THE SUBSTITUTABILITY AND SEPARABILITY OF MONETARY ASSETS

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by

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ABSTRACT

In this thesis, I have three objectives: (1) to estimate the substitutability/complementarity relationship between monetary assets; (2) to systematically test the empirical significance of the separability (aggregation) assumptions implicit in broad definitions of money, and (3) to compare the use of simple-sum and Divisia monetary quantity indices as data in an empirical demand system.

The theoretical part of the thesis consists of deriving a system of monetary asset demand equations from an individual model of utility maximizing behaviour. This model takes advantage of a number of important advances in the theory of models of utility maximization in this context, such as: the duality between direct and indirect utility functions; and the theory of two-stage optimization.

For our empirical work, we use a demand system which is derived from a flexible functional form interpretation of the indirect translog utility function. The use of such an approximate demand system enables us to impose theoretical restrictions through explicit side constraints on the parameters, thus permitting statistical

tests of their validity. Moreover, as Denny and Fuss (1977) have shown, the approximate translog model permits a less restrictive test of separability than the Berndt-Christensen exact translog framework.

The model was applied to quarterly Canadian data for the period 1968I-1982IV, and was estimated using Barten's (1969) F.I.M.L. method for estimating singular systems.

The results obtained from the study lead to a number of important conclusions and are also of value as indicators of potential future research.

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CHAPTER I

INTRODUCTION

I.1 INTRODUCTORY REMARKS

The main purposes of this thesis are to examine the substitutability/complementarity relationship between monetary assets; to establish the advantages of applying the theory of economic and statistical index numbers to the problem of defining monetary aggregates; and to test the separability assumptions implicit in previous studies and in various money measures. This chapter discusses some of the issues on which the results will shed light and provides a brief summary of previous studies that are most similar in approach.

The issue of monetary asset substitutability has attracted a great deal of attention and has been extensively explored in the literature during the past two decades. Despite, however, the volume of research undertaken the results are still inconclusive as to the degree of substitutability between money and near money and leave much room for further empirical investigation. As Feige and Pearce (1977) put it:

> .... the issue of substitutability between money and near-monies is likely to continue to be an important question for monetary economics just as it has been since the days of the currency-banking school controversy[1].

Knowledge of the substitutability of monetary assets is essential in order to understand the potential effects of monetary policy actions. In particular, the stability of a narrowly defined demand for money function, the appropriate definition of money[2], and the effects of the growth of financial intermediation, are closely linked to the degree of substitutability that exists between monetary assets. For example, if these assets are close substitutes for money, their inclusion into a broader measure of money could provide a more stable demand for money function. On the other hand, the existence of such substitutes could increase the interest elasticity of the liquidity preference schedule for a narrowly defined monetary aggregate and reduce the effectiveness of monetary policy (targeting) based on

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[1] Feige and Pierce (1977, p. 465).

[2] There are two major approaches to defining the money stock. The "a priori" approach and the "empirical" approach. The former defines money in a theoretical sense by appeal to its functions while the latter defines money as that collection of financial assets that has the most predictable effect on nominal income. Both approaches implicitly make the assumption that aggregation over different financial assets by simple addition, is feasible.

such a narrow aggregate as individuals could very easily shift out of money into the higher interest earning financial assets[3].

The issue of monetary asset aggregation has, in recent years, attracted a great deal of attention in the literature. Much of the attention derives directly or indirectly from Barnett's challenging paper (1980a) in which he voiced objections to simple-sum aggregation procedures and argued instead for applying aggregation theory and statistical index number theory to monetary aggregation. As he and co-authors argued in a later paper:

> By equally weighting components, simple-sum aggregation can badly distort an aggregate. For example, if one wished to obtain an aggregate of transportation vehicles, one would never aggregate by simple summation over the physical units of, say, subway trains and roller skates. Instead, one could construct a quantity index (such as the Commerce Department's many Laspeyres quantity indexes), using weights based upon the values of the different modes of transportation[4].

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[3] Gurley and Shaw (1960) take this position and argue that the growth of near-bank liabilities was the main reason that monetary authorities failed to reduce the liquidity in the economy during the post-war period.

[4] Barnett, Offenbacher and Spindt (1984).

These authors argue that a more satisfactory approach to monetary aggregation must involve consideration of the utility function underlying the demand for monetary assets. For example, the appropriate form of aggregation (simple-sum as opposed to other possibilities) will be determined by the relationship monetary assets bear to one another and their contribution to total "money". Simple-sum aggregation (the usual procedure) is justified, when viewed in this framework, only if the component assets are perfect substitutes (implying linear indifference surfaces). With perfect substitutability a change in the monetary components within the aggregate, that leaves the level of the aggregate constant, would be completely internalized and have no effect on other variables in the system. If this condition of perfect substitutability is violated, it is inappropriate to form a quantity index by giving an equal weight of unity to each asset component. Barnett has shown that consistent and satisfactory approach to aggregation results if aggregation is performed using a superlative quantity index (Barnett 1980a, 1981a). Such an index attaches different weights to assets according to their degree of "moneyness" or "liquidity," and uses monetary asset user costs to calculate these weights.

Separability

A totally unresolved problem is the method by which various monetary assets are selected to be included in the aggregate. As Barnett (1982a) notes, this has to do with the separability properties of the underlying utility function. In particular, an aggregate exists in aggregation theory if the utility function defined over the items of the aggregate and other items as well, is weakly separable in the components of the aggregate. If this separability condition is violated, stable preferences cannot exist over the aggregate in the sense that varying the relative quantities of the elements within the aggregate (while holding the aggregate level constant) will affect consumer preferences over other assets or goods. Further, knowledge of the change in the level of the aggregate will say little about the change in the demand for the monetary elements within the aggregate in the absence of additional conditions. Only under certain assumptions will an aggregate behave like an elementary good, in the sense that a change in a price of an asset outside the separable group is the same for all assets in the given aggregate and hence the same for the aggregate. Homothetic weak separability with respect to the monetary assets within the aggregate is necessary and sufficient for the monetary aggregate to be a meaningful functional quantity index. Neither weak separability nor homothetic weak separability have yet been tested with monetary assets.

This thesis will provide new evidence bearing on these issues by adopting a theoretical framework which allows for sophisticated modelling of the demand for money (and money substitutes) and which is empirically interesting. The utility theory (or demand system) approach to money demand modelling has been chosen, in the present application, because of the desire to consider the monetary problems in a framework that takes account of the different relationships monetary components bear to one another and, also, because this approach is currently the basis for rapidly expanding empirical research in the literature.

In addition to the utility theory approach to modelling the demand for money (and money substitutes), suggested above, and which we adopt in this thesis, we may distinguish two alternative frameworks: a) the mean-variance (Markowitz (1952), Tobin (1958)) approach, and b) the transactions cost (Baumol (1952), Tobin (1956)) approach. Each of these approaches captures different aspects of the role of money. We briefly discuss the advantages and disadvantages of these three alternative theoretical frameworks to the demand for money.

The mean-variance approach deals with the division between riskless assets (money) and risky assets (bonds). It is based on the assumption that utility depends on risk and expected returns and it views the speculative (and the precautionary) demand for money in the context of a portfolio choice problem. This approach, however, has not been frequently used in empirical work because of the difficulties involved in measuring the risk of holding bonds.

The transactions cost (or inventory-theoretic) approach, ignores uncertainty and focuses on the transactions motive for holding money. The major novelty of this approach is that it takes account of inventory holding costs as well as brokerage costs, since it is based on the assumption that money is the means of exchange in the economy and that there are transaction costs in switching between money and interest-earning assets. However, most transaction cost models are usually presented in a two-asset (money-bond) framework and as such do not easily apply to the modelling of the choice among liquid assets. Also, the empirical application of these models is restricted due to the lack of consistent data series on transaction costs (both pecuniary and non-pecuniary in nature).

The demand system approach, sometimes regarded as a generalization of the transactions cost approach, views money as a durable good (or monetary assets as durable goods) yielding a flow of non-observable services (either transaction services or store of wealth services) which enter as arguments in aggregator functions (either utility or production functions). This approach possesses a number of advantages over the other approaches, in particular, with respect to empirical implementation. First, it premits imposition of symmetry restrictions that are sufficient for integrability, i.e., the demand system can be shown to be derivable from an aggregator function. The existence of this aggregator function is important because substitution is thought of as being a property of an aggregator function. Moreover, this aggregator function itself, once estimated, would imply the appropriate aggregation procedure. Second, the demand system approach allows for interdependence between the real and financial decisions (of the economic unit) by modelling the demand for monetary assets simultaneously with the demand for consumption goods and leisure. Finally, the demand system approach allows the testing of functional form restrictions such as homotheticity and/or separability and provides, also, a suitable framework for analyzing data sets that appear in a disaggregated form.

We now return to the theoretical framework adopted in this thesis. The model used is based on the demand theory approach and takes advantage of a number of important advances in the theory and application of models of utility maximization in this context, such as: the duality between direct and indirect utility functions; the theory of multistage optimization; the flexible functional forms method of approximating aggregator functions; and constrained estimation and hypothesis testing techniques.

My approach also assumes a common aggregator function for all economic agents and I am mainly concerned with the problem of monetary asset aggregation and the application of empirical demand analysis. Questions of aggregation over economic agents are beyond the scope of this study, although I recognize that the data used are observations of expenditure on monetary assets made by a group of economic agents (both households and firms). Future work could usefully investigate possible differences across the groups.

The issue of monetary asset aggregation is addressed by attempting to provide a suitable setting for a demand analysis of a data set that appears in a too disaggregated form. As the number of monetary assets is large (n = 19, in the present application), the estimation of a highly disaggregated demand system encompassing this many assets is econometrically intractable, both because in many cases separable returns could not be identified and because of computational difficulties in the parameter space. We reduced the number of variables by forming quantity subaggregates and price indices for these subaggregates.

The next problem with regard to monetary asset aggregation is to investigate whether these monetary subaggregates can be further aggregated into a single aggregate that could be used as an aggregate for the system as a whole. The majority of empirical studies have been based on relationships involving broad-based monetary aggregates (e.g., M2, M3). It is appropriate, therefore, that we examine the empirical significance of the separability (aggregation) assumption.

The major differences between this study and related previous studies are now discussed. We only review, briefly, those studies that take a demand system approach. As was pointed out, this approach is arguably superior to the single demand equation approach, since it can be shown to be grounded in a neoclassical aggregator function. Before turning to the studies we note that as yet no clear view has emerged in these studies as to the degree of substitutability between money and near monies.

Feige (1964), was the first to estimate simultaneously a system of monetary asset demand equations. He regressed quantities of financial assets, excluding currency, on income and on own and other asset returns. He used U.S. data, that involved a pooling of cross-section and time series observations, and he could not reject the symmetry conditions[5] and found low substitutability between money and near money. (Chetty (1969) suggested the derivation of money demand equations directly from an explicit model of consumer utility maximization. He extended the Feige model and applied it to U.S. time series data. His framework without symmetry imposed, indicates a high degree of substitutability between monetary assets. Gramlinch and Kalchbrenner (1970) later used a quadratic utility function with arguments being the end of period liquid asset values. Maximi-

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[5] The conditions he imposed were not the usual symmetry restrictions (on the compensated cross price partial derivatives of the demand functions), but rather restrictions on the cross price elasticities.

zation subject to a constraint, and estimation of money demand for U.S. time series indicated very low substitution effects. Kemp (1975) using Feige's data, and a translog direct utility function, again found low substitution elasticities. He imposed the symmetry restriction but did not test it.

Donovan (1978) used Canadian data for the period 1947-1974, and under the assumption that the services of money are proportional to the stocks, derived user costs for monetary assets. These user costs were used as the price variables in the budget constraint where the products of the stocks times corresponding user costs define expenditure on the services of the stocks. Three models were examined empirically. The first contained only consumption goods and leisure. The second included money along with consumption goods and leisure while the third contained only liquid assets, i.e., disaggregated components of money and near money. The demand equations for each model were derived from a Gorman polar form representation of the indirect utility function. The model indicated low substitutability between money and near-money.

Barnett (1980a), assumed intertemporal weak separability to acquire a current period utility function. Then using a C.E.S. specification, he estimated a model of liquid financial assets with U.S. data. The results indicate high substitutability among the various passbook (savings and time deposits at different institutions) accounts and low substitutability between transaction balances and the passbook account aggregate. Tests for the empirical validity of the chosen aggregates were not performed. More recently, Barnett (1983a), in a model which includes aggregate money along with consumption goods, compared simple-sum and Divisia monetary aggregates in terms of estimation performance. He concluded in favour of Divisia monetary aggregation.

Finally, Offenbacher's (1979) work is quite similar to Barnett's on both theoretical and empirical grounds. The major difference is that Offenbacher takes a different approach to commercial bank behaviour using an implicit rate of return to bank deposits (while Barnett treats regulated own rates of return on monetary assets as the true own rates). This implies an explicit, meaningful distinction between currency and deposits and allows the estimation of substitution elasticities between these assets. He uses the linear logarithmic expenditure system (a homogeneous of degree minus one indirect translog function). The major findings are complementary to Barnett's in spite of the widely disparate assumptions on the own rates and the differences in functional form.

Hence the state of the art is to develop a monetary asset expenditure system that is capable of generating some new results and is also theoretically more satisfactory than the monetary asset demand functions now in use. The principal motivation is the possibility of shedding some light on the problem of measuring the degree of substitutability among alternative liquid assets and, also, on the problem of constructing an appropriate definition of money.

I.2 SYNOPSIS OF THE DISSERTATION

In the theoretical section of this thesis, I derive a system of monetary asset demand equations from an individual model of utility maximizing behaviour. I exploit the literature on multistage budgeting and aggregation theory to this end. I use a flexible second-order approximation to an arbitrary indirect utility function, and estimate it with quarterly Canadian data for the period 1968I-1982IV. In the empirical section, I investigate the degree of substitutability between monetary assets and I compare two forms of monetary subaggregation -- simple sum and Divisia -- in terms of satisfying the integrability conditions of the demand functions based on the indirect utility function. Lastly, I systematically test the weak separability restrictions implicit in various money measures.

The remainder of this thesis is organized as follows: I begin Chapter II with a discussion of the "price of money." Next, I investigate the conceptual shortcomings of conventional monetary aggregates and I emphasize the need for a more satisfactory method of aggregation. I distinguish between the economic approach and the statistical approach to the aggregation problem. This leads me to Diewert's (1976) contribution which pulls together the two approaches and describes how and when statistical index numbers provide parameter-free approximations to economic aggregates. Finally, I conclude with a discussion of which indices are more appropriate for the purpose of our analysis.

In Chapter III, I outline a general model of individual utility maximization. I assume a two-stage optimization procedure and focus on the details of the demand for monetary assets ignoring other types of goods. The salient issues in the two-level optimization literature relevant to our monetary assets demand model are explicitly stated. I then argue that the specification of the translog indirect utility function is the best approximation to the true indirect utility function for monetary assets. I distinguish between the homothetic and quasi-homothetic versions of the translog, and based on the latter, I derive price and expenditure elasticities as well as the elasticities of substitution. The rest of the chapter is devoted to econometric considerations. In particular, a stochastic version of the share equation system is specified and the error term is given an interpretation. The various tests of the theory of demand are outlined and finally we formulate the approximate weak separability hypotheses to be tested, while distinguishing between exact and approximate separability.

The first part of Chapter IV is concerned with data. I discuss the main issues regarding the monetary components used and I provide an appendix with data sources. In the second part the results are presented and interpreted.

Finally, in Chapter V, I summarize the main findings of my research and examine their importance with respect to the carrying out of monetary policy. In addition, I outline further useful directions of enquiry and conclude the thesis.

CHAPTER II

THE MEASUREMENT OF MONEY: AN APPLICATION OF INDEX NUMBER THEORY

This chapter is concerned with the problems of aggregating quantities of different monetary assets in a non-parametric manner in the context of estimating demand systems. That our empirical analysis deals with subaggregates could hardly be otherwise. There are simply too many monetary assets ($n = 19$, see Table IV.1) with their corresponding prices, for simultaneous estimation of a demand system encompassing the full range of assets. Such estimation, although theoretically conceivable, is not possible in practice, because of computational difficulties in the large parameter space. Some degree of prior aggregation is inevitable in order to reduce the number of parameters to be estimated.

We, therefore, reduced the number of variables by forming quantity subaggregates and price indices for these subaggregates, based on prior knowledge of the particular assets being analyzed. By taking this approach, we work directly with a small system of aggregated monetary assets (i.e., money (M), checkable deposits (C), savings deposits (S), and time deposits (T), see Table IV.1), abstracting from the obvious fact that there are many types of checkable, savings and time deposits.

In constructing these monetary subaggregates, we obviously have to use an index (either economic or statistical). In what follows, we present a survey of the theoretical results on aggregation and index number theory. We discuss these results systematically from the point of view of empirical demand analysis. Our particular focus is on indices which may be consistent with the underlying assumptions about preferences.

The Sections are organized as follows: Section II.1 investigates the conceptual shortcomings of conventional monetary aggregates and emphasizes the need for a more satisfactory method of aggregation. In Section II.2, a discussion of the price of money is provided. Then in Section II.3, I take up the economic approach and in Section II.4 the statistical approach to the aggregation problem. This will lead us to Section II.5 which pulls together the two approaches and describes how and when statistical index numbers provide parameter-free approximations to the economic aggregates. Finally, in Section II.6, I conclude as to which indices are more appropriate for the purpose of my analysis.

II.1 SIMPLE SUMMATION MONETARY QUANTITY INDICES

Conventional monetary aggregates are simple-sum indices in which all monetary components are assigned a constant and equal (unitary) weight. This summation index

$$(2.1) \qquad Q = \sum_{i=1}^{n} x_i$$

implies that all monetary components (x_i) contribute equally to the money total (Q) and it views all components as dollar for dollar perfect substitutes. Such an index, there is no question, represents an index of the stock of nominal monetary wealth, but cannot, in general, represent a valid structural economic variable for the services of the quantity of money.

Empirical evidence accumulated over the last decade on substitutability within money markets indicates that monetary assets are not perfect substitutes[1]. In fact, these assets contain differing degrees of moneyness or liquidity[2]. For example, money (currency plus demand deposits) has by assumption perfect liquidity, while near-monies (financial assets that are excluded from the definition of money but are very similar to some assets that are included[3]) although very liquid, are not as liquid as money since there is cost and trouble in turning them into money.

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[1] See Feige and Pearce (1977) and Offenbacher (1979).

[2] The liquidity of an asset depends on how easily it can be bought or sold and on the transaction cost of selling or buying it.

[3] The distinction between money and near-monies is not clear cut but depends on the definition of money that we accept.

Hence, if we are to construct a monetary aggregate that captures the contribution of its components to the economy's flow of monetary services, simple-sum aggregation seems unsatisfactory. As Irving Fisher put it:

> The simple arithmetic (simple-sum index) should not be used under any circumstances[4].

Similarly, Friedman and Schwartz noted:

> This procedure (simple-sum aggregation) is a very special case of the more general approach discussed earlier. In brief, the general approach consists of regarding each asset as a joint product having different degrees of "moneyness," and defining the quantity of money as the weighted sum of the aggregate value of all assets, the weights for individual assets varying from zero to unity with a weight of unity assigned to that asset or assets regarded as having the largest quantity of "moneyness" per dollar of aggregate value. The procedure we have followed implies that all weights are either zero or unity[5].

Over the years, there have been a series of attempts at properly weighting monetary components within a simple-sum aggregate[6]. However, it is only recently that Barnett (1980a) applied the literature on aggregation theory and index number theory to

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4 I. Fisher (1922, p. 361).

5 Friedman and Schwartz (1970, p. 151-152).

6 See, for example, Friedman and Meiselman (1963) and Chetty (1969).

monetary aggregation. He constructed monetary aggregates based upon Diewert's class of superlative index numbers[7] and in particular advocated the use of Divisia quantity indices.

Our approach in this chapter is to provide a sharp qualitative assessment of the relative merits of monetary quantity indices based on the rigorous application of economic aggregation and index number theory. The application, however, of aggregation theory and index number theory to monetary aggregation requires information about the price of money. I take up this problem in the section that follows.

II.2 THE MEANING OF THE PRICE OF MONEY

The meaning of the price or value of money is not obvious in monetary theory. Usually this price has been viewed to vary inversely with the general price level. In this sense the price of money is, in fact, its purchasing power in terms of real goods and services. In the recent literature, however, money

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[7] A quantity index is superlative if it is exact for an aggregator function which can provide a second-order approximation to a linear homogenous function.

is treated as a durable good having an infinite life and it is assumed that money retains at least some value beyond the holding period. Under such assumptions it would be wrong to attribute a price of unity -- the full purchase price -- to a unit of the stock of money, simply because this one dollar price represents the price of a unit of the stock over an infinite holding period. There is no question that money is a stock (at an instant of time). But money is also an economic good which provides a variety of services (i.e., liquidity, safety, convenience). These services of money are better described in a flow dimension (per period of time).

Donovan (1978) asserted that a "Jorgensonian" user cost concept, rather than the full purchase price is more appropriate for pricing money. Donovan derived the user cost formula through general economic reasoning and under the assumption that the services of money are proportional to the stocks with a unitary factor of proportionality. Barnett (1978) derived the same formula through an intertemporal consumption allocation model, without any assumptions regarding the factor of proportionality. The user cost formula is

$$p_i = \frac{P^*(BR - r_i)}{1 + BR} \tag{2.2}$$

where p_i = user cost of monetary component i

P^* = aggregate price index of consumption goods

BR = the highest rate of interest (benchmark rate)

r_i = own rate of return on the i^{th} component

Observe that the user cost is the discounted foregone interest by holding one dollar of the i^{th} asset. Equivalently, it is the price of the quantity of services provided by a unit of the stock during a finite holding period.

The remainder of this chapter is mainly composed of two sections devoted to aggregation theoretic procedures for constructing monetary quantity indices. The first section discusses the economic (or functional) approach which considers the utility function and its properties, while the second section discusses the statistical (or atomistic) approach to index numbers. We also, briefly, review the recent contributions by Diewert (1976).

II.3 THE ECONOMIC APPROACH TO MONETARY QUANTITY INDICES

This approach is concerned with rational individuals who act according to their scale of preferences and is related to aggregation theory. Monetary aggregates (e.g., M1, M2) are regarded as functional quantity indices providing a mapping from the set of monetary assets into the set of real numbers. The existence of a functional quantity aggregator

$$u = u(x) \tag{2.3}$$

is important in understanding functional quantity indices, where u is "utility" (or "output" depending on the case) and x represents the vector of monetary asset quantities. Notice that (2.3) involves only the unknown aggregator function, u, the quantities, and any unknown parameters. The economic approach then suggests the selection of a differentiable functional form for the functional quantity aggregator (2.3). Then, using Wold's (1944) theorem[8], the inverse demand functions

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[8] See Diewert (1976).

$$(2.4) \qquad \phi_i(x) = \frac{u_i(x)}{\sum u_i x_i} \qquad i = 1,\ldots,n$$

can be derived[9].

Using equation (2.4) and detailed data we can estimate the parameters and replace the unknown parameters by their estimates. The resulting function is called an economic (or functional) index and its function value is an economic quantity index number.

Using such an index simply means the aggregation of a well defined set of quantities and, of course, the loss of some information. A clear concept of an economic index exists only in the case of a linearly homogeneous utility function, in which case equation (2.4) becomes

$$(2.5) \qquad \phi_i(x) = \frac{u_i(x)}{u(x)} \qquad i = 1,\ldots,n$$

and the resulting economic index (aggregate) is called "consistent". The use of any particular functional form for the monetary quantity aggregator function, when viewed in this framework, reflects a particular set of preferences among monetary assets. For example, the use

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[9] Ordinary demand functions could be derived from the indirect utility function by Roy's theorem.

of a weighted linear aggregator function

$$(2.6) \qquad u(x) = \sum_i a_i x_i$$

implies perfect substitutability among the assets and hence should logically lead to specialization in consumption of the least expensive asset[10]. If this is inaccurate, obviously, we commit a specification error by using this functional form.

As a second example, the use of a Cobb-Douglas functionsl form

$$(2.7) \qquad u(x) = \prod_i x_i^{a_i}$$

imposes an elasticity of substitution equal to unity, ($\sigma = 1$), between every pair of assets and its use implies that each asset always accounts for a constant share of the expenditure. Again, if this proposition is at odds with the facts, the use of the Cobb-Douglas seems inappropriate.

Thirdly, a Constant Elasticity of Substitution

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[10] The more restrictive unit-weighted ($a_i = 1, \forall i$) aggregator function implies dollar for dollar perfect substitutability.

aggregator function

$$(2.8) \qquad u(x) = \sum_i \left[a_i x_i^r\right]^{1/r} \qquad 0 < a_i < 1, -\infty < r < 1$$

relaxes the unitary elasticity of substitution restriction imposed by the Cobb-Douglas, but imposes the restriction that the elasticity of substitution between any pair of assets is always constant ($\sigma = 1/(1-r)$).

Finally, the Leontief aggregator function (a special case of the C.E.S. when $r \to -\infty$)

$$(2.9) \qquad u(x) = \min\left(\frac{x_1}{a_1}, \ldots, \frac{x_n}{a_n}\right)$$

imposes the restriction of zero substitutability ($\sigma = 0$) between any pair of assets, and its use would lead us to assume that relative monetary asset demands are totally unresponsive to changes in other asset prices.

These functional forms, as we have just pointed out, all imply serious limitations on behaviour if they are used as aggregator functions. An alternative way to view these limitations is to note that these functional forms impose (rather than test) potentially undesirable functional form restrictions such as linear homogeneity and/or strong separability. The main disadvantage of strongly separable utility functions is that they cannot model in a general way the interaction of different assets. A more flexible approach is to use a function from the class of quadratic utility functions which do not impose separability or homogeneity. Some commonly used quadratic functional forms are:

the generalized quadratic

$$(2.10) \qquad u(x) = a_0 + \sum_i a_i f_i(x_i) + 1/2 \sum_i \sum_j \beta_{ij} f_i(x_i) f_i(x_j)$$

where $\beta_{ij} = \beta_{ji} \quad \forall\ i,j$, and

the quadratic mean of order r

$$(2.11) \qquad u(x) = \left[\sum_i \sum_j \beta_{ij} (x_i \ x_j)^{r/2} \right]^{1/r}$$

where $\beta_{ij} = \beta_{ji} \quad \forall\ i,j \qquad r \neq 0$. This latter class of functions is of considerable interest becuase it contains a number of interesting special cases including: the generalized Leontief ($r = 1$), the quadratic mean function ($r = 2$), and the Constant Elasticity of Substitution function ($\beta_{ij} = 0$ for $i \neq j$, and $r \neq 1$ or 2).

But the most popular specification appears to be the transcendental logarithmic utility function, which is a quadratic function in the logarithms of the variables[11].

(2.12) $$u(x) = a_0 + \sum_i \ell n\, x_i + \frac{1}{2} \sum_i \sum_j \beta_{ij}\, \ell n\, x_i\, \ell n\, x_j$$

where $\beta_{ij} = \beta_{ji} \quad \forall i,j$

The functional forms (2.10)-(2.12) imply neither separability nor homotheticity with the exception of (2.11) and the class of functions generated by it, which, without any further restrictions, are homogeneous of degree one.

So far the quadratic utility functions have been interpreted here as utility functions in their own

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[11] See Christensen, Jorgenson and Lau (1975).

right. Their use then implies that we are willing to make strong exact a priori restrictions about preferences among monetary assets. A more flexible approach is to view such functions as flexible functional forms[12]. By using a flexible functional form as opposed to an exact form, on the one hand, we increase the possibility of approximating the true functional form, but on the other, we face a difficult procedure of estimating systems of demand equations which are nonlinear in a large number of parameters. Hence, theoretical purity is gained only at the expense of econometric inconvenience.

So far we have been using aggregation theory to argue that aggregator functions defined over monetary assets cannot be adequately approximated by simple-sum functions. Aggregation theory suggests the use of nonlinear aggregator functions in aggregating over monetary assets. However, this economic approach to index numbers is not frequently used. As Barnett puts it:

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[12] A functional form is said to be flexible if it can provide a second-order approximation to an arbitrary twice differentialbe aggregator function.

> Estimates of the unknown parameters depend upon the specified model, the data, and the estimator. Hence, aggregator functions, although important in theory and in hypothesis testing are not generally useful in constructing index numbers which are publishable as data by governmental agencies[13].

For precisely these reasons Frisch (1936), in one of the most important papers on index number theory, distinguished between the statistical and the economic approach to index numbers and we turn now to the statistical approach.

II.4 THE STATISTICAL APPROACH TO MONETARY QUANTITY INDICES

Statistical indices are simply descriptive statistics that treat prices and quantities as independent variables and measure the variations in

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[13] Barnett (1981a, p. 220).

these variables without any reference to economic theory. They are widely used since they can be computed from price and quantity data alone, eliminating the need to estimate an underlying structure. Well known examples of statistical price indices are:

$$(2.13) \qquad P_L(p^0, x^0, p^1, x^1) = \frac{p^1.x^0}{p^0.x^0} = \sum w_i^0(p_i^1/p_i^0)$$

"Laspeyres"

$$(2.14) \qquad P_P(p^0, x^0, p^1, x^1) = \frac{p^1.x^1}{p^0.x^1} = 1/\sum w_i^1(p_i^0/p_i^1)$$

"Paasche"

$$(2.15) \qquad P_{Id}(p^0, x^0, p^1, x^1) = \left[p^1.x^0.p^1.x^1/p^0.x^0.p^0.x^1\right]^{1/2}$$

$$= \left[\sum_i w_i^0(p_i^1/p_i^0)/\sum_i w_i^1(p_i^0/p_i^1)\right]^{1/2}$$

"Fisher's Ideal"

where the weights

$$w_i^r = p_i^r \cdot x_i^r \; / \; \sum_i p_i^r \cdot x_i^r$$

are user cost shares and the superscripts on values refer to time periods.

The Laspeyres price index weights prices by quantities in the base year, the Paasche price index weights prices by quantities in the final year and the Fisher's Ideal price index is the geometric average of the Laspeyres and Paasche price indices.

The Laspeyres, Paasche and Fisher's Ideal quantity indices are defined in a similar manner by simply interchanging quantities and prices in the above formulas.

Statistical indices are mainly characterized by their properties. These properties were examined in great detail by I. Fisher (1922) and serve as tests for assessing the quality of a particular index. They have been named after Fisher as "Fisher's System of Tests"[14].

Fisher's tests for statistical price indices are[15]:

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14 For a detailed analysis as well as a comprehensive bibliography on Fisher's test (or axiomatic) approach, see Eichhorn and Voeller (1976) or Eichhorn (1976).

15 These tests can equally be applied to the corresponding quantity indices.

The Proportionality Test

If all base period prices have changed in the same proportion, say by λ-fold, then the value of the price index should equal the common factor of proportionality λ.

$$(2.16) \qquad P(p^0, x^0, \lambda p^0, x^1) = \lambda,$$

The Circular Test

If in the first time period all prices and quantities change from (p^0, x^0) to (p^1, x^1) and in the second time period they change from (p^1, x^1) to (p^2, x^2), then the value of the price index for the entire period should be given by

$$(2.17) \qquad P(p^0, x^0, p^1, x^1)\, P(p^1, x^1, p^2, x^2)$$

$$= P(p^0, x^0, p^2, x^2)$$

The following tests are consequences of the Circular Test:

$$(2.18) \qquad P(p^0, x^0, p^0, x^0) = 1 \qquad \text{(Identity Test)}$$

$$(2.19) \qquad P(p^0, x^0, p^1, x^1) \cdot P(p^1, x^1, p^0, x^0) = 1$$

(Time Reversal Test)

The Determinateness Test

If any individual price or quantity in the price index, $P(p^0, x^0, p^1, x^1)$, becomes zero, then the price index must tend to a unique positive real number (must not go to zero, become infinite, or indeterminate) depending on the set of the remaining prices and quantities.

The Commensurability Test

If there is a change in the units of measurement, the price index must not change.

The Factor Reversal Test

The product of the price and quantity indices must be equal to the expenditure ratio for the two periods under consideration.

$$(2.20) \qquad P(p^0, x^0, p^1, x^1)\ Q(x^0, p^0, x^1, p^1)$$

$$= p^1.x^1/p^0.x^0$$

where the quantity index, Q, satisfies the same tests as those satisfied by the price index.

It has been shown that Fisher's system of tests is inconsistent in the sense that there does not exist any index satisfying all the tests. There exist even inconsistent subsets of these tests. The Laspeyres and Paasche indices satisfy the Proportionality, Determinateness and Commensurability tests but not the Circular and Factor Reversal tests. Similarly, Fisher's Ideal index satisfies all Fisher's tests except the Circular test[16].

From the axiomatic point of view it is not clear which is the most appropriate index number formula. For this reason Eichhorn and Voeller (1976) have considerably weakened Fisher's system of tests by replacing the Circular test and the Factor Reversal test by weakened tests. These tests are

<u>The Base Test</u>

Instead of the Circular Test (2.17), one would require

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16 For exactly this reason Fisher (1922) termed this index "Ideal".

$$(2.21) \qquad \frac{P(x^0, p^0, x^1, p^1)}{P(x, p, x^1, p^1)} = R(x^0, p^0, x, p);$$

that is, the comparison between base period quantities and prices and current period quantities and prices vis-a-vis a third point of time, must be independent of this third point.

<u>The Product Test</u>

Instead of the Factor Reversal Test (2.20) one only requires

$$(2.22) \qquad P(p^0, x^0, p,x) \, . \, Q(p^0, x^0, p,x) = \frac{p.x}{p^0.x^0}$$

Note that Eichhorn and Voeller (1976) have shown that every subset of the system of weakened tests is consistent while the whole system is inconsistent. For instance, Fisher's Ideal index satisfies all Fisher's tests except the Circular Test and the Base Test (a weakened form of the Circular Test).

In connection with the statistical approach to index numbers, the Divisia index deserves special mention[17]. This index was first introduced by Divisia (1925) and is based on the chain approach to index numbers. To derive the Divisa indices[18], we assume that a value

$$(2.23) \qquad p.x = \sum_i p_i x_i$$

is differentiated with respect to time and divided by the value. That is

$$(2.24) \qquad \frac{\sum(x_i dp_i + p_i dx_i)}{\sum p_i . x_i} = \sum_i w_i \left(\frac{dp_i}{p_i} + \frac{dx_i}{x_i}\right) \equiv \frac{dp}{p} + \frac{dx}{x}$$

where

$$w_i = p_i . x_i / \sum_i p_i . x_i$$

is the expenditure share of the i^{th} component.

The index problem consists of a decomposition of (2.24). The only reasonable decomposition results

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17 This index recently has been used extensively in the literature on measurement of productivity change.

18 See Jorgenson and Griliches (1967).

from the requirement that "quantity changes without price changes should not change the price index and vice versa"[19]. The Divisia price and quantity indices would then be defined in differential form as

$$(2.25) \qquad \frac{dp}{p} = \sum_i w_i \frac{dp_i}{p_i} \quad \text{and} \quad \frac{dx}{x} = \sum_i w_i \frac{dx_i}{x_i}$$

The price and quantity Divisia indices (2.25) reflect the price and quantity changes at all times between two situations. Their values can be given by

$$(2.26) \qquad \begin{aligned} P_D &= P^0 \exp\left[\int_{t_0}^{t_1} \sum_i w_i \frac{dp_i}{p_i}\right] \quad \text{and} \\ Q_D &= Q^0 \exp\left[\int_{t_0}^{t_1} \sum_i w_i \frac{dx_i}{x_i}\right] \end{aligned}$$

where P^0 and Q^0 are constants of integration which scale the indices. An undesirable property of the Divisia indices, which is often emphasized in the literature

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[19] This has been applied by Divisia (1925), see Vogt (1976).

is that these indices, depending on the path that runs from the base point to the observed point, do not compare two price-quantity situations but instead reflect the price and quantity changes at all times under two alternative situations.

Since economic data take the form of observations at discrete points in time, equation (2.26) can be approximated by the following forms in discrete differences (see Hulten (1973)).

$$P_D^1 = P_D^0 \prod_i \left(\frac{p_i^1}{p_i^0}\right)^{1/2(w_i^1+w_i^0)} \quad \text{and}$$

(2.27)

$$Q_D^1 = Q_D^0 \prod_i \left(\frac{x_i^1}{x_i^0}\right)^{1/2(w_i^1 + w_i^0)}$$

Taking logarithms of each side of (2.27) the Divisia indices can therefore be written as

$$\log P_D^1 - \log P_D^0 = \sum w_i^*(\log p_i^1 - \log p_i^0)$$

(2.28) and

$$\log Q_D^1 - \log Q_D^0 = \sum w_i^*(\log x_i^1 - \log x_i^0)$$

where

$$w_i^* = 1/2(w_i^1 + w_i^0)$$

are the weights depending upon all prices and all quantities.

From (2.28) we observe that the growth rates of the Divisia indices, measured as first differences in the logged values, are weighted averages of the growth rates of the components, the weights being the shares of each component's user cost in total user cost. A change in the user cost (in our case, the cost of a component monetary asset due to a change, say, in the interest rate) will induce changes in the asset's weight in the same direction. However, a combination of price (quantity) changes that leaves the total expenditure unchanged will be represented by no change in the Divisia index. Hence, Divisia indices perfectly internalize pure substitution effects and change only if the change in price has an expenditure effect. This is known as the Invariance Property of Divisia indices and it is exactly this property that speaks for the Divisia indices.
In addition, Divisia indices satisfy the Proportionality Test, the Time Reversal Test and they approximately satisfy

the Factor Reversal Test[20], but they satisfy neither the Circular Test nor the Base Test.

Two related log-change statistical indices recently discovered are: (i) The Vartia (1974) log-change quantity index[21], given by

$$(2.29) \qquad \log Q_r^1 - \log Q_r^0 = \sum_i \bar{w}_i (\log x_i^1 - \log x_i^0)$$

where the weights are

$$\bar{w}_i = L(p_i^1 x_i^1,\ p_i^0 x_i^0) / L(\sum p_i^1 x_i^1,\ \sum p_i^0 x_i^0)$$

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20 The Factor Reversal Test requires that the sum of the log-changes in the price and quantity indices be equal to the log-change in total expenditure (Theil (1973)). Note that:

$$\sum_i w_i^* \log(p_i^1/p_i^0) + \sum_i w_i^* \log(x_i^1/x_i^0) = \sum w_i^* (\log p_i^1/p_i^0$$

$$+ \log x_i^1/x_i^0) = \sum w_i^* \log(p_i^1 . x_i^1/p_i^0 . x_i^0) = \sum w_i^* \log(w_i^1/w_i^0)$$

$$(m^1/m^0) = \sum w_i^* \log(w_i^1/w_i^0) + \log(m^1/m^0)$$

hence, $\sum w_i^* \log(w_i^1/w_i^0)$ is the discrepancy (relative to the Factor Reversal test holding) which as Theil (1973) shows, is quite small.

21 The price index is defined in an analogous manner.

and

$$\text{(2.30)} \quad L(a,b) = \begin{cases} (a-b)/(\log a - \log b), & a \neq b \\ a & a = a \end{cases}$$

is the logarithmic average defined for positive a and b. The Vartia index satisfies the Factor Reversal Test always, and the Proportionality Test only when $w_i^1 = w_i^0 \; \forall i$[22].

(ii) The Vartia (1974)-Sato (1976) log change quantity index, given by

$$\text{(2.31)} \quad \log Q_{rs}^1 - \log Q_{rs}^0 = \sum \bar{w}_i (\log x_i^1 - \log x_i^0)$$

where

$$\bar{w}_i = L(w_i^1, w_i^0) / \sum L(w_j^1, w_j^0)$$

and L is defined as in (2.30)

The Vartia (1974)-Sato (1976) index satisfies the Proportionality Test always and exactly satisfies the Factor Reversal Test[23].

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[22] See Vartia (1976)

[23] See Diewert (1976)

II.5 DIEWERT'S (1976) SUPERLATIVE CLASS OF INDICES

In the recent literature on index numbers, Diewert (1976) provides a link between the economic and statistical approach to index numbers, by attaching economic properties to statistical indices. These properties are defined in terms of the indices' ability to approximate a particular functional form for the aggregator function. What Diewert does is to show that, instead of estimating a functional form for an aggregator function, we can use a statistical index which provides both a parameter-free as well as a specification-free approximation to this functional form. To ensure that this statistical index approximates the functional form for the aggregator function, it is required that the following relation is satisfied

$$(2.32) \qquad u(x^r)/u(x^0) = Q(p^0, x^0, p^r, x^r) \qquad \forall\ r = 1 \ldots T$$

whenever $x^r > 0$ is the solution to the following aggregator maximization problem:

$$\max_{x}\{u(x):\ p^r.x \leq p^r.x^r, \quad x \geq 0\} \qquad r = 0,\ldots T$$

For a base period normalization $u(x^0) = 1$, equation (2.32) implies that the quantity index at time t equals the aggregator function evaluated at that point. If equation (2.32) is satisfied the quantity index, Q, is said to be exact for the aggregator function u[25]. It is obvious that the definition of exact statistical indices depends upon microeconomic maximizing behaviour and is completely independent of the form or properties the aggregator function might have. However, if we do not know the true functional form for the aggregator function (that is, if we do not have a priori information about preferences) it would be wise to choose a statistical index which is exact for a flexible functional form. Diewert termed statistical indices that are exact for flexible aggregator functions "superlative".

Following Diewert (1976) we will demonstrate how an exact index for the homogenous translog functional form can be derived. Consider the homogenous (of degree one) translog functional form:

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[25] Exact in the sense that its use is equivalent to the use of the functional form for the aggregator function.

$$\ell n\ u(x) = a_0 + \sum_i a_i \ell n\ x_i + 1/2 \sum_i \sum_j \beta_{ij} \ell n\ x_i \ell n\ x_j$$

where

$$\sum a_i = 1 \quad \text{and} \quad \sum_j \beta_{ij} = 0 \text{ (by homogeneity)}$$

Next, we make use of the quadratic approximation lemma[26] of Theil (1967, p. 222-23), and Kloek (1966).

$$(2.33) \qquad u(z^1) - u(z^0) = 1/2\left[\nabla u(z^1) + \nabla u(z^0)\right](z^1 - z^0)$$

where $\nabla u(z)$ is the gradient vector of u evaluated at z. Now, since for the translog

$$z_i^r = \ell n\ x_i^r \quad \text{and} \quad u(z^r) = \ell n\ u(x^r)$$

$$(2.34) \qquad \nabla u(z^r) = \partial u(x^r)/\partial x = \partial\left[\ell n\ u(x^r)\right]/\partial x$$

$$= \partial u(x^r)/\partial x \cdot x^r/u(x^r) = \hat{x}^r\ \nabla u(x^r)/u(x^r)$$

where $\hat{x}^r$ is the vector x diagonalized into a matrix. Then if we substitute (2.34) into (2.33), we obtain

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26 It is this lemma which provides the underpinning for the superlative index numbers.

$$\ln u(x^1) - \ln u(x^0) = 1/2\Big[\partial u(x^1)/\partial x . x^1/u(x^1)$$

$$+ \partial u(x^0)/\partial x . x^0/u(x^0)\Big](\ln x^1 - \ln x^0)$$

(2.35)

$$= 1/2\Big[\hat{x}^1 \ \nabla u(x^1)/u(x^1) + \hat{x}^0 \ \nabla u(x^0)/u(x^0)\Big]$$

$$(\ln x^1 - \ln x^0)$$

where

$$\ln x^r = \Big[\ln x_1^r \ \ln x_2^r .. \ln x_n^r\Big] \quad \text{for } r = 0,1$$

Using Wold's theorem (for a linearly homogeneous function)

$$p^r/p^r.x^r = \nabla u(x^r)/u(x^r)$$

and substituting the last two relations into (2.35) we obtain

$$\ln\Big[u(x^1)/u(x^0)\Big] = 1/2\Big[p^1.\hat{x}^1/p^1.x^1$$

$$+ p^0.\hat{x}^0/p^0.x^0\Big](\ln x^1 - \ln x^0)$$

$$= \sum_i 1/2(w_i^1 + w_i^0)(\ln x_i^1 - \ln x_i^0)$$

or

$$(2.36) \qquad u(x^1)/u(x^0) = \prod_i (x_i^1/x_i^0)^{1/2(w_i^1 + w_i^0)}$$

The right hand side of (2.36) is the discrete time Divisia index which, as we have just shown is exact for the homogenous translog. Since the homogenous translog is a flexible functional form the Divisia index is a superlative index[27].

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[27] In a strictly analogous manner, Diewert shows that the Fisher's Ideal Index number is exact, if, and only if, the function is the square root of a homogenous quadratic function (see also, Lau (1978)).

$$u(x) = \left[x' A x/2\right]^{1/2}$$

He also shows that the (homogenous) quadratic mean of order r flexible functional form is the only differentiable function which is exact for the following quadratic mean of order r quantity index

$$Q_r(p^0, x^0, p^1, x^1) = \left[\sum w_i^0 (x_i^1/x_i^0)^{r/2}\right]^{1/r} \cdot \left[\sum w_k^1 (x_k^1/x_k^0)^{-r/2}\right]^{-1/r}$$

where

$$w_i^0 = p_i^0 . x_i^0 / p^0 . x^0 \quad \text{and} \quad w_k^1 = p_k^1 x_k^1 / p^1 x^1$$

Note that the quadratic mean of order r index satisfies all Fisher's tests except the Circular test and the Factor Reversal test. It was also shown that the Vartia (1974) index is exact for the Cobb-Douglas aggregator function and that the Vartia (1974)-Sato (1976) index is exact for a homogenous of degree one C.E.S. functional form (see Sato (1976) and Lau (1978)).

II.6 SUMMARY OF THE PRINCIPAL ISSUES

If we have to use an index to measure the quantity of services of money, we obviously have to make a choice among alternative indices and we face a hard task. The difficulty is not that plausible indices are hard to find, but rather that there are too many plausible choices.

The economic approach, although it permits a wide variety of indices, provides no answer to the question: What is the proper (economic) index to be used in empirical demand analysis? The use of any index would reflect our *a priori* information about the structure of preferences among monetary assets. To fail to adopt the appropriate index is to commit a specification error. This problem might be partially overcome by introducing an economic index which is consistent with a large class of utility functions.

The statistical approach provides axioms or tests which serve as criteria in developing indices and in examining their characteristics. These tests reduce the number of conceivable indices and help us to identify the best possible index. This approach provides a strong rationale for using the Fisher Ideal index, since it best satisfies Fisher's system of tests. The Divisia index, on the other hand, is very appealing because of the Invariance property. This property characterizes no other index (see Hulten (1973)).

Diewert (1976) provides a new basis for selecting an index. He shows that both the Divisia index and the Fisher Ideal index are superlative (provide second-order approximations) and he argues that there is no quantitative difference between superlative indices, since they approximate each other to the second order for small changes in prices and quantities[28]. It is however, apparent that superlative indices are restricted to the class of functions that are linearly homogenous, flexible and quadratic in the logarithms.

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[28] Diewert (1978), however, shows that there are qualitative differences between these indices. In particular, only the Laspeyres and Vartia (1974) indices are consistent in aggregation, while superlative indices have an approximate consistency in aggregation property. Consistency in aggregation refers to a situation where the set of assets is partitioned into subsets and subindices for these subsets are calculated. Then the total index is calculated using the subindices and the same index formula. If this two-stage aggregation procedure gives the same results as a single stage aggregation procedure then we say that the index formula is consistent in aggregation.

In this thesis, in order to provide a quantitative assessment of the relative merits of simple-sum versus superlative monetary aggregation, we follow Barnett's (1980a) and Barnett, Offenbacher and Spindt's (1984) proposal and we use the Divisia quantity index. Hence, simple-sum and Divisia indices are constructed with the same components and component groupings and are then used as data in an empirical demand[29] system. We then examine these subaggregates with a view to asking whether or not satisfactory conditions for aggregation (over these subaggregates) hold. In this regard, we focus on flexible functional forms, and in particular, the quasi-homothetic translog.

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29 Barnett, Offenbacher and Spindt (1984) systematically compared the empirical performance of simple-sum and Divisia monetary aggregates relative to various policy criteria. They concluded that neither simple-sum nor Divisia aggregates were uniformly best relative to the considered criteria. However, they have shown that Divisia aggregates outperform the sum aggregates at high levels of aggregation. This is so, because the Sum aggregates at high levels of aggregation, heavily weight high yielding component assets while the Divisia aggregates attach lower weights to such high yielding assets.

CHAPTER III

THE DEMAND FOR MONETARY ASSETS IN A TWO-LEVEL MODEL OF DEMAND

III.1 INTRODUCTION

In this chapter, in order to investigate consumer monetary expenditure decisions in a utility maximizing framework, we develop a demand system that is capable of generating new results and is also, we argue, theoretically more satisfactory than the monetary asset demand functions now in use.

The approach adopted is the Strotz-Gorman two-stage optimization framework. This approach can be rationalized by certain assumptions regarding consumers' preferences. These assumptions and their analytical consequences are made explicit to emphasize the restrictive nature of familiar approaches which do not build upon a choice theoretic model.

As will be discussed below, a pleasing feature of our approach is that we replace the homotheticity assumption with the far more reasonable assumption of quasi-homotheticity, i.e., homotheticity with respect to a point other than the origin. Quasi homothetitic preferences imply linear Engel curves (income-consumption paths for fixed prices) which do not pass through the origin, while ordinary homotheticity forces linear Engel curves through the origin.

Another main feature of our approach is the use of a flexible second-order approximation, to an arbitrary indirect utility function, as a framework for empirically investigating the aggregation of monetary assets. To our knowledge no empirical study has systematically tested the parametric restrictions implied by the hypothesis of functional weak separability between monetary assets. Our approach is to use the Translog (Christensen, Jorgenson and Lau (1975)) functional form. This form is relatively attractive in that it does not restrict the value of the elasticities of substitution and it does not impose separability. In contrast, traditional forms such as the Cobb-Douglas and the C.E.S. utility functions are strongly separable and in addition, constrain the partial elasticities of substitution to be equal to each other (and in the case of the Cobb-Douglas to be equal to one). Moreover, a flexible functional form interpretation of the translog permits less restrictive tests of separability than the Berndt-Christensen exact translog framework.

The plan of the chapter is as follows: In the section that follows we present a general theoretical model, discuss the conditions which allow two-level optimization and finally indicate the tradeoff in choosing between a homothetic form and a quasi-homothetic form for monetary assets. In Section III.3, we specify a flexible functional form for the monetary services indirect utility function and determine the budget share system to be used in empirical work. Section III.4 spells out the stochastic specification and the method of estimation. Section III.5 presents the expenditure, price and elasticity of substitution formulas. Finally, Section III.6 outlines the various tests of the theory of demand and formulates the approximate weak separability hypotheses to be tested.

III.2 THE THEORETICAL FRAMEWORK

Cosnider an economy with identical individuals, having three types of goods: consumption goods, leisure and the services of monetary assets. It is assumed that the services of consumption goods, as well as the services of monetary assets and leisure enter as arguments in the representative individual's utility function

(3.1) $$u = u(c, \ell, m)$$

where

c = a vector of consumption goods

ℓ = leisure time, and

m = a vector of the services of monetary assets (assumed to be proportional to the stocks)

The utility function (3.1) is assumed to be maximized subject to the constraints:

(3.2) the Budget Constraint: $p_c . c + w . \ell + p . m = Y$

(3.3) the Time Constraint: $\ell \leq T$, and the

(3.4) Non-Negativity Constraints: $c \geq 0$; $\ell \geq 0$;

$m \geq 0$,

where

p_c = vector of prices of consumption goods, c

w = wage rate

p = vector of user costs of monetary assets

Y = full income (i.e., any exogenous income plus w.T)

T = total number of hours available

The model (3.1) to (3.4) is too general to be applied empirically. Furthermore, our interest is to study the demand for the services of monetary assets, if possible, without reference to other decisions of the representative individual[1]. For these two reasons we assume two-stage optimization[2]. The

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[1] This follows the tradition of Feige (1964), Chetty (1969), Donovan (1977), Offenbacher (1979), Barnett (1981a) and Ewiss and Fisher (1984).

[2] The notion of two-stage optimization was investigated in the context of consumer theory by Strotz (1957, 1959) and Gorman (1959); it refers to a sequential expenditure allocation, where in the first-stage the consumer allocates his expenditure among broad categories, and then in the second stage, he maximizes utility within each category.

first stage is "Budgeting" or "Price Aggregation," under which the consumer allocates full income[3] among consumption goods, leisure and foregone interest on monetary services[4]. The second stage is "Decentralization," under which the consumer allocates the foregone interest expense across all the monetary assets (and similarly consumption expenditures across consumption goods).

Before we proceed, a brief summary of some background material in both Budgeting and Decentralization is required[5]. Suppose that n elementary quantities

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[3] Full income is the amoung of income that would be available if all leisure were given up for work.

[4] It is the demand pattern of aggregate quantities which is of prime interest in this stage.

[5] For an extensive review of the subject, see Pollack (1970) and Blackorby, Primont and Russell (1975a)

are partitioned into R nonoverlapping subsets $(I_1,...,I_R)$. A superscribed vector indicates a subvector with respect to this partition. For example, x^r (or p^r) is the subvector of x (or p) corresponding to the r^{th} partition. Let x_i^r and p_i^r denote the quantity and the price of the i^{th} good in subset r. Assume individuals maximize a weakly separable utility function

$$u(x) = g[u_1(x^1), \ldots, u_R(x^R)] \tag{3.5}$$

subject to the budget constraint.

$$\sum_{r=1}^{R} p^r . x^r = Y \tag{3.6}$$

The solution to this problem can be written as

$$x^r = x^r(p^r, Y_r) \qquad r = 1,\ldots,R \tag{3.7}$$

where

$$(3.8) \qquad Y_r = \theta_r(p,Y) \qquad r = 1,\ldots,R$$

i.e., the demand for each group depends only on prices and expenditure for that group, while the group expenditures, Y_r, depend on all prices and on total expenditure (see Gorman (1959)).

It is obvious from (3.7) that weak separability does not imply that the demand for goods within a separable sub-group is independent of prices of goods outside of this group or of total income. As Phlips put it

> We do not say that the quantities in one branch are independent of the prices of commodities in other branches or of total expenditure. What we say is that total income and the prices of goods outside the branch enter the demand function for goods in the branch only through their effect on Y_r, the budget allotment to that branch. And that, when the budget allotment to the branch is known, we can ignore prices of goods outside the branch[6].

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6 Phlips (1974, p. 73).

Definition 1.

The consumer exhibits "Strong Budgeting" (or "Strong Price Aggregation") if there exists positive linear homogenous functions

$$(3.9) \qquad \{P_1(p^1),\ldots,P_R(p^R)\}$$

$$\text{where } P_r(p^r) \quad = \quad P_r(p_1^r\ldots p_k^r) \qquad r = 1,\ldots,R$$

such that the income allocation functions, θ_r, can be written as

$$(3.10) \qquad Y_r = \theta_r(p,Y) = \theta_r(P_1(p^1),\ldots,P_R(p^R),Y) \qquad r=1,\ldots,R$$

That is, the optimal category expenditures are functions of category price indices and income. The functions θ_r are called "perfect price aggregates," and the P_r, are called "perfect price indices."

Definition 2.

The consumer exhibits "strong decentralization," if the demand functions can be written as in (3.7), that is, the intracategory allocation depends only on group expenditure and group prices[7].

Weak (direct) separability, as illustrated in (3.5), implies Strong decentralization[8]. Strong budgeting simplifies the first stage allocation from (3.8) to (3.10)[9]. Gorman (1959) established necessary and sufficient conditions for Strong budgeting. He has shown that, given a weakly separable (direct) utility function with more than two categories, Strong budgeting is possible if and only if the utility function is homothetically separable, or strongly separable with category functions restricted by the generalized Gorman polar form (see below), or a mixture of the two structures, such as

$$(3.11) \qquad u(x) = \sum_{S=1}^{d} u_S(x^S) + F\left[u_{d+1}(x^{d+1}) \ldots u_R(x^R)\right]$$

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[7] For a discussion of "weak budgeting" and "weak decentralization" -- which are relevant in the contexts of indirect utility trees -- see Blackorby, Primont and Russell (1975a).

[8] In fact, weak (direct) separability is both necessary and sufficient for Strong decentralization, see Deaton and Muelbauer (1980).

[9] Strong budgeting is, simply, a structural characteristic of the income allocation functions, and is relatively attractive in the way that it simplifies estimation of (3.8).

where each $u_r(x^r)$ is homothetic in x^r for $r > d$ and each $u_S(x^S)$, $S = 1,\ldots,d$, can be written indirectly as

$$(3.12) \qquad V_S(p^S, Y_S) = G_S(Y_S/h_S(p^S)) + H_S(p^S)$$

where G_S is a strictly increasing function of its single argument, $h_S(p^S)$ is a linear homogeneous function of p^S, and $H_S(p^S)$ is homogenous of degree zero in p^S (see Anderson (1979) or Blackorby, Primont and Russell (1975a)). The indirect utility function (3.12) is known as "generalized Gorman polar form"[10].

We return now to discussing our model defined by (3.1)-(3.4). Our principal interest is to focus on the details of demand for the services of monetary assets, ignoring other types of goods. Two-stage optimization permits decentralization of this sort, if the representative individual's utility function (3.1) is weakly separable (implying a utility tree), in the services of monetary assets. That is, it must be possible to write the utility function as:

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10 In the two-group case (i.e., R = 2) and in the case where all but one of the aggregator functions are homothetic (i.e., d = 1) the structure (3.1) is only a sufficient condition for the existence of perfect price aggregates (see Blackorby, Primont and Russell (1978)).

(3.13) $u = \hat{u}(c, \ell, f(m))$

This separability condition, which implies the assumption that asset demands are independent of relative prices outside the monetary group, is treated as a maintained (untested) hypothesis in this thesis. Such treatment, although not entirely satisfactory, appears necessary for the type of empirical demand analysis with which we are concerned. As Parks (1983, p. 221) puts it:

> Some form of separability, as an a priory and untested hypothesis, underlies the great majority of empirical demand analysis. It is invoked implicitly when we consider the allocation of total expenditure in a static context; when we consider the choice of goods, ignoring the labour supply issues, and when we focus on the details of demand for particular groups of goods ignoring others.

The model that we want to estimate is derived from the second stage of the optimization, where the consumer allocates the foregone interest expenses among elementary monetary quantities. The maximization problem at this stage is:

(3.14) $\underset{m}{\text{Max}}\ f(m_1,\ldots,m_n)$

subject to

(3.15) Budget constraint: $\sum_{i=1}^{n} p_i m_i = Y_m$

(3.16) Non-Negativity constraints: $m_i \geq 0, \quad i = 1\ldots n$

where Y_m(we will drop the m subscript hereafter for convenience) has been determined in the first stage of the two-stage process.

Given that our interest lies in monetary aggregation, and in particular, in forming an aggregate that could be thought of as being determined at the first stage of a two-stage budgeting process, it is necessary to make the Strong budgeting assumptions outlined above. This requires that we place the monetary assets into either a homothetic or a generalized Gorman polar form subfunction. Suppose we assume a homothetic structure over monetary assets. Homotheticity implies that the monetary assets' cost or expenditure function takes the form

(3.17) $C(p,u) = u.f(p)$

where $f(p)$ is linearly homogenous and concave in p. Such preferences are quite restrictive as can be seen by application of Shephard's Lemma to give (Marshallian) demand functions of the form

$$(3.18) \qquad m_i = \frac{1}{f(p)} \cdot \frac{\partial f(p)}{\partial p} \cdot Y, \qquad i = 1,\ldots,n.$$

These demand equations (3.18) impose unitary expenditure elasticities, i.e.,

$$(3.19) \qquad \partial \ell n\ m_i / \partial\ \ell n\ Y = 1 \qquad i = 1,\ldots,n.$$

As a consequence, the assumption of a homothetic structure over monetary assets does not seem very attractive for empirical purposes.

The alternative is to assign a generalized Gorman polar form branch to monetary assets. This structure is a good deal more flexible than the homothetic structure, because it puts no restrictions on within-group price effects, and in addition, permits non-linear Engel curves[11]. On the other hand, however,

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[11] This can be seen by applying Roy's identity to (3.12).

due to additivity across generalized Gorman polar form branches, the inter-category substitution possibilities imply rather restricted behaviour[12].

We choose a specification for the monetary services utility function on the basis of a compromise between the conflicting criteria of a homothetic structure which is empirically restrictive but simplifies the estimation considerably, and of implying a generalized Gorman polar form which is empirically more interesting but can make the estimated demand system deeply nonlinear. We assigned to monetary services, a Gorman polar form -- the best known subclass of the generalized Gorman polar form, for which G_S (in

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12 Goldman and Uzawa (1964) have shown that direct weak separability implies a special structure on the between-group blocks of the Slutsky matrix, namely for good i of branch r and good j of branch s, $r \neq s$, the Slutsky term takes the form:

$$S_{ij}^{rs} = \phi_{rS}(\partial x_i^r/\partial Y).(\partial x_j^s/\partial Y)$$

where ϕ_{rs} summarizes the interrelation between categories r and s. However, additive direct utility implies $\phi_{rs} = \bar{\phi}$, $\forall$ r,s. That is, additive direct utility implies the absence of any special relationship between particular pairs of groups. This is somewhat restrictive though its consequences can be reduced by specifying only a few branches as in our case.

equation 3.12) is the identity function[13]. Preferences represented by a Gorman polar form are said to be quasi-homothetic, i.e., homothetic to a point other than the origin[14].

Given this assumption, all that remains, in order to obtain an expenditure system for empirical work, is to choose a specific functional form for the indirect utility function. We have chosen a flexible functional form[15]. Flexible functional forms do not impose any restrictive constraints a priori, thus pvoviding a suitable framework for estimating complementarity and substitutability.

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13 See Blackorby, Boyce, Nissen and Russell (1973).

14 Quasi-homotheticity is exploited extensively by Gorman (1961, 1976) and Deaton and Muellbauer (1980). Well known examples of quasi-homothetic utility functions are the Stone-Geary utility function (Stone (1954)), which is the quasi-homothetic version of the Cobb-Douglas, the S-Branch utility function (Brown and Heien (1972)) which is the quasi-homothetic version of the C.E.S., and Barnett's (1977) quasi-homothetic version of the generalized quadratic mean of order r function which gives rise to the g-hypo model.

15 The indirect utility function approach is preferred because it simplifies the estimation considerably, since prices are exogenous in explaining consumer behaviour. In Appendix III.B (at the end of this chapter), we point out the duality relation that exists between direct and indirect utility functions and we prove three theorems on the structure of indirect utility functions.

III.3 SPECIFICATION OF THE TRANSLOG FLEXIBLE FUNCTIONAL FORM

In recent years a number of empirical studies have made use of the flexible functional forms method to approximate aggregator functions. The advantage of this method is that the corresponding expenditure system will adequately approximate systems resulting from a broad class of aggregator functions. We adopt the translog flexible functional form which represents a second-order Taylor series approximation to an arbitrary twice differentiable aggregator function[16]. Our starting point is to express the indirect utility function in logarithmic form

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[16] Most of the flexible functional forms used to date have used a Taylor's expansion as the approximating mechanism and are capable of approximating an arbitrary aggregator function only locally (at a point). Their global properties are not well known (see Simmon and Weiserbs (1979) and Caves and Christensen (1980)). Due to this fact, Gallant (1981), introduced the Fourier flexible form--using the Fourier expansion as the approximating mechanism--which has global properties and is regarded to be an unbiased form. Also, Barnett (1983a) has used the Laurent series expansion--a generalization of the Taylor series expansion, possessing a better behaved remainder term --to generate the Laurent demand model which has known global properties. These new demand systems provide, obviously, new capabilities and their adoption in empirical research is clearly warranted. We leave the exploration of these new structures to future work.

$$\ell n\ V = f(\ell n\ q_1, \ldots, \ell n\ q_m) \tag{3.20}$$

where

$$q_i = p_i/(Y - \sum p_k \gamma_k) \qquad i = 1, \ldots, m,\ \gamma_k > 0, \text{ and} \tag{3.21}$$
$$y - \sum p_k \gamma_k > 0.$$

It is this definition of q_i that gives us the quasi-homothetic form. A second-order Taylor series expansion around q^* yields:

$$\ell n\ V^* = f(q^*) + \sum_{i=1}^{m} \partial f/\partial \ell n\ q_i \Big|_{q^*} \cdot (\ell n\ q_i - \ell_n\ q_i^*)$$
$$+ 1/2 \sum_{i=1}^{m} \sum_{j=1}^{m} \partial^2 f/\partial \ell n\ q_i\ \partial \ell_n q_j \Big|_{q^*} \cdot (\ell n\ q_i$$
$$- \ell n\ q_i^*) \cdot (\ell n\ q_j - \ell n\ q_j^*) \tag{3.22}$$

where V^* is the second-order approximation of V.

Assuming that the derivatives are locally constant, the translog function

$$\ell n\ V = a_0 + \sum_i a_i\ \ell n\ q_i + 1/2 \sum_i \sum_j \beta_{ij}\ \ell n\ q_i\ \ell n\ q_j \tag{3.23}$$

with symmetry imposed ($\beta_{ij} = \beta_{ji}$) is a second-order approximation to the indirect utility function (3.20) at the point $q^*[(= q_1^*, \ldots, q_m^*) = 1]$

where $\quad a_0 = f(q^*)$

(3.24) $\quad a_i = \partial f/\partial \ell n\, q_i \Big|_{q^*}$

$$\beta_{ij} = \partial^2 f/\partial \ell n\, q_i\, \partial \ell n\, q_j \Big|_{q^*}$$

Applying Roy's identity (and using the definition of q given in (3.21)), we can generate the corresponding budget shares as

(3.25) $$s_i = \frac{p_i . m_i}{Y} = \frac{p_i \cdot \gamma_i}{Y} + \left(1 - \frac{\sum p_k \gamma_k}{Y}\right) . \frac{a_i + \sum \beta_{ij} \ell n\, q_j}{a_M + \sum \beta_{jM} \ell n q_j}$$

where $a_M = \sum\limits_i a_i$ and $\beta_{jM} = \sum\limits_i \beta_{ji}$

In equation (3.25) we can think of the a's, β's and γ's as parameters to be estimated. However, equation (3.25) represents a non-homothetic (and also a non-quasi-homothetitic) version of the translog. Pollack and Wales (1980) refer to this model as the 'generalized translog model' (GTL). The quasi-homothetic translog model can be derived by imposing on the GTL the following restriction on the sums of the second-order parameters:

(3.26) $$\sum_i \beta_{ij} = 0 \quad \text{for all } j$$

The budget share equations corresponding to the quasi-homothetic translog model then are

$$(3.27) \qquad S_i = \frac{p_i \cdot m_i}{Y} = \frac{p_i \cdot \gamma_i}{Y} + (1 - \frac{\sum p_k \gamma_k}{Y}) \cdot (a_i + \sum_j \beta_{ij} \ln p_j), \qquad \forall i$$

where the normalization $a_M = \sum a_i = 1$ has been imposed. The form in (3.27) is nonlinear in parameters and variables though it has linear Engel curves which need not pass through the origin. If $\gamma = (\gamma_1 \ldots \gamma_n) = 0$ the quasi-homothetic model reduces to the homothetic model which is linear in the parameters:

$$(3.28) \qquad S_i = a_i + \sum_j \beta_{ij} \ln p_j, \qquad \forall i$$

The budget share equations in (3.27) form the basis of our empirical estimations. Once the model is estimated, a number of issues of interest can be investigated. First, the theoretical restrictions that are not maintained hypotheses, i.e., monotonicity, nonnegativity and quasi-convexity can be checked. Second, the price and expenditure elasticities as well as the elasticities of substitution can be calculated and compared with those in the literature. Finally, we can examine whether any subgroups of the monetary assets can be further aggregated (i.e., are separable from the others).

III.4 STOCHASTIC SPECIFICATION AND THE METHOD OF ESTIMATION

In order to estimate the share equation system given by equation (3.27) a stochastic version must be specified and the error term given an interpretation. Since equation (3.27) is in share form and only exogenous variables appear on the right hand side, it is reasonable to assume that the observed shares deviate from the true shares by an additive disturbance term u_i. The system of budget share equations can then be rewritten vectorially at time t as

$$(3.29) \qquad S_t = f_t(X_t, \theta) + u_t$$

Here X_t are the exogenous variables, θ represents the vector of unknown parameters and the vector u_t is interpreted as being the result of errors in the individual's optimization process in determining monetary asset holdings. It is assumed that u_t is distributed normally, independently of the exogenous variables. Furthermore, it is assumed, at least initially, that u_t is a "classical" disturbance term with the following properties:

$$E(u_t) = 0$$

(3.30)

$$E(u_t u_s') = \begin{cases} \Sigma \text{ for } s = t \\ 0 \text{ for } s \neq t \end{cases} \quad \text{all } s,t$$

where Σ is symmetric and positive definite covariance matrix.

Assumption (3.30) permits correlation among the disturbances at time t but rules out the possibility of autocorrelated disturbances. This assumption and the fact that the S_t (and therefore the u_t) satisfy an adding up condition (because this is a singular system) imply that the disturbance covariance matrix is also singular. If autocorrelation in the disturbances is absent, Barten (1969) has shown that full information maximum likelihood estimates of the parameters can be obtained by arbitrarily deleting one equation in such a system. The resulting estimates are invariant with respect to the equation deleted. The parameter estimates from the deleted equation can be recovered from the restrictions.

We initially estimated the regression model

(3.29). Early results, however, led us to believe that the disturbances were serially correlated[18]. We therefore assumed first-order autocorrelation so that

$$(3.31)\qquad u_t = Ru_{t-1} + v_t$$

where $R = (R_{ij})$ is an $n \times n$ matrix of unknown parameters and v_t is a "classical" disturbance term.

Writing equation (3.29) for period t-1 and multiplying by R we obtain

$$(3.32)\qquad RS_{t-1} = Rf_{t-1}(X_{t-1}, \theta) + R\, u_{t-1}$$

Subtracting (3.32) from (3.29) and rearranging (using (3.31)) we obtain the following final model:

$$(3.33)\qquad S_t = f_t(X_t, \theta) - R\, f_{t-1}(X_{t-1}, \theta) + R\, S_{t-1} + v_t$$

We estimated the difference equation system

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18 The computed equation-by-equation Durbin-Watson statistics were very low.

(3.33) using the procedure described above. However, we incorporated a result developed by Berndt and Savin (1977). They have shown that the adding up property of a singular system with autocorrelation, imposes additional restrictions on the parameters of the autoregressive process. In particular, if one assumes no autocorrelation across equations (i.e., R is diagonal) the autocorrelation coefficients for each equation must be identical (i.e., $R_{11} = R_{22} = \ldots = R$). Consequently, this is what we have assumed[19].

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19 When these restrictions are not imposed, any estimation and any hypothesis testing are conditional on the equation deleted. Moreover, a non-diagonal version would involve a much larger estimation problem. So we have taken the case of identical coefficients.

III.5 EXPENDITURE, PRICE AND SUBSTITUTION ELASTICITIES

The system of equations in (3.27) provides a complete characterization of consumer preferences over monetary services and can be used to estimate the price and income elasticities as well as the Allen partial elasticities of substitution. As will be shown below, these elasticities are particularly useful in judging the validity of the parameter estimates. These elasticities are derived from (3.27) by writing the left-hand side as

$$X_i = \frac{S_i . Y}{p_i}$$

or logarithmically as

$$\ell n\ X_i = \ell n\ S_i - \ell n\ p_i + \ell n\ Y$$

The income elasticities can then be calculated as

$$(3.34) \qquad n_i = \frac{\partial \ell n\ X_i}{\partial \ell n\ Y} = 1 + \frac{\partial \ell n\ S_i}{\partial \ell n\ Y}$$

$$= 1 - \frac{p_i \cdot \gamma_i}{S_i \cdot Y} + \frac{\sum p_k \gamma_k}{S_i \cdot Y}(a_i + \sum_j \beta_{ij}\ \ell n\ p_j), \ \forall i$$

Similarly, the own-price elasticities are given as

$$(3.35) \qquad n_{ii} = \frac{\partial \ \ell n \ X_i}{\partial \ \ell n \ p_i} = -1 + \frac{\partial \ \ell n \ S_i}{\partial \ \ell n \ p_i}$$

$$= -1 + (1 - \frac{\sum p_k \gamma_k}{Y}) \frac{\beta_{ii}}{S_i}$$

$$+(1-a_i \sum_j \beta_{ij} \ \ell n \ p_j) \frac{p_i \cdot \gamma_i}{S_i \cdot Y}, \quad \forall i$$

and the cross-price elasticities as

$$(3.36) \qquad n_{ij} = \frac{\partial \ \ell n \ X_i}{\partial \ \ell n \ p_j} = \frac{\partial \ \ell n \ S_i}{\partial \ \ell n \ p_j}$$

$$= (1 - \frac{\sum p_k \gamma_k}{Y}) \frac{\beta_{ij}}{S_i} - \frac{p_j \gamma_j}{S_i \cdot Y} (a_i + \sum_j \beta_{ij} \ \ell n \ p_j), \ \forall i$$

If $n_{ij} > 0$ the assets are (gross) substitutes, if $n_{ij} < 0$ they are (gross) complements, and if $n_{ij} = 0$ they are independent.

To derive the elasticities of substitution we make use of Slutsky's (1915) equation rewritten in elasticity terms[20].

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[20] See Allen and Hicks (1934, p. 201-202).

$$(3.37) \qquad n_{ij} = S_j(\sigma_{ij} - n_i)$$

where σ_{ij} is the Allen partial elasticity of substitution that measures the response of derived demand to a price change holding utility and all other prices fixed. The product $S_j\ \sigma_{ij}$ gives the income compensated cross-price elasticity (the percent change in X_i resulting from a one percent change in p_j if income is allowed to vary so that utility is held constant). From (3.37) we can solve for the elasticity of substitution as

$$(3.38) \qquad \sigma_{ij} = n_{ij}/S_j + n_i$$

$$= 1 - p_i \cdot \gamma_i/S_i \cdot Y + (1 - \sum p_k\gamma_k/Y)\beta_{ij}/S_i \cdot S_j$$

$$+ (\sum p_k\gamma_k - p_j\gamma_j/S_j) \cdot \frac{1}{S_i \cdot Y}\ (a_i$$

$$+ \sum_j \beta_{ij}\ \ell n\ p_j)$$

Estimates of all these elasticities are reported in chapter IV.

III.6 HYPOTHESES TESTING

Assuming utility maximization, the estimated demand system (3.27) must satisfy integrability. Following Christensen, Jorgenson and Lau (1975), we restrict the parameters in (3.27) (as part of the maintained hypothesis) to satisfy adding up and symmetry of the substitution matrix. Adding up requires that the budget shares sum to unity in our model, thus the adding up condition requires

$$\sum_i a_i = 1$$

and

$$\sum_i \beta_{ij} = 0, \qquad \forall\, i,j$$

while symmetry requires that

$$\beta_{ij} = \beta_{ji}, \qquad \forall\, i,j$$

These restrictions on equation (3.27) are not tested in what follows. However, we do test the theoretical restrictions that are not part of the maintained hypothesis, i.e., non-negativity, monotonicity and the curvature conditions on the indirect utility function -- as well as some functional form restrictions. We briefly outline the nature of these restrictions in what follows.

The non-negativity condition (which requires that the values of the fitted demand functions be nonnegative; $x_i \geq 0$, $\forall$ i) can be easily checked by direct computation of the fitted budget shares, S_i. The monotonicity condition (which requires that $\partial V/\partial p_i < 0$, $\forall$ i) can also be checked by direct computation of the gradient vector of the estimated indirect utility function. The curvature conditions require quasi-convexity of the indirect utility function (and hence, quasi-concavity of the direct utility function). This implies that the Allen partial elasticities of substitution must provide a negative semi-definite matrix[21,22] (the principal minors of

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[21] See Diewert (1977)

[22] Berndt, Diewert and Darrough (1977) have shown that a necessary (but not sufficient condition) for negative semi-definiteness of (σ_{ij}) is that the own-price elasticities of substitution be non-positive.

(σ_{ij}) must alternate in sign with the first-order minor negative).

Before we come to the functional form restrictions another area of concern consists of investigation of first-order autocorrelation. Since the translog specification with autocorrelation is taken as the maintained hypothesis, we also test

$$H_0 : R = 0$$

vs

$$H_1 : \text{Not } H_0$$

i.e., there is no autocorrelation.

Lastly, we consider the aggregation conditions (separability). To our knowledge, these conditions in the case of monetary assets have not yet been <u>systematically</u> tested. Only Offenbacher (1979), using an exact interpretation of the Linear Expenditure System[23] has carried out separability tests in the case of U.S. data and these failed to support the existence of monetary aggregates.

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[23] This system is derived by assuming that the indirect utility function is a homogenous (of degree minus one) transcendental logarithmic function.

The tests we carry out for separability (i.e., aggregation) are based on the translog function as a second-order approximation to an arbitrary indirect utility function[24]. This approach has been proposed by Jorgenson and Lau (1975) and Denny and Fuss (1977). The latter distinguishes between the translog as an approximation to a functional form from the more restrictive notion of the translog as an exact functional form, and show that the latter approach provides a more restrictive test of weak separability, and that this test is nested in the approximate test[25].

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24 In the production context, Berndt and Christensen (1973a, 1973c, 1974) have attempted to test for weak separability and the possible existence of consistent aggregates of labour and capital, using an exact interpretation of the translog function. However, as was shown by Denny and Fuss (1977) and Blackorby, Primont and Russell (1977), the exact interpretation of the translog cannot provide a flexible second-order approximation to separable preferences (or to a production function) and that the Berndt-Christensen test is a joint test of weak separability and a linear logarithmic aggregator function.

25 The Berndt-Christensen exact test is a test for global separability (separability at all points of the utility surface) while the Denny-Fuss approximate test is a test for local separability (separability only at the point of expansion $\ln p = 0$).

To derive our approximate tests for weak separability we consider the separability restrictions associated with restrictions on the functional form. With four variables, three separability patterns exist: the separability of two variables from the other two variables; the symmetric separability of two variables from the other two variables; and the separability of three variables from the fourth. These possibilities and the corresponding parametric restrictions[26] are shown in Table III.1.

Since we have four variables, with the first weak separability pattern $(f(G(\ell n\ q_i,\ \ell n\ q_j),\ \ell n\ q_k,\ \ell n\ q_\ell))$, there are six ways of chossing a group of two variables to be separable from the two other variables. Corresponding to each possibility there are two parametric restrictions analogous to those in Table III.1. Under the second type of weak separability $(f(G(\ell n\ q_i,\ \ell n\ q_j),\ H(\ell n\ q_k,\ \ell n\ q_\ell)))$, there are three ways of placing two variables in each group. Corresponding to each possibility there are four

[26] The derivation of these restrictions is based on the apparatus developed by Denny and Fuss (1977). The parameters are those in the share equations (3.27). These restrictions hold under the maintained hypothesis of quasi-homotheticity ($\sum_i \beta_{ij} = 0$).

TABLE III.1

PARAMETRIC RESTRICTIONS FOR APPROXIMATE WEAK SEPARABILITY

Separability Pattern	Parametric Restrictions
$F(G(\ell n\ q_i,\ \ell n\ q_j),\ \ell n\ q_k,\ \ell n\ q_\ell)$	$a_i/a_j = \beta_{ik}/\beta_{jk} = \beta_{i\ell}/\beta_{j\ell}$
$F(G(\ell n\ q_i,\ \ell n\ q_j),\ H(\ell n\ q_k,\ \ell n\ q_\ell))$	$a_i/a_j = \beta_{ik}/\beta_{jk} = \beta_{i\ell}/\beta_{j\ell}$
	$a_k/a_\ell = \beta_{ik}/\beta_{i\ell} = \beta_{jk}/\beta_{j\ell}$
$F(G(\ell n\ q_i, \ell n\ q_j,\ \ell n\ q_k),\ \ell n\ q_\ell)$	$a_i/a_j = \beta_{i\ell}/\beta_{j\ell}$, $a_i/a_k = \beta_{i\ell}/\beta_{k\ell}$,
	$a_j/a_k = \beta_{j\ell}/\beta_{k\ell}$

parametric restrictions. However, only three of these restrictions are independent, for if we add any three we can obtain the fourth. Finally, under the third type of weak separability ($f(G(\ell n\ q_i, \ell n\ q_j, \ell n\ q_k), \ell n\ q_\ell)$), we can distinguish among four possible ways with three variables in one group. Corresponding to each possibility there are three restrictions, though only two are independent.

Next we want to express the conditions for weak separability in terms of the free parameters of the model. Our model has thirteen free parameters (i.e., parameters that are estimated directly). Under each separability type we must be able to eliminate a number of free parameters equal to the number of independent parametric restrictions corresponding to that separability type. However, depending on the equation deleted, we cannot always independently eliminate a free parameter[27]. To avoid this problem we have treated each set of parametric

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27 Consider the approximate separability restrictions for $F(G(\ell n\ q_1, \ell n\ q_2), \ell n\ q_3, \ell n\ q_4)$ weak separability, in terms of the parameters of the translog function

$a_1\beta_{23} = a_2\beta_{13}$ and $a_1\beta_{24} = a_2\beta_{14}$.

restrictions as a simultaneous system of equations and then used the non-trivial solutions of this system to impose the separability restrictions[28]. The parameter restrictions which were used to impose the separability types listed in Table III.1 are presented in Appendix III.A to this chapter.

In the following chapter, we describe the data used and consequently estimate the share equations specified in equation (3.27). Based on the parameters obtained from the regression, the price and expenditure elasticities as well as the elasticities of substitution are calculated. Finally, weak separability tests are carried out.

Suppose we have chosen to delete the S_1 equation. We then want to express these conditions in terms of our thirteen free parameters -- a_2, a_3, a_4, β_{22}, β_{23}, β_{24}, β_{33}, β_{34}, β_{44} and the four γ's. By substituting into the above conditions the quasi-homotheticity restriction, $\sum_i \beta_{ij} = 0$, and the normalization, $\sum_i a_i = 1$, and rearranging we obtain

$$\beta_{23} = -a_2(\beta_{33} + \beta_{34})/(1-a_3-a_4) \text{ and } \beta_{24} = -a_2(\beta_{34} + \beta_{44})/(1-a_3-a_4)$$

it is not possible, to make all these substitutions, however, when we delete either the S_3 or the S_4 equation.

[28] The results were checked by estimating the model more than once, deleting different equations and using different non-trivial solutions each time.

APPENDIX III.A

In Table III.A in this appendix we indicate how the restrictions in Table III.1 were imposed in the estimation. To see how the table works, consider row 1 of Table III.A. The 13 elements of this row (continued on the second and third pages of the table) indicate what restrictions must be placed on a_M if a_M appears as a parameter in the system to be estimated. Where the table entry simply reads a_M, no restriction is required.

TABLE III.A

MEANS OF IMPOSING SEPARABILITY RESTRICTIONS

Unrestricted Parameter	SEPARABILITY TYPES [(M,C),S,T]	[(M,S),C,T]	[(M,T),C,S]	[(C,S),M,T]	[(C,T),M,S]
a_M	$\frac{a_C(\beta_{MM} + \beta_{MC})}{\beta_{MC} + \beta_{CC}}$	$\frac{a_S(\beta_{MM} + \beta_{MS})}{\beta_{MS} + \beta_{SS}}$	a_M	a_M	a_M
a_C	a_C	a_C	a_C	$\frac{a_S(\beta_{CC} + \beta_{CS})}{\beta_{CS} + \beta_{SS}}$	a_C
a_S	a_S	a_S	a_S	a_S	a_S
β_{MM}	β_{MM}	β_{MM}	β_{MM}	β_{MM}	β_{MM}
β_{MC}	β_{MC}	$\frac{\beta_{CS}(\beta_{MM} + \beta_{MS})}{\beta_{MS} + \beta_{SS}}$	$\frac{a_M(\beta_{CC} + \beta_{CS})}{a_C + a_S - 1}$	$\frac{\beta_{MS}(\beta_{CC} + \beta_{CS})}{\beta_{CS} + \beta_{SS}}$	$\frac{a_C(\beta_{MM} + \beta_{MS})}{a_M + a_S - 1}$
β_{MS}	$\frac{\beta_{CS}(\beta_{MM} + \beta_{MC})}{\beta_{MC} + \beta_{CC}}$	β_{MS}	$\frac{a_M(\beta_{CS} + \beta_{SS})}{a_C + a_S - 1}$	β_{MS}	β_{MS}
β_{CC}	β_{CC}	β_{CC}	β_{CC}	β_{CC}	β_{CC}
β_{CS}	β_{CS}	β_{CS}	β_{CS}	β_{CS}	$\frac{a_C(\beta_{MS} + \beta_{SS})}{a_M + a_S - 1}$
β_{SS}	β_{SS}	β_{SS}	β_{SS}	β_{SS}	β_{SS}

MEANS OF IMPOSING SEPARABILITY RESTRICTIONS

Unrestricted Parameter	SEPARABILITY TYPES [(S,T),M,C]	[(M,C),(S,T)]	[(M,S),(C,T)]	[(M,T),(C,S)]
a_M	a_M	a_M	a_M	a_M
a_C	a_C	a_C	a_C	a_C
a_S	a_S	a_S	a_S	a_S
β_{MM}	β_{MM}	β_{MM}	β_{MM}	β_{MM}
β_{MC}	β_{MC}	β_{MC}	$\frac{-a_C(\beta_{MM} + \beta_{MS})}{1-a_M - a_S}$	$\frac{-a_M a_C(\beta_{CS} + \beta_{SS})}{a_S(1-a_C-a_S)}$
β_{MS}	$\frac{a_S(\beta_{MM} + \beta_{MC})}{a_M + a_C-1}$	$\frac{-a_S(\beta_{MM} + \beta_{MC})}{1 - a_M - a_C}$	β_{MS}	$\frac{-a_M(\beta_{CS} + \beta_{SS})}{1 - a_C - a_S}$
β_{CC}	β_{CC}	$\frac{a_C(\beta_{MM} + \beta_{MC})}{a_M} - \beta_{MC}$	β_{CC}	$\frac{a_C(\beta_{CS} + \beta_{SS})}{a_S} - \beta_{CS}$
β_{CS}	$\frac{a_S(\beta_{MC} + \beta_{CC})}{a_M + a_C - 1}$	$\frac{-a_C a_S(\beta_{MM} + \beta_{MC})}{a_M(1-a_M-a_C)}$	$\frac{-a_C a_S(\beta_{MM} + \beta_{MS})}{a_M(1-a_M-a_S)}$	β_{CS}
β_{SS}	β_{SS}	β_{SS}	$\frac{a_S(\beta_{MM} + \beta_{MS})}{a_M} - \beta_{MS}$	β_{SS}

MEANS OF IMPOSING SEPARABILITY RESTRICTIONS

Unrestricted Parameter	SEPARABILITY TYPES $[(M,C,S),T]$	$[(C,S,T),M]$	$[(M,C,T),S]$	$[(M,S,T),C]$
a_M	a_M	a_M	$\frac{-(1 - a_S)\beta_{MS}}{\beta_{SS}}$	$\frac{-(1 - a_C)\beta_{MC}}{\beta_{CC}}$
a_C	a_C	$\frac{-(1 - a_M)\beta_{MC}}{\beta_{MM}}$	$\frac{-(1 - a_S)\beta_{CS}}{\beta_{SS}}$	a_C
a_S	a_S	$\frac{-(1 - a_M)\beta_{MS}}{\beta_{MM}}$	a_S	$\frac{-(1 - a_C)\beta_{CS}}{\beta_{CC}}$
β_{MM}	β_{MM}	β_{MM}	β_{MM}	β_{MM}
β_{MC}	β_{MC}	β_{MC}	β_{MC}	β_{MC}
β_{MS}	β_{MS}	β_{MS}	β_{MS}	β_{MS}
β_{CC}	$\frac{a_C(\beta_{MM} + \beta_{MC} + \beta_{MS})}{a_M} - (\beta_{MC} + \beta_{CS})$	β_{CC}	β_{CC}	β_{CC}
β_{CS}	β_{CS}	β_{CS}	β_{CS}	β_{CS}
β_{SS}	$\frac{a_S(\beta_{MM} + \beta_{MC} + \beta_{MS})}{a_M} - (\beta_{MS} + \beta_{CS})$	β_{SS}	β_{SS}	β_{SS}

NOTE: No restrictions were imposed on the γ's.

APPENDIX III.B

SOME REMARKS ON DUALITY AND THE STRUCTURE OF UTILITY FUNCTIONS

The structure of preferences can be represented by either a direct or an indirect utility function. However, a structural property, such as separability, of the direct utility function does not necessarily carry over to the indirect utility function. In general direct separability implies a different preference ordering than indirect separability. In what follows, we briefly review some background material of neoclassical consumer theory and consequently derive a group of results clarifying the behavioural implications of direct and indirect separability for the practice of applied demand analysis.

UTILITY MAXIMIZATION

Let the vector $x = (x_1, \ldots, x_n)'$ represent n elementary quantities and $p = (p_1 \ldots p_n)'$ represent the corresponding price vector. Defined on quantities is a utility function u(x) which is twice differentiable and quasi-concave. The classical consumer problem is

$$\text{(1)} \qquad \max_x u(x) \quad \text{subject to } p'.x \leq Y^{1}$$

where Y is total expenditure.

The necessary and sufficient conditions for utility maximization are

$$\text{(2)} \qquad u_i(x) = \lambda p_i \qquad \text{and} \qquad p'.x = Y$$

with solutions

$$\text{(3)} \qquad x = x(p,Y) \qquad \text{and} \qquad \lambda = \lambda(p,Y),$$

where x = x(p,Y) is homogenous of degree zero in p and Y, and represents the system of (Marshallian) demand functions.

An alternative method of deriving the system of Marshallian demand functions is from the indirect utility function (defined on prices and total expenditure) which is the dual of the direct utility

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[1] Assuming non-satiety the inequality in the budget constraint reduces to equality, or else the constraint is redundant.

function when the latter is maximized subject to a budget constraint. The indirect utility function defined as

$$(4) \qquad \hat{V}(p,Y) = \max_{x}\{u(x) : p'.x \leq Y\} ,$$

and is continuous, quasiconvex in p, increasing in Y and homogenous of degree zero in p and Y. The last property enables us to write

$$(5) \qquad V(q) = \max_{x} \{u(x) : q . x \leq 1\}$$

where $q = p/Y$ and $V(q) = \hat{V}(p,Y)$ is continuous, quasi-convex and increasing.

Applying Lau's (1970) Roy's identity we get

$$(6) \qquad x_i = -1/\lambda . \frac{\partial V}{\partial q_i} , \qquad i = 1 \ldots n$$

where λ is the Lagrange multiplier.

FUNCTIONAL SEPARABILITY

The notion of separability was first introduced by Sono (1947, English translation 1961), and Leontief (1947). It is of considerable importance, because it provides a means of justifying the use of aggregates (of goods or assets) and also resolves the statistical problem caused by the lack of degrees of freedom, since it rationalizes the estimation of a smaller set of demand equations. In the context of preference structures there are five different separability concepts: direct, indirect, implicit (or quasi), direct pseudo, and indirect pseudo separability. These different concepts give rise to both different grouping patterns and different behavioural implications[2]. Here we examine only direct and indirect separability. In doing so, some basic notation needs to be defined.

Suppose that the n elementary quantities are partitioned into R non-overlapping subsets $(I_1 \ldots, I_R)$. A superscribed vector indicates a subvector with respect to this partition. For example, x^r and p^r are the subvectors of x and p corresponding to the r^{th} partition.

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2 For a good exposition of alternative forms of separability and their behavioural implications, see Pudney (1981).

Definition 1. Preferences are directly weakly separable if, and only if, the direct utility function can be written as

$$(7) \qquad u(x) = F(u_1(x^1),\dots,u_R(x^R))$$

where F is strictly increasing in each argument and each u_r is continuous.

The requirement of direct weak separability is that the marginal rate of substitution between any two goods in a separable component group be invariant with respect to any commodity outside the group. Algebraically

$$(8) \qquad \partial(\partial u/\partial x_i/\partial u/\partial x_j)/\partial x_k = 0 \qquad i,j\varepsilon I_r, \quad k \not\varepsilon I_r$$

A sufficient condition for direct weak separability is, of course, perfect substitutability. Under perfect substitutability the ratio of marginal utilities is constant and hence invariant to any commodity change.

Definition 2. Preferences are directly strongly separable if, and only if, the direct utility function can be written as

$$u(x) = F(u_1(x^1) + \ldots + u_R(x^R)) \tag{9}$$

$$= F(\sum_{r=1}^{R} u_r(x^r))$$

where F is strictly increasing in each argument and each u_r is continuous. The requirement of direct strong separability can be expressed algebraically as

$$\partial(\partial u/\partial x_i/\partial u/\partial x_j)/\partial x_k = 0 \text{ for all } i \in I_r,\ j \in I_s,\ k \notin I_r \cup I_s \tag{10}$$

Definition 3. Preferences are indirectly weakly separable if, and only if, the indirect utility function can be written as

$$V(q) = V(V_1(q^1), \ldots, V_R(q^R)) \tag{11}$$

where V_r, $r = 1,\dots,R$, is continuous, quasiconvex and a nonincreasing indirect aggregator function (price index), while V is quasiconcave continuous and increasing. The algebraic requirement of indirect weak separability is that[3] :

$$(12) \qquad \partial(\partial V\, q_i/\partial V/q_j)/\partial q_k = 0, \quad \forall i,j \in I_r, \quad k \notin I_r$$

or using Roy's identity.

$$(13) \qquad \partial(x_i/x_j)/\partial q_k = 0, \quad \forall\, i,j \in I_r, \quad k \notin I_r$$

and since $V(q) = V(p,Y)$, due to homogeneity of degree zero in p and Y, equation (13) can be written as

$$(14) \qquad \partial(x_i/x_j)/\partial p_k = 0 \quad \forall\, i,j \in I_r, \quad k \notin I_r$$

which implies that the optimal consumption ratios within I_r are independent of the k^{th} price (outside I_r).

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3 See Blackorby, Primont and Russell (1975b).

Definition 4. Preferences are indirectly strongly separable iff the indirect utility function can be written as

$$V(q) = V(V_1(q^1)+\ldots+V_R(q^R)) = V(\sum_{r=1}^{R} V_r(q^r)) \tag{15}$$

The requirement of indirect strong separability can be expressed algebraically as

$$\partial(x_i/x_j)/\partial p_k = 0 \quad i, \varepsilon\ I_r, \quad j\ \varepsilon\ I_S, \quad k \not\in I_r \cup I_S \tag{16}$$

Leaving aside the choice between direct and indirect separability, if the category functions are homothetic, quasi-homothetic or homogenous, the utility function is said to be homothetically, quasi-homothetically or homogenously weakly (or strongly) separable, respectively. Also, if there are only two groups (i.e., $R = 2$) weak and strong separability coincide (see Blackorby, Primont and Russell (1975a)).

The structure of preferences can be represented by either a direct or an indirect utility function. The indirect utility function is more appealing, because it simplifies the estimation procedure considerably, since it has prices exogenous in explaining consumer behaviour.

However, as noted earlier, a structural property of the direct utility function does not imply the same property on the indirect utility function. In order to implement a model of demand based on the indirect function that satisfies properties of the direct utility function, a correspondence between direct and indirect properties is needed. In order to proceed in this manner, we state some relationships between direct and indirect utility functions in the propositions that follow.

<u>Proposition 1</u>. (Samuelson (1965), Lau (1970), Katzner (1970)): Positive direct homogeneity implies negative indirect homogeneity and vice versa.

<u>Proposition 2</u>. (Samuelson (1965), Lau (1970)): Positive direct homotheticity implies negative indirect homotheticity and vice versa.

<u>Proposition 3</u>. (Blackorby, Primont and Russell (1975b)): $V(q)$ is weakly (strongly) separable in a partition if, and only if, $\hat{V}(p,Y)$ is weakly (strongly) separable in the same partition.

<u>Proposition 4</u>. (Lau (1970), Lemma III): A weakly separable utility function is homothetic only if each category function is homothetic.

Proposition 5. (Lau (1970), Theorem VI): A homothetic direct utility function is weakly separable if, and only if, the indirect utility function is weakly separable in the same partition. Homotheticity, however, is not a necessary condition.

In the remaining part of this section we state and prove two theorems. The first relates to indirect separability and the equality conditions implied on the cross price elasticities while the second relates to indirect homothetic separability and the equality on the Allen partial elasticities of substitution.

Theorem 1. The indirect utility function is weakly separable in the partition I_r, if

$$(17) \qquad e_{ik} = e_{jk} \qquad i,j \in I_r, \quad K \notin I_r$$

where e_{ik} is the cross price elasticity.

Proof. The Leontief-Sono separability condition implies

$$\partial(x_i/x_j)/\partial p_k = 0 \qquad i,j \in I_r, \quad k \notin I_r$$

or, equivalently,

$$x_j \frac{\partial x_i}{\partial p_k} = x_i \frac{\partial x_j}{\partial p_k} \quad ;$$

this in turn implies (17).

Corollary 1. Indirect strong separability is equivalent to

$$e_{ik} = e_{jk} \qquad i \in I_r, \quad j \in I_s, \quad k \notin I_r \cup I_s$$

Proof. The corollary is implied by the theorem[4].

Theorem 2. If the indirect utility function is weakly separable with homothetic aggregator functions, then the Allen partial elasticities of substitution σ_{ik}, σ_{jk} $(i,j \in I_r,\ k \notin I_r)$ are equal.

---

[4] Houthaker (1960) has shown that indirect additivity (complete strong separability) implies that all commodities have equal percentage responses to the change in any single commodity price $e_{ik} = e_{jk} (i \neq k;\ i,j,k = 1,\dots,n)$.

Proof. By theorem 1, indirect weak separability implies the equality condition (17). Using the Slutsky (1915) equation reqritten in elasticity terms,

(18) $$e_{ik} = s_k(\sigma_{ik} - e_i) \quad \text{and} \quad e_{jk} = s_k(\sigma_{jk} - e_j)$$

and using $e_i = e_j = 1$, the equality condition (17) reduces to

(19) $$\sigma_{ik} = \sigma_{jk} \qquad i,j \in I_r, \quad k \notin I_r$$

Corollary 2. Indirect strong separability is equivalent to

(20) $$\sigma_{ik} = \sigma_{jk} \qquad i \in I_r, \quad j \in I_s, \quad k \notin I_r \cup I_s$$

Proof. The corollary is implied by the theorem.

Theorem 3. If the indirect utility function is homothetic, then (19) holds, if the indirect utility function is weakly separable.

Proof. If the indirect utility function is weakly separable and homothetic, it is homothetically separable[5,6]. Hence the equality condition (19) follows from theorem 2.

Corollary 3. If the indirect utility function is homothetic, then (20) holds if, and only if, the indirect utility function is strongly separable.

Proof. The proof is implied by the theorem.

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5 See Lau (1970).

6 Homotheticity of the indirect utility function and weak separability implies unitary sectoral expenditure elasticities, in which case overall expenditure elasticities are equal within each group.

CHAPTER IV

DATA AND EMPIRICAL RESULTS

In this chapter we first describe the data and the construction of quantity and price indices for the four monetary categories. Then in Section IV.2 we report parameter estimates and estimates of the price and expenditure elasticities as well as the partial elasticites of substitution. We also report on tests of the validity of the restrictions implied by the hypotheses of homotheticity and first-order autocorrelation. Finally, in Section IV.3 we test for weak separability among different combinations of the monetary categories.

IV.1 DATA

Our data consists of quarterly Canadian data, for the period 1968I-1982IV, on four monetary categories. The four categories (henceforth referred to as monetary subaggregates) are: (i) money (M), (ii) checkable deposits (C), (iii) savings deposits (S), and (iv) time deposits (T). The classification scheme used to generate these monetary subaggregates is presented in Table IV.1[1].

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[1] A more detailed description of the data is provided in the appendix to this chapter.

TABLE IV.1

MONETARY ASSETS AND THE GROUPING PATTERN

Group Number	Group Name	Variable Number	Variable
1	Money (M)	1	Currency outside banks
		2	Demand deposits at banks
2	Chequable deposits (C)	3	Chequable, personal savings deposits at banks
		4	Daily interest chequing accounts at banks
		5	Chequable, non-personal deposits at banks
		6	Chequable demand deposits at TML companies
		7	Chequable demand deposits at credit unions
3	Savings deposits (S)	8	Non-chequable, personal savings excluding daily interest deposits at banks
		9	Non-chequable, personal daily interest savings deposits
		10	Non-chequable, non-personal deposits at banks
		11	Non-chequable demand deposits at TML companies
		12	Non-chequable demand deposits at credit unions
4	Time deposits (T)	13	Deposits at Quebec savings banks other than those of the Federal Government
		14	Personal fixed term deposits at banks
		15	Less than one year term deposits at TMLs
		16	Greater than one year term deposits at TMLs
		17	Credit union shares
		18	Credit union term deposits
		19	Canada Savings bonds

NOTE: The monetary assets are taken from an M2 measure proposed by Cockerline and Murray (1981).

Three main issues regarding the components listed in Table IV.1 are: (1) the inclusion of near-bank liabilities, (ii) the method of aggregation of the components, and (iii) data adjustments. In what follows we take each of these issues separately.

NEAR-BANK LIABILITIES

In the empirical approach to defining money, the definition of money may change as the economic structure changes. In the last decade important changes took place that have eroded the distinction between banks and near-banks. This suggests that conventional money measures, independently of the method of aggregation, may have lost their significance and that we should extend monetary definitions to include near-bank liabilities and money market instruments which function like bank liabilities (and which have grown rapidly in recent years). In this study, in an effort to reflect the declining differentiation between banks and near-banks, we include certain near-bank liabilities. We include four main new types; the liabilities of Trust and Mortgage Loan (TML) companies[2], the liabilities of Credit unions and Caisses Populaires[3], the liabilities of Quebec Savings Banks and Canada Savings Bonds. The last three types of liabilities typically have not been considered in studies on the substitutability of monetary assets owing to the unavailability of information on the interest rates offered. Here, following Cockerline and Murray (1981) we assume that Credit Unions and Caisses Populaires, and Quebec Savings Banks offer the same interest as TML companies.

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[2] Trust companies on the one hand and Mortgage Loan Companies on the other are quite different financial institutions by their origin but their liabilities have similar characteristics and have been treated in the literature as one homogenous stock. These liabilities fall into three categories, checkable demand deposits, non-checkable demand deposits, and term deposits of various maturities. Short and Villanueva (1975) discovered that TML demand deposits are the best substitutes for money (narrowly defined as currency and demand deposits) and suggested that the definition of money should be expanded to include these liabilities.

[2] Credit Unions and Caisses Populaires, usually referred to as Local Credit Unions, have advanced from their early years of providing basic savings and loan services and today vigorously compete with large banks issuing liabilities that are more similar to bank deposits and hence, to money. These liabilities fall into three categories: demand deposits, term deposits and Credit Union shares. The first group is further divided into checkable and non-checkable components.

AGGREGATION

In aggregating over the monetary components we are necessarily defining a quantity index which replaces the distinct quantities of the separate assets and which is treated as if it were a single asset. The construction of this index involves two operations. First, we must select the individual assets to be included in the index and second, we must add them together in some way.

With regard to the first problem we assume that the monetary assets-monetary subaggregates mapping, provided in Table IV.1 is given. This *a priori* categorization of the assets is subjective and is based on the assumption that either the separability assumption is empirically valid or Hicks' price aggregation condition (relative prices remain in fixed proportions) hold[4]. We are forced to appeal to such a maintained hypothesis because the estimation of a highly disaggregated demand system, encompassing the full range of assets, although theoretically conceivable is not possible in practice for at least

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4 An alternative sufficient condition for aggregation over monetary assets is the Leontief condition (assets are assumed to be consumed in fixed proportions).

the following two reasons: (i) the estimation would be plagued by extreme multicollinearity, and (ii) the number of parameters to be estimated would be extremely large. In any event, it is the demand pattern of the monetary subaggregates which is of prime interest in this work, and one of the above-named aggregation assumptions allows us to treat these subaggregates as if they were single assets.

With regard to the method of aggregation, we construct the monetary subaggregates through the rigorous application of quantity aggregation theory, which is dual to the well known price aggregation theory. For purposes of providing a sharp quantitative assessment of the relative merits of simple sum versus superlative monetary aggregation we use both a simple sum and a Divisia monetary quantity aggregation procedure[5]. Simple sum and Divisia monetary subaggregates are then used as data in estimating the demand system outlined

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5 We prefer the Divisia index from the Diewert superlative class of index numbers, because this index has been widely used in the literature, and has been recently advocated by Barnett as the most attractive quantity index for measuring money. In any event, as Barnett (1980a) has shown, selection between index numbers from the superlative class is of little empirical importance, since these indices move very closely together.

in Chapter 3. Corresponding to each of the two monetary quantity aggregation methods we construct price indices for the monetary subaggregates. Following Barnett (1983a) when those monetary subaggregates are computed as Divisia indices we make use of Fisher's (1922) weak factor reversal test, to compute the corresponding price indices. The test states that the product of the values of the price and quantity indices should equal the ratio of total expenditures in the two periods. On the other hand when the monetary subaggregates are computed as simple sum indices, we use as the corresponding price indices, the Laspeyres indices[6].

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[6] Barnett, instead, uses the Leontief indices (the smallest element of the vector of component user costs) arguing that "simple sum quantity aggregation implies perfect substitutability of components and hence consumption, only of the least expensive good." (Barnett, 1983a, p. 18). There are considerable problems, however, with using such an index with the translog. This is so, because one user cost is always zero (since the benchmark rate is determined in each period as the maximum among all own rates). The translog function is then incapable of handling the model. Because we do not use the rigorously correct price index we treat the simple-sum here not as a rigorously correct alternative to the Divisia, but simply as a commonly used alternative approach to the aggregation.

DATA ADJUSTMENTS

In an attempt to preserve consistency, we make a number of adjustments to our disaggregated data.

The first adjustment concerns the derivation of quarterly average figures. The problem arises because we have three types of data: monthly (average of Wednesdays), month-end, and quarter-end data. Where average monthly data were collected we constructed the quarterly series by taking the arithmetic average of these monthly figures. In the case, however, of month-end data, we first computed an average for each month by taking the arithmetic average of month-end figures for the month in question and the immediately preceding month, and then we used these monthly averages to obtain the quarterly series. Similarly, when quarter-end figures were collected, we constructed quarterly averages by taking the arithmetic average of quarter end figures for the quarter in question and the immediately preceding quarter.

The second adjustment concerns the holding periods. The problem arises because the rates on all the different types of instruments are clearly influenced by the investor's expectations about the future of interest rates. To compare a bond (an investment running over, say, 10 years) and a 90-day Treasury bill (an investment running for 90 days) we need to know the interest rate that will prevail when the Treasury bill will mature. In other words, it is the holding period yield that is relevant and not the yield to maturity. It was necessary therefore to adjust the own rates to a base maturity. Following Cockerline and Murray (1981), all the own rates with a maturity greater than one year were 'yield-curve adjusted' to a 91-day Treasury bill rate as follows:

$$r_i^a = r_i^u - (r_G - r_{TB})$$

where

r_i^a = the yield-curve adjusted rate

r_i^u = the unadjusted rate

r_G = the ordinate on the yield curve of Canada Savings Bonds with the same maturity as i

r_{TB} = the 91-day treasury bill rate

A third adjustment is concerned with the conversion to real per capita balances. Given that the theoretical model is based on an individual decision making problem, each asset stock in Table IV.1 was divided by the population of Canada aged 15 years and over to get per capita quantity series. Also, we have divided each asset stock by the GNE implicit price deflator (which is a weighted average of the prices of all items that enter the measure of GNE) to convert nominal quantities to real terms.

Finally, we consider some specific points regarding the construction of user costs. The user cost formula was presented in chapter II and is repeated here for convenience:

$$p_{it} = \frac{P^*(BR_t - r_{it})}{1 + BR_t}$$

In order to apply this formula we require data on the general price level, P^*, the own rates r_i and the benchmark rate, BR. For BR_t issues exist in the operational definition of the rate. Term structure theory and interest arbitrage dictates use of the maximum available short-term rate. However, recent empirical work by Shiller (1979) does not support this conclusion and dictates maximization over available rates at all maturities. Ideally, we look for a highly marketable asset accumulated for the purpose of transferring wealth between multiperiod planning horizons rather than to provide monetary services. A long term bond, obviously, fulfills this criterion but it is extremely difficult to choose a single rate for the benchmark because no single rate exceeds all rates on monetary assets for all time periods. We determined the benchmark rate in each period as the maximum of all rates[7].

Finally, prior to estimation, the price indices were all scaled to be equal 1.0 in 1968I for both sets of data. Furthermore, we multiplied the quantity indices by the original base period value of the corresponding price indices, in order to ensure that the product of the values of the price and quantity indices remained unchanged by our rescaling.

This completes the discussion of the data used and we now turn to empirical results.

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[7] Note that relative prices are more sensitive to own rates, r_i, than to the benchmark rate, since the latter appears in all user costs.

IV.2 EMPIRICAL RESULTS AND INTERPRETATION

In what follows we will be pursuing a dual objective. On the one hand we wish to compare the use of simple-sum and Divisia monetary quantity indices as data in empirical demand systems, and on the other we wish to examine the empirical results to shed some light on the controversy of whether monetary services are substitutes or complements.

In Table IV.2 we present parameter estimates, based on FIML regressions, for the quasi-homothetic translog model, after correction for autocorrelation, together with the value of the log likelihood at the optimum[8]. In the first two columns we present the symmetry restricted quasi-homothetic translog estimates (thirteen free parameters) and in the last two columns the symmetry restricted homothetic translog estimates (nine free parameters)[9]. These results are discussed in two stages. First, we consider whether the results are consistent with an individual's optimization model

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[8] The results were checked for convergence to the global maximum by estimating the model more than once, deleting a different equation each time.

[9] We report in the first two columns, for example, eighteen rather than just the thirteen free parameters. The implied parameter estimates can be calculated from the restrictions.

TABLE IV.2

PARAMETER ESTIMATES, TRANSLOG FORMS, 1968I-1982IV

For (M) money, (C) chequable deposits, (S) savings deposits, (T) time deposits; Symmetry imposed; standard errors in parentheses

	QUASI-HOMOTHETIC VERSION		HOMOTHETIC VERSION	
Parameter	With Divisia Monetary Aggregation	With Simple Sum Monetary Aggregation	With Divisia Monteary Aggregation	With Simple Sum Monetary Aggregation
γ_M	0.0147649 (0.00658790)	0.581488 (0.0144968)		
γ_C	0.00203715 (0.00217)	0.298145 (0.0106800)		
γ_S	0.0200430 (0.00265138)	1.38076 (0.0964925)		
γ_T	0.0207871 (0.00105781)	2.70624 (0.132339)		
a_M	0.5636964 (0.0685885)	0.125685 (0.0362698)	0.348465 (0.061888)	0.134519 (0.0241281)
a_C	0.202945 (0.033978)	0.261560 (0.0299639)	0.0481525 (0.0618472)	0.17722 (0.0392025)
a_S	0.166673 (0.051775)	0.240100 (0.0465964)	0.329293 (0.0566581)	0.305871 (0.0382261)
a_T	0.0666816 (0.0403225)	0.372648 (0.0857157)	0.274089 (0.078633)	0.541888 (0.05474)
β_{MM}	0.21144869 (0.0541157)	-0.0721869 (0.0753582)	0.2059652 (0.0168968)	0.1788088 (0.0268018)
β_{MC}	-0.1386572 (0.0559386)	0.0901228 (0.0741421)	-0.0509937 (0.0158369)	-0.0471927 (0.025835)
β_{MS}	-0.0710575 (0.0254347)	0.0125269 (0.0197559)	-0.07439 (0.00707489)	-0.0520255 (0.00371833)
β_{MT}	-0.00173399 (0.0227196)	-0.0304734 (0.0174757)	-0.0805815 (0.00529477)	-0.0795906 (0.0023715)
β_{CC}	0.168999 (0.0513509)	-0.0396474 (0.0738095)	0.108159 (0.0156103)	0.149215 (0.0262047)

...continued

TABLE IV.2 (continued)

Parameter	QUASI-HOMOTHETIC VERSION Divisia	QUASI-HOMOTHETIC VERSION Simple Sum	HOMOTHETIC VERSION Divisia	HOMOTHETIC VERSION Simple Sum
β_{CS}	-0.0113937 (0.0125775)	-0.0304019 (0.0174775)	-0.0209634 (0.00481868)	-0.0367202 (0.00290829)
β_{CT}	-0.0189481 (0.00931101)	-0.0200824 (0.0140146)	-0.0362019 (0.00410459)	-0.0653021 (0.00302675)
β_{SS}	0.0567697 (0.0371058)	0.289243 (0.0399346)	0.113169 (0.00570878)	0.146778 (0.00469406)
β_{ST}	0.0256815 (0.0154247)	-0.271547 (0.0210434)	-0.017856 (0.00401729)	-0.0580323 (0.00311278)
β_{TT}	-0.00499941 (0.0264465)	0.321900 (0.0330839)	0.134599 (0.00430602)	0.203925 (0.00363374)
ℓn L	663.125	670.534	617.627	615.117
D - W_M	1.7716	1.8765	2.1579	2.1072
D - W_C	2.236	1.7332	2.1057	2.2925
D - W_S	1.4558	1.8230	1.8664	2.1072
D - W_T	1.3683	1.9323	2.1869	2.5898

NOTES: ℓn L refers to the log of the likelihood function. D-W_M, D-W_C, D-W_S, and D-W_T refer to Durbin-Watson statistics for the equations M, C, S and T, respectively.

by looking at the integrability conditions. Second, we present income elasticities, price elasticities and partial elasticities of substitution.

When Divisia monetary quantity indices are used, positivity, monotonicity, and the appropriate curvature conditions on the indirect utility function (i.e., the matrix of elastiticities of substitution should be negative semidefinite) are satisfied by the symmetry restricted quasi-homothetic model at each observation and for no observations by the homothetic model. On the other hand with simple-sum monetary quantity indices these conditions are satisfied by the quasi-homothetic model for the observations 1973III-1982IV only, and for no observations by the homothetic form. These results favour Divisia monetary quantity aggregation over simple-sum aggregation.

Next, we report on testing the validity of the restrictions, implied by the hypotheses of first-order autocorrelation and homotheticity. We employed the likelihood ratio method which requires estimation of both restricted and unrestricted versions. The likelihood ratio test statistic is two times the difference of the logs of the likelihood functions of unrestricted and restricted estimates, and is distributed as chi-square with degrees of freedom equal to the number of restrictions. We set the level of significance at .01. This implies that the probability of rejecting a true hypothesis in our tests is .01. Results of these tests are presented in Table IV.3.

The null hypothesis of no autocorrelation could not be accepted[10]. The likelihood ratio test statistics are 405.42 (Divisia Aggregation) and 240.70 (simple-sum aggregation) while the .01 chi-square critical value is 6.63. Next the hypothesis of homotheticity was tested conditional on first-order autocorrelation. The likelihood ratio test statistics in this case are 90.99 (Divisia aggregation) and 110.83 (simple-sum aggregation) while the .01 chi-square critical value is 13.28. We conclude that conditional on first-order autocorrelation we reject the homotheticity restrictions.

We now turn to examine the implications for consumer behaviour of the estimated budget share equations, and to measure monetary asset substitution

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[10] See chapter III for a discussion of this test.

TABLE IV.3

LIKELIHOOD RATIO TEST RESULTS FOR THE QUASI-HOMOTHETIC TRANSLOG

Test	Log Likelihood	Test Statistic 2(V-R)	D.F.	Critical Value (1%)
H_0 : No autocorrelation				
(H_1 : Not H_0)				
Divisia Aggregation	460.415	405.42	1	6.63
Simple-Sum Aggregation	550.184	240.70	1	6.63
H_0 : Homotheticity				
(H_1 : Not H_0)				
Divisia Aggregation	617.627	90.99	4	13.28
Simple-Sum Aggregation	615.117	110.83	4	13.28

TABLE IV.4

ESTIMATED EXPENDITURE ELASTICITIES: QUASI-HOMOTHETIC TRANSLOG WITH SYMMETRY IMPOSED

Elasticity	ESTIMATE					
	1968I		1975I		1982I	
	Divisia	Simple-Sum	Divisia	Simple-Sum	Divisia	Simple-Sum
η_{MY}	1.07	1.00	1.06	0.99	1.06	0.99
η_{CY}	1.01	0.97	1.01	1.01	1.01	1.01
η_{SY}	0.96	0.98	0.98	1.01	0.98	1.02
η_{TY}	0.95	0.81	0.94	0.89	0.92	0.92

possibilities. The expenditure elasticities (n_{iY}), the price elasticities (n_{ij}), and the partial elasticities of substitution (σ_{ij}) as formulated in Section III.5 are calculated from the coefficients shown in Table IV.2 and evaluated at different sample periods (near the beginning, middle and end of the period). These elasticities are displayed in Tables IV.4, IV.5 and IV.6.

The estimated expenditure elasticities in Table IV.4 reveal a clear-cut pattern. All asset services are "normal goods" (i.e., all expenditure elasticities are positive. Under Divisia monetary quantity indices, clearly money (M1) and checkable deposits are "luxury goods" ($n_{iy} > 1$) with money being more so; the other two assets -- savings deposits and time deposits -- are expenditure inelastic. For the simple-sum monetary quantity indices, the results are almost the same with some small differences in the magnitudes of the expenditure elasticities.

Several important conclusions emerge from Table IV.5. All own-price elasticities are negative though cross-price elasticities vary between positive and negative. Under Divisia monetary quantity indices, all assets are own-price inelastic ($|n_{ii}| < 1$); with

TABLE IV.5

ESTIMATED OWN-AND CROSS-PRICE ELASTICITIES: QUASI HOMOTHETIC TRANSLOG WITH SYMMETRY IMPOSED

	ESTIMATE					
	1968I		1975I		1982I	
Elasticity	Divisia	Simple-Sum	Divisia	Simple-Sum	Divisia	Simple-Sum
n_{MM}	-0.765	-0.547	-0.755	-0.851	-0.744	-0.932
n_{CC}	-0.361	-1.360	-0.487	-0.840	-0.554	-0.956
n_{SS}	-0.875	-1.530	-0.870	-1.194	-0.855	-1.028
n_{TT}	-0.952	-0.949	-0.942	-1.014	-0.940	-0.975
n_{MC}	-0.141	-0.543	-0.147	-0.149	-0.155	-0.049
n_{MS}	-0.115	-0.081	-0.106	-0.031	-0.103	-0.018
n_{MT}	-0.049	0.170	-0.053	0.033	-0.053	0.0005
n_{CM}	-0.525	0.809	-0.423	-0.349	-0.369	-0.065
n_{CS}	-0.047	-0.262	-0.039	0.108	-0.035	0.004
n_{CT}	-0.075	-0.161	-0.065	0.066	-0.061	-0.002
n_{SM}	-0.098	-0.041	-0.128	-0.033	-0.157	-0.028
n_{SC}	-0.015	0.059	-0.020	0.013	-0.025	-0.010
n_{ST}	0.022	0.532	0.036	0.204	0.049	0.039
n_{TM}	-0.005	-0.017	-0.004	-0.004	-0.004	-0.003
n_{TC}	-0.027	-0.004	-0.027	-0.003	-0.028	-0.003
n_{TS}	0.032	0.161	0.033	0.126	0.036	0.053

TABLE IV.6

ESTIMATED PARTIAL ELASTICITIES OF SUBSTITUTION: QUASI-HOMOTHETIC TRANSLOG WITH SYMMETRY IMPOSED

	ESTIMATE					
	1968I		1975I		1982I	
Elasticity	Divisia	Simple-Sum	Divisia	Simple-Sum	Divisia	Simple-Sum
σ_{MM}	-1.01	-3.91	-0.93	-3.89	-0.89	-3.76
σ_{CC}	-2.75	19.15	-2.80	10.48	-2.52	-5.40
σ_{SS}	-2.13	-4.06	-2.91	-3.00	-3.53	-2.34
σ_{TT}	-2.79	-0.62	-2.52	-1.33	-2.50	-1.84
σ_{MC}	-0.39	8.26	-0.93	-1.04	0.05	0.67
σ_{MS}	0.66	0.73	0.58	0.89	0.51	0.93
σ_{MT}	0.87	1.25	0.86	1.07	0.86	1.00
σ_{CS}	0.84	0.10	0.83	1.38	0.83	1.03
σ_{CT}	0.71	0.72	0.77	1.16	0.79	1.01
σ_{ST}	1.05	1.78	1.11	1.45	1.16	1.13

the demand for checkable deposits being much more own-price inelastic than the demand for money, savings deposits and time deposits. Also, the own price elasticity of checkable deposits appears to have increased over time which could be interpreted as a result of the increasing substitutability of these deposits for money. All monetary assets are found to be (gross) complements ($n_{ij} < 0$) with the exception of n_{ST} and n_{TS}; here, although the sizes of these elasticities are less than 0.1 it appears that they rise slightly over time.

The use of simple-sum monetary quantity indices, on the other hand, does not reveal such a clear-cut substitutability pattern. Some of the elasticities fluctuate over time, switching from substitutability to complementarity -- as, for example, n_{cm}, n_{CS}, n_{CT}, and n_{SC}. This instability of the substitution relation again suggests support for the use of Divisia monetary aggregation over simple-sum aggregation.

The estimated partial elasticities of substitution in Table IV.6 show quite different patterns of substitution from the uncompensated price elasticities. All own elasticities of substitution are negative, except for checkable deposits (σ_{CC}) which is positive in the case of simple-sum data thus violating the curvature conditions. On the whole the assets appear to be net substitutes -- with savings and time deposits being more so -- with the exception of the elasticity of substitution between money and checkable deposits, which fluctuates over time switching from complementarity in the early years to substitutability in the later years. The estimated substitutability relationship, however, among the monetary assets is weak, and is inconsistent with the view that the substitutability between money (M1) and the nested "like assets" group (checkable deposits) should be stronger than the substitutability between money and any other "less like assets" group (either savings or time deposits). Our estimated relationship is[11].

$$\sigma_{MT} > \sigma_{MS} > \sigma_{MC}$$

This result raises a serious question about the aggregation of monetary assets which, itself, is

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[11] This result is consistent with the within-M1 low substitutability findings by Offenbacher (1979) and Ewiss and Fisher (1984), in U.S. data.

linked with the problem of the definition of money. The problem is that the degree of substitutability among monetary assets used to be interpreted as providing -- explicitly or implicitly -- a rationale for the choice of assets to be included in the monetary aggregate. The argument was that, if different monetary assets are close substitutes, then there might be a summary measure of money which could be obtained simply by adding together in some way the different monetary assets in a functional group. Our results concerning the low degree of substitutability among the services of monetary assets -- a finding that corroborates previous results using other consumer demand models -- suggest that the conventional wisdom regarding the definition of money, requires re-examination. The two major approaches to defining money -- the "a priori" approach which defines money in a theoretical sense by pointing to its functions, and the policy oriented "empirical" approach which defines money as that collection of monetary assets that has the most predictable effect on nominal income -- implicitly make the assumption that aggregation over monetary assets is feasible, whereas the approach taken here raises some doubts in that regard.

IV.3 SEPARABILITY HYPOTHESIS TESTS

In this section we have two objectives. The first objective is to discover the structure of preferences over monetary assets by empirically testing for the appropriateness of the weak separability assumptions. The second objective is to elaborate the results of the empirical tests and their implications for monetary theory and the conduct and effectiveness of monetary policy. Table IV.7 gives the results for formal tests of the separability restrictions imposed on the quasi-homothetic translog model. This is done twice for each separability type: firstly using Divisia data and secondly using simple-sum data. The first column of Table IV.7 describes the maintained hypothesis.

With regard to the first approximate weak separability pattern ($f[G(\ell n q_i, \ell n\ q_j),\ \ell n\ q_k,\ \ell n\ q_\ell]$, see discussion in Chapter III), there are six null hypotheses (the first six entries of Table IV.7). We find that the [(M,C), S,T] and [(S,T), M, C] types of weak separability are consistent with our Divisia data while only the [(M,C), S,T] type of weak separability is consistent with the simple-sum data. The test statistics decisively reject all the other possible separability types (rows 2, 3, 4 and 5), with the margins of rejection generally

TABLE IV.7

SEPARABILITY HYPOTHESES TESTS UNDER THE QUASI-HOMOTHETIC TRANSLOG

	Hypothesis	D.F.	Divisia Aggregation Log of Likelihood Function	Divisia Aggregation Test Statistic 2(U - R)	Simple-Sum Aggregation Log of Likelihood Function	Simple-Sum Aggregation Test Statistic 2(U - R)	CV (1%)
	Unrestricted		663.125		670.534		
1.	[(M,C),S,T]	2	660.218	5.81*	668.923	3.22*	9.21
2.	[(M,S),C,T]	2	657.829	10.57	662.377	16.31	9.21
3.	[(M,T),C,S]	2	654.831	16.58	664.080	12.90	9.21
4.	[(C,S),M,T]	2	654.291	17.66	631.528	78.01	9.21
5.	[(C,T),M,S]	2	655.823	14.60	659.248	22.57	9.21
6.	[(S,T),M,C]	2	660.015	6.22*	635.934	69.20	9.21
7.	[(M,C),(S,T]	3	659.968	6.31*	667.810	5.44*	11.34
8.	[(M,S),(C,T)]	3	657.101	12.04	660.342	20.38	11.34
9.	[(M,T),(C,S)]	3	652.702	20.84	628.824	83.42	11.34
10.	[(M,C,S),T]	2	653.832	18.58	631.865	77.33	9.21
11.	[(C,S,T),M]	2	661.432	3.38*	668.117	4.83*	9.21
12.	[(M,C,T),S]	2	655.345	15.56	657.382	26.30	9.21
13.	[(M,S,T),C]	2	661.010	4.23*	668.726	3.61*	9.21

* indicates that we cannot reject the null hypothesis

being smaller under Divisia monetary subaggregation.

With regard to the second approximate weak separability pattern ($F[G(\ell n\ q_i, \ell n\ q_j), H(\ell n\ q_k, \ell n\ q_\ell)]$), the only separability condition which our data (either Divisia or simple-sum) cannot reject is the [(M,C), (S,T)] weak separability restriction. This implies that we cannot reject the conditions for the further aggregation of M and C and of S and T, i.e.,

$$u = F[f_1(M,C),\ f_2(S,T)]\ .$$

where $f_1(M,C)$ is an index of M and C and $f_2(S,T)$ is an index of S and T[12]. Thus, we can establish the two intermediate quantity aggregator functions

$$u_1 = f_1(M,C) \qquad \text{and} \qquad u_2 = f_2(S,T)$$

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12 These indices measure the total service flow produced by different categories of monetary assets. For example, $f_1(M,C)$ represents an index of monetary assets closely associated with the transactions or liquidity function, while $f_2(S,T)$ represents an index of monetary assets associated with the store of wealth function.

and deal independently with the M-C and S-T substitutions inherent within these functions.

Finally, with regard to the last approximate weak separability pattern $(f[G(\ell n\ q_i,\ \ell n\ q_j,\ \ell n\ q_k),\ \ell n\ q_\ell])$, there are four null hypotheses (the last four entries of Table IV.7). The test statistics for these four hypotheses indicate that the [(C,S,T),M] and [(M,S,T),C] types of weak separability are consistent with our data (either Divisia or simple-sum). On the basis of these results we argue that the independent estimation of the following three-argument quantity aggregator functions

$$f_1(C,S,T) \qquad \text{and} \qquad f_2(M,S,T)$$

is possible. Alternatively, the values of these functions could be candidates for a definition of a monetary aggregate suggested by this approach.

We now turn to examine the implications for monetary theory of the results of the separability tests. The question is whether our results provide a rationale for the choice of assets to be included in a monetary aggregate. As indicated in the previous section, the conventional wisdom, regarding this problem, suggests that the means of payment type assets money (M1) and checkable deposits, are the most likely candidates for inclusion followed by the next best substitute.

Looking at the test results in Table IV.7, it would appear that a narrow definition of money can be employed in empirical studies since the [(M,C), S,T] weak separability type cannot be rejected throughout. This is certainly a separability type which would have been selected a priori without having to exploit the sample to find other more suitable grouping patterns. The results, however, reject the hypothesis that money could be defined more broadly to include savings deposits (see row 10). A surprising result is that other admissible groupings exist (see rows 11 and 12). These groupings, of course, are not in line with the conventional wisdom which has it that broad-based monetary aggregates should be nested about monetary components that can normally be used to make payments (money and checkable deposits in our case). As a consequence these groupings may be entirely spurious but it would be appropriate to treat this finding skeptically. For example, non-rejection of the separability type [(C,S,T),M] (row 11 of Table IV.7) strongly suggests that the inclusion of money (M1) in a broad-based monetary aggregate can be questioned. Even here nothing unambiguous can be said. Cagan raised a similar objection:

> The traditional inclusion of currency in monetary aggregages can be questioned. Currency is used primarily to service retail trade and is issued in the short run largely on the demand of the public[13].

Furthermore, the weak separability types [(C,S,T),M] and [(M,S,T),C] are consistent with the low substitutability between M and C (i.e., σ_{MC} is the smallest σ) established in the previous section. However, a problem of further analysis exists in that there are no theoretical guidelines to aid the interpretation of these results. Someone, for example, might argue that a monetary aggregate of C,S and T is inadequate because it misses the essence of money, and contradicts conventional views from monetary theory. My own view is that one would need more test results, using different host models, and a clear view of what

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[13] P. Cagan (1982, p. 673).

one is trying to accomplish via monetary aggregation, before claiming the use of any monetary aggregate.

To briefly sum up, the results of the separability hypothesis tests suggest that a narrow definition of money (currency, demand, and checkable deposits) could be employed, but cast substantial doubt on the assumption that broader-based monetary aggregates should be nested about the means of payment assets. Of course, adoption of a narrow definition of money should not preclude the introduction into a monetary model of other monetary assets. Our findings strongly suggest, however, that these assets should not be introduced by means of defining money more broadly. The use of a broad monetary aggregate would imply that these assets and money, as narrowly defined, are perfect substitutes, or that other suitable aggregation conditions hold. These considerations suggest that a useful approach to the monetary aggregation problem, would be to think in terms of monetary sub-aggregates.

APPENDIX IV.A

ASSET STOCKS AND CORRESPONDING OWN RATES

Our objective in this appendix is to provide a brief description of each asset stock listed in Table IV.1 as well as a description of each own rate. The asset stocks and the corresponding own rates are given in Table IV.A.1, in mnemonic pairs.

Asset Stocks

CUR: Monthly (average of Wednesdays) figures are listed in Selected indicators of money and credit (series B 2001), published by the Bank of Canada in the Bank of Canada Review. Quarterly figures were derived by taking the arithmetic mean of the monthly figures, Series with a B prefix, mentioned hereafter are taken from the same source which will not be repeated.

BDD: Monthly (average of Wednesdays) figures are tabulated in Chartered bank selected liabilities (series B478) published by the Bank of Canada (hereafter referred to as Selected Liabilities). These figures apply to our familiar checking accounts, de facto

TABLE IV.A.1

MNEMONICS OF MONETARY ASSET STOCKS AND OWN RATES

Component Number	Mnemonic for Asset	Monetary Asset	Mnemonic for Rate
1	CUR	Currency outside banks	RZER
2	BDD	Demand deposits at banks	RZER
3	BCPSD	Checkable, personal savings deposits at banks	RBCPSD
4	BDICA	Daily interest checking accounts at banks	RBDICA
5	BCNPD	Checkable, non-personal deposits at banks	RBCNPD
6	TMCDD	Checkable demand deposits at TML Companies	RTMCDD
7	CUCDD	Checkable demand deposits at Credit Unions	RCUCDD
8	BNCPSXDI	Non-checkable, personal savings excluding daily interest deposits at banks	RBNCPSXDI
9	BNCPDI	Non-checkable, personal daily interest savings deposits	RBNCPDI
10	BNCNPD	Non-checkable, non-personal deposits at banks	RBNCNPD
11	TMNCDD	Non-checkable demand deposits at TML Companies	RTMNCDD
12	CUNCDD	Non-checkable demand deposits at Credit Unions	RCUNCDD
13	QSBD	Deposits at Quebec savings banks other than those of the Federal Government.	RQSBD
14	BPFTD	Personal fixed term deposits at banks	RBPFTD

...continued

Component Number	Mnemonic for Asset	Monetary Asset	Mnemonic for Rate
15	TML1TD	Less than one year term deposits at TMLS	RTML1TD
16	TMG1TD	Greater than one year term deposits at TMLS	RTMG1TD
17	CUSHA	Credit Union shares	RCUSHA
18	CUTD	Credit Union term deposits	RCUTD
19	CSB	Canada Savings Bonds	RCSB

withdrawable upon demand. The majority of these accounts are made up of current accounts, although personal checking accounts are also included.

BCPSD: This series applies to deposits formerly found in Selected Liabilities, series B452 (monthly, average of Wednesdays). In early 1982 when daily interest accounts were introduced the series was terminated and replaced with two separate series, B485 and B484, with the former corresponding to regular checkable personal savings deposits.

BDICA: Monthly (average of Wednesdays) figures are found in Selected Liabilities, as series B484.

BCNPD: Monthly (average of Wednesdays) figures are listed in Selected Liabilities as series B472.

TMCDD: End of quarter figures are listed in Trust and Mortgage Loan Companies, published by the Bank of Canada.

CUCDD: End of quarter figures are listed in Statistics Canada publication, Financial Institutions (Catalogue 61-006), 1975 to date. No data is available prior to 1975. However, we do have data on the aggregate volume of demand deposits at Credit Unions and Caisses Populaires. The amount of checkable deposits as of 1975I was $2,522 million amounting to 51.638 per cent of aggregate deposits at that date. In an attempt to preserve consistency and to obtain data on checkable deposits for 1968I-1974IV we multiplied the aggregate deposits series by the factor .51638 for the earlier period.

BNCPSXDI: Monthly (average of Wednesdays) figures are listed in Selected Liabilities as series B453. When daily interest accounts were introduced in late 1979, the series was terminated and replaced with the series B479 and B480, corresponding to regular non-checkable and daily interest non-checkable personal savings accounts.

BNCPDI: These accounts were introduced in 1979III and are reported in Selected Liabilities as series B479 (monthly, average of Wednesdays).

BNCNPD: Monthly (average of Wednesdays) figures are listed in Selected Liabilities as series B473.

TMNCDD: End of quarter figures are listed in Trust and Mortgage Loan Companies.

CUNCDD: End of quarter figures are listed in Financial Institutions, 1975I to date. We constructed the series prior to 1975 by subtracting checkable demand deposits from demand deposits at Credit Unions and Caisses Populaires.

QSBD: Month end figures are listed in Quebec Savings Banks (series B2255), published by the Bank of Canada. We excluded Federal Government deposits since these deposits have no influence on Federal Government expenditure which is determined by the behaviour of the economy (high deposit holdings do not necessarily imply an increase in government spending).

BPFTD: Monthly (average of Wednesdays) figures are listed in Selected Liabilities as series B454.

<u>TML1TD</u>: End of quarter figures are listed in <u>Trust and Mortgage Loan Companies</u>.

<u>TMG1TD</u>: This series was constructed by adding two separate series, TML 1-5 years term deposits and TML over 5 years term deposits. Month end figures for these series are available in <u>Trust and Mortgage Loan Companies</u>.

<u>CUSHA</u>: End of quarter figures are listed in <u>Financial Institutions</u>.

<u>CUTD</u>: End of quarter figures are listed in <u>Credit Unions and Caisses Populaires</u> (series B3919) published by the Bank of Canada.

<u>CSB</u>: Month end figures are listed in <u>Government of Canada Direct and Guaranteed Securities and Loans: Distribution of Holdings</u> (series B2406) published by the Bank of Canada.

Own Rates (used to calculate user costs)

RZER: Zero rate. It is assumed that for currency, the rate of return is always zero. For demand deposits, however, although banks do not pay interest explicitly, in effect they do pay imputed interest by providing free, or below cost, services to holders of these deposits. For example, banks may provide free safety deposit boxes or clear a certain number of cheques without charge for depositors who maintain a minimum balance in their accounts. It is difficult to say what is the imputed interest, since statistics on bank service charges are not available. Here we ignore the problem by assuming that the value of these services equals the cost of holding demand deposits[1].

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[1] In the Offenbacher (1979) U.S. study, where the substitutability relation between currency and demand deposits is examined, an implicit rate of return is imputed to demand deposits. This is Klein's (1974) rate of return based on the assumption that in a competitive system banks must pass on all marginal profits from demand deposits.

RBCPSD: The rate is available on a monthly basis from Cansim B14035. It was further adjusted for minimum monthly balance to obtain the effective rate. The adjustment was made as follows:

$$\text{effective rate} = \text{quoted rate} \cdot \left[\frac{\text{checkable, personal savings deposits (monthly, minimum of Wednesdays)}}{\text{checkable, personal savings deposits (monthly average of Wednesdays)}}\right]$$

RBDICA: Averages of the rates paid by banks on balances above a certain level (usually $2,000) were provided by the Department of Banking and Financial Analysis of the Bank of Canada (henceforth referred to as DBFA).

RBCNPD: 90-day personal fixed term deposit rate, available in Selected Canadian and International Interest Rates Including Bond Yields and Interest Arbitrage (series B14043) published by the Bank of Canada (henceforth referred to as Interest Rates).

RTMCDD: Monthly (average of weekly data) figures were provided by the DBFA. The quarterly series (obtained according to the rule outlined in the text) was further adjusted for minimum monthly balance to obtain the effective rate of return. The adjustment was made as follows[2]:

effective rate = quoted rate-checkable personal savings deposit rate + effective checkable personal savings deposit rate.

RCUDD: RTMCDD

RBNCPSXDI: Monthly figures (series B14019) are available in Interest Rates. The quarterly series was obtained according to the rule outlined in the text. The rate was further adjusted for minimum monthly balance in a manner similar to the adjustment on RBCPSD.

RBNCPDI: Quarter end figures were provided by the DBFA.

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[2] See Cockerline and Murray (1981).

RBNCNPD: RBCNPD

RTMNCDD: Monthly (average of weekly data) figures were provided by the DBFA.

RCUNCDD: RTMNCDD

RQSB: RTMNCDD

RBPFTD: Unfortunately we could not construct a representative rate weighted by the amount of deposits in each maturity class, since data on the dollar value of deposits are not available, although representative rates are reported for the 90-day, one year and five year maturities. However, since personal term deposits are, apart from ownership, very similar to non-checkable, non-personal deposits, we used the rate appropriate for the latter, the chartered bank 90-day personal fixed term deposit rate.

RTML1TD: Yield curve adjusted, 1-2 years Trust Company Guaranteed Investment Certificates (G.I.C.)

rate. The 1-2 years Trust Company G.I.C. rate was provided by the DBFA in monthly (average of weekly data) observations.

RTMG1TD: Yield curve adjusted 5 years Trust Company G.I.C. rate. The 5 years Trust Company G.I.C. rate is reported in Interest Rates as series B14023.

RCUSHA: 90-179 day Trust Company G.I.C. rate provided by the DBFA in (average) monthly figures.

RCUTD: RCUSHA

RCSB: Maturity adjusted first year coupon rate on Canada Savings Bonds. The first year rate on Canada Savings bonds is available on a monthly basis from Cansim B14040.

CHAPTER V

SUMMARY AND CONCLUSION

V.1 SUMMARY AND CONCLUSIONS

This thesis presented a model for estimating the substitutability of monetary assets and for examining the empirical significance of the separability (aggregation) assumptions implicit in broad definitions of money. Our approach was to estimate translog budget share equations using Barten's (1969) F.I.M.L. method for estimating singular equation systems. We used Canadian quarterly data for the period 1968I-1982IV. Our results lead to a number of conclusions, and we take each of these separately, in what follows.

MONETARY ASSET SUBSTITUTABILITY

The most interesting finding is the low degree of substitution among the monetary assets. Furthermore, money (M1) and checkable deposits have the lowest partial elasticity of substitution in absolute value. These results provide no justification for broad money measures and suggest that we can only identify the true monetary aggregate by testing for the necessary and sufficient separability conditions in the underlying utility function.

MONETARY AGGREGATION

We have compared two forms of aggregation -- simple-sum and Divisia -- in terms of satisfying the integrability conditions of the demand system based on the indirect utility function. Our results favour Divisia monetary aggregation and there does not seem to be any basis for justifying simple-sum monetary aggregation on *a priori* grounds. Furthermore, the instability of the substitutability relationship between monetary assets under simple-sum aggregation disappears when Divisia indices are used. This could reflect greater theoretical consistency, or simply that the assumptions under which the Divisia index is constructed are satisfied by our data.

Our results are consistent with all other empirical evidence accumulated over the last five years in the debate on the measurement of money. Barnett and

Spindt (1979), and Barnett, Offenbacher and Spindt (1984), compared the empirical performance of Divisia and simple-sum monetary aggregates in terms of policy criteria, such as causality, information content of an aggregate, and stability of money demand equations, and found no basis for preferring the simple-sum aggregates. Similarly, Barnett (1983), comparing the simple-sum and Divisia aggregates in terms of the fit of a joint sector-wide demand system, concluded in favour of the Divisia aggregates.

THE QUASI-HOMOTHETICITY ASSUMPTION

In exploring the quasi-homothetic version of the translog we have found it a very attractive functional form. Furthermore, the hypothesis of homotheticity of the translog form is decisively rejected by the statistical test. Our results highlight the important role that could be played by the quasi-homotheticity assumption in multistage budgeting and aggregation theory. Unfortunately much recent work on the specification of utility-tree structures rests on the assumption that preferences are homothetically separable. The major limitation of homothetic utility functions is the implied unitary expenditure elasticity at all price and expenditure configurations. However, homotheticity imposes testable restrictions on the structure of demand systems, so that its form remains an empirical issue. But if adopted in contexts where it is not warranted, there may be specification errors. Here, again, the quasi-homotheticity assumption provides a generalization. It eliminates the possibility of misspecification of unitary expenditure elasticities implied by the use of the homotheticity assumption, thus allowing for more complicated interdependencies among monetary assets. In addition, quasi-homotheticity, like homotheticity, is a sufficient condition in the theory of consistent multi-stage optimization. As a consequence, quasi-homotheticity is clearly an attractive feature to build into a model.

SEPARABILITY HYPOTHESIS TESTS

The separability tests carried out here cannot be considered as conclusive, but rather should be viewed as a first step toward providing rigorous microeconomic and aggregation theoretic foundations for monetary aggregation. Our principal motivation has been the possibility of shedding some light on the problem of choosing one aggregate as the appropriate definition of money. The complication is that a number of different definitions of money are now available and while this may be encouraging for the econometrician who is unsure which definition of money to assume, it is not encouraging for someone interested in designing optimal economic policies for whom different definitions of money have very different implications. As a result, it is particularly important to be consistent with the theory. Further research is clearly needed in this area.

V.2 SUGGESTIONS FOR FUTURE WORK

The results of this work suggest a number of interesting avenues for further research. While we have investigated the quasi-homothetic translog, further research could be carried out using more flexible functional forms such as the basic translog or the generalized translog. However, since the translog flexible functional form family provides only a local approximation, a particularly constructive approach would be based on the use of flexible functional forms that possess global properties.

Two such forms are the Fourier flexible functional form (see Gallant 1981) and the Laurent flexible functional form (see Barnett 1983a). Separability hypothesis tests could also be carried out using these globally flexible functional forms.

One issue which has not been examined is the question of stability of monetary asset demand functions. To my knowledge the tests usually proposed do not apply in the context of systems of equations (especially when they are nonlinear in parameters).

Another issue relates to the benchmark rate. In theory this rate is the maximum expected yield on an asset which provides no monetary services except for its financial yield. In the present study, however, the benchmark rate was determined in each period as the maximum among all own rates, because of difficulties in finding a single rate series that exceeds all other rates on monetary assets for all time periods. Experimentation, therefore, with different benchmark rates might be fruitful.

Weak separability is a hypothesis of consumer behaviour so it would be preferable to test it against consumer data rather than aggregate data. This would require disaggregation by sector and, therefore, construction of two sets of monetary asset holding -- one for consumers and another for business -- and also, computation of sector specific monetary aggregates.

Finally, the model can be extended to consider the choice among risky financial assets or to incorporate uncertainty. As Feige and Pearce (1977, p. 4) put it:

> None of the empirical studies... explicitly introduced risk factors.

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