MARKET STRUCTURE AND
CONTRACTIONARY DEVALUATION

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ABSTRACT

In practice, devaluation has been prescribed as a major policy instrument in many countries, on the presumption that devaluation is expansionary. However, whether devaluation is contractionary is a matter of considerably theoretical and practical importance. This thesis investigates whether and under what conditions devaluation is contractionary under a wide range of assumptions concerning market structures. The investigation is conducted at both micro and macro levels, and for both short-run and long-run time horizons.

In general, contractionary devaluation is not an unusual phenomenon; instead, it can easily occur as long as the demand and supply structures in the market satisfy a condition which we derive and interpret.

In short-run micro models with homogeneous goods, our findings show that perfect competition, monopoly and oligopoly do not change the nature of the conditions required for expansionary, neutral or contractionary devaluation. However, these three market structures do have an impact through altering the magnitude (not sign) of the output change following a devaluation. Thus both theorists and policymakers should be aware that the existence of imperfect competition can largely reduce the power of a devaluation (if not the direction of that effect).
Our partial equilibrium micro model with monopolistic competition provides results which challenge the theory of neutrality of devaluation in the long-run. Also, the micro foundations reveal the mechanism through which the long-run real effects of devaluation occur. It is the devaluation which causes the changes in the number of varieties and it is the change in the number of the varieties which alters the market structure and thus causes the long-run effects on real output.

As a key variable, the number of varieties also generates some seemingly odd results which are not seen in perfect competition: following a devaluation, the aggregate price and aggregate output, welfare and employment, may change in the opposite directions.

To establish the linkage between the microeconomic foundations and the macroeconomic formulations, we extend an existing closed macro model with monopolistic competition to a small open economy version. The neutrality of devaluation is derived in this context. This macro result should not be considered inconsistent with the result of non-neutrality in the micro model with monopolistic competition. The main reasons is that: the number of varieties is an exogenous variable in the macro model but endogenous variable in the micro model; and the money wage is flexible in the former but fixed in the latter.
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* * * * *

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CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION TO THE TOPIC

If devaluation of the home currency stimulates an increase in domestic output, it is called expansionary devaluation; if, instead, a devaluation causes a decrease in domestic output, it is called contractionary devaluation.

In practice, devaluation has commonly been prescribed as a major policy instrument for quite a long period in many countries, due to the traditional belief in expansionary devaluation. It has nearly always been routinely administered by the IMF to improve a nation's economic performance. However, in the past decade or more, while the industrial world turned to adopt a floating exchange rates system, the developing countries have become cautious in resorting to devaluation. The main reason is that the effect of devaluation on output seem more and more controversial both theoretically and empirically.

While expansionary devaluation was the orthodox view, the possibility of contractionary devaluation and perverse outcomes have been identified and discussed intermittently by
economists for more than three decades. The earliest study may be traced back to Hirschman (1949). After the influential paper by Krugman and Taylor (1978), however, the literature on this subject has started to grow rapidly.

In a Keynesian model, Krugman and Taylor demonstrated that several conditions can lead to contractionary devaluation. These conditions are: (1) initial trade balance is negative; (2) wage earners have higher propensities to consume than profit earners do; (3) certain commercial policy is imposed to increase the government revenue, such as significant export taxes. They also showed how contractionary devaluation occurs through the channels of both real balances and the nominal money supply reduction.

Later, a number of papers expanded on Krugman and Taylor by exploring a variety of macroeconomic channels through which a nominal devaluation could cause real output to contract. Some of these papers are surveyed in the next chapter.

Despite the fact that numerous macro models have been built based on the different specifications to examine contractionary devaluation, the difficulty economists are facing today is that a general model involving all the features that have been stressed in this is not analytically tractable. In this context, the task of macro formulations has not yet been accomplished, and further advance is expected.
Moreover, in recent publications attention has been drawn to the micro foundations of contractionary devaluation. This is not a surprising situation. The historical trend seems obvious: New Classical scholars would not be convinced by any macroeconomic modelling without micro underpinnings; and New Keynesian scholars have also been concerned about clarifying the micro basis of their macro models. In short, both schools of thought consider micro foundations as essential parts in economic modelling. This common insight, of course, should apply to any specific policy questions such as contractionary devaluation.

Unfortunately, the microeconomic foundations of contractionary devaluation have not attracted enough attention which they deserve. This situation provides us with a good opportunity to develop the micro foundations of contractionary devaluation, and thus establish a sound theoretical linkage with its macroeconomic formulations.

The need for conducting further research on devaluation effects is not only from the theoretical but also from the empirical aspect. Empirical work on the effects of devaluation does not abound. Nevertheless, through limited empirical evidence, we can still see clearly that the traditional understanding of devaluation effects, i.e. expansionary devaluation, has been challenged.

Historically, currency devaluation in the 1930s' Depression is almost universally condemned or indicted despite
other factors involved. Empirical analysis indicates that the competitive devaluations pushed the recession of 1930 into an unprecedented Depression (Friedman & Schwartz 1963, Meltzer 1976 and Saint-Etienne 1984). This is one of the most serious lessons in the economic history of devaluations.

In their 1978 paper, Krugman and Taylor found that, in a numerical example, devaluation of the home currency by 25% could cause output of home goods to fall by 6.5%. Even though the numbers chosen in their study seem arbitrary, they are actually designed to fall within a reasonable range for semi-industrial country cases conducted by Abel et al (1976) on Portugal and Taylor (1974) on Chile.

In a study of 18 devaluation episodes in 14 mostly developing countries for the period of 1959-1970, Connoly and Taylor (1976) suggested that devaluations appear on average not to have a significant effect on output even though at the same time generally improving the balance of payments. However, Krueger's (1978) findings based on the analysis of 22 devaluations in 10 third world countries showed that in 3 cases a significant recession occurred. In addition, by using a one sector general equilibrium model, Gylfasson and Schimid (1983) tested a group of 10 countries (5 industrialized and 5 developing) and found that contractionary devaluation emerged in 2 countries (India and U.K.).

By and large, the empirical work done so far has not provided assured conclusions on either expansionary or
contractionary devaluation. One important reason is the difficulty of isolating the devaluation effects from other parallel influences in the economy. This gives economists a challenging area in terms of overcoming the difficulty of specifications in empirical modelling. More importantly, the empirical findings have confirmed the ambiguity of devaluation effects on output and raised serious questions concerning the robustness of the existing theoretical models.

As a consequence of the controversy about the effects of devaluation, conscientious policy-makers are no longer confident in their ability to determine whether devaluation is an appropriate instrument. Needless to say, further research in this field has important significance both for the theory of devaluation effects and for practical policy.

1.2 INTRODUCTION TO OUR RESEARCH

Whether devaluation is contractionary is a matter of considerably theoretical and practical importance. The matter can be addressed at the level of the individual firm or the level of the economy as a whole. Also, the matter can be addressed in the context of different market structures, and focusing on the different time horizons.

In the context of a competitive model, a devaluation has positive effects on output through the demand side but has negative effects on output through the supply side. Thus the
net effects of devaluation on output would be jointly determined by both sides, specifically, by both parameters of the aggregate demand function and the parameters of the supply function. Three possible results can be shown in the following diagram:

Figure 1.1
Panel 1.1a shows expansionary devaluation, Panel 1.1b neutral devaluation and Panel 1.1c contractionary devaluation.

In this thesis, we investigate whether devaluation is contractionary under a wider range of market structures such as perfect competition, monopoly, oligopoly and monopolistic competition. The investigation is conducted at both micro and macro levels, and in both short-run (impact period) and long-run.

To examine the effects of devaluation on output in the short-run at the micro level, we analyze the case of homogeneous goods in three market structures (perfect competition, monopoly and oligopoly), in a partial equilibrium framework. The methodology is to consider a Cournot-Nash equilibrium model which contains perfect competition and monopoly as special cases, and a Stackelberg duopoly model. We address and solve the related optimization problems, and then derive the conditions for expansionary, neutral and contractionary devaluation.

Our findings show that contractionary devaluation is not an unusual phenomenon; instead, it may easily occur as long as the demand structure in the markets and the cost structure of the firms satisfy the conditions we derive and interpret. However, different market structures have virtually no effect on the nature of the conditions required for expansionary, neutral or contractionary devaluation.
Our findings also show that, despite the essentially same nature of the conditions for the sign of output effect following a devaluation across different market structures, the magnitude of the change in output is dependent on different market structures. Under the same set of parameters in the market demand and supply sides, and the same initial value of the exchange rate, devaluation is most effective in perfect competition and least effective in monopoly. This is not only theoretically important but also empirically significant. In particular, there is no unique model which can be universally applied to different market structures either for the purpose of estimating or forecasting the effects of a devaluation on output. Therefore it is a serious issue that different market structures are ignored in the modelling practice of devaluation.

Our findings have two important implications for public policy. First, the existence of imperfect competition can largely reduce the change in output following a devaluation. This idea may be illustrated by Figure 1.2.

In Figure 1.2, a devaluation shifts the marginal cost curve from MC to MC' and shifts the average cost curve from AC to AC' (assuming for convenience the demand side effect has been neutralized by some adjustment in policy). In monopoly, the initial equilibrium output is Qo, the new equilibrium point is reached at the intersection of MC' and MR. As a result, output changes from Qo to Qi. In perfect competition,
the initial equilibrium output is $Q_0'$. The new intersection of $MC'$ and $D$ is $A$ (constant cost industry is implicitly assumed). As a result, price increases by $OA$ and output changes from $Q_o$ to $Q_1$. Obviously $Q_o - Q_1 < Q_0' - Q_1'$ since $MR$ is steeper than $D$, i.e. devaluation has smaller output effects with monopoly than with perfect competition. Of course, this rudimental illustration does not tell all the stories but at least provides an intuition.

Therefore policy makers should not overestimate the power of the instrument (devaluation) in their hands. Instead, they need to identify how effective devaluation would be in their particular circumstances (perfect competition or imperfect competition), even though the condition for expansionary devaluation is met.
Second, devaluation is not an inexhaustible resort. Our result shows a quadratically diminishing effect of devaluation on output in respect to the pre-devaluation exchange rate level. Even if devaluation is used for the first time as a policy action, it is necessary to assess how effective a devaluation would be at the initial exchange level. Therefore devaluation should not be abused.

To examine the effects of devaluation on output in the long-run at the micro level, we take the case of differentiated goods featuring monopolistic competition in a partial equilibrium framework. S-D-S varieties (Spence-Dixit-Stiglitz, 1976, 1977) are adopted in our model. Each individual producer has monopoly power in the market for his own variety and the whole market leaves enough room for the producers to compete since all varieties of goods are imperfect substitutes. Also, firms are allowed to enter or exit.

The general results show that the effect of devaluation has its long-run effects on output. The effects are expansionary, neutral or contractionary if the ratio of the foreign income to the global income is greater than, equal to, or smaller than the elasticity of aggregate price with respect to foreign exchange rate. The income ratio is a fixed fraction at the initial equilibrium. However, the elasticity of aggregate price with respect to foreign exchange rate is affected by devaluation through two channels: the change in
the price of each individual variety and the change in the aggregate price. The former is the **direct effect** through the change in income and the latter is the **indirect effect** through the change in the number of firms in industry. Therefore the outcome will be again determined by both demand side and supply side.

It is interesting to see that various parameters in demand and supply side give several combinations of the changes in the aggregate price and the aggregate output: following a devaluation, when the aggregate price goes up, the aggregate output may increase, stay unchanged or decrease; when the price level stays unchanged or decreases, aggregate output may increase. Some of these results seem odd and difficult to imagine in the context of perfect competition. They follow from certain features of our model with monopolistic competition and from the assumption of free entry/exit in our model.

Since symmetry is assumed, the results from the special version of our model reveal the mechanism of the long-run effects of devaluation: it is the devaluation which causes a change in the number of varieties and it is the change in the number of varieties which alters the market structure and thus causes the long-run effects on real output. Moreover, when assuming there are no intermediate inputs, the results show an expansionary output and a lower aggregate price. This is because more firms enter the
industry and more varieties become available, and a larger number of varieties raises the level of competitiveness and drives the industry price down. However, we do not obtain this result in models of perfect competition.

The assumption of symmetry also allows us to make approximations to examine the effects of devaluation on employment and measured price level. Our findings show that employment may increase, stay unchanged or decrease, but measured price level goes up following a devaluation. This is consistent with our knowledge in macroeconomic theory. Our findings also show that devaluation may have an expansionary effect on welfare while a contractionary effect on employment.

To examine the effects of devaluation on output at the macro level, we take the case with the features of monopolistic competition (S-D-S varieties) similar to those at the micro level. But now the number of firms is assumed to be an exogenous variable. Using an extension of the Blanchard-Kiyotaki model (1987) to a small open economy version, the effects of devaluation on the price level and aggregate output are investigated on the assumption that each individual maximizes his net utility subject to his budget constraint.

The results show a one to one relationship between the percentage increase in exchange rate and the price level and therefore the neutrality of devaluation on real output. Specifically, for a small open economy, a certain percentage
of devaluation in the domestic currency raises the price level by exactly the same percentage and does not change real aggregate output. This result involves the assumption that the ratio of the original money holdings in the rest of the world to the global money holdings is approximately equal to one (small open-economy case). This confirms the conventional knowledge of neutrality by modelling a different type of market structure — monopolistic competition.

By using the feature of monopolistic competition (S-D-S varieties), we build two models at the micro and macro levels. However, the two models give quite different results of devaluation on real output. Apparently, the assumption of S-D-S varieties in monopolistic competition does not contribute to such a difference in two models. As we shall see later on, the general conditions for expansionary, neutral or contractionary devaluation are virtually the same. The possible reasons may be as follows:

1. Our micro model is a partial equilibrium model with fixed income. This implies money wage rigidity of labour. But in our macro model, each agent’s income is endogenous and his elasticity of disutility with respect to output is not assumed as a constant. Thus a flexible money wage of labour is implicitly assumed. An exchange rate shock can be passed to real output through money wage rigidity but cannot through a flexible money wage. Therefore the different assumptions about money wage may contribute to the different results in
real output following a devaluation.

2. The number of varieties is endogenous in the micro model but is exogenous in the macro model. We have pointed out early how free entry and exit alter the market structure and therefore the long-run effects on real output following a devaluation.

To summarize, in this thesis we analyze several models and investigate the effects of devaluation on output in different market structures, at both the micro and macro levels, and in both short-run and long-run. As we know, models which are very general are simply not analytically tractable. For this reason, our research only attempts to advance certain issues at each stage. Therefore we adopt a partial equilibrium framework and a simplified general equilibrium framework in our analysis. Despite the fact that each model in this thesis is treated separately, our comparative analysis provides coherence throughout the whole study.

The contents of this thesis are arranged as follows: The next chapter, Chapter 2, reviews the most closely related literature on macro formulations and micro foundations of contractionary devaluation. Chapter 3 gives a systematic treatment of devaluation effects on output, by analyzing three different market structures with intermediate imports (perfect competition, monopoly and oligopoly), thus deriving the micro underpinnings of contractionary devaluation. These are
partial equilibrium models with homogeneous products. In Chapter 4, monopolistic competition is analyzed in order to further examine the microeconomic foundations of contractionary devaluation. This is a partial equilibrium model with differentiated products. In Chapter 5, we present an open-economy macro version with the feature of monopolistic competition by extending the closed-economy model developed by Blanchard and Kiyotaki. This is a new type of macro formulation which links macro and micro modelling. Thus the long-run effects of devaluation on output are examined in a macro model with the feature of monopolistic competition. The final chapter summarizes the conclusions and suggests direction for further research.
CHAPTER 2

A SURVEY OF THE LITERATURE

2.0 INTRODUCTION

In this chapter, we survey the literature most closely related to macro formulations and micro foundations of contractionary devaluation.

Numerous macro models have been built based on different specifications and most of them have implicitly assumed perfect competition. These models are the major part of the literature in the field of contractionary devaluation. Our research focus is on the effects of devaluation in various market structures. However, before we turn to models with imperfect competition, it is appropriate to have a brief overview of the macro literature involving perfect competition. This gives a relatively complete picture of the literature and shows the context and significance of our work.

This is the content of Section 1 in Chapter 2.

There have not been many complete models on the micro foundations of contractionary devaluation. In Section 2 of this chapter, two papers are reviewed with special attention to the role of industrial market structure. This section
establishes the need for a systematic treatment of the micro foundations of devaluation effects.

Section 3 introduces a new type of literature, which combines market structures within a macro model in a simple style. This type of modelling opens a new avenue for us to advance our research in this area.

2.1 MACRO MODELS WITH IMPORTED INPUTS

It is the consideration of intermediate imports that brought more controversial results to the traditional conclusion of expansionary devaluation. It started with questioning the robustness of the Marshall-Lerner (M-L) condition (the sum of the export and import elasticities must exceed 1) in the context of an open economy with an imported input.

Coppock (1971) may be the first author who questioned the generality and accuracy of the M-L condition for the analysis of devaluation under the assumption of trade flows involving an import content. He suggested that the basic elasticities $\varepsilon_x$ and $\varepsilon_{E,IM}$ in the M-L condition should be redefined to be consistent with the condition of improving the balance of payments derived in his paper.

Later, disagreeing with Coppock's view, Shea (1976) defended the generality of the M-L condition for improving balance of payments by modelling intermediate goods in a macro
structure although his analysis was limited to the specific assumption of Cobb-Douglas production function. However, these authors did not examine the effects of devaluation on output.

The major literature on expansionary/contractionary devaluation may be classified according to the use of imported goods. Specifically, imported goods may be assumed to be used as: (1) intermediate input only; (2) intermediate input and consumption goods as well; (3) content of investment goods. We analyze these different cases as follows.

Actually, assuming that all imports are an input into final goods production does not change the result of expansionary devaluation. In a simplified macro model with only intermediate imports, Scarth (1988, P.139) showed that the M-L condition implies expansionary devaluation.

\[2.1\] \[ Y = C(Y^d) + I(r^f) + G + X(E\Pi^x/P) \]
\[2.2\] \[ Y^d = (1 - E\Pi^m/P)Y \]
\[2.3\] \[ M/P = L(Y, r^f) \]
\[2.4\] \[ Y = F(N) \]
\[2.5\] \[ W = (P - E\Pi^m)F_N \]

where perfect capital mobility is assumed.

\[
\frac{dY}{dE} = \frac{(1 - E)(X_E - C_{yd}Y)}{1 - C_{yd}(1 - E)} > 0 \text{ iff } X_E - C_{yd} > 0.
\]
But the relevant M-L condition is $X_E - Y > 0$ (partial derivative of the trade balance with respect to exchange rate is positive), which is sufficient for $dY/dE > 0$. Hence devaluation is expansionary.

However, using this model, it is possible to give a different approach to the second proposition concerning contractionary devaluation by Krugman and Taylor that devaluation is contractionary when wage earners have higher propensities to consume than profit earners do.

Following Krugman-Taylor’s hypothesis, we let $c_R$ and $c_W$ be the propensities to consume out of profit income and wage income; and we let $Y^d_R$ and $Y^d_W$ be disposable income of the two classes. We define

$$Y^d = Y^d_R + Y^d_W$$
$$C(Y^d) = c_R Y^d_R + c_W Y^d_W$$

$$Y^d_W = WN/P$$

Substituting these definitions into equation (2.1), (2.2) and (2.5), and the same manipulation as in Scarth yields:

$$\begin{bmatrix}
1-c_R(1-E-W)-c_W\hat{W} & 0 \\
-L & 1
\end{bmatrix}
\begin{bmatrix}
dY \\
dM
\end{bmatrix}
= 
\begin{bmatrix}
Y[c_R (E-1+W)-c_W \hat{W}] + X_E (1-E) \\
M
\end{bmatrix}
dE$$
which implies
\[
\frac{dY}{dE} = \frac{YW(c_w-c_R)+(1-E)(X_k-Yc_R)}{1-c_R(1-E)+W(c_R-c_w)}
\]

The M-L condition \(X_k - Y > 0\) guarantees \(X_k - Yc_R > 0\) since \(c_R < 1\). If \(c_w < c_R\), then \(dY/dE > 0\) for sure. But it is widely accepted that propensities to consume of wage earners are higher than of profit earners (Krugman and Taylor, 1978), i.e. \(c_w > c_R\). This implies that the sufficient condition for expansionary devaluation, \(c_w < c_R\), contradicts the conventional wisdom, \(c_w > c_R\). If \(c_w > c_R\), \(dY/dE\) is ambiguous, i.e. contractionary devaluation is possible. Krugman and Taylor showed that \(c_w > c_R\) is sufficient for contractionary devaluation. In our analysis, \(c_w > c_R\) is a necessary but not sufficient condition for contractionary devaluation in a model with only imported inputs. Nevertheless, our result, even if it does not completely match that of Krugman and Taylor, shows that their concerns are valid in a more general macro model.

Modelling both intermediate imports and imported finished goods and using a fraction parameter of intermediate imports over total imports, Nielsen (1987) derived a general form of the M-L condition, which treated the case with only imported finished goods as a special case, where the defined fraction parameter of intermediate imports is zero. Thus Nielsen claimed that as long as the modified M-L condition
holds, the expansionary devaluation will occur in a small open economy with both imported finished goods and intermediate imports.

In addition, according to Buffie (1986a), it is still possible to have contractionary devaluation despite Nielsen's condition holding. When production functions are non-separable between domestic factors and imported inputs, contractionary devaluation may occur. For instance, in a case of three inputs (capital K, labour L and imported input IM), condition \( \sigma_{K, IM} > \sigma_{L, IM} \) on the elasticities of technical substitution ensures contractionary devaluation.

Under the assumption of importing both inputs and finished goods, Hanson (1983) extended Krugman-Taylor's model by incorporating substitution between domestic and foreign goods in production and consumption. As a result, he derived the condition for contractionary devaluation:

\[
\frac{m}{x} \left[ 1 - (m_i \sigma_i + m_c \sigma_c) + (1-\delta)m_c(1-\sigma_c)/\delta \right] > 1
\]

where \( m_i, m_c \) are proportion of imported input and of imported finished goods, \( \sigma_i \) and \( \sigma_c \) are corresponding elasticities.

The condition depends on three items: the ratio of imports to exports, \( m/x \), the weighted sum of the elasticities, and a correcting factor which is generally small and positive. Thus devaluation will more likely be contractionary when elasticities are very low or and the trade deficits are large.
We can find that Krugman-Taylor’s first proposition concerning contractionary devaluation (Chapter 1, P.2) is just a special case where $m_c = 0$, $\sigma_i = 0$, $m/x > 1$.

Buffie (1986b) assumed that imported goods are just one of the contents in one input. He specified a production function with two inputs: capital goods $K$ and labour $L$. Moreover, the capital good $K$ is defined as output of composite goods from both domestic and imported components in fixed proportions. In order to capture the nature of imported goods in Buffie’s model, we may want to call this type of imported goods as "sub-intermediate" imports, i.e. intermediate goods to produce input for domestic production of final goods. Introducing sub-intermediate imports alters a number of results. For instance, Buffie concluded that the M-L condition is neither necessary nor sufficient for expansionary devaluation; it is altogether irrelevant. It is the import shares, the share of domestic output in aggregate investment, the export elasticities and the individual import elasticities that determine the impact of devaluation on the balance of payments and the demand for domestic output. The M-L condition is unnecessarily stringent for expansionary devaluation when import elasticities are sufficiently low; the M-L condition is not sufficient to guarantee expansionary devaluation when import elasticities are high. The larger the investment elasticity is, the greater the likelihood of contractionary devaluation is.
By and large, the import-export ratio (thus the trade deficits) still remains an important factor which affects the condition for contractionary/expansionary devaluation in models with imported inputs. Furthermore, the different uses of imported inputs and the relevant elasticities also affect the condition.

The macro literature surveyed in this section is based on the implicit assumption of perfect competition. Our research focus is on models with imperfect competition. However, it is useful to provide a brief survey on models with perfect competition and then later on concentrate on a specific survey on models with imperfect competition.

Among the macro models with imperfect competition, Harris (1984) conducted an applied general equilibrium analysis of small open economies. Startz (1989) built a general equilibrium macro model based on monopolistic competition. Mankiw (1988) developed a macro model with a parameter which features three different market structures. Blanchard and Kiyotaki (1987, 1989) constructed a macro framework with monopolistic competition. However, the issue of contractionary devaluation was not the interests of the above-mentioned authors. In this thesis, however, we only intend to examine the issue of contractionary devaluation based on the work by Blanchard and Kiyotaki, which is surveyed separately in Section 2.3.
2.2 PARTICULAR LITERATURE ON MICRO MODELS

It is only recently that microeconomists have turned their attention to the issue of contractionary devaluation. Several papers focusing on the relationship between devaluation and prices have been published. Among them, the work by Dornbusch (1987) and Baldwin (1988) are presented here.

In an imperfect competition framework, Dornbusch found a common feature of various models: they all predict that devaluation should lead to an increase in the price of imports, and in the relative price of imported goods vs. domestic products. Dornbusch's approach is a short-run analysis because entry and exit are not taken into account.

In a simple monopolistic competition model with sunk costs, Baldwin showed that a sufficiently large exchange-rate change can have persistent effects on prices, in contrast with the earlier insight of neutral exchange-rate effects on prices in the long run.

Neither author was interested in the effects of devaluation on output. However, as we show in this thesis, their conclusions are related to contractionary/expansionary devaluation analysis: the price change will directly affect demand, hence real output. These papers have led to a growing literature on devaluation effects in various models with imperfect competition.
2.2.1 Devaluation and Prices: Short-run

Dornbusch investigated the exchange-rate effects on prices in four different models of imperfect competition: the Cournot-Nash model, the Dixit-Stiglitz model, the extended Dixit-Stiglitz model and a model of competition on the circle.

First, a Cournot model was formulated. The basic assumptions in the home market were: a linear demand function \( Q = a - bP \); \( n \) identical domestic suppliers with each output \( q \) and unit labour cost \( W \), \( n' \) identical foreign suppliers with each output \( q' \), and unit labour cost \( eW' \). From a Cournot-Nash equilibrium solution, the relationship between price and the exchange rate \( e \) is written as \( \frac{d \ln P}{d \ln e} = \frac{n'}{N} \frac{eW'}{P} \), where \( N = n + n' + 1 \). A small home country case can be characterized by \( n'/N \) approaching 1, therefore a devaluation will raise price by the same proportion. A large home country case can be characterized by \( n'/N \) approaching 0, therefore a devaluation has no effects on domestic price. In short, market share determines the quantity of equilibrium price elasticity with respect to the exchange rate. This is a simple "group duopoly" or oligopoly form.

Second, the Dixit-Stiglitz model (1977) of monopolistic competition was employed. Maximize consumer's utility function \( U(Z, X) \), in which \( Z \) is homogeneous goods and \( X \) is an index of \( i \) brands of differentiated goods in CES form. The demand function for each brand of \( X \) is derived in a non-linear form. \( n \) domestic firms and \( n' \) foreign firms supply
goods X. By assuming that each supplier does not affect industry price, the maximization of domestic firm i's and foreign firm j's profit yields the constant mark-up pricing equation $P_i = \alpha W, P_j = \alpha eW^*$, where $\alpha = 1/(1 - 1/\sigma)$ and where $\sigma$ is the elasticity of substitution among brands. Given the unit cost of labour, the only input, the implication of a devaluation of home currency is obvious: prices of domestic varieties of goods X, $P_i$, remain unchanged but prices of foreign varieties of goods X, $P_j$, go up proportionally. Hence import prices will increase relative to domestic prices.

Third, the Dixit-Stiglitz model was extended by relaxing the assumption that the individual firm does not affect industry price. As a result, profit maximization no longer yields the constant mark-up pricing equation, instead, the demand curve facing the individual firm becomes function $P_i = \alpha' W, P_j = \alpha' eW^*$, where $\alpha' = 1/[1-1/\sigma(1-\epsilon)]$, which captures the price elasticity term $\epsilon = dlnP/dlnP_{i,j}$ reflecting the strategic interaction between firms. Furthermore, the price reaction functions can be written as:

$$P_i = F(P_j/P_i, \mu, \sigma)W$$

$$P_j = F'(P_i/P_j, \mu, \sigma)eW^*$$

where $\mu$ is the conjectural variation parameter.

A devaluation shifts reaction function $P_j$ but leaves $P_i$ in the same position. Ultimately, at the new equilibrium,
both $P_i$ and $P_j$ are higher than before, and the relative price of imported products, $P_j/P_i$, is higher than before as well. This is an extended monopolistic competition form with price interaction.

Finally, a model of competition on the circle was developed. The characteristics of each firm's products are described by its position on a circle. Unlike the assumption in Dixit-Stiglitz model that consumers buy some of each brand, now consumers are assumed to buy only one brand of differentiated goods from the most adjacent supplier along the circle where domestic and foreign firms are alternately located and compete with each other by using a Cournot strategy. This is an extension of the Hotelling model (1929).

Both domestic and foreign firms' reaction functions were derived:

$$P_i = \sigma/n + (eW^* + 2W)/3$$

$$P_j = \sigma/n + (2eW^* + W)/3$$

from which we can see that a devaluation will raise prices of both domestic and foreign products, and that the magnitude of price increase depends on the total number of firms and substitutability between the brands i and j. Further derivation of the elasticities of prices with respect to the exchange rate shows that devaluation increases the relative price of imported goods $P_j/P_i$. This will lead a demand shift
from foreign firms to home firms as consumers weigh the trade-off between the increase in foreign price and the farther distance of the home firm.

Intuitively, one would expect that a devaluation will raise the price of imported goods, thus the general domestic price level. Dornbusch has proved this conclusion in an imperfect competition framework. However, he did not investigate the effects of devaluation on output. Besides, the assumption of one input labour is restrictive in excluding intermediate imports.

In the next chapter, we shall extend a part of Dornbusch’s analysis to a model with intermediate inputs and investigate the conditions for contractionary devaluation on output in three different market structures (perfect competition, monopoly and oligopoly). A partial equilibrium framework is used in our analysis as in Dornbusch’s model.

2.2.2 Devaluation and Import Prices: Long-run

A sufficiently large, temporary exchange-rate change may have persistent effects on import prices and quantities through altering domestic market structure. This phenomenon is what Baldwin (1988) describes as hysteresis, or beachhead effects.

In an intertemporal setting with an infinite time horizon, Baldwin used a modified S-D-S framework to model the industry structure and imperfect competition. The important
assumptions in this partial equilibrium model are: (1) each home and foreign firm produces a distinct variety of a particular good and they engage in Cournot competition in the domestic market; (2) there is a fixed, sunk market-entry cost $F$ which determines the entry and exit conditions for each firm; and (3) total varieties $N$ are sold and $N$ is negatively related to the price level of each variety.

The firm's optimization problems are to maximize:

$$
\pi = \sum_{t=0}^{\infty} R^t (P[N_t, X_t]X_t - c_tX_t) - F \quad \text{for home firms,}
$$

$$
\pi^* = \sum_{t=0}^{\infty} R^t (P[N_t, Y_t]Y_t - c_*te_tY_t) - F \quad \text{for foreign firms,}
$$

where $t = 0, 1, \ldots \infty$ is time period, $R$ is the discount factor, $X,Y$ are sales of domestic and foreign products, $P$ is inverse demand function for any variety, $c$ is unit cost and $F$ is sunk cost.

Let $S_t = \pi + F$ and $S'_t = \pi^* + F$ be the operating profit of domestic and foreign firms respectively. The key point is that as long as operating profit is within the range from 0 to $F$, i.e. $0 \leq S_t \leq F$, $0 \leq S'_t \leq F$, neither entry nor exit will occur. This implies there exist multiple equilibria between entry and exit conditions, which, generally speaking, are not sensitive to a small shock, say, a small devaluation. However, if a devaluation is large enough to violate the
condition on profits which was give above, then either entry or exit will certainly occur. Consequently, market structure will be changed permanently.

Baldwin showed the relationship between price, number of varieties and elasticities in the following form:

\[
P_t = \frac{1}{1 - 1/\varepsilon [N_t, Y_t]} c_t e_t
\]

where \( P_t \) is import price and \( \varepsilon \) is the perceived elasticity. A decrease in the number of varieties \( N \) makes less elastic the demand curve for each variety.

A small devaluation will indeed affect the price level immediately. But the price level will go back to the original value when the exchange rate returns to the original value (which is the 'stylized' devaluation pattern assumed by Baldwin). A large devaluation not only changes the \( \varepsilon \) value in the price equation, but also affects the value of \( M_t \) since such a devaluation is assumed to be large enough to violate the profit condition and to cause exit. After a devaluation, the exchange rate returns back to the original level, but \( N_t \), due to exit, is still lower than before. Therefore the post-devaluation price is permanently higher than the pre-devaluation price.

Baldwin's insight is of significance to our research. Even though Baldwin did not intend to examine the output change, the idea of the structural change in the long-run can
be extended to the analysis of devaluation effects on output in the monopolistic competition case, as we shall see, in Chapter 4.

2.3 A MACRO MODEL WITH MONOPOLISTIC COMPETITION

Blanchard and Kiyotaki examined the importance of monopolistic competition to an understanding of the effects of aggregate demand on output in a closed-economy model. The choice of monopolistic competition has the advantage of endogenizing the price setting decision yet still keeping the manipulation of the model tractable.

In a general equilibrium model with goods, labour and money, monopolistic competition is assumed in both goods and labour markets. Therefore price setting and wage setting can be modeled in both markets. To avoid the assumption that the supply of goods produced by the monopolistically competitive firms automatically generates its own demand, real money demand is introduced to the model. Thus money represents non-produced goods and other services. Money also plays the role of the numeraire since producers and labourers quote prices and wages in terms of money.

There are m firms employing n labourers and producing m varieties of a differentiated good. n labourers consume m varieties of goods and supply n types of differentiated labour. The specification of constant elasticity of
substitution (CES) by Spence-Dixit-Stiglitz (S-D-S) is adopted in both goods and labour, i.e. there is a CES parameter, say, \( \sigma \), among any varieties of goods; and there is a CES parameter, say, \( \theta \), among any types of labour. The general equilibrium will be reached when market clearing conditions hold in both goods and labour markets. Walras' law guarantees clearing in the money market.

Firm \( i \) \((i = 1, \ldots, m)\) has profit function

\[
\pi = P_i Y_i - \sum_{j=1}^{n} (W_j N_{ij})
\]

where \( P_i \) is the price of variety \( i \) which firm \( i \) charges and \( Y_i \) is its output. \( N_{ij} \) is the quantity of labour of type \( j \) used in the production of output \( i \), and \( W_j \) is the wage of labour type \( j \).

Firm \( i \) has CES production function:

\[
Y_i = \left( \sum_{j=1}^{n} \frac{\theta - 1}{\theta} N_{ij} \right)^{\frac{\theta}{\theta - 1}} \cdot \left( \frac{1}{\theta} \right)^{\frac{1}{\theta - 1}} \cdot \frac{1}{a}
\]
where parameter $\theta$ is the elasticity of substitution of labour inputs in production and $\theta > 1$. Parameter $1/\alpha$ characterizes the degree of returns to scale.

Firm $i$ maximizes its profit $\pi_i$ subject to the production function $Y_i$, taking wages and prices of other varieties as given and assuming the total number of firms is large enough that taking prices of other varieties as given is equivalent to taking the general price level as given.

Labourer $j$ ($j = 1, \ldots, n$) has utility function

$$U_j = \left( m \frac{1}{1-\sigma} C_j \right)^\gamma \left( \frac{M_i}{P} \right)^{1-\gamma} - N_j^\beta$$

where

$$C_j = \left( \sum_{i=1}^m \frac{\sigma-1}{\sigma-1} \right)^{\frac{\sigma}{\sigma-1}}$$

is a CES form of $c_{ij}s$ and $c_{ij}$ is the consumption of variety $i$ by labour $j$. Parameter $\sigma$ is the elasticity of substitution between any pair of varieties. We assume $\sigma > 1$. $M_i/P$ is labourer $j$'s demand for real money balances and $N_j$ is the amount of labour supplied by labourer $j$. Parameter $\gamma$ is the budget share on consumption goods ($0 < \gamma < 1$) and parameter $\beta-1$ is the elasticity of marginal disutility of labour ($\beta-1 \geq 0$). In short, the labourer gains utility by enjoying
consumption goods and real money balance in a Cobb-Douglas form and loses utility by supplying labour.

The general price level is again in a CES form

\[ P = \left( \frac{1}{m} \sum_{i=1}^{m} P_{i}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \]

Labourer \( j \) has the budget constraint

\[ \sum_{i=1}^{m} P_{i} C_{ij} + M_{j} = W_{j} N_{j} + \bar{M}_{j} + \sum_{i=1}^{m} \pi_{ij} \]

where \( M_{j} \) bar is j's initial money holding and \( \pi_{ij} \) is his profit share from firm \( i \). The wage level is again defined in a CES form:

\[ W = \left( \frac{1}{n} \sum_{j=1}^{n} W_{j}^{1-\theta} \right)^{\frac{1}{1-\theta}} \]

Labourer \( j \) maximizes his utility \( U_{j} \) subject to his budget constraint, taking all prices and other wages as given, and assuming the total number of labourers is large enough that taking other wages as given is equivalent to taking the wage level as given; and from his indirect utility function each labourer chooses his wage and labour supply, taking all
prices and other wages as given. Finally, solve for equilibrium output and labour.

However, the matter can be simplified if we think the situation as an economy of worker-producers, where each individual is both worker and producer. Then there is only one set of price setters and the analysis is simplified. This is the version in Blanchard and Fisher (1989).

Blanchard and Kiyotaki investigated three questions: First, using perfect competition as a benchmark, can monopolistic competition, by itself, explain why aggregate demand movements affect output? Second, can monopolistic competition, together with some other imperfections, generate effects of aggregate demand in a way that perfect competition cannot? Third, taking as given that aggregate demand movements affect output, can monopolistic competition give a more accurate account of the response of the economy to aggregate demand shocks? The Blanchard-Kiyotaki model is a closed-economy model. In Chapter 5, we adopt and extend the simplified version of their framework to a small open economy and examine the effects of devaluation on real output.

2.4 SUMMARY

Now we are closing our survey of the literature. In the light of the literature just surveyed, we are in a better position to state our research problems and methodology.
From the partial equilibrium literature we have learned that, in most imperfect competition models, devaluation increases price level in both short-run and long-run. However, we have not found much direct analysis related to the effects on output.

Therefore our research task is to provide a systematic analysis of the effects of devaluation on output in a variety of market structures. These market structures can also be classified into two categories: markets with homogeneous goods and markets with differentiated goods.

In the homogeneous goods category, we consider a Cournot-Nash and a Stackelberg equilibrium models. The former contains perfect competition and monopoly as special cases. We address and solve the related optimization problems, and then examine the effects of devaluation on output at equilibrium in three market structures. This is done in Chapter 3.

In the differentiated goods category, we extend Baldwin's model with persistent effect of temporary exchange rate fluctuations on import prices. We focus on the goods market and demonstrate the hysteresis in both the general price level and real output. Also, the mechanism for such hysteresis is explored. This is done in Chapter 4.

Although our analysis at the micro level is based on a partial equilibrium framework, it provides meaningful
results, ceritus paribus, avoiding the intractable situation in the models incorporating all the aspects.

From the macro literature we have learned that contractionary devaluation is possible in macroeconomic models. However, all the open macro models we have surveyed are actually based on the assumption of perfect competition. Imperfect competition is still a rarely touched issue in the area. Therefore, our research interest focuses on developing a macro model with the feature of imperfect competition, and thus we are able to examine the effects of devaluation on output in this type of macro formulation. The methodology is to extend the model by Blanchard and Kiyotaki to an open version. By examining a small open-economy version, we investigate robustness of the conventional open-economy prediction: the long-run neutrality of devaluation. The model is a simplified general equilibrium model, which keeps the manipulations tractable. This is done in Chapter 5.
CHAPTER 3
MICROECONOMIC FOUNDATIONS:
CASE OF HOMOGENEOUS GOODS

3.0 INTRODUCTION

It seems indisputable that a devaluation affects the output in the short-run. Whether and under what conditions the effects are expansionary, neutral or contractionary are the questions to be investigated.

We now give a systematic treatment of the effect of devaluation on output by analyzing three traditional forms of market structures with intermediate imports. These forms are: perfect competition, monopoly and oligopoly. All these forms are the case of homogeneous products.

Actually, perfect competition and monopoly can be viewed as two special cases in a Cournot-Nash oligopoly model by assuming an infinite number of firms and a single firm respectively. Thus a general Cournot-Nash oligopoly model would serve our purpose to analyze three market structures. However, this will ignore the Stackelberg behaviour in the oligopoly model. To overcome this shortcoming, we model the
Stackelberg behaviour separately.¹ Therefore in this chapter we take the Cournot-Nash strategy and the Stackelberg behaviour to derive the corresponding conditions for expansionary, neutral and contractionary devaluation in a partial equilibrium framework.

The different market structures determine what roles a devaluation plays in the demand side through the price mechanism. The intermediate imports determine how a devaluation affects the supply side through the different types of technology (production functions). To concentrate on the differences of devaluation effects on output in the different market structures, we choose the linear demand function and Leontief production function, which are commonly assumed by economists to simplify the matters.

We discuss the Cournot-Nash oligopoly and the Stackelberg oligopoly in two separate sections. In the final section of this chapter, the results derived from the previous sections are summarized and compared. Also, the policy implications are discussed.

3.1 COURNOT-NASH OLIGOPOLY

We assume that there are two countries in the world economy. There are n firms in the home country and n' firms

¹ Perrakis (1990, p.133) suggested that the Stackelberg model can be integrated with other models using conjectural variations. We are aware of this literature, but have chosen not to pursue it.
in the foreign country. Both home and foreign firms produce homogeneous consumer goods and compete with each other in the world market. To produce one unit of finished goods, a typical home firm $i$ uses $L_i$ units of domestic labour with money wage $W$ and $k_i$ units of imported input (from the foreign country) with the unit price $P^M$; a typical foreign firm $j$ uses $L_j$ units of foreign labour with money wage $W'$ and $k_j$ units of other input with unit price $P^M$. $P^M$ and $W'$ are in foreign currency. The fixed costs are ignored. A typical home firm $i$ produces output $q_i$ and a typical foreign firm $j$ produces output $q_j$. There are no substitutes for these consumer goods. There are no trade barrier between the two countries.

The demand curve in the domestic market is

$$Q = a - bP,$$

where $P$ is the price in domestic currency.

The demand curve in the foreign market is

$$Q' = a' - b'P',$$

where $P'$ is the price in foreign currency.

We have the law of one price:

$$P = eP',$$

where $e$ is the nominal exchange rate, i.e. the price of the foreign exchange in domestic currency. This actually assumes there is a costless arbitrager. Thus [3.2] can be written as

$$Q' = a' - \frac{b'}{e} \cdot P$$
The aggregate demand curve can be derived by adding up equation [3.1] and [3.4]:

\[
[3.5] \quad \hat{Q} = (a + a') - (b + \frac{b'}{e})P
\]

\[
[3.6] \quad P = \frac{a + a' - \hat{Q}}{b + \frac{b'}{e}}
\]

or simply

\[
[3.7] \quad P = P(\hat{Q})
\]

A typical home firm i has profit function

\[
[3.8] \quad \pi = P(\hat{Q}) \cdot q_i - C_i(q_i)
\]

where

\[
C_i(q_i) = (WL_i + eP^m_{i}k_i)q_i
\]

The first order condition (FOC) of profit maximization is

\[
[3.9] \quad P(\hat{Q}) + P'(\hat{Q}) \cdot q_i - C_i'(q_i) = 0
\]
At the equilibrium we should have goods market clearing condition

\[
\hat{\zeta} = q_i + \sum_{i=1}^{n} q_i + q_j^* + \sum_{j=1}^{n^*} q_j^*
\]  

Substituting [3.6] and [3.10] into [3.9] and rearranging it, we have

\[
\begin{align*}
3.11 \quad & a + a' - Q - Q^* - q_i - (b + \frac{b'}{e}) (WL_i + eP^*k_i) = 0 \\
3.12 \quad & n(a + a') - (n+1)Q - nQ^* - \left( b + \frac{b'}{e} \right) \sum_{i=1}^{n} (WL_i + eP^*k_i) = 0
\end{align*}
\]

Taking summation of [3.11], we have

\[
\begin{align*}
3.13 \quad & \pi^* = P(\hat{\zeta}) \cdot q_j^* - eC_j^*(q_j^*)
\end{align*}
\]

where

\[
C_j^*(q_j^*) = (WL_j + P^*k_j^*) q_j^*
\]
Similarly, for the FOC of profit maximization we have

\[ 3.14 \quad n^*(a+a') - (n^*+1)Q^* - n^*Q - \left( b + \frac{b'}{e} \right) \sum_{j=1}^{n^*} e(W^*L_j^* + P^k_j^*) = 0 \]

To simplify the matter, we assume symmetry, i.e.

\[ L_i = L, \quad k_i = k, \quad L_j = L^*, \quad k_j^* = k^* \text{ for all } i's, j's. \]

Solving equation [3.12] and [3.14] for \( Q \) and differentiating it with respect to \( e \), we have

\[ 3.15 \quad Q = \frac{1}{n+n^*+1} \left[ n(a+a') + \left( b + \frac{b'}{e} \right) \left[ nn^*e(W^*L^* + P^k^*) - n(n^*+1)(WL+eP^k^*) \right] \right] \]

\[ 3.16 \quad \frac{dQ}{de} = \frac{n}{(n+n^*+1)e^2} \cdot \left[ b'(n^*+1)WL + be(n^*e(W^*L^* + P^k^*) - (n^*+1)eP^k^*) \right] \]

which gives condition

\[ 3.17 \quad \frac{dQ}{de} \geq 0 \quad \text{iff} \quad \frac{b'/e}{b} \geq \frac{eP^k^*}{WL} - \frac{n^*eC^*}{n^*+1WL} \]
where

\[ ec^* = e(W^*L^* + P^Mk^*) \]

is the unit cost of the foreign product in domestic currency.

[3.17] is the general condition for expansionary, neutral and contractionary devaluation in the Cournot-Nash oligopoly model. On the LHS of condition [3.17], the numerator \( b'/e \) is the slope of the foreign demand curve in equation [3.4] and the denominator \( b \) is the slope of the domestic demand curve in equation [3.1]. On the RHS, the \( eP^Mk \) is the unit cost of the imported content and \( WL \) is the unit cost of the domestic content of home product; and \( e(W^*L^* + P^Mk^*) \) is the unit cost of foreign product. In short, the LHS represents the effects of devaluation on the demand side and the RHS represents the effects of the devaluation on the supply side.

The precise verbal description of condition [3.17] is somewhat awkward. Nevertheless, generally speaking, devaluation is expansionary, neutral or contractionary if and only if its effect on the demand side is greater than, equal to or smaller than its effect on the supply side.

Taking the duopoly case, we have condition:

\[[3.17'] \quad \frac{dQ}{de} < 0 \quad \text{iff} \quad \frac{b'/e}{b} < \frac{eP^Mk}{WL}\]
Condition [3.17'] is based on the assumption that there are no foreign firms \( (n^* = 0) \). Condition [3.17''] is based on the assumption that there are one home firm and one foreign firm \( (n = 1, n^* = 1) \).

As we mentioned earlier, perfect competition and monopoly can be considered as two special cases in the Cournot-Nash oligopoly model. To see the devaluation effects on domestic output in these different market structures, we examine these cases in the following subsections. Besides, the effects of devaluation on the world output and price level are also examined.

3.1.1 Perfect Competition

When the number of firms is sufficient large in the Cournot-Nash oligopoly model we view the model as the version of perfect competition. From [3.16] we know that both the number of home firms, \( n \), and the number of foreign firms \( n^* \), determine the magnitude of devaluation effect on output. From [3.17], we know only the number of foreign firms affects the direction of devaluation effect on output. Letting \( n^* \to \infty \), [3.17] and [3.16] can be written as:

\[
[3.17''] \quad \frac{dQ}{de} > 0 \quad \text{iff} \quad \frac{b'/e}{b} > \frac{eP^Mk}{WL} - \frac{1}{2} \frac{ec^*}{WL}
\]
Devaluation is expansionary, neutral or contractionary if and only if the ratio of the slope of the foreign demand curve to the slope of the domestic demand curve is greater than, equal to or smaller than the ratio of the unit cost of the imported content to the unit cost of the domestic content of home product minus the ratio of the unit cost of foreign product to the unit cost of domestic content of home product.

[3.19] \[
\frac{dQ}{de} = \frac{n}{e^*} \cdot [b'WL + be^*(ec^* - eP^Mk)]
\]

[3.18] \[
\frac{dQ}{de} \geq 0 \quad \text{iff} \quad \frac{b'/e}{b} \geq \frac{eP^Mk - ec^*}{WL}
\]
Q either goes to infinity or zero following a devaluation.

If we assume \( n = 0 \), the magnitude of devaluation effect on domestic output would be zero. However, it is not sensible to assume \( n = 0 \) since our model is built to examine the effect of devaluation on domestic output.

### 3.1.2 Monopoly

When there is only a single firm in the Cournot-Nash oligopoly model, we view the model as the version of monopoly. We may either assume a domestic monopoly firm or a foreign monopoly firm. But it is not sensible to assume a foreign monopoly firm if our model is to examine the devaluation effect on domestic output. Hence we assume a domestic monopoly firm, i.e. \( n = 1, n' = 0 \). Then we have:

\[
\frac{dQ}{de} \geq 0 \quad \text{iff} \quad \frac{b'/e}{b} \geq \frac{eP^*k}{WL}
\]

\[
\frac{dQ}{de} = \frac{1}{2e^2} \cdot [b'WL - be \cdot eP^*k]
\]

Devaluation is expansionary, neutral or contractionary if and only if the ratio of the slope of the foreign demand curve to the slope of the domestic demand curve is greater than, equal to or smaller than the ratio of the unit cost of the imported content to the unit cost of the domestic content of home product.
Now we discuss the intuition behind condition \([3.20]\).

The monopolistic firm's strategy is to produce at \(MR = MC\).

We know

\[
MR = \frac{a + a' - 2Q}{b'/e + b}
\]

\[
MC = WL + eP^nk
\]

In Figure 3.1, a devaluation shifts \(MC\) up and rotates \(MR\) up. At the new equilibrium, output may increase (Panel 1), remain the same (Panel 2) or decrease (Panel 3). The results will depend on how much \(MC\) shifts and how much \(MR\) rotates. The relatively high imported contents would shift \(MC\) more and thus would more likely cause a contraction in domestic output, ceteris paribus; a more elastic foreign demand curve, thus a more elastic aggregate demand curve, would rotate \(MR\) more and thus would more likely cause an expansion in domestic output, ceteris paribus. The rotation of \(MR\) is generated by the force in the demand side due to a devaluation, and the shift of \(MC\) is generated by the force in the supply side due to the same devaluation. Condition \([3.20]\) gives the precise quantitative relationship, which determines whether the change in \(MR\) or the change in \(MC\) would dominate, namely, whether the expansionary or the contractionary devaluation would occur.
Figure 3.1

(Panels 1 and 2)

(Panels 1 and 2)
3.1.3 World Output

We might be interested in the effects of devaluation of domestic currency on the foreign output and world output. Solving equation [3.12] and [3.14] for $Q^*$ and differentiating it with respect to $Q^*$ and $e$, we have

$$[3.22] \quad Q^* = \frac{1}{n+n^*+1} \left[ n(a+a') + \left( b + \frac{b'}{e} \right) [nc - (n+1)ec^*] \right]$$

where $c = WL + eP^Mk$ is the unit cost of domestic product.

$$[3.23] \quad \frac{dQ^*}{de} = \frac{n^*}{(n+n^*+1)e^2} \cdot \left[ be\left[nEP^Mk - (n+1)ec^*\right] - b'nWL \right]$$

Thus we have condition:

$$[3.24] \quad \frac{dQ^*}{de} \geq 0 \quad \text{iff} \quad \frac{b'/e}{b} \geq \frac{eP^Mk}{WL} - \frac{n+1}{n} \cdot \frac{ec^*}{WL}$$

For the world output, we have

$$[3.25] \quad \frac{dQ}{de} = \frac{1}{(n+n^*+1)e^2} \cdot \left[ b'nWL - b'enEP^Mk - ben^* ec^* \right]$$
The higher proportion of foreign products in the world market more likely contributes to contractionary devaluation in world output, since \( n'/n \) is large on the RHS of condition [3.26]. If the home country has a very small share in world production (i.e. \( \frac{n}{n+n^*} \to 0 \), thus \( n^* \to \infty \)), devaluation is certainly contractionary since

\[
\lim_{n^* \to \infty} \frac{d\hat{Q}}{de} = -bc^* < 0
\]

This is plausible since under these conditions devaluation shifts the world demand curve but does not affect production significantly.

3.1.4 Price Level

We might be interested in the effect of devaluation on the price level. This can be derived by differentiating equation [3.6] with respect to \( P \) and \( e \).
\[
\frac{dP}{de} = -\frac{\left(b + \frac{b'}{e}\right) \cdot \frac{dQ}{de} + \left(a + a' - Q - Q^*\right) \cdot \frac{b'}{e^2}}{\left(b + \frac{b'}{e}\right)^2}
\]

Using [3.25], [3.5] and [3.22] to substitute \(dQ/de\), \(Q\) and \(Q^*\) respectively, after some lengthy manipulations, we can show that

\[
\frac{dP}{de} > 0
\]

This demonstrates that devaluation is always inflationary in our Cournot-Nash oligopoly model.

### 3.2 Stackelberg Behaviour

The basic assumption of the Stackelberg behaviour is that one firm takes the other firms' reaction function as given. This is useful in the industry with a natural leader who has a first move advantages, and in the two-period game modelling. For convenience, we consider Stackelberg duopoly. The Stackelberg equilibrium can be reached only when one firm decides to be a follower and the other a leader. We eliminate two other cases: both firms want to be follower (which is the Cournot-Nash equilibrium); and both firms want to be leader (which is the indeterminate case called the Stackelberg warfare).
Three different cases are discussed in this section. They are: (1) the home leader and the foreign follower; (2) the foreign leader and the home follower; and (3) the home leader and the home follower.

3.2.1 Home Leader and Foreign Follower

We assume that the home firm is the leader and the foreign firm is the follower. Their output are $q$ and $q^*$ respectively.

Two duopoly firms' profit functions are

\[
\pi = (P - WL - eP^Mk) \cdot q
\]

\[
\pi^* = (P - eW^*L^* - eP^Mk^*) \cdot q^*
\]

Using the same aggregate demand function as before in [3.5] and deriving the FOC's, we have the reaction functions of two firms as follows:

\[
q = \frac{1}{2} \cdot \left[ a + a' - q^* - (WL + eP^Mk) \left( \frac{b'}{e} + b \right) \right]
\]

\[
q^* = \frac{1}{2} \cdot \left[ a + a' - q - e(W^*L^* + P^Mk^*) \left( \frac{b'}{e} + b \right) \right]
\]
As the leader, the home firm takes the foreign firm's reaction function as given in his profit maximization, which gives the home firm's output at equilibrium:

\[ q = \frac{1}{2} \cdot \left[ a + a' + [e(W^*L^* + P^*k^*) - 2(WL + eP^*k)](\frac{b'}{e} + b) \right] \]  

from which we derive:

\[ \frac{dQ}{de} = \frac{1}{2e^2} \cdot [be^2e(W^*L^* + P^*k^*) + 2(b'WL - be^2eP^*k)] \]

Then we have following conditions:

\[ \frac{dq}{de} \geq 0 \quad \text{iff} \quad \frac{b'/e}{b} \geq \frac{eP^*k}{WL} - \frac{1}{2} \frac{ec^*}{WL} \]

The home firm's output is the total output of the home country, i.e. \( q = Q \). Therefore, the necessary and sufficient condition for expansionary, neutral of contractionary devaluation in this model, condition [3.33], is the same as condition [3.17''] in the Cournot-Nash duopoly model (\( n = 1 \) and \( n'' = 1 \)).

Moreover, we can show that \( dP/de > 0 \), i.e. devaluation is inflationary in the Stackelberg duopoly model with home leadership and foreign followership.
3.2.2 Foreign Leader and Home Follower

We assume that the foreign firm is the leader and the home firm is the follower.

Substituting the home firm's reaction function [3.29] into the foreign firm's profit function [3.27], we have the foreign firm's optimal output at the equilibrium:

\[ q^* = \frac{1}{2}(a + a' + (WL + P^n k)(b'/e + b) - 2(W'L' + eP^n k')(b' + be)) \text{ .} \]

Substituting [3.34] into [3.29], we have the home firm's optimal output at equilibrium:

\[ q = \frac{1}{2}(a + a' + (WL + P^n k)(b'/e + b) - 3(W'L' + eP^n k')(b' + be)) \text{ ,} \]

thus,

\[ dq/de = \frac{1}{4}e^{-2}[2be \cdot e(W'L' + P^n k') + 3(b'WL - be \cdot eP^n k')] \text{ .} \]

\[ \frac{dq}{de} > 0 \text{ iff } \frac{b'/e}{b} > \frac{eP^n k}{WL} - \frac{2}{3} \frac{ec^*}{WL} \text{ .} \]

which is less stringent in terms of the expansionary devaluation or more stringent in terms of the contractionary devaluation than condition [3.33] in the case of home leadership and foreign followership.
Again, we can show that $dP/de > 0$, i.e. devaluation is also inflationary in the Stackelberg duopoly model with foreign leader and home follower.

3.2.3 Home Leader and Home Follower

Now let us look at the situation where both Stackelberg behaving firms are home firms. It does not matter to us which firm is leader or follower, since we examine the effect of devaluation on the total domestic output and both firms are home firms.

Firm 1 is the leader and its profit function is the same as [3.27]. Firm 2, as the follower, has profit function

$$\pi' = (P - WL' - e P^m k') q' .$$

Similarly, we derive firm 1's output at equilibrium:

$$q = \frac{1}{3} [a + a' + (WL' + P^m k')(b'/e+b)$$

$$- 2(WL + e P^m k)(b'/e+b)],$$

also, we have firm 2's output:

$$q' = \frac{1}{6} [2a + 2a' - (WL' + P^m k')(b'/e+b)$$

$$+ 2(WL + e P^m k)(b'/e+b)$$

$$- 3(b'WL'e^{-1} + bWL' + b' P^m k' + bP^m k'e)].$$

Therefore,
A devaluation would affect the total domestic output by:

\[ dQ/de = dq/de + dq'/de \]

\[ dQ/de = 1/3 \cdot e^{-2} [(b'WL - be \cdot \text{eP}^*k) + (b'WL' - be \cdot \text{eP}^*k')] \]

from which we have condition:

\[ \frac{dQ}{de} > 0 \iff \frac{b'/e}{b} > \frac{\text{eP}^*(k+k')}{W(L+L')} \]

If we assume two firms have identical cost function, condition \[3.45\] becomes exactly the same as condition \[3.17'\] in the Cournot-Nash equilibrium model (n' = 0).

Similarly, we can show that \( dP/de > 0 \), i.e. inflationary devaluation exists in the Stackelberg duopoly model with only home firms.

The results we derived in this section have confirmed:

(1) Regardless the different strategies, the Cournot-Nash oligopoly model provides one set of the common conditions
for the expansionary, neutral or contractionary devaluation in the different market structures.

(2) On one hand, this set of conditions can be viewed as the 'augmented' conditions of those in the monopoly model; on the other hand, the conditions derived from the monopoly model can be viewed as a special case of the conditions in a general model. In short, the different strategies only affect the "augmenting" term in the general condition.

3.3 SUMMARY AND IMPLICATIONS

Now we are closing Chapter 3. So far we have discussed the different market structures with homogeneous goods and algebraically derived the conditions for expansionary, neutral or contractionary devaluation. Under both perfect competition and imperfect competition, we have seen that contractionary devaluation is not an unusual phenomenon. Instead, it occurs whenever the negative effect of increased domestic costs offsets the positive effect of increasing demand. Although this is well known in the case of perfect competition, our results show it holds under a wide range of market structures.

It is possible and also important to compare the results we derived from the different market structures. Among the results, there are two types: the type involving only the home firms and the type involving both the home firm and the foreign firms. However, the latter shall not be
considered in our comparative studies. The reason is that there is no common basis to compare the two types.

In addition, as we see earlier, the Cournot-Nash strategy and the Stackelberg strategy in the duopoly models do not make any difference in terms of the devaluation conditions, with the exception of foreign leadership in the Stackelberg model, which we do not pursue here. Therefore there is only one set of conditions for all duopoly models to identify expansionary, neutral or contractionary devaluation.

On the other hand, different market structures significantly affect the magnitude of the change in output under the same scale of devaluation. For example, assume there are only home firms in the consumer goods market and derive the following equations from [3.16]:

\[ dQ/de = e^{-2}(b'WL - be'P^Hk) \]
\[ dQ/de = 1/2\cdot e^{-2}(b'WL - be'P^Hk) \]
\[ dQ/de = 1/3\cdot e^{-2}[b'W(L+L') - be'P^H(k+k')] \]

which are for perfect competition, monopoly and duopoly respectively.

\[ ^2 \text{Actually, the Stackelberg model with foreign leadership is relevant to the Canadian case. It is an interesting issue for future research.} \]
These equations can also be expressed as

\[3.46'] \quad \frac{dQ}{de} = Z ,
\[3.47'] \quad \frac{dQ}{de} = \frac{1}{2}Z ,
\[3.48'] \quad \frac{dQ}{de} = \frac{2}{3}Z ,

if we let \( Z = e^{-2(b'WL - b\cdot e^{P'k})} \), and \( L = L' \) and \( k = k' \) for two identical duopoly firms.

The absolute magnitude of the change in output due to a devaluation is determined by both the parameters in the demand function in the world market and the parameters such as input contents, input prices, also the initial level of the exchange rate.

More importantly, the comparative results show that devaluation is most effective in altering output under perfect competition and least effective under monopoly. This is not only theoretically important but also empirically significant. In particular, there is no unique model which we can apply to different market structures either for the purpose of estimating or forecasting the effects of a devaluation on output. However, different market structures have been ignored in the modelling practice in the context of devaluation as far as we can see. It is a serious issue.

In the policy arena, our results suggest that policymakers should be cautious in using devaluation as a policy instrument.
First of all, contractionary devaluation, as we pointed out, is not an unusual phenomenon. Only after they investigate the nature of the demand curve in the world market and the cost structure of the domestic and foreign industries, will policy-makers be able to judge whether devaluation can be used as a useful policy tool to stimulate the domestic economy or whether can bring a perverse outcome to the economy.

Secondly, even though the condition for expansionary devaluation is met, policy-makers may often overestimate the power of the instrument in their hands. As we have shown, the existence of imperfect competition can largely reduce the effects of devaluation on output, compared to the effect in perfect competition. For instance, in a monopoly economy, devaluation has just half the power it has in an economy with perfect competition.

Thirdly, devaluation is not an inexhaustible resort. This can be seen from the factor $e^{-2}$ in equation \[3.46\] -- [3.48], which shows a quadratically diminishing effect of devaluation on output as $e$ value increases. Even if devaluation is used for the first time as a successful policy action, further uses of the policy will have progressively smaller effects.

Finally, devaluation is inflationary. Even though devaluation has positive effect on output, there is still a trade-off between the higher level of output (thus higher
level of employment) and inflation. When devaluation is 
contractionary, stagflation occurs. This is consistent with 
our knowledge in macroeconomic theory. However, in the next 
chapter we shall see that devaluation is not necessarily 
inflationary in the case of differentiated goods.
4.0 INTRODUCTION

The conventional view involves the belief that a nominal devaluation has no effects on real output in the long run. This is questionable in the case of non-homogeneous goods. In this chapter, we build a model with the features of monopolistic competition and free entry/exit, and demonstrate the possibilities and conditions for the expansionary, neutral and contractionary effects of a devaluation on real output.

Monopolistic competition is a special market structure which we encounter frequently in our economy. The major characteristic of monopolistic competition is that all the firms in the market produce similar but heterogeneous products, i.e. differentiated products. We refer to the types of differentiated products as varieties. This class of models represents an intermediate case and partially fills the gap between perfect competition and monopoly: on one hand, each individual producer has monopoly power in the market for his own variety of products; and on the other hand, the whole
market leaves enough room for the firms to compete since all the varieties of products are imperfect substitutes. Also, firms are allowed to enter or exit. In this chapter, we try to form such a framework and to examine the effect of devaluation on real output.

First, to avoid any possible confusions, a number of assumptions are clarified and two specifications are justified in Section 4.1. One specification is that the inverse demand function for an individual variety i is the function of its own output, aggregate price and the foreign exchange rate. Another specification is the consumer goods market clearing condition, in which dual price-quantity indices are justified in order to examine aggregate price and output in an economy with differentiated products.

In Section 4.2, we lay out the model and derive the results from the model. Our model is a partial equilibrium model. To concentrate on the goods market, we neglect the labour market and the foreign exchange market. Thus we assume national income is fixed and do not consider the trade balance. We show that the possibilities of expansionary and contractionary devaluation exist in the model. Also, the directions of changes in aggregate price and output following a devaluation show some unusual and interesting outcomes. For example, an expansion in aggregate output and a fall in aggregate price may coexist.
Section 4.3 presents a special version of our general model by assuming symmetry of firms and products. This allows us to examine the effects of devaluation on the number of varieties and the output of each firm. It also allows us to formulate alternative price and quantity indices which are closer in spirit to those that would be used by statistical agencies. This special version reveals the mechanism of the effect of a devaluation on real output: devaluation causes a change in the number of the varieties and thus altering market structure. This causes the effect on real output. It is also interesting to note that devaluation may affect the alternative price and output indices differently.

Finally, a summary of major findings is presented in Section 4.4.

4.1 THE ASSUMPTIONS, MODEL AND SPECIFICATIONS

We assume that there are two countries in the world: home country and foreign country. There is one industry in each country. The home industry produces consumer goods to sell in both domestic and foreign markets. The foreign industry does not produce consumer goods but produces intermediate goods. To produce consumer goods, the home industry imports intermediate inputs from foreign country, which are produced by the foreign industry. Consumer goods are differentiated products and are described by varieties of
products. The intermediate goods are homogeneous products. Thus the home industry is characterized by monopolistic competition. The foreign industry is under the assumption of perfect competition. Furthermore, we assume that there are \( N \) firms in the home industry and each firm produces a distinct variety. Therefore \( N \) varieties of consumer goods are available in the world market. Of course, all varieties of differentiated products are imperfect substitutes.

Consumers in both home and foreign countries consume \( N \) varieties of goods and they have the same tastes. There are \( m \) identical consumers in the home country and \( n \) identical consumers in the foreign country. To each consumer, the elasticity of substitution between any varieties is a constant, \( \sigma \). This specification is essentially that of Spence (1976) and Dixit-Stiglitz (1977), hereafter S-D-S.

There are no trade barriers between two countries. We define \( e \) as the price of the foreign exchange in domestic currency. A fixed exchange rate system is assumed.

Our model consists of three sets of equations:

\[ 4.1 \quad p_i(X_i, P, e) \cdot X_i - c_i(e) \cdot X_i - F = 0 \]

\[ 4.2 \quad p_i(X_i, P, e) + X_i \cdot p_{ix_i}(X_i, P, e) - c_i(e) = 0 \]
Equation [4.1] is the long-run zero-profit condition for a typical home firm i, which captures the possibilities of entry and exit. $c_i(e)$ is the unit variable cost function, which has imported content. $F$ is the fixed cost.

Equation [4.2] is the profit maximization condition, i.e. the first order condition (FOC) of the long-run profit function.

Equation [4.3] is the goods market clearing condition. $E$ is the domestic expenditure and $eE^*$ is the foreign expenditure. $P$ and $Q$ are aggregate price index and aggregate quantity index to be defined below.

The endogenous variables are $X_i$, $P$ and $Q$.

Two important specifications in our model need to be justified. They are the inverse demand function $P_i(X_i, P, e)$ and dual aggregate price-quantity indices $P$ and $Q$.

**THE INVERSE DEMAND FUNCTION:**

The inverse demand for an individual variety of consumer goods, $p_i$, is a function of the aggregate demand for that variety, $X_i$, the aggregated price index $P$, and the foreign exchange rate, $e$, i.e. $p_i = p_i(X_i, P, e)$. This may be justified as follows.
A typical domestic consumer maximizes his utility function featuring S-D-S varieties:

\[ U = \left( \frac{\sum_{i=1}^{N} x_i^\alpha}{N} \right)^{\frac{1}{\alpha}} \]  

subject to his budget constraint

\[ \sum_{i=1}^{N} (p_i x_i) = I \]

where

- \( x_i \) = consumption of variety \( i \), \( i = 1, 2, ..., N \),
- \( p_i \) is the price of variety \( x_i \),
- \( I \) is the individual's expenditure,
- \( \alpha = (\sigma-1)/\sigma \), \( \sigma \) is the constant elasticity of substitution among varieties and \( \sigma > 1 \).

The constrained maximization gives the domestic demand function for variety \( i \) (see Appendix I):

\[ x_i = \frac{I}{p_i^\alpha \sum_{i=1}^{N} p_i^{1-\alpha}} \]
Similarly, a typical foreign consumer maximizes his utility function:

\[ U^* = \left( \sum_{i=1}^{N} (x_i^*)^a \right)^{\frac{1}{a}} \]  

subject to his budget constraint

\[ \sum_{i=1}^{N} (p_i^* x_i^*) = eI^* \]  

under the law of one price

\[ p_i = e p_i^* \]

We derive the foreign consumer's demand function for variety i:

\[ x_i^* = \frac{eI^*}{\sum_{i=1}^{N} p_i^* \sum_{i=1}^{N} p_i^{1-a}} \]

The aggregate demand for variety i is

\[ X_i = mx_i + nx_i^* \]

which is
where $E = mI$ is national expenditure for the home country and $eE' = nE'$ is national expenditure for the foreign country in domestic currency. In this partial equilibrium model, we assume both domestic expenditure $E$ and foreign expenditure $E'$ are fixed. Later on in Chapter 5, we will allow expenditure to be a variable in a general equilibrium framework.

Define the aggregate price index

\[ P = \left[ \sum_{i=1}^{N} \left( \frac{1}{\sigma} \right) \right]^{\frac{1}{1-\sigma}} \]

The aggregate price index not only captures the feature of differentiated goods but also describes the feature of homogeneous goods as a special case of differentiated goods. Assuming symmetry of firms and hence equality of all equilibrium prices $p_i$s, we have

\[ P = N^{\frac{1}{1-\sigma}} p_i \]
Moreover, if we have homogeneous goods, i.e. \( \sigma = \infty \),
then [4.13] becomes:
\[
P = p_i
\]
This is true in the case of homogeneous goods.

[4.12] can be written as:

\[
[4.14] \quad p^{1-\sigma} = \sum_{i=1}^{N} p_i^{1-\sigma}
\]

Substituting [4.14] into [4.11], we have

\[
[4.15] \quad X_i = \frac{E + eE^*}{p_i^{\sigma} p^{1-\sigma}}
\]

Thus the inverse demand function for variety \( i \) is

\[
[4.16] \quad p_i = \left( \frac{E + eE^*}{X_i p^{1-\sigma}} \right)^{\frac{1}{\sigma}}
\]

or simply in a general form:

\[
[4.17] \quad p_i = p_i(X_i, P, e)
\]

For convenience, dropping the subscript \( i \), we write [4.16] and [4.17] as
Hence we justified that the inverse demand function used in equations [4.1] - [4.3] is a function of the aggregate demand for this variety X, the aggregate price index P and the foreign exchange rate e.

Furthermore, we can derive the following first and second partial derivatives by differentiating [4.16'] with respect to X, P and e:

\[ p_X = \frac{p}{\sigma X} < 0 \]

\[ p_P = \frac{p(\sigma-1)}{\sigma P} > 0 \]

\[ p_e = \frac{p^{1-\sigma}E^*}{\sigma XP^{1-\sigma}} > 0 \]

\[ p_{XX} = \frac{p(\sigma+1)}{\sigma^2 X^2} > 0 \]

\[ p_{XP} = -\frac{p(\sigma-1)}{\sigma^2 XP} < 0 \]
These partial derivatives will be used later on to illustrate our general results in such a way that we are able to see the intuition behind the general results.

THE AGGREGATE PRICE AND QUANTITY INDICES:

The goods market clearing condition [4.3] states that the product of aggregate dual price and quantity indices $P$ and $Q$, $PQ$, is equal to the total expenditures in two countries. Thus we assume such indices exist. In this section we derive precise forms for these indices.

According to Leontief's theorem (1947) (see Green 1964, p.12), a necessary and sufficient condition for the grouping of variables is that the marginal rate of substitution between any two variables in a group is a function only of the variables in that group. In our utility function with a group of elementary commodities $X_i$, the condition for existence of an aggregate quantity index is that the marginal substitution between any two commodities is a function only of those commodities in the group. That this condition holds can be justified as follows:

Utility function \[ U = (\Sigma x_i^e)^{1/\varepsilon}, \ i = 1,2, \ldots N \]

Take any $k, r$ for $i$ ($k$ is not equal to $r$), we have

\[ p_{xe} = \frac{\varepsilon E^*-p^1}{\sigma^2 x^2 p^{1-\varepsilon}} < 0 \]
\[ \frac{\partial U}{\partial x_k} = \left( \frac{x_k}{x_r} \right)^{\alpha - 1} \]

which is a function of \( x_k \) and \( x_r \). This shows that the marginal utility of substitution between elementary commodity \( k \) and \( r \) is a function only of the two elementary commodities \( x_k \) and \( x_r \). Moreover, \( k \) and \( r \) are arbitrarily chosen for \( i \), therefore by Leontief's theorem an aggregate quality index for the group of elementary commodities \( x_i \) must exist.

The above theorem also allows us to write the maximum utility (indirect utility function) as a function of the total expenditure and the aggregate price index in our simple one group case (Green, p.20). This is shown by the following manipulations:

Substituting demand function \([4.6]\) into the utility function \([4.1]\), we have the indirect utility function:

\[
V = \left[ \sum_{i=1}^{N} \left( \frac{1}{p^i} \sum_{i=1}^{N} \left( \frac{1}{p^i} \right)^{\frac{1-\sigma}{\sigma}} \right) \right]^{\frac{\sigma}{\sigma-1}}
\]
which can be simplified as

\[
V = \frac{I}{\left(\sum_{i=1}^{N} p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}
\]

But from [4.12] we know

\[
P = \left(\sum_{i=1}^{N} p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}
\]

Therefore we have

\[
V = I/P, \quad \text{or}
I = PV = PU
\]

If there exists a quantity index such that \( I = PQ \), we must have:

\[
Q = U - \left(\sum_{i=1}^{N} X_i^{\frac{1}{\sigma}} \right) - \left(\sum_{i=1}^{N} \frac{X_i^{\sigma-1}}{\sigma} \right)^{\frac{1}{1-\sigma}}
\]

Moreover, if quantity index \( Q \) is a function homogeneous of degree one in its elementary commodities \( X_i \) (Green, p.25), then we have

\[
I = PQ = \sum_{i=1}^{N} p_i X_i
\]
A function is said to be homogeneous of degree one, if multiplication of each of its independent variables by a constant $k$ will alter the value of the function by the proportion $k$. It is easy to show

$$Q(kX_i) = kQ(X_i).$$

$$Q(kX_i) - \left( \sum_{i=1}^{N} \left( kX_i \right)^{\frac{\sigma-1}{\sigma-\alpha}} \right)^{\frac{1}{1-\sigma}}$$

$$- k \left( \sum_{i=1}^{N} X_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{1-\sigma}} = kQ(X_i)$$

Thus quantity index $Q$ is homogeneous of degree one is $X_i$ and $I = PQ$ holds.

In the integrated two markets, the total expenditure is $E + eE'$, we therefore have the goods market clearing condition:

$$E + eE' = PQ,$$

which is the second specification in our model.

From the derivations of dual price-quantity indices, we see that $P$ and $Q$ are aggregate price index and aggregate
quantity index respectively in a model with differentiated goods. Both are adjusted for the number of varieties offered. They represent the aggregate price level and aggregate output level in a differentiated goods market. Q also represents the utility in our model and thus can also be viewed as a welfare index. Accordingly, the reciprocal of the aggregate price, $1/P$, can be viewed as the marginal utility of income.

However, price and output indices reported by statistical agencies usually do not account for changes in the number of varieties. Later on, in Section 4.3, we introduce approximate price index and approximate quantity index, which also do not account for the number of varieties and thus are closer to the indices reported by statistical agencies.

4.2 THE RESULTS OF THE GENERAL MODEL

Based on the assumptions and specifications in Section 4.1, we have the following general model:

\[ p(X, P, e)\cdot X - c(e)\cdot X - F = 0 \]
\[ p(X, P, e) + X\cdot p_{x}(X, P, e) - c(e) = 0 \]
\[ E + eE' = PQ \]

Model [4.24] - [4.26] are virtually the same as [4.1] - [4.3]. To avoid the double and triple subscripts in the first and second derivatives, we dropped subscript $i$ in $p_i$ and $X_i$ in [4.1] - [4.3], i.e. $X$ is the output of variety $i$, or
firm output index; p is the price of variety i, or firm price index; and c is the unit cost of variety i.

Equation [4.24] is the long-run zero-profit condition for a typical home firm i, \(c(e)\) is the unit variable cost function, which has imported content. \(F\) is its fixed cost.

Equation [4.25] is the profit-maximization condition, i.e. the first order condition (FOC) of the long-run profit function. And equation [4.26] is the goods market clearing condition.

The endogenous variables are \(X, P\) and \(Q\).


Totally differentiating equation [4.24] and [4.25] with respect to \(X, P\) and \(e\), we have the following matrix form:

\[
\begin{bmatrix}
0 & XP_p \\
2p_pXP_{XX} & p_fp_pXP_{XX}
\end{bmatrix}
\begin{bmatrix}
\frac{dX}{de} \\
\frac{dP}{de}
\end{bmatrix}
= \begin{bmatrix}
X(c_e - p_e) \\
(c_e - p_e - XP_{XX})
\end{bmatrix}
\frac{de}{de}
\]

Applying Cramer's rule, we have:

\[
[4.27] \quad \frac{dX}{de} = \frac{X[(c_e - p_e)P_{XP} + P_PP_{XX}]}{-p_p(2p_pXP_{XX})}
\]
where

\[ 2p_x + xp_{xx} < 0 \]

is the second order condition.

Totally differentiating equation [4.26] with respect to \( P, Q \) and \( e \), we have

\[ \frac{dQ}{de} = \frac{1}{P} \left( E^* - Q \frac{dP}{de} \right) \]

Our interest is to know what would happen to the output of an individual firm, the aggregate price index and the aggregate quantity index following a devaluation. Hence we derive the following conditions

\[ \frac{dX}{de} > 0 \quad \text{iff} \quad \frac{P_e - C_e}{P_{xe}} \leq \frac{P_p}{P_{xp}} \]

\[ \frac{dP}{de} < 0 \quad \text{iff} \quad P_e - C_e > 0 \]
In the next three segments, we discuss these three conditions further in order to see the intuition behind and provide sensible interpretations.

**DISCUSSION OF CONDITION [4.30]:**

\[
\frac{dQ}{de} \geq 0 \quad \text{iff} \quad \frac{dP}{de} \leq \frac{E^*}{Q}
\]

\[
\frac{dX}{de} \geq 0 \quad \text{iff} \quad \frac{P_e - C_e}{P_{Xe}} \leq \frac{P_p}{P_{XP}}
\]

In the LHS of [4.30], \( P_e - C_e \) is the change in the unit profit and \( P_{Xe} \) is the change in the slope of the inverse demand curve following a devaluation. In the RHS of [4.30], \( P_p \) is the change in the price of the individual variety and \( P_{XP} \) is the change in the slope of the inverse demand curve due to the change in the aggregate price index.

Therefore a devaluation may leave an individual firm's output expand, stay unchanged or contract. The result depends on whether the ratio of the change in the unit profit to the change in the slope of the inverse demand curve directly following a devaluation is greater than, equal to, or smaller than the ratio of the change in the price of its variety to the change in the slope of the inverse demand curve due to the
change in the aggregate price index following a devaluation.

To comprehend the mechanism of devaluation, let us assume there is no imported content, i.e. $c_e = 0$. Then condition [4.30] becomes:

$$\left[4.30'\right] \quad \frac{dX}{de} > 0 \text{ iff } \frac{P_e}{P_{Xe}} < \frac{P_p}{P_{XP}}$$

On the LHS of condition [4.30'], both $p_e$ and $p_{Xe}$ capture the **direct effects** of devaluation on the inverse demand curve of variety $i$, which are from the change in income following a devaluation. $p_e$ measures how much the inverse demand curve would shift and $p_{Xe}$ measures how much the inverse demand curve would rotate in $p$-$X$ dimension.

On the RHS of condition [4.30'], both $p_p$ and $p_{XP}$ capture the **indirect effects** of devaluation on the inverse demand curve of variety $i$, which are from the change in the aggregate price index following the devaluation. $p_p$ measures how much the inverse demand curve would shift and $p_{XP}$ measures how much the inverse demand curve would rotate in $p$-$X$ dimension from the entry of exit of firms. Later on in the special version of our model we shall show that the change in the aggregate price index is due to the change in the number of varieties or firms' entry or exit.

Alternatively, condition [4.30'] can be written as:
Both the LHS and the RHS of \([4.30''']\) are in terms of absolute value. Thus condition \([4.30''']\) simply says that a devaluation leaves an individual firm's output expansionary, unchanged or contractionary if and only if the direct effects of a devaluation are greater than, equal to or smaller than the indirect effects of a devaluation. To see the intuition, a geometric demonstration is provided in Section 4.3.

Under the assumption that there is no imported content in the domestic product, although the effect of devaluation on output of each variety is indeterminate in general, devaluation can be shown to have no effect in the specific case of an S-D-S utility function form \([4.4]\). Our constrained maximization of utility function \([4.4]\) gives the specific results of the partial derivatives for \(p_e, p_v, p_{xp}\) and \(p_{xs}\) in \([4.19] - [4.23]\). Substituting these results into condition \([4.30''']\), we have an equality sign. Therefore \(dX/de = 0\), i.e. a devaluation will not affect the output of variety \(i\). If the total number of firms remains unchanged, the total output would not be affected by a devaluation, i.e. we have the result of neutrality. This seems to confirm the traditional view that in the long run an exchange rate shock can have no effects on real output if there is no imported content in the domestic product (Lizondo and Montiel, 1988, p.2). However,
the conclusion of neutrality ignores the change in market structure following an exchange shock. In our case, the change in market structure is the change in the number of varieties of goods. Therefore, it is too early for us to conclude the neutrality only based on the effects of devaluation on the output of an individual variety of goods. Later on we shall see that a devaluation in general does affect the total output through altering the number of firms in the market.

Relaxing the assumption of no intermediate imports adds the change in cost into the direct effect of devaluation in our model, but it does not complicate the model very much. In Section 4.3, we also provide a geometric demonstration to explain the results under the assumption of intermediate input.

DISCUSSION OF CONDITION [4.31]:

\[ [4.31] \quad \frac{dp}{de} \geq 0 \quad \text{iff} \quad p_e - c_e \leq 0 \]

This condition says that the aggregate price index will increase, remain unchanged or decrease if and only if the change in the unit profit of variety \( i \) is negative, zero, or positive following a devaluation. To see the intuition behind, take the case of \( p_e - c_e < 0 \). According to condition
This guarantees \( \frac{dX}{de} < 0 \) since LHS > 0 and RHS < 0. This means that each firm reduces its output. As a consequence of the shrinking supply, the price level naturally goes up. \( p_e \) measures the effect of devaluation from the demand side and \( c_e \) measures the effect of devaluation from the supply side. Whether the change in the unit profit is positive, zero or negative depends on the strength of two forces from both demand and supply sides.

In the case of no intermediate imports (\( c_e = 0 \)), the aggregate price index will fall following a devaluation. This result seems unusual but its intuition can be explained from the long-run zero-profit condition \([4.24]\). The positive effect on the unit profit will drive the existing firms to produce more and attract new firms to enter. At the new long-run equilibrium, the number of firms will increase but the output of each firm will go back to the original level (from the case of \( c_e = 0 \) in the discussion of condition \([4.30]\)). If a devaluation does not affect the firm's output and does not affect the firm's cost structure (no intermediate goods), then the aggregate price index \( P \) in the inverse demand function \( p(X, P, e) \) must come down to satisfy the zero-profit condition at the new equilibrium.

Another approach would also explain this phenomenon. Recall that the reciprocal of the aggregate price index, \( 1/P \), represents the marginal utility of income in our model. A greater number of varieties available in the market offers
consumers more choice and thus a higher marginal utility of income. A higher marginal utility of income 1/P implies a lower aggregate price index P.

DISCUSSION OF CONDITION [4.32]:

\[
[4.32] \quad \frac{dQ}{de} < 0 \quad \text{iff} \quad \frac{dP}{de} \leq \frac{E^*}{Q}
\]

Condition [4.32] can be written as:

\[
[4.32'] \quad \frac{dQ}{de} < 0 \quad \text{iff} \quad \varepsilon_{po} = \frac{d\ln P}{d\ln e} \geq \frac{eE^*}{E + eE^*}
\]

where \( \varepsilon_{po} \) is the elasticity of aggregate price index with respect to foreign exchange rate.

Condition [4.32'] says that the devaluation is expansionary, neutral or contractionary if and only if the ratio of foreign income to world income is greater than, equal to, or less than the elasticity of aggregate price index with respect to foreign exchange rate.

Alternatively, substituting \( dP/de \) in condition [4.32] by using equation [4.28], we have:

\[
[4.33] \quad \frac{dQ}{de} > 0 \quad \text{iff} \quad c_0 < \frac{p_e E^*}{Q}
\]
from which we see: if \( c_e = 0 \), then \( \frac{dQ}{de} > 0 \) for sure. Thus we have shown that an exchange shock has effect on aggregate output. This happens because the market structure (the number of varieties of goods) has been changed following an exchange rate shock. To reveal the mechanism and to see the intuition, we demonstrate explicitly in Section 4.3 that under the assumption of no intermediate imports a devaluation does not affect the output of each variety but does increase the total number of varieties thus increasing aggregate output.

Also, we may substitute the partial derivative \( p_p \) and \( p_e \) derived in [4.19] and [4.20], which we derived from our specific S-D-S utility function form, into condition [4.33]:

\[
\frac{dQ}{de} \geq 0 \quad \text{iff} \quad \frac{eC_e}{p} \leq \frac{eE^*}{E + eE^*}
\]

Condition [4.34] can be verbally expressed as: a devaluation is expansionary, neutral or contractionary if only if the foreign share in world income is greater than, equal to, or less than the proportion of imported content in the price of a variety.\(^1\)

Therefore we have the following observations, ceteris

\(^1\) We assume unit variable cost function \( c = c_0 + ep^nk \), where \( c_0 \) is the domestic cost, and \( p^n \) is the import price and \( k \) is the technology coefficient. \( c_0, p^n \) and \( k \) are constant. Then \( c_e = p^n k \). Setting \( p^n = k = 1 \), we have \( c_e = 1 \) and \( e = ep^n k \) which is the unit cost of imported content. Then \( ec_e \) can be viewed as unit cost of imported content.
paribus:

(1) A smaller foreign share in world income would more likely contribute to contractionary devaluation.

(2) A higher proportion of the imported value in the price of each variety would more likely contribute to contractionary devaluation.

We may want to see what conclusion we can reach if a small open economy is assumed. Under the assumption of small open economy, we consider the RHS of condition [4.34], $eE'/(E+eE') - 1$. However, the LHS of the condition is less than 1 since there is domestic added value in the price. Therefore $dQ/dE > 0$, we have expansionary devaluation for a small open economy.

It is interesting to compare conditions [4.31] and output condition [4.33]. For convenience, we put two conditions together:

$$\frac{dP}{dE} > 0 \text{ iff } c_e > p_e$$

$$\frac{dQ}{dE} > 0 \text{ iff } c_e < p_e + \frac{p_E'}{Q}$$

The RHS of [4.31] is smaller than the RHS of [4.33] since $p_E'/Q > 0$. Therefore it is possible to have the following combinations of the changes in the aggregate price index and the aggregate quantity index:
(1) if \( c_* < p_* \), then \( dP/de < 0 \) and \( dQ/de > 0 \);
(2) if \( c_* = p_* \), then \( dP/de = 0 \) and \( dQ/de > 0 \);
(3) if \( c_* = p_* + p_r E'/Q \), then \( dP/de > 0 \) and \( dQ/de = 0 \);
(4) if \( c_* > p_* + p_r E'/Q \), then \( dP/de > 0 \) and \( dQ/de < 0 \);
(5) if \( p_* < c_* < p_* + p_r E'/Q \), then \( dP/de > 0 \) and \( dQ/de > 0 \).

In the case of \( c_* = 0 \), we have \( dP/de < 0 \) and \( dQ/de > 0 \).

These combinations clearly show the important role the firm's cost structure plays, despite other factors such as the inverse demand function for its output, the foreign income and the initial aggregate output level.

4.3 A SPECIAL VERSION OF THE MODEL

To see the roles of the number of varieties \( N \) and the constant elasticity \( \sigma \), we assume a special case of our general model.

As we mentioned earlier, if we assume symmetry, i.e. all \( p_i 's \) are equal and all \( X_i 's \) are equal, then the aggregate price index becomes

\[
[4.13'] \\
P = N^{1/(1-\sigma)}p
\]

Now it is obvious that the aggregate price index is the function of the number of varieties. Chamberlain (1933) argued that an increase in the number of varieties in an industry shifts down and makes more elastic the demand curve.
for each variety. From equation [4.13'], we see the mechanism Chamberlain argued is through the industry price level (i.e. the aggregate price index $P$) and thus affects the inverse demand curve for each variety. Actually, in a general (asymmetry) model, the inverse demand curve $p(X, P, e)$ implicitly captures the role of the number of varieties through the aggregate price index $P$ which is determined by a number of individual prices.

Equation [4.13'] also shows that the aggregate price index $P$ is not equal to the firm price index $p$ even though we assume symmetry. This is due to the nature of heterogeneity of products.

Accordingly, the aggregate quantity index becomes

$$[4.13''] \quad Q = N^{\alpha \over \alpha-1} X$$

which shows that the aggregate quantity index $Q$ is not a simple arithmetic summation of each individual firm's output indices $NX$. The explanation is quite clear: we can not add up the products in physical units due to the nature of heterogeneity.

If we assume both symmetry and perfect substitution ($\sigma = \infty$), the aggregate price index $P$ will become exactly the same as the individual firm's price index $p$, and the aggregate quantity index will simply be a summation of each individual firm's output index,$NX$. This is the case of homogeneous goods.
From [4.13'] and [4.13''], the value of the aggregate output is

\[ PQ = N^{1-\sigma} P \cdot N^{\sigma-1} X = pNX \]

In this special case our general model can be written as

\[ p(X, N, e) \cdot X - c(e) \cdot X - F = 0 \]

\[ p(X, N, e) + X \cdot p_x(X, N, e) - c(e) = 0 \]

The endogenous variables are \( X \) and \( N \).

From [4.36] and [4.37], we have:

\[ \frac{dX}{de} = \frac{X \left[ (c_e - p_e) P_{XN} + P_{N} P_{X0} \right]}{- P_N (2 P_X + X P_{XX})} \]

\[ \frac{dN}{de} = \frac{c_e - p_e}{P_N} \]

Hence the following conditions:

\[ \frac{dX}{de} \geq 0 \quad \text{iff} \quad \frac{P_e - C_e}{P_{X0}} \leq \frac{P_N}{P_{XN}} \]
Comparing the RHSs of conditions [4.41] and [4.30], it is clear that the change in the aggregate price index $P$ is due to the change in the number of varieties or firms' entry/exit. In condition [4.30], the number of varieties, $N$, is implicitly modeled in the aggregate price index $P$. In condition [4.41], the assumption of symmetry makes it possible for us to model $N$ explicitly and $N$ reveals the mechanism of indirect effect of devaluation on each individual firm's output.

Thus the LHS of condition [4.41] measures the direct effects of devaluation on the inverse demand curve of a variety, which are from the change in income following a devaluation. The RHS of condition [4.41] measures the indirect effects of a devaluation, through free entry/exit, on the inverse demand curve of a variety.

Condition [4.40] is consistent with condition [4.31]. When a devaluation makes a variety profitable, new firms will enter ($N$ increases) and the aggregate price index will be driven down ($P$ decreases).

Furthermore, if we assume there are no intermediate imports, then conditions [4.41] and [4.42] become:

\[
\frac{dN}{de} \geq 0 \quad \text{iff} \quad p_e - c_e \geq 0
\]
\[ [4.41'] \quad \frac{dX}{de} \geq 0 \quad \text{iff} \quad \frac{P_e}{P_{Xe}} \leq \frac{P_N}{P_{XN}} \]

\[ [4.42'] \quad \frac{dN}{de} \geq 0 \quad \text{iff} \quad p_e \geq 0 \]

In the specific case of an S-D-S utility function form, we have the following partial derivatives under the assumption of symmetry:

\[ [4.43] \quad p_x = - \frac{P}{X} < 0 \]

\[ [4.44] \quad p_n = - \frac{P}{N} < 0 \]

\[ [4.45] \quad p_e = \frac{E^*}{XN} > 0 \]

\[ [4.46] \quad p_{xx} = \frac{P}{X^2} > 0 \]

\[ [4.47] \quad p_{xn} = \frac{P}{XN} > 0 \]
By substituting the above partial derivatives into condition \( [4.41'] \) and \( [4.42'] \), we have \( \frac{dX}{de} = 0 \) and \( \frac{dN}{de} > 0 \), i.e. a devaluation does not affect the individual firm's output but attracts new firms to enter the industry. This can be demonstrated in Figure 4.1.

In Figure 4.1, the initial equilibrium point is \( A \). A devaluation shifts and rotates the inverse demand curve from \( p \) to \( p' \). The average cost curve stays since there are no intermediate inputs. An existing firm makes profit and new firms enter. As a result, the inverse demand curve shifts and rotates back to the zero-profit equilibrium point \( A \).

\[ p_{xe} = - \frac{E^*}{X^2 N} < 0 \]
To relax the assumption of no intermediate imports, we use Figure 4.2 to illustrate the situation.

Figure 4.2

4.2a

4.2b
In Figure 4.2, the initial equilibrium point is A. A devaluation shifts and rotates the inverse demand curve from p to p'. The average cost curve shifts up to AC' due to a cost increase in intermediate imports. In Panel 4.2a, an existing firm makes positive profit and new firms enter. As a result, the inverse demand curve shifts and rotates back onto the tangent of AC' with zero-profit. In Panel 4.2b, an existing firm makes loss. As a result, some existing firms exit and the inverse demand curve shifts and rotates onto the tangent of AC' with zero-profit. It is not clear that the individual firm's output will increase, stay unchanged or decrease. The precise quantitative description is condition [4.41].

Our analysis has shown that the assumption of symmetry allows us to explicitly model the number of varieties and thus to reveal the mechanism of indirect effects of devaluation on individual firm's output. Moreover, the assumption of symmetry also allows us to make approximations to examine the effects of devaluation on employment and the price level.

If each variety has the same domestic content (say labour), then NX represents the employment level. NX may also be viewed approximately as the output level, which is not adjusted for different varieties and thus close to that used by statistical agencies. Comparing to the aggregate quantity index Q, NX is an unadjusted quantity index. We may call it as measured output. We derive:
which gives condition

\[ d(NX) \leq 0 \quad \text{iff} \quad e \in E^* \]

\[
(p_e - c_e)(Np_{XN} - 2P_X - Xp_{XX}) - Np_N p_{Xe} \geq 0
\]

From condition [4.42], we know \( p_e - c_e \geq 0 \) is the necessary and sufficient condition for \( dN/\de \geq 0 \). Now in condition [4.50], \( p_e - c_e \geq 0 \) is a necessary but not sufficient condition for \( d(NX)/\de > 0 \).

Furthermore, substituting the partial derivatives [4.43] - [4.48], which we derived from our specific S-D-S utility function form, into condition [4.50], we have:

\[ d(NX) \leq 0 \quad \text{iff} \quad \frac{e c_e}{p} < \frac{1}{2} \frac{e E^*}{E + e E^*} \]

Recall that \( ec_e \) may be interpreted as the unit cost of imported content in condition [4.34]. Therefore condition [4.51] shows that devaluation has an expansionary, neutral or contractionary effect on employment or measured output if and
only if the imported content in domestic production is less than, equal to, or greater than half the foreign share of world income.

If the home country is a very large economy, then employment or measured output contracts since the RHS of condition [4.51] approaches to 0. When there are no intermediate imports, employment or measured output stays unchanged. This is the result called neutrality.

If the home country is a very small economy, then employment or measured output may expand, stay unchanged or contracts. When there are no intermediate imports, employment or measured output expands.

Under the assumptions of symmetry and identical cost structure for each firm, another approximation we may make is \( P = p \), i.e. the aggregate price index can be approximately represented by the firm price index. This is like the price index a statistical agency would use. Therefore the approximation we made has its practical significance. Hence we may also call this approximate aggregate price index as measured price index. We derive

\[
\frac{dp}{de} = \frac{P_X X [ (c_e - P_e) P_{XX} + P_Y P_{Xe} ]}{-P_N (2P_X X + P_{XX})} + c_e
\]
From condition [4.42], we know that \( p_e - c_e \leq 0 \) is the necessary and sufficient condition for \( dN/de \leq 0 \). Now in conditions [4.53], \( p_e - c_e \leq 0 \) is a sufficient but not necessary condition for \( dp/de > 0 \).

Furthermore, substituting the partial derivatives [4.43] - [4.48], which we derived from our specific S-D-S utility function form, into condition [4.53], we have

\[
\begin{align*}
[4.54] \quad \frac{dp}{de} &= 2c_e, \quad \therefore \frac{dp}{de} \geq 0 \quad \text{iff} \quad c_e \geq 0
\end{align*}
\]

Therefore devaluation causes inflation when there are intermediate imports and devaluation does not affect the price level (measured price index) when there are no intermediate imports. This result is consistent with our knowledge of inflationary devaluation in macroeconomic theory, but it seems to contradict the possible result of deflationary devaluation in the previous condition [4.31] in our model. The contradiction comes from the approximation, which fails to adjust the price index for the change in the number of varieties. As we pointed earlier, strictly speaking, \( P = p \) and \( Q = NX \) are only true in the case of homogeneous goods.
Considering conditions [4.51] and [4.54] together, we have the following results:

(1) If the home country is very large, devaluation causes employment or measured output to contract and the measured price level to rise. When there are no intermediate imports, devaluation does not affect employment or measured output and the measured price level.

(2) If the home country is very small, devaluation has an indeterminate effect on employment or measured output, but raises the measured price level. When there are no intermediate imports, devaluation expands employment or measured output but does not cause inflation.

Also, considering condition [4.34] and [4.51] together, and letting Q and NX represent welfare and employment respectively, we find that employment and welfare need not move together.

(1) Devaluation may have expansionary effects on both welfare and employment;

(2) Devaluation may have expansionary effect on welfare but contractionary effect on employment;

(3) Devaluation may have contractionary effect on both welfare and employment.

4.4 SUMMARY OF THE MAJOR FINDINGS

Our model describes an open economy in which monopolistically competitive firms produce differentiated
goods with imported content. We focus on the differentiated goods market by assuming domestic income is fixed and thus provide a partial equilibrium analysis. The major findings are summarized as follows:

(1) Devaluation expands domestic aggregate output if and only if the foreign share of world income exceeds the exchange elasticity of the domestic price. In general, a devaluation may expand, leave unchanged, or contract domestic aggregate output. The conditions for these possibilities are expressed in [4.32']:

\[ [4.32'] \quad \frac{dQ}{de} > 0 \quad \text{iff} \quad \epsilon_{Pe} = \frac{d\ln P}{d\ln e} < \frac{eE^*}{E + eE^*} \]

The LHS is the elasticity of the aggregate price index with respect to the foreign exchange rate. The RHS is the ratio of foreign income to global income. Verbally, the condition says if the ratio of foreign income to global income is greater than, equal to, or smaller than the elasticity of the aggregate price index with respect to the foreign exchange rate, then the devaluation will be expansionary, neutral or contractionary.

(2) When we assume there are no intermediate imports \((c_e = 0)\), devaluation is expansionary. This shows the expansionary effects of a devaluation on real output in a model with the feature of monopolistic competition and free
entry. Thus it challenges the conventional view that a devaluation can have no real effects (Lizondo and Montiel, 1988).

(3) When there are no intermediate imports, devaluation expands aggregate output and reduces aggregate price. By assuming $c_e = 0$ and using the partial derivatives [4.18] - [4.23] derived from a specific S-D-S utility function form [4.4], and the demand function [4.16], we have expansionary effects of a devaluation on real output and a lower aggregate price than the initial equilibrium price following a devaluation. As a result, we see the situation that a higher output and a lower price level coexist. This happens because the increased number of varieties raises the level of competitiveness and thus lowers the aggregate price level in the monopolistic competition market.

(4) The assumption of symmetry allows us to reveal the mechanism of how devaluation affects real output. In the monopolistic competition framework, the change in the number of varieties implies a change in the market structure. Therefore it is the devaluation which alters the market structure and it is the change in the market structure which causes the effects on real output.

(5) The assumption of symmetry also allows us to make approximations to examine the effects of devaluation on employment or measured output and measured price level. Furthermore, by using the partial derivatives [4.43] - [4.48],
which we derived from a specific S-D-S utility function form, our findings show that while employment or measured output may increase, stay unchanged or decrease, the measured price level definitely rises following a devaluation. Our findings also show that devaluation may raise welfare while at the same time have contractionary effect on employment or measured output.

Among our findings, result (3) shows the possibility of deflationary devaluation, where the price level $P$ is the "precise" aggregate price index $P_i$; however, result (5) shows that devaluation is inflationary, where the price level is approximately represented by the price of an individual variety. These two different results tell us that different aggregations (precise vs. approximate) matter.

Aggregation is one of the most difficult issues in macroeconomic theory. If we make comparisons, the aggregate price in macroeconomic theory is more like the approximate price index than the "precise" aggregate price index in our model. As we know, the aggregate price in macroeconomic theory does not consider the number of varieties and neither does the approximate price index in our result (5) here. Perhaps this is why inflationary devaluation in result (5) with the approximate price index is consistent with our knowledge of inflationary devaluation in macroeconomic theory. Moreover, from the viewpoint of statistical agencies, price level is not affected by the number of varieties. Therefore inflationary devaluation in our result (5) should be expected
by statistical agencies, which is usually true.

Among our findings, result (5) shows the possibility that welfare increase and employment decrease may coexist. This is an interesting result generated in the model with monopolistic competition.

The summary effects of devaluation on price and output are as follows:

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<th>General Case</th>
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<td>P</td>
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<td>Q</td>
<td>?</td>
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</tr>
<tr>
<td>P</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>NX</td>
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<td>+</td>
</tr>
</tbody>
</table>

It is important to emphasize that our findings are based on two crucial assumptions: free entry/exit and fixed income in a partial equilibrium model of monopolistic competition. Later on in Chapter 5 we shall see that, in a general equilibrium model of small open economy with monopolistic competition and no free entry or exit, there are no effects of a devaluation on real output. This should not surprise us.

The model in this chapter has the number of varieties as an endogenous variable, which, as we have seen, is the major mechanism which causes effects on real output following a devaluation. But the model in Chapter 5, as we shall see, has the number of the varieties as an exogenous.
The model in this chapter assumes fixed income, which implies money wage rigidity of labour. But in Chapter 5, each agent's income is endogenous and his elasticity of disutility with respect to output is not assumed constant. Thus a flexible wage is implicitly assumed. Whether we assume money wage rigidity or flexibility will obviously make difference in the demand side following a devaluation. Therefore the different results in the two models with monopolistic competition are not contradictory but complementary.
CHAPTER 5
MACROECONOMIC FORMULATION:
CASE OF DIFFERENTIATED GOODS

5.0 INTRODUCTION

In Chapter 4, we assumed fixed income in a partial equilibrium micro model with monopolistic competition. To overcome this shortcoming, we may endogenize income. In addition to the goods market, we may want to incorporate other markets into the model. Therefore there is a need to develop a macro model with monopolistic competition.

It is a new avenue to examine the effects of devaluation on output by formulating macroeconomic models based on microeconomic foundations. In this chapter, we extend the simplified version of the closed-economy model of Blanchard-Kiyotaki (Blanchard and Fisher, 1989) to an open-economy version (hence named as extended Blanchard-Kiyotaki model). An interesting treatment in Blanchard-Kiyotaki model is its macro formulation based on micro foundations. Our model characterizes the case of differentiated goods and is a direct open macro extension of the partial equilibrium model.

We model the home country as a small open economy, which produces N varieties of differentiated goods to supply the world market, and which imports intermediate goods from the rest of the world. N varieties of differentiated goods are not produced in the rest of the world. The N agents in the home country are both producers and consumers. Each of them produces a distinct variety and consumes a bundle of N varieties of goods. N is an exogenous variable in the model. An agent gains utility from enjoying consumption goods and his real money balances; but loses utility from producing goods by sacrificing his leisure. The effects of a devaluation on the price level and output in the home country have been investigated based on the microeconomic assumption that each agent maximizes his utility subject to the budget constraint.

The results show a one to one relationship between the percentage increases in exchange rate and the price level. As a result, there is the usual neutrality property of devaluation on real output. Specifically, for a small open economy, a certain percentage of devaluation in domestic currency would increase the price level by exactly same percentage and would not change the aggregate output, under the assumption that the ratio of the original money holdings in the rest of the world to the global money holdings is equal to one. This confirms the conventional knowledge of
neutrality by modelling the different type of market structures -- monopolistic competition without free entry/exit. This result is consistent with the non-neutrality result derived in the previous chapter by modelling monopolistic competition with free entry/exit.

5.1 EXTENDED BLANCHARD-KIYOTAKI MODEL

We assume there are two parties in the world economy: the home country and the rest of the world. Also, we assume that the home country is a small open economy. On one side, the home country provides consumer goods to both home and foreign markets. Its products are differentiated goods, which we refer to as the varieties of finished goods. In order to produce these finished goods, the home country imports intermediate inputs from the rest of the world. On the other side, the rest of the world imports the different varieties of finished goods for its consumers from and exports intermediate goods to the home country. Therefore trade pattern is essentially an exchange of finished goods for intermediate goods.

In most models of a small open economy, the home country is a price taker. However, in our model the home country is only a price taker for intermediate goods. For finished goods exported to the rest of the world, each home firm has some monopoly power in the market since it is the
only producer of its own variety. Thus it behaves as a monopolistic competitor in the market.

A typical individual in the home country is a double agent: as a consumer, the individual consumes varieties of finished goods, which are imperfect substitutes; as a producer, the individual produces one particular variety of finished goods (variety i) for the markets. We label the individual as consumer-producer i. There are N individual consumers-producers in the home country, i.e. $i = 1, 2, \ldots, N$. Each of them produces one different variety of the finished goods and then the total number of varieties is N. Production in the home country is characterized by monopolistic competition.

The rest of the world (ROW) consumes N varieties of imported goods from the home country and provides a homogeneous intermediate good to the home country. We assume that the ROW consumes its homogeneous intermediate good as well. Moreover, there are other non-traded goods produced and consumed in the ROW (non-traded with home country but traded within the ROW). These assumptions make it possible for us to consider the small open economy as a price taker for intermediate goods. For explanatory convenience, a summary of the equation systems and variables is presented later in the middle of Section 5.2.

The specification of our model is as follows:

A domestic consumer-producer has net utility function
\[ U_i = \left( \frac{C_i}{\alpha} \right)^\alpha \left( \frac{M_i/P}{1-\alpha} \right)^{1-\alpha} - \frac{\mu}{\beta} (Y_i)\beta \]

where \( 1 > \alpha < 0 \), \( \mu > 0 \), and \( \beta \geq 1 \), and the budget constraint is

\[ \sum_{j=1}^{N} P_j C_{ji} + M_i + eP^x X_i = P_i Y_i + M_i = I_i, \]

where \( i = 1, 2, \ldots, N \).

The net utility function \( U_i \) is positively related to the consumption of all varieties of goods \( C_i \) and to the services of real money balance \( M_i/P \), but negatively related to the level of production of goods \( Y_i \).

\( C_i \) is the aggregate consumption of \( N \) varieties of consumer goods and is defined as a CES form with constant elasticity of substitution between any pair of varieties, \( \sigma \):

\[ C_i = N^{1-\sigma} \left( \sum_{j=1}^{N} \frac{\sigma-1}{\sigma} C_{ji}^{\sigma} \right)^{\frac{1}{\sigma-1}} \]

where \( \sigma > 1 \), \( C_{ji} \) is the amount of variety \( j \) consumed by individual \( i \) in the home country.

\( M_i/P \) is individual \( i \)'s demand for the real money balance and the price level \( P \) is defined as
where $P_j$ is the price of variety $j$.

\[ P = \left( \frac{1}{N} \sum_{j=1}^{N} P_j^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \]  

[5.4]

is the technology equation. $X_i$ represents intermediate goods used by producer $i$, $Y_i$ finished goods produced by producer $i$ and $k$ technology coefficient.

In the net utility function [5.1], $a$ is the share of individual $i$'s budget spent on consumption goods and $1 - a$ is the share spent on money holdings. One may ask why an individual demands real money balances? According to Blanchard and Kiyotaki, a full treatment of the role of money would require a dynamic model, such as Rotemberg's (1987) model with a cash-in-advance constraint. Blanchard and Kiyotaki did not want to elaborate this issue in their paper. As an extension of the Blanchard-Kiyotaki model, we choose not to pursue it either. However, the role of money probably can be simply viewed as for transaction purpose and for consumption of other goods (not modeled) other than the $N$ varieties specified in our model.

The second term of the RHS in equation [5.1] represents disutility from production activity. This is because production of $Y_i$ requires individual $i$'s labour input which reduces his leisure. It is useful to explore parameters
β and μ in order to understand their economic meanings. Using $U_i^d$ to represent individual i's disutility:

$$U_i^d = \frac{1}{\beta} (Y_i^s)^\beta$$

It is easy to show that

$$\frac{\partial \ln U_i^d}{\partial \ln Y_i^s} = \beta,$$

which reveals that β is the elasticity of disutility with respect to output. Alternatively, we have

$$\frac{\partial \ln \frac{d U_i^d}{d Y_i^s}}{\partial \ln Y_i^s} = 1 - \beta,$$

which reveals that $1 - \beta$ is the elasticity of marginal disutility with respect to output. When $\beta = 1$, or $1 - \beta = 0$, we have constant marginal disutility μ. Moreover, if we consider that output is a function of labour, L, i.e.

$$Y_i^s = Y_i^s (L),$$

then the constant marginal disutility μ can be expressed as

$$\mu = \frac{d U_i^d / d L}{d Y_i^s / d L}$$

which is the ratio of marginal disutility of labour to the
marginal product of labour. The marginal disutility of labour captures individual's preference between consumption and leisure, and the marginal product of labour captures individual's productivity.

Evidently, $\mu > 0$ since disutility is the negative part of the net utility function. Besides, we must have $\beta > 1$. $\beta > 1$ reflects the normal situation of labour supply, and $\beta = 1$ implies that, at the margin, labour supply is inexhaustible. $\beta < 1$ would imply that the more labour is supplied, the less utility the individual loses, which is an absurd assumption of the labour market.

In the budget constraint equation [5.2], $x_i$ is the units of the intermediate good imported by individual $i$ in order to produce $Y_i$ units of finished goods. $P_k$ is the unit price of intermediate good in terms of foreign currency and is assumed to be constant. $e$ is the price of the foreign exchange in domestic currency. $M_i$ is the money demand and $\overline{M}_i$ is the initial money holdings by individual $i$ in nominal term.

Equation [5.5] describes the constant returns to scale of intermediate good in the production. It says that in order to produce one unit of finished goods, $k$ units of intermediate goods are required. In other words, parameter $k$ is the imported content per unit of output.
5.2 SOLVING PROCEDURES AND RESULTS

To solve the model and derive the conditions for contractionary devaluation, we take five steps:

First, we derive consumer-producer $i$'s demand function for variety $j$, $C_{j,i}$, and consumer-producer $i$'s demand function for money, $M_i$. This is done by solving the constrained optimization of individual $i$'s net utility function. In the meantime, we simply assume the demand function for variety $j$ in the ROW.

Second, we derive the demand function for variety $i$ in the world economy, or the demand function facing producer $i$, $Y_i$. This is done by summing up the domestic and foreign demand for variety $i$.

Third, we derive the indirect net utility function of consumer-producer $i$ by substituting its demand functions for variety $j$s, $C_{j,i}$s, and its demand function for money, $M_i$, into the utility part of his net utility function, and by substituting the demand function facing producer $i$, $Y_i$, into the disutility part of his net utility function.

Fourth, we let consumer-producer $i$ choose the optimal price he can charge by maximizing his indirect net utility function with respect to the relative price $P_i/P$.

Finally, we examine the effect of devaluation on the domestic price level and aggregate output by assuming general equilibrium and symmetry in prices for different varieties.
STEP I:

The constrained optimization problem of consumer-producer i is:

\[
\text{MAX } U_i = \left( \frac{C_i}{\alpha} \right)^{\alpha} \left( \frac{M_i}{P} \right)^{1-\alpha} - \frac{\mu}{\beta} (Y_i^s)^\beta
\]

with respect to \( C_{ji} \) and \( M_i \),

\[
\text{S.T. } \sum_{j=1}^{N} P_j C_{ji} + M_i + eP^x X_i = P_i Y_i^s + \bar{M}_i = I_i,
\]

where

\[
C_i = N^{\frac{1}{\alpha}} \left( \sum_{j=1}^{N} C_{ji}^{\frac{\alpha-1}{\alpha-1}} \right)^{-\frac{1}{\alpha-1}}
\]

\[
P = \left( \frac{1}{n} \sum_{j=1}^{N} P_j^{1-\alpha} \right)^{\frac{1}{1-\alpha}}
\]

\[
X_i = kY_i
\]

Forming the Lagrange function and differentiating it with respect to \( C_{ji} \) and \( M_i \), we derive individual i's demand function for variety j, \( C_{ji} \), and his demand function for money, \( M_i \) (see Appendix II):
Since our model is a small open economy, we simply assume the demand function for variety $j$ in the ROW is:

$$\text{(5.8)} \quad C_{j}^* = \left( \frac{P_j}{P} \right)^{-\sigma} \frac{\alpha(I_i - e^{PX_i})}{NP}$$

where we assume the same constant elasticity of substitution between any pair of varieties, $\alpha$, and

$$\text{(5.9)} \quad I' = e^{P_X} + e^{M'}$$

In equation [5.9], $e^{P_X}$ is the export income of the ROW and $e^{M'}$ is the original money holdings in the ROW. Both are in domestic currency.

**STEP II:**

The total demand for variety $j$ is the sum of the domestic and foreign demand

$$\text{(5.10)} \quad Y_j = \sum_{i=1}^{N} C_{ji} + C_{j}^*$$
Substituting [5.6], [5.8] into [5.10], we have

\[ [5.11] \quad Y_j = \left( \frac{P_j}{P} \right)^{-\sigma} \left[ \frac{\alpha (I - eP^X) + \alpha eI^*}{NP} \right] \]

where \( I = \sum_{i=1}^{N} I_i \), \( X = \sum_{i=1}^{N} X_i \).

Substituting [5.9] into [5.11], we have

\[ [5.12] \quad Y_j = \left( \frac{P_j}{P} \right)^{-\sigma} \frac{\alpha (PY + \bar{M} + e\bar{M}^*)}{NP} \]

where \( PY = \sum_{j=1}^{N} P_j Y_j \),

\( \bar{M} = \sum_{i=1}^{N} \bar{M}_i \).

But the aggregate demand can be written as

\[ [5.12'] \quad Y = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{P_j C_{ji}}{P} + \sum_{j=1}^{N} \frac{P_j C_j^*}{P} \]

Substituting [5.6] and [5.8] into [5.12'], we have

\[ Y = \alpha (Y + \bar{M}/P - eP^X/P) + \alpha (eP^X/P + e\bar{M}^*/P) , \]

which is
\[ Y = \alpha \left( Y + \frac{\bar{M} + e\bar{M}^*}{P} \right) = \alpha \left( Y + \frac{\bar{M}}{P} \right) \]

or

\[ Y = \frac{\alpha \bar{M}}{(1-\alpha)P} \]

where \( \bar{M} \) is the original global money holdings, i.e.

\[ \bar{M} = \bar{M} + e\bar{M}^* . \]

[5.13] is the aggregate demand function.

Substituting [5.13] into [5.12], we have

\[ [5.14] \quad Y_i = \left( \frac{P_i}{P} \right)^{-\sigma} \frac{\hat{M}}{NP} \]

where

\[ \hat{M} = \alpha \bar{M} = \frac{\alpha}{1-\alpha} (\bar{M} + e\bar{M}^*) \]

Equation [5.14] is the demand function facing producer \( i \), which is the function of the relative price of its output and the global money holdings.

**STEP III:**

To derive consumer-producer \( i \)'s indirect net utility function, we substitute demand functions [5.6], [5.7] and
[5.14] into his net utility function [5.1]. After some manipulations, individual i's indirect net utility function can be written as a function of the relative prices of all varieties of consumption goods:

\[ \hat{U}_i = N^{1-\sigma} P^{-1} \left[ (P_i - eP^xk) \left( \frac{P_i}{P} \right)^{-\sigma} \frac{\bar{M}}{NP} + \sum_{j=1}^{N} \left( \frac{P_j}{P} \right)^{1-\sigma} \right]^{\frac{\sigma\alpha}{\sigma-1}} \]

- \left( \frac{\mu}{\beta} \right) \left( \frac{P_i}{P} \right)^{-\sigma\beta} \left( \frac{\bar{M}}{NP} \right)^{\beta} \]

Alternatively, only substituting [5.6] and [5.7] into [5.1], individual i's indirect net utility function can be written as a function of his own output \( Y_i \). But as a monopoly firm in his variety, producer i's output is subject to the demand function facing him, which is equation [5.14]. Therefore,

\[ \hat{U}_i = N^{1-\sigma} P^{-1} \left[ (P_i Y_i - eP^xkY_i + \bar{M}_i) \sum_{j=1}^{N} \left( \frac{P_j}{P} \right)^{1-\sigma} \right]^{\frac{\sigma\alpha}{\sigma-1}} \]
STEP IV:

Consumer-producer i's optimization problem now becomes:

Maximize [5.15] with respect to $P_i/P$, or

Maximize [5.16] with respect to $P_i/P$ and $Y_i$.

These two problems are equivalent.

Taking the first derivative of [5.15] with respect to $P_i/P$ and setting it to zero, we have the first order condition of equation [5.15]:

\[
\frac{\partial \hat{U}_i}{\partial \left( \frac{P_i}{P} \right)} = -\frac{\alpha \sigma}{N^{1-\sigma} P^{-1}} \left[ -\sigma \left( \frac{P_i}{P} \right)^{-\sigma-1} \left( P_i - eP^x_k \right) \frac{N}{NP} \right. \\
\left. + \left( \frac{P_i}{P} \right)^{-\sigma} \frac{\hat{M}}{N} \right] \sum_{i=1}^{N} \left( \frac{P_i}{P} \right)^{\frac{\alpha \sigma}{\sigma-1}} + \frac{\alpha \sigma}{N^{1-\sigma} P^{-1}}
\]
In deriving [5.17], we have assumed that \( N \) is large enough that producer \( i \) takes the price level \( P \) and the prices of other varieties \( P_j \) \((j \neq i)\) as given, when choosing his price. Equation [5.17] shows that the optimal price \( P_i \) is a function of the price level \( P \), the prices of other varieties \( P_j \) and other exogenous variables.

Up to this step, let us summarize the markets and the equation systems.

The finished goods market clears. This is guaranteed by equations [5.15], in which the supply function of each individual producer is replaced by the total demand for his variety.

The intermediate good market clears by assuming [5.8] and [5.9].

The budget constraints of each consumer-producer,
equations [5.2], imply that the money market clears if trade is balanced. This can be seen by doing a little manipulations:

Taking the summation of budget constraint equations [5.2] with respect to \( i \), we have the budget constraint for the whole domestic economy:

\[
\sum_{i=1}^{N} \sum_{j=1}^{N} P_{ij}C_{ij} + M + eP^X = \sum_{i=1}^{N} P_{i}Y_{i} + \bar{M}
\]

which can be rearranged as

\[
\bar{M} - M = \sum_{i=1}^{N} P_{i}Y_{i} - \sum_{i=1}^{N} \sum_{j=1}^{N} P_{ij}C_{ij} - eP^X
\]

or

\[
[5.2'] \quad \bar{M} - M = \sum_{j=1}^{N} P_{j}C_{j} - eP^X
\]

The LHS of [5.2'] is the net money balance and the RHS is the trade balance for the whole home economy. Under a fixed exchange rate regime, [5.2'] guarantees that the money market clears if the balance of trade is zero. A policy of sterilization is assumed in order to insulate the domestic money supply \( \bar{M} \).

To see the total number of equations and the total number of variables in our system, we give the following table:
The total number of equations is 5N+1 and the total number of variables is also 5N+1. So the system is consistent.

STEP V:

Our interest is to examine the effect of devaluation on the whole domestic economy: the price level and real output. Thus, assuming symmetry and general equilibrium, i.e. $P_i / P = 1$ for all i's, we have equation [5.17] written as:

\[ [5.18] \quad \ddot{M}P(\alpha - \sigma - \sigma N + N + \sigma \mu \dot{M}^{\alpha - 1}N^{2 - \beta}P^{1 - \beta}) = \alpha \sigma \ddot{M}P - \sigma eP^x \ddot{M}(\alpha + N) \]

Differentiating equation [5.18] with respect to P and e, we have

\[ [5.19] \quad \ddot{M}P[\sigma \mu \dot{M}^{\alpha - 1}N^{2 - \beta}(1 - \beta)P^{-\beta}dP + \sigma \mu (\beta - 1)\dot{M}^{\alpha - 2}N^{2 - \beta}P^{1 - \beta} \ddot{M}^x e] + \dot{M}(\alpha - \sigma - \sigma N + N + \sigma \mu \dot{M}^{\alpha - 1}N^{2 - \beta}P^{1 - \beta})dP \]
Thus

\[ \frac{dP}{de} = \frac{\Gamma_1}{\Gamma_2} \]

where

\[ \Gamma_1 = - \sigma eP^\alpha k\alpha' M^\beta (\alpha + N) - \sigma \mu M^\delta N^2 \beta \]

\[ - \sigma \mu (\beta - 1) M^\delta N^2 \beta \]

\[ - \sigma \mu (\beta - 1) M^\delta N^2 \beta \]

\[ + \dot{M} (\alpha + N) - \sigma \mu (\beta - 1) M^\delta N^2 \beta \]

From [5.18], we know

\[ (-\alpha \sigma - \sigma N + N + \sigma \mu M^\delta N^2 \beta \]

\[ = \frac{\alpha \sigma \overline{M} P - \sigma eP^\alpha k\dot{M}(\alpha + N)}{\dot{M} P} \]

Substitute [5.23] into [5.21] and [5.22],

\[ \Gamma_1 = - \sigma P^\alpha k\dot{M}(\alpha + N) - \sigma \mu (\beta - 1) M^\delta N^2 \beta \]

\[ - \sigma \mu (\beta - 1) M^\delta N^2 \beta \alpha' M^\beta \]

\[ - \alpha' M^\beta \alpha \sigma \overline{M} P / \dot{M} < 0 \]
\[
\Gamma_2 = -\sigma\bar{M}\bar{N}^{3-B}(B-1)P^{1-B}
\]
\[
-\sigma eP^s k\bar{M}(\alpha+N)/P < 0
\]

Therefore
\[
\frac{dP}{de} = \frac{\Gamma_1}{\Gamma_2} > 0 \text{ always.}
\]

Condition [5.26] says that a devaluation at equilibrium will unambiguously increase the price level.

Now we examine the effect of devaluation on aggregate output. For convenience, recall aggregate demand function [5.13]
\[
Y = \frac{a\bar{M}'}{(1-\alpha)P}
\]

Differentiating the demand function with respect to \(Y, P\) and \(e\), we have
\[
dY = \frac{a\bar{M}'}{(1-\alpha)P} \quad de = \frac{a\bar{M}'}{(1-\alpha)P^2} \quad dP
\]

where \(P\) is the initial equilibrium price level.

From [5.27], we have
or

\[ [5.29] \quad \frac{dY}{de} = \frac{\alpha \bar{M}'}{(1-\alpha)eP} \left( \frac{e\bar{M}'}{\bar{M}'} - \frac{dP}{e} \right) \]

where

\[ r = \frac{e\bar{M}'}{\bar{M}'} \] is the ratio of the original money holding in the ROW to the money holding in the world,

\[ \varepsilon = \frac{dP/P}{de/e} \] is the elasticity of price with respect to foreign exchange rate.

Obviously,

\[ [5.30] \quad \frac{dY}{de} > 0 \quad \text{iff} \quad \frac{e\bar{M}'}{\bar{M}'} > \frac{dlnP}{dlne} \]

or

\[ [5.30'] \quad \frac{dY}{de} > 0 \quad \text{iff} \quad r - \varepsilon > 0 \]

To see the sign of condition [5.30'] in our model, we need to pursue further. From [5.20],[5.24], [5.25] and [5.26], we have
where

\[ \Phi_1 = \sigma e^{P^{*}k\bar{M}(\alpha+N)}(e\bar{M}'-\bar{M}') - \bar{M}'\alpha'e\bar{M}'\alpha(\bar{M}'-e\bar{M}')P \]

Furthermore, by factoring \( M' \) out, it can be rearranged as:

\[ \Phi_1 = M'[\sigma e^{P^{*}k\bar{M}(\alpha+N)}(e\bar{M}'/\bar{M}'-1) - \bar{M}'\alpha'e\bar{M}'\alpha(1-e\bar{M}'/\bar{M}')P] \]

It is reasonable to assume \( r = e\bar{M}'/\bar{M}' = 1 \) for a small open economy, which implies that the money holdings in the home country can be negligible compared to the money holdings in the ROW. This makes \( \Phi_1 = 0 \), i.e. the numerator of \( r - \epsilon \) is zero. According to [5.31], \( \Phi_1 = 0 \) implies \( r = \epsilon = 1 \). Therefore, under the assumption of the small open economy, we have the same percentage of increase in the price level following a certain percentage of devaluation at the initial equilibrium in the home country, i.e.

\[ \frac{d\ln P}{d\ln \epsilon} = 1 \]
Furthermore, this will give
\[ \frac{dY}{de} = 0 \]
which means that a devaluation has no effects on real output. It is a rather interesting result.

The above findings can be summarized as follows: devaluation will have a one to one relationship between the percentage increase in exchange rate and the price level but have no effects on real output in a small open economy with monopolistic competition but without free entry/exit.

5.3 INTERPRETATIONS

The result of neutrality may be explained by proving that the indirect utility function is homogeneous of degree zero in price \( P_i, P \) and foreign exchange \( e \) under the assumption of a small open economy.

The indirect utility function of consumer-producer \( i \), equation [5.15], contains the finished goods market clearing condition, intermediate good market clearing condition and money market clearing condition. As we pointed out earlier, the finished goods market clearing condition is guaranteed by equating the supply and demand functions in the disutility term of the utility function. We also know that we derive the indirect utility function [5.15] by substituting demand functions [5.6], [5.7] and [5.14] into net utility function [5.1]. But these demand functions are derived based on
equations [5.8] and [5.9], which imply intermediate good market clearing. Also these demand functions are derived based on the budget constraints of each consumer-producer, equations [5.2], which implies the money market clearing and the balance of trade.

Therefore the homogeneity of degree zero of the indirect utility function in price $P_i$, $P$ and exchange rate $e$ would explain the neutrality of real output and the one-to-one relationship between the percentage increase in $e$ and $P_i/P$. Substituting $P_i$, $P$ and $e$ by $\lambda P_i, \lambda P$ and $\lambda e$ into [5.15], we have

$$[5.15'] \dot{U}_i = N^{1-\sigma} \lambda^{-1} P^{-1} \left[ \lambda (P_i - e P_i)^{x_k} \left( \frac{P_i}{P} \right)^{-\sigma} \frac{M}{NP} + \bar{M}_i \right] \left[ \sum_{k=1}^{N} \left( \frac{P_i}{P} \right)^{1-\sigma} \right]^{\frac{\alpha \sigma}{\sigma - 1}}$$

$$- \left( \frac{\mu}{\beta} \right) \left( \frac{P_i}{P} \right)^{-\sigma/\beta} \left( \frac{M}{NP} \right)^{\beta}$$

In general, the homogeneity of degree zero in price and exchange rate does not hold. If looking at [5.15'], carefully, we can find it is the term $\dot{M}/NP$ and the term $\bar{M}_i$, which cause the problem. However, if we assume a small open economy, $\dot{M}/NP$ becomes homogeneous of degree zero in $P$ and $e$.

---

1 A function is said to be homogeneous of degree zero, if multiplication of each of its independent variables by a constant will not alter the value of the function. We need to show

$$\frac{\dot{M}}{NP} (\lambda P, \lambda e) = \frac{\dot{M}}{NP} (P, e)$$

For a small open economy, we assume $e\dot{M} = \bar{M}'$, then
Also, the assumption of a small open economy allows us to treat $M_i \rightarrow 0$. Thus we have proved the homogeneity of degree zero of the indirect utility function in price $P$, $P$ and exchange rate $e$ under the assumption of a small open economy.

Comparing the results in Chapter 4 and 5, there are two major differences. One difference is the effects of devaluation on real output and another difference is on the price level. It is worth elaborating the intuition behind the possible reasons for these two differences.

According to Lizondo and Montiel (1988), in a model of perfect competition, when the other exogenous variables do not change, a nominal devaluation can be neither expansionary nor contractionary in the long-run. Now we see this is true in our small open-economy macro model with the feature of monopolistic competition but without free entry/exit.

However, from a partial equilibrium micro model in the previous chapter we see that the feature of monopolistic competition with free entry/exit does alter the conventional results of neutrality in models with perfect competition. Even though we assume a small open economy, the result of non-

\[
\frac{\dot{M}}{NP}(\lambda P, \lambda e) = \frac{\alpha' M^*}{NP}(\lambda P, \lambda e) - \frac{\alpha' eM^*}{NP}(\lambda P, \lambda e) = \frac{\alpha' \lambda eM^*}{N\lambda P} = \frac{\dot{M}}{NP}(P, e),
\]

thus $\frac{\dot{M}}{NP}$ is homogeneous of degree zero in $P$ and $e$. 
neutrality still holds. Therefore, in terms of the effects of devaluation on real output, the degree of competition is not the essential feature, the important issue is whether there is free entry or exit.

There are two reasons, which cause the different results. One reason is the different assumptions about the number of varieties. Even though assuming the same feature of the S-D-S monopolistic competition, we endogenized the number of varieties in the model in the previous chapter but exogenized the number of varieties in the model of the present chapter. As we have shown in Chapter 4, it is the change in the number of varieties which alters the market structure and thus causes the long-run effects on real output.

Another reason is the different assumption about money wage in the two models. The micro model is a partial equilibrium model with fixed income, which implies money wage rigidity of labour. But in the macro model, each agent's income is endogenous and his elasticity of disutility with respect to output is not assumed as constant. Thus a flexible wage is implicitly assumed. Under the assumption of money wage rigidity, an exchange rate shock affects the price level and thus alters real wage. For profit-maximizing agents, the real wage is set equal to the marginal product of labour. As a consequence, real output is affected. However, if we allow the money wage to be flexible, the exchange rate shock will be fully absorbed by the change in the money wage through the
change in price. As a result, the real wage and therefore the marginal product of labour are not affected. Thus real output is not affected.

As for the price level, it goes up unambiguously by the same percentage of devaluation in the macro model. In other words, the elasticity of price with respect to exchange rate is equal to one. However, the price level in the micro model may go up, remain unchanged or come down following a devaluation.

Again, we may use the two different assumptions in our model to explain the different effects of devaluation on the price levels.

In the micro model, we assume a variable number of varieties. Recall the special version of the model in Section 4.3. By assuming symmetry, we revealed that the price level is a function of the number of varieties. This gives direct evidence that the variable number of varieties affects the price level. And we also showed that a devaluation will alter the number of firms. Thus we conclude that the variable number of varieties is one of the channels through which devaluation can affect the price level. Alternatively, we may use Chamberlain's argument to explain the mechanism. Chamberlain argued that a change in the number of varieties in an industry shifts and rotates the demand curve for each variety. In other words, the number of varieties is an argument in the inverse demand function of each variety. But
the price level is an index of all individual prices of varieties. Thus a devaluation which alters the number of varieties must change the prices of each variety and consequently affect the overall price level. This virtually is the (indirect) supply side effects on the price level.

In the micro model, we assumed fixed nominal income. A devaluation affects fixed nominal income in domestic currency. The change in nominal income affects the price level. This is the direct demand side effects on the price level.

The net effects of devaluation on the price level is determined jointly by these two forces. This is why we have three possible results: the price level increases, stays unchanged or decreases.

In the macro model, we assumed fixed number of varieties. Therefore the number of varieties no longer is a source which generates a change in the price level.

In the macro model, total income is an endogenous variable. A devaluation affects the price level through affecting the total income. But total income eventually is only a function of the original global money holdings (see [5.13]). Total income increases by the same percentage as the exogenous devaluation, since we assume that for a small open economy its money holdings are a trivial proportion of the world money supply (see[5.30]).
Therefore the different results in the two models with the features of monopolistic competition are due to the differences in the assumptions in the two models. Moreover, these results are not contradictory but complementary.
CHAPTER 6
CONCLUSIONS

Even though the orthodox view of expansionary devaluation was questioned several decades ago and the possibility of contractionary devaluation has been discussed in the past ten years or so, the remaining problems are: Do the different market structures matter? Does the neutrality of devaluation hold in the long-run? What are the micro foundations of expansionary, neutral or contractionary devaluation?

We try to give a theoretical treatment of these issues in this thesis. Our answers to the above questions are: different market structures matter; the neutrality of devaluation does not necessarily hold in the long-run model; and the demand function facing the producers and the cost or/and variety structure of producers jointly determine the outcomes of devaluation.

In the short-run micro models with homogeneous goods, three traditional markets are studied in order to see what differences in the results they contribute to. We concluded that perfect competition, monopoly and oligopoly do not change the nature of the conditions required for expansionary, neutral or contractionary devaluation even though some
augmenting terms are added in the oligopoly results. However, we stressed that the three market structures do alter the magnitude of devaluation on output significantly, since imperfect competition can largely limit the effects of devaluation on output. Therefore, while there are not qualitatively different results across the three market structures, the results are quantitatively different.

To appreciate the significance of our findings, some implications were elaborated. From the theoretical view, the econometrician may have to be aware whether they should apply the same theoretical model to different market structures either for the purpose of estimating or forecasting the effects of devaluation on output and price; from the practical view, policy makers may have to distinguish the differences of market structures when they exercise devaluation; and policy makers also have to realize that devaluation does not have the same effect at all times: a higher initial exchange rate weakens the effect of devaluation on output.

There is no difficulty in explaining why the conditions for expansionary, neutral or contractionary devaluation in a micro model with differentiated goods are different from those in models with homogeneous goods. The reason is that the specifications of the two models are quite different. Consequently, we further believe that different market structures matter in the effects of devaluation on output.
Our micro foundations revealed the mechanism which causes the possible outcomes: devaluation affects individual demand thus aggregate demand, and devaluation affects the supply side through the cost structure or/and the variety structure in models with differentiated goods. Ultimately, the outcomes are determined by the net effect of the positive effect from the demand side and the negative effect from the supply side.

In the literature that exists to date, it seems indisputable that devaluation has no long-run effect on real output. Our analysis, however, argues it is not necessarily true in a model with monopolistic competition.

It is convenient to build a long-run model by using the feature of monopolistic competition. The partial equilibrium micro model provides the results which challenge the theory of neutrality of devaluation in the long-run. Therefore, not only the traditional view of expansionary devaluation is disputable in the short-run but also the neutrality theory of devaluation is disputable in the long-run.

We have shown the mechanism of the long-run effects of devaluation: it is the devaluation which causes a change in the number of varieties and it is the change in the number of varieties which alters the market structure and thus causes the long-run effect on real output. The long-run effect may
be described as hysteresis of devaluation in output following Baldwin's hysteresis of devaluation in import prices.

Further interesting findings are that aggregate price and aggregate output may move towards the opposite directions following a devaluation. These results may be unusual in the context of perfect competition but they are justified in a model with monopolistic competition. Again, the key variable which generates these seemingly odd results is the number of varieties. However, if we assume symmetry and an identical cost structure for each firm, we may make the approximations that the price level is represented by the price of an individual variety and the employment level is represented by the arithmetic summation of the quantities of each individual varieties. In addition, the aggregate quantity index represents the utility in our model and thus can be viewed as a welfare index. The results based on the approximations confirm the conventional knowledge of inflationary devaluation, and show that the possibility of expansionary effect on welfare and contractionary effect on employment exists.

In our small open economy model with monopolistic competition, the neutrality of devaluation on output has been derived. However, this result is not inconsistent with our result of non-neutrality in the micro model with monopolistic competition. The major difference between the two analyses is that, the key variable — the number of varieties, is an
exogenous variable in the macro model but is an endogenous variable in the micro model. Therefore the result of neutrality in the macro model has actually proved the legitimacy of the result of non-neutrality in the micro model from the opposite point.

Of course there are other assumptions which are also important to the derivation of the neutrality result. For instance, if we do not assume that the small domestic economy's money supply is a trivial proportion of the world money supply, then the conditions for expansionary, neutral or contractionary devaluation are similar to those in the micro model. This is an evidence for consistency of the results in the two models. Another important assumption, which contributes toward the neutrality result in the macro model, is flexible wage.

As we pointed out earlier, it is ideal to build models which incorporate all important features at once, to examine the effects of devaluation on output and price. Unfortunately, it is simply not analytically tractable. Our research only attempts to discuss certain issues at each stage. Nevertheless, some suggestions for further research seem feasible to achieve in the future.

First, a macro model may be developed based on the micro foundations in the models of homogeneous goods in Chapter 3. The conjectural variations in Eaton and Grossman (1986) may be adopted to develop such a model.
Mankiw (1988) presented a simple general equilibrium model which incorporates imperfect competition in a homogeneous goods market. Imperfect competition in his model includes monopoly and oligopoly, and perfect competition is treated as a special case of imperfect competition.

Although Mankiw’s model is not formulated for an open economy, it highlights the assumption about industrial structure in a macro model and brings us to a new dimension of linkage between macro formulations and micro foundations.

Recall Chapter 3 in this thesis, the case of homogeneous goods is analyzed at the micro level to examine the effect of devaluation on output. Thus it is possible for us to extend Mankiw’s closed economy model to an open version based on the micro foundations provided in our Chapter 3. Likely, we are able to define a parameter of conjectural variations which captures the features of perfect competition, monopoly, and Cournot-Nash and Stackelberg oligopoly (Perrakis, 1990), and to incorporate it into an open macro framework, then to derive the effects of devaluation on output and price.

Second, a macro model may be directly developed based on the micro foundations in the model with monopolistic competition in Chapter 4.

Actually, the goods market is already modeled in our analysis. We need to model labour market and money market to relax the assumption of fixed income and thus have a general
equilibrium framework. This is certainly not an easy task both analytically and technically. However, it is still possible to work on it.

As we mentioned earlier, Startz built a general equilibrium model based on monopolistic competition. Like the Blanchard-Kiyotaki model, it is a closed-economy model. However, Startz’s macro analysis is interesting and may be introduced to our proposed general equilibrium open-economy model.

Third, the homogeneous goods case and the differentiated goods case do not have to be treated separately. In the analysis of tariff protection and imperfect competition, Brander and Spencer (1984) pointed out that their homogeneous framework can be easily extended to a differentiated product case. This interesting treatment opens a new avenue for us to search a more general framework in the analysis of devaluation effects on output and price.

Finally, we can still think about how to improve the extended Blanchard-Kiyotaki model in Chapter 5.

One problem is the implicit assumption of no free entry/exit. This is a hereditary imperfection from the Blanchard-Kiyotaki model and is a major shortcoming especially in a long-run model with monopolistic competition. However, so far we have not found any better option to relax this assumption.
An alternative to the assumption of small economy is to have a two-country model, which, of course, will bring more complications to the model. Moreover, the trade pattern of exchange between finished goods and intermediate goods may be replaced by intra-industry trade, which is considered as a typical trade pattern between industrialized countries.
Form Lagrange function

\[ L = (\sum_{i} x_i^\alpha)^{1/\alpha} + \mu[I - \sum_{i} (p_i x_i)] \]

The first order condition is

\[ \frac{\delta L}{\delta x_i} = \frac{1}{\alpha} (\sum_{i} x_i^\alpha)^{1-\alpha} \alpha x_i^{\alpha-1} - \mu p_i = 0 \]

\[ (\sum_{i} x_i^\alpha)^{1-\alpha} x_i^{\alpha-1} = \mu p_i \]

\[ U^{1-\alpha} x_i^{\alpha-1} = \mu p_i \]

\[ x_i = U(\mu p_i)^{1/(\alpha-1)} \]

\[ \frac{\delta L}{\delta \mu} = I - \sum_{i} (p_i x_i) = 0 \]

Substituting [I-2] into [I-3], we have

\[ I = \sum_{i} [p_i U(\mu p_i)^{1/(\alpha-1)}] \]

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\[ I = U \mu^{1/(\varepsilon-1)} \sum [p_i^{\varepsilon/(\varepsilon-1)}] \]

\[ U \mu^{1/(\varepsilon-1)} = \frac{I}{\sum [p_i^{\varepsilon/(\varepsilon-1)}]} \]

\[ X_i = \frac{I}{\sum [p_i^{\varepsilon/(\varepsilon-1)}]} p_i^{1/(\varepsilon-1)} \]

or

\[ X_i = \frac{I}{p_i^\alpha} \sum p_i^{1-\alpha}, \quad \text{since} \quad \alpha = (\sigma-1)/\sigma \]

The industry price index is

\[ P = [\sum p_i^{1-\alpha}]^{1/(1-\alpha)} \]

Substituting the industry price into \([I-5]\), we have:

\[ X_i = \frac{I}{p_i^\alpha p^{1-\alpha}} \]
Appendix II

Transform the maximization question into a logarithm form:

\[
\text{MAX} \quad \frac{\alpha \sigma}{\sigma - 1} \log \sum_{j=1}^{N} C_j^{\frac{\sigma - 1}{\sigma}} + (1-\alpha) \log M
\]

\[
\text{S.T.} \quad \sum_{j=1}^{N} P_j C_j + M = I
\]

where we dropped subscript i for convenience, and we also dropped those terms which do not affect our constrained maximization.

Form a Lagrange function:

\[
L = \frac{\alpha \sigma}{\sigma - 1} \log \sum_{j=1}^{N} C_j^{\frac{\sigma - 1}{\sigma}} + (1-\alpha) \log M + \lambda \left( \sum_{j=1}^{N} P_j C_j + M - I \right)
\]

FOCs are:
(II-1) \[ \frac{\alpha}{C_k^\sigma \sum_{j=1}^{N} C_j^\sigma} = \lambda P_k \]

(II-2) \[ (1-\alpha) \frac{1}{M} = \lambda \]

(II-3) \[ \sum_{j=1}^{N} P_j C_j + M = I \]

Multiplying (II-1) by \( C_k \), we have

(II-4) \[ \frac{\alpha C_k^\sigma}{\sum_{j=1}^{N} C_j^\sigma} = \lambda P_k C_k \]

Multiplying (II-2) by \( M \), we have

(II-5) \[ 1 - \alpha = \lambda M \]

From (II-4) and (II-5) we have:

\[ \frac{\alpha \sum_{k=1}^{N} C_k^\sigma}{\sum_{j=1}^{N} C_j^\sigma} + (1 - \alpha) - \lambda \sum_{k=1}^{N} P_k C_k + \lambda M \]
which gives

\[(II-6) \quad \lambda = \frac{1}{I}\]

Substituting (II-6) into (II-5), we have

\[(II-7) \quad M = (1 - \alpha)I\]

In our case (II-7) can be written as

\[M_i = (1 - \alpha)(I_i - \epsilon p^k x_i)\]

which is individual i's demand function for money [5.6].

Substituting (II-6) into (II-1), we have

\[(II-8) \quad C_k = \frac{1}{P_k^\sigma} \left( \frac{\alpha I}{\sum_{j=1}^{N} C_j^\sigma} \right)^\sigma\]

which can be written as

\[\sum_{k=1}^{N} C_k^{\sigma-1} = \sum_{k=1}^{N} P_k^{1-\sigma} \left( \frac{\alpha I}{\sum_{j=1}^{N} C_j^\sigma} \right)^{\sigma-1}\]
or

\[ \sum_{k=1}^{N} C_k^{\alpha-1} - (\alpha I)^{\alpha-1} \sum_{k=1}^{N} P_k^{1-\alpha} \left( \sum_{j=1}^{N} C_j^{\alpha-1} \right)^{1-\alpha} \]

which can be simplified as:

\[ (II-9) \quad \left( \sum_{k=1}^{N} C_k^{\alpha-1} \right)^{\alpha} = (\alpha I)^{\alpha-1} \sum_{k=1}^{N} P_k^{1-\alpha} \]

Substituting (II-9) into (II-8) and making some manipulations, we have:

\[ (II-10) \quad C_k = \frac{\alpha I}{p_k^a \sum_{k=1}^{N} P_k^{1-\alpha}} \]

We know that the price index is defined as

\[ P = \frac{1}{N} \left( \sum_{j=1}^{N} P_j^{1-\alpha} \right)^{1-\alpha} \]

which gives

\[ (II-11) \quad \sum_{j=1}^{N} P_j^{1-\alpha} = NP^{1-\alpha} \]
Substituting (II-11) into (II-10), we have

\[ C_k = \frac{\alpha I}{P_k NP^{1-\sigma}} \]

or

\[ C_k = \left( \frac{P_k}{P} \right)^{-\sigma} \frac{\alpha I}{NP} \]

In our case, we have

\[ C_{j_1} = \left( \frac{P_k}{P} \right)^{-\sigma} \frac{\alpha (I - eP^X_I)}{NP} \]
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