the distribution of k16 is Poisson

k16	O_i	E_i
0	375	378.47
1	46	41.07
2	7	8.46

$\chi^2(1) = 0.935$, $p(1) = 33.62\%$. H_0 is not rejected by k16.

Frequency distribution of the old child variable (k618) in the sample for working women

k618	O_i
0	149
1	99
2	97
3	58
4	17
5	7
6	1

H_0: the distribution of k618 is Poisson

k618	O_i	E_i
0	149	144.86
1	99	122.49
2	97	81.42
3	58	44.83
4	17	20.99
≥5	8	13.41

$\chi^2(4) = 14.41$, $p(4) = 0.61\%$, H_0 is rejected by k618.

Frequency distribution of the young child variable (k16) in the whole sample

k16	O_i
0	606
1	118
2	26
3	3

H_0: the distribution of k16 is Poisson

k16	O_i	E
0	606	621.90
1	118	96.09
≥2	29	35.01

$\chi^2(1) = 6.435$. $p(1) = 1.12\%$. H_0 is not rejected by k16.

Frequency distribution of the old child variable (k618) in the whole sample

k618	O_i
0	258
1	185
2	162
3	103
4	30
5	12
6	1
7	1
8	1

H_o: the distribution of k618 is Poisson

k618	O_i	E_i
0	258	256.49
1	185	213.23
2	162	142.32
3	103	79.31
4	30	37.59
≥5	15	24.06

$\chi^2(4) = 18.488$, $p(4) = 0.099\%$, H_o is rejected by k618.

CHAPTER FOUR

FIXED COSTS AND EQUIVALENCE SCALES

1. Introduction

Households differ in their characteristics, and the needs of their individual members vary with their ages, work status and other attributes. Therefore expenditure behaviour across households is expected to be different. Household expenditure patterns are observed in household expenditure surveys, but not the welfare levels of members in households. In terms of designing or implementing government welfare policies, it is essential to know how well off the members of one household are relative to those of another. In the literature, the notion of equivalence scales is developed to bridge demand behaviour and welfare comparisons across households of differing composition. Equivalence scales can be defined as the ratio of the cost of a household obtaining a given level of utility to the cost of a different household with a different set of characteristics obtaining the same level of utility. There are several ways of estimating equivalence scales, most of them are based on empirical analyses of household demand behaviour. The estimation of equivalence scales from analysis of a system of demand equations was started many years ago. In recent years, many constraints on the way that demographic variables affect household demand behaviours have been proposed so that equivalence scales can be estimated from demand systems (see Ray (1986), Blackorby and Donaldson (1988), Lewbel (1989), Browning (1988), Deaton, Ruiz-Castillo and Thomas (1989)). However, not all the information required to make welfare comparisons is identifiable from demand analysis alone (see Pollak and Wales (1979), Blundell and Lewbel (1991)). To completely identify equivalence scales from demand data alone for welfare comparisons, Blackorby and Donaldson (1988) assumed that an equivalence scale is independent of the utility level (or income), which they termed equivalence scale exactness (ESE). Its imposition on household preferences enables us to determine the equivalence scale from demand behaviour alone unless preferences are homothetic. The restrictions implied by ESE are tested by Blundell and Lewbel (1991), and Dickens, Fry and Pashardes (1992). Their empirical results rejected ESE. These empirical rejections of restrictions implied by ESE put some doubt on the validity of the identified equivalence scales. Therefore, instead of restricting equivalence scales to be independent of income (or equivalently, utility), we generalize the ESE assumption by allowing equivalence scales to be a function of income in addition to prices and demographic variables. We call such an extension of ESE as Generalised ESE (GESE).

In order to examine the validity of GESE empirically, we need a demand system on which the testable restrictions implied by GESE can be imposed. As far as we are aware, the existing demand systems cannot be used to carry out the test for GESE. Many studies indicate that the widely used rank two demand system, such as AIDS, is not flexible enough to capture the complicated curvatures existing in household expenditure behaviour (see Lewbel (1991), Xie (1992), Fry and Pashardes (1992), Banks, Blundell and Lewbel (1993)). Hence in this paper, a new demand system with rank equal three is developed (which nests AIDS as a special case) so that consumer behaviour can be modelled more accurately and GESE can be tested. This demand system is termed fixed cost AIDS.

Briefly, our objective in this paper is first to generalize ESE and derive the implications of ESE and GESE concerning the budget share for 'children only' goods. The second aim is to propose a new rank three demand system to allow for modelling of both fixed expenditure and flexible curvature in budget share equations. The third goal is to examine empirically the testable implications of GESE and compute the equivalence scales. The data used in this paper is extracted from five Canadian family expenditure surveys for 1978, 1982, 1984, 1986 and 1990.

The structure of this paper is as follows. In addition to generalising ESE, we describe Blackorby and Donaldson's ESE and provide an alternative way of proving the uniqueness of equivalence scales when preferences are not homothetic and equivalence scales are ESE, and present the implications of ESE and GESE concerning 'children only' goods (i.e., children's clothing) in section 2. The data used is briefly described, and the non-parametric and semi-parametric tests for the implications of ESE and GESE regarding 'children only' good are carried out in section 3. A new demand system called fixed cost AIDS is developed in section 4. The method of econometrics involved in estimating the fixed cost AIDS and testing the GESE restrictions is discussed, and the empirical results are reported in section 5 and conclusions are summarized in section 6.

2. Identifying Equivalence Scales

In demand analysis, we investigate the effects of household characteristics on demand patterns of households by using household expenditure survey data. Interpersonal comparison is not needed. However, in welfare analysis, comparison of the well-being of members of households with different demographics is unavoidable. Equivalence scales enable us to compare the welfare of different households, because they claim to answer the iso-welfare question such as "what is the minimum expenditure level that would make a family with children as well off as a comparable family without children"? (see Browning, 1992). The fact that different family types are treated differently in government welfare policies can be justified by such comparison. However, it must be recognized that it is individual members and not households that experience welfare (see Blackorby and Donaldson (1988)). In order for the welfare comparison to be possible, we adopt the assumption[1] that the intra-household allocation is optimal in the way that everyone in a given household enjoys the same level of utility. Hence, the welfare level of a given household is equal to the level of utility enjoyed by each member in a given household.

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[1] Consideration of more realistic intra-household allocation is beyond the scope of this paper.

Equivalence scales can be defined as the cost required to achieve a certain level of utility by a household relative to that of a reference household enjoying the same level of utility. Let the cost function $c(\mathbf{p}, u, \mathbf{z})$ represent the preferences of a household where u is the level of utility, $\mathbf{p}$ is a vector of commodity prices, and $\mathbf{z}$ is a vector of demographic characteristics. For a reference household, we let $\mathbf{z} = \mathbf{0}$ for convenience. An equivalence scale for a household with characteristics $\mathbf{z}$ is defined by

$$d = c(\mathbf{p}, u, \mathbf{z}) \,/\, c^r(\mathbf{p}, u) \qquad (2.1)$$

where $c^r(u, \mathbf{p})$ is the cost function for a reference household. In addition to demographics $\mathbf{z}$ and the price vector $\mathbf{p}$, equivalence scale d is a function of utility level u, namely $d = d(\mathbf{p}, u, \mathbf{z})$. Letting $c^r(\mathbf{p}, u) = y^r$ (income for a reference household), then dy^r is defined as an equivalent income (denoted by y) for a household with characteristics $\mathbf{z}$. That is, $y = dy^r$. Alternatively, the equivalence scale in (2.1) can be defined in terms of the indirect utility function:

$$v(\mathbf{p}, y, \mathbf{z}) = v^r(\mathbf{p}, y/d) \qquad (2.2)$$

where v(.) is the indirect utility function of households. Both (2.1) and (2.2) suggest that a household with (equivalent) income y and characteristics $\mathbf{z}$ is equally as well off as a reference household with income y/d.

It is well known that the demand data only provide information on preferences conditional on demographic characteristics. It is unconditional preferences that are required to recover the impact of demographic characteristics on welfare (see Pollak and

Wales (1979), Blundell and Lewbel (1991)). Without making further assumptions on the structure of household preferences, equivalence scales cannot be completely identifiable through demand behaviour alone and therefore cannot be meaningful. Among others, Pollak and Wales (1979) and Blackorby and Donaldson (1988) argue that the normalization of the utility function which includes household characteristics as a separate argument does not alter demand behaviour. However, its implication towards equivalence scales may be meaningless if the normalization of the utility function reverses ordinal orders made by using the pre-normalized utility function. Inevitably, meaningful interpersonal comparisons of welfare need more information than analysis of systems of demand behaviour.

The idea that demand patterns can be used as an indirect indication of welfare was pioneered by Engel. He observed that poorer households spend a large share of their income on food than richer ones; and larger families spend higher share of their income on food than the smaller ones. It seems plausible to infer that households who behave identically in terms of spending an equal share of their income on food (or some other bundle of goods) enjoy the same level of welfare. This approach is termed the Engel or iso-prop method. An alternative version of this method is the Rothbarth method which suggests that households who spend the same amount of money on adult-only goods experience the same level of welfare. Deaton and Muellbauer (1986) estimated the cost of children using both the Engel method and Rothbarth method. They found that the true costs of children are generally overestimated by the Engel approach and underestimated by the Rothbarth method using Sri Lankan and Indonesian data. Many researchers have estimated equivalence scales from their demand analysis by proposing restrictions on the way that demographic variables affect the demand behaviour of households. These include Generalised Cost Scaling (GCS) (Ray, 1986), Partial Engel for individual commodity and Rothbarth, and Full Engel (Browning, 1988). Browning (1990) used a PIGLOG utility function to test the conditions implied by all the structures except the Rothbarth method mentioned above. He found that all of the restrictions are rejected for some strata from the Canadian family expenditure surveys. Deaton, Ruiz-Castillo and Thomas (1989) proposed a concept of demographic separability[2] to estimate equivalent scales in a demand system using Spanish data. None of these constraints mentioned above warrant the unique identification of equivalence scales from demand analysis alone. However, the restriction in Blackorby and Donaldson (1988) does so. They propose such a restriction (namely, ESE[3]) that equivalence scales are independent of utility level (or income), and show that ESE holds if and only if the cost function has the following form:

$$c(\mathbf{p}, u, \mathbf{z}) = d(\mathbf{p}, \mathbf{z})c^r(\mathbf{p}, u) \qquad (2.3)$$

where $d(\mathbf{p}, \mathbf{0}) = 1$. A direct implication of ESE is that the equivalent income y of a household $\mathbf{z}$ is a linear function of reference household income y^r without an intercept.

[2] It means that there are groups of goods with little or no relationship to a specific set of demographic variables.

[3] Lewbel (1989) calls this restriction independence of base level of utility (IB), but he does not provide an explicit proof of the uniqueness result.

Namely, $y = d(\mathbf{p}, \mathbf{z})y^r$. Due to the duality between the cost function and indirect utility function, (2.3) is equivalent to

$$v(\mathbf{p}, y, \mathbf{z}) = v^r(\mathbf{p}, y/d(\mathbf{p}, \mathbf{z})) \qquad (2.4)$$

Blackorby and Donaldson proved that if a household utility function is not homothetic and ESE holds, then the equivalence scale is determined uniquely from demand analysis. We regard this result as being the most important proposition in all of the equivalence scale literature. A simple, alternative proof of their result is provided below.

Lemma 1. If ESE holds and there exist two different scales, $d(\mathbf{p}, \mathbf{z})$ and $d^*(\mathbf{p}, \mathbf{z})$, that are consistent with the same household behaviour, and there exists a transformation $G(v, \mathbf{z})$, increasing in its first argument, such that

$$v^r(\mathbf{p}, y/d(\mathbf{p}, \mathbf{z})) = G(v^r(\mathbf{p}, y/d^*(\mathbf{p}, \mathbf{z})), \mathbf{z}) \qquad (2.5)$$

for all $\mathbf{p}$, y and $\mathbf{z}$, then preferences are homothetic.

Proof. See Appendix 3.

Theorem 1. If preferences are not homothetic and ESE holds, then the equivalence scale is uniquely determined from demand behaviour.

Proof. The result follows directly from **Lemma 1**.

The ESE assumption imposes testable restrictions on the way household characteristics enter a system of demand equations. Blundell and Lewbel (1991), and

Dickens, Fry and Pashardes (1992) tested the restrictions implied by ESE using demand systems, and their empirical results rejected ESE. These empirical rejections of the restrictions implied by ESE put the validity of the identified equivalence scales in doubt. There is a need to be able to relax the ESE restrictions. Before generalising ESE, we present ESE's implication regarding the 'children only' good. To our knowledge, this has not been examined before.

Suppose there exists one such good, children's clothing (= good 1 for presentation convenience), that a reference household never purchases, then $c(\mathbf{p}_{-1}, u)$ is the cost function for a reference household where $\mathbf{p}_{-1}$ is the vector of price without the price of good 1. ESE implies that the cost function for any household $c(\mathbf{p}, u, \mathbf{z})$ can be expressed by $d(\mathbf{p}, \mathbf{z})c(\mathbf{p}_{-1}, u)$. As a result, we have the following:

Corollary 1. If ESE holds and there exists a 'children only' good 1 (that is, $c(\mathbf{p}, u, \mathbf{z}) = d(\mathbf{p}, \mathbf{z})c(\mathbf{p}_{-1}, u)$), then the budget share of the 'children only' good for all households is independent of income.

Proof: Since $c(\mathbf{p}, u, \mathbf{z}) = d(\mathbf{p}, \mathbf{z})c(\mathbf{p}_{-1}, u)$, $\ln(c(\mathbf{p}, u, \mathbf{z})) = \ln(d(\mathbf{p}, \mathbf{z})) + \ln(c(\mathbf{p}_{-1}, u))$.

$w_1 = \partial \ln(c)/\partial \ln(p_1) = \partial \ln(d(\mathbf{p}, \mathbf{z}))/\partial \ln(p_1)$.

Since reference households do not consume the 'children only' good, the budget share on it is naturally zero which is certainly independent of income. ESE suggests that the Engel curve of the 'children only' good for households with children has unitary income elasticity. This implication seems unlikely. In section 3, we find that it is rejected.

It seems to us that the basic premise of ESE is questionable since all the testable implications of ESE are rejected by the data. Therefore, instead of restricting equivalence scales to be independent of income (or equivalently, utility), we generalize the ESE assumption by incorporating a fixed cost into the cost function where ESE holds. We call this extension generalised ESE (GESE). It is expressed by

$$c(\mathbf{p}, u, \mathbf{z}) = D(\mathbf{p}, \mathbf{z})c^r(\mathbf{p}, u) + F(\mathbf{p}, \mathbf{z}), \qquad (2.6)$$

where F(.) represents a fixed cost and is a linear homogenous function in **p**. D(.) is zero homogenous in **p**. In terms of equivalent income, (2.6) can be written as $y = D(\mathbf{p}, \mathbf{z})y^r + F(\mathbf{p}, \mathbf{z})$. That is, equivalent income for a household with characteristics **z** is an affine function of a reference household income. Thus, when GESE holds the equivalence scale is a nonlinear function of income as described below:

$$d(\mathbf{p}, y, \mathbf{z}) = y/y^r = D(\mathbf{p}, \mathbf{z}) + F(\mathbf{p}, \mathbf{z})/y^r. \qquad (2.7)$$

In terms of the indirect utility function, (2.6) can be written as

$$v(\mathbf{p}, y, \mathbf{z}) = v^r(\mathbf{p}, (y - F(\mathbf{p}, \mathbf{z}))/D(\mathbf{p}, \mathbf{z})). \qquad (2.8)$$

We show below that GESE also leads to a unique determination of the equivalence scale from demand analysis.

Lemma 2. If GESE holds and there exist two different $D(\mathbf{p}, \mathbf{z})$ and $D^*(\mathbf{p}, \mathbf{z})$, and two different $F(\mathbf{p}, \mathbf{z})$ and $F^*(\mathbf{p}, \mathbf{z})$, namely, $D(\mathbf{p}, \mathbf{z}) \neq D^*(\mathbf{p}, \mathbf{z})$, or $F(\mathbf{p}, \mathbf{z}) \neq F^*(\mathbf{p}, \mathbf{z})$

or both and there exists a transformation $G(v, \mathbf{z})$, increasing in the first argument, such that

$$v^r(\mathbf{p}, (y - F(\mathbf{p}, \mathbf{z})) / D(\mathbf{p}, \mathbf{z})) = G(v^r(\mathbf{p}, (y - F^*(\mathbf{p}, \mathbf{z})) / D^*(\mathbf{p}, \mathbf{z})), \mathbf{z}) \qquad (2.9)$$

for all $\mathbf{p}$, y, and $\mathbf{z}$, then preferences are quasi-homothetic.

Proof. See Appendix 3.

Theorem 2. If preferences are not quasi-homothetic and GESE holds, then the equivalence scale is uniquely determined from demand behaviour.

Proof. The result follows directly from **Lemma 2**.

Clearly, this weakening of the ESE assumption is introduced at the price of widening the class of preferences that is ruled out. Specifically, the ESE result states that either the equivalence scale is unique or preferences are homothetic while the GESE result suggests that either the equivalence scale is unique or preferences are quasi-homothetic.

Like ESE, GESE also has an implication concerning the 'children only' good. Corollary 2 indicates that this implication is weaker than the corresponding one implied by ESE.

Corollary 2. If GESE holds and there exists a 'children only' good, i.e., cost function $c(\mathbf{p}, u, \mathbf{z}) = D(\mathbf{p}, \mathbf{z})c(\mathbf{p}_{-1}, u) + F(\mathbf{p}, \mathbf{z})$ where $\mathbf{p}$ and $\mathbf{p}_{-1}$ are defined as before, then the Engel curve for the 'children only' good is a straight line with an intercept for all households except for reference households.

Proof:

$$q_1 = \frac{\partial c}{\partial p_1} = D_1(\mathbf{p}, \mathbf{z})c^r(\mathbf{p}_{-1}, u) + F_1(\mathbf{p}, \mathbf{z}) \tag{2.10}$$

From the cost function, we have $c^r(\mathbf{p}_{-1}, u) = (y - F(\mathbf{p}, \mathbf{z}))/ D(\mathbf{p}, \mathbf{z})$ (2.11)

Substituting (2.11) in (2.10), we have

$$q_1 = \frac{D_1(\boldsymbol{p}, \boldsymbol{z})}{D(\boldsymbol{p}, \boldsymbol{z})} y + F_1(\boldsymbol{p}, \boldsymbol{z}) - \frac{D_1(\boldsymbol{p}, \boldsymbol{z})}{D(\boldsymbol{p}, \boldsymbol{z})} F(\boldsymbol{p}, \boldsymbol{z})$$

where

$$D_1(\boldsymbol{p}, \boldsymbol{z}) = \frac{\partial D(\boldsymbol{p}, \boldsymbol{z})}{\partial p_1}, \quad F_1 = \frac{\partial F(\boldsymbol{p}, \boldsymbol{z})}{\partial p_1}$$

In summary, when GESE holds, the unique determination of the equivalence scale rules out the possibility of preferences of households being quasi-homothetic; and GESE implies that the Engel curve for the children only good is affine in income. Similarly, when ESE holds, the unique determination of the equivalence scale rules out the possibility of preferences of households being homothetic; and ESE implies that the budget share for the 'children only' good is constant. The implications in Corollaries 1 and 2 are necessary conditions for ESE and GESE, respectively. The acceptance of these implications does not necessarily lead to the acceptance of ESE and GESE. However, their rejection does lead to the rejection of ESE and GESE. In the next section, we are going to investigate the validity of Corollaries 1 and 2.

3. Nonparametric Test of ESE and GESE

In this section, we first examine the implications of ESE and GESE regarding the 'children only' good presented in the previous section using a non-parametric graphical method. We then semi-parametrically test the implications of GESE for the budget share of the 'children only' good given in Corollary 2.

To investigate the implications of ESE and GESE concerning the 'children only' good, specifically, children's clothing, we use Canadian family expenditure (FAMEX) survey data of 1982, 1984, 1986 and 1990. In each survey, five regions (i.e., Atlantic, Quebec, Ontario, Prairie and British Columbia) are covered. The distinction between the clothing bought for adults and for older children becomes unclear, hence we use only the households with children under age 13 to test the implications of ESE and GESE on the budget share for children's clothing. The expenditure on children's clothing is constructed by summing expenditures on clothing for infants (children < 4 years), and girls and boys (aged from 4 to 13). The 1978 data is not used in this section since there is no way to separate the households with children under age 13 from those with older children due to the design of the 1978 FAMEX survey[4]. In addition, our sample is restricted to households of married couples with and without children, with husband in full time employment, and no other adult living in the household. FAMEX records annual purchases of goods, so only a very small percentage of our sample records zero purchases for any good. We leave these households in the sample and treat zero purchase as one element in the choice set. More detailed descriptions of data selection are provided in Part a of Appendix 1.

Based on the Corollaries 1 and 2 in the last section, for given prices and demographic variables, ESE indicates that the budget share of the children's clothing for families with children is constant, whereas GESE implies that the budget share of children's clothing for them is an affine function of the inverse of real expenditure. Total expenditure is obtained by summing spending on the non-durable goods, and is used to compute the budget share for children's clothing. Since total expenditure variable is considered endogenous for demands, net income is used to construct real expenditure (RY), which is obtained as the ratio of net income to the Stone price index[5].

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[4] The data from 1978 Famex will be used to increase price variation when we estimate our demand system in section 5.

[5] The log of the Stone price index is defined as sum of products of budget shares and corresponding logged commodity prices.

In order to concentrate on the analysis of the relationship between budget share for children's clothing and the inverse of RY, the influences of demographic attributes and prices of commodities need to be removed from the budget share for children's clothing and the inverse of RY. This is accomplished by regressing the budget share for children's clothing and the inverse of RY respectively on a set of non-children demographic variables[6] (i.e., dw, dct, dca, dhf, dho, dhe, dhp, ha, ha2, dsf, dso, dse, dsp, dslfp1, dslfp2, sa, sa2), and children variables and their dummies (i.e., ych[7], och, dych doch) and year-region dummies; and the residuals from each regression are used in the graphical analysis. Since there are 5 regions and 4 years of data being used, 20 region-year dummy variables can be generated, which capture all the possible price differences across regions and years. One year-region dummy is excluded from the regressions to avoid the perfect multicollinearity problem.

The residual from the regression of the budget share for children's clothing is graphed against the residual from the regression of the inverse RY in Figure 1. The cubic spline fitting in Figure 1 seems to have a downward slope; this leads us to reject ESE. The fitted line looks approximately linear, which is consistent with the implication of GESE on the budget share of children's clothing. Hence, the finding from the graphical

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[6] For details of definitions of these variables, see the part b of Appendix 1.

[7] ych denotes the number of young children aged from 0 to 3, och represents the number of old children aged from 4 to 12 in this section. dych is the young children dummy, doch is the old children dummy.

analysis is that the implication of GESE in Corollary 2 is not rejected and the implication of ESE in Corollary 1 is rejected.

Corollary 2 implies that in addition to some terms involving prices and demographics, the inverse real expenditure should be on the right hand side (RHS) of the budget share equation for children's clothing. To examine the validity of Corollary 2 alternatively, we run the regression of the budget share for children's clothing on regional dummies, the logged price ratios, the same set of non-children demographic variables, and children variables and their dummies as used in the graphical analysis, and the inverse of real total expenditure (denoted as RX = total expenditure/Stone price index) as well as other functions of real total expenditure , specifically, ln(RX) and $(\ln(RX))^2$. An F~test is used to examine the joint significance of ln(RX) and $(\ln(RX))^2$. Since total expenditure is often considered endogenous, net income is used as an instrument for the total expenditure in the regression analysis, and the two stage least square method is used. The coefficients and t-ratios of the inverse of real total expenditure (i.e., 1/RX), log real total expenditure (i.e., ln(RX)) and the square of log real total expenditure (i.e., $(\ln(RX))^2$), and F statistic for the joint significance of ln(RX) and $\ln(RX))^2$ are reported in column 2 of Table 1A.

We also run the above regression by imposing the implication of GESE, namely, excluding ln(RX) and $(\ln(RX))^2$ on the RHS of the regression. The coefficient and t-ratio of the inverse of real total expenditure are shown in column 3 of Table 1A.

Column 2 of Table 1A tells us that t-ratios for the three expenditure variables are not significant. And the F~statistic on the joint significance of ln(RX) and $(\ln(RX))^2$ is not significant, either. However, the t-ratio of inverse real total expenditure is "significant" when imposing the implication of GESE concerning the budget share for children's clothing as shown in column 3 of Table 1A. This result suggests that the hypothesis implied by GESE that the budget share of children's clothing is affine in inverse income is not rejected, and the implication of ESE is rejected. All of these regression results are consistent with the finding of the graphical analysis. Therefore, we conclude that ESE is rejected by the data. Even though the implication of GESE as in Corollary 2 is not rejected by the data, other implications of GESE need to be investigated. To carry out the investigation, a suitable demand system is required. In the next section, we will develop a flexible new demand system which allows us to test other implications of GESE and estimate equivalence scales.

4. The Fixed Cost Almost Ideal Demand System

4.1. A New Demand System

In order to examine GESE empirically, we need a demand system on which the testable restrictions implied by GESE can be imposed. Many studies indicate that the widely used rank[8] two demand system, such as the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980), is not flexible enough to capture the complicated non-linear curvatures in household expenditure behaviour (for example, Lewbel (1991), Xie (1992), Fry and Pashardes (1992), Banks, Blundell and Lewbel (1993)). Lewbel (1991) and Banks, Blundell and Lewbel (1993) investigated the rank of demand systems using non-parametric methods. Both indicated rank two models including PIGLOG are not adequate for use in the analysis of demand behaviour of all households. Xie (1992) developed a rank three demand system which nests AIDS as a special case to test the conditions for aggregation across individuals and aggregation across commodities. She found that both conditions are rejected and a rank three demand system is required to model consumer behaviour. Banks, Blundell and Lewbel (1993), and Fry and Pashardes (1992)[9] proposed a rank three demand system (called quadratic AIDS), which includes AIDS as a special case. Both studies analyzed the U.K. micro-data using this quadratic AIDS and found significant quadratic log expenditure effects. It seems that a rank three[10]

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[8] The rank of a demand system is defined as the maximum dimension of the function space spanned by the Engel curves of the demand system. For rank one and rank two demand system, the necessary and sufficient conditions are : (1) A demand system has rank equal to one if and only of the demands are homothetic; (2) a demand system has rank equal to two if and only demands are generalised linear. For rank three demand system, the sufficient conditions are provided on page 219-220 by Lewbel (1990, International Economic Review).

[9] The parameterisation of the quadratic AIDS in these two papers is a bit different. Nevertheless, their demand systems are of rank three.

[10] Gorman (1981) showed that the matrix of coefficients of Engel curves for demands that are linear in functions of nominal income is at most rank three.

demand system fits micro-data better than a rank two system. As far as we are aware, the existing rank three demand systems cannot be used to test the restriction implied by GESE. Therefore, we develop a new rank three demand system on which the GESE restriction can be tested and imposed, and equivalence scales can be computed.

To develop this model, we generalize the almost ideal (AI) cost function by incorporating fixed expenditure. We term this cost function the fixed cost almost ideal function (FCAI). There are several reasons for choosing the AI cost function as a base for expansion: (1) the AIDS is a popular demand system, which is used quite frequently in literature of the budget share analysis; (2) most of the rank three demand systems developed in previous studies nest the AIDS as a special case; (3) GESE cannot be tested through a system with rank less than three; and (4) GESE is a extension of ESE whose testable restrictions are examined through the AIDS by Blundell and Lewbel (1991), so it seems natural to use an extension of the AIDS to test the GESE. Even though our system, quadratic AIDS and the demand system used by Xie (1992) all are generated from the AIDS, they are non-nested structures.

The parameterisation for the FCAI cost function is specified as follows:

$$c(\mathbf{p}, u, \mathbf{z}) = \phi(\mathbf{p}, \mathbf{z})G(\mathbf{p}, u, \mathbf{z}) + \varphi(\mathbf{p}, \mathbf{z}) \qquad (4.1)$$

where $\mathbf{p}$, u, and $\mathbf{z}$ are defined as before. A childless couple is chosen as a reference household, for whom, $\mathbf{z} = \mathbf{0}$. G($\mathbf{p}$, u, $\mathbf{z}$) and $\varphi(\mathbf{p}, \mathbf{z})$ are linear homogenous function of $\mathbf{p}$, and $\phi(\mathbf{p}, \mathbf{z})$ is zero homogenous in **p**. Their specifications are given as follows:

$$\ln(\phi(\boldsymbol{p}, \boldsymbol{z})) = \sum_i \sum_m \alpha_{im} z_m \ln(p_i) \tag{4.2}$$

with restriction

$$\sum_i \alpha_{im} = 0$$

$\ln(G(\mathbf{p}, u, \mathbf{z})) = \ln(\alpha(\mathbf{p})) + u\beta(\mathbf{p}, \mathbf{z})$, where $\alpha(\mathbf{p})$ is linear homogenous in **p**, and $\beta(\mathbf{p}, \mathbf{z})$ is zero homogenous in **p**:

$$\ln(\alpha(\boldsymbol{p})) = \alpha_0 + \sum_i \alpha_i \ln(p_i) + (1/2)\sum_i \sum_j \gamma^*_{ij} \ln(p_i)\ln(p_j) \tag{4.3}$$

with restrictions:

$$\sum_i \alpha_i = 1, \qquad \sum_i \gamma^*_{ij} = \sum_j \gamma^*_{ij} = 0$$

$$\ln(\beta(\boldsymbol{p}, \boldsymbol{z})) = \sum_i (\beta_i + \sum_k \beta_{ik} z_k) \ln(p_i) \tag{4.4}$$

with restrictions:

$$\sum_i \beta_i = \sum_i \beta_{ik} = 0$$

The fixed expenditure is specified to allow for substitutions among different goods[11],

[11] If fixed expenditure is specified as a linear function of prices of individual commodities, then the system allows for no substitutions among goods.

$$\varphi(\boldsymbol{p},\boldsymbol{z}) = (\theta_0 + \sum_l \theta_{ol} z_l) \prod p_i^{\theta_i} \tag{4.5}$$

with restrictions:

$$\sum_i \theta_i = 1,\ \theta_o > 0,\ \theta_{ol} > 0 \textit{ for all } l.$$

Note that if there is no fixed expenditure term, i.e., $\varphi(\mathbf{p}, \mathbf{z}) = 0$, then the FCAI cost function (4.1) is reduced to $\phi(\mathbf{p}, \mathbf{z})G(\mathbf{p}, u, \mathbf{z})$ which represents the almost ideal cost function. So we can interpret the cost function (4.1) as a linear extension of the AI cost function.

The budget share for good i associated with the preferences expressed by the cost function (4.1) is given by:

$$w_i = [\alpha_i + \sum_m \alpha_{im} z_m + \sum_j \gamma_{ij} \ln(p_i)](1-\frac{\varphi}{x})$$

$$+ (\beta_i + \sum_k \beta_{ik} z_k)(1-\frac{\varphi}{x})[\ln(x-\varphi)-(\phi+\alpha)] + \theta_i \frac{\varphi}{x} \tag{4.6}$$

We term this demand system the fixed cost AIDS. Note that the budget share equations in the demand system (4.6), unlike those in the AIDS, do not possess the monotone relation with total expenditure x. When estimating, homogeneity (i.e., $\Sigma_j \gamma_{ij} = 0$ for all i, where $\gamma_{ij} = 0.5(\gamma_{ij}^* + \gamma_{ji}^*)$) is imposed since it is not usually rejected by household expenditure data (see Browning and Meghir (1991), and Fry and Pashardes (1992)) .

When there exists a 'children only' good (i.e., children's clothing), the number of commodities consumed by families with children and families without children is different. To childless couples, the consumption of children's clothing is zero. If we conduct our empirical investigation of testable GESE restrictions using a sample containing both households with children and households without children, we run into a technical problem because of the difference in the number of budget share equations for these two types of households. To avoid this problem, we stratify the data on the basis of the presence of children in a given household. Further stratification of the data will be described in the next section.

For a reference household (i.e., a childless couple), the cost function is expressed by

$$c^r(\mathbf{p}_{-1}, u) = \phi^r(\mathbf{p}_{-1})G^r(\mathbf{p}_{-1}, u) + \varphi^r(\mathbf{p}_{-1}) \qquad (4.1r)$$

where $\mathbf{p}_{-1}$ denotes the price vector $\mathbf{p}$ without element p_1 (which denotes the price for children's clothing). $\ln(G^r(\mathbf{p}_{-1}, u)) = \ln(\alpha^r(\mathbf{p}_{-1})) + u\beta^r(\mathbf{p}_{-1})$, the functional forms for $\phi^r(\mathbf{p}_{-1})$, $\ln(\alpha^r(\mathbf{p}_{-1}))$, $\beta^r(\mathbf{p}_{-1})$, and $\varphi^r(\mathbf{p}_{-1})$ are defined similarly as those in (4.2) - (4.5) except that the vector $\mathbf{z}$ is replaced by $\mathbf{0}$ and the price vector $\mathbf{p}$ by $\mathbf{p}_{-1}$. Given the specification for each component of the cost function (4.1r), the expression for the budget share equation for good i (i > 1) derived from (4.1r) is in (4.6r), which is very similar to the one in (4.6).

$$w_i^r = [\alpha_i^r + \sum_j \gamma_{ij}^r \ln(p_j)](1 - \frac{\varphi^r}{x^r})$$

$$+ \ (\beta_i^r)\left(1-\frac{\varphi^r}{x^r}\right)\left[\ln(x^r-\varphi^r)-(\phi^r+\alpha^r)\right] \ + \ \theta_i^r\frac{\varphi^r}{x^r} \qquad (4.6r)$$

where parameters in (4.6r) are superscripted so that they can be differentiated from those without superscripts in (4.6).

4.2. The Testable GESE Restriction

As discussed in section 2, for the case of the existence of a 'children only' good, if GESE is imposed, the relationship between the cost functions for households with children and households without children can be expressed by:

$$c(\mathbf{p}, u, \mathbf{z}) \ = \ D(\mathbf{p}, \mathbf{z})c^r(\mathbf{p}_{-1}, u) + \ F(\mathbf{p}, \mathbf{z}) \qquad (4.7)$$

Since the functional forms of $c(\mathbf{p}, u, \mathbf{z})$ and $c^r(\mathbf{p}_{-1}, u)$ are given above, the expressions for $D(\mathbf{p}, \mathbf{z})$ and $F(\mathbf{p}, \mathbf{z})$ can be obtained by substituting them in (4.7). By simple manipulation, we obtain $D(\mathbf{p}, \mathbf{z}) = \{\phi(\mathbf{p}, \mathbf{z})\alpha(\mathbf{p})\exp[u\beta(\mathbf{p}, \mathbf{z})]\}/\{\phi^r(\mathbf{p}_{-1})\alpha^r(\mathbf{p}_{-1})\exp[u\beta^r(\mathbf{p}_{-1})]\}$ and $F(\mathbf{p}, \mathbf{z}) = \varphi(\mathbf{p}, \mathbf{z}) - D(\mathbf{p}, \mathbf{z})\varphi^r(\mathbf{p}_{-1})$. GESE restrictions indicate that both $D(\mathbf{p}, \mathbf{z})$ and $F(\mathbf{p}, \mathbf{z})$ are independent of u. As a result, $\beta(\mathbf{p}, \mathbf{z})$ should equal $\beta^r(\mathbf{p}_{-1})$. This leads to $D(\mathbf{p}, \mathbf{z}) = \{\phi(\mathbf{p}, \mathbf{z})\alpha(\mathbf{p})\}/\{\phi^r(\mathbf{p}_{-1})\alpha^r(\mathbf{p}_{-1})\}$and $F(\mathbf{p}, \mathbf{z}) = \varphi(\mathbf{p}, \mathbf{z}) - \varphi^r(\mathbf{p}_{-1})\{\phi(\mathbf{p}, \mathbf{z})\alpha(\mathbf{p})\}/\{\phi^r(\mathbf{p}_{-1})\alpha^r(\mathbf{p}_{-1})\}$. The equality restriction means that the first partial derivative of $\beta(\mathbf{p}, \mathbf{z})$ with respect to $\mathbf{z}$ should equal zero for households with children and $\beta_1 = 0$, and β_is (for i > 1) should be same for those with children and those without children. Hence, the testable restriction from GESE is:

$\partial\beta(\mathbf{p}, \mathbf{z})/\partial\mathbf{z} = \mathbf{0}$, and $\beta_1 = 0$ and $\beta_i = \beta_i^r$ for all $i > 1$ (4.8)

Like ESE, GESE implies the testable restriction with regard to the way demographic variables, in particular children variables entering the equations of the demand system. Had Blundell and Lewbel (1991) paid attention to a 'children only' good, the ESE restriction tested by them using the AIDS would be similar to the GESE restriction in (4.8).

4.3. Identifying the Equivalence Scale

In light of Theorem 2, we know that the equivalence scale can be uniquely identified from estimates of parameters of the fixed cost AIDS developed in this section when the GESE restrictions in (4.8) are imposed. For a given household, the equivalence scale is defined as a ratio of its cost function to a cost function of a reference household. Therefore, from (4.7) we have the formula for the equivalence scale:

$d(\mathbf{p}, \mathbf{z}) = c(\mathbf{p}, u, \mathbf{z})/c^r(\mathbf{p}_{-1}, u) = D(\mathbf{p}, \mathbf{z}) + F(\mathbf{p}, \mathbf{z})/y^r$ (4.9)

where $D(\mathbf{p}, \mathbf{z}) = \{\phi(\mathbf{p}, \mathbf{z})\alpha(\mathbf{p})\}/\{\phi^r(\mathbf{p}_{-1})\alpha^r(\mathbf{p}_{-1})\}$, and

$F(\mathbf{p}, \mathbf{z}) = \varphi(\mathbf{p}, \mathbf{z}) - \varphi^r(\mathbf{p}_{-1})\{\phi(\mathbf{p}, \mathbf{p})\alpha(\mathbf{p})\}/\{\phi^r(\mathbf{p}_{-1})\alpha^r(\mathbf{p}_{-1})\}$.

When **p = 1**, the equivalence scale is expressed by

$$d(.) = \exp(\alpha_o - \alpha_o^r) + [(\theta_o + \theta_{o1} z_1 + \theta_{o2} z_2) - \theta_o^r \exp(\alpha_o - \alpha_o^r)] / y^r \qquad (4.10)$$

where z_1 and z_2 are children variables, whose definitions can be found in the next section. To estimate the value of the equivalence scale and the parameters in our model, we use the FAMEX data. In next section, we describe the econometric method used for estimation.

5. Empirical Results

To investigate the restrictions of GESE using our demand system, we extract a sample on household expenditure from the Canada family expenditure survey for 1978, 1982, 1984, 1986 and 1990 according to a set of criteria (see part a in Appendix 1). To model the impact of children on household expenditure behaviour, several children variables are included in the budget share equations of our system. They are defined as the number of children in different age ranges: ych denotes the number of young children in a given household, och the number of old children, chh the total number of children. Due to changes in how children's age bands are classified in the surveys, for years 1982, 1984, 1986 and 1990, ych (och) is defined as the number of children aged from 0 to 3 (from 4 to 15), and for year 1978, ych (och) is defined as the number of children aged from 0 to 4 (from 5 to 15). Clearly, chh = ych + och.

We stratify the selected sample according to the employment status of wife and presence of children in a household. We do so because the labour force status of wife and presence of children in a household affect household demands (see Browning and Meghir, 1991). There are three possibilities for wife's employment status (full time, part time and not in labour force) and two possibilities of children situations in a given household (having children or not). Hence, six strata can be generated. Three of them consist of households without children with wife being in full time, part time employment and not in labour force (denoted as NC_F, NC_P and NC_NW respectively). The other three contain households with children with wife being in full time, part time employment and not in labour force (denoted as C_F, C_P and C_NW respectively). The basic descriptive statistics of each data set are given in the part c of Appendix 1. We choose to work with eight non-durable commodities for households with children: children's clothing, adult clothing, food at home, food at restaurant, services, recreation, transportation and vices (i.e., alcohol beverages and tobacco products), and seven goods for childless couples since they do not consume children's clothing. By doing so, we implicitly assume that these goods and services are weakly separable from any other non-durables and durables.

In the analysis of the demand system, it is important to allow enough variation in parameters to capture the preference differences caused by differences in household characteristics. Hence, in addition to children variables, we also include dummies for regions, home ownership, car ownership, area with population not less than 100,000, and other demographic variables in the system[12].

To estimate (4.6) or (4.6r), a random component e_i is added to each equation in the system. This captures any random differences in individual decision-making. We drop the vice equation so that adding up restrictions can be satisfied. Our fixed cost AIDS (4.6) becomes:

$$w_i = [\alpha_i + \sum_m \alpha_{im} z_m + \sum_j \gamma_{ij} \ln(p_j)] (1 - \frac{\varphi}{x})$$

$$+ (\beta_i + \sum_k \beta_{ik} z_k) (1 - \frac{\varphi}{x}) [\ln(x - \varphi) - (\phi + \alpha)] + \theta_i \frac{\varphi}{x} + e_i$$

where z_m corresponding to α_{im} can be different from z_k corresponding to β_{ik} for all i. The structure of the budget share equations for households with children and that for those without children are similar except for the number of equations in the system.

Note that the budget share equations of our fixed cost AIDS model are nonlinear in parameters. Two linear homogenous price indices appear in all the budget share equations of our system, which depend on the estimated parameters and is common across all equations, and hence inevitably give rise to cross-equation restrictions. Given all these, estimation of our demand system by the nonlinear full information maximum likelihood method can be computationally expensive. Such estimation methods can be further

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[12] For the complete list of the demographic variables included in the system for each stratum, see estimation results in Tables 2.1-2.6 in Appendix 2.

complicated by the need to instrument household expenditure which is often treated as an endogenous variable in the analysis of micro expenditure data. For these reasons we consider an iterative linear procedure for the estimation of our system.

The parameters in our demand system can be divided into two groups: one group contains linear parameters, the other nonlinear parameters. Conditional on the nonlinear parameters and some explanatory variables, our demand system is a linear equation system. Estimation of the linear parameters in the system is carried out through an iterative linear procedure in which five steps are followed: (1) set nonlinear parameters at certain values; (2) set price index φ at a column vector and use the Stone price index to approximate $(\phi + \alpha)$; (3) estimate the linear parameters in the system; (4) update the value of φ and $(\phi + \alpha)$ using the estimated parameters; (5) repeat (3) and (4) until there is no significant change in the estimated parameters. The values of the nonlinear parameters are grid-searched by repeating (1) to (5), the final values of nonlinear parameters are the ones that correspond to the minimised value of the objective function. The iterative linear procedure is started with estimates from the linear approximation of the AIDS. At each iteration the system is estimated by the 3SLS method with household total expenditure on non-durable goods and all other terms which involve this variable treated as endogenous variables, and heteroscedasticity corrected (see Appendix 4 for the brief description of the 3SLS method with endogenous variable treated and heteroscedasticity corrected). We report in Tables 2.1 - 2.6 of Appendix 2 estimation results of our fixed cost AIDS model for all strata: estimates and standard errors of the linear parameters, and the values of nonlinear parameters (through grid-search), Sargan statistic[13], and a list of instrumental variables used. For all strata, variables z_ms in each budget share equation are chosen to be dummies for regions, some non-children demographic variables (see tables 2.1 - 2.6 for details); and variables z_ks are chosen to be dummies for home ownership, car ownership, area with population no less than 100,000. Furthermore, for all the data sets with children, z_ms also include two children variables: the number of young children (ych), and the number of old children (och), and z_ks include the number of children (chh).

Given the unrestricted estimates, we can test the GESE restrictions (and estimate the restricted parameters) using the minimum chi-square (MCS) method (see Blundell (1988)). As we pointed out before, we estimate the demand systems for households with children and for those without children separately. g denotes the estimates of the parameters from the demand system for households with children, and g^0 the estimates of the parameters from the demand system for those without children. In order to test the restrictions implied by GESE using the MCS method, we assume the independence between g and g^0. Let $gg^0 = g \mid$[14]g^0, $\sum$ be the covariance matrix for gg^0. Based on the

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[13] Sargan statistic examines the covariances between the estimated residuals from the instrument variables method and a set of instruments that need not have been used in the estimation of the model under the null hypothesis (see Godfrey, 1988).

[14] | is the sign for vertical concatenation.

independence assumption, $\Sigma = [\, \Sigma_g \sim$[15] $O] \mid [O' \sim \Sigma_g o]$, where $\Sigma_g = var(g)$, $\Sigma_g o = var(g^o)$. O is a matrix with zeros as its elements. qq^o denotes the restricted parameters, and R is a matrix which enables $gg^o = Rqq^o$ under the null hypothesis suggested by (4.8). The rank of R is equal to the number of restrictions. The estimates of the restricted parameters can be obtained by minimising $\chi^2 = (gg^o - Rqq^o)' \Sigma^{-1} (gg^o - Rqq^o)$. From the first order condition, we have $qq^o = (R' \Sigma^{-1} R)^{-1} R' \Sigma^{-1} gg^o$. An estimate of the covariance matrix of qq^o is $(R'\Sigma^{-1}R)^{-1}$. The minimum value of χ^2 follows a chi-squared distribution with degrees of freedom equal to the rank of R. The test results are reported in Table 1B. χ^2 statistics in Table 1B indicate that the testable restrictions implied by GESE are rejected by the data with the wife's employment status being either not in the labor force or part-time. The GESE restrictions are not rejected for the data where wife works full-time. Since the testable restrictions of GESE are only necessary conditions for the satisfaction of GESE, their acceptance does not necessarily guarantee that GESE actually holds, their rejection is enough to conclude that GESE does not hold.

By imposing the GESE restrictions on the fixed cost AIDS, we can compute the equivalence scale according to the formula (4.10) at unit prices. The scales for one young child, one old child, one young child plus one old child, and two old children are graphed in Figures 2 - 4 for households with spouse's employment status being not in the labor force, part-time, and full-time, respectively. We report the values of equivalence scales

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[15] $\sim$ is the sign for horizontal concatenation

at selective income levels in Tables 2 - 4. Figures 2 - 4 suggest that when income level approaches to positive infinity, the equivalence scale reaches the limit of 1. The equivalence scale is inversely related to the income. That is, the higher the income, the lower the equivalence scale. The rich households' equivalence scales for both a young and old child is lower than that for poor households, ceteris paribus. This suggests that a child costs more for poor households than for rich ones. Blundell and Lewbel (1991) have estimated the equivalence scales using the AIDS with UK family expenditure survey data. Due to the imposition of ESE, their equivalence scales estimated do not change with income levels of the household.

We find that costs of a child increase with the age of a child. Setting the scale at 1 for a childless couple, for example, the cost of a young child (aged 0 to 3 or 0 to 4 if 1978 survey) is .130 of that of a couple, that of a old child (aged 4 to 15 or 5 to 15 if 1978 survey) is .132 of that of a couple at income equal to $60,000 for households where both adults work full-time (see Table 4). Like ours, the results obtained by Blundell and Lewbel (1991) also suggest that costs of children increase with their ages. For instance, the cost of a child aged 0 to 2 is 0.091 of that of a couple, the cost of a child aged 3 to 5 is 0.144 of that of a couple, the cost of a child aged 6 to 10 is 0.164 of that of a couple, the cost of a child aged 10 above is 0.180 of that of a couple.

In our results, holding income constant, the cost of a child increases with the labor force involvement of female adults. For example, with income level held at $40,000, the cost of a young child is .008 of that of a couple when the female adult is not in labor force; 0.115 of that of couple when the female adult works part time, 0.143 of that of a couple when the female adult works full time.

6. Summary and Conclusions

This paper generalises Blackorby and Donaldson's Equivalence-Scale Exactness (ESE) condition to GESE. We show that both GESE and ESE can lead to the unique determination of equivalence scales from analysis of demand data alone. GESE allows equivalence scales to depend on household income, whereas ESE is more restrictive and does not permit equivalence scales to vary with household income. The paper also presents the implications of ESE and GESE concerning the budget share for the 'children only' good, and tests them using non-parametric methods. Evidence is found that the restriction of ESE regarding the 'children only' good is rejected, whereas that of GESE is not.

To further investigate the testable restrictions of GESE, a demand system is needed on which the restrictions of GESE can be imposed. For this purpose, a new rank three demand system is developed, which is called fixed cost AIDS. This new system generalises the almost ideal demand system to allow for modelling of fixed cost and flexible curvature in budget shares. It is used to examine empirically the testable implications of GESE, which are found to be rejected. Since the testable restrictions of GESE are only necessary conditions for the satisfaction of GESE, their rejection is enough to conclude that GESE does not hold. The data used in this paper is extracted from the Canadian family expenditure surveys for 1978, 1982, 1984, 1986 and 1990. Finally, equivalence scales are estimated under the assumption of GESE. It is found that the cost of a child for a household with children increases with the age of a child; a child costs more for poor households than for rich ones; the cost of a child increases with the labor force involvement of female adults. Based on this result, our fixed cost AIDS can be useful in the analysis of policy changes affecting households with children.

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Figure 1

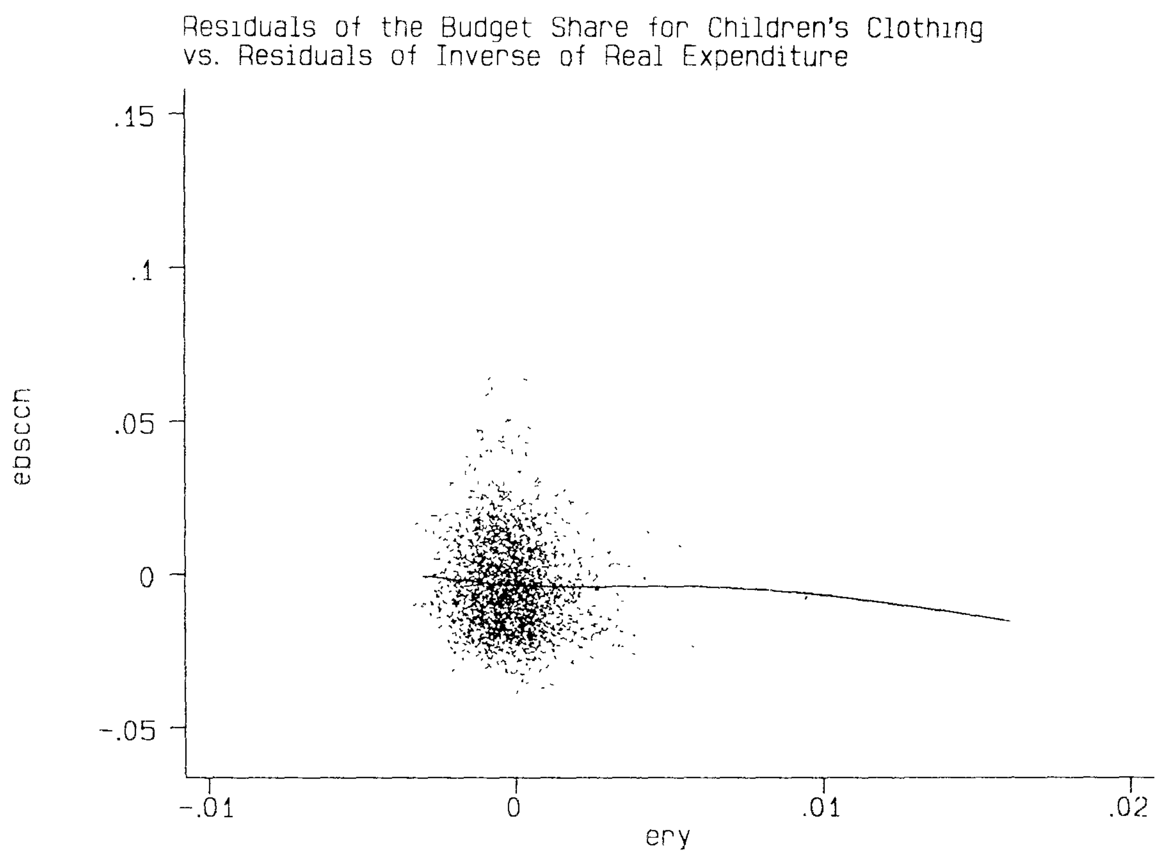
Residuals of the Budget Share for Children's Clothing
vs. Residuals of Inverse of Real Expenditure
.15
.1
.05
0
-.05
ebscch
-.01
0
.01
.02
ery

Table 1A. Tests of GESE for Children's Clothing

Variable	Coefficient (GESE tested)	Coefficient (GESE imposed)
1 / RX	0.46 (0.16)	-0.96 (-6.10)
ln(RX)	0.08 (0.41)	
(ln(RX))2	-0.0067 (-0.38)	
F(2, 3780) statistic probability	0.30 74.42%	

Note: t ratios are in parentheses.

Table 1B. Parametric Test Results for GESE

Spouse's employment status	test statistic
not in labor force	$\chi^2 = 53.43$ DF = 35 p = 2.38%
part time	$\chi^2 = 46.08$ DF = 28 p = 1.71%
full time	$\chi^2 = 47.63$ DF = 35 p = 7.54%

Figure 2

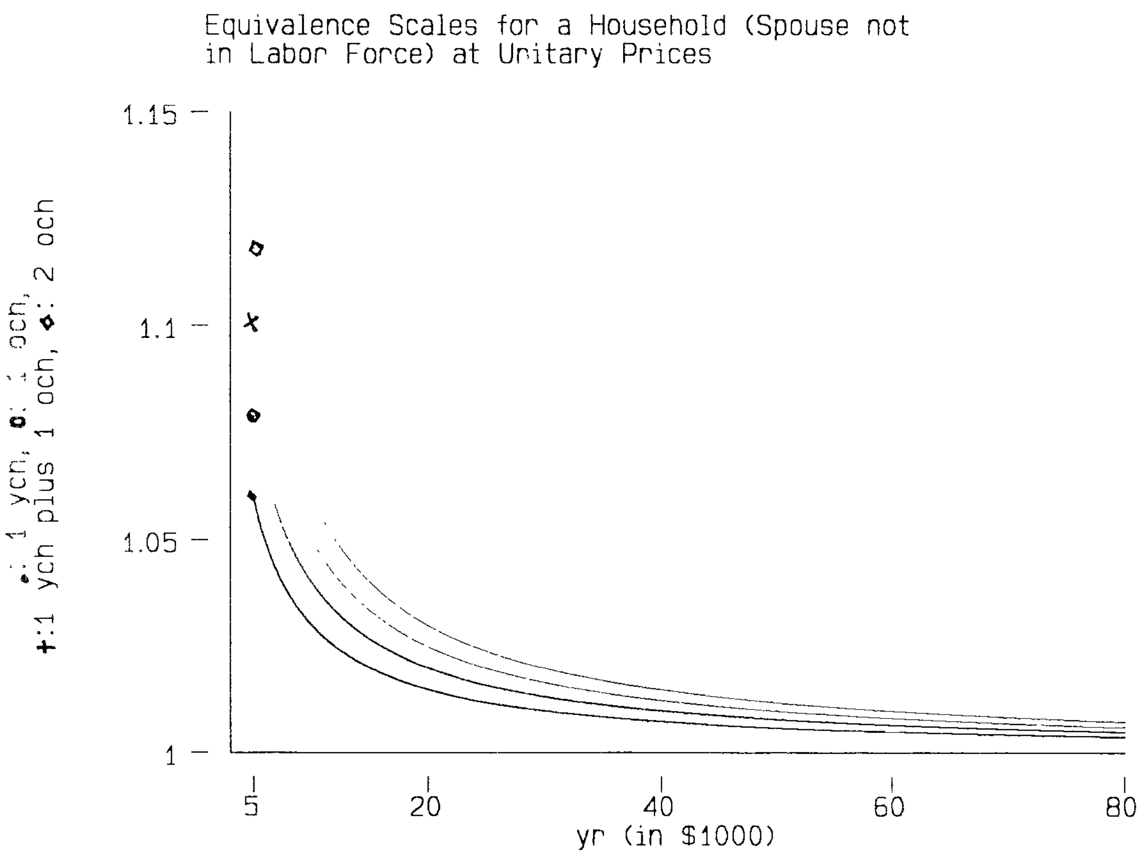
Equivalence Scales for a Household (Spouse not in Labor Force) at Unitary Prices
1.15
1.1
1.05
1
5
20
40
60
80
yr (in $1000)
: 1 ych, : 1 och,
+:1 ych plus 1 och, : 2 och

Table 2. Equivalence Scales

(for households with spouses not being in the labor force)
(income in $1000)

Income level	1 YCH	1 OCH	1 YCH+1 OCH	2 OCH
5	1.06	1.08	1.10	1.12
10	1.03	1.04	1.05	1.06
20	1.015	1.02	1.025	1.03
40	1.008	1.01	1.013	1.015
60	1.005	1.007	1.008	1.01
100	1.003	1.004	1.005	1.006

Figure 3

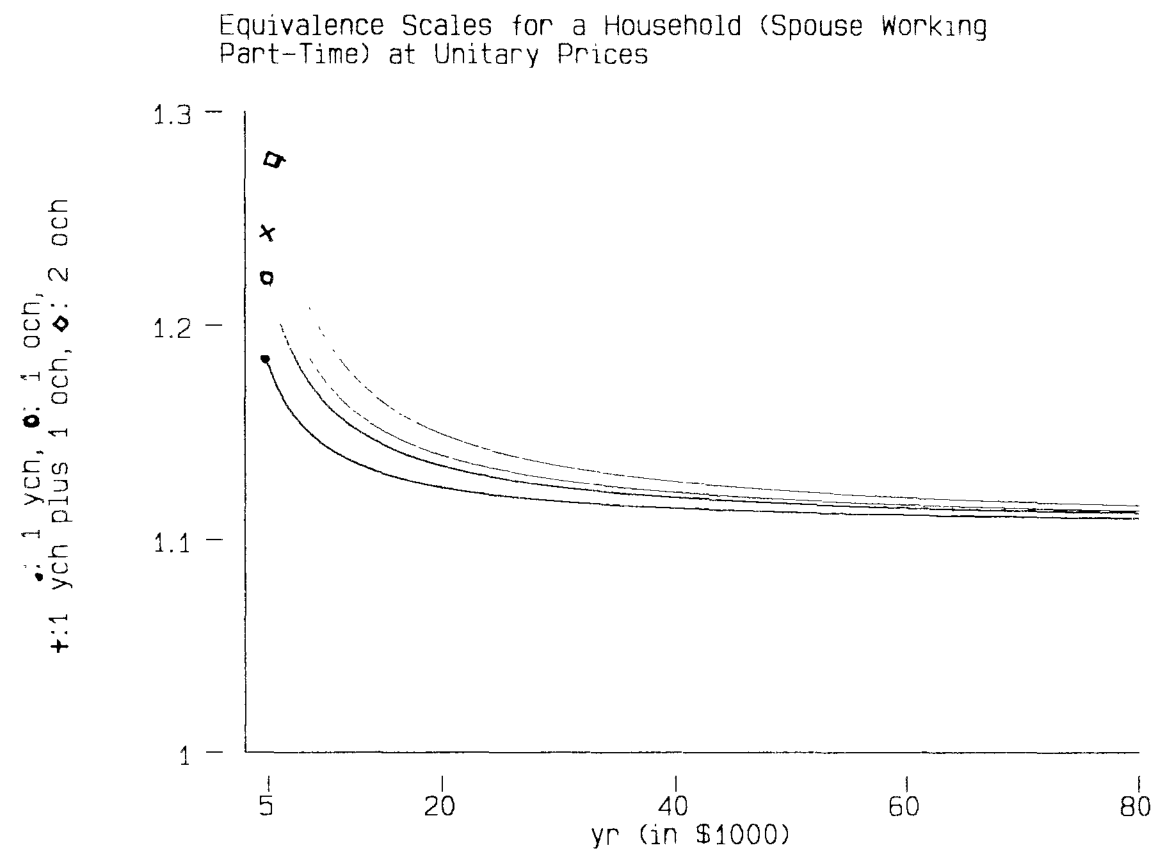
Equivalence Scales for a Household (Spouse Working
Part-Time) at Unitary Prices
1.3
1.2
1.1
1
5
20
40
60
80
yr (in $1000)
•: 1 ych, o: 1 och,
+:1 ych plus 1 och, ◇: 2 och

Table 3. Equivalence Scales

(for households with spouses working part time)
(income in $1000)

Income level	1 YCH	1 OCH	1 YCH+1OCH	2 OCH
5	1.182	1.222	1.242	1.282
10	1.144	1.164	1.174	1.194
20	1.124	1.134	1.139	1.149
40	1.115	1.120	1.122	1.127
60	1.112	1.115	1.117	1.120
100	1.109	1.111	1.112	1.114

Figure 4

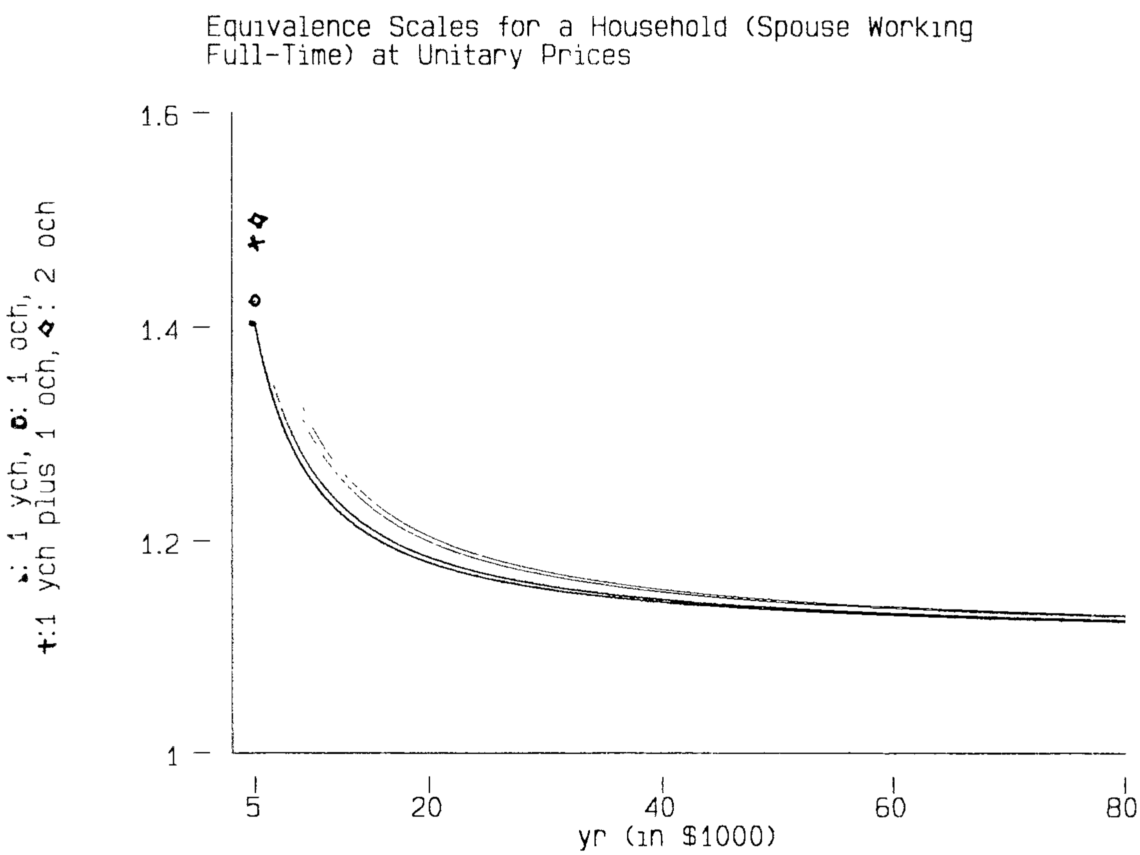
Equivalence Scales for a Household (Spouse Working Full-Time) at Unitary Prices
1.6
1.4
1.2
1
5
20
40
60
80
yr (in $1000)
: 1 ych, : 1 och,
+:1 ych plus 1 och, : 2 och

Table 4. Equivalence Scales

(for households with spouses working full time)
(income in $1000)

Income level	1 YCH	1 OCH	1 YCH+1 OCH	2 OCH
5	1.404	1.424	1.484	1.504
10	1.255	1.265	1.295	1.305
20	1.180	1.185	1.200	1.205
40	1.143	1.145	1.153	1.155
60	1.130	1.132	1.137	1.138
100	1.120	1.121	1.124	1.125

Appendix 1. Data

The data used in this paper is from the Canadian family expenditure survey for 1978, 1982, 1984, 1986 and 1990, which were conducted by the Family Expenditure Survey Section, Household Division of Statistics Canada. For details, see its publication on the Survey of Family Expenditure -- Public Use Micro Data.

a. Sample Selection Criteria

A sample is chosen, which contains two-adult households without or with children aged from 0 to 15, spending no negative amount on any goods and services, based on the following criteria:

Tenure of Household (tenure): 1 = owner without mortgage, 2 = owner with mortgage, 3 = renter

Net Household Income (nety) > 0

Age of Head of a Household (hage) < = 59

Labour Force Status of Head of a Household (hlfp): hlfp = 2, full time employment

Spouse's Labour Force Status (slfp): 0 = not in labour force, 1 = part time employment, 2 = full time employment

Gross Income of Spouse (sgy) > = 0;

Education of Head (Spouse) of a Household (h(s)educ): 1 = less than 9 years, 2 = secondary, 3 = some post-secondary, 4 = post-secondary certificate, 5 = university degree

Number of Children Aged from 0 to 3 (0 to 4 for 1978) (ych) < = 2

Number of Children Aged from 4 to 15 (5 to 15 for 1978) (och) < = 3.

Occupation of Head (Spouse) of a Household (h(s)ocup): for 1978, 1 = management and administration, 2 = professional and technician, 3 = clerical, 4 = sales, 5 = services, 6 = farming, fishing, forestry, 7 = other, 8 = not working including retired; for 1982, 84, 86 and 90, 1 = management and administration, 2 = professional and technician, 3= teaching, 4 = clerical, 5 = sales, 6 = services, 7 = farming, fishing, forestry, 8 = mining processing, 9 = product fabricating, 10 = construction, 11 = other, 12 = not working including retired.

Mother Tongue of Head in a Household (h(s)lang): 1 = English, 2 = French, 3 = other.

The chosen data set contains eight nondurables and services: children's clothing, adult's clothing, food at home, food at restaurant, services, transport, recreation and vices (alcohol and tobacco).

b. Definition of Variables

Year = 1978, 1982, 1984, 1986, 1990

Province (pro): 1 = Atlantic, 2 = Quebec, 3 = Ontario, 4 = Prairie, 5 = British Columbia dpv1 = 1 if pro = 1; 0 otherwise

dpv2 = 1 if pro = 2; 0 otherwise

dpv3 = 1 if pro = 3; 0 otherwise

dpv4 = 1 if pro = 4; 0 otherwise

dpv5 = 1 if pro = 5; 0 otherwise

dw = 1 if tenure < = 2; 0 otherwise

dct = 1 if area = 1; 0 otherwise

dca = 1 if gas > 0; 0 otherwise

dh(s)f = 1 if h(s)lang = 2; 0 otherwise

dh(s)o = 1 if h(s)lang = 3; 0 otherwise

dh(s)e = 1 if h(s)educ = 5; 0 otherwise

dh(s)p = 1 if h(s)ocup < 4 (3 for 1978)

dslfp1 = 1 if slfp = 1; 0 otherwise

dslfp2 = 1 if slfp = 2; 0 otherwise

Age of Head (Spouse) of a household (h(s)age):

ha = (hage - 40) / 100, sa = (sage -40) / 100

Square of h(s)a: ha2 = ha2, sa2 = sa2

chh = ych + och

pcc: price of children's clothing

pca: price of adult's clothing

pfh: price of food at home

pfr: price of food at restaurant

ps: price of services

pt: price of transport

pr: price of recreation

pv: price of vices

xcc: expenditure on children's clothing

xca: expenditure on adult's clothing

xfh: expenditure on food at home

xfr: expenditure on food at restaurant

xs: expenditure on services

xt: expenditure on transport

xr: expenditure on recreation

xv: expenditure on vices

ly = ln(nety)

lysq = ly^2

ry = 1/nety

Note: when estimating, all prices are divided by 100; and all expenditure and net income are divided by 1000.

c. Descriptive Statistics for the Stratified Data Sets from Famex

Table c1
Data set: C_NW
N = 2128

Variable	Mean	Std. Dev.	Min	Max
year	81.942	3.717	78.000	90.000
prov	2.757	1.276	1.000	5.000
slfp	0.000	0.000	0.000	0.000
ych	0.633	0.697	0.000	2.000
och	1.395	0.949	0.000	3.000
dw	0.788	0.409	0.000	1.000
dct	0.677	0.468	0.000	1.000
dca	0.944	0.230	0.000	1.000
dhf	0.268	0.443	0.000	1.000
dho	0.133	0.340	0.000	1.000
dhe	0.175	0.380	0.000	1.000
dhp	0.367	0.482	0.000	1.000
dsf	0.273	0.446	0.000	1.000
dsc	0.128	0.334	0.000	1.000
dse	0.064	0.245	0.000	1.000
dsp	0.000	0.000	0.000	0.000
ha	-0.040	0.071	-0.200	0.190
ha2	0.007	0.007	0.000	0.040
sa	-0.068	0.065	-0.220	0.210
sa2	0.009	0.009	0.000	0.048
dpv1	0.201	0.401	0.000	1.000
dpr2	0.264	0.441	0.000	1.000
dpr3	0.212	0.409	0.000	1.000
dpr4	0.225	0.417	0.000	1.000
dpr5	0.099	0.298	0.000	1.000
nety	25.362	12.437	5.548	121.110
hgy	30.974	17.397	6.011	183.400
sgy	0.810	2.984	0.000	96.501
xcc	0.535	0.435	0.000	3.984
xca	1.152	1.037	0.000	11.550
xfh	4.001	1.514	0.798	15.360
xfr	0.805	0.808	0.000	8.340
xs	1.636	1.005	0.219	15.390
xt	3.016	1.906	0.000	12.954
xr	1.366	1.352	0.000	14.639
xv	0.770	0.851	0.000	18.670
xtot	13.281	5.739	3.319	63.480
pcc	1.346	0.274	1.013	1.971
pca	1.272	0.268	0.883	1.881
pfh	1.426	0.349	0.980	2.096
pfr	1.409	0.367	0.949	2.234
ps	1.449	0.371	0.962	2.214
pt	1.678	0.581	0.849	3.119
pr	1.375	0.350	0.938	2.203
pv	1.673	0.721	0.912	5.066
dych	0.506	0.500	0.000	1.000
chh	2.027	0.761	1.000	5.000

Table c2
Data set: NC_NW
N = 581

Variable	Mean	Std. Dev.	Min	Max
year	82.936	3.688	78.000	90.000
prov	2.905	1.312	1.000	5.000
slfp	0.000	0.000	0.000	0.000
ych	0.000	0.000	0.000	0.000
och	0.000	0.000	0.000	0.000
dw	0.726	0.446	0.000	1.000
dct	0.716	0.451	0.000	1.000
dca	0.910	0.286	0.000	1.000
dhf	0.260	0.439	0.000	1.000
dho	0.146	0.354	0.000	1.000
dhe	0.120	0.326	0.000	1.000
dhp	0.305	0.461	0.000	1.000
dsf	0.256	0.437	0.000	1.000
dso	0.148	0.355	0.000	1.000
dse	0.040	0.195	0.000	1.000
dsp	0.000	0.000	0.000	0.000
ha	0.086	0.109	-0.210	0.190
ha2	0.019	0.011	0.000	0.044
sa	0.070	0.121	-0.220	0.240
sa2	0.020	0.013	0.000	0.058
dpv1	0.174	0.379	0.000	1.000
dpr2	0.253	0.435	0.000	1.000
dpr3	0.203	0.403	0.000	1.000
dpr4	0.234	0.424	0.000	1.000
dpr5	0.136	0.343	0.000	1.000
nety	25.810	14.035	5.841	120.000
hgy	31.633	19.413	3.185	200.000
sgy	1.232	3.367	0.000	30.280
xca	1.351	1.294	0.000	9.065
xfh	3.172	1.332	0.720	8.800
xfr	0.933	1.144	0.000	16.000
xs	1.550	0.857	0.284	6.588
xt	3.059	1.936	0.000	16.000
xr	0.976	1.210	0.000	10.164
xv	1.019	1.057	0.000	7.520
xtot	12.061	5.515	3.161	43.031
pca	1.344	0.257	0.892	1.877
pfh	1.524	0.333	0.980	2.096
pfr	1.516	0.360	0.949	2.234
ps	1.555	0.354	0.960	2.239
pt	1.829	0.548	0.840	3.026
pr	1.468	0.344	0.938	2.203
pv	1.873	0.810	0.915	5.173

Table c3
Data set: C_P
N = 1973

Variable	Mean	Std. Dev.	Min	Max
year	82.958	3.889	78.000	90.000
prov	2.900	1.272	1.000	5.000
slfp	1.000	0.000	1.000	1.000
ych	0.542	0.648	0.000	2.000
och	1.332	0.979	0.000	3.000
dw	0.811	0.391	0.000	1.000
dct	0.710	0.454	0.000	1.000
dca	0.966	0.182	0.000	1.000
dhf	0.205	0.404	0.000	1.000
dho	0.110	0.313	0.000	1.000
dhe	0.239	0.427	0.000	1.000
dhp	0.416	0.493	0.000	1.000
dsf	0.209	0.407	0.000	1.000
dso	0.097	0.296	0.000	1.000
dse	0.138	0.345	0.000	1.000
dsp	0.343	0.475	0.000	1.000
ha	-0.050	0.063	-0.200	0.170
ha2	0.006	0.007	0.000	0.040
sa	-0.072	0.058	-0.220	0.150
sa2	0.009	0.009	0.000	0.048
dpv1	0.192	0.394	0.000	1.000
dpr2	0.192	0.394	0.000	1.000
dpr3	0.237	0.425	0.000	1.000
dpr4	0.284	0.451	0.000	1.000
dpr5	0.096	0.294	0.000	1.000
nety	32.034	14.078	5.915	158.202
hgy	31.265	16.231	1.440	175.000
sgy	8.799	7.381	0.030	55.166
xcc	0.607	0.495	0.000	4.460
xca	1.548	1.215	0.000	11.816
xfh	4.171	1.551	0.910	13.000
xfr	1.138	0.992	0.000	7.275
xs	2.000	1.096	0.344	13.392
xt	3.647	2.138	0.000	18.527
xr	1.668	1.514	0.030	27.169
xv	0.936	0.871	0.000	5.950
xtot	15.715	6.265	3.318	59.657
pcc	1.420	0.280	1.013	1.971
pca	1.344	0.271	0.879	1.882
pfh	1.519	0.346	0.980	2.096
pfr	1.509	0.375	0.949	2.234
ps	1.536	0.364	0.962	2.239
pt	1.791	0.573	0.840	3.085
pr	1.463	0.364	0.938	2.203
pv	1.840	0.773	0.912	5.173
dych	0.457	0.498	0.000	1.000
chh	1.874	0.729	1.000	4.000

Table c4
Data set: NC_P
N = 685

Variable	Mean	Std. Dev.	Min	Max
year	83.177	3.916	78.000	90.000
prov	3.036	1.284	1.000	5.000
slfp	1.000	0.000	1.000	1.000
ych	0.000	0.000	0.000	0.000
och	0.000	0.000	0.000	0.000
dw	0.549	0.498	0.000	1.000
dct	0.736	0.441	0.000	1.000
dca	0.936	0.245	0.000	1.000
dhf	0.220	0.415	0.000	1.000
dho	0.098	0.297	0.000	1.000
dhe	0.166	0.373	0.000	1.000
dhp	0.328	0.470	0.000	1.000
dsf	0.219	0.414	0.000	1.000
dso	0.085	0.279	0.000	1.000
dse	0.118	0.323	0.000	1.000
dsp	0.242	0.429	0.000	1.000
ha	-0.020	0.125	-0.200	0.190
ha2	0.016	0.010	0.000	0.040
sa	-0.041	0.128	-0.220	0.230
sa2	0.018	0.012	0.000	0.053
dpv1	0.158	0.365	0.000	1.000
dpr2	0.200	0.400	0.000	1.000
dpr3	0.223	0.417	0.000	1.000
dpr4	0.286	0.452	0.000	1.000
dpr5	0.133	0.340	0.000	1.000
nety	30.477	14.395	6.001	123.550
hgy	28.047	15.683	2.259	162.000
sgy	9.930	7.572	0.048	60.000
xca	1.870	1.557	0.000	12.158
xfh	2.915	1.256	0.200	9.570
xfr	1.424	1.407	0.000	12.750
xs	1.808	1.024	0.269	8.000
xt	3.553	2.270	0.000	21.489
xr	1.548	1.448	0.000	11.440
xv	1.212	1.167	0.000	9.140
xtot	14.330	6.375	3.702	50.539
pca	1.359	0.271	0.887	1.880
pfh	1.539	0.348	0.980	2.096
pfr	1.538	0.375	0.949	2.234
ps	1.561	0.364	0.965	2.211
pt	1.823	0.570	0.854	3.119
pr	1.483	0.364	0.938	2.203
pv	1.885	0.799	0.912	5.173

Table c5
Data set: C_F
N = 1290

Variable	Mean	Std. Dev.	Min	Max
year	83.709	3.837	78.000	90.000
prov	2.787	1.221	1.000	5.000
slfp	2.000	0.000	2.000	2.000
ych	0.360	0.535	0.000	2.000
och	1.361	0.903	0.000	3.000
dw	0.823	0.382	0.000	1.000
dct	0.757	0.429	0.000	1.000
dca	0.951	0.216	0.000	1.000
dhf	0.196	0.397	0.000	1.000
dho	0.167	0.373	0.000	1.000
dhe	0.176	0.381	0.000	1.000
dhp	0.349	0.477	0.000	1.000
dsf	0.203	0.402	0.000	1.000
dso	0.144	0.351	0.000	1.000
dse	0.131	0.338	0.000	1.000
dsp	0.358	0.480	0.000	1.000
ha	-0.035	0.063	-0.190	0.190
ha2	0.005	0.006	0.000	0.036
sa	-0.059	0.059	-0.200	0.170
sa2	0.007	0.007	0.000	0.040
dpv1	0.198	0.399	0.000	1.000
dpr2	0.196	0.397	0.000	1.000
dpr3	0.309	0.462	0.000	1.000
dpr4	0.214	0.410	0.000	1.000
dpr5	0.083	0.276	0.000	1.000
nety	40.024	16.682	9.564	128.008
hgy	30.053	15.904	2.552	150.217
sgy	20.774	11.454	1.169	100.360
xcc	0.708	0.601	0.000	4.465
xca	2.162	1.690	0.000	21.635
xfh	4.472	1.724	0.900	12.660
xfr	1.603	1.315	0.000	9.485
xs	2.388	1.399	0.410	16.259
xt	4.322	2.483	0.000	19.216
xr	2.105	1.748	0.015	16.805
xv	1.181	1.096	0.000	7.880
xtot	18.942	7.573	4.349	67.011
pcc	1.475	0.273	1.013	1.971
pca	1.384	0.255	0.879	1.883
pfh	1.587	0.333	0.980	2.096
pfr	1.575	0.369	0.949	2.234
ps	1.616	0.348	0.964	2.235
pt	1.920	0.552	0.840	3.090
pr	1.543	0.361	0.938	2.203
pv	1.995	0.815	0.912	5.133
dych	0.332	0.471	0.000	1.000
chh	1.721	0.699	1.000	4.000

Table c6
Data set: NC_F
N = 1389

Variable	Mean	Std. Dev.	Min	Max
year	83.425	3.863	78.000	90.000
prov	3.014	1.227	1.000	5.000
slfp	2.000	0.000	2.000	2.000
ych	0.000	0.000	0.000	0.000
och	0.000	0.000	0.000	0.000
dw	0.630	0.483	0.000	1.000
dct	0.808	0.394	0.000	1.000
dca	0.945	0.228	0.000	1.000
dhf	0.220	0.415	0.000	1.000
dno	0.104	0.306	0.000	1.000
dhe	0.181	0.386	0.000	1.000
dhp	0.377	0.485	0.000	1.000
dsf	0.212	0.409	0.000	1.000
dso	0.096	0.294	0.000	1.000
dse	0.164	0.371	0.000	1.000
dsp	0.340	0.474	0.000	1.000
ha	-0.040	0.109	-0.200	0.190
ha2	0.013	0.010	0.000	0.040
sa	-0.061	0.108	-0.220	0.210
sa2	0.015	0.011	0.000	0.048
dpv1	0.145	0.352	0.000	1.000
dpr2	0.202	0.401	0.000	1.000
dpr3	0.258	0.438	0.000	1.000
dpr4	0.287	0.452	0.000	1.000
dpr5	0.109	0.312	0.000	1.000
nety	38.573	16.722	8.148	184.000
hgy	28.765	16.285	0.600	172.000
sgy	20.610	10.695	1.064	87.215
xca	2.461	1.903	0.000	20.110
xfh	2.981	1.218	0.000	11.300
xfr	1.902	1.580	0.000	13.100
xs	2.078	1.240	0.220	10.698
xt	4.013	2.393	0.000	17.185
xr	1.889	1.845	0.000	23.387
xv	1.294	1.233	0.000	14.534
xtot	16.619	7.113	3.355	61.718
pca	1.377	0.267	0.881	1.881
pfh	1.560	0.342	0.980	2.096
pfr	1.558	0.374	0.949	2.234
ps	1.583	0.355	0.960	2.226
pt	1.870	0.556	0.846	3.140
pr	1.511	0.360	0.938	2.203
pv	1.914	0.782	0.912	5.173

Appendix 2. Estimation Results

Table 2.1
Data set: C_NW, N = 2128

********* AI with fixed cost Demand System Estimation **********

Homogeneity Imposed

Instrumental Variable Estimation

Included Goods = CC CA FH FR S R T
Good Dropped for Estimation = V

Instruments = intercept, non-homogeneous prices and
DPV1 DPV3 DPV4 DPV5 DW DCT DCA YCH OCH DYCH DHF
DHO DHE DHP HA HA2 HGY DSF DSO DSE SA SA2
SGY ly lysq ry

coeff	estimate	se	t-value
alINTCC	-0.159	0.113	-1.412
alDPV1CC	0.002	0.004	0.422
alDPV3CC	-0.005	0.003	-1.481
alDPV4CC	0.001	0.006	0.215
alDPV5CC	-0.001	0.006	-0.226
alDCACC	0.043	0.086	0.493
alDHFCC	-0.003	0.003	-1.114
alDHOCC	-0.001	0.002	-0.520
alDHECC	0.000	0.002	0.147
alDHPCC	0.004	0.002	2.555
alHACC	-0.046	0.013	-3.696
alDSECC	-0.005	0.003	-1.824
alSA2CC	-0.080	0.104	-0.767
alYCHCC	0.052	0.028	1.846
alOCHCC	0.063	0.029	2.219
gaCCCC	0.078	0.044	1.778
gaCCCA	-0.066	0.039	-1.666
gaCCFH	0.068	0.028	2.432
gaCCFR	-0.059	0.033	-1.805
gaCCS	-0.046	0.028	-1.640
gaCCR	0.009	0.022	0.383
gaCCT	0.017	0.018	0.966
beINTCC	0.101	0.067	1.496
beDWCC	0.000	0.001	0.184
beDCTCC	-0.001	0.001	-1.428
beDCACC	-0.037	0.059	-0.626
beCHHCC	-0.026	0.016	-1.635

siINTCC	0.069	0.033	2.127
alINTCA	-0.196	0.238	-0.826
alDPV1CA	-0.003	0.008	-0.415
alDPV3CA	-0.015	0.007	-2.081
alDPV4CA	0.003	0.012	0.225
alDPV5CA	-0.016	0.014	-1.184
alDCACA	0.209	0.183	1.142
alDHFCA	0.001	0.006	0.232
alDHOCA	-0.005	0.004	-1.120
alDHECA	0.005	0.005	0.946
alDHPCA	0.010	0.003	2.895
alHACA	-0.002	0.029	-0.071
alDSECA	-0.002	0.006	-0.361
alSA2CA	0.130	0.233	0.559
alYCHCA	-0.040	0.059	-0.678
alOCHCA	-0.042	0.060	-0.705
gaCACC	0.092	0.093	0.990
gaCACA	-0.161	0.085	-1.889
gaCAFH	0.162	0.059	2.733
gaCAFR	-0.038	0.069	-0.546
gaCAS	-0.150	0.060	-2.489
gaCAR	0.074	0.045	1.639
gaCAT	0.048	0.038	1.262
beINTCA	0.234	0.141	1.660
beDWCA	0.001	0.002	0.505
beDCTCA	-0.002	0.002	-0.864
beDCACA	-0.194	0.124	-1.566
beCHHCA	0.013	0.033	0.401
siINTCA	0.128	0.072	1.784
alINTFH	0.593	0.399	1.487
alDPV1FH	-0.038	0.012	-3.159
alDPV3FH	-0.035	0.011	-3.066
alDPV4FH	-0.063	0.020	-3.237
alDPV5FH	-0.040	0.024	-1.662
alDCAFH	-0.023	0.321	-0.073
alDHFFH	0.017	0.009	1.900
alDHOFH	0.034	0.008	4.079
alDHEFH	-0.004	0.008	-0.505
alDHPFH	-0.006	0.006	-1.126
alHAFH	0.160	0.045	3.564
alDSEFH	-0.019	0.010	-1.819
alSA2FH	-1.237	0.405	-3.054
alYCHFH	-0.022	0.103	-0.213
alOCHFH	-0.020	0.103	-0.197
gaFHCC	0.235	0.160	1.467
gaFHCA	-0.075	0.145	-0.518
gaFHFH	0.035	0.094	0.375
gaFHFR	0.076	0.126	0.602
gaFHS	0.034	0.102	0.332
gaFHR	-0.087	0.078	-1.116
gaFHT	-0.154	0.064	-2.419
beINTFH	-0.147	0.238	-0.618
beDWFH	-0.001	0.004	-0.163
beDCTFH	-0.000	0.003	-0.046
beDCAFH	-0.017	0.217	-0.077
beCHHFH	0.027	0.057	0.477
siINTFH	0.500	0.122	4.097
alINTFR	-0.427	0.175	-2.441
alDPV1FR	-0.000	0.008	-0.055
alDPV3FR	0.000	0.007	0.038

alDPV4FR	0.020	0.011	1.756
alDPV5FR	0.010	0.014	0.745
alDCAFR	0.232	0.174	1.328
alDHFFR	-0.007	0.005	-1.303
alDHOFR	-0.007	0.004	-1.600
alDHEFR	0.010	0.005	2.094
alDHPFR	0.009	0.003	2.871
alHAFR	-0.040	0.025	-1.642
alDSEFR	-0.009	0.006	-1.644
alSA2FR	-0.427	0.214	-1.997
alYCHFR	0.064	0.059	1.100
alOCHFR	0.062	0.059	1.043
gaFRCC	-0.007	0.088	-0.074
gaFRCA	-0.079	0.077	-1.017
gaFRFH	0.036	0.060	0.604
gaFRFR	-0.077	0.071	-1.085
gaFRS	0.035	0.057	0.617
gaFRR	0.020	0.044	0.459
gaFRT	0.041	0.031	1.345
beINTFR	0.327	0.110	2.978
beDWFR	-0.003	0.002	-1.615
beDCTFR	0.008	0.002	4.860
beDCAFR	-0.196	0.116	-1.691
beCHHFR	-0.043	0.033	-1.309
siINTFR	0.102	0.047	2.166
alINTS	-0.153	0.194	-0.789
alDPV1S	0.018	0.007	2.503
alDPV3S	0.007	0.007	1.137
alDPV4S	0.038	0.012	3.102
alDPV5S	0.029	0.015	1.966
alDCAS	0.147	0.187	0.784
alDHFS	-0.021	0.005	-4.123
alDHOS	-0.016	0.004	-3.645
alDHES	0.006	0.005	1.232
alDHPS	-0.000	0.003	-0.046
alHAS	-0.019	0.025	-0.757
alDSES	0.002	0.006	0.392
alSA2S	0.411	0.238	1.726
alYCHS	0.027	0.059	0.458
alOCHS	0.018	0.060	0.302
gaSCC	0.057	0.089	0.645
gaSCA	-0.142	0.082	-1.724
gaSFH	0.047	0.064	0.738
gaSFR	-0.114	0.076	-1.495
gaSS	-0.036	0.058	-0.617
gaSR	0.062	0.049	1.286
gaST	0.075	0.030	2.476
beINTS	0.185	0.123	1.503
beDWS	-0.002	0.002	-0.979
beDCTS	-0.001	0.002	-0.691
beDCAS	-0.139	0.126	-1.103
beCHHS	-0.014	0.033	-0.435
siINTS	0.174	0.049	3.522
alINTR	-0.066	0.299	-0.222
alDPV1R	0.018	0.011	1.670
alDPV3R	0.031	0.010	3.223
alDPV4R	0.021	0.017	1.254
alDPV5R	0.025	0.020	1.243
alDCAR	0.121	0.266	0.454
alDHFR	0.005	0.008	0.627

alDHOR	-0.014	0.007	-2.112
alDHER	-0.003	0.007	-0.363
alDHPR	0.001	0.005	0.123
alHAR	-0.072	0.040	-1.801
alDSER	0.030	0.009	3.247
alSA2R	-0.157	0.301	-0.520
alYCHR	-0.106	0.086	-1.234
alOCHR	-0.103	0.086	-1.189
gaRCC	0.034	0.135	0.256
gaRCA	0.044	0.121	0.366
gaRFH	-0.163	0.087	-1.870
gaRFR	0.132	0.104	1.273
gaRS	-0.104	0.085	-1.226
gaRR	-0.078	0.071	-1.085
gaRT	0.085	0.050	1.680
beINTR	0.100	0.182	0.547
beDWR	0.005	0.003	1.569
beDCTR	-0.000	0.003	-0.077
beDCAR	-0.117	0.179	-0.651
beCHHR	0.055	0.047	1.157
siINTR	0.209	0.088	2.369
alINTT	0.816	0.497	1.642
alDPV1T	0.035	0.017	2.034
alDPV3T	0.012	0.015	0.813
alDPV4T	0.017	0.027	0.612
alDPV5T	0.026	0.033	0.785
alDCAT	-0.291	0.417	-0.696
alDHFT	0.005	0.012	0.397
alDHOT	0.028	0.011	2.683
alDHET	0.004	0.011	0.328
alDHPT	-0.004	0.008	-0.480
alHAT	0.036	0.059	0.603
alDSET	0.005	0.013	0.404
alSA2T	1.147	0.520	2.206
alYCHT	0.024	0.133	0.182
alOCHT	0.017	0.134	0.128
gaTCC	-0.291	0.214	-1.357
gaTCA	0.139	0.195	0.712
gaTFH	-0.065	0.132	-0.490
gaTFR	0.135	0.170	0.795
gaTS	0.015	0.138	0.111
gaTR	0.177	0.101	1.755
gaTT	0.029	0.083	0.354
beINTT	-0.527	0.302	-1.745
beDWT	0.009	0.005	1.817
beDCTT	-0.003	0.004	-0.755
beDCAT	0.422	0.282	1.495
beCHHT	-0.009	0.074	-0.128
siINTT	-0.110	0.140	-0.783

$\alpha_0 = 0$, $\theta_0 = 1.9$, $\theta_{11} = 0.1$, $\theta_{02} = 0.2$

Over-Identifying Restrictions Tests

Sargan chi-squared =	57.5678041
degrees of freedom =	49.0000000
probability (%) =	18.7779620

Table 2.2
Data set: NC_NW, N = 581

********* AI with fixed cost Demand System Estimation **********

Homogeneity Imposed

Instrumental Variable Estimation

Included Goods = CA FH FR S R T
Good Dropped for Estimation = V

Instruments = intercept, non-homogeneous prices and
DPV1 DPV3 DPV4 DPV5 DW DCT DCA DHF DHO DHE
DHP
HA HA2 HGY DSF DSO DSE SA SA2 SGY ly
lysq
ry

coeff	estimate	se	t-value
alINTCA	0.529	0.287	1.843
alDPV1CA	-0.029	0.021	-1.425
alDPV3CA	-0.035	0.019	-1.876
alDPV4CA	-0.050	0.024	-2.084
alDPV5CA	-0.028	0.027	-1.052
alDCACA	-0.462	0.189	-2.442
alDHFCA	-0.008	0.016	-0.497
alDHOCA	0.007	0.011	0.662
alDHECA	0.013	0.016	0.832
alDHPCA	0.003	0.009	0.307
alHACA	-0.026	0.039	-0.667
alDSECA	0.008	0.019	0.431
alSA2CA	0.179	0.248	0.725
gaCACA	0.054	0.124	0.435
gaCAFH	0.351	0.137	2.551
gaCAFR	-0.131	0.131	-0.994
gaCAS	-0.228	0.121	-1.879
gaCAR	-0.061	0.114	-0.533
gaCAT	0.027	0.050	0.528
beINTCA	-0.179	0.139	-1.289
beDWCA	0.007	0.005	1.275
beDCTCA	-0.004	0.005	-0.665
beDCACA	0.243	0.112	2.172
siINTCA	-0.033	0.146	-0.224
alINTFH	0.348	0.370	0.941
alDPV1FH	0.005	0.026	0.174
alDPV3FH	-0.018	0.024	-0.769
alDPV4FH	-0.024	0.029	-0.856
alDPV5FH	-0.060	0.033	-1.827
alDCAFH	-0.091	0.247	-0.369
alDHFFH	0.023	0.019	1.216
alDHOFH	0.080	0.015	5.164
alDHEFH	-0.007	0.017	-0.448
alDHPFH	-0.010	0.011	-0.939
alHAFH	0.199	0.051	3.928
alDSEFH	-0.038	0.023	-1.679
alSA2FH	-0.054	0.356	-0.152
gaFHCA	0.037	0.179	0.208
gaFHFH	-0.319	0.197	-1.624

gaFHFR	0.273	0.177	1.546
gaFHS	0.061	0.187	0.325
gaFHR	0.176	0.146	1.211
gaFHT	-0.114	0.062	-1.825
beINTFH	-0.030	0.181	-0.168
beDWFH	0.012	0.007	1.767
beDCTFH	-0.002	0.007	-0.278
beDCAFH	-0.008	0.143	-0.057
siINTFH	0.573	0.170	3.363
alINTFR	0.428	0.233	1.837
alDPV1FR	-0.020	0.015	-1.300
alDPV3FR	-0.005	0.012	-0.422
alDPV4FR	-0.016	0.017	-0.931
alDPV5FR	-0.023	0.018	-1.246
alDCAFR	-0.272	0.168	-1.622
alDHFFR	-0.029	0.010	-2.817
alDHOFR	-0.028	0.008	-3.543
alDHEFR	0.017	0.011	1.546
alDHPFR	-0.003	0.008	-0.371
alHAFR	-0.006	0.030	-0.199
alDSEFR	0.003	0.017	0.167
alSA2FR	-0.167	0.251	-0.666
gaFRCA	-0.062	0.115	-0.542
gaFRFH	0.057	0.128	0.446
gaFRFR	0.142	0.113	1.258
gaFRS	-0.119	0.099	-1.200
gaFRR	-0.128	0.099	-1.290
gaFRT	0.097	0.038	2.561
beINTFR	-0.162	0.119	-1.363
beDWFR	-0.006	0.005	-1.179
beDCTFR	0.003	0.005	0.644
beDCAFR	0.154	0.099	1.562
siINTFR	-0.125	0.099	-1.252
alINTS	0.088	0.210	0.420
alDPV1S	0.010	0.017	0.608
alDPV3S	-0.002	0.015	-0.137
alDPV4S	0.043	0.020	2.136
alDPV5S	0.053	0.022	2.476
alDCAS	0.149	0.149	0.998
alDHFS	-0.010	0.013	-0.769
alDHOS	0.001	0.009	0.124
alDHES	0.007	0.011	0.593
alDHPS	0.004	0.008	0.512
alHAS	-0.050	0.031	-1.628
alDSES	0.015	0.014	1.057
alSA2S	0.338	0.244	1.385
gaSCA	-0.072	0.109	-0.662
gaSFH	0.083	0.112	0.740
gaSFR	-0.239	0.116	-2.064
gaSS	0.107	0.107	0.997
gaSR	0.103	0.086	1.200
gaST	-0.037	0.038	-0.964
beINTS	0.081	0.107	0.756
beDWS	0.005	0.004	1.176
beDCTS	-0.002	0.004	-0.380
beDCAS	-0.139	0.089	-1.572
siINTS	0.006	0.095	0.063
alINTR	0.732	0.325	2.255
alDPV1R	-0.045	0.021	-2.218
alDPV3R	-0.007	0.020	-0.361

alDPV4R	-0.033	0.025	-1.324
alDPV5R	-0.001	0.030	-0.040
alDCAR	-0.608	0.249	-2.442
alDHFR	-0.037	0.014	-2.626
alDHOR	-0.020	0.014	-1.470
alDHER	-0.002	0.014	-0.147
alDHPR	0.008	0.010	0.728
alHAR	-0.062	0.045	-1.369
alDSER	0.042	0.025	1.656
alSA2R	0.252	0.294	0.856
gaRCA	0.063	0.153	0.409
gaRFH	0.107	0.141	0.764
gaRFR	0.031	0.136	0.227
gaRS	-0.285	0.142	-1.999
gaRR	0.097	0.136	0.713
gaRT	-0.026	0.055	-0.472
beINTR	-0.327	0.165	-1.983
beDWR	-0.004	0.006	-0.793
beDCTR	0.011	0.006	1.729
beDCAR	0.349	0.148	2.353
siINTR	-0.071	0.144	-0.489
alINTT	-0.654	0.393	-1.666
alDPV1T	0.082	0.034	2.410
alDPV3T	0.040	0.030	1.306
alDPV4T	0.086	0.036	2.365
alDPV5T	0.019	0.039	0.494
alDCAT	0.939	0.293	3.200
alDHFT	0.038	0.028	1.356
alDHOT	0.001	0.016	0.054
alDHET	-0.006	0.021	-0.291
alDHPT	0.007	0.015	0.488
alHAT	-0.013	0.056	-0.224
alDSET	0.010	0.032	0.313
alSA2T	0.113	0.430	0.263
gaTCA	-0.249	0.212	-1.175
gaTFH	-0.439	0.230	-1.907
gaTFR	0.195	0.208	0.939
gaTS	0.398	0.201	1.978
gaTR	0.113	0.180	0.632
gaTT	0.083	0.070	1.184
beINTT	0.295	0.195	1.512
beDWT	-0.005	0.008	-0.641
beDCTT	-0.002	0.009	-0.227
beDCAT	-0.367	0.176	-2.083
siINTT	0.366	0.201	1.823

$\alpha_5^r = 0.0,\ \theta_5^r = 1.7$

Over-Identifying Restrictions Tests

Sargan chi-squared = 51.2422780
degrees of freedom = 42.0000000
probability (%) = 15.5226581

Table 2.3
Data set: C_P, N = 1973

********* AI with fixed cost Demand System Estimation **********

Homogeneity Imposed

Instrumental Variable Estimation

Included Goods = CC CA FH FR S R
T
Good Dropped for Estimation = V

Instruments = intercept, non-homogeneous prices and
DPV1 DPV3 DPV4 DPV5 DW DCT DCA YCH OCH DYCH
DHF
DHO DHE DHP HA HA2 HGY DSF DSO DSE DSP
SA
SA2 SGY ly lysq ry

coeff	estimate	se	t-value
alINTCC	-0.305	0.238	-1.278
alDPV1CC	0.002	0.004	0.460
alDPV3CC	-0.002	0.004	-0.520
alDPV4CC	-0.003	0.007	-0.424
alDPV5CC	0.002	0.008	0.304
alDWCC	0.219	0.098	2.243
alDCTCC	-0.010	0.106	-0.098
alDCACC	-0.011	0.005	-2.033
alDHECC	-0.000	0.002	-0.096
alDHPCC	0.001	0.002	0.354
alHACC	-0.037	0.014	-2.686
alDSFCC	-0.000	0.003	-0.086
alDSOCC	-0.001	0.003	-0.543
alDSECC	0.001	0.002	0.268
alYCHCC	0.031	0.049	0.637
alOCHCC	0.039	0.048	0.806
gaCCCC	-0.013	0.052	-0.259
gaCCCA	-0.086	0.047	-1.815
gaCCFH	0.082	0.033	2.517
gaCCFR	-0.051	0.063	-0.809
gaCCS	-0.060	0.026	-2.340
gaCCR	0.066	0.038	1.705
gaCCT	0.042	0.045	0.918
beINTCC	0.157	0.109	1.438
beDWCC	-0.120	0.054	-2.246
beDCTCC	0.004	0.056	0.063
beCHHCC	-0.015	0.026	-0.601
siINTCC	0.217	0.156	1.396
alINTCA	-1.095	0.474	-2.309
alDPV1CA	0.003	0.009	0.302
alDPV3CA	0.005	0.009	0.559
alDPV4CA	-0.000	0.015	-0.000
alDPV5CA	0.003	0.016	0.193
alDWCA	0.186	0.197	0.944
alDCTCA	0.427	0.233	1.832
alDCACA	-0.031	0.012	-2.644
alDHECA	0.001	0.005	0.202
alDHPCA	0.009	0.004	2.048

alHACA	0.009	0.030	0.304
alDSFCA	0.014	0.007	1.861
alDSOCA	-0.006	0.006	-0.931
alDSECA	-0.003	0.005	-0.664
alYCHCA	0.074	0.114	0.648
alOCHCA	0.054	0.111	0.481
gaCACC	-0.232	0.125	-1.854
gaCACA	-0.114	0.100	-1.138
gaCAFH	0.154	0.069	2.239
gaCAFR	-0.232	0.128	-1.822
gaCAS	-0.065	0.054	-1.194
gaCAR	0.186	0.085	2.183
gaCAT	0.225	0.092	2.430
beINTCA	0.592	0.218	2.713
beDWCA	-0.105	0.107	-0.983
beDCTCA	-0.231	0.123	-1.871
beCHHCA	-0.060	0.059	-1.017
siINTCA	0.822	0.289	2.843
alINTFH	0.431	0.714	0.603
alDPV1FH	-0.048	0.012	-4.101
alDPV3FH	-0.039	0.013	-3.026
alDPV4FH	-0.053	0.020	-2.567
alDPV5FH	-0.030	0.024	-1.215
alDWFH	-0.242	0.307	-0.787
alDCTFH	-0.035	0.339	-0.103
alDCAFH	-0.054	0.017	-3.166
alDHEFH	0.001	0.007	0.171
alDHPFH	-0.009	0.006	-1.463
alHAFH	0.207	0.047	4.455
alDSFFH	0.008	0.011	0.758
alDSOFH	0.035	0.009	3.837
alDSEFH	-0.005	0.006	-0.755
alYCHFH	0.233	0.166	1.404
alOCHFH	0.236	0.163	1.449
gaFHCC	0.177	0.171	1.035
gaFHCA	0.036	0.149	0.242
gaFHFH	-0.056	0.099	-0.564
gaFHFR	0.047	0.192	0.242
gaFHS	-0.039	0.080	-0.488
gaFHR	-0.008	0.123	-0.066
gaFHT	-0.074	0.138	-0.539
beINTFH	-0.070	0.326	-0.213
beDWFH	0.131	0.168	0.778
beDCTFH	0.019	0.179	0.104
beCHHFH	-0.097	0.085	-1.134
siINTFH	0.391	0.472	0.828
alINTFR	-0.178	0.500	-0.355
alDPV1FR	0.006	0.008	0.802
alDPV3FR	0.007	0.008	0.832
alDPV4FR	0.020	0.014	1.473
alDPV5FR	0.011	0.016	0.692
alDWFR	0.184	0.213	0.863
alDCTFR	0.015	0.228	0.068
alDCAFR	-0.028	0.013	-2.187
alDHEFR	0.001	0.004	0.156
alDHPFR	0.009	0.004	2.262
alHAFR	-0.076	0.027	-2.770
alDSFFR	-0.003	0.007	-0.455
alDSOFR	-0.011	0.006	-1.705
alDSEFR	0.009	0.004	2.197

alYCHFR	-0.094	0.101	-0.933
alOCHFR	-0.097	0.099	-0.981
gaFRCC	-0.053	0.111	-0.474
gaFRCA	-0.164	0.094	-1.740
gaFRFH	0.033	0.061	0.534
gaFRFR	-0.038	0.132	-0.288
gaFRS	0.010	0.051	0.195
gaFRR	0.082	0.080	1.022
gaFRT	0.104	0.098	1.059
beINTFR	0.115	0.226	0.507
beDWFR	-0.102	0.116	-0.874
beDCTFR	-0.002	0.120	-0.016
beCHHFR	0.035	0.053	0.662
siINTFR	0.320	0.342	0.937
alINTS	-0.037	0.435	-0.086
alDPV1S	0.018	0.006	2.788
alDPV3S	0.005	0.006	0.765
alDPV4S	0.035	0.012	2.923
alDPV5S	0.007	0.016	0.472
alDWS	0.079	0.170	0.463
alDCTS	-0.017	0.194	-0.090
alDCAS	-0.047	0.009	-4.992
alDHES	0.007	0.004	1.678
alDHPS	-0.002	0.003	-0.548
alHAS	0.037	0.024	1.549
alDSFS	-0.014	0.005	-2.760
alDSOS	-0.011	0.005	-2.389
alDSES	-0.001	0.004	-0.229
alYCHS	0.005	0.099	0.053
alOCHS	-0.019	0.097	-0.193
gaSCC	-0.089	0.094	-0.953
gaSCA	-0.248	0.088	-2.804
gaSFH	-0.031	0.057	-0.533
gaSFR	0.046	0.116	0.393
gaSS	0.147	0.048	3.032
gaSR	0.113	0.070	1.611
gaST	0.034	0.080	0.420
beINTS	0.078	0.199	0.395
beDWS	-0.048	0.093	-0.520
beDCTS	0.006	0.103	0.058
beCHHS	-0.000	0.051	-0.003
siINTS	0.366	0.271	1.349
alINTR	-0.108	0.661	-0.163
alDPV1R	0.017	0.011	1.548
alDPV3R	-0.006	0.011	-0.538
alDPV4R	0.040	0.018	2.196
alDPV5R	0.029	0.022	1.359
alDWR	-0.350	0.291	-1.203
alDCTR	-0.232	0.305	-0.760
alDCAR	-0.072	0.017	-4.157
alDHER	-0.001	0.007	-0.177
alDHPR	0.011	0.006	1.819
alHAR	-0.077	0.042	-1.832
alDSFR	0.001	0.009	0.090
alDSOR	-0.021	0.008	-2.694
alDSER	-0.006	0.007	-0.859
alYCHR	0.449	0.150	2.996
alOCHR	0.463	0.146	3.161
gaRCC	0.178	0.158	1.124
gaRCA	-0.262	0.128	-2.051

gaRFH	-0.046	0.091	-0.503
gaRFR	0.240	0.175	1.367
gaRS	0.046	0.075	0.604
gaRR	-0.018	0.112	-0.166
gaRT	-0.056	0.126	-0.444
beINTR	0.162	0.305	0.531
beDWR	0.188	0.159	1.182
beDCTR	0.118	0.161	0.729
beCHHR	-0.225	0.078	-2.893
siINTR	-0.352	0.416	-0.848
alINTT	1.482	0.954	1.553
alDPV1T	0.035	0.015	2.284
alDPV3T	0.019	0.015	1.233
alDPV4T	0.003	0.029	0.106
alDPV5T	0.017	0.034	0.505
alDWT	0.423	0.378	1.117
alDCTT	-0.092	0.457	-0.202
alDCAT	0.258	0.018	14.702
alDHET	0.013	0.010	1.301
alDHPT	-0.004	0.008	-0.455
alHAT	0.070	0.062	1.120
alDSFT	-0.007	0.013	-0.556
alDSOT	0.028	0.011	2.496
alDSET	0.010	0.009	1.077
alYCHT	-0.658	0.233	-2.830
alOCHT	-0.643	0.227	-2.830
gaTCC	0.299	0.230	1.298
gaTCA	0.304	0.204	1.492
gaTFH	-0.127	0.141	-0.899
gaTFR	0.039	0.274	0.143
gaTS	-0.163	0.109	-1.495
gaTR	-0.192	0.172	-1.116
gaTT	-0.112	0.186	-0.602
beINTT	-0.712	0.437	-1.630
beDWT	-0.210	0.206	-1.018
beDCTT	0.055	0.242	0.228
beCHHT	0.341	0.121	2.828
siINTT	-0.446	0.594	-0.752

$\alpha = 0.0$, $\theta = 1.5$, $\theta = 0.1$, $\theta = 0.3$

Over-Identifying Restrictions Tests

Sargan chi-squared = 70.3433892
degrees of freedom = 56.0000000
probability (%) = 9.40668545

Table 2.4
Data set: NC_P, N = 685

********* AI with fixed cost Demand System Estimation **********

Homogeneity Imposed

Instrumental Variable Estimation

Included Goods = CA FH FR S R T
Good Dropped for Estimation = V

Instruments = intercept, non-homogeneous prices and
DPV1 DPV3 DPV4 DPV5 DW DCT DCA DHF DHO DHE
DHP
HA HA2 HGY DSF DSO DSE DSP SA SA2 SGY
ly
lysq ry

coeff	estimate	se	t-value
alINTCA	-0.569	0.423	-1.343
alDPV1CA	0.049	0.020	2.379
alDPV3CA	0.030	0.020	1.527
alDPV4CA	0.019	0.019	1.003
alDPV5CA	0.005	0.021	0.242
alDWCA	0.080	0.115	0.693
alDCTCA	0.675	0.324	2.081
alDCACA	-0.072	0.014	-5.090
alDHECA	0.005	0.010	0.555
alDHPCA	0.006	0.008	0.753
alHACA	-0.073	0.034	-2.133
alDSFCA	0.036	0.016	2.288
alDSOCA	0.003	0.009	0.363
alDSECA	0.003	0.011	0.281
gaCACA	-0.084	0.132	-0.637
gaCAFH	-0.002	0.163	-0.013
gaCAFR	0.111	0.178	0.627
gaCAS	0.044	0.147	0.298
gaCAR	-0.085	0.156	-0.549
gaCAT	0.002	0.061	0.026
beINTCA	0.383	0.182	2.107
beDWCA	-0.035	0.054	-0.649
beDCTCA	-0.355	0.167	-2.118
siINTCA	-0.031	0.394	-0.078
alINTFH	0.915	0.479	1.909
alDPV1FH	-0.022	0.024	-0.925
alDPV3FH	0.005	0.023	0.213
alDPV4FH	-0.033	0.023	-1.473
alDPV5FH	0.003	0.024	0.108
alDWFH	-0.204	0.121	-1.684
alDCTFH	-0.171	0.357	-0.477
alDCAFH	-0.011	0.016	-0.713
alDHEFH	-0.010	0.009	-1.212
alDHPFH	-0.004	0.008	-0.589
alHAFH	0.201	0.038	5.349
alDSFFH	0.016	0.018	0.887
alDSOFH	0.041	0.014	2.860
alDSEFH	-0.011	0.010	-1.155
gaFHCA	0.178	0.151	1.181

gaFHFH	-0.034	0.201	-0.169
gaFHFR	-0.033	0.189	-0.175
gaFHS	-0.109	0.168	-0.651
gaFHR	0.110	0.169	0.652
gaFHT	-0.166	0.070	-2.365
beINTFH	-0.295	0.206	-1.428
beDWFH	0.100	0.057	1.744
beDCTFH	0.088	0.186	0.474
siINTFH	0.051	0.447	0.114
alINTFR	0.710	0.372	1.909
alDPV1FR	-0.038	0.018	-2.140
alDPV3FR	-0.034	0.018	-1.888
alDPV4FR	0.003	0.016	0.212
alDPV5FR	0.013	0.019	0.697
alDWFR	0.073	0.093	0.780
alDCTFR	-0.296	0.275	-1.079
alDCAFR	-0.058	0.014	-4.261
alDHEFR	-0.002	0.009	-0.201
alDHPFR	0.013	0.007	2.027
alHAFR	-0.042	0.030	-1.384
alDSFFR	-0.022	0.013	-1.785
alDSOFR	-0.037	0.008	-4.567
alDSEFR	-0.007	0.010	-0.684
gaFRCA	-0.196	0.110	-1.787
gaFRFH	0.116	0.139	0.830
gaFRFR	-0.301	0.159	-1.896
gaFRS	-0.099	0.121	-0.819
gaFRR	0.367	0.132	2.786
gaFRT	-0.065	0.051	-1.264
beINTFR	-0.225	0.160	-1.405
beDWFR	-0.036	0.044	-0.825
beDCTFR	0.173	0.142	1.216
siINTFR	-0.782	0.331	-2.361
alINTS	0.232	0.341	0.679
alDPV1S	0.029	0.015	1.853
alDPV3S	-0.014	0.015	-0.908
alDPV4S	0.036	0.015	2.379
alDPV5S	0.004	0.016	0.253
alDWS	0.060	0.084	0.709
alDCTS	-0.400	0.273	-1.466
alDCAS	-0.042	0.010	-4.071
alDHES	0.009	0.008	1.122
alDHPS	0.003	0.006	0.428
alHAS	0.009	0.025	0.370
alDSFS	-0.011	0.012	-0.987
alDSOS	-0.003	0.010	-0.296
alDSES	-0.015	0.009	-1.691
gaSCA	-0.157	0.105	-1.497
gaSFH	0.010	0.122	0.081
gaSFR	0.126	0.135	0.931
gaSS	-0.038	0.116	-0.327
gaSR	0.131	0.126	1.040
gaST	0.079	0.047	1.684
beINTS	-0.081	0.148	-0.545
beDWS	-0.028	0.040	-0.709
beDCTS	0.199	0.142	1.403
siINTS	0.538	0.321	1.676
alINTR	-0.634	0.462	-1.371
alDPV1R	0.022	0.020	1.098
alDPV3R	0.011	0.020	0.527

alDPV4R	0.023	0.018	1.332
alDPV5R	-0.009	0.021	-0.435
alDWR	-0.040	0.113	-0.355
alDCTR	0.500	0.345	1.449
alDCAR	-0.027	0.015	-1.814
alDHER	0.015	0.011	1.331
alDHPR	-0.006	0.009	-0.688
alHAR	-0.068	0.039	-1.756
alDSFR	0.002	0.015	0.160
alDSOR	-0.006	0.013	-0.428
alDSER	0.016	0.012	1.338
gaRCA	-0.244	0.139	-1.757
gaRFH	0.070	0.185	0.377
gaRFR	0.296	0.195	1.522
gaRS	0.257	0.161	1.594
gaRR	-0.212	0.163	-1.299
gaRT	-0.077	0.065	-1.186
beINTR	0.352	0.199	1.775
beDWR	0.017	0.053	0.315
beDCTR	-0.268	0.178	-1.501
siINTR	0.402	0.442	0.910
alINTT	0.069	0.634	0.109
alDPV1T	0.000	0.031	0.004
alDPV3T	-0.011	0.031	-0.369
alDPV4T	-0.008	0.028	-0.272
alDPV5T	-0.038	0.035	-1.100
alDWT	-0.104	0.172	-0.603
alDCTT	0.412	0.527	0.782
alDCAT	0.222	0.017	13.247
alDHET	-0.001	0.014	-0.073
alDHPT	-0.010	0.011	-0.912
alHAT	-0.088	0.051	-1.742
alDSFT	-0.023	0.024	-0.961
alDSOT	0.044	0.017	2.522
alDSET	0.022	0.018	1.262
gaTCA	-0.067	0.201	-0.336
gaTFH	-0.254	0.250	-1.015
gaTFR	0.038	0.279	0.137
gaTS	0.261	0.246	1.063
gaTR	-0.066	0.261	-0.251
gaTT	0.071	0.090	0.794
beINTT	0.003	0.279	0.010
beDWT	0.057	0.082	0.699
beDCTT	-0.201	0.272	-0.739
siINTT	0.022	0.536	0.041

$\alpha_5^F = -0.1,\ \theta_5^F = 1.1$

Over-Identifying Restrictions Tests

Sargan chi-squared =	59.1374741
degrees of freedom =	48.0000000
probability (%) =	13.0094233

Table 2.5
Data set: C_F, N =1290

********* AI with fixed cost Demand System Estimation **********

with children data set, full time work

Homogeneity Imposed

Instrumental Variable Estimation

Included Goods = CC CA FH FR S R
T

Good Dropped for Estimation = V

Instruments = intercept, non-homogeneous prices and
DPV1 DPV3 DPV4 DPV5 DW DCT DCA YCH OCH DYCH
DHF
DHO DHE DHP HA HA2 HGY DSF DSO DSE DSP
SA
SA2 SGY ly lysq ry

coeff	estimate	se	t-value
alINTCC	-0.096	0.205	-0.469
alDPV1CC	0.006	0.005	1.194
alDPV3CC	0.001	0.005	0.152
alDPV4CC	-0.008	0.007	-1.061
alDPV5CC	-0.006	0.007	-0.778
alDWCC	0.200	0.192	1.039
alDCACC	-0.171	0.141	-1.212
alDHOCC	0.007	0.005	1.587
alDHECC	-0.002	0.002	-0.617
alDSFCC	0.001	0.003	0.285
alDSOCC	-0.003	0.004	-0.706
alDSPCC	0.007	0.002	3.158
alSACC	-0.083	0.020	-4.056
alYCHCC	0.100	0.069	1.437
alOCHCC	0.108	0.070	1.543
gaCCCC	-0.033	0.059	-0.554
gaCCCA	0.035	0.055	0.629
gaCCFH	0.025	0.042	0.581
gaCCFR	0.016	0.044	0.373
gaCCS	-0.080	0.045	-1.773
gaCCR	0.036	0.031	1.156
gaCCT	0.013	0.021	0.600
beINTCC	0.045	0.069	0.651
beDWCC	-0.073	0.072	-1.025
beDCTCC	-0.001	0.001	-1.781
beDCACC	0.059	0.052	1.145
beCHHCC	-0.030	0.024	-1.237
siINTCC	-0.014	0.060	-0.228
alINTCA	-0.786	0.598	-1.313
alDPV1CA	0.002	0.012	0.133
alDPV3CA	-0.003	0.012	-0.251
alDPV4CA	-0.021	0.019	-1.094
alDPV5CA	-0.018	0.020	-0.908
alDWCA	0.004	0.497	0.009
alDCACA	0.426	0.413	1.032
alDHOCA	0.011	0.012	0.924

alDHECA	0.006	0.007	0.892
alDSFCA	0.009	0.008	1.143
alDSOCA	0.010	0.010	0.945
alDSPCA	0.007	0.006	1.224
alSACA	0.012	0.055	0.226
alYCHCA	0.031	0.197	0.160
alOCHCA	0.029	0.199	0.147
gaCACC	-0.152	0.167	-0.911
gaCACA	0.002	0.155	0.014
gaCAFH	0.400	0.114	3.515
gaCAFR	-0.135	0.118	-1.138
gaCAS	-0.337	0.116	-2.896
gaCAR	0.159	0.080	1.975
gaCAT	0.088	0.057	1.552
beINTCA	0.332	0.200	1.661
beDWCA	0.001	0.185	0.007
beDCTCA	-0.002	0.002	-0.940
beDCACA	-0.163	0.150	-1.085
beCHHCA	-0.024	0.067	-0.365
siINTCA	0.242	0.184	1.313
alINTFH	1.366	0.766	1.784
alDPV1FH	-0.038	0.017	-2.213
alDPV3FH	-0.014	0.018	-0.784
alDPV4FH	-0.085	0.026	-3.306
alDPV5FH	-0.061	0.027	-2.314
alDWFH	-1.053	0.692	-1.522
alDCAFH	-1.012	0.522	-1.937
alDHOFH	0.014	0.016	0.911
alDHEFH	-0.007	0.009	-0.750
alDSFFH	-0.001	0.013	-0.043
alDSOFH	0.019	0.015	1.305
alDSPFH	-0.011	0.008	-1.339
alSAFH	0.211	0.072	2.940
alYCHFH	0.519	0.240	2.165
alOCHFH	0.530	0.242	2.186
gaFHCC	-0.116	0.203	-0.571
gaFHCA	0.405	0.192	2.111
gaFHFH	-0.217	0.144	-1.510
gaFHFR	0.245	0.154	1.586
gaFHS	-0.001	0.156	-0.005
gaFHR	-0.189	0.111	-1.701
gaFHT	-0.033	0.073	-0.447
beINTFH	-0.392	0.259	-1.515
beDWFH	0.384	0.257	1.493
beDCTFH	-0.002	0.003	-0.879
beDCAFH	0.342	0.191	1.786
beCHHFH	-0.174	0.083	-2.099
siINTFH	0.436	0.203	2.143
alINTFR	-0.127	0.473	-0.268
alDPV1FR	0.001	0.011	0.117
alDPV3FR	0.009	0.011	0.826
alDPV4FR	0.054	0.018	3.086
alDPV5FR	0.038	0.017	2.215
alDWFR	0.752	0.439	1.714
alDCAFR	-0.380	0.348	-1.092
alDHOFR	-0.020	0.010	-1.880
alDHEFR	0.011	0.006	1.913
alDSFFR	0.007	0.009	0.821
alDSOFR	0.012	0.010	1.286
alDSPFR	0.007	0.005	1.370

alSAFR	-0.097	0.047	-2.045
alYCHFR	-0.054	0.158	-0.340
alOCHFR	-0.058	0.160	-0.360
gaFRCC	0.079	0.144	0.550
gaFRCA	-0.124	0.138	-0.903
gaFRFH	0.017	0.095	0.176
gaFRFR	-0.206	0.095	-2.176
gaFRS	-0.019	0.101	-0.182
gaFRR	0.140	0.070	1.999
gaFRT	0.069	0.048	1.432
beINTFR	0.075	0.159	0.476
beDWFR	-0.282	0.163	-1.727
beDCTFR	0.007	0.002	3.455
beDCAFR	0.140	0.128	1.095
beCHHFR	0.016	0.055	0.297
siINTFR	-0.003	0.132	-0.026
alINTS	-0.296	0.508	-0.582
alDPV1S	0.013	0.010	1.313
alDPV3S	0.016	0.009	1.781
alDPV4S	0.033	0.015	2.148
alDPV5S	0.007	0.016	0.459
alDWS	0.778	0.471	1.652
alDCAS	-0.406	0.341	-1.191
alDHOS	-0.009	0.010	-0.930
alDHES	0.006	0.006	1.029
alDSFS	-0.008	0.006	-1.242
alDSOS	-0.010	0.009	-1.130
alDSPS	0.000	0.005	0.048
alSAS	0.086	0.043	1.985
alYCHS	-0.122	0.166	-0.737
alOCHS	-0.146	0.168	-0.872
gaSCC	-0.068	0.125	-0.540
gaSCA	-0.102	0.112	-0.911
gaSFH	-0.131	0.080	-1.642
gaSFR	0.090	0.089	1.004
gaSS	0.003	0.095	0.027
gaSR	0.077	0.061	1.255
gaST	0.049	0.045	1.086
beINTS	0.132	0.172	0.767
beDWS	-0.294	0.175	-1.683
beDCTS	-0.004	0.002	-2.444
beDCAS	0.145	0.124	1.173
beCHHS	0.042	0.057	0.724
siINTS	0.392	0.132	2.978
alINTR	0.471	0.846	0.557
alDPV1R	0.002	0.013	0.172
alDPV3R	-0.011	0.013	-0.852
alDPV4R	0.002	0.021	0.083
alDPV5R	-0.014	0.022	-0.641
alDWR	-0.637	0.698	-0.911
alDCAR	0.001	0.549	0.002
alDHOR	0.001	0.015	0.044
alDHER	0.006	0.008	0.801
alDSFR	-0.011	0.009	-1.139
alDSOR	-0.028	0.014	-1.975
alDSPR	0.007	0.007	0.962
alSAR	0.003	0.067	0.041
alYCHR	0.164	0.233	0.705
alOCHR	0.175	0.236	0.742
gaRCC	-0.063	0.184	-0.342

gaRCA	-0.229	0.160	-1.431
gaRFH	0.087	0.118	0.743
gaRFR	0.255	0.133	1.908
gaRS	0.170	0.151	1.122
gaRR	0.006	0.088	0.071
gaRT	-0.127	0.062	-2.068
beINTR	-0.089	0.288	-0.311
beDWR	0.240	0.259	0.927
beDCTR	-0.002	0.003	-0.588
beDCAR	-0.031	0.199	-0.157
beCHHR	-0.055	0.080	-0.691
siINTR	-0.107	0.197	-0.540
alINTT	0.161	1.105	0.146
alDPV1T	0.042	0.023	1.792
alDPV3T	-0.002	0.022	-0.073
alDPV4T	0.058	0.035	1.681
alDPV5T	0.065	0.034	1.905
alDWT	0.771	0.947	0.814
alDCAT	1.051	0.775	1.356
alDHOT	0.001	0.021	0.031
alDHET	-0.001	0.012	-0.086
alDSFT	0.005	0.016	0.316
alDSOT	0.019	0.019	0.989
alDSPT	0.001	0.011	0.100
alSAT	0.043	0.099	0.430
alYCHT	-0.670	0.334	-2.007
alOCHT	-0.683	0.338	-2.020
gaTCC	0.496	0.288	1.721
gaTCA	-0.351	0.260	-1.348
gaTFH	-0.004	0.188	-0.022
gaTFR	-0.107	0.210	-0.509
gaTS	-0.043	0.213	-0.200
gaTR	-0.004	0.142	-0.030
gaTT	0.025	0.099	0.249
beINTT	-0.061	0.374	-0.162
beDWT	-0.276	0.352	-0.784
beDCTT	0.005	0.004	1.421
beDCAT	-0.288	0.284	-1.016
beCHHT	0.241	0.115	2.092
siINTT	0.088	0.291	0.303

$\alpha_{[illegible]} = -0.8$, $\theta_{[illegible]} = 2.3$, $\theta_{[illegible]} = 0.3$, $\theta_{[illegible]} = 0.4$

Over-Identifying Restrictions Tests

Sargan chi-squared =	67.3983658
degrees of freedom =	56.0000000
probability (%) =	14.1495146

Table 2.6

Data set: NC_F, N = 1389

********* AI with fixed cost Demand System Estimation *********

Homogeneity Imposed

Instrumental Variable Estimation

Included Goods = CA FH FR S R T
Good Dropped for Estimation = V

Instruments = intercept, non-homogeneous prices and
DPV1 DPV3 DPV4 DPV5 DW DCT DCA DHF DHO DHE
DHP
HA HA2 HGY DSF DSO DSE DSP SA SA2 SGY
ly
lysq ry

coeff	estimate	se	t-value
alINTCA	0.792	0.568	1.395
alDPV1CA	0.017	0.012	1.409
alDPV3CA	0.005	0.011	0.498
alDPV4CA	0.009	0.013	0.701
alDPV5CA	0.015	0.015	1.010
alDWCA	-0.187	0.200	-0.936
alDCACA	-0.442	0.452	-0.979
alDHOCA	0.015	0.008	1.737
alDHECA	0.012	0.006	2.073
alDSFCA	0.021	0.009	2.285
alDSOCA	0.004	0.009	0.426
alDSPCA	-0.003	0.005	-0.510
alSACA	-0.044	0.022	-2.052
gaCACA	-0.109	0.076	-1.440
gaCAFH	0.415	0.124	3.333
gaCAFR	-0.179	0.094	-1.893
gaCAS	-0.206	0.085	-2.417
gaCAR	-0.075	0.109	-0.692
gaCAT	0.051	0.040	1.273
beINTCA	-0.155	0.157	-0.986
beDWCA	0.056	0.063	0.878
beDCTCA	-0.004	0.002	-1.994
beDCACA	0.107	0.130	0.825
siINTCA	-0.509	0.317	-1.604
alINTFH	-0.137	0.606	-0.227
alDPV1FH	-0.028	0.011	-2.500
alDPV3FH	-0.013	0.009	-1.386
alDPV4FH	-0.043	0.012	-3.689
alDPV5FH	-0.027	0.016	-1.693
alDWFH	-0.132	0.162	-0.813
alDCAFH	0.294	0.486	0.606
alDHOFH	-0.005	0.008	-0.621
alDHEFH	-0.009	0.005	-1.865
alDSFFH	-0.000	0.008	-0.039
alDSOFH	0.018	0.009	2.062
alDSPFH	-0.004	0.005	-0.839
alSAFH	0.193	0.022	8.620
gaFHCA	0.214	0.078	2.754

gaFHFH	-0.230	0.138	-1.671
gaFHFR	0.204	0.113	1.809
gaFHS	-0.044	0.087	-0.506
gaFHR	0.181	0.115	1.571
gaFHT	-0.082	0.043	-1.921
beINTFH	0.085	0.169	0.505
beDWFH	0.045	0.051	0.887
beDCTFH	-0.001	0.002	-0.765
beDCAFH	-0.087	0.141	-0.619
siINTFH	1.250	0.375	3.338
alINTFR	0.673	0.580	1.160
alDPV1FR	0.008	0.012	0.625
alDPV3FR	-0.002	0.011	-0.208
alDPV4FR	0.010	0.012	0.803
alDPV5FR	0.014	0.015	0.897
alDWFR	0.363	0.203	1.789
alDCAFR	-0.900	0.427	-2.110
alDHOFR	0.006	0.008	0.787
alDHEFR	0.015	0.006	2.763
alDSFFR	0.009	0.009	1.065
alDSOFR	-0.012	0.008	-1.459
alDSPFR	-0.001	0.005	-0.189
alSAFR	-0.088	0.022	-3.994
gaFRCA	-0.106	0.075	-1.418
gaFRFH	-0.043	0.132	-0.328
gaFRFR	-0.022	0.098	-0.224
gaFRS	-0.022	0.081	-0.272
gaFRR	0.081	0.104	0.779
gaFRT	0.075	0.036	2.113
beINTFR	-0.160	0.159	-1.005
beDWFR	-0.114	0.064	-1.761
beDCTFR	0.009	0.002	4.736
beDCAFR	0.258	0.121	2.134
siINTFR	-0.150	0.318	-0.472
alINTS	0.076	0.409	0.185
alDPV1S	0.000	0.008	0.052
alDPV3S	0.000	0.007	0.004
alDPV4S	0.013	0.008	1.594
alDPV5S	-0.010	0.010	-1.025
alDWS	-0.264	0.141	-1.870
alDCAS	0.125	0.332	0.377
alDHOS	-0.002	0.006	-0.368
alDHES	0.002	0.004	0.586
alDSFS	-0.017	0.006	-2.916
alDSOS	-0.007	0.006	-1.323
alDSPS	0.003	0.003	0.977
alSAS	0.056	0.015	3.718
gaSCA	-0.042	0.058	-0.730
gaSFH	-0.191	0.090	-2.123
gaSFR	0.104	0.072	1.453
gaSS	0.190	0.052	3.625
gaSR	-0.038	0.073	-0.512
gaST	-0.012	0.029	-0.414
beINTS	0.017	0.115	0.146
beDWS	0.085	0.045	1.913
beDCTS	-0.003	0.001	-2.375
beDCAS	-0.048	0.097	-0.495
siINTS	0.292	0.243	1.203
alINTR	1.874	0.645	2.906
alDPV1R	0.003	0.013	0.218

alDPV3R	0.009	0.011	0.797
alDPV4R	0.013	0.014	0.934
alDPV5R	0.028	0.017	1.690
alDWR	-0.181	0.250	-0.726
alDCAR	-1.391	0.494	-2.813
alDHOR	0.000	0.009	0.030
alDHER	0.006	0.006	0.886
alDSFR	-0.006	0.009	-0.620
alDSOR	0.015	0.010	1.517
alDSPR	0.004	0.006	0.606
alSAR	-0.130	0.024	-5.438
gaRCA	-0.108	0.087	-1.243
gaRFH	0.257	0.131	1.963
gaRFR	-0.055	0.113	-0.484
gaRS	-0.120	0.091	-1.308
gaRR	-0.142	0.124	-1.147
gaRT	-0.044	0.041	-1.072
beINTR	-0.485	0.178	-2.723
beDWR	0.057	0.080	0.717
beDCTR	-0.001	0.002	-0.677
beDCAR	0.389	0.141	2.756
siINTR	-0.783	0.360	-2.178
alINTT	-0.574	0.904	-0.635
alDPV1T	0.013	0.019	0.703
alDPV3T	-0.009	0.016	-0.556
alDPV4T	0.012	0.020	0.577
alDPV5T	-0.010	0.024	-0.431
alDWT	0.493	0.269	1.836
alDCAT	0.851	0.748	1.137
alDHOT	0.012	0.014	0.862
alDHET	-0.013	0.008	-1.633
alDSFT	-0.014	0.013	-1.071
alDSOT	-0.001	0.014	-0.063
alDSPT	0.005	0.007	0.624
alSAT	-0.021	0.032	-0.650
gaTCA	-0.122	0.125	-0.975
gaTFH	0.007	0.195	0.038
gaTFR	-0.004	0.155	-0.028
gaTS	0.015	0.126	0.120
gaTR	0.091	0.172	0.531
gaTT	0.055	0.062	0.888
beINTT	0.189	0.254	0.743
beDWT	-0.158	0.085	-1.859
beDCTT	0.001	0.003	0.525
beDCAT	-0.195	0.219	-0.891
siINTT	0.085	0.528	0.160

$\alpha_o^r = -0.9$, $\theta_o^r = 1.0$

Over-Identifying Restrictions Tests

Sargan chi-squared = 48.922253243
degrees of freedom = 48.000000000
probability (%) = 43.583744640

Appendix 3. Proof

Proof of Lemma 1:

Applying the Roy's identity to (2.5), we have the following (a):

$$x_i^r = -\frac{\dfrac{\partial v^r}{\partial p_i}}{\dfrac{\partial v^r}{\partial y^r}} = -\frac{\dfrac{\partial G}{\partial p_i}}{\dfrac{\partial G}{\partial y^r}} = -\frac{\dfrac{\partial G}{\partial v^{r*}}\dfrac{\partial v^{r*}}{\partial p_i}}{\dfrac{\partial G}{\partial v^{r*}}\dfrac{\partial v^{r*}}{\partial y^r}} = -\frac{\dfrac{\partial v^{r*}}{\partial p_i}}{\dfrac{\partial v^{r*}}{\partial y^r}} = x_i^{r*}$$

If ESE holds, then we have

$$V(\boldsymbol{p}, y, \boldsymbol{z}) = V^r(\boldsymbol{p}, \frac{y}{d(\boldsymbol{p}, \boldsymbol{z})}) \tag{1}$$

$$V(\boldsymbol{p}, y, \boldsymbol{z}) = V^r(\boldsymbol{p}, \frac{y}{d^*(\boldsymbol{p}, \boldsymbol{z})}) \tag{2}$$

Applying the Roy's identity to (1), we have

$$x_i = -\frac{\dfrac{\partial v}{\partial p_i}}{\dfrac{\partial v}{\partial y}} = \frac{-\left[\dfrac{\partial v^r}{\partial p_i} - \dfrac{\partial v^r}{\partial (y/d)}\dfrac{y}{d^2}\dfrac{\partial d}{\partial p_i}\right]}{\dfrac{\partial v^r}{\partial (y/d)}\dfrac{1}{d}} = -d\,\frac{\dfrac{\partial v^r}{\partial p_i}}{\dfrac{\partial v^r}{\partial (y/d)}} + \frac{y}{d}\frac{\partial d}{\partial p_i}$$

$$x_i = dx_i^r + \frac{y}{d}\frac{\partial d}{\partial p_i}$$

$x_i^r = [x_i - y\partial \ln(d)/\partial p_i)]/d$

Similarly, from (2),

$$x_i = d^* x_i^r{*} + \frac{y}{d^*}\frac{\partial d^*}{\partial p_i}$$

$x_i^{r*} = [x_i - y\partial \ln(d^*)/\partial p_i)]/d^*$

Based on (a) and if $d \neq d^*$, then

$x_i = [y/(d^* - d)][d^*\partial \ln(d)/\partial p_i - d\partial \ln(d^*)/\partial p_i]$

Therefore, x_i is a linear function of y without intercept. We conclude that v(**p**, y, **z**) is homothetic, namely, c(**p**, u, **z**) = uβ(**p**, **z**) or v(**p**, y, **z**) = y/(**p**, **z**). In other words, if d = d*, v(**p**, y, **z**) has to be non-homothetic.

Proof of Lemma 2

Applying the Roy's identity to (2.9), we also have (a)

If GESE holds, we have

$$v(\mathbf{p}, y, \mathbf{z}) = v^{r}\left(\mathbf{p}, \frac{y-F(\mathbf{p}, \mathbf{z})}{D(\mathbf{p}, \mathbf{z})}\right) \tag{3}$$

$$v(\mathbf{p}, y, \mathbf{z}) = v^{r}\left(\mathbf{p}, \frac{y-F^{*}(\mathbf{p}, \mathbf{z})}{d^{*}(\mathbf{p}, \mathbf{z})}\right) \tag{4}$$

From (3):

$$x_i = Dx_i^r + \frac{y-F}{D}\frac{\partial D}{\partial p_i} + \frac{\partial F}{\partial p_i}$$

and from (4):

$$x_i = D^{*}x_i^{r*} + \frac{y-F^{*}}{D^{*}}\frac{\partial D^{*}}{\partial p_i} + \frac{\partial F^{*}}{\partial p_i}$$

Using (a) and if $D \neq D^*$,

$$(D-D^*)x_i^r = \frac{y-F^*}{D^*}\frac{\partial D^*}{\partial p_i} + \frac{\partial F^*}{\partial p_i} - \left(\frac{y-F}{D}\frac{\partial D}{\partial p_i} + \frac{\partial F}{\partial p_i}\right)$$

$$(D-D^*)x^r = \left(\frac{1}{D^*}\frac{\partial D^*}{\partial p_i} - \frac{1}{D}\frac{\partial D}{\partial p_i}\right)y - F^*\frac{\partial \ln(D^*)}{\partial p_i} + \frac{\partial F^*}{\partial p_i} + F\frac{\partial \ln(D)}{\partial p_i} - \frac{\partial F}{\partial p_i}$$

Clearly, x_i^r is a linear function of y with intercept. Since x_i is a linear function of x_i^r, v(.) has to be quasi-homothetic. In other words, if $D = D^*$, v(.) has to be non-quasi-homothetic.

Appendix 4

Brief Description of 3SLS with endogenous variables treated and heteroscedasticity corrected

Suppose we have a system expressed as

$w_k = f(x_k, \delta) + e_k$

where wk (e_k) is the column vector of dependent variables (error terms) for the kth observation. That is, $w_k = (w_{1k}, w_{2k}, \ldots, w_{mk})'$, $e_k = (e_{1k}, e_{2k}, \ldots, e_{mk})'$ where m is the number of the equations in the system. δ is a set of parameters to be estimated in the system. Some variables on the right hand side of the equation are endogenous, hence we adopted instrument variable approach to estimate the system. We assume there exists a set of instrument variables W, such that plim(W'e/N) = 0, and plim[(1/N)W'X] = M, where N is the number of observations, $e = (e_1, e_2, \ldots, e_N)'$. Assume f(.) is a linar function. X is the matrix of independent variables for all observations. M is the matrix with rank equal to the dimension of δ. Given all these information, the estimates of δ can be estimated in the following stages (see Gallant, 1987):

Stage 1:

Estimate δ from minimizing $\{ e'(I_m \otimes P)e \}$ with respect to δ, assuming the error terms are of homoscedasticity, where $P = W(W'W)^{-1}W'$. Since the f(.) is linear, we have an explicit solution for the estimates of the parameters δ such that $[X'PX]^{-1}X'Pw$, where $w = (w_1, w_2, \ldots, w_N)'$.

Stage 2:

Compute $V = \Sigma[\mathbf{e}_k \otimes W_k][\mathbf{e}_k \otimes W_k]'$ and then V^{-1}, the summation runs from $k = 1$ to N.

Stage 3:

Use V^{-1} and estimate heteroscedasticity-corrected δ by minimizing $[\Sigma e_k \otimes W_k]V^{-1}[\Sigma e_k \otimes W_k]'$. For linear structure, we have explicit solution for the estimates of δ such as $[XW^*V^{-1}*XW']^{-1}*\ XW^*\ V^{-1}*Ww$, where $XW = I_m \otimes X'W$, $Ww = vec(W'w)$