Number, Language, and Cognitive Development

Number, Language and Cognitive
Development

by

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Abstract

This thesis explores various aspects of young children's concepts of number and length. The intentions of the author were: to examine some of the factors which affect young children's choices of quantitative strategies; to explore their understanding of comparative terms such as "longer" and "more"; to consider some questions about the nature of concepts and cognitive development.

The first part of the introduction describes theories of number, both formal and psychological, with particular emphasis on Piaget's theory of number. Six main issues are derived from the theories. The author argues that these issues might all benefit from a research paradigm in which the young child's concept of number is studied in conjunction with his concept of length. For example, most number theorists assign the small numbers in the natural number series a unique status. The author argues that while it is possible that small numbers of elements will enable numerical operations, they may make length operations more difficult when they are arranged in a linear formation. This can only be determined by asking the child to evaluate both number and length on the same stimulus arrays. The next three sections of the introduction review previous research relevant to: factors influencing children's choice of strategy; the nature of their understanding of quantitative terms; theories of concept acquisition and cognitive development. The final section restates the six main theoretical issues in light of previous research findings.

The second chapter presents six experiments. In all these experiments the author asks young children between three and seven to

make judgments of the equivalence of the number and length of two rows of elements. However, in each experiment different aspects of the task are varied, amongst them: the set size; training; the means of presentation of the rows; the nature of the terms used, whether positive, for example "more", or negative, for example "less"; presentation of two or three dimensional arrays.

The major finding of the research is that some children consistently employ a length strategy to judge both number and length; other children consistently employ a number strategy to judge both number and length. These strategy biases can be altered through changes in task variables such as set size and training. For example, small sets produce a number bias, large sets a length bias. Strategy biases also occur in response to negative terms such as "shorter', and on three dimensional as well as two dimensional arrays.

In the final chapter the author discusses the implications of the research findings for the six main issues outlined in the introduction. The author concludes that children's early quantity concepts are multidimensional. Initially the child may begin with a superordinate concept of bigness. In the course of cognitive development the child acquires various plans for discriminating dimensions, estimating quantitatives, and differentiating situations appropriate to the application of these strategies. In the absence of appropriate plans for any of the above, the child may substitute inappropriate plans in some consistent fashion. The inappropriate, consistently applied plans account for systematic strategy errors. In the course of time and experience these plans come to act as referents for the linguistic system. The author also concludes that, while some of Piaget's insights and theoretical arguments have been invaluable in aiding psychologists' understanding of the development of quantity concepts, various aspects of his theories such as the notion of centering and the role of transformation in the development of conservation, require further specification or modification.

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Nunc dimittis:

After attaining the presidency Abe Lincoln was asked by an old acquaintance from Springfield how it felt to get the presidency. Lincoln replied: "I feel rather like the man who was tarred, feathered, and ridden out of town on a rail. If it weren't for the honour of the thing, I'd rather have walked".

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INTRODUCTION

The general purpose of the following work was to study some aspects of the development of young childrens' concept of number, including some of the basic comparative language related to the number and length concepts.

It is perhaps a commonplace to state that the study of the development of concepts is one of the most difficult yet interesting endeavours that a psychologist can undertake. There are important questions to answer concerning the means whereby the variety of stimuli, in many domains and on many levels of abstraction, are processed, organized into concepts or categories, and utilized by the organism (Clark, E., 1973; Garner, 1974; Rosch, 1976). To many psychologists, most notably Piaget in recent years, the study of quantity concepts has promised to increase their knowledge of the development of basic reasoning processes, and the study of the terminology associated with quantity concepts their understanding of some important aspects of the development of language.

During approximately the past fifteen years most psychological research on quantity concepts has been influenced by Jean Piaget's substantial work in this area (S. Miller, 1976). However, Piaget's influence, at least in North America came late. In 1951, for example, a North American review of the literature available on number concepts gives sixty-four references, none with Piaget's name (Martin, 1951).

Long before Piaget, psychologists and educators viewed the clarification of the psychological processes involved in the development of quantity concepts, of number concepts in particular, as important. As stated previously, on one level this reflected the belief that an understanding of the processes involved in the use of the number concept offered to reveal something of the nature of high level reasoning abilities. However, on another level it reflected the great value placed upon mathematical skills in western culture and education. Pragmatic concerns for improved educational practices, and theoretical concerns about the nature of intelligence, motivated the interest in number concepts of early psychologists such as Alfred Binet (1890), John Dewey (1897-8), and Max Werthheimer (1912).

The reason for the impact of Piaget's work on quantity concepts appears to be the substantial role these concepts played in the development of his theory of intelligence. He created his theory of intelligence in an attempt to produce a genetic epistemology, i.e., to produce a theory of knowledge based upon data concerning the psychological origins of such knowledge. Although Piaget has disclaimed any intention of arguing for the necessity of such psychological data for a formal epistemology, he has argued that the psychological data could be of value to the epistemologist in clarifying certain issues, for example, the criteria for concepts (Beth and Piaget, 1966). Philosophers and psychologists both have questioned and debated the legitimacy of his claim (Hamlyn, 1971; Toumlin, 1971; Kaplan, 1971). This is not an issue to which this dissertation will address itself; however, it might be noted that Piaget has not been alone in asserting the value of psychological data for clarifying formal questions (Brainerd, 1973).

In constructing one of the only comprehensive theories of the development of intelligence in modern psychology, Piaget filled a vacuum. Within this theory the quantity concepts, particularly the number concept, have achieved an important status as indicators of the developmental changes in operations and structures which constitute the fundamental quality of intelligence. For example, the failure of a child to conserve number, i.e., his failure to maintain that the numerical value of two rows of dots as presented in Figure 1(a)

(a)	(b)
. . . .	. . . .
. . . .	. . . .

Figure 1

remains the same irrespective of irrelevant changes in the sets, such as a change of length as in the figure 1(b), supposedly indicates the absence of certain logical operations basic to combining elements, i.e., A + A = 2A, and ordering sets of elements, i.e., if A > B, and B > C, then A > C. Such a failure places the child in the "pre-operational" period of thought, a period qualitatively different from the concrete operational period, where the child will view the conservation of number as a logical necessity (Flavell, 1963).

In addition to the above, Piaget's concept of <u>the concept</u> has had an important influence on psychology. He has claimed that concepts and numbers have an important common property, i.e., ..."the additive operations which bring together the scattered elements into a whole, or divides these into parts" (Piaget, 1952). However, his major contribution to the study of concepts has been to emphasize "within things" variability when most psychologists have been interested in "between things" variability (Elkind, 1969). That is, Piaget, in his studies of the development of quantity concepts, has focused interest on changes in the characteristics of a single instance or examplar, as in the number example presented in Figure one. He has pointed out the necessity for having rules, or operations, or principles for distinguishing between real and apparent changes. In contrast, most developmental and cognitive psychologists of the associationistic tradition have been interested in the processes involved in grouping according to similarities and differences along discrete continua. For example, they have been interested in establishing dimensions for distinguishing "dogs" from "cats" and including these together as "animals". These psychologists have tended to neglect the development of the rules or operations for dealing with "within" category or concept variation.

The differences between the traditional and the Piagetian approaches reflect the Aristotelian - Gallilean distinction regarding concepts (Elkind, 1969; Rosch, 1976). The Aristotelian regards properties as discrete, the function of the concepts as classificatory, and the content of the concept as derived from the properties of the class or category. The Galilean emphasizes the variations in the properties of a single exemplar, the function of the concept as predictive and explanatory, and the content of the concept as discovered in the variations in a single instance.

While it is true, as Rosch has recently argued, that Piaget and his followers have neglected the internal structure of categories in studies of classification in favor of studying reasoning processes, the Piagetian work on quantity concepts has pointed to the importance of developing means for dealing with some kinds of changes in some kinds of single instances. Piaget has argued that it is the necessity for dealing with this "within things" variability which leads the individual to develop quantity concepts. This seems to have its parallel in the history of science; the interest in states, processes, relations and transformation, rather than abstract classes has led to the refinements of methods of quantification (Cohen, 1956).

Piaget's work on quantity concepts, then, has had a considerable impact on psychologists' understanding of the development of thinking and the nature of concepts. It has certainly been influential in forming the specific questions posed in the research presented in this thesis. However, the specific intention of this author was not to produce data to refute or support aspects of Piaget's theory, although some of the data may do this. Rather, in light of certain common issues addressed by those interested in quantity concepts, and through the use of a particular method, it was the author's intention:

(1) to examine some of those factors which influence a child's quantitative judgments on linear, numerical arrays;

(2) to further delineate the nature of the young child's understanding of comparative terms.

(3) to consider some general questions on the nature of concepts and

cognitive development;

Throughout the research presented in this thesis the author asked children to judge exemplars such as those in Figure 1(a) and (b) in terms of both number and length. It was her belief that such an approach had its own particular value for realizing the above three general intentions. The reasons for that belief will be argued in chapter one.

This thesis will be presented in three chapters. Chapter one will provide pertinent background literature and a statement of the general issues of concern in this research. Chapter two will consist of a presentation of the research findings. The conclusion, presented in chapter three, will attempt to indicate the relevance of the main research findings to the major issues raised in chapter one.

Chapter One

This chapter consists of five subsections. The first section will present some of the theories of the origin of number with an emphasis on Piaget's theory. From certain aspects of these theories it will be argued that there are reasonable theoretical grounds for maintaining that young children possess initially a multi-dimensional or global quantity concept from which develop specific concepts, such as number and length. Part of the development of specific concepts such as number and length involves either learning to attend to appropriate dimensions or learning the operations which allow those dimensions to be treated differently, or both. Whether the child does attend to the dimensions, or is able to perform the operations may depend on aspects of the array such as set size and configuration, and methods of presentation. Observing the child's responses on a quantity task requiring a verbal response to length and number questions should provide information about: (1) the meaning which the child assigns to comparative terms such as "more" and "longer"; and (2) some of the general processes of concept acquisition. This section will conclude with a statement of the main theoretical issues of interest in this thesis. In section two some of the empirical data will be presented which indicate that factors such as set size, configuration, the nature of the method of presentation, for example, static or transformable arrays, do affect childrens' quantitative strategies.

Section three will contain the literature available on young children's comprehension and production of comparative terms. The literature in this section raises the important issue of the relationship between cognitive and linguistic status. As well, the literature indicates that from children's judgments on quantity tasks the researcher can infer what the referents and meaning of quantity terms may be. The material presented here provides evidence for the gradual emergence of a hierarchy of comparative terms and for a "loose relationship" between cognitive and linguistic status (Holland and Palermo, 1975). In Palermo's terms the relationship is "loose" in this sense: evidence indicates that while success on a number test such as conservation correlates positively with a test of a child's understanding of basic terms such as "more" and "less" (Harasym et al., 1971; Sinclair de Zwart, 1969), some children conserve who do not understand the terms, and some children do not conserve who do understand the terms.

The fourth section will outline briefly some general approaches to the study of concept acquisition. It will be argued: first, that by studying over different ages the ways in which the young child uses quantity concepts one may be able to evaluate various aspects of the general approaches; second, that although the actual mechanisms for the acquisition of the number concept and for quantitative terminology are not clear, it may be helpful to view acquisition as the assembling of plans, or strategies, for example, "small number, count; large number, use length". These plans or strategies constitute the meaning of the concept and may be referred to as units or components of meaning. The addition of such components may result in the child attending to different dimensions of an array. In turn this could result in new bases for judgments and referents for terminology.

The fifth section will conclude with the major questions of this thesis. These are based on the main theoretical issues presented in section one, now supported by the literature reviewed in sections two, three, and four.

Section One: Theories of the Origin of Number

Psychologists, Brainerd (1973) notes, should be careful not to equate mathematics with the science of number. In fact, one persistent question in mathematics is the relationship of number to other areas of mathematics, for example, geometry. Nevertheless, most mathematicians have occupied themselves with number and some of their general and specific occupations have been reflected in psychology.

Three main approaches to number are found in the history of mathematics. The earliest view, but one still present in modern times, is that the natural numbers, the positive integers occurring early in the series, are the foundation of mathematics. Those who accord the natural number this primary status have believed either: that the "everyday" integers have an independent existence, i.e., a reality independent of any construction in men's minds [Pythagoras]; or, more recently, for example, [Brouwer] - that the series of natural numbers are known intuitively, i.e., are not constructed but given [Poincaré] (Beth and Piaget, 1966; Brainerd, 1973). The natural numbers are contrasted with negative numbers and fractions which, since they apparently lack any correspondence to physical reality, are believed to be constructed. The higher positive integers are also viewed in some sense as being "unnatural", "abstracted", or "constructed", as they cannot be intuitively grasped but must be constructed from knowledge of the number series (Lanczos, 1968; Lovell, 1961; Piaget, 1952).

The view that the natural numbers are the foundation of mathematics has been contested by modern logicians and mathematicians such as Dedekind, Frege, Russell, and Whitehead. The above have argued that logic is at the foundation of mathematics, but have expressed two opposing views. Dedekind argued that number is ordinally based, i.e., the basic concept is relation; the basic operation, that of ordering a series according to principles of transitivity and asymmetry. Frege, Russell, and Whitehead, on the other hand proposed that the basis of number is the concept of class, and the basic operation that of correspondence - a cardinal number is "class of classes" (Beth and Piaget, 1966; Brainerd, 1973).

The third school, called the nominalistic or formalistic school, is represented by D. Hilbert. Followers of the formalistic school not only rejected natural numbers as the foundation of mathematics, they argued that both logic and mathematics simply reflect linguistic devices (Beth and Piaget, 1966; Brainerd, 1973; Cohen, 1956).

Over the years various psychologists have emphasized different aspects of the formal theories of number outlined above. Some psychologists, Piaget pre-eminently among them, have claimed that an understanding of the psychological origins of number (or geometric concepts) might be helpful in resolving some of the formal debates. Although not all the psychological theories of the origin of number can be presented here, some of the historically more important theories will be. The purpose of this presentation is twofold; first, to indicate the counterparts in psychological theories of certain concepts and concerns found in the formal theories; second, to show how different psychologists have conceived of the emergence of the number concept. The emphasis will be on Piaget's theory and will include a comparison of aspects of this theory to his length theory.

Helmholtz claimed that ordinal number could be reduced to the sense of time, derived from successive states of consciousness (Beth and Piaget, 1966). In a somewhat similar vein to Helmholtz, Phillips argued in 1897 that successiveness is the fundamental experience of consciousness and is derived from all sensory modalities. Eventually the series idea itself is abstracted. Children, Phillips maintained, show the series idea well in advance of the number idea. They find an intrinsic pleasure in any kind of series, including, eventually, the series of number names independent of any application to objects. Eventually the symbols are applied to objects. At first, however, the child does not perceive an object and the number name he applies to it as independent. The child is not able to count vicariantly, i.e., to apply the numbers independent of any order of the objects. The number name becomes an intrinsic part of the object to which it is attached. Phillips took strong exception to the notion that the child derives number from the necessity to measure, "the rational process"; or to make quantitative judgments (Dewey); or from the apprehension or discrimination of number as an attribute of a group of objects (Lefevre); or from enumeration itself. "Number is derived from the experience of successiveness". An interesting feature of his argument is his assertion that too much emphasis has been placed on sight in the studies of number, with the result that the series idea has been neglected:[1]

> "Number in its genesis is independent of all quantity, and the science of number is essentially the relation of one number in the series idea to the other." (Phillips, pg. 243)

Dewey strongly disagreed with Phillips, mainly because Phillips did not take account of magnitude, or intensive concepts of degree such as "little, much, more, less", etc.

> "The point is that the savage and the child begin with equally vague concepts of plurality corresponding to the series, and with an equally vague sense of the totality or unity (quantity) which is split up into this plurality. By the application of one factor to the other, each is defined.--- We begin with an equally vague "many" and an equally vague "much --- number is absolutely essential to the measurement of quantity." (Dewey, 1897).

The series only becomes numerical when its parts are ordered with reference to place and value in constituting the whole group. The group can only be counted when it is perceived as made up of a plurality. Because of this interdependence, Dewey claimed that the series idea and the quantitative or magnitude idea are "logically correlative". The child's problem is to distinguish relative place from absolute place, and from spatial form.

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1. Marion Blank has recently argued that the role of language in cognition has been undervalued because of an emphasis in modern cognitive psychology on visual discrimination rather than on auditory discrimination, which of necessity involves temporal-serial discrimination (1974).

Dewey's theory presupposes active mental processes in the child which bring about number concepts through dealing with objects (McLellan and Dewey, 1896). The basic processes are those of discrimination and abstraction. The child has to isolate individual units and reject all the qualities of the objects but their individuality. He must then group these individuals into classes (McLellan and Dewey, 1896; Lovell, 1961). However, the grouping is not perceptual, but an abstract cognition built up from activity. Dewey's ideas in this respect are similar to Piaget's which will be presented shortly. However, as Lovell points out, Dewey, unlike Piaget, did not develop a comprehensive theory to account for various aspects of the transition from physical to mental activity.

Werthheimer, as might be expected of a Gestaltist, emphasized the importance of the structured and natural groupings as the basis of the number. The grouping, "non-quantified" plurality, must be considered genetically prior to counting in the development of number. "Plurality is not genetically a quantity of identical items, but an articulate whole" (Werthheimer, 1912). There are, however, different types of groups in the environment. There are groups such as "family", and the fingers on the hand, which are natural groups. There are other groups such as "the group of eight watermelons", or "the group of nine watermelons", which are not natural groups; these are groups only by virtue of their numerical total. It is these groups which Werthheimer believed to be the foundation of mathematics.

According to Werthheimer children and "savages" are bound to concrete objects and to natural groups. The problem the child has in learning about number is overcoming the power of the natural groups in his thinking. He must do this before he can master number, for the main feature of number is its transferability. To use the number five mathematically the child must be able to perceive the number as not intrinsically part of the group "fingers", but as transferable to any group of objects. In addition to the ability to separate the number from the object, proper counting requires the ability to recognize the basic unit for the count; to apprehend the total group to be counted; and, to perceive the distance from the origin of the count.

With respect to the requirement that the child must learn to distinguish relative place from absolute place, and from spatial form, Dewey's and Werthheimer's theories appear to be similar. The major difference lies in the role they assigned to perceptual and cognitive processes. Werthheimer emphasized the important role of perception in grouping; Dewey, it should be recalled, perceived grouping as a cognitive process evolving out of activity.

Pre First World War psychologists such as Phillips, Dewey and Werthheimer developed their number theories either through naturalistic observations of children in school settings, or, in the case of Werthheimer, of "savages". The study of individual children was also popular (Court, 1920). After the war, theories of number tended to be based on findings from studies of groups of children in which experimenters manipulated variables, even though the methods they employed were as disparate as Piaget's clinical method and the standardized test procedures of Douglas (1925).

Despite the post war changes in approach, there is still no agreement on a psychological theory of number. However, the most influential number theorist of the period has been Jean Piaget. His theory of number will now be examined in some detail.

Piaget

Briefly, Piaget states the observable stages in the development of number are three. In stage one, when judging number on an array which consists of two rows of equal numbers of items, the child appears to base his judgments on a single attribute, usually length. He is not able to perform one to one correspondence. A child performs one to one correspondence when he matches one object in one group or series with one object in another group or series. In stage two, the child can establish one to one correspondence but this is perceptually based; in the face of changes in length or density he will surrender a judgment of equivalence. For example, if a child has matched eggs in a row to egg cups laid out in a row of equal length, and judged them to be equal in number, he will change his judgment of equivalence if the experimenter extends one of the rows. Judgments tend to be inconsistent at this stage since they may be based alternately on length or density. In stage three, the child conserves number, i.e., will judge the number to be equivalent even in the face of spatial distortions. The ability to conserve number and explain why the numbers remain the same despite spatial changes is the criterion for the possession of the concept of number. This ability usually appears at about age seven (Piaget, 1952).

In Piaget's view, the activity of the young child in relation to certain environmental events, for example, the sorting and arranging of groups of objects, is important in the genesis of his concept of number. Certain characteristics of these activities are abstracted, becoming mental activities or internal operations. These internal operations are identified with logical operations.

The internal operations that form the basis for the construction of number fall into two categories; operations of class and operations of relations. Operations of class depend on similarity; operations of relation on differences. An example of simple classification is a child's assigning roses (A) and daisies (B) to the same class, flowers (C). The development of simple classification operations allow the child to establish the equivalence of sets - the cardinal aspect of number, i.e., two is always two, always belongs to the class of two. Basic to establishing cardinal equivalence is the ability to perform 1 to 1 correspondence operations. The child must disregard differences in the objects and take one for one. A child can establish cardinal equivalence this way even without counting. Also basic to establishing cardinal equivalence is the recognition that the value of the set is not altered except by addition or subtraction.

An example of simple operations of relation or order is a child's arranging a series of sticks of different lengths in ascending or descending order. The development of ordering operations enables the child to order sets, or elements within sets, according to differences - the ordinal aspect of number, i.e., first, second, third,.... Basic to the ordinal concept of number is the ability: to order the sets according to a unit difference, "Number is an additive union of units" (Piaget, 1952); to apply transitivity, i.e., given that A > B, B > C, then A > C; and to maintain one-to-one correspondence between two orders, for exampl·, between the sequence of the numerals themselves and the counting of the elements. The child must count in sequence and no object more than once. The ordinal value is assigned by the order in which enumerated.

Piaget proposes that the operations of class or similarity, and the operations of relation or difference develop in parallel. For the child to construct the infinite number series, he must coordinate both sets of operations. The child must treat each element of a set as the same, i.e., as one unit irrespective of other qualities. He must also treat each element as different, i.e., with regard only to its arbitrary position in a series. If the child attended only to similarity, he could not establish the sequence of elements, if to differences, the equality of the elements.

For many years now Piaget has represented the set of mental operations underlying simple classification and ordering described above with logico-algebraic models called Groupings (Piaget, 1952; Beth and Piaget, 1966). If a child shows that he is able to perform simple addition and subtraction of classes, for example, $A + A_1 = B$, $B - A_1 = A$. or A (roses) + A_1 (daisies) = B (flowers), he is said to possess Grouping I. If the child shows he is able to do basic ordering operations on differences in value along a dimension, for example, given a row of objects of different lengths the child is able to order them correctly from longer to shorter, he is said to possess Grouping V. The additive and ordering operations represented by Groupings I and V are intensive, i.e., in terms of performance this means the child is able to judge that something is "less", or "more", or "longer", or "shorter" than something else, without quantifying the degree of difference (Flavell, 1963; Ginsburg & Opper, 1969).

When the child shows he is able to conserve number, he is said to possess the Group. The Group represents formally the integrated set of class and ordering operations, the..."additive group of the whole numbers" (Flavell, 1963). Iteration, i.e., A + A = 2A, has been acquired. In terms of performance this means the child can transform the degree of difference into iterable units resulting in arithmetic operations.

Until the two sets of class and relations operations emerge and are integrated, Piaget assigns the child to the preoperational or prelogical period of thought. In this period he will judge number, (and other experiences) according to simple, isolated perceptions. For example, he will judge the value of number of two rows of dots as in Figure 1 (a) as equal, then change his judgment if one row is changed in length. He will not take into account the change, or transformation itself. When the operations are coordinated, and the child is able to take account of both similarity and difference simultaneously, he will conserve number. With this, Piaget assigns the child to the concrete operational or logical period. Piaget, then, views logic not as developmentally prior to the emergence of number, but as concomitant with it.

In the concrete operational period the child's judgments are not bound to isolated perceptions. The internal operations are now flexible; they allow the child to combine successive isolated states into a kind of continuous and reversible motion picture. The flexibility of the operations is due to their reversibility. For example, an operation of adding A + B to form class C can be negated by taking subclass B away from C. Operations also allow for reciprocity, i.e., they allow the child to recognize that a change in one dimension is compensated for by reciprocal change in another. For example, a change in length in a row of discrete elements produces a reciprocal change in density. The only limitation still present on the child's ability to judge number in this period is that the operations cannot be performed in the abstract; he can only perform these operations on objects actually present.[1]

As indicated several times in the discussion above, the child's ability to conserve number, i.e., to maintain a judgment of numerical equivalence despite apparent changes in an irrelevant dimension, is Piaget's main criterion for stating that the child possesses the number concept. It is as well an important milestone in the child's intellectual development. Indeed, Piaget considers the ability to conserve a quantity, i.e., to maintain a judgment of equivalence despite apparent changes in an irrelevant dimension, to be the chief criterion for the possession of any quantity concept, be it number, length, volume, density, etc. The child's progress through the concrete operational period is marked by the acquisition of these various concepts. The principle of conservation is therefore important both in Piaget's theory

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[1]In the preoperational period before the basic operations are integrated there is some flexibility attained when individual operations are acquired. Piaget calls this "intuitive regulation". (Flavell, 1963).

of the development of number and in his general theory of the development of intelligence.

The notion of identity and conservation should not be confused. Identity refers to permanence in the definitions of a term: a watermelon must not be assigned the properties of an orange in the course of a discussion, nor four the properties of five. Conservation refers to permanence in the exemplars; a group of four watermelons must not be judged as a group of five watermelons unless another watermelon is added. Logically, an idea or term which does not retain its identity in an argument, i.e., does not have any permanence of definition, cannot be used in any meaningful reasoning process (Elkind, 1969). However, Piaget maintains that the notion of identity is not sufficient for reasoning; it is necessary to have the principle of conservation. A major question for Piaget is how the child acquires the principle of conservation; his answer rests on the concepts of structure, centration, and equilibrium, in his theory of intellectual development. These concepts will now be considered.

In Piaget's theory of the development of intelligence the idea of structure is extremely important. It is structure which changes with development and which determines the characteristics of thought at different periods. It is, therefore, of the greatest interest. For Piaget, structures are "the organizational properties of intelligence, organizations created through functioning and inferrable from the behavioral contents whose nature they determine." (Flavell, 1963, p.17)

In the discussion of Piaget's theory of number mention was made of the changes in the characteristics of the child's judgments from the preoperational to the concrete operational period. If the child makes contradictory statements about the value of number when changes are made in an array of dots, Piaget infers that the child's thought at this stage is organized or structured in such a way that he responds to isolated perceptual features. He then assigns him to the pre-operational or prelogical period of thought. When the child conserves number, Piaget infers that his thought is now organized or structured by the integrated operations of class and relations. He assigns him now to the concrete operational or logical period of thought.

Centration is a characteristic of thought prior to the concrete operational period. It consists of attending to individual features or dimensions of objects or elements without taking account of other important features or dimensions. Decentration, on the other hand, consists in attending to other possibly relevant features. This is made possible when the child achieves flexible operations which allow him to consider the possibility of negation and reciprocity.

Decentering may be viewed as a cognitive process akin to the broadening of attention. Piaget assigns it an important general role in the development of intelligence. Noteworthy is Piaget's statement:

> "Here as indeed in every other realm of human knowledge, intelligence starts from an unconsciously egocentric outlook which is a very congeries of unwarranted centerings, and which hinders grouping because it leads to irreversible assimilations. The very fact of overcoming this egocentric outlook, is by definition, we may say, the first and necessary step towards the logical combinations of relations. The process of decentering reaches its final state with logical (or sublogical) grouping and the mathematical group".
>
> (Piaget, Inhelder, and Szeminska, 1960, p.25)

Centration and Decentration are important in the equilibrium process.

Piaget employs the equilibrium model to account for changes in the structure of the child's intelligence. Different states of equilibrium exist, which can be compared according to the following three main properties:

(1) field of application. As the child develops intellectually the field of application of the system of actions or operations expands, and the state of equilibrium improves. Early in development the field of application may be based on single perceptions, single centrations.

To illustrate: When the child does not possess the integrated class and relations operations necessary to number, he judges numerical equivalence according to a single visual perception of the length or density of rows in an array. He "centers" on length or density. His judgments of equivalence can therefore alter easily depending on his momentary perceptions. When he does possess the integrated operations he will be able to take into account the relationship between length and density.

(2) mobility. With the extension of the field of application the child's judgments are freed from the spatio-temporal constraints of single perceptions. This provides mobility - an important feature in the development of greater equilibrium. Referring to the illustration above, the child is able to take account of: changes in the past, such as addition or subtraction; constancies in the past, such as the equivalence in the two rows.

(3) stability or permanence. A more stable equilibrium state is present when new input into the system is not apt to alter the balance of the

structures: When the child is bound to single perceptions, new perceptual input will lead him to alter values he assigns to the objects or attributes he perceives. For example, in the absence of operations for number, changes in the length of an array cause the child to alter the value of "number". When the system is cognitively balanced, changes in length will not result in such changes in value.

When a system is equilibrated, it is balanced or stable. Intellectual development for Piaget consists of the development of successive systems of mental operations which are in a balanced and stable state of interaction with different aspects of reality. A child is only able to deal with environmental events in terms of the actions and operations which he has available to him. What motivates intellectual development is the need the child feels to establish a more stable interaction between himself and the environment. That is, the systems are modified as the child grows aware of the discrepancies in values which he successively assigns to the same objects or elements. The child's movement from one state of equilibrium to another is described by Piaget in terms of the probability of his adopting a particular strategy. For example, in the development of the number concept, the probability of the child using length, density, or both to judge number alters. (Flavell, 1963; Ginsburg and Opper, 1969).

The notion of equilibrium is thus important in Piaget's theory of intelligence, for it is the basic mechanism for transition. True intellectual development is based upon this mechanism, in particular upon the child's experience of disequilibrium. Specific learning, i.e., learning specific responses to specific events, can take place, but will not be generalized correctly to new situations in the absence of the necessary mental structures. That is:

> "If there is too great a disparity between the type of experience presented to the child and his current cognitive structures, one of two things is likely to happen. Either the child transforms the experiences into a form which he can readily assimilate and consequently does not learn what is intended, or else he merely learns a specific response which has no strength or stability, cannot be generalized, and will disappear soon" (Ginsburg and Opper, 1969).

That being the case, another important criteria for the possession of a concept for Piaget is that it can be generalized across a wide variety of applicable situations. Such generalization indicates true intellectual development.

Having reviewed Piaget's concepts of structure, centration, and equilibrium, it is possible to present his view of the development of the principle of conservation of number. (The process is comparable for other quantity concepts such as length).

Piaget maintains that conservation of number develops through equilibriation of cognitive processes in four steps. The strategies the child adopts reflect a probability based on subject-object relations. For example, given a linear, numerical, display with two dimensions, density and length, in the first step, on a chance basis, the child will center on one of the dimensions of the display and make that the basis of his judgments. Supposedly, both dimensions have an equal probability of being "centered." In step two, the child attends to, or centers on, the other dimension, for example, density. Piaget maintains that two factors account for this shift: First, the child is subjectively dissatisfied with his response because of constant changes in his perception, i.e., he experiences disequilibrium. Second, changes in the saliency or prominence of contrasting features will attract the child's attention, i.e., an increase in the length of a linear array of discrete elements will make density a more prominent feature. After both features are noticed the child begins to alternate between them, until eventually he notices both at the same time - step 3. When he has noted both features over a series of trials, he begins to observe the transformations which relate to the inverse changes in the dimensions. With this integration of states and transformations, conservation is achieved. Since conservation is a principle which guarantees a stable interaction with a wide range of events in the environment there is no further need for changes in strategy. A stable cognitive structure has been achieved. (Flavell, 1963; Ginsburg and Opper, 1969).

It is appropriate now to consider the relationship of Piaget's theory of number to: the three main formal approaches to number; his own theory of length.

Because Piaget does not view logic as preceding number in the genesis of the number concept, but rather as emerging concomitant with it, he rejects the formal number theories of Dedekind, Russell, and Whitehead. These theorists argued that logic is the basis of number. His position on the natural numbers is less clear.

In The Child's Conception of Number, which was published originally in 1941, he appeared to accord an intuitive status to the early positive integers of the natural number series.

> "To sum up, we can now see why the additive hierarchy of classes, seriation of relationships, and operational generalization of number (i.e., the construction of numbers above 1, 2 ... 5, which are intuitive) appear at approximately the same time, about the age of 6 or 7." (Piaget, 1952)

However, in 1966 he seemed to change his position on the status of these natural numbers.

> "When we say that the series of positive integers is "natural", we generally mean by this that the early part of this infinite series corresponds to a group of everyday concepts expressed in language by the number one, two, three, etc., either in speech or writing....the early members of the series of positive integers....correspond to distinct concepts in the subject's conscious thought, and this has been sufficient for many writers to regard them as being "natural" (which will not however be sufficient for us)....psychologically the construction of the first natural numbers is the result of a "synthesis" in a single system of a series of class inclusions and of seriations." (Beth and Piaget, 1966 pp 166-167.)

The latter statement appears to be a clear rejection of the position of the formal theorists such as Brouwer who argued that the natural numbers are not constructed but given. However, again in 1952 he wrote:

> "But apart from the numbers 1 to 3 at about the age of 3, 1 to 4 at about the age of 4, and 1 to 5 at about the age of 5, the construction of number cannot remain within the field of perceptual intuition and can therefore be completed only on the operational plane...It follows that at this level the intermingling of

> the cardinal and ordinal processes that constitute number is only in its early stage, and there is not as yet true co-ordination." (Piaget, 1952, p.154)

This last statement may help resolve the apparent contradiction of the first two. Piaget may accord a peculiar status to the early natural numbers, but he still retains the notion that they are constructed. The argument appears to be the following: Because the child is able to grasp perceptually the similarities and differences between groups of small numbers, he is able to use intuitively the beginning class and relations operations to construct a small number series. The small numbers then may enable the beginning integration of the class and relations operations. However, until these operations are truly integrated on the cognitive plane the remaining number series cannot be constructed.

Finally, what does Piaget have to say about the possibility that both logic and mathematics simply reflect linguistic devices? Briefly, because in Piaget's system operations are directly derived from actions, the thought processes involved in logic and mathematics do not depend upon linguistic abilities. Piaget therefore rejects the "nominalistic or linguistic" theories of mathematicians such as Hilbert.

In Piaget's system the child's thought processes are simply reflected in his language - "language is the dependent variable (Flavell, 1963)". Even though the child may have learned to use the socially transmitted linguistic symbols, the information he transmits via language will be bound to his level of understanding. Similarly, the information he derives from others speech will be interpreted in terms of his underlying available mental structures (Piaget, 1955). For example, when a child says "there are more glasses here" to the longer of two parallel rows, after having said that both have six, he is indicating something about his logical status. According to Piaget, the use of "more" here reflects the child's lack of operations which would allow him to differentiate number and the space occupied (Piaget, 1952). The young child may also not have certain words available to him. However, for Piaget it is the logical underpinnings of the words which are of interest, and he believes that the logical underpinnings are reflected in the child's verbal responses.

Piaget also argues that verbal labels, such as those employed in counting, can be a purely rote accomplishment; they have no bearing on the processes involved in the development of number. As in the example above, the child might be able to count to six by rote but this has little to do with his grasp of number. Verbal competence does not guarantee success at a task. The only role allowed language in the early development of thought is as a stimulant, for example, noun class names may help a child distinguish discrete classes (Piaget, 1952). Only later, in the adult period of formal operations does Piaget assign language a more important role as one of the useful notational systems; algebraic symbols are another.

As stated in the introduction to this thesis the basic method of study employed throughout the research was to ask a child to judge both number and length on the same exemplars. The rationale for this method rests, in part at least, on the very close relationship between Piaget's theories of number and length. It is now appropriate to consider what this relationship is.

Piaget has proposed that three elementary structures of thought develop in the preoperational period. Two of the structures are based on operations relevant to discrete objects or elements: structures of class operations and structures of relations operations. These are the structures basic to number. The third structure is based on operations relevant to objects as continous wholes. These operations are basic to spatial concepts such as length. Taken together then the number and length concepts include the three elementary structures of thought. And, Piaget believes, all future intellectual development rests upon these three structures - including the high level reasoning exemplified by mathematicians and logicians. To quote Piaget"

> "Accepting the hypothesis that the three elementary G (Group) structures alone cover all the natural structures, and the hypothesis that the mathematician independently of the formalization which always occurs a posteriori, only constructs mathematical entities by using "natural thought", simply refined by an uninterrupted series of progressive abstractions originating not from empirical objects (perception, etc.), but from the actions and operations which he performs on these objects, it then follows that this construction of mathematical entities will be conditioned by the characteristics of three elementary G structures." (Beth and Piaget, 1966, p.18)

Piaget's theory of the development of the concept of length is clearly and explicitly parallel to his theory of the development of number (Piaget et al., 1960). The main stages in the development of the concept of length correspond to those for number. In the first stage, the child appears to attend only to the single end-points of two equal-lengthed objects. If one end protrudes beyond the other, he will judge one stick as longer than the other, Figure 2(a). Or, if one string is straight and the other is curved, he is apt to judge the curved string with matching end-points as equal in length, because

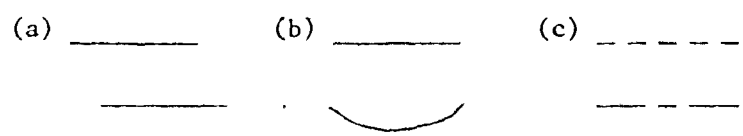

Figure 2

he disregards rectilinearity, Figure 2(b). At stage two, the child may take account of the overall interval, i.e., attend to both sets of end-points and attempt to compensate for lack of rectilinearity. However, if the length is segmented, he may attend to the segments of the line, ignoring the overall extension, Figure 2(c). Sometimes he may treat these segments as equal units, i.e., count them, or he may attend to a "privileged segment" - i.e., to one segment which is longer or shorter than the others, and use it as the basis of his judgments. In stage three-A the child attains qualitative conservation of length. If he has first judged two lines as equal in length, he is able to maintain the judgment of equivalence, even if the lines are segmented, bent, or changed in position. At stage three-B, for which there is no parallel in number, he attains quantitative conservation of length. He can apply spontaneously some standard unit of measure, for example, his own finger, to establish equivalence.

Piaget reserves the term logical operations for mental actions performed on discrete objects. He calls the mental actions performed on geometric spatial concepts such as length, sublogical operations (Piaget et al, 1960). The sublogical operations which he believes to be basic to the development of length are: (1) operations relating parts and wholes; and (2) operations relating orders. For example, if a child is to judge the length of a strip of paper cut into pieces, he must be able to reconstitute the whole; and this must be done by placing the pieces in a systematic relationship, one piece beside the other.

As in the case of number, the systems of mental operations for relating parts to wholes, and spatial and temporal orders, are represented by formal models called Groupings. Only when the operations represented by the Grouping for sublogical operations relating parts to wholes, and the operations represented by the Grouping for spatial and temporal orders are coordinated will the child conserve length qualitatively. He must attend both to the subdivision and to the order of the end-points. This is at about age 7. The child will conserve length quantitatively when the sublogical elements can be quantified, i.e., transformed into iterable units. This makes measurement possible; however, Piaget states that measurement operations lag behind arithmetic operations "...because the application of the notion of a metric unit involves an arbitrary disintegration of continuous wholes" (Piaget et al., 1960). Children, therefore, conserve length quantitatively at about age 8. The development of the principle of conservation of length is brought about through the equilibrium process, as in the case of number.

Although the logical and sublogical operations basic to the development of the concepts of number and length are formally similar, and the psychological development parallel, Piaget argues that they must be distinguished, because they are in several important ways the inverse of each other: (1) Logical operations are performed on discrete sets of objects; sublogical operations on continuous wholes. When discrete objects are combined they remain discrete, for example, daisies and roses into the class of flowers. When parts of a whole are combined, a whole is produced, for example, pieces of paper into a strip of paper. (2) Logical operations are not dependent on the proximity of elements, i.e., classes and relations can be established independent of spatial and temporal factors, for example, the class of flowers. Sublogical operations are performed on wholes - if the whole is partitioned spatially or temporally, the whole no longer exists. For example, the strip of paper.

Of importance to the conceptualization of the research presented in this thesis is the following: According to Piaget and Inhelder (1959) children in the preoperational period may not distinguish logical from sublogical operations. For example, in classifying a set of objects they may group some of the objects according to an attribute such as colour or shape and at the same time group some of the objects to make a figure (Flavell, 1963, p.198). In the course of the preoperational period, therefore, the child must learn to distinguish situations appropriate to the use of sublogical operations. In terms of the concepts of interest to this thesis, he must learn to dintinguish situations appropriate to the use of numerical operations from those appropriate to the use of length operations.

To conclude this discussion of Piaget and this section: It was stated earlier that there is as yet no agreement on a psychological theory of number. Piaget's theory is not excepted. Given the broad role the quantity concepts play in Piaget's theory of intelligence and in his genetic epistemology his number theory is open to a broad range of criticism. There have been questions concerning: the appropriateness of his methods to assess the child's status on a concept (Miller, 1976; Siegel, 1977b); the true order of emergence of cardinal, ordinal, and natural numbers (Brainerd, 1973); the accuracy of his mathematical models (MacNamarra, 1975); and the legitimacy of his epistemological claims (Hamlyn, 1971).

It was not a primary goal of the research presented in this thesis to add to the literature specifically critical of Piaget's theory of number. Rather, certain unresolved issues found in the formal and psychological theories, particularly Piaget's, and in the related research literature, generated the research questions presented in Chapter Two. It was hoped that in providing empirical data relevant to these questions the general intentions of this thesis could be met. The main theoretical issues will now be presented. On these issues, a rationale is given for considering the number and length concepts of the preoperational child conjointly. Section five will formulate these issues as general hypotheses supported by the literature reviewed in sections two, three, and four.

(1) There is a broad agreement among number theorists that the child infers, constructs, or abstracts some aspects of number from experiences

in the real world, whether perceptual or based on action. Initially the child may consider number to be an actual part of a group of objects (Werthheimer, 1912; Dewey, 1897) or of a particular object (Phillips, 1897). If the child derives number from perceptual discrimination of, or actions upon real objects, then his concept of number could partake of other dimensions which these objects possess: their aggregate qualities, for example, space occupied by a group; or their particular qualities, for example, one is the index finger, two the middle finger, etc. It seems reasonable to argue that, whatever the mechanism, the child will have to develop some means for responding to the dimensions relevant to the particular concept, number; until he acquires these means, quantity concepts such as number will be perforce multidimensional. As well, other dimensions such as length, which were originally part of the child's real world experience of number, should themselves reflect the development of the child's numerical strategies. One would expect to find this multidimensional concept in children in Piaget's preoperational period (ages 3-7).

(2) As outlined previously, Piaget's theory of the development of the length concept is clearly parallel to his theory of the development of the number concept. He proposes some communality in the logical and sublogical operations underlying the two concepts (Piaget et al., 1960). He and his collaborators have reported that children in the pre-operational period do not always distinguish the situations appropriate to one set of operations from those appropriate to the other (Flavell, 1963, p.195). Further, at some stage in the development of the concepts, children use length to judge number and number of

segments to judge length. It is possible that a child only has one set of operations available at the time when he uses a strategy inappropriately. For example, he may have only the operations appropriate to continuous elements and apply these uniformly. This would not be Piaget's position. A conjoint study of the length and number concepts might demonstrate the degree to which children treat the concepts as independent and have operations appropriate to both concepts available.

(3) According to Piaget's equilibrium model a child's current cognitive structure determines the probability of adopting a particular strategy to judge number (or length). Prior to reaching full conservation status his strategy may depend on the dimension to which his attention is drawn. Different dimensions may be more salient or noticeable at different times. The dimension he attends to may depend on perceptual factors, or cognitive factors, or both. It could be that a child is able to identify attributes of an array relevant to length or number, but for some reason finds a particular attribute overpowering and makes his judgment accordingly. Altering certain features of an array while asking for judgments of length and number should allow one to identify some of the factors controlling dimensional saliency.

(4) An important feature of an array which may determine a child's choice of strategy may be the size of the number set. Both formal and psychological theories of number assign a peculiar status to the early numbers in the natural number series. The possibility of perceiving small groups of objects as a group, but at the same time composed of easily perceptible different elements, allows for the child employing

different response strategies to small number sets than large number sets. As well, the child might show different response strategies to length according to changes in set size. In terms of the preceding discussion of saliency, small numbers may make the attributes of the array appropriate to number more salient. This may facilitate the use of numerical strategies, but make length strategies more difficult.

(5) Piaget's view of the language related to quantity concepts is that it reflects current cognitive structure. If current cognitive structure determines the probability of a strategy and a child is not able to differentiate dimensions clearly, in the preoperational period, the referents or meanings of a child's quantity terms should reflect this confusion. That is, the child could assign words appropriate to different dimensions, for example, "longer" and "more", equivalent meaning. Also, the same words could change meaning according to the nature of the task, for example, judging small or large sets.

(6) In keeping with the tradition of psychologists such as Werthheimer, Dewey, and Piaget, who believed that examination of the number concept could shed some light on the general processes of concept acquisition and cognitive development, it is argued here that an examination of various aspects of the number concepts simultaneously with the length concept may provide new insight into some of the general processes involved in concept acquisition and cognitive development.

In summary, some clarification of the above issues, through examining the pattern of responses of young children (children in Piaget's preoperational period) to number and length questions on the same exemplars, could increase psychologists' understanding of number, language, and cognitive development. A child may respond correctly or incorrectly to a request to make a quantitative judgment. The pattern of such responses allows the experimenter to infer that the child is employing a particular strategy. In this introductory section mention has been made of some of the possible quantitative strategies, for example, counting, and the factors which may influence their use, for example, set size. Section two, which follows, outlines the research literature bearing on the quantitative strategies a child may employ in judging linear, discrete, arrays; and the factors which may influence the choice of strategy.

Section Two: Quantitative strategies and the factors influencing the choice of strategy.

This section will review the various quantitative strategies children (and adults) use to establish the numerical value of a set of discrete items and the equivalence of the number or length of two sets. There are, of course, nonquantitative strategies which a child may employ in response to quantitative questions. For example, some very young children always point to the top row of two rows of dots. Of major concern here, however, are the quantitative strategies various investigators have proposed as the basis of children's evaluation of the numerosity of arrays. Amongst these are: (1) estimation of numerical value based on other qualities of the array besides numerosity, for example, length or density; (2) perceptual strategies including pattern recognition or subitizing; (3) counting; (4) inferential cognitive strategies involving the use of a rule. In discussing these strategies attention will be given to factors which seem to influence the child's choice of strategy. The section will close with a brief consideration of the strategies employed in making length judgments.

There is evidence that at some stages in the development of the number concept, length acts as an important cue for children's numerical judgments (Binet, 1890; Gelman, 1969; Piaget, 1952; Pufall and Shaw, 1973; Siegel, 1974a). It is inferred that a child is using a length strategy by examining his response pattern to different arrangements of rows of dots. For example, the experimenter might conclude that a child who says that Figure 3(a) top row has more, Figure 3(b) bottom row has more, Figure 3(c) the two rows have the same, is using the length dimension.

(a) . . .   (b) ....   (c) . . .
    ...   .

Figure 3

This strategy shows changes with age and with set size. For example, Gelman found that no nursery school children used length for their judgments of set sizes 2-3, whereas 50% of them used length on set sizes 5 and 9; no grade 2 child used length on any set size (Gelman, 1972). She found no evidence for a density strategy.

Schaeffer, Eggleston, and Scott (1974) have recently argued that the estimation of small numbers (set size 1-4) is founded on pattern recognition, the result of perceptual learning. For example, the child learns that two dots imply a straight line, three a triangle.

It is their contention that pattern recognition is eventually integrated with counting as the child attempts to estimate the numerosity of larger sets. However, recognition itself constitutes the basis of the growth of estimation skills. These investigators support their argument with findings from a study in which pattern recognition was credited when a child did not point at, or count, figures in an array, but simply gave a number. They used set sizes from 1-7 and found that, initially, young children (3.8) relied primarily on pattern recognition to establish the value of small sets (1-4). The same children were unable to establish with any accuracy the value of sets 5-7. There were indications that as the children got older (4-5 yrs.) they were more apt to rely on counting to establish the 5-7 set sizes. However, since the investigators discouraged the children from counting out-loud, their assumption that a child was not counting when he gave the numerical value of the small sets seems unwarranted.

Over the years a variety of studies on adult's span of apprehension have established that subjects under time constraints are only able to accurately report the number of elements in an array when these fall below seven, although the "magic" number seven could be extended to seven groups of more than one element.[1] These studies were usually tachistoscopic studies with limited presentation time. They produced set size related curves which show very rapid responding

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1. Sperling's studies in the early 1960's indicated that the limitation on the "magic" number seven was imposed by the speed with which a subject could read off a rapidly decaying visual image. Subjects actually had a great deal more information available to them than the reporting limits indicated (Neisser, 1966).

on sets below 6 elements and progressively slower responding on sets above. These findings from adult span of apprehension studies have led to the argument that the process for estimating the numerosity of small sets is a primary immediate perceptual mechanism "subitizing", a kind of rapid pattern recognition. The process for estimating large sets, over 6 elements, is counting (Kaufman et al., 1949).

The findings from adult studies have led to suggestions that the subitizing mechanism is the mechanism employed by young children when they estimate the numerosity of small sets - a kind of primitive intuition (Klahr and Wallace, 1973). It should be recalled in this regard that Piaget has stated that "apart from numbers 1 to 3 at about the age of 3, 1 to 4 at about the age of 4, and 1 to 5 at the age of 5, the construction of number cannot remain within the field of perceptual intuition." (Piaget, 1952).

However, the advocates of subitizing have trouble accounting for the developmental changes in accuracy of estimation and the type of strategies employed. In a recent study of estimation accuracy in relation to set size by Gelman and Tucker (1975), time constraints of varying stringency were imposed, i.e., 1 sec., 5 sec., and 1 min. The children were ages 3 to 5 and the set size 2 to 5. Older children were less likely to count than younger children, and accuracy in estimating the larger numbers increased with age. Counting increased with the amount of time allowed for estimation.

Gelman disagrees with the advocates of subitizing, arguing that the young child may actually employ counting in his estimate of the numerosity of small sets. She has proposed that this counting is a true counting, indicating the proper coordination of actions or mental operations needed to bring a set and a verbal series into relationship (Gelman, 1975). Beokman and Descoudres and Gelman have provided evidence that children do count when making either absolute (Gelman & Tucker, 1975) or relative numerical judgments (Gelman, 1972).[1] This, combined with evidence that these same young children possess some operator, for example, they are aware that the numerosity of a set is changed through the addition or subtraction of one unit, has led Gelman to assert that counting may be the basic mechanism which eventually leads to an apparent subitizing, i.e., rapid counting in adults; or, alternatively, both counting and subitizing may take place together.

Whether a child counts or in some other way estimates numerosity, (When he does count, he appears to be more accurate in his estimates of numerosity. Gelman, 1975), seems also to depend on other aspects of the stimulus array besides set size, like the arrangement, or heterogeneity of the items in the set. With regard to the arrangement of items, Potter and Levy (1968) showed that when very young children (age 2½ to 4 years) were simply asked to point once to each member of a set of identical items without counting, performance was better with a single, horizontal row when there were more than 6 items, as compared to performance with two rows of three each.

The effects of configuration were also demonstrated by Beckwith and Restle (1966) in a reaction time study with older children (7-9) in

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1. An absolute judgment consists of assigning the numerical value to a single set.
   A relative judgment consists of deciding which of two sets has more or less.

which reaction time was greater for larger sets; this was related to the configuration arrangment, i.e., the most rapidly counted arrangement was the rectangle, then a single horizontal line, a circle, and a random pattern. These findings on the effect of configuration appear to indicate an increasing reliance on grouping as a means of counting as children get older. Beckwith and Restle propose that a subitizing mechanism may be employed on small "good groups" and these then summed.

With regard to the heterogeneity of items, the Potter and Levy study showed that the youngest children in their sample (2½) were better able to point once to each member of a set when it consisted of randomly arranged heterogeneous rather than identical items. However, there is some evidence that heterogeneity of set items may distract somewhat older children from numerosity. Gelman & Tucker (1975) cite a study by von Gast with evidence that preschoolers, who could give accurate estimates of numerosity on homogeneous displays, could not do so on heterogeneous displays. In contrast, Gelman & Tucker did not find heterogeneity to have an effect in children age 3-5.

Siegel, however, like von Gast, found that in preschoolers heterogeneity of items was a powerful distractor in numerical tasks. Siegel (1973) required that children age 4-8 match sets of 1-9 items for equivalence. The arrays varied according to linearity of arrangement and heterogeneity of items. She found that there were significant differences in task difficulty in the 4-5 year olds. For these children, the matching of heterogeneous items in linear arrangement was most difficult, followed by matching of nonlinear arrangements of homogeneous sets.

In a 1974 study, Siegel examined the ability of young children 4-5 to associate numerals with the correct set size (1-3 items) when linear arrangement and heterogeneity of items were varied. She found that heterogeneity created poorer performance in these children while spatial arrangement did not. Siegel speculated that the difference in findings on spatial arrangement in the two studies probably reflects the difference in the set size employed. On small sets children may be able to ignore spatial arrangement. However, even on such small set sizes heterogeneity had a substantial effect (Siegel, 1974b).

The variable effects of heterogeneity may relate to the nature of the task and the age of the child. Siegel employed a match-to-sample task; Gelman, absolute estimation. In the match-to-sample task, the presence of a variety of geometric forms in both the sample and the alternatives could have resulted in children forming subgroups according to form, across arrays. Such subgrouping could interfere with the identification of the actual groups or sets required to make the numerical comparison. On absolute estimation tasks like Gelman's, heterogeneity facilitates the count. Support for such an argument comes from findings by Schaeffer et al. (1974) which showed that in children 4-9 absolute estimation was better on sets with subgroups of different types of items than on sets with identical items.

If "grouping" is a basic mechanism which develops in some way, it is possible that the effects of heterogeneity alter with age. In the Potter and Levy experiment (1968) heterogeneity appeared to assist young children under three in keeping track of items. However, if as children get older they tend to group by patterns, or common features, or define dimensions by "higher order invariants" (Gibson, 1969), heterogeneity could interfere with the estimation of number as in the Siegel studies. Such a course of development would seem to presuppose an early predisposition to attend to variability or novelty, and development to consist of learning to isolate appropriate dimensions and to take variability into account in an appropriate fashion. At the same time, while the homogeneity of items in linear arrangement seemed to facilitate the extraction of numerosity in Siegel's study, it is possible that homogeneity of items, by facilitating the extraction of group dimensions, for example, length, could at some ages or stages or with some set sizes, make judgments of numerosity more difficult.

According to Piaget, children in the preoperational period will necessarily be dependent on perceptual factors and perceptual strategies because they lack the integrated set of logical operations necessary for true quantitative strategies. Counting in this period cannot be considered true counting. It might be recalled that from his equilibrium model Piaget argues that the child's choice of strategy in the preoperational period is predicated on the saliency of a particular dimension of an array, i.e., in the attention attracting power of a dimension. Changes in the child's strategy then reflect changes in the attention attracting power of the dimension. Unfortunately, psychologists do not know much about the basis of saliency; saliency may result from physiological, perceptual or cognitive factors.

Rosch (1976) has suggested that there may be cases where saliency is physiologically determined, for example, certain colours may be more salient because the visual system is constructed so as to respond maximally to these colours. In other cases, a class exemplar may be more salient because it more closely corresponds to a category prototype. P. Miller, Grabowski, and Heldmeyer (1973) have suggested that saliency may be perceptually or cognitively determined, although tearing the two apart may be extremely difficult. Some features of a stimulus array may be more salient because they show "good form" in the Gestalt sense; other features may be salient because of the child's knowledge of the world. Given that Piaget clearly distinguishes between perceptual and cognitive processes, he probably would accept that both kinds of saliency are operative. He has, however, paid scant attention to saliency effects of any kind.

Perceptual saliency has been the most widely studied. It is usually defined in terms of a child's (or adult's) preferred choice of a dimension on a classification task (Odom, R. and Guzman, 1972). The child is typically given 3 items which vary on two dimensions, say colour and form: for example, two circles and one triangle - one triangle red, one circle red, and one circle white. The experimenter then suggests to the child that he puts the ones together that belong together. Typically, preferences for different dimensions are established. Odom has suggested that hierarchies of dimensional saliency exist and these are subject to changes with development. Some experimenters have found that these preferences operate in such a way as to affect performance on learning discrimination tasks (Suchman & Trabasso, 1966). If the child is attending to one dimension, and he must learn to respond to another, he will perform more poorly than if he is attending to the dimension which is positive in the learning situation.

In several recent studies Miller has explored the role of stimulus dimensions in children's quantitative judgments. She has demonstrated that Piaget's assumptions concerning the equal probability of attending to both dimensions in making quantity judgments do not always hold. Some dimensions may be more salient than others. Testing for children's (5-6 yrs.) attention to the dimensions of length and width in a conservation of continuous quantity task, she found that in spite of the experimenters' attempts to call attention to the width dimension, the children persisted in attending to height (P. Miller, 1973). In a follow up study on a conservation of substance task, (children, age 5-6 yrs.) Miller found that length was the primary determinant of the child's judgment, with very little evidence for any period involving width (P. Miller et al., 1973). Miller suggests that the child who is about to become a conserver may break away from a "perceptual" use of a dimension, but this does not mean that he necessarily will attend to another dimension. In line with the above, Gelman (1969) suggested that there may be a hierarchy of response strategies which the child uses in making his quantitative judgments. Amongst the factors important in controlling this hierarchy of response strategies may be an as yet undelineated hierarchy of dimensional saliency.

Not everyone agrees with Piaget that the child in the pre-operational period will depend on perceptual strategies because he lacks logical structures. Bruner, for example, maintains that indeed the child does rely on perceptual strategies, but this is not indicative of an absence of a logical structure. The child probably possesses this structure but is overwhelmed by his perceptions (Bruner, 1964).

Despite differences in their theoretical positions, both Piaget and Bruner predict changes in the value a child assigns a dimension after he observes a transformation or movement of a stimulus configuration. Their position is supported by considerable evidence that young children do not conserve length or number in the face of observed transformation, or changes in an irrelevant dimension of a stimulus array (Piaget, 1952; Piaget et al., 1960). Where a child has made a correct judgment concerning the equivalence of two equal rows of elements, or of the equivalence of two equal sticks, he will frequently change his judgments in the face of either the extension of a row of discrete elements, or a movement of one of the sticks.

However, unlike both Piaget and Bruner, some psychologists, amongst them Baron (1974) and Bryant (1974), have considered the possibility that inferential strategies based on the child's ability to use a rule may be available to Piaget's preoperational child and under certain circumstances contribute to his performance. The availability of such inferential strategies is usually assessed in terms of the possible, or actual difference in performance on static or transformed arrays. A static or "quasi-conservation" number array consists of two rows of dots in various arrangements, stuck on a white card. Transforming an array requires that the dots be moveable. Consequently, a transformed array is an array where the experimenter can move the dots. The transformation of interest is the transformation of a dimension irrelevant to the quantity under study, for example, length on a numerical array. Whether observing an "irrelevant" transformation results in improvement or decrement in performance is the question.

Both Baron (1974) and Bryant (1974) have speculated on the nature of the strategies available for estimating number on static and transformed arrays. Under the static condition the only strategies available to the child would appear to be those relying on estimation techniques such as counting, subitizing, or the use of perceptual correspondence of some kind. Under the change condition an historical strategy may be available which depends on the child estimating the opening and subsequent configurations, and assessing any changes made in them. If the child possesses a number invariance rule, i.e., he understands that in the absence of any addition or subtraction of elements the number of an adjudged row remains the same, he may be able to infer changes or lack of changes in the relative values of the rows without counting. In the absence of information on which to base such inferences, i.e., in judgments based on static arrays, performance might be more subject to perceptual influences.

Bryant (1974) has presented evidence that children's scores (age 3-6) may improve or deteriorate under the transformation condition on a numerical task depending on the configurations employed. If the transformation is from a configuration which allows for a numerical estimation strategy, i.e., where one to one correspondence is perceptually visible, to one which does not allow for the same estimation strategy, it should be possible to observe the use of an inference because of either below or above chance level performance on the second configuration. Figure 4 illustrates with the appropriate configurations.

Figure 4

If the child transfers information about a judgment of A_1 A_2, where A_1 has more, to C_1 C_2 he should judge C_1 as having more even if he cannot count properly. If the transformation is from a configuration which does not allow the 1 to 1 estimation strategy to another which does not allow the 1 to 1 estimation strategy the child may employ a length strategy and give only chance level performance on a number question. Figure 5 illustrates the appropriate configurations in this case.

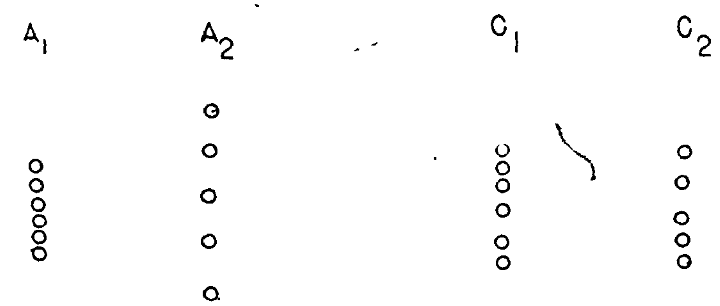

Figure 5

Bryant's findings led him to conclude that the main deficit of a young child who fails to conserve is to be found in the lack of a criterion for choosing between different estimation techniques, not in his inability to make an historically based inference.

In a study of number using both transformed and static arrays, Zimiles showed that children (Kindergarten to Grade 2) made more errors on transformed than on static arrays, although the differences were not large (Zimiles, 1966). Conservers were more apt to give a correct number response to the static arrays. However, the correlation between conservation status and the number of correct number responses on static arrays was not perfect. In contrast to Zimiles, Beilin found that children of this age made more correct responses on transformed arrays than on static arrays (Beilin, 1969); however, in a study of 3 to 5 year olds he found the reverse, i.e., the children made more correct responses on the static arrays (Beilin, 1968). Beilin suggests that the discrepancy between his two age groups may indicate not an absence of the necessary strategies in the young child but some inability to "correctly utilize the information imparted by the transformation" (Beilin, 1968). That, of course, does not explain why their performance was worse on the transformed arrays.

If the child does not have an invariance rule, or is unable to infer, or lacks the necessary estimation techniques like counting, he may then rely on the same strategy on both static and transformed arrays. For example, the child may always use length for his judgments. Indeed, if the transformation itself does not attract the child's attention, then it might be expected that there would be no effect of the transformation, i.e., performance would be equivalent on transformed or static arrays. Within Piaget's equilibration scheme, the transformation per se is not attended to and coordinated with pre and post transformation states until steps 3-4 in the process.[1] i.e., late in the development of the concept of number. As stated previously, changes in strategy prior to steps 3-4 are predicated on changes in dimensional saliency.

To summarize this discussion of numerical strategies: There is evidence that young children have available a number of strategies for estimating the numerosity of a set. The strategy they employ

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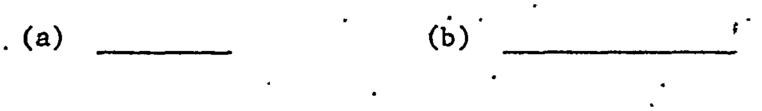
[1] Piaget does propose, however, that the child may attend to the row or object being manipulated and make his judgment on that row. If that row always has "more" in it as it did in Mehler and Bever's (1967) study, the child will appear to be always right.

seems to depend on such factors as set size; the arrangement or configuration of an array, similarity of set items, saliency of dimensions, the presence of a transformation, and their cognitive status. Whatever the mechanism, even young children (younger than 5) are able to give fairly accurate estimates of the numerosity of small sets. When they do not give accurate estimates of numerosity, children appear to rely fairly consistently on another dimension of the array. In the case of homogeneous, linear arrays, they appear to rely on length. The studies also show developmental trends, i.e., children appear to be able to give more accurate estimates of larger sets as they get older, relying less on non-numerical strategies.

The strategies which a child employs when making length judgments, and some of the factors which may influence the choice of strategy will now be considered briefly. As in the case of number, Piaget's theory of the development of length predicts changes in strategy based on changes in cognitive status. In the preoperational period the child judging length will rely on perceptual strategies. These strategies may be greatly influenced by factors affecting dimensional saliency. Of particular interest is evidence about those factors which may influence the child's choice of a numerical or nonnumerical strategy.

Length is, of course, a spatial concept. In his theory of the development of length Piaget indicates the various strategies which the child may employ in judging length. He argues that the child's first spatial concepts are topological, dependent on such factors as proximity, separation, etc. (Piaget, 1968). Theoretically, when evaluating length, the very young child will initially employ such features as "heaping" (density). Therefore, the "more heaped" row would be the more salient row for a length judgment. The child then proceeds to an ordinal consideration of length based on points of departure and arrival, i.e., the child will then judge as longer a line or stick which sticks out after displacement. At this stage the child may also attend to segments or intervals. This is followed by a time when the child coordinates information about end-points and intervals. The final strategy is the use of an arbitrary unit of measurement, a proper numerical strategy.

One of the factors which appears to influence a child's length strategy is the absolute length of the lines used when the child is asked to make a relative judgment. If the absolute length of one set of sticks is greater than the absolute length of another set of sticks, then, after displacement, the young child is more apt to judge the two longer sticks as the same length, rather than the two shorter sticks. For example, the experimenter asks a child to judge whether two sticks are the same length, or one is longer, and then arranges the sticks as in Figure 6(a) and 6(b). However, he employs one set of sticks 5 cm. long and one 7-10 cm. long. The child is more apt to judge the sticks in Figure 6(a) as unequal in length than the sticks in 6(b).



(Piaget et al., 1960).

Figure 6

Piaget also reports that the size or number of internal segments, or partitions, or units influence the child's length judgments. For example, the experimenter cuts two equal length strips of paper, such as those in Figure 7(a) into segments as in 7(b), and asks a young child to compare their lengths. The child may judge the top strip in

Figure 7

7(b) as longer because of the longer internal segment in the top strip, or the bottom strip in 7(b) as longer because of the greater number of segments. The children who judged according to the number of segments appeared to respond to numerosity when asked to judge length, i.e., they may have been employing a counting strategy.

A counting strategy has been found to operate in other spatial judgments, i.e., area judgments. Of particular interest here are the findings of a relationship between number of segments and strategy. In a study of children's (age 5-11) area concepts (Beilin, 1964) found that in making comparative judgments on displays like those in 8(a) and (b) children used two different strategies on a segmented configuration. Where the number of segments was small (four), they used a counting strategy; however, where the number of segments was large (nine), they used a "translocative" strategy, that is, judged in terms of compensation; "here's a piece missing, but here's another piece more to fit in". The younger children were less apt

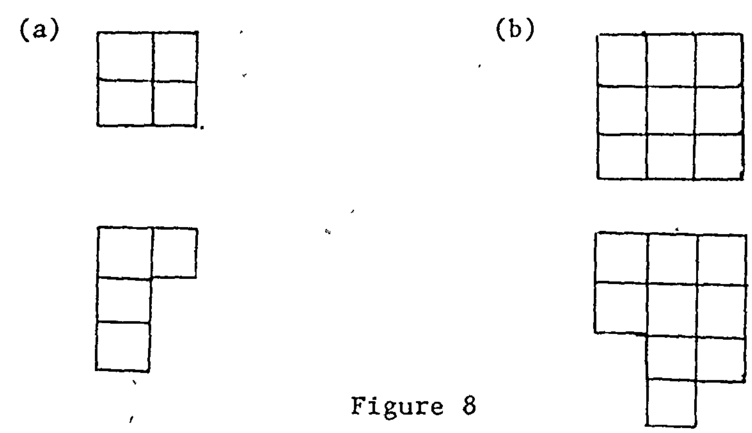


Figure 8

to use a counting strategy on the display with the large number of segments.

To summarize the above discussion on length: There is evidence that a preoperational child's choice of strategy to judge length may depend on such factors as the length of a line or row and whether the line is segmented. Also, Piaget's theory predicts developmental changes from a strategy where the child may attend to the number of segments without assessing end-points, to a true numerical strategy where a true numerical measurement is applied. Beilin provided some evidence for developmental changes in the application of a numerical strategy to a spatial concept. He also provided evidence for the effect of set size. A small number of segments is apt to provoke a counting strategy to compare spaces.

Section Two has reviewed some of the evidence concerning the types of strategies children have available to make number and length judgments. It has emphasized developmental changes, and the factors which may influence the choice and change of choice of strategy. On the basis of the evidence presented here, there is reason to expect a child's use of numerical, nonnumerical, or inferential strategies in judging length and number to vary according to: the saliency of a dimension, for example, length of a row, or number of elements; arrangement or configuration of rows; homogeneity or heterogeneity of elements; mode of presentation, i.e., static or transformed, and the age of the child. Of interest in this thesis is the relationship which exists between a child's strategies for judging number and for judging length. How do changes in factors such as set size alter the relationship between the length and number strategies?

Section Three: The Child's Language

Amongst those factors which influence the child's performance on a quantity task, one of the most important may be the level of his language. Difficulties with either receptive or productive language, relating to either syntax or semantics, could result in the child responding inappropriately. Those inappropriate responses would not necessarily indicate that the child lacked the logical or cognitive skill being evaluated; however, they could indicate something about the child's understanding of the words involved. Also, unless the experimenter assesses the relationship between the questions asked and the features of the stimulus arrays at the time of the questions, he may wrongly evaluate the child's abilities. Rothenberg (1969), for example, found that if the child had only to judge "same" on a conservation of numerical equality task his score was much higher than if he had to judge "more"; the opposite was true for a conservation of inequality task.

Ultimately, however, the interpretation of such data as the above depends on what one believes to be the nature of the relationship between the linguistic and cognitive domains, i.e., between language and thought. It was pointed out in the introduction that Piaget believes the child's language reveals his thought processes but does not directly influence them. Because of his position Piaget has rarely felt it necessary to systematically explore the child's comprehension of terms or syntax.[1] Bruner (1964), on the other hand, views cognitive development as dependent upon the modes of representation available to the child for structuring reality: language frees the child's thought processes from dependence upon his perceptions. Others have argued that a child's responses on verbal questions do not necessarily reflect his cognitive level; therefore, a non-verbal approach is preferred (Miller, 1976; Siegel, 1977b).

In studies in response to the above problems and positions four general approaches have appeared: a nonverbal approach; attempts to "correlate" the child's linguistic abilities with performance on a conservation task; attempts to demonstrate either dependence/or independence of language and thought; and studies of the child's comprehension of terms which procede from some form of formal linguistic analysis. These four approaches will be considered in turn.

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[1] One of the occasions on which he has felt it necessary to assess the role of the language was in his debate with Mehler and Bever (1968). In his reply he indicated that he did assess the childrens language to determine the comprehension of "more". The younger children did not understand the term, but did appear to understand "a lot", which Piaget then used in his task. Mehler and Bever questioned the use of this term, as "a lot" is an absolute rather than a relational term.

Nonverbal Approach

In the nonverbal approach to Piagetian tasks, efforts are made to reduce language requirements to a minimum. The studies taking this approach are important in the following way: They allow assessment of concepts in such a way as to make it possible to demonstrate that some form of the concept is available at an earlier point in time than could be demonstrated on a language task. However, for the purposes of this thesis, such studies provide limited information about the relationship between these nonverbal concepts, the verbal concepts, and the role of language when the task requires its use. Therefore, the other approaches will be examined in more detail.

Comprehension of terms and conservation status

Some general findings have emerged from those studies which attempt to assess a child's comprehension or production of relevant language and relate this to some independent evaluation of his conservation status. These are:

(1) According to Sinclair de Zwart children who conserve show the following characteristics to their language: they use more relational terms; more highly differentiated terms, e.g., "long-short", "fat-thin", as compared to "big-small"; and more coordinated descriptions, i.e., they refer to more than one dimension at a time. (Sinclair-de-Zwart, 1969).

(2) Success on the conservation task is a good predictor of the child's ability to comprehend, and spontaneously use, basic terms such as "same", "less", "more", "longer", (Beilin, 1965; Harasym et al., 1971; Sinclair de Zwart, 1969; Palermo, 1973), as well as to make a

correct verbal justification for his performance (Beilin, 1965; Sinclair de Zwart, 1969). Success on verbal tasks, on the other hand, is not as powerful a predictor of performance (Beilin, 1965; Siegel and Goldstein, 1969).

(3) "Same" is, of course, a basic term for making a conservation judgment. Knowledge of "same" appears to emerge later than the knowledge of "more", while it is not clear whether it precedes or follows knowledge of "less" (Siegel & Goldstein, 1969; Harasym et al., 1971; Beilin, 1965; Palermo, 1973; Griffiths, Shantz and Siegel, 1967). However, it does not emerge in a unitary fashion. Of interest is that Griffiths et al., (1967) found that correct usage on the number, length, and weight dimensions corresponded to the order of emergence of conservation, i.e., length, number, weight.

Some important aspects of the above findings which relate to specific terms will now be discussed.

Griffiths et al., (1967) have argued that three possibilities exist to account for the difficulties with the word "same". (1) The word is ambiguous; it can refer to the identity of an object or to the equality or equivalence of two objects; also, there is the possibility that children interpret it on a physical plane - i.e., to look alike, rather than to refer to a specific attribute. (2) Children may differ in their criterion for allowing "sameness", - i.e., category width may vary, and (3) children may attend to differences rather than similarities. Along these same lines Beilin speculates that some children start with a "set" to search for discriminating details (Beilin, 1965). For these children, the judgment of "same" is a negative one - applied only to stimuli which are not different in any discriminable way. A child may find it easier to judge in this way than to identify a conceptual basis for similarity. Mehler and Bever (1967) have also pointed out the difficulty that young children seem to have in verbalizing the concept of equality. Given that the basic requirement on a conservation task is to maintain that some quantity remains the same despite apparent change, a child's inability to understand the word "same" would seem to make verbal conservation assessment impossible. (Siegel & Goldstein, 1969).

Harasym et al. (1971), using a semantic differential analysis with children, grades one to three, were able to relate the degree of differentiation of the terms "more", "less", "same", and "different" to the child's status as a nonconserver, transitional, or logical conserver. The semantic differential is a technique for measuring the meaning of individual words via subject rating scales. In this study the experimenters employed scales such as "long-short", "low-high", where the child could choose one of five positions between those poles. There were six conservation tasks, such as conservation of weight and number. The child's combined scores over tasks conferred his conservation status. The investigators found that children who conserved treated "same" and "different" very much alike while clearly distinguishing "less" and "more". Nonconservers, on the other hand, made the "more" - "less" distinction less well but made a much greater distinction between "same" and "different". Transitional conservers were somewhere in between, according to their results. The profile for "less" remained constant with "more" emerging from it.

Palermo (1973) also attempted to relate children's (kindergarten - grade two) performance with the terms "more" and "less" on quantity tasks to their understanding of these terms as revealed on the semantic differential. The experimenter required the children to make comparative judgments on two quantity tasks, number and liquid quantity, under various conditions. Almost all of the children responded correctly to the term "more" on the quantity tasks; however, many children did not respond correctly to the term "less". The profiles of this latter group of children on the semantic differential showed a confusion of "more" with "less" which is in keeping with Harasym et al., (1971). However, in Palermo's study "more"'s profile tended to remain constant with "less" emerging from it. The direction was opposite to that found by Harasym et al. Like Griffiths et al., (1967), Palermo found that the children's performance on the quantity tasks with the term "same" was better than with the term "less".

Problematic for the interpretation of discrepancies in studies such as Palermo's and Harasym et al.'s are possibly important age and task differences. For example, children in the Harasym et al.'s study were older on the average, were given quantity tasks, and were assigned conservation status on the basis of their response to questions about "more" and "same". In Palermo's study the children were also asked about "less" on the quantity tasks, and the number of correct responses made to the term "less" was the basis for grouping for comparison on the semantic differential.

While the above findings do point to some important relationships between verbal performance and conservation performance, the relationship is by no means straightforward. Given that knowledge of abilities with relevant verbal terms on a nonconservation task does not allow one to completely predict performance on a conservation task, and that the emergence of a clear understanding of the terms seems to coincide with the emergence of conservation abilities, one may propose one of the following:

(1) Conservation is solely dependent on the acquisition of nonverbal logical operators. In line with Piaget's position, verbal terms such as "longer" and "more" simply reflect present cognitive status.

(2) Conservation is dependent on the activation of verbal processes, i.e., a verbal mediation hypothesis (Beilin, 1965; Kendler and Kendler, 1969).

(3) The connection between conservation and verbal ability is limited to what Holland and Palermo describe as "an alternative to the dependency hypothesis - that is, the loose interrelationship of conservation, and "more-less" comprehension as part of the same cognitive structure" (Holland and Palermo, 1975)."

The above three proposals will now be considered in turn.

Piaget, of course, argues for the primacy of the acquisition of the nonverbal logical operators. However, it is difficult for him to explain why children may understand "same" in relation to length before "same" in relation to number, and "more" before "less" and "same". In fact, his theory would seem to require that the terms "more", "less", and "same" be understood at the same time, given that a proper understanding of any one of them requires the same reorganized structure of logical operators. The Piagetians might, of course, take refuge in the argument they have used to explain that conservation is not acquired in all areas at once, i.e., some physical dimensions are more difficult to deal with than others. However, that is a problematic argument given the notion of structural unity and the absence of independent evidence for dimensional difficulty. A more acceptable argument might have to do with the frequency of exposure to different dimensions. An additional problem, however, is that Piaget's theory makes no predictions concerning the <u>direction</u> of differentiation of terms, i.e., "more" from "less"; "less" from "more".

The verbal mediation hypothesis maintains that being able to verbally label a dimension will facilitate the performance of certain cognitive tasks (Kendler and Kendler, 1969). However, the verbal mediation hypothesis must be able to account for the fact that while certain terms seem to be available to a child, he only succeeds in some tasks requiring the use of those terms, and fails in others requiring the use of the same term. One may argue, of course, that activation of the verbal process is necessary, and that the requirements in some tasks are not such as to activate the verbal processes. Such an argument needs documentation. At the present it would seem as appropriate to argue that word meaning grows slowly and that some of the semantic components of the meaning of the words might be missing, even though others are present. Also, the verbal mediation hypothesis would seem to require some principle which would account for the order of emergence of terms: "more" before "same"; and "same" on one dimension, not on another. For example, one might propose a perceptual hierarchy, i.e., children will actively use verbal mediators for dimensions which are perceptually salient to them; or, a Piagetian type cognitive hierarchy, i.e., children will be able to actively use verbal mediators to the extent that they have appropriate "operators" available to them.

One seems left with the "loose" integrated structure hypothesis, which, as Holland and Palermo state, simply indicates the lack of precise knowledge about cognitive and language development (Holland & Palermo, 1975). The relationship between cognitive and linguistic status is "loose" in this sense: evidence indicates that while a child's success on a number test such as conservation correlates positively with his understanding of basic terms such as "more" and "less" (Harasym et al., 1971; Sinclair de Zwart, 1969), some children conserve who do not understand the terms, and some children do not conserve who do understand the terms.

Attempts to demonstrate dependence/or independence of language and thought

The third group of studies to be discussed in this section are those in which experimenters have attempted to provide more precise knowledge about the relationship, dependent or independent, between language and thought. In these studies efforts have been made to distinguish between a position like Bruner's which assigns language an important role in developing thought process, and Piaget's which does not.

Frank (1966), in an experiment designed to confirm Bruner's position on the important role of language in cognitive tasks like conservation, devised a screened conservation task. After first viewing two beakers with equal amounts of water and judging them to be the same, children aged 4-5 were then shown a wider beaker of the same height. Instead of pouring from one of the standard beakers to the wider beaker in front of the child, the usual procedure in conservation tasks, the experimenter poured behind the screen. She then asked the children if both beakers had the same amount of water or one had more. performance of children who had the screened task was considerably better than the performance of the children who did not. When the experimenter removed the screen the four year old children changed their judgments, whereas the five year olds did not.

Frank argues that screening the display allowed the child to represent the situation verbally before being subjected to a confusing perceptual effect, i.e., change in the height of the water. The study does provide some evidence that verbal mediation may be important in cognitive performance at some stage; however, a number of issues are left unresolved: (1) One can only infer that language was involved - what language is not clear (Dale, 1972). (2) Piaget argues that the study merely demonstrates the concept of identity, not true conservation, i.e., the child, when he asserts that the water is the same, is merely maintaining that the object has remained the same. Conservation requires that the child maintain sameness in the face of perceptual changes (Piaget, 1967). (3) There is no explanation as to why language is completely activated in the 5 year olds but not in the four year olds.

If, as Bruner asserts, language assumes a crucial role in developing cognitive skills, then teaching a child appropriate language might make him more successful on conservation tasks. A number of attempts at language training have been made, but the benefits have been minimal. One of these attempts was made by Sinclair de Zwart (1969). Sinclair de Zwart had observed that children who conserved tended in their free speech to describe objects by using more relational and differentiated terms, and by making reference to several dimensions rather than one. De Zwart gave nonconservers training in the kind of language used by conservers. However, although the nonconservers now had access to the appropriate language they still failed to conserve. Piaget and De Zwart view this study as supportive of Piaget's position on the relationship between language and thought, and on the nature of specific learning. True learning requires an alteration in the underlying structures; calling the child's attention verbally to the two dimensions, in the absence of real changes in the underlying structure, does not enable him to use the information from the two dimensions.

Anglin (1973) and Townsend (1973) have pointed out two potential problems with de Zwart's interpretation of her findings:

Anglin (1973) has argued that it is possible that inappropriate language terms were trained. The term "same" was not trained and since the notions of identity and equivalence are basic for the development of the conservation, training on the term "same" might have produced improvement in conservation. In keeping with this suggestion it might be noted that one of the few successful conservation training studies was Gelman's, in which she asked the children to respond to directions using the words same/and different (Gelman, 1969).

Townsend has argued that it is possible that the nonconservers in Sinclair de Zwart's study do have a proper grasp of the relationships involved but do not show that "grasp" because of the form of the comparative questions on conservation tasks (Townsend, 1973). He bases his argument on findings that children could make a two dimensional comparison when there was a subject noun in the second clause of the sentence, but not a pronoun referent. That is, he found that "Who has more oranges than Johnny has apples", gave better performance than "Who has more oranges than he has apples". In answering the questions, a child's response strategy varied depending on his age, whether he knew the meaning of the adjectives, or how to interpret the pronoun referent. For example, young children who did not know the meaning of the adjective "less", chose the stimulus array with the most elements; older children guessed. If the child understood the adjective but could not use pronoun referents, he chose according to the first clause referent.[1] Evaluating the effect of language training then and inferring something of the nature of the relationship between language and the cognitive processes require that more be considered than a simple understanding of terms.

Investigators such as Holland and Palermo (1975), Beilin (1966), have also attempted training studies. Beilin taught the child a verbal rule; Holland and Palermo taught the single term "less". Beilin found that the verbal rule method was more successful in changing

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[1] Note that Siegel & Goldstein found that young children used a recency strategy, choosing the last alternative (1969).

conservation performance than nonverbal reinforcement, equilibriatory or verbal orientation. He argues that the distinguishing characteristic of the verbal rule method is its algorithmic quality. However, the training was not completely successful and there was no transfer. Holland and Palermo found that training on "less" did not improve conservation performance. At this time then the evidence from the training studies is not conclusive concerning the benefits of language training in improving conservation performance.

Siegel has taken a somewhat different approach toward determining the relationship between the cognitive and linguistic systems (Siegel, 1977b). She has employed a nonverbal concept attainment paradigm in which the child is rewarded for the selection of a stimulus meeting a criterion such as "more" or "less" dots and then tested independently for comprehension: Which is the little one?" In variations on this paradigm Siegel has compared children on: (1) how easily they acquire a quantity concept when they are given a verbal cue and when they are not; (2) the language they use to explain why they pick a particular stimulus after the concept attainment task. Results of the studies indicated that the concepts of big and small number could be learned prior to the child's being able to respond appropriately to the verbal terms "big" and "small"; that cuing did not assist the three year olds, but did assist the four year olds; and that there was little relationship between production of language and success on the concept attainment task.

The above results led Siegel to conclude that language and cognition are at first independent systems, with the cognitive components necessary for a particular term preceding language in development. Unlike Piaget, Siegel believes that the early quantity concepts which can be acquired in a specific learning situation are proper quantity concepts. With age the cognitive and linguistic systems become increasingly integrated; with integration the verbal term may become useful.

An implication of this with reference to word meaning is that for the young child the meaning of terms does not necessarily include those components or characteristics considered part of the adult definition. By studying the ways in which the child responds to these terms one might infer which components of the adult term the child possesses. With increasing age, the referents for the child's term should come to incorporate more of the components of the adult term. However, the young child's failure to respond to the terms in ways in which adults or older children might, cannot be used as evidence for his inability to perform operations which have been incorporated in the adult's language.

The evidence from the above studies suggest the possibility of a developmental trend in which the linguistic and cognitive systems come into closer and closer correspondence (Frank, 1966; Siegel, 1977b). However, the evidence is not substantial enough to allow a confident choice of either Bruner's or Piaget's position on the nature of the dependency existing between language and thought (Anglin, 1976; Beilin, 1966; Frank, 1966; Sinclair de Zwart, 1969; Townsend, 1973). The interpretation of gains which have been made through language training has actually been in terms of changing the child's attention (Beilin, 1966; Dale, 1972). That interpretation is not more precise than Palermo's "loose dependency" hypothesis.

These first three groups of studies make it possible to speculate on: (1) limits on the inferences one can draw concerning cognitive abilities when assessed through verbal tasks; (2) the conditions which may facilitate performance on quantity tasks at different ages. For example, in the young child, changes in dimensions such as set size, configuration and heterogeneity may facilitate performance on a cognitive task; in the older child, verbal cues; training then would need to be geared to the age of the child; (3) the information such studies may provide about changes in word meaning.

If the child does not respond to key words in the same way as an adult does in the same context, this may tell something about what the word means to him. If a child responds in a systematic way, correctly, or incorrectly, it is possible to make some inferences about the meaning of the words to the child. For example: If a child on a large set always picks the longer row of dots when asked about "more", and on a small set the more numerous row, one might infer that for this child the word "more" includes aspects of spatial extension <u>and</u> numerosity. The meaning assigned depends on the set sizes. Verbal studies of quantity concepts, therefore, may allow one to discover the nature of the changes in the meaning of comparative terms. If one can look at these changes over different ages, one might also be able to study the development of word meaning.

In order to get a comprehensive picture of word meaning, however, words must be studied in relation to other words. Of particular interest are the child's errors. A child's failure to extend a word to the full range of permissible situations or contexts is one type of error, underextension; his use of a word in a non-permissible context is another, overextension. When a child does not respond appropriately to the request to pick the row with "more", and says "this row has more because it's longer", he is not only underextending the term "more", but overextending the word "longer". He reflects a reasonable hypothesis, i.e., that longer rows often have more, but he is indicating that the boundaries on his words are not the same as an adult's.

It might be recalled that there is evidence that some quantity terms precede other quantity terms in being used appropriately (Griffiths et al., 1967); that overextension and underextension can take place with terms referring to the same dimension, for example, "more" and "less" (Palermo, 1973). Is it possible there are verbal hierarchies expressed in children's verbal errors, and do these verbal hierarchies reflect linguistic hierarchies, hierarchies of perceptual saliency, changing cognitive structure, or all three? A fourth group of "quantity relevant" language studies can be identified, pertinent to these questions. In these studies investigators have observed young children's use of comparative terms within the framework of a formal linguistic analysis. They have attempted to discover regularities in the linguistic system itself, and then to develop a theory to explain how comparative terms develop and how they may relate to the young child's perceptual and cognitive systems. This fourth approach to understanding young children's quantitative terms will now be examined.

The formal linguistic approach to children's comprehension of comparative terms

In studies of quantity concepts children are asked to make judgments in response to questions or commands using the words "same", (sometimes "different") and either the positive forms of the comparatives, i.e., "longer", "more", "wider", or, less frequently, the negative forms "less", "shorter", "narrower", etc. Proceeding from a linguistic analysis of the comparatives, a number of investigators have noted the peculiar properties of the positive and negative forms (H. Clark, 1970; E. Clark, 1973; Donaldson & Wales, 1970). The positive form of the comparative can be designated by fewer semantic features than the negative form. Only the positive term appears in the nominal form, i.e., where it designates the dimension being referred to, for example, "the board is six feet long", a form which requires a less complicated syntactic structure than the contrastive forms "shorter" and "longer". In addition, the contrastive use of the negative form, for example, "shorter", requires a more complex syntax than the contrastive use of the positive form (H. Clark, 1970).

Of interest to psycholinguists and cognitive psychologists are the results of a number of studies which indicate that the positive form of the adjective is understood and produced earlier than the negative form (Donaldson & Balfour, 1968; Palermo, 1973; Klatzky, R.L., Clark, E.V., and Macken, M., 1973). More specifically, there is some evidence that the negative form of the adjective is understood as if it were the positive form, for example, "less" is responded to as if it referred to "more" by children about $4\frac{1}{2}$. (Donaldson and Balfour, 1968; Palermo, 1973; Klatzky et al., 1973; Weiner, 1974). The matter is made more complex, however, by some evidence that "less" is not always responded to as if it were "more". Weiner (1974) found that children at three picked the smaller array in response to "less"; Townsend (1973) that they picked randomly in a three choice situation.

The discussion of the above phenomena will centre on the terms "more" and "less", as the literature available on other comparative terms is not so extensive. However, there is some evidence that subjects perform less well using the term "shorter" than "longer" (Donaldson and Wales, 1970; Townsend, 1973) although there is no evidence that the negative form in this case is responded to as if it were the positive form, i.e., that "shorter" is responded to as if it meant "longer".

It is not clear why the positive form should emerge earlier than the negative form - even though linguistically "simpler" (Clark, H., 1970), nor is it clear why the negative form should go through a period of being responded to as the positive form (Donaldson & Balfour, 1968; Donaldson & Wales, 1970). Explanations range from the influence of frequency characteristics of adult language, i.e., "less" occurs less often in adult speech, to response bias, i.e., for some reason the object having the most extent may be more salient or viewed as the good one (H. Clark, 1970; Huttenlocher, 1974; Palermo, 1973; Weiner, 1974), to limitations on cognitive and perceptual processes, i.e., the development of the child's understanding and correct usage of the terms reflects processes which allow information to be extracted only in a certain order (E. Clark, 1973b).

The purely linguistic accounts (H. Clark, 1970; McNeill, 1970) of the order of development of the polar terms and of the failure in differentiation are, of course, not adequate psychological explanations, for it is necessary to establish by what processes the words come to have their meaning. While it may be true linguistically that the meaning of a negative term can be derived from the meaning of a positive term, it is necessary to establish that it is psychologically "true". That is, that the child acquiring the terms necessarily derives the negative term from the positive term. Also, while it is possible to speculate on the original and changing meanings of words, it is necessary to establish by what processes these meanings are acquired and why they change (Weiner, 1974).

The frequency effect which argues for the influence of the characteristics of adult language, has not been given a great deal of support. Although it is possible that the child hears positive forms more frequently than negative forms, accounting perhaps for the more frequent usage of such terms as "more" in child language (Bellugi and Brown, 1964), the frequency effect would not seem to account for the assimilation of "less" to "more". A bias in the language towards the more frequent use of the positive numerical term, does not account for a bias in the strategy which gives the term its referent or meaning, for example "more" = "longer" or "less" = "more". Donaldson and Balfour point to the children's unhesitating choice of the more numerous set in response to "less", i.e., the children do not act as if they do not know the word, but as if they do (1968). As well, Klatzky et al., in a study requiring that children substitute nonsense syllables for positive and negative poles in a concept learning task, found that the asymmetry was observed under these conditions, arguing against a direct frequency effect (1973).

The response bias argument has been put forth mainly by Huttenlocher (1974). She argues that for some reason, the young child views the more extended choice as the more desirable, although he is quite able to discriminate and understand the difference between, for example, large and small. Holland and Palermo believe that some support for a response bias argument comes from a study in which they taught young children the meaning of "less" (Holland and Palermo, 1975). They point out that the children learned the term much too rapidly for the "less-more" confusion to be anything more than a superficial difficulty. Klatzky et al., have argued, however, that if all that is involved is a bias to the "big one", then on their nonsense-syllable task they might have expected that the "proportion of correct responses" would be greater on the positive nonsense syllables. They were not. Klatzky and Clark also point out that response bias offers a description, not an explanation.

Klatzky et al., base their explanation of the order of emergence of the terms on the notion of inherent limitations on cognitive and perceptual processes, which only allow information to be extracted in a certain order. There is evidence, for example, that the natural implicit standard in a comparative judgment is the "small one". The requirement, therefore, to judge the "small one" compells that another standard be established, and a reversal in judgment - a more complex cognitive task (Donaldson and Wales, 1970; Farnham-Diggory, 1972; Klatzky et al., 1973).

Although neither the frequency effect nor the response bias accounts are adequate to explain the order of emergence or confusion of comparative terms, a strictly independent cognitive argument has its problems also. Donaldson and Wales maintain that it is pointless to demonstrate comparable performances on linguistic and nonlinguistic tasks and then argue from that, that the underlying cognitive structure is responsible for both. Their point is that it would be necessary to demonstrate that the linguistic and cognitive systems are unrelated, and that "the apparent convergence of the language performance and other cognitive performance misleadingly reflects two quite unrelated systems of competence." The question is whether there is anything to be gained in trying to integrate information about features of the world, which acts as referents for a child's language, information about the child's changing cognitive structure, and information about the formal properties of the linguistic system, ..."what usefulness there may be in trying to map certain cognitive relations in ways that are consistent with linguistic relations." (Donaldson and Wales, 1970).

The major attempt to perform such a task of integration in this area has been made by Eve Clark (1973b). Clark has developed a component theory of verbal concept acquisition; she argues that children's language develops through the acquisition of semantic units or components rather than whole words. These components "which are used in interpreting any sort of input to the human organism, whether linguistic or not, are the semantic primitives, i.e., categories or principles according to which real, fictitious, perceived, and imagined situations are structured or classified." According to Clark the more general component is acquired first. For example, the superordinate concept of "big" is acquired first, with additional components resulting in the refinement of the superordinate, such as a component specifying the opposite of "big". Then comes a component specifying a dimension, such as length as compared to width. After the acquisition of the component specifying the dimension, positive extension is specified according to a basic cognitive encoding mechanism which uses the smaller object as the standard (see Klatzky et al., above). Only then is the negative component acquired which allows for the emergence of the negative term. [1]Clark argues that the phenomena of overextension in children's speech, i.e., children using terms in wider contexts than considered acceptable by adults, is best explained by such a general to specific component theory (E. Clark, 1973(a), 1973(b); Klatzky et al., 1974). Siegel (1977a) has pointed out that an assumption of Clark's theory (and Donaldson & Wales, 1970; Holland and Palermo, 1975) is that word meaning reflects non-linguistic strategies and that the growth of word meaning shows the gradual acquisition of such strategies.

Clark's theory has been criticized in a number of ways. Anglin (1977) has argued that Clark's theory does not deal with the phenomena of underextension, i.e., it does not account for those findings in

[1]Support for Clark's position concerning "big" comes from studies by Donaldson & Balfour (1968) and Siegel (1977a).

early childhood language which indicate that some terms are not applied in all the appropriate contexts. Huttenlocher (1974) has argued that overextension does not necessarily reflect any "primitive semantic categories"; the child may well perceive and understand appropriate referents and applications but be biased to respond in terms of "the big one". As noted earlier, Klatzky et al., have replied that response bias is simply a description of empirical findings not an explanation of them (1973).

An important question raised by the above is, whether, in fact, it is true that the child acquires concepts and words piece by piece and processes the world feature by feature (Rosch, 1976). The perceptual world is not so tidily constructed, and the cognitive process available for analyzing the world could work according to prototype rather than feature or undergo important changes depending on the nature of the task demands (Brooks, 1976). Nonetheless, Clark's theory stands as perhaps the most comprehensive effort to account for the order of emergence of dimensional, positive and negative terms, while presupposing some kind of integrated, cognitive-linguistic structure. As will be seen in the following section, Clark's component theory serves as the model for Baron's semantic component theory which (amongst other things) attempts to explain children's acquisition of quantity concepts and comparative terms, and their cognitive development, according to the gradual accumulation of habitual plans and strategies, i.e., the semantic components.

Section Four - Concepts and Cognitive Development

By studying how children at different ages understand the number concept and quantity terms, and the factors which relate to their success or failure, one can hope to gain some insight into more general processes of conceptual or cognitive development. Different approaches to, or theories of, concept acquisition make different assumptions concerning the important processes in cognitive development. Findings from the studies presented in this thesis should permit speculation on the validity of a few of these assumptions. The following is a brief review of some of the general approaches to concept acquisition.

To briefly restate Piaget's position: Piaget argues that activity is at the foundation of all cognitive development. The child internalizes actions and these become logical operations. The acquisition of any concept requires the additive operation; the acquisition of the quantity concepts requires the coordination of a related group of logical operations, a structure. By asking a child to make, and justify, comparative judgments of quantity under a variety of task conditions, Piaget believes he can determine the status of a child's concept and his general stage of cognitive development.

Piaget says a child has acquired a quantity concept when: (1) an invariance rule has appeared, i.e., a rule which indicates that only that variability which is criterial to the concept should be assessed, all other variability ignored; (2) the child is able to produce verbal statements justifying the reasons for his judgment;

(3) the concept shows evidence of generality, i.e., is not bound in application to a specific situation. He states that when these criteria for the number concept are met, a basic structure of logical thought is present. The quality of thinking is now different from that prior to the formation of the structure when thought was prelogical or preoperational.

Since, in Piaget's view language reflects thought, but does not influence it, he views word meaning as restricted by the level of cognitive processes. Meanings change with changes in cognitive structure. Both the child's comprehension and production of speech in relation to a concept task will tell something about his cognitive level, and in studying the same child at different ages, or different children at different ages, one may be able to make inferences about the nature and direction of cognitive development. Since Piaget proposes that the preoperational period is the period of the acquisition of logical structures and of language, developmental studies of quantity concepts in this period should be particularly productive for revealing something about the general processes of cognitive and linguistic growth.

Studies of the type suggested in the preceding paragraph could be helpful in clarifying some aspects of Piaget's theory. Everyone does not agree with his inferences concerning cognitive development, drawn from the studies of quantity concepts. Some researchers for example, have pointed to the ability of children to demonstrate certain logical operations earlier in development than Piaget has allowed for (Gelman, 1972; Siegel, 1977). The order of acquisition of operations has been disputed, as well as Piaget's basic conception of the nature of number (Brainerd, 1973; MacNamarra, 1975). Many aspects of the theory, therefore, need clarification. As well, not all aspects of Piaget's theory have been extensively studied, for example, the role of dimensional saliency (Ginsburg and Opper, 1969), a matter of some interest in this thesis.

As stated in the opening preamble to the thesis, one of Piaget's major contributions to psychology has been to encourage some thinking about the adequacy of the stimulus-response association paradigm, which has been the basis of the traditional North American approach to concept acquisition. According to the stimulus-response psychologist's view, a concept is acquired through the formation of associative bonds between particular stimuli and particular responses.

In the S-R paradigm studying concepts has frequently involved tracing the acquisition of discriminatory response to a set of stimuli constructed of combinations of discrete attributes. Subsets of these are designated as concepts, for example, yellow, blue; triangles, circle; concept = red triangles. Such a concept is designated by Neisser (1966) as "well defined". Concept acquisition, or identification, involves the abstraction of the criterial attributes. One says that an individual has acquired a concept when his responses meet the criterion for successful acquisition, for example, so many successful trials. For physical concepts, at least, the basic processes involved in successful abstraction are assumed to be generalization and discrimination (Hull, 1920). However, the notion of covert, mediating responses was found to be necessary to account for the derivation of concepts where similarities and relations were not based on physical dimensions (Pollio, 1974; Rosch, 1976).

According to the mediating response hypothesis, a single instance of a concept supposedly arouses a number of associations and responses; for example, carrot = orange, vegetable, dislike, long; spinach = green, mushy, vegetable, like. The common response in these two examples is "vegetable". An individual subject eventually classifies instances as belonging to the same concept via the same mediating response, which in this case would be "vegetable" - a common association. An important part of concept acquisition, then, is identifying the mediator, which is normally assumed to be a verbal label. The mediator is believed to reduce the number of individual associations which must be learned and thereby facilitates performance (Osgoode, 1953; Pollio, 1974).

Within the S-R learning tradition there is no reason to suppose that the basic underlying processes of concept acquisition should differ across ages, although rates of learning might change as more associations are built up and more mediators are available. However, the mediating response notion has been responsible for the development of a paradigm which has served as a vehicle to explore possible discontinuities in cognitive development: the reversal shift paradigm (Kendler and Kendler, 1969).

To illustrate the paradigm: A stimulus set is constructed out of a combination of squares: large/small; black/white; a child is trained to pick the large one. On a second discrimination the child is either required to learn to pick the small one, i.e., same dimension (a reversal shift), or the black one, i.e., different dimension (a nonreversal shift). In rats and young children (below five) Kendler and Kendler found that the nonreversal shift was relatively easier than the reversal shift; in older children the opposite was true.

They give the following explanation to account for the way different age children perform on the second shift: Animals and young children do not employ mediators; they learn by the accumulation of single stimulus-response associative bonds. Older children and adults learn via mediating responses. On a nonreversal shift, some of the noncriterial attributes associated with the first discriminatory response will remain the same. In this example where colour was irrelevant on the first discrimination, a positive response to the large black stimulus and a negative response to the small white stimulus will still be correct on the second discrimination. Therefore, the subject has only to establish two new associative bonds. On the reversal shift the subject has to establish four new associative bonds. For rats and young children, who learn through the accumulation of single associative bonds, a nonreversal shift is easier than a reversal shift, as only two new bonds are required. Older subjects (about six and up), who actively employ verbal mediators, find it easier to simply reverse on the size dimension; a nonreversal shift is more difficult for them as it requires a new mediator, "colour".

Examining the performance of children at different ages on concept acquisition tasks of the S-R discrimination type could theoretically provide information about: any changes in development in the ease with which various attributes or dimensions can be abstracted;

changes in the availability of verbal mediators; and possibly, therefore some information about the development of word meaning. However, while the paradigm may work well for unrelated dimensions such as size and colour, it may not show the same type of mediation effects for dimensions such as number and length, where density always varies in relation to these two dimensions.

An outcome of formulating concept acquisition in S-R terms, employing well-defined concepts, was the production of a rather rigid paradigm. This paradigm seemed adequate to account for the learning of certain kinds of information but not adequate to account for certain important characteristics of human beings and of the world. Human beings frequently seem to be actively engaged in solving problems or identifying concepts; many categories in the world cannot be specified according to combinations of discrete attributes. Approaches which deal with these important aspects of the organism and of the world are now considered in turn.

With the arrival of the computer (Pollio, 1974) psychologists interested in concept acquisiton and utilization began to discuss cognitive operations in information processing terms such as strategies, plans, decision trees, coding and decoding, capacity (Bruner, 1956; G. Miller, Galanter and Pribram, 1960). They viewed human beings as active processors of information, possessing some means of encoding, for example, language; an ability to employ decision rules; built in limitations on capacity. Under the influence of the stimulus response association approach, psychologists had tended to view human beings as passive receivers. One of the earliest and most influential psychologists to show the change in view concerning the human being was Jerome Bruner (1956).

Bruner views the processes of concept acquisition, identification, and utilization, as active processes in which subjects use information to make a series of decisions. Even though the experimenter structures the situation, subjects define the nature of the tasks for themselves. They bring with them their own notions about what may be important attributes for defining a concept, how to achieve success, the acceptable criteria for attainment of the concept, and the consequences of their choices. Bruner sees the unit of behavior employed in the S-R approach, the single response, as too small to reveal the important features of decision making. These have to be revealed in the sequences of responses made by subjects, i.e., in their strategies. "Regularities in decision making we shall call strategies." Strategies need not be conscious to be called strategies; subjects might frequently be able to distinguish exemplars of concepts without being able to name criterial attributes.

During the 60's the methods, terms, and theories of the information processors became more sophisticated than Bruner's, but the basic idea of an active, hypothesis-testing organism was retained. The course of cognitive growth was discussed in terms of an increase in capacity (Pascul-Leone, 1969), the development of more efficient and comprehensive strategies or plans (G. Miller et al., 1960) and the acquisition of different means of encoding. With reference to the last, Bruner, not unlike Piaget, sees children acquiring different modes of representation; a motoric mode, an ikonic mode, and finally a linguistic mode, each one more flexible than the other. Unlike Piaget, however, Bruner views language as the transformer of thought (Bruner, 1964).

From the information processing perspective, children's performances at different ages on tasks that involve the utilization or acquisition of concepts might well show changes in: "attribute predilection" (preference); in patterns of responding (strategies); in the role of language as a code; and in the ability to deal with increasingly complex rules. In general, the information processing view does not presuppose necessary discontinuities in the course of development, although in Bruner's theory at least, the change in mode of encoding should bring about some qualitative change in cognitive processes. Prior to the full appearance of the linguistic period, word meaning would be expected to reflect predominantly a sensitivity to perceptual attributes.

The inadequacy of the traditional "well-defined" concept employed in the S-R paradigm in replicating real world categories has been noted by Bruner and others (Bruner, 1956; Rosch, 1976). Bruner was amongst those who pointed out that in the real world categories frequently were not Aristotelian in character but probabilistic-relational - Gallilean. In any single exemplar of a natural category, for example, dogs, there are any number of attributes which could be relevant. These attributes do not vary discretely, but continuously, and probabilistically. However, in spite of this kind of acknowledgment of differences between the real world concepts or categories and those traditionally employed in the laboratory, most studies of concepts tend to be with artificially constructed concepts. The experimenters construct concepts from dimensions which can be defined discretely and in which the relations between the dimensions have equal probability of occurrence.

Very recently Eleanor Rosch has presented a comprehensive argument for a different approach to the acquisition of natural concepts (Rosch, 1976). She maintains that, while the world is made up of an infinite variety of stimuli, the child does not gradually isolate dimensions and attributes and then combine these to form concepts. In fact, the child confronts a world of "intrinsically separable things."

Rosch gives three principles which she believes account for the organisms confronting a world of "intrinsically separable things." First, in the real world, certain attributes are frequently found together, i.e., correlated, and other attributes are not. For example, "soft" and "furry" are correlated attributes which will tend to produce a separate category response. Second, there is an optimal level at which category formation may take place; this level has maximum cue validity. The cues with maximum cue validity are those which operate to maintain maximum predictive value, while being the most economic in terms of not making unnecessarily fine distinctions. For example, "soft", "furry", "four legs", and "moves" may be cues with high validity. "Curly" has probably little validity for separating categories although it may be highly correlated with "fur". Third, the means whereby the organism processes real world information is such as to heighten the discrete characteristics of categories - for organisms process real world information by means of prototypes, not by hierarchies of discrete features. A prototype is a concrete instance which is closest to the average of the correlated attributes. The use of the prototype will encourage category distinctions, as the prototype enhances the major, correlated, valid cues. The prototype may be the most salient member of a category by virtue of some central tuning. Where no underlying physiological base for saliency could be expected, prototypes are believed to represent the central tendency of the category, the "category" of the category, with new instances being assigned to categories in an analog fashion.

According to the above model, an organism would derive a dimension after it had experienced a sufficient number of instances. Superordinate categories would be built up from subordinate categories, i.e., in the reverse direction to that proposed by Eva Clark. Errors in categorization would be expected to occur more frequently as one moved toward the periphery of the categories, i.e., where the correlation of attributes found in the instance was at some distance to the average found in the prototype, for example, sparrow = prototypes for bird, penguin = ? bird? To the extent that the development of meaning reflects the above, it would be expected to proceed from less to greater generality (Anglin, 1977).

It is not clear how the prototype-analog theory would account for the acquisition of the number concept, number names, or relational terms. Since it seems that a child may achieve all of these quantity concepts in perceiving and interacting with the real world, his early understanding of number, and the relational terms, might be expected to reflect the real world correlations of dimensions. There are real groups of things in the real world. Where there are "more" things in the real world, the group is also usually larger, covers more area, etc. If one of the spatial dimensions of the group is altered and number is not altered, density is. It is also possible that the earliest conception of numbers are built on some kind of real world prototype of a set, say <u>two</u> (hands), <u>five</u> (fingers) (Werthheimer, 1912). Acquisition of other <u>twos</u> or <u>fives</u> could theoretically take place according to an analog mechanism. The intuitive early numbers, representing readily perceivable groups, provide the possibility for such prototypes. However, there are problems with the model in dealing with numbers. Numbers seem to be either/or - what would the "category" of the category of five be? What is the "average" of five? It is also difficult to conceive of the prototype for a relational concept such as "more". Such a term has no "real world" referent. Numbers, unlike natural concepts are not intrinsically separable things of the world, rather, they represent ordered relationships.

It is possible that there are developmental changes in the mechansims for the acquisition of all concepts - a movement from an analog process to an analytic process. In studying number concepts in well-defined concept type paradigms, i.e., where all logically possible combinations of length/number/ and density are employed, the analytic mechanism may be revealed, but the analog mechanism obscured. Possibly the paradigm should be altered to attempt to demonstrate the analog prototype mechanism. However, if ultimately the child must move towards analytic, rule directed operations in order to construct number, then studying the early responses in well-defined paradigms may in fact reveal the beginning of the analytic process.

Possibly a component theory of concept acquisition like that of Eve Clark's could at this time provide an adequate account of the presently available data on the acquisition of numerical and other quantity concepts. Jon Baron (1973) has proposed a theory of both verbal and nonverbal concept acquisition which is a modification of Eve Clark's component theory of verbal concept acquisition (E. Clark, 1973b), and of information processors, Miller, Galanter and Pribram's notion of plans (1960). Like Clark, Baron argues that concepts may be acquired a component at a time, but proposes to define concepts as "habitual plans", and components as subplans, i.e., as ordered actions or strategies in the manner of G. Miller et al. Baron states that one of the major requirements for any theory of concept of acquisition is to account for systematic errors. Systematic errors could result from missing plans, confusion of plans, or lack of rules for ordering. The component missing on a task could be verbal or nonverbal or both.

For example, in relation to quantity concepts, Baron has argued that the child may begin with the concept "big", a set of synonyms for "big" such as "more", "longer", etc., and some strategies suitable for assessing bigness on different dimensions. The child might make systematic errors for the following reasons. First, he might not have one of the important strategies for one of the dimensions, for example, number, in which case he might substitute an available strategy appropriate to another dimension, for example, length. Second, he might have both strategies available to him but not distinguish one from the other in terms of appropriate application. For example, he may have strategies available for assessing both number and length but consider them as equivalent because he lacks a component specifying the dimension to which he should attend in applying the strategy. Third, he may lack the part of the plan which specifies order of application for the plan, or when to stop. For example, asked for number, he uses the length strategy, then counts, then adjusts count to reflect length judgment. If any of the above conditions held, the child would treat one concept as another i.e., would assimilate one concept to another. Missing components could be verbal or nonverbal or both. For example, the child could have appropriate strategies in a nonverbal context, but not have a subplan for attaching the strategy to the appropriate word. A concept could not be considered as mature if it could not be applied to a variety of appropriate situations, nor could it be considered mature if it was applied to a variety of inappropriate situations.

According to the theory, components could be acquired in an associative fashion, but being conceived as plans, once acquired they would have considerable generality. Order of acquisition might vary for different children, although there could be hierarchies of components, because some tasks might be more difficult than others. The difference from Piaget's theory lies in the less extended notion of the component. In Piaget's theory, acquisition of a concept requires a total reorganization of the cognitive structures. According to component theory the acquisition of a component of a concept could be reflected in more than one concept, resulting in spurts of growth, but there would be no need to argue for a wholly reorganized cognitive structure.

To summarize this section from the perspective of this thesis: By observing how children utilize concepts in a variety of situations, and how they interpret their instructions, one may infer something about the processes whereby the child has acquired the concept and its related terminology. By comparing performance at different ages one may infer something about the course of cognitive development and the acquisition of word meaning. All of the approaches outlined above have been helpful in providing methodology and conceptual tools to deal with the issues of interest in this thesis. Study of these issues may, in turn, allow for an evaluation of certain aspects of these approaches.

The conclusion of section one outlined the theoretical issues which form the foundation of this thesis. With the support of the literature reviewed in sections two, three, and four, these issues will be restated as general hypotheses, or expected outcomes, in the concluding section of this introduction, section 5.

Section Five: The Thesis

(1) A child's early concept of number is probably multidimensional (Dewey, 1897; Werthheimer, 1912; Zimiles, 1966). He should, therefore, show some confusion, systematic or otherwise, in evaluating dimensions which are part of his real life experience of number. Piaget maintains

that the child in the preoperational period confuses strategies appropriate to discrete and spatial concepts (Piaget et al., 1960). There is evidence that children in the appropriate age group (3-7) do confuse number with length, and that there are developmental trends to this confusion (Gelman, 1972; Piaget, 1952; Pufall and Shaw, 1973). There is little evidence on confusions of length with number (Piaget et al., 1960), and no evidence concerning the systematic or unsystematic nature of these confusions.

In the experiments in this thesis the experimenter asked children to judge both length and number on the same array. The hypothesis was that many young children would confuse length with number or number with length in some systematic way and that the choice of strategy would show developmental trends.

(2) Piaget's theory of the development of number parallels his theory of the development of length. He proposes some communality in the logical and sublogical operations underlying the two concepts (Piaget et al., 1960). If a preoperational child does judge number incorrectly according to length, and length correctly, this may indicate he has only a single strategy available to him. On the other hand this may only indicate that he has a bias to respond with this particular strategy. Possibly the other strategy is available.

The hypothesis was: If only a single strategy is available a child will employ that strategy regardless of changes in task variables which might enable, or call attention to the possibility of another strategy. These task variables might be changes in set size on training. If the child does have strategies appropriate to both dimensions, he may be induced to use them.

(3) According to Piaget's equilibrium model, a child's current cognitive structure determines the probability of his employing a particular strategy to judge number (or length). In the preoperational period the child's thought is characterized by centering on a dimension, excluding information he might obtain if he attended to two dimensions at once, or to states and transformations. From the viewpoint of this thesis, the centering supposedly characteristic of a preoperational child's thought may influence his performance not only within, but across concepts.

Important factors which may determine which strategy the child employs are dimensional saliency and the transformation of an array (Ginsburg and Opper, 1969; Piaget, 1952). P. Miller (1973) has shown that one dimension may be more salient than another in judgments of certain quantities. However, there is no information about the role of dimensional saliency in numerical judgments, nor about the possible basis of that saliency. Saliency could be due to the ease of perceptual discrimination, or to the ease with which one can perform operations on a particular dimension. With respect to the effect of the transformation, Beilin (1968) and Zimiles (1966) have provided evidence that the effects of a transformation may vary with age. Bryant (1974) has argued that the effects of a transformation may vary according to the nature of the configurations.

Attempts were made in the thesis to obtain information about the basis of dimensional saliency through: a discrimination learning paradigm comparing rate of acquisition of length and number concepts;

variations in set size.

Attempts were also made to assess the effects on a child's judgments of length and number of: varying modes of presentation, i.e., static vs. transformal arrays; different configurations; and age.

It was expected that:

(1) Children would find length an easier dimension than number when very young (age 3), since end points are easily discriminable, but that this would change with age in relation to the emergence of a numerical strategy. If the child has a bias to number he might discount end points.

(2) Set size, transformation, configuration, and age, would all affect performance in the preoperational child. However, it was not clear what the direction of influence might be. For example, the same configuration might or might not be equally difficult for length or number judgments.

(4) The small numbers in the number series are assigned a unique role in some formal theories of number, and an ambiguous role in Piaget's theory. There is evidence in the literature that numerical tasks employing small numbers may reveal early operational abilities (Gelman, 1972; Weiner, 1974). It is possible that on small number sets, the child may be able to perform numerical operations which he could not perform on the large sets. Small numbers may make the attributes of an array appropriate for number more salient. This could make it difficult for a child with a numerical strategy to attend to the other dimension and perform the operations appropriate to that dimension. If he overextends the numerical strategy to the length

dimension, one might argue that his number concept is immature; for, even if there is evidence that certain numerical operations are available, the child apparently lacks a criterion for setting appropriate limits on the application of the numerical strategy. Such evidence might allow one to argue that small numbers do have a unique status which enables certain operations, but the numbers themselves are not fully constructed.

It was hypothesized that on small numbers some young children would be able to correctly judge the relative numerical value of two sets and also to conserve number. Some of these children would also overextend the numerical strategy to the length dimension. On large sets, these same children, or children of comparable age would give no indication of the availability of a numerical strategy.

(5) The child's cognitive status may well influence his comprehension of language (Sinclair de Zwart, 1969; Donaldson and Wales, 1970) and his language, his problem solving abilities (Frank, 1964; Kendler & Kendler, 1969 Siegel, 1977b). There is empirical data available indicating that the referents for the young child's words are not the same as the referents for the adult's words, and that there are some regularities and asymetries in development (Clark, 1972; Donaldson & Wales, 1970; Holland & Palermo, 1975). These investigators have suggested that the growth of word meaning reflects developmental changes in cognition. Siegel (1977b) has pointed out that an assumption of these theories is that word meaning reflects nonlinguistic strategies and that the growth of word meaning shows the gradual acquisition of such strategies.

In the preoperational period, when the child's linguistic and cognitive systems are supposedly converging, studies of comparative terms for concepts which have been shown to be interdependent should provide information about the growth of word meaning.

In the various experiments in this thesis the experimenter explicitly asked children to respond to questions containing terms such as "longer", "more", and on occasion to "shorter" and "less". It was expected that examination of the child's strategies in response to these terms and to the particular experimental conditions would shed some light on the changes in referents for comparative terms.

(6) Different approaches to concept acquisition make different assumptions concerning the evaluation of the maturity of a concept, the role of language, and the best methods of characterizing the mechanisms of acquisition (Baron, 1973; Bruner, 1956; Kendler & Kendler, 1969; Piaget, 1952; Rosch, 1976). The investigator expected that a simultaneous comparison of performance on related quantity concepts under different task conditions would allow some reasonable speculation concerning: the appropriate means of evaluating a concept; the role of language in concept tasks; the most suitable model for characterizing concept acquisition and conceptual development.

In the research which will now be presented in chapter two, the same basic paradigm was used throughout: young children between 3-7, children in Piaget's preoperational period, were asked to make number and length judgments on a variety of linear numerical arrays. It was hoped that in this research, which attempts to shed some light on the issues outlined above, the three general intentions of this thesis would be met:

(1) to examine some of those factors which influence a child's quantitative judgments on linear, numerical arrays.

(2) to further delineate the nature of the young child's understanding of comparative terms.

(3) to consider some general questions on the nature of concepts and cognitive development.

Chapter Two

The first two experiments presented in this chapter have appeared in publication under joint authorship. They are not presented, therefore as the research proper of the thesis, but as the foundation of the four following experiments. They will be given in a summary form, with the exception of the method section of the first paper. The method devised for this experiment is the basic method employed in all the following research.

Experiment I

The Role of Number and Length Cues in Children's Quantitative Judgments (Lawson, Baron, and Siegel, 1974).

The authors noted the fact that young children's errors in response to number questions were frequently attributed to a tendency to attend to length, (Gelman, 1969, 1972; Zimiles, 1966). They proposed to examine: the ability of young children to respond to length and number as independent dimensions; the degree of consistency in their choice of cues for making length and number judgments. In order to determine the consistency and independence of a child's response to length and number questions, the experimenter must employ configurations such as those shown in Figure 1-1.

Incongruent configurations 3, 4, and 5, where length and number cues are in conflict, allow the experimenter to determine the nature

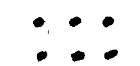
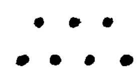
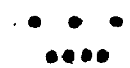
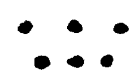
CONFIGURATION	EXAMPLE	LENGTH STRATEGY RESPONSE	NUMBER STRATEGY RESPONSE	UNEXPLAINED ERROR
1 CONGRUENT		"SAME"	"SAME"	TOP OR BOTTOM
2 CONGRUENT		BOTTOM	BOTTOM	TOP OR "SAME"
3 INCONGRUENT		TOP	BOTTOM	"SAME"
4 INCONGRUENT		"SAME"	TOP	BOTTOM
5 INCONGRUENT		TOP	"SAME"	BOTTOM

Figure 1-1 Examples of configurations used and the results of applying different strategies in response to questions about relative length of or number of dots in the rows.

of the child's response strategy. For example, if in response to a question about length on Configuration 3, a child said the bottom row was longer, the experimenter might conclude he was using a number cue. If the child consistently used the number cue in response to length and number questions the experimenter might conclude he was using a number strategy. This pattern of responses would allow the statement that the child does not treat the length and number dimensions independently, but he is consistent in his choice of cues. Other children might respond to length cues on number, both length and number cues correctly, or some unknown or unexplained cues. In this last case one might say of a child who replied with the answer to "same" either a length or number question on configuration 3 that his cue and his error were unexplained. The congruent configurations 1 and 2 allow an evaluation of the child's basic ability to differentiate "same" from a comparative.

The authors also hoped to determine whether a child's choice of cue on the main task, which employed static arrays, related to his performance on a conservation type task, which employed transformable arrays. They expected that a child who used a number strategy on the main task would be more apt to be a conserver than a child who used a length strategy on the main task.

Method

Twenty seven children were tested who ranged in age from 2.10 to 5.0 years with a median age of 4.2. There were 11 male subjects and 16 female subjects. The static configurations, referred to as the number-length assimilation test, and the conservation test were presented in balanced order.

Number-Length assimilation test

This test consisted of the presentation of static stimulus configurations to the child as shown in Figure 1-1. Variations of configurations 1 and 5 were included, which had four dots in each row. The arrays consisted of black discs 5/8 inch pasted on 5 by 8 inch white cards. They were designed so as to employ combinations of length, density, and number cues. Those arrays on which it is possible to score number to length or length to number errors are labeled incongruent; those arrays on which these dimensions are not in conflict are labeled congruent. In configurations 1 and 2 numerical and spatial cues were redundant and congruent. In configurations 4 and 5 either number or length was held constant (number in 5; length in 4) and the other dimension was varied. Configuration 3 had conflicting cues, that is, the longer row was the least numerous, whereas the shorter row was the most numerous. After each card was presented, the child was asked: "Do these rows have the same number of dots, or does one row have more dots?" If the child answered that one row had more, he was asked which had more. A second question was then asked: "Are these rows the same length, or is one row longer?" If the child said one row was longer, he was asked which row was longer. All possible positions of the independent clauses were employed for both number and length questions. The order of the questions was varied. The result was eight different question sets, which were randomized for each stimulus presentation. Each configuration, with the exception of configuration 1, was shown to the child three times with counter-balancing of position and number (three or four) in the equality sets.

Configuration 1 was shown six times so that there would be an equal probability of correct same and different answers. In all, then, the child saw 18 cards and responded to 36 questions. There were two orders of stimulus presentation.

Conservation task

In the conservation task the child was asked three questions. After being shown two parallel rows of equal length each with four pennies, the child was asked these questions (1), "Are there the same number of pennies in each row or does one row have more pennies than the other row;" While the child watched, the row of pennies closest to him was pushed together. Then the child was asked question (2), which was either the same question as question (1) or the same question with the clauses reversed: "Does one row have more pennies than the other row, or are there the same number of pennies in each row"? Prior to question (3), the altered row was returned to its original position identical with, and parallel to, the other row. To be judged a conserver, the child must answer "same" to all three questions.

Results

If a child confused length and number questions but employed a consistent length response strategy (See Figure 1-1), that is, he was generally correct when asked about length and incorrect when asked about number, he was said to have made number-to-length errors, or to be a length responder. If a child confused length and number, but employed a consistent number response strategy (See Figure 1-1), that is, he was generally correct on number and incorrect on length, he was said to have made length-to-number errors, or to be a number responder. The nine children who made more than four incorrect responses on the congruent stimuli were not included in further analysis of the number-to-length or length-to-number errors. This left 18 children.

Twelve out of 18 children were characterized as number responders. These children made 0 or only 1 error on number questions, out of a possible 9, but at least 3 errors attributable to a number strategy on length, out of a possible 9. Six children out of 18 were characterized as length responders, with 0 to 1 errors on length, but at least 3 errors attributable to a length strategy on number. There was a significant negative correlation between subjects length to number errors and number to length errors (Kendall rank coefficient = -.57, $p < .002$). There was no significant difference in error rate on the different configurations. A graph of the age trends for the 18 children (Figure 1-2) showed that unexplained errors, that is, errors which could not be attributed to the use of the length or number strategy remained constant over the age groups, but the relationship between the two systematic types of errors changed. Until about age four, the number to length and length to number errors remained approximately equal. From four onward this changed, with number to length errors reducing to 0 at age five and length to number errors increasing.

Seven out of the twenty-seven children conserved on the conservation task; all of these children were number responders, that is, they were usually correct on number questions but answered length questions as if they were about number. Conservers were significantly

MEAN NUMBER OF ERRORS ON
CONFIGURATIONS 3, 4, AND 5

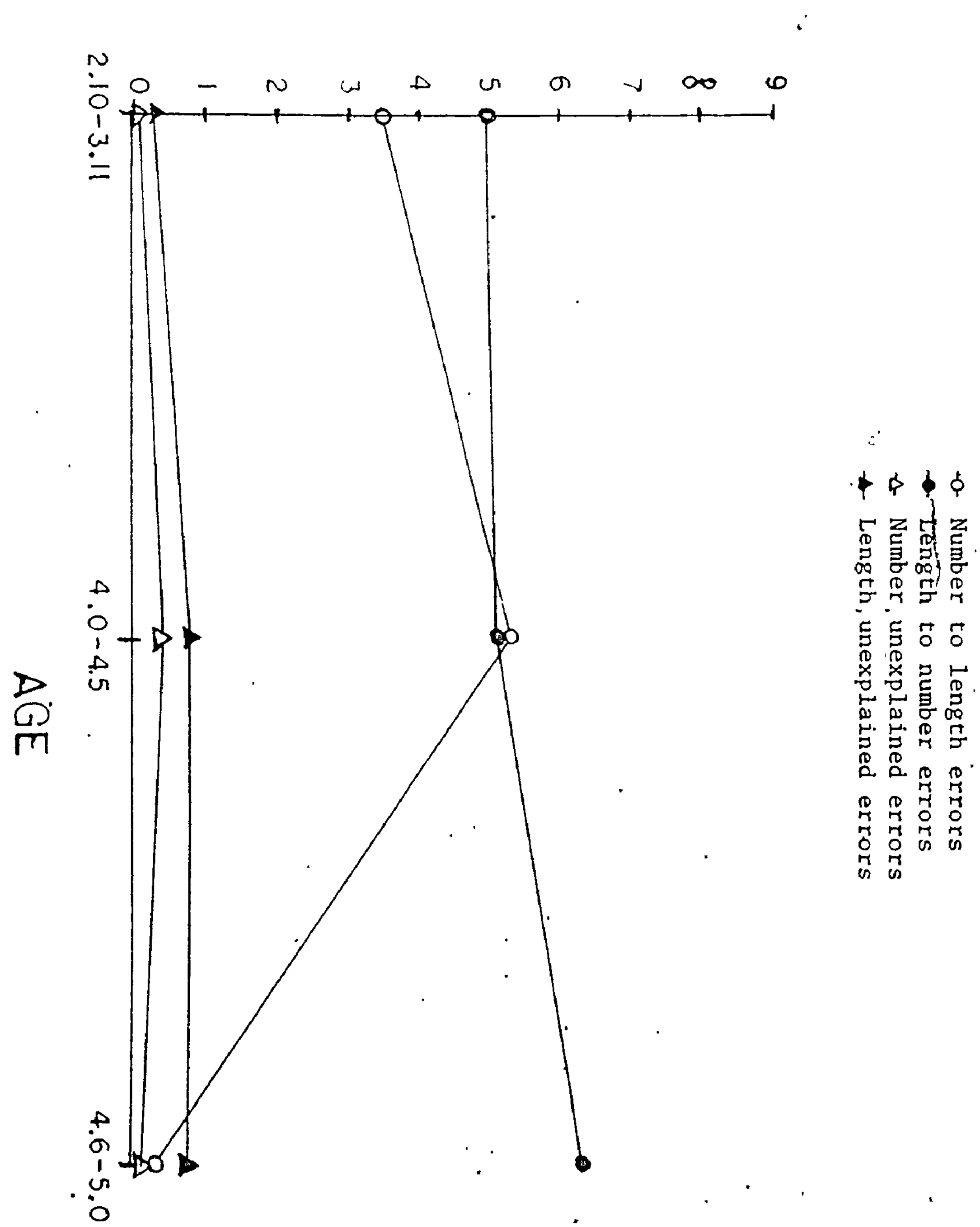


Figure 1-2 Mean number of errors on configurations 3, 4, and 5 as a function of age and error type.

different from nonconservers in their preference for number (Fisher's Exact test, $p < .05$). Density did not appear to have an important effect on the children's response strategy. For example, there were children who judged the rows in configuration 5 as the same length. In configuration five length and density differ, only number is the same. Also, some children indicated they were counting in their responses to length questions about the configuration. They would say, "One, two, three. The rows are the same".

Discussion

The results indicated that number could be a powerful cue provided the set size was small enough. The discrepancy between this study and those studies which show the power of the length cue was explained in terms of the different set sizes; most studies which have shown the power of the length cue have used large numbers - probably numbers beyond the estimation range of the subjects (Pufall and Shaw, 1972). The number cue in this task was, in fact, so powerful that it enabled numerical operations, but disabled length operations, for the majority of children. All of the children who employed number for the length judgments on this task, were able to judge continuous length correctly three times on a subsidiary task included at the end of the experiment.

The consistency of the 18 children's responses pointed to the existence of some kind of stable cognitive structure. They did not make both length to number and number to length errors at the same time. However, although the children did have consistent strategies enabling them to respond differentially to "same", and to the comparative terms "more" and "longer", their concepts of length and number had still to be construed as immature for they applied the strategies in inappropriate contexts. Baron has argued that the children might best be characterized as possessing a superordinate concept of "bigness", missing some of the components necessary to allow them to treat length and number as independent dimensions (Baron, 1974).

The findings suggested that the development of conservation might procede best on the small number sets, where the child could easily check on invariance. On large number sets, he would be apt to use a length strategy and be unable to check his estimates. The findings also indicated that success on a conservation task could not be taken as sure evidence of a mature concept. The children who succeeded on the conservation task were the children who misapplied number to length.

Experiment 2

Effects of Training and Set Size on Children's Judgments of Number and Length (Baron, Lawson, and Siegel, 1975).

The findings from Experiment One indicated that children had difficulty differentiating the dimensions of length and number when asked to make comparative judgments on both dimensions. However, these children were consistent in their choice of cues attending to either length or number. The second experiment was undertaken to attempt to establish:whether the children understood only one concept and did not understand the other, and therefore only had one strategy available; or whether neither concept was fully understood but the strategies appropriate to both were available. In this last case, the child might lack a criterion for the application of the appropriate strategy and employ a strategy in an inappropriate situation simply because it was easier, or because his attention was attracted to some salient features of the array. If the child does have both length and number strategies available and is biased to use one rather than the other because of certain features of the task, it should be possible to induce him to use the other strategy by altering the features of the task. If he does not have both strategies available, he should continue to use the same strategy no matter what changes are made in the task.

Two factors were introduced in this experiment which could alter the child's response strategy, if it were simply due to a bias. These were variations in set size, and training.

The results of Experiment One showed that small number sets could produce numerical strategies on length questions. Other investigators had shown that large number sets produce length strategies on number questions (Pufall and Shaw, 1972). Suppose one were to ask the same child to answer length and number questions on both large and small sets, and found that the child, in response to both questions, used a number strategy on the small set and a length strategy on the large set. One might conclude that the child had both length and number strategies available, but was biased by certain features of the task to employ one strategy or the other. Changes in set size could make one strategy easier than the other, for example, number on the small set. Or, changes in set size could make one dimension more salient than the other, for example, number may be easier on the small set because the individual dots stand out.

Training a child to employ an appropriate strategy on the one dimension where he employed an inappropriate strategy could result in the child using appropriate strategies on both dimensions. For example, if a child had used a number strategy appropriately on number questions, but inappropriately on length questions, he might be trained to use the appropriate length strategy on length questions. If the child has a proper number concept, that is, not only has the strategy but the criterion for when to apply it, he should not give up his number strategy when trained on length. If he lacks the criterion, training him on length could simply bias him toward the use of the length strategy, which he might then employ in response to number questions.

Method

Eighty-four children between the ages of 3-5 were tested on two separate days in a pretest-training-post-test design. On day one the experimenter presented the children with arrays like those in Figure 1-1. The same method was employed as in Experiment One, except that the children were tested with both large sets (6-8) and small sets (3-4). The order of the sets was counterbalanced across children. There were 10 stimulus cards in each set, each configuration presented twice. One week later, the children were trained with the small sets on the dimension on which they had made the most errors on the pretest. Training consisted of asking the child questions only on this dimension, and then giving him corrective feedback. He was given 12 training trials, the three incongruent configurations were each presented four times. The training was followed by a post-test identical to the pretest, only employing the small arrays. A control group was also employed in which procedures were identical in training except that the children received no feedback.

Results

Subjects who answered incorrectly on the pretest on the congruent configurations were again excluded from analysis of the length to number and number to length errors on the incongruent configurations. The results from the pretest indicated that row size affected choice of strategy. Length to number errors occurred more often on the small set than on the large set, $t(37) = 4.32$, $p < .001$; number to length more often on the large set than on the small set,

-t(37) = 4.34, $p < .001$. There was no significant difference between the two set sizes in the number of errors overall. The findings were consistent with those of Smithers, Smiley, and Rees (1974). There was, however, an order effect for set size. Children who had the large set first, the set which provoked the length strategy, were more apt to persist in the use of the length strategy on the small set than children who had the small set first.

Training proved to be effective in altering children's response strategies. Subjects in the experimental group made fewer systematic errors[1] on the trained dimension in the post-test than in the pretest. These subjects also made significantly fewer errors on post-test than those in the control group who showed no change, $t(29) = 3.14$, $p < .005$. The interesting result, however, was that while subjects in the control group made no more errors on the untrained dimension on the post-test than on the pre-test, subjects in the experimental group did make more errors, $t(28) = 2.86$, $p < .01$. There was, then, evidence for negative transfer. However, there was some positive benefit from training which could not be simply attributed to altering a bias: the amount of increase in error on the untrained dimension was not as great as the decrease on the trained dimension, $t(28) = 1.91$, $p < .05$.

Conclusion

Many of the children in this experiment showed that they had both length and number strategies available to them, but could be

[1] Systematic errors: This term will be used to facilitate the discussion of "length to number" and "number to length" errors in those cases where a general term is necessary. Systematic errors can be contrasted with unexplained errors.

biased to use one of the strategies by training, changes in set size, and the order of presentation of set sizes. These children were not simply limited to a single concept and a single strategy. Rather they had both strategies available, or parts of both concepts, but appeared to lack an appropriate criterion for the application of their strategies. The choice of strategy seemed to reflect a response bias induced by particular features of a task. As far as language is concerned the results suggested that the referents of the comparative terms "longer" and "more" can be both numerical and spatial for a preoperational child. The meaning of the term will reflect the particular response bias produced by the features of the task.

The findings spoke to two issues: the evaluation of the status of a concept and training.

The authors concluded that a demonstration that a child has an appropriate strategy for a concept is not sufficient to allow one to say that the child has a mature concept. To say that a child has a mature concept one must demonstrate that the child does not misapply the concept, for example, use number to judge length, and that one cannot easily manipulate his response strategy. Success on a conservation task, therefore, might not be the best evidence for acquisition of a quantity concept. Such success may only indicate that the child has a number bias.

With respect to training, the results suggested that non-differential training of the type employed here may have little value, as such training may simply alter response biases. Also, the rather rapid effect of the training in altering children's response strategies and producing a negative transfer, suggests the possibility that training did not simply alter single associative bonds, but a larger, bound unit, a subplan or component, as suggested by Baron (1973).

Experiment 3

Effects of Set Size, Static and Transformed Presentations, Configurations, and Age on Children's Judgments of Length and Number

Findings from Experiments One and Two showed that some young children, age 3-5, did not respond differentially to length and number questions; rather, they consistently used a length or number strategy regardless of the question asked. Of particular relevance to this third experiment were the following findings: First, in children of this age, the small number set provoked the use of a number strategy, the large number set a length strategy. Second, children who succeeded on the number conservation task, that is, on the task using transformed arrays, were number responders on the main task using static arrays. They used the numerical strategy to make length judgments. Third, on the small sets the use of the number strategy to make length judgments appeared at about the age of 4½-5. Fourth, although the finding was not significant there was a slight tendency for children to make more length errors on configuration four (length same, number different) and number errors on configuration five (number same, length different). The intention of the following experiment was to explore further the relationship between children's response strategies in making comparative judgments of length and number and the factors of set size, static versus transformed presentations, configurations, and age.

In Experiment Two every child judged both number and length on both small (3-4 dots) and large (6-8 dots) sets. On small sets the children were more apt to use a number strategy to judge both length and number, on large sets, a length strategy. The same child might use a number strategy on the small sets and a length strategy on the large sets. In Experiment One there was evidence that it was the older children (4½-5 yrs) who were apt to use a number strategy. This finding is consistent with findings by Gelman (1972) and Smithers et al. (1974) that children of this age are skilled in judging numbers up to five. In the following experiment, Experiment Three, the author presented large and small sets to different groups of children over the 3-7 age range. A question of particular interest was whether children 5 to 7 years of age, who should be very accurate in number judgments of arrays of three to four dots use the number strategy to judge length.

In Piaget's theory of number the ability of the child to withstand a change, or transformation, in an irrelevant dimension such as length, that is, to conserve number; is the main indicator of the child's possession of a mature number concept. Early in the preoperational period the child attends to single dimensions such as length or density. He may change the basis of his judgment in light of a transformation because a different perceptual cue becomes more salient. Only in the later stages of the development of the number concept is the child able to use the information imparted by the transformation itself. Piaget does point out that some children in this preoperational period appear able to conserve small numbers;

however, he believes this judgment is based on perceptual intuition rather than on true operational understanding of the concept.

Other investigators besides Piaget believe that the child receives valuable information from witnessing a transformation if he attends to it (Baron, 1974; Bryant, 1974; Pufall and Shaw, 1972; Rose and Blank, 1974). The difference between Piaget's position and that of those cited above is that the latter group believe that the child actually has true inferential strategies available to him prior to achieving conservation. He may not succeed at number conservation because of language problems or a misunderstanding about the requirements of the task. For example Rose and Blank have shown that a young child may view the repetition of a question as suggestion that he should change his judgments (Rose and Blank, 1974).

Some investigators of the factors influencing success on the conservation task have proposed comparing children's performance under transformed and static conditions. This comparison supposedly allows an assessment of the role of the transformation. In Experiment One an association between conservation and the use of the number strategy was established. Zimles (1966) in a study of older children, Kindergarten to Grade two, found that this age group made more errors on number judgments on transformed than on static arrays. That is consistent with the findings in Experiment One for the 3-5 age groups. Those children who were successful in the transformed arrays used number on the static arrays, but not all children who were successful in using number on the static arrays succeeded on the transformed arrays. Beilin (1969), too, found that children in the 3-5 age group tended to make more errors on the transformed than on the static arrays. On the other hand, he found in a study of children of the same age as those in Zimiles sample, that performance was generally better on the transformed arrays than on the static arrays.

In the experiment, 3 to 7 year old children were given both large and small sets under both static and transformed conditions. They were asked both length and number questions over a series of fixed, random presentations of static and transformed arrays. The only dimension transformed was length. The author expected that the effect of the transformation could vary according to set size, age and the individual child's preferred cue. On the small set, where number is salient, the transformation might have less effect than on the large set, where length is salient. Alternatively, on the large set, the transformation might have less effect simply because length is such a powerful cue that the child may make no attempt to assess numerosity or invoke an invariance rule. Also, since children under four seem more apt to attend to length regardless of set size (Smithers et al., 1974) they might be more apt to change judgments in light of a transformation in length than older children. However, it did seem possible that set size, rather than presentation condition would be the major determinant of strategy, particularly within a context where the transformation itself does not stand out as the only change. In the method employed here the child saw static and transformed arrays interspersed and was asked two different questions with each stimulus configuration that appeared.

The final question of interest in this study concerned the possibility of differential error rates on the different configurations in response to length and number questions. There is evidence that judgments of equality are more difficult than judgments of inequality (Beilin, 1969; Rose and Blank, 1974). Possibly children seek differences (Pufall & Shaw, 1972). Therefore, one might expect a higher error rate on configuration one (all cues equal), than on configuration two (all cues congruent). In Experiment Two there were indications that configuration five (number same, length different) was the most apt to produce an error on number questions, configuration four (length same, number different) an error on length questions. Pufall and Shaw (1972) also found that with static arrays children made the most errors on number questions on configuration five.

In this experiment the children were given all five configurations in both set sizes and asked both length and number questions. Of interest was whether the same order of difficulty on configurations might occur on both set sizes in spite of overall differences in numbers of errors. For example, on the large set, there might be many more errors on number questions than on length questions, but the same configuration might still be the most difficult on both set sizes. Also of interest was order of difficulty of configurations where the children were asked about length. If young children attend to differences, then configuration four (length same, number different) should produce the greatest number of errors on length questions.

In summary: This experiment was designed to look at the following: (a) the age range over which children employ a numerical strategy to judge length; (b) the relationship between the factors of transformation, set size, a child's individual strategy and his age, and (c) differential error rates on configurations in relation to both set size and question asked.

Method

In this study, children 3.7 to 7.1 years of age were shown congruent and incongruent configurations (Figure 1-1) under both static and transformed conditions. One group of children was shown a small number set (3 to 4 dots); another a large number set (7 to 8 dots). A mixed, balanced order of presentation was employed. All children were asked to make judgments of both number and length.

Tests included number and length questions on both static and transformed arrays. As in the previous studies, the purpose of the tests was to determine the child's ability to judge whether two simultaneously presented rows of dots had the same number of dots or one had more, and whether they were the same length or one was longer. Subjects were tested individually.

Subjects

One hundred and forty subjects were tested. They ranged in age from 3.7 to 7.1 with a median age of 6.4. There were 74 male subjects and 66 female subjects. The kindergarten and grade one children in the study were attending a parochial school and were of mixed class and ethnic backgrounds. The pre-school children attended either a private or a co-operative nursery school and were of middle-class background.

Materials

The Number-length Test in the Experiment was given on both static and transformed arrays presented as the stimulus configurations shown in Figure 1-1. These were the same configurations used in Experiments One and Two. The one change was the introduction of a millimetre standard unit to determine distance between dots; for example, on configuration one, the dots were placed 12 mm. apart. Arrays of two sizes were used, with the same configurations, combinations, orders, etc. used in each.

The configurations for the static arrays were made by pasting 5/16" black discs on 5" by 8" white cards. The configurations for the transformed arrays were formed with black dots, identical to those pasted on the static arrays, i.e., 5/16" black discs. These discs, however, were attached to very small magnets. The transformation took place on a 5" by 8" white card, which had small pin pricks in it so that the experimenter could give the dots spacings identical to those found in the static configurations when she formed the configurations. This card was placed on top of a small metal sheet.

Procedure

The school age children were tested one at a time in a session lasting approximately twenty minutes. There were two kindergarten classes and two grade one classes. The nursery school children were also tested one at a time; however, the youngest group, those born in 1971 (N = 20) were tested in two sessions of about ten minutes each in order to keep them attentive. Sixty-eight children were randomly assigned to receive the 3-4 dot set of configurations, seventy-two the 6-8 dot set.

Each child received three transformations, in each of which there were three states, the initial and final state being identical. The three transformations were chosen so that there was one number equality condition (using configurations 5 and 1) and two number inequality conditions (using configurations 2,3,4). The sets were arranged so that for each subject transformations included changes from: congruent to incongruent to congruent (e.g, 1,5,1 or 2,4,2, or 2,3,2) incongruent to congruent to incongruent (4,2,4 or 5,1,5, or 3,2,3) and incongruent to incongruent to incongruent (e.g, 4,3,4, or 3,4,3). The experimenter alternated between moving the row closer to the child and the row further away. The nine static configuration cards were chosen to match the transformable configurations presented to the child. They were presented one at a time in the same order as the transformed configurations, three in sequence, for example, cards with configurations 1, 5, and 1 were presented in a sequence. There were four different combinations of configurations employed across children and two set orders of presentation - static (3 cards), transformed (3 configurations), transformed (3 configurations), static (3 cards), transformed (3 configurations), static (3 cards), and the converse. The order was such that the identical sequence of transformed and static arrays never followed each other.

The questioning procedure was identical for both the static and transformed conditions. After the experimenter formed each configuration or presented the static cards, she asked the child, "Do these rows have the same number of dots or does one row have more dots?" If the child answered that one row had more he was asked which had more. A second question was then asked: "Are these rows the same length or is one row longer?" If the child said one row was longer he was asked which row was longer. Following each transformation the same questions were asked. All possible positions of the independent clauses were employed for both number and length questions and the order of the questions was varied. The result was eight different question sets, one of which was randomly chosen for each stimulus presentation.

Results

The children's protocols were scored for errors. A total of 36 errors was possible: 18 on the static arrays; 18 on the transformed arrays; 9 on length and number questions under both the transformed and static presentation conditions. As in Experiments One and Two errors were classified as attributable to systematic length or number response strategies, or to irrelevant, unexplained factors. Systematic errors can only be assigned on incongruent configurations 3,4, and 5 (see Figure 1-1). The criteria employed are described in the introduction to Experiment One and indicated in Figure 1-1. Each subject received 6 congruent configurations (1 and 2), and 12 incongruent configurations (3, 4, and 5). Therefore, a total of 24 systematic errors was possible: 12 under each presentation condition: 6 on length and number questions under the two transformation conditions.

Those subjects who made 5 or more errors (out of a possible 12) on the congruent configurations were classified as guessers. As in Experiments One and Two, a high error rate on configurations 1 and 2 was believed to indicate the child's inability to understand, in some basic way, "same", or the comparative terms, or both. A subject who made 8 or more correct responses on the congruent arrays was classified as a nonguesser. The criterion was reduced slightly from that employed in the previous experiments, so as not to exclude from analysis those children who might have been affected by the transformation. Sixteen children were classified as guessers, one hundred and twenty-three as nonguessers. Guessers were excluded from any analysis of systematic errors. Systematic errors (which show a child's response strategy) were of primary interest in this experiment, as they were in the others in this thesis. However, it seemed possible that the effects of presentation could be revealed in changes in both systematic and unsystematic errors. Consequently, for most purposes, separate analyses of systematic errors and all errors were carried out. In the analyses of all errors guessers were included, as were all unsystematic errors on both congruent and incongruent configurations.

To assess the overall effect of set size, transformed vs. static presentation, and question, a three factor mixed design Anova was performed on all subjects' errors (Tables A3-1 and 3-2, Appendix A). The between groups factor was set size, the within factors: transformed and static presentation, length and number questions. In order to have an equal N in the cells, four subjects were dropped on a random basis from the large set where N = 72. The main effects of set size, $F(1,134) = 5.55$, $p < .01$ and the double interaction of set size and question, $F(1,134) = 69.76$, $p < .001$ were significant (Source Table B3-1, Appendix B).

The means and total scores are presented in Table 3-1. There

Table 3-1

Total number of errors and means by presentation condition, question, and set size.

Grand total, both sets = 1403, $\underline{N}$ = 136, $\bar{X}$ = 10.32

Presentation				Question			
static,	Σ=691, $\bar{X}$=5.08	transformed,	Σ=712, $\bar{X}$=5.2	number,	Σ=813, $\bar{X}$=5.98*	length,	Σ=590, $\bar{X}$=4.338*

Question x Presentation

number				length			
static,	Σ=408, $\bar{X}$=3.00	transformed,	Σ=405, $\bar{X}$=2.98	static,	Σ=283, $\bar{X}$=2.08	transformed,	Σ=307, $\bar{X}$=2.26

Grand total, small set = 619, $\underline{N}$ = 68, $\bar{X}$ = 9.10*

Presentation				Question			
static,	Σ=303, $\bar{X}$=4.46	transformed,	Σ=316, $\bar{X}$=4.65	number,	Σ=231, $\bar{X}$=2.40*	length,	Σ=388, $\bar{X}$=5.706*

Question x Presentation

number				length			
static,	Σ=116, $\bar{X}$=1.71	transformed,	Σ=115, $\bar{X}$=1.69	static,	Σ=187, $\bar{X}$=2.75	transformed,	Σ=201, $\bar{X}$=2.96

Grand total, large set = 784, $\underline{N}$ = 68, $\bar{X}$ = 11.53*

Presentation				Question			
static,	Σ=388, $\bar{X}$=5.71	transformed,	Σ=396, $\bar{X}$=5.82	number,	Σ=582, $\bar{X}$=8.56*	length,	Σ=202, $\bar{X}$=2.97*

Question x Presentation

number				length			
static,	Σ=292, $\bar{X}$=4.29	transformed,	Σ=290, $\bar{X}$=4.26	static,	Σ=96, $\bar{X}$=1.41	transformed,	Σ=106, $\bar{X}$=1.56

* $p < .05$ Duncan's multiple range test.

were 36 errors possible overall: 18 under the transformed and static condition: 18 for the number and length questions: 9 for each presentation by question cell.

On the average, children made 10 out of a possible 36 errors. They made significantly more errors on the large set than on the small set, and overall made more number errors than length errors. However, the interaction of set size and question indicated that the number of errors the children made on the length and number questions depended on the set size. The Duncan's multiple range test, $p < .05$, comparing the question means on the two set sizes revealed that children who received the large set made their greatest number of errors in response to number questions. Children who received the small set made their greatest number of errors in response to length questions.

To assess the effect of set size, presentation condition, and question on children's systematic errors, the same analysis was performed on systematic errors, guessers excluded. (Tables A3-3 and A3-4, Appendix A). The main effect of question, $F(1,122) = 10.44$, $p < .005$, and the set size by question interaction, $F(1,122) = 62.7063$, $p < .001$ were significant (Source Table B3-2, Appendix B).

The means and total scores for the above analysis of systematic errors are presented in Table 3-2. Twenty-four errors were possible overall: 12 under the static and transformed presentation; 12 for the number and length questions; 6 for each presentation by question cell. On this measure there was no significant difference in the mean number of errors made on the two set sizes, although the results are in the

Table 3-2

Total systematic errors and means by presentation condition, question, and set size.

Grand total, both sets = 854, N = 124, $\bar{X}$ = 6.89

Presentation		Question	
static, Σ=424, $\bar{X}$=3.419	transformed, Σ=430, $\bar{X}$=3.47	number, Σ=538, $\bar{X}$=4.34*	length, Σ=330, $\bar{X}$=2.66*

Question x Presentation

number		length	
static, Σ=261, $\bar{X}$=2.10	transformed, Σ=363, $\bar{X}$=2.12	static, Σ=163, $\bar{X}$=1.31	transformed, Σ=167, $\bar{X}$=1.35

Grand total, small set = 389, N = 62, $\bar{X}$ = 6.274

Presentation		Question	
static, Σ=195, $\bar{X}$=3.15	transformed, Σ=194, $\bar{X}$=3.13	number, Σ=125, $\bar{X}$=2.02*	length, Σ=264, $\bar{X}$=4.26*

Question x Presentation

number		length	
static, Σ=62, $\bar{X}$=1	transformed, Σ=63, $\bar{X}$=1.02	static, Σ=133, $\bar{X}$=2.145	transformed, Σ=131, $\bar{X}$=2.11

Grand total, large set = 465, N = 62, $\bar{X}$ = 7.50

Presentation		Question	
static, Σ=229, $\bar{X}$=3.69	transformed, Σ=236, $\bar{X}$=3.806	number, Σ=399, $\bar{X}$=6.44*	length Σ=66, $\bar{X}$=1.06*

Question x Presentation

number		length	
static, Σ=199, $\bar{X}$=3.209	transformed, Σ=200, $\bar{X}$=3.23	static, Σ=30, $\bar{X}$=.48	transformed, Σ=36, $\bar{X}$=.58

* $p < .05$ Duncan's multiple range test.

same direction as in the analysis of all errors. The only significant main effect was attributable to the question asked; more errors were made in response to number questions than length questions. The interaction indicated that the question which produced the greatest number of systematic errors varied according to set size. Duncan's multiple range test on the means in the question by set size cells showed that the children who received the small set made more systematic errors on the length question, whereas the children who received the large set made more systematic errors on the number question. The greatest number of systematic errors occurred in response to the number question on the large set. Within the sets there were significant differences in response to the two questions.

The difference in the mean number of errors made under the transformed or static presentation conditions was not significant for either the large or small sets in either of the above analyses. However, it seemed possible that the effects of a transformation might interact with a child's preferred strategy. Subjects (only nonguessers) were, therefore, classified according to the direction of their maximum number of systematic errors. Fifteen subjects could not be classified, as they made equal numbers of systematic errors on both questions. There were 66 length responders or strategists over both sets, and 44 number responders or strategists. The data analyses included all errors. Seperate treatment (presentation condition) by treatment (question) by subject Anovas were carried out on the error data for length strategists and number strategists. There were no significant effects except the effect of questions for length strategists, $F(1,195) = 399.0210$, $p < .0001$; for number strategists, $F(1,126) = 187.8740$, $p < .0001$ (Source Tables B3-3, B3-4, Appendix B), which simply indicated that children who used a length strategy made more errors on number questions than on length questions and children who used a number strategy made more errors on length questions than on number questions.

Of some interest in this experiment was the relationship between age, type of error, and method of presentation. To simplify this age analysis, the children were grouped according to school grade. The three grade groups, nursery school, kindergarten, and grade one were equalized by adding several of the oldest nursery school children to the kindergarten group and dropping several of the grade ones. Small and large sets were analyzed separately, on both all errors and systematic errors. In each case a three factor mixed Anova was used, with Grade the between factor, and presentation condition and question the within factors.

The results of the analysis on all errors for the small set yielded significant main effects of grade, $F(2,63) = 12.72$, $p < .001$, and question $F(1,63) = 11.7266$, $p < .005$ (Source Table B3-5, Appendix B). The means and total scores are presented in Table 3-3. There were 36 errors possible overall: 18 under the transformed and static condition; 18 for the number and length questions; 9 for each presentation by question cell. Significantly more errors were made in response to the length question than the number question (see Table 3-1). Of more interest here, however, was the age effect. A Duncan's multiple range test ($p < .05$) on the mean grade differences showed

Table 3-3

Total number of errors and means on small set by grade, presentation condition, and question (N=68)

			Grade		
			Nursery S(25)	Kindergarten(18)	Grade one(25)
Total			311	150	158
$\bar{X}$			12.44*	8.33*	6.32*
presentation:					
static	Σ	=	159	68	76
	$\bar{X}$	=	6.36	3.78	3.04
trans.	Σ	=	152	82	82
	$\bar{X}$	=	6.08	4.56	3.28
question:					
number	Σ	=	156	54	21
	$\bar{X}$	=	6.24	3.00	.84
length	Σ	=	155	96	137
	$\bar{X}$	=	6.2	5.33	5.48
ques. x pres.:					
numb.stat.	Σ		84	24	8
	$\bar{X}$	=	3.36	1.33	.32
trans.	Σ	=	72	30	13
	$\bar{X}$	=	2.88	1.67	.52
leng.stat.	Σ		75	44	68
	$\bar{X}$	=	3.00	2.44	2.72
trans.	Σ	=	80	52	69
	$\bar{X}$	=	3.20	2.88	2.76

Table 3-4

Total systematic errors and means on small set by grade, presentation condition, and question

			Grade		
			NurseryS(20)	Kindergarten(18)	Grade one(24)
Total			168	102	119
$\bar{X}$			84.*	5.6*	4.96
presentation:					
static	Σ	=	86	46	63
	$\bar{X}$	=	4.3	2.56	2.63
trans.	Σ	=	82	56	56
	$\bar{X}$	=	4.1	3.11	2.333
question:					
number	Σ	=	84	30	11
	$\bar{X}$	=	4.2	1.67	.458
length	Σ	=	84	72	108
	$\bar{X}$	=	4.2	4.00	4.50
ques. x pres.:					
numb,stat,	Σ	=	43	14	5
	$\bar{X}$	=	2.15	.778	.208
numb,trans	Σ	=	41	16	6
	$\bar{X}$	=	2.05	.89	.25
leng.stat.	Σ	=	43	32	58
	$\bar{X}$	=	2.15	1.78	2.42
leng.trans.	Σ	=	41	40	50
	$\bar{X}$	=	2.05	2.22	2.083

that all three grades differed from each other. There was a decrease in the mean number of errors made by each Grade.

The results of the analysis of the systematic errors by grade on the small set yielded a significant main effects of grade $F(2,57) = 5.0256$, $p < 01$, and question, $F(1,57) = 10.4263$, $p < .001$ (Source Table B3-6, Appendix B). The means and total scores are presented in Table 3-4. There were 24 errors possible overall: 12 under the transformed and static condition; 12 for the number and length questions; 6 for each presentation by question cell. More systematic errors were made in response to length questions than to number questions (see Table 3-2). The grade means were tested with the Duncan's multiple range test. The nursery school children made significantly more systematic errors than either the kindergarten or grade one children. These last two grades did not differ from each other.

The analyses on all errors on the large set yielded significant main effects of grade, $F(2,63) = 14.359$, $p < .001$ and question, $F(1,63) = 89.4935$, $p < .0001$ (Source Table 3B-7, Appendix B). Table 3-5 presents the means and total scores. As with the small set there were 36 errors possible overall. The children made more errors on the number question than the length question. The Duncan's multiple range test on the grade means indicated that the nursery school children did not differ in the mean number of errors; however, the grade one children made significantly fewer errors. This result is different from that on the small set where all grades differed.

The analysis of the systematic errors on the large set showed a main effect of grade, $F(2,57) = 11.5639$, $p < .001$ and question,

Table 3-5

Total number of errors and means on large set
by grade, presentation condition, and question

			Grade		
			Nursery S(25)	Kindergarten(18)	Grade one(25)
Total			378	234	172
$\bar{X}$			15.12	13.00	6.88*
presentation:					
static	Σ	=	179	117	92
	$\bar{X}$	=	7.16	6.5	3.68
trans.	Σ	=	199	117	80
	$\bar{X}$	=	7.96	6.5	4.44
question:					
number	Σ	=	267	191	124
	$\bar{X}$	=	10.68	10.61	4.96
length	Σ	=	111	43	48
	$\bar{X}$	=	4.44	2.39	1.92
ques. x pres.:					
numb,stat.	Σ		131	98	63
	$\bar{X}$	=	5.24	5.44	2.52
numb,trans.	Σ		136	93	61
	$\bar{X}$	=	5.44	5.17	2.44
leng.stat.	Σ		48	19	29
	$\bar{X}$	=	1.92	1.06	1.16
leng.trans.	Σ		63	24	19
	$\bar{X}$	=	2.52	1.33	.76

Table 3-6

Total systematic errors and means on large set by grade, presentation condition and question

		Grade	
	Nursery S(24)	Kindergarten(15)	Grade one(24)
Total	221	143	101
$\bar{X}$	9.208	9.533	4.208*
presentation:			
static, Σ =	104	70	50
$\bar{X}$ =	4.333	4.667	2.08
trans., Σ =	117	64	51
$\bar{X}$ =	4.88	4.267	2.126
question:			
number, Σ =	185	134	80
$\bar{X}$ =	7.708	8.93	3.333*
length, Σ =	36	9	21
$\bar{X}$ =	150	.60*	.875
ques. x pres.:			
numb.,stat. Σ	89	70	40
$\bar{X}$ =	3.708	4.67	1.67
numb,trans, Σ	96	64	40
$\bar{X}$ =	4.00	4.267	1.67
leng.,stat. Σ	15	5	10
$\bar{X}$ =	0.625	.333	.417
leng.trans. Σ	21	4	11
$\bar{X}$ =	.875	.27	.458

F(1,57) = 97.0586, p < 001 and a significant grade by question interaction. F(2,57) = 9.3877, p < 001 (Source Table B3-8, Appendix B). Table 3-6 presents total systematic error scores and means for the large set by grade. As with the small set, a total of 24 systematic errors is possible overall.

On the large set children made more systematic errors on the number question than the length question (see Table 3-2). The grade means were tested with Duncan's multiple range test, p < .05. The grade one children made fewer systematic errors than either the kindergarten or nursery school group. As in the analysis of all errors, the kindergarten and nursery school children did not differ from each other. The interaction found between grade and question indicated that on number questions, there was a significant decrease in the grade one group, whereas on length questions a significant decrease took place in the kindergarten group.

The children's systematic errors were summarized and graphed according to age and set size, pooling the transformation and static conditions. Figure 3-1 presents the data for the small set. The figure suggests that while overall performance improved as a function of age, the main change in performance took place at about age 5 and is apparently attributable to a decline in the number of errors made in judging number according to length cues. The slope for length to number errors remains constant until age 7, indicating that at all ages examined there are some children for whom number is the preferred strategy for judging length.

Figure 3-2 presents systematic errors plotted against age for the large set. The graph reveals that on the large set, judging number according to a length strategy remains at a high level until about age six when this strategy appears to fall off rapidly.

In Experiments One and Two, a significant negative correlation was established between subjects' systematic errors. Children who used a number strategy on length questions did not use the length strategy on number questions; children who used the length strategy on number questions, did not use a number strategy on length qeustions. In this study, no significant correlations between error types were established overall on either the small or large set, Kendall's tau small set = -.20, large set = 0.06. Scatter plots of the data are presented in figures 3-3 and 3-4. The plots show that the association between subject's systematic errors can probably not be represented by a straight line. In both figures the nursery school sample corresponding in age range to those in Experiments One and Two are represented by X's. Kendall's tau on the small set nursery school children tau = -.36 which is significant at the $p < 01$ level. This is consistent with the findings in Experiment One. On the large set tau was not significant, which is not consistent with the findings in Experiment Two.

The proportion of errors made on each configuration was calculated for each set size and dimension, collapsed across the static and transformed condition. The results are presented in Table 3-7 by proportions and in Table 3-8 according to rank. Since there is no appropriate statistic for related proportions based on unequal N's, no statistical analysis was performed. The unequal N's resulted from

MEAN NUMBER OF ERRORS ON

CONFIGURATIONS 3, 4, AND 5

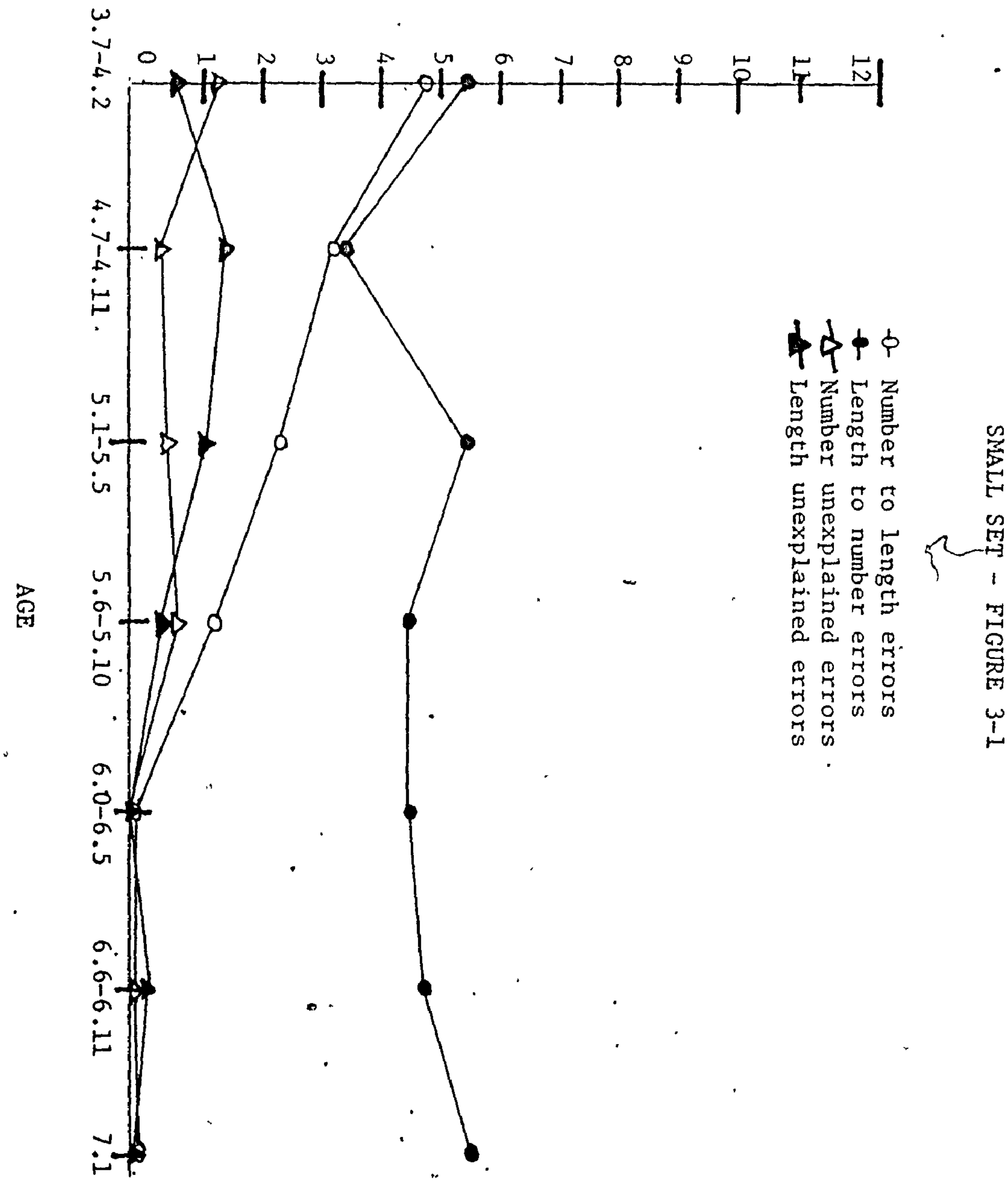


Figure 3-1 Mean number of errors on configurations 3,4, and 5 as a function of age and error type.

MEAN NUMBER OF ERRORS ON

CONFIGURATIONS 3, 4, AND 5

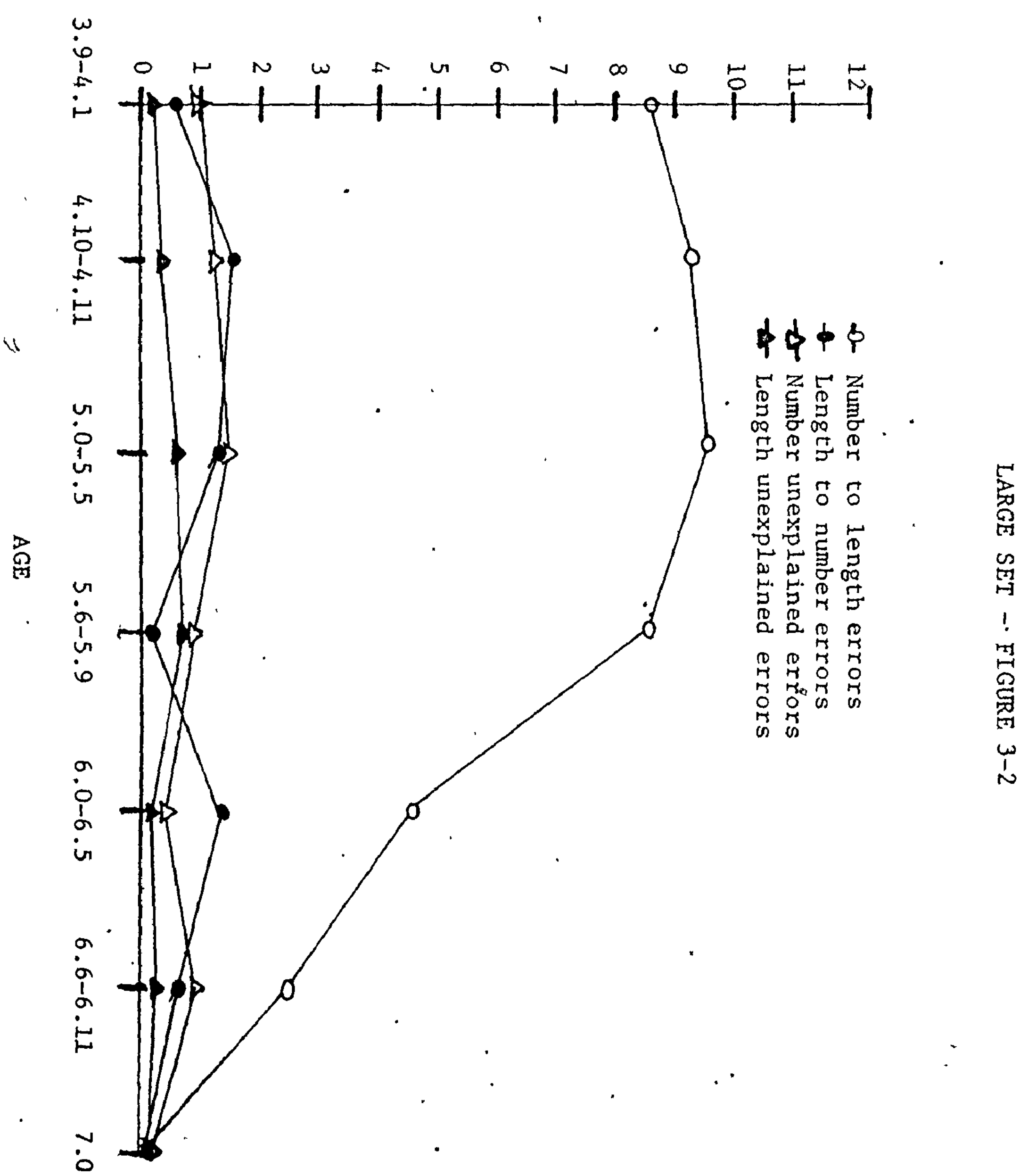


Figure 3-2 Mean number of errors on configurations 3, 4, and 5 as a function of age and error type.

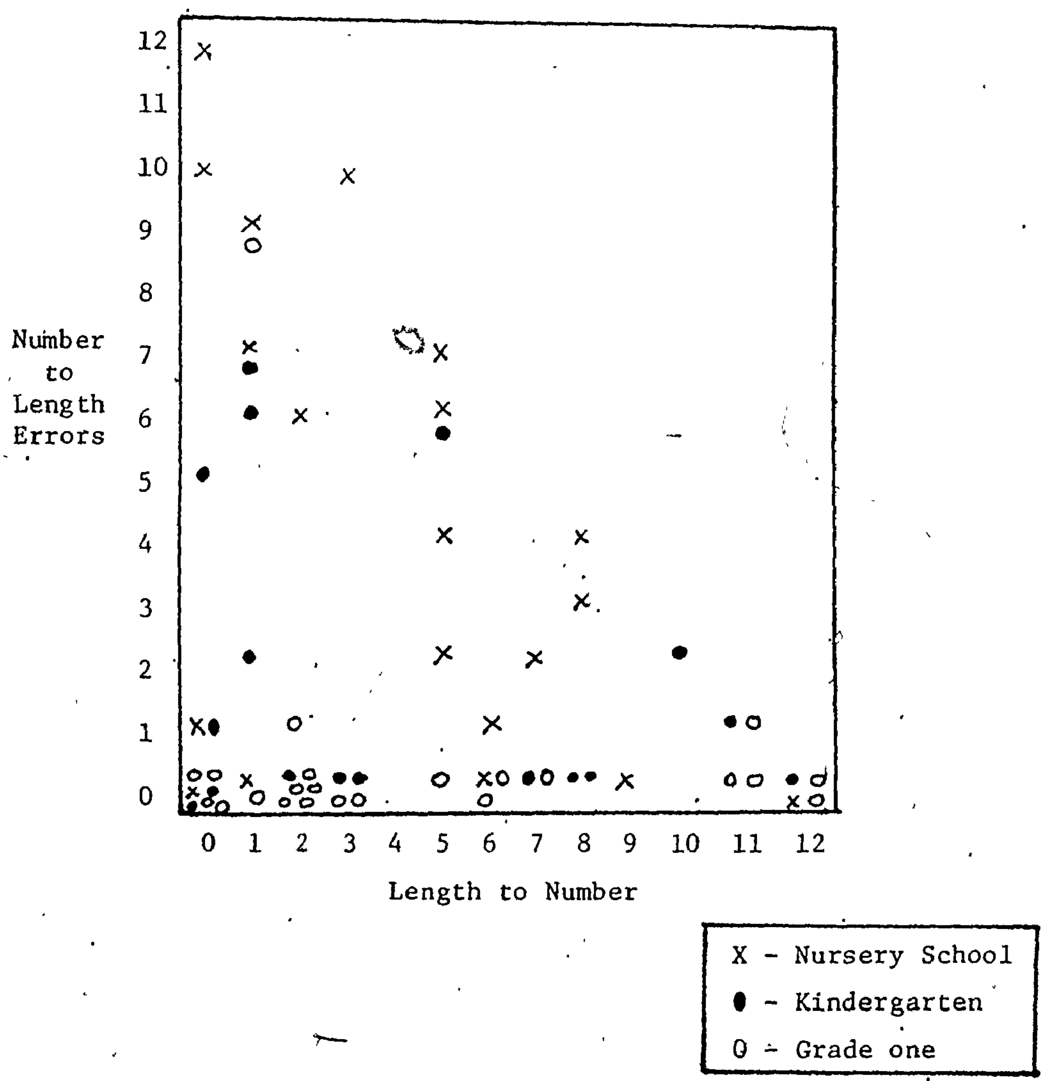


Figure 3-3 Scatter plot of association of subjects' length to number, number to length errors, small set.

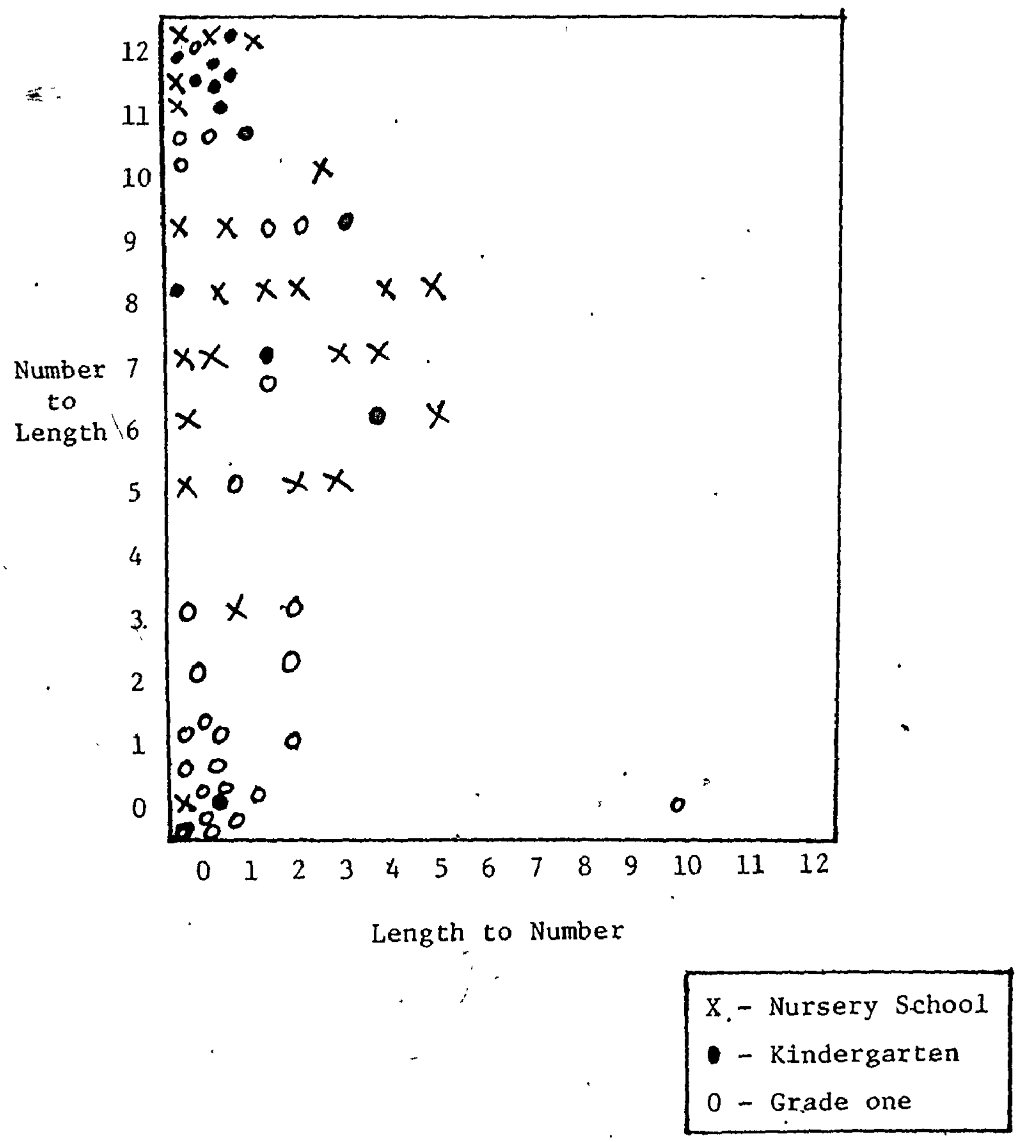


Figure 3-4 Scatter Plot of association of subjects' length to number, number to length errors, large set

Table 3-7

Proportion of Errors by Set Size and Configuration

Small Set

	Number					Length				
Config.	1	2	3	4	5	1	2	3	4	5
	33	13	73	35	77	33	11	126	197	42
N	204	204	314	298	204	204	204	314	298	204
prop.	.16	.06	.23	.12	.38	.16	.05	.40	.66	.21

Large Set

	Number					Length				
Config.	1	2	3	4	5	1	2	3	4	5
	43	14	226	184	152	37	19	38	101	23
N	214	218	334	312	218	214	218	334	312	318
prop.	.20	.06	.68	.59	.70	.17	.09	.11	.32	.10

Table 3-8 Ranks - from least to most to difficult

	Number		Length	
Set Size	Large	Small	Large	Small
	2	2	2	2
	1	1	1	1
	4	4	5	5
	3	3	3	3
	5	5	4	4

the attempt to balance the various possible types of configurations: congruent, incongruent, congruent. On both set sizes configuration two is easiest. For number judgments configuration 5 (number equal, length different) produces the most errors. For length judgments configuration 4 (length equal, number different) produces the most errors. This is true for both set sizes. Configurations 3 and 1, all cues in conflict, and all cues in agreement tend to be intermediate in difficulty.

Discussion

The results indicate a very strong effect of set size on the number of errors a child makes and on the response strategy he employs. The large set tends to produce more errors overall, and a very powerful bias towards length. The small set tends to provoke a number bias. Interesting here is the strength of this bias on small sets in children as old as 6.6 - 7.00.

The reduction with age in the overall number of errors indicates that the children are becoming progressively more able to respond with appropriate strategies to questions about quantities. On the small set there was a steady grade by grade reduction in the number of errors made. On the large sets, the grade one children made fewer errors than the nursery school or kindergarten children. The greatest change took place on number judgments. By grade one few children were making number errors on either set size. On both large and small sets, the drop in systematic errors seems to reflect an increasing ability to use number appropriately. The bias towards length persists longer on the large sets, probably reflecting the greater difficulty the child has in discriminating and labelling large number sets. At the same time, at least for some children, the emergence of a numerical strategy results in its overextension, its application to length, at least in those situations where number is more easily discriminable. It is possible that the overuse represents the addition of a new "component" to the child's understanding of the relation between number and length, that is, the recognition that number may be the best strategy for estimating length under some circumstances; for example, the number of books may be the best indicator of the spatial extent necessary to shelve them. The relation of this component to the use of a number strategy requires further study, possibly with adults.

Combined with findings from Baron, Lawson, and Siegel (1975) that the same child may change response strategy in a consistent fashion when confronted with different set sizes, the above findings lend support to a response bias interpretation of children's responses to quantitative questions. Correct answers cannot be assumed to indicate mature concepts unless tested against a situation in which a response bias may be revealed. Testing a child's concept of length or number in a situation where a strong response bias is present may also provide incomplete information concerning the availability and stability of cognitive operations, and their range of application.

The failure to find significant negative correlations between the alternative strategies, as in Experiments One and Two, does not require the rejection of the argument that at least some of the children are responding with a consistent bias. The negative correlation was present on the small set with the nursery school children and while not significant, the correlation on the large set was also negative. The older children on the small set made almost all their errors on length questions and none on the number question, that is, they were consistently employing a number strategy. The older children on the large set were making almost all their errors on the number questions and none on the length questions.

The absence of the significant negative correlation overall simply seems to indicate that the age factor results in a noncurvilinear negative relationship between length and number errors. This may be related to the extended age range in this study, resulting in a large number of subjects who make very few errors of any kind. In the case of the large set an additional factor which may reduce the possibility of establishing a linear correlation is the strength of the length response bias on the large set. This strong bias tends to produce a ceiling on the number of length to number errors.

The findings of no differential effects of the static and transformed conditions provide no support for either Zimiles' (1965) or Beilin's (1969) findings with respect to differences in performance under transformed and static conditions. One might conclude either that the transformation had no effect, or, that the order of viewing of the static configurations had the same effect as the order of viewing of the transformations. With regard to the first alternative; it is possible that the method of presentation employed here operates to eliminate any differences in performance on the static and transformed arrays. This interposing of static and transformed trials may result in the child not relying too heavily on information from the transformation trials but rather on his preferred strategy.

With respect to the second alternative; it is possible that the presentation of the static configurations in a particular order provokes a child to attend to a particular dimension. Pufall and Shaw (1972) provided evidence for such order effects. They found that a configuration which provoked attention to a length cue tended to produce a number judgment based on length on a configuration which followed it. A configuration which provoked attention to a density cue tended to provoke a judgment based on density. The effects of a transformation could operate in a similar way. That is, the child may not attend to the transformation but simply to salient dimensions and there may be order effects. However, the power of the sets in producing a response bias, and the absence of any evidence for the effect of the transformation, would seem to suggest that at least under these presentation conditions, the transformation is not itself an important source of information for any age group.

It is important to consider the role of response bias and dimensional saliency in discussions of the effects of a transformation and the development of invariance rules. An invariance rule requires the integration of information over time; a transformation is only a source of information if the dimension on which the transformation takes place is attended to. A child may only be able to integrate information on a dimension when it is salient to him. An important aspect of that act of integration is the recognition of variance as well as invariance; the child attending to length (defined by end points) appears able to integrate information about variance in length, and not able to integrate information about invariance in number. The child attending to number may be able to integrate information about invariance in number, but unable to utilize information about variance in length. In this experiment there is no way of completely evaluating the number condition; such an evaluation would require that items be added or taken away.

In the development of the number and length concepts it is possible that children do not move through stages of alternating attention to different dimension over all set sizes, then integrating this information with the information from the transformation, as in Piaget's equilibrium model. Rather, they may attend to different dimensions on different set sizes, learn invariance and variance rules in relation to the salient dimensions on different set size, that is length on large sets beyond their easy estimation range, number on small sets within their estimation range, and then integrate this information.

The differential difficulty of the various configurations may reflect a primary attentional mechanism - a bias toward differences (as suggested by Pufall and Shaw, 1972); it may also reflect a dependent developmental sequence, that is, the later development of the concept of "sameness". The findings may be interpreted in the light of findings by Donaldson and Balfour (1970) and Siegel (1977a), that young children understand "big" and "little" before they understand "same". Beilin (1969) found that judgments of equality tended to be poorer than judgments of inequality.

If the child does not understand "same" as well as he understands "more" or "longer", or he is biased to attend to differences, he will then tend to make errors on configuration one as compared to configuration two where there is no conflict in the cues and where there is a difference on both the number and length dimensions. This was found to be the case.

The pattern of the proportion of errors on configurations 3, 4, and 5, seems also to indicate the child's sensitivity to differences. Whether on the large or the small set, children seem to find it most difficult to make a judgment of numerical equality when only the length cue differs, as in configuration five, or a judgment of length equality when only the number cue differs. In judging configuration three (all cues in conflict) the child may, as with configuration four and five, attend to differences, but here all cues vary, and the child can attend to either difference.

The order of the difficulty for the configurations for number judgments fits with Piaget's theory of number. Children find it easier to compare the numerosity of two sets where length is equal, number not equal, or where neither are equal. Most difficult is the judgment of equivalence where number is equal but length is not. However, according to Pufall and Shaw (1972) Piaget's theory does not explain why configuration four (the length equal, number not equal) is easier than configuration three. Nor does his theory explain why a judgment of length equivalence would be most difficult on configuration four (length equal, number not equal). However, it is evident that the spatial cues are not coordinated with number in either case. Consequently, according to Piaget, the concepts are not operational.

The findings on the configurations in relation to the findings on presentation condition do raise questions about the appropriate means for evaluating a child's inferential or logical abilities. Piaget uses the equal number configurations to assess conservation status. He argues that failure on this configuration indicates a lack of certain logical abilities. This is the 1 5 1 configuration pattern employed in this experiment, a pattern which produces a very high error rate on number questions irrespective of any transformation (see Figure 1-1). Bryant has used a 2 4 2 configuration pattern (see Figure 1-1), and found evidence that even on large number sets, young children perform above chance on number judgments on configuration four (see Figure 1-1). He argues this demonstrates the presence of inferential abilities. However, these configurations are least likely to produce errors in number judgments irrespective of any transformation. If some attentional factor is determining the child's differential performance on these configurations, then attempts to demonstrate the presence or absence of logical abilities with different configurations without assessing their role, will produce an impasse.

Experiment IV

As outlined in Chapter one - section three, there has been a substantial controversy about the nature and order of development of the negative and positive forms of the comparative adjectives. In the three previous experiments the children were asked to make their judgments in response to questions formed with the positive forms of the comparative objectives, i.e., "longer" and "more". It is appropriate at this point to ask whether the young child's strategy in response to number and length would be as consistent, and of the same type, if he was asked to respond to the negative forms of the adjectives.

If the child uses a number strategy to judge number and length when asked for "more" and "longer", will he employ a number strategy when asked to judge "less" and "shorter"? If the child does not distinguish the negative and positive terms he should give the same kind of performance with "less" and "shorter" as with "longer" and "more". On the other hand the child may distinguish "less" from "more" and "longer" from "shorter" while confusing length and number. In that case he might, if he had a length strategy, indicate in his judgment that he recognized "less" as the "small one", but judge it according to spatial extension. If he had a numerical strategy, he might judge "less" correctly and "shorter" as the "small one" according to set size. It also seemed possible that preferred estimation strategies might not be as tightly connected to the negative poles of the adjectives perhaps because those negative terms are less frequent in adult language. In this case strategies may be less consistently employed in conjunction with the negative terms.

Findings from previous studies (Donaldson and Wales, 1970; Palermo, 1973; Weiner, 1974) led to the expectation that the understanding of the term "less" would lag behind that of "more", as would possibly the understanding of "shorter" behind that of "longer". However, there was no information in these studies as to whether a child might employ a length strategy on "less" or a number strategy on "shorter". The author expected that some of the children would employ their preferred strategy in response to negative terms if they did not have fully differentiated concepts of length and number. An additional expectation was that there would be individual differences in strategies which might be age related.

Of some interest also in this study was the extent to which children might employ quantitative terms in their answers different from those employed in the questions put to them. One might regard these new terms as intrusions. It seemed possible, for example, that if a child did not understand a term such as "less", he might use other terms, for example, "more", or refer to other dimensions, for example, the length of the row, "longer", "shorter". The intrusions could provide additional information about which dimensions are salient or the nature of the relationships between terms.

Method

Subjects

Thirty one subjects ranging in age from 3.3. to 5.2, were tested on two occasions. The median age of the subjects tested was 4.6. There were 16 females and 15 males. The children in this study attended either private or cooperative nursery schools and were of predominantly middle class background.

Stimuli

The stimuli were identical to those described in Experiment Two, with the exception that configuration one appeared twice. There were twelve configurations presented on the first occasion and twelve on the second. Small size arrays were used because findings from the previous experiments had shown that children use both length and number strategies with these set sizes, whereas, on the large number sets, almost all children use the length strategy.

Procedure

Testing sessions were conducted individually on two separate days. On one of the days the experimenter gave each child a length and number test employing the terms "longer" and "more"; on the other day, at least one week intervening, the experimenter gave the child a similar set of configurations but used "shorter" and "less" in the questions. Sixteen children received "longer" and "more" in the first session and "shorter" and "less" in the second session. The remaining fifteen children received "shorter" and "less" first, and "longer" and "more" second. Subjects were randomly assigned.

At the end of the second session the children were given the conservation task described in Experiment One. They were also given the following length test.

Length Test

The length test consisted of the presentation of 3 cards, each with two black lines. On one card the lines were equal; on the other two cards the lines differed in length. The differences matched the minimal differences in length in the large and small number sets. The experimenter asked the child: "Are these lines the same length or is one line longer"? The order of the clauses varied as did the order of presentation of the three cards.

Results

The children's protocols were scored for errors on each of the critical terms. They could make a total of 12 errors on each term, 48 overall.

To assess the effects of order of testing and the differences in numbers of errors on the critical terms, a two factor mixed design analysis was performed on all errors for all subjects (Tables A 4-1 & A 4-2, Appendix A). The between factor was order of testing, the within factor, the critical term. There were no significant effects (Source Table B 4-1, Appendix B). The means and total scores are presented in Table 4-1.

Since order of testing had no effect, the subjects' data could be pooled. As in Experiments One, Two, and, Three, the performance of those children who show a comprehension of the basic terminology

Table 4-1

Total number of errors and means according to critical term and order of testing.

Order of first testing	Critical Term: more	less	longer	shorter	
longer and more (N=16)	Σ = 80 $\bar{X}$ = 5.0	Σ = 91 $\bar{X}$ = 5.69	Σ = 75 $\bar{X}$ = 4.69	Σ = 83 $\bar{X}$ = 5.19	329 20.56
shorter and less (N=15)	Σ = 64 $\bar{X}$ = 4.27	Σ = 88 $\bar{X}$ = 5.87	Σ = 85 $\bar{X}$ = 5.67	Σ = 87 $\bar{X}$ = 5.8	324 21.6
	Σ = 144 $\bar{X}$ = 4.65	Σ = 179 $\bar{X}$ = 5.77	Σ = 160 $\bar{X}$ = 5.16	Σ = 170 $\bar{X}$ = 5.48	653 21.00

$\bar{X}$ for term = 5.27

was of particular interest. These were the children who could be classified as nonguessers according to their performance on the congruent configurations in response to questions with "longer" - "same length", "more" - "same number". A child had to answer 8 out of the 12 questions on the congruent configurations correctly to be classified as a nonguesser. According to this criterion, 16 children were nonguessers, 15 were guessers (Tables A 4-3, A 4-4, Appendix A).

The nonguessers had been classified on the basis of their performance on "more" and "longer" on the congruent configurations. Therefore, the differences in the errors for each critical term for these subjects were tested only on the incongruent configurations. There was a significant effect of the critical term, $F(3,45) = 3.748$, $p < .05$ (Source Table B 4-2, Appendix B). The means and total scores are presented in Table 4-2. A total of 6 errors on each critical term was possible, 24 errors overall. The Duncan's multiple range test, $p < .05$ showed that fewer errors were made on "more" than on "longer", or "shorter". "Less" did not differ from "more", "longer", nor "shorter".

The previous experiments had established that most of the nonguessers' errors in response to "more" and "longer" resulted from their consistent use of an opposite dimension strategy. Again in this experiment the author classified nonguessers' errors on "longer" and "more" as due to the use of either the opposite dimension strategy or to an unexplained factor. However, in this experiment, there was an interest in when, or if, children used an opposite dimensional,

Table 4-2

Total number of errors and means on incongruent configurations according to critical term, nonguessers only.

	more	less	longer	shorter
Total N=16	23	42	56	49
$\overline{X}$ N=16	1.44	2.73	3.53	3.01

Grand total = 170

Subject $\overline{X}$ = 10.63

Term $\overline{X}$ = 2.66

polar, or guessing strategy in response to the negative terms "less", and "shorter". Therefore, the errors on these terms were subjected to finer analysis than those on "longer" and "more". One pattern of response was identified which could indicate that a child was responding to "less" as if it meant "more", another as if it meant "shorter". Similarly, one pattern of strategies was identified which could indicate that a child was responding to "shorter" as if it meant "less", another as if it meant "longer". The coding used for assessing these patterns is given below in Table 4-3. In relation to Table 4-3 the reader should note that configuration 3 does not, alone, allow one to decide whether a child is using a polar or opposite dimensional strategy. In order to assess these strategies it was necessary to include an analysis of a child's errors on congruent configuration two.

Table 4-4 presents the strategies employed by the individual nonguessers on the incongruent configurations. A total of six responses was possible on each term. A summary table of these strategies is included in the appendix (Table B 4-3, Appendix B). The nonguessers are grouped in three ways according to their performance on "more" and "longer" on the incongruent stimuli. The high performers (Group I, N = 5) include those children who made fewer than 4 errors on the 12 questions on the incongruent configurations with the terms "longer" and "more". The number strategists (Group II, N = 7) consist of those children who made more than four errors on "longer" and "more" combined, but who demonstrated a very powerful number

Table 4-3

Key for coding "less" and "shorter" according to a polar (less = more) or an opposite dimensional (less = shorter) strategy.

Less

Config.	Polar less = more (number)	Opposite Dimension less = shorter (length)
3	shorter more numerous row (U)	shorter more numerous row (U)
4	more numerous row(P)	same number (D)
5	same number (P) (P correct)	shorter row (D)
2	longer more numerous row (P)	shorter, less numerous row (D) (D correct)

Shorter

Config.	Polar shorter = longer	Opposite Dimension shorter = less
3	longer less numerous row (U)	longer less numerous row (U)
4	same length (P) (P correct)	less numerous row (D)
5	longer row (P)	same length (D)
2	longer more numerous row (P)	shorter, less numerous row (D) (D correct)

U = uncertain
P = polar, (P correct) but fits polar pattern
D = opposite dimension (D correct) but fits OD pattern
O = other, for example numerals alone

Table 4-4

Types of responses of nonguessers on the incongruent configurations.

CRITICAL TERM

		more			less						longer			shorter					
Group I	S	C	L	O	C	L	O	P	U	2	C	N	O	C	N	O	P	U	2
	13	6	-	-	2	1	3	-	-	OP	6	-	-	6	-	-	-	-	-
high	14	6	-	-	1	1	1	2	1	-	6	-	-	2	3	-	-	1	-
performers	23	4	-	2	2	-	-	2	2	PP	5	1	-	6	-	-	-	-	-
(N=5)	27	5	-	1	6	-	-	-	-	-	4	1	1	6	-	-	-	-	-
	29	6	-	-	6	-	-	-	-	-	6	-	-	6	-	-	-	-	-
Group II																			
	2	4	-	2	6	-	-	-	-	-	-	6	-	-	4	-	-	2	-
number	5	4	-	2	3	1	1	-	1	-	-	6	-	1	3	-	-	2	-
strategists	6	5	-	1	5	-	1	-	-	-	-	6	-	-	4	-	-	2	-
(N=7)	10	6	-	-	3	-	2	-	1	-	-	6	-	-	3	1	-	2	-
	11	5	-	1	4	1	-	1	-	P	-	6	-	3	3	-	-	-	-
	12	3	1	2	4	1	1	-	-	-	1	4	1	2	2	-	-	2	-
	28	6	-	-	2	-	-	4	-	PP	-	6	-	-	5	-	-	1	DN
Group III																			
	1	4	-	2	4	1	1	-	-	O-	2	3	1	3	2	1	-	-	-
inconsistent	8	4	2	-	1	1	2	1	1	LP	3	3	-	4	1	1	-	-	O
strategists	20	1	2	3	3	1	-	1	1	PP	4	2	-	4	-	2	-	-	PP
(N=4)	31	4	1	1	5	1	-	-	-	-	3	3	-	2	3	1	-	-	-

C = correct
L = length
N = Number

O = other - unexplained
P = polar
U = uncertain

2 = congruent configuration 2

bias on "longer". The inconsistent strategists (Group III, N = 4) include those nonguessers who showed neither of the above patterns. Responses on "longer" and "more" are classified as either correct (C), opposite dimensional (L or N) or other (O) unexplained. The responses for "less", and "shorter" are presented as either correct (C); opposite dimensional (L or N); unexplained (O); polar (P); or uncertain (U). "Uncertain" resulted when the child's response could be classified as either polar or opposite dimensional. Errors made on configuration two in response to the negative terms are included in the table and classified according to the same code, Figure 4-1 illustrates the overall relationship between the different types of terms and strategies.

Noteworthy about the children's strategies are the following: A length strategy was very seldom employed in judging "more"; it was most often used by the inconsistent strategists. The high performers and number strategists, both nonguessers, gave evidence of having some consistent quantitative strategies, correct or incorrect. If some form of consistent quantitative strategy can be taken as evidence of emerging cognitive skills or operations, then it might be expected that these children would tend to be older than the children who do not demonstrate such consistency. The high performers and number strategists were compared on age to the inconsistent strategists nonguessers, and the guessers on the Median Test (Table A 4-5, Appendix A). The children with indications of consistent quantitative strategies were older than the other group of children ($X_1^2 = 4.32$, $p < .01$).

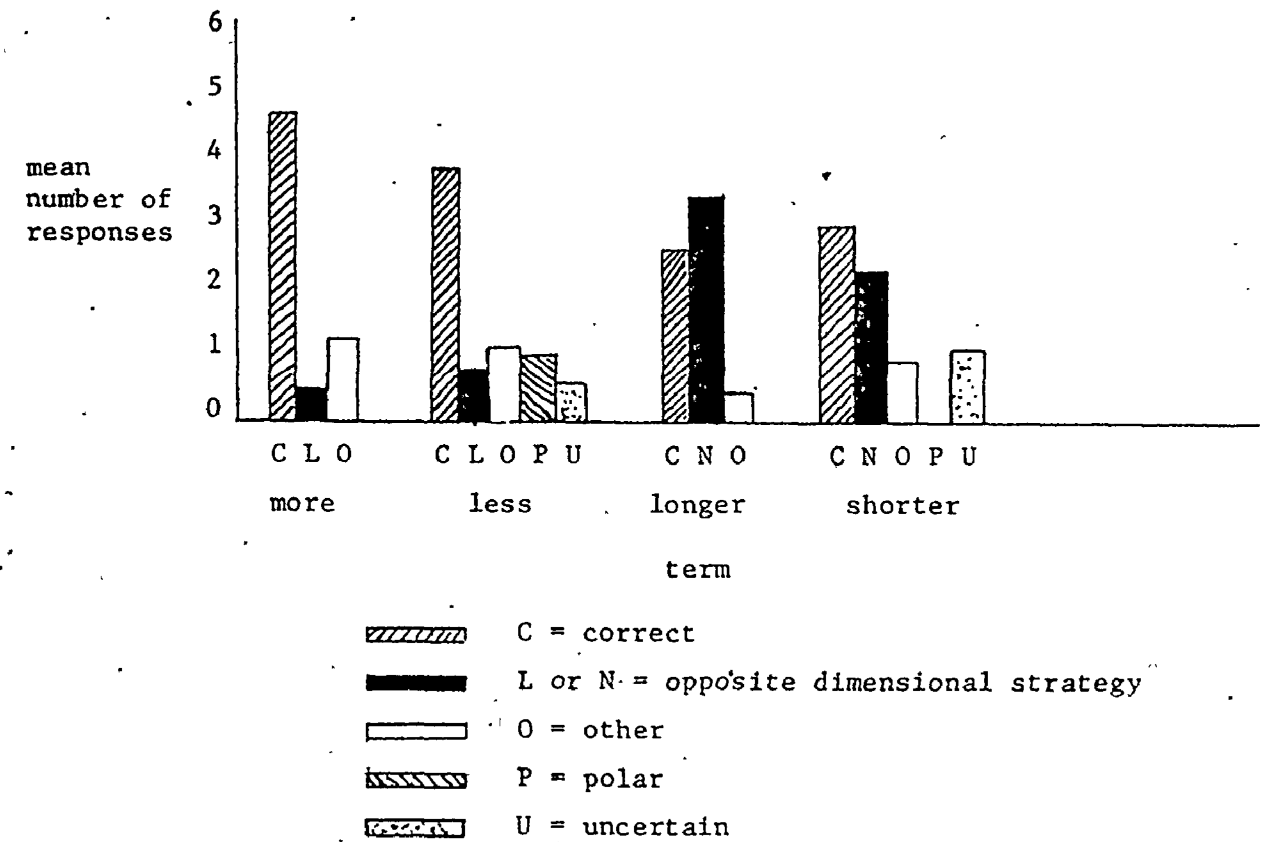


Figure 4-1

Relationship between different types of responses and terms, nonguessers only.

Having a numerical strategy available and being able to apply it consistently should be related to conservation status. Those children who showed some facility with the numerical strategy were in the nonguesser category. Nonguessers and guessers were compared for their performance on the conservation task (Table A 4-5, Appendix A). Being a conserver was associated with being a nonguesser ($X_1^2 = 11.51$, $p < 001$). Amongst the guessers (N = 15) there was only one conserver; amongst the nonguessers (N = 16) there were 10. It might be noted that among the inconsistent strategists (nonguessers, but no identifiable consistent strategy) there were no conservers.

The performance of the nonguessers and guessers was compared on the length test. The length test required three comparative judgments of continuous length, one of equality, two of inequality. This test was included primarily to provide information about the children's ability to judge length according to end points. However, it also provides some information about the validity of the nonguesser-guesser classification procedure. Children were classified as guessers because they failed to make the "same" - comparative distinction. If this failure is not simply related to discrete arrays, one might expect to find that these same children had problems with judgments of continuous length. The raw data can be found in Tables A 4-5 and 6, Appendix A. Table 4-5 presents a summary of the results of the length test.

The results indicate that both guessers and nonguessers were very accurate in making length judgments of inequality according to end

Table 4-5

Numbers of subjects correct and percentage of correct responses on the length test for nonguessers and guessers.

Numbers of subjects correct

condition	inequality		equality
	1	2	3
nonguessers (N = 16)			
Group I	5/5	5/5	4/5
Group II	7/7	7/7	5/7
Group III	4/4	4/4	4/4
	16/16	14/16	13/16
Guessers (N = 15)	15/15	13/15	5/15
Total =	31/31	27/31	18/31

Percentages

Nonguessers	100%	88%	81%	90%
Guessers	100%	87%	33%	69%
	100%	87.5%	57%	
	93.75%		57%	

points. Approximately 94% of their judgments were correct. This high level of performance was also found in group II children who used a number strategy to make length judgments. Judgments of equality appeared to be more difficult, approximately 57% of the judgments were correct. The discrepancy between equality and inequality was mainly effected by the guessers, who were very poor at making judgments of same length. Only 5 of the 15 guessers succeeded at this task.

In response to questions about "less" there is little evidence for an opposite dimensional strategy (less=shorter) and only a slight indication that a polar strategy (less=more) might be present in some children. Only two children, subjects 23, and subjects 28 responded consistently as if "less" meant "more". Subject 28 responded to "less" questions with the word "longer" and judged according to numerosity, which makes his strategy a debatable case. The errors of the other children who had 3 or fewer correct responses on "less" do not appear to fall into any consistent patterns. Figure 4-1 shows that, over all subjects, errors on "less" tend to be equally distributed amongst the error categories.

The apparent lack of a consistent incorrect strategy in the errors in response to questions about "less" contrasts with the pattern found in the errors in response to questions about "shorter", particularly amongst the number strategists. There is evidence for a consistent but incorrect numerical strategy. Figure 4-1 shows that, overall subjects, errors on "shorter" seemed to fall mainly into the opposite dimensional category. Amongst the number strategists several children who used a numerical strategy for "shorter", (shorter= less) failed to use a numerical strategy on "less" (subjects 5,10, and 28).

If a child is uncertain about the meaning of a term, this could appear in his productive language: he might search for other terms which seem related in meaning. The child's use of additional terms, terms other than those employed in the questions, had been recorded. These additional terms were labelled as intrusions. An intrusion might be: on the same dimension as the question asked, for example, "shorter", when "longer" was the critical term: on an opposite dimension, for example, the child might use the term "longer" in reply to a question about "more"; neither of these, but other terms such as "smaller", or simply numerals without a judgment. The child was assigned a score of 1 for a trial on which an intrusion occurred even though sometimes more than one intrusion occurred. All configurations were included, therefore, a child could obtain a score of 12 on each term, 48 over all terms (Tables A 4-7, A 4-8, Appendix A).

To assess the relationship between the total number of intrusions and the subjects' classification as guesser or nonguesser the data was subjected to the Mann Whitney U-test. There was a significant difference in the median number of intrusions of the guessers and nonguessers. $U = 73.6$, $p < .05$, two-tailed. The median number of intrusions for guessers was 15, for nonguessers, 10. Table 4-6 presents the total scores and sum of ranks within subjects, for each

Table 4-6

Total number of intrusions and sum of ranks for each term by subject classification.

	more	less	longer	shorter
Nonguesser				
Total	47	58	36	36
Sum of ranks	43.5	49.5	34	33
Guesser				
Total	55	82	60	61
Sum of ranks	35.5	47	32	35.5
Grand Total	102	140	96	97
Grand Sum of Ranks	79	95	66	68

term according to subject classification.

To determine whether there were any differences in the number of intrusions on the critical terms the Friedman test for matched Groups was used. The test yielded a $X_3^2 = 9.974$, $p < .025$. This indicated that at least two of the terms differed. Inspection of Table 4-6 shows that, overall, the two terms which differed most in number of intrusions were "less" and "longer". However, the difference in sum of ranks between "longer" and "shorter" is minimal. The pattern for the nonguessers and guessers appears to be similar with the exception of intrusions in response to "more". Relative to the other terms, nonguessers answered with an intrusion more often than guessers in response to questions with "more".

The nonguessers' intrusions were then categorized as either from the same dimension, opposite dimension, or other. The same dimension was further subdivided into same dimension, words repeated, for example "longer", "same length", together; mixed, for example more-less; and polar, for example "more" in response to "less". The distribution of these errors can be seen in Table 4-7.

These statistics are only descriptive. However, the following should be noted: Intrusions were least likely to occur in response to questions about "longer" and "shorter"; they were most likely to occur in response to questions about "less". When these intrusions on "less" questions occurred they were most likely to be from the same dimension and to consist of, or include, the polar term "more". On "less", "longer", and "shorter" the intrusions were most likely

Table 4-7

Percentage of responses with linguistic intrusions by critical word and type of intrusion, nonguessers.

	more	less	longer	shorter
Overall Percentage of Intrusions	27.42	37.66	25.806	26.00
Type of Intrusion				
(1) Same dimension				
(a) word repeated	8%	1.8%	14%	2.8%
(b) mixed	6%	20.69%	11%	8%
(c) polar	21%	24%	36%	27%
Total	35.42%	46.55%	61%	38.8%
(2) Opposite dimension	31%	15.5%	5.5%	19%
(3) Other	33%	37.9%	33%	51.67%

to come from the "same dimension" or "other" categories rather than from the "opposite" dimension category. This is interesting in the case of "longer" and "shorter", where the predominant strategy employed by some children was numerical, an opposite dimensional strategy. When intrusions occurred on "longer" and "shorter" they were least likely to be numerical. Only in response to questions about "more" did intrusions appear to be equally distributed amongst the three main intrusion categories.

Discussion

The absence of a finding of a difference in the overall level of performance on the critical terms "more", "less", "longer", and "shorter" is not in accord with findings from other studies (Donaldson and Balfour, 1968; Klatzky et al., 1973; Palermo, 1973, 1974). However, in these studies comparisons of performance were made between two terms only, for example "more" and "less", "longer" and "shorter". Perhaps when all four terms are compared, as in this experiment, the variability contributed by differences in individual children's strategies (or lack of strategies) is simply too great relative to the variability resulting from the terms. "Less" compared to "more" may give a relatively low level of performance. "Less" compared to "longer" and "shorter", where systematic errors abound for some children, presents another picture.

The nonguessers alone also did not perform better on "more" than on "less" on the incongruent configurations. They did perform better on "more" than on "longer" and "shorter". However, if one considers the number of nonguessers who made 4 or more correct responses (out of 6 possible) on the incongruent configuration, performance was better on "more", than on "less", "longer", or "shorter" (14 on "more"; 8 on "less"; 6 on "longer", 6 on "shorter").

There were important differences in the strategies which resulted in the erroneous responses to the critical terms. Children used the length strategy infrequently in judging "more", perhaps because of the small set size. Only amongst the inconsistent strategists, nonguessers, was there evidence for this strategy and none of these children conserved. The findings on "less" do not give any strong support to the Donaldson and Wales (1970), Palermo (1973) findings that "less" was responded to as if it meant "more". Only two children employed a consistent polar strategy, that is, responded to "less" as if it meant "more". The other children who made at least three errors on the incongruent stimuli had no consistent strategy for "less"; they appeared to be guessing. These findings are more in keeping with those of Townsend (1973). In all the studies comparing "more" and "less", different age groups, methods, and set size have been employed. It would seem that the parameters determining the nature of the child's response to "less" still need to be explored. Whether one finds polar strategies or guessing in response to "less" seems very dependent on the methodology employed (Kavanaugh, 1976).

One of the parameters which could be explored is set size. In judging large sets for "more" the young child frequently resorts to a length strategy (Baron, Lawson, & Siegel, 1975). If "less" means "more", the child should resort to the length strategy on the larger set, "less" being equal to the "longer" row. If he did not do this, and he also did not pick the more numerous array on a small set, it would seem safe to conclude that he was guessing and did not employ a polar strategy. If he did use a polar strategy on the small set, it would be interesting to see if he might switch this to the length dimension when the number set is beyond his estimation range.

One subgroup of the nonguessers employed a powerful numerical strategy in response to both length questions, although they were apparently capable of judging length according to end points. There was no evidence that any of these children responded as if "shorter" meant "longer"; rather, they responded as if "longer" meant "more", and as if "shorter" meant "less". It is possible that some children, when asked to judge "shorter" on continuous lines would use a polar strategy. In this case an opposite dimension numerical strategy would seem to be unlikely. (It might be noted, however, that one child, when asked to judge continuous length, moved his fingers down the lines, counting, in order to make his judgment). It would be interesting to compare responses to "shorter", and "longer", when used for judgments of different size number sets and of continuous length.

The results of this study bear on the controversy concerning the cognitive-noncognitive basis for the reported lag in young children's comprehension of "less". (Donaldson & Wales, 1970; Holland and Palermo, 1975; Klatzky, et al., 1973). Although the high performers and number strategists were able to use the number strategy and the number strategists used it to judge "longer" and "shorter", some of these children did not use an appropriate numerical strategy to judge "less". Over both groups, 6 of these 12 children had only chance level performance on "less". Apparently these children could make the large-small distinction, and they did have a numerical strategy available. They were not, therefore, lacking the basic cognitive components to make a judgment of "less". The two basic components were simply not attached to the word "less". Perhaps the most parsimonious explanation of the "less" phenomena is simply the low frequency of "less" in the language. With respect to individual differences it should be noted here that one child judged "less" correctly, "longer" and "shorter" according to a numerical strategy, but did not seem to understand the meaning of "more".

The nonguessers' types of intrusions suggest an asymmetry between length and number terms. Overall, the child was less likely to use an alternate term when making any length judgment than when making any numerical judgment. Further, "longer" evoked the fewest number terms and "more" the most length terms. This could indicate a stronger perceptual basis to the child's original understanding of "longer", and a confidence in his understanding of the term even when he may clearly misunderstand it, as in the case of those children who employed a numerical strategy. Number and its relational terms require abstraction and differentiation from related dimensions such as length. Dimensions such as length may be retained permanently as part of the meaning of number's relational term "more". Alternately, if "more" emerges as the first coordinate for proper quantitative judgments, then, the child who has "more" firmly in hand may begin to refer to other dimensions such as length in a contrastive way - this one is 'more' and 'longer'.

Finally, this experiment provides evidence for stable cognitive structures and for the increasing stability of these structures with age. The number strategists showed that strong number bias on the second time of testing a week to two weeks later. Children who had consistent strategies were also more able to resist irrelevant changes in an array; it was the high performers and the number strategists who conserved. And, the older the child the more apt he was to give evidence of a consistent cognitive structure.

Experiment V

Four main questions were explored in this experiment. First, is length a more easily discriminable dimension than number? Second, are there asymmetries in nonverbal discriminations on the above dimensions such that judgments on the positive poles "more" and "longer" are easier than judgments on negative poles "shorter" and "less"? Third, what is the relationship between a child's performance on a nonverbal discrimination task and the verbal task which has been used throughout the experiments in this thesis? Fourth, do cognitive-linguistic mediators primarily tag specific dimensions, for example, "number", or do they primarily tag poles, for example "big" or "small" in relation to which dimensional strategies are altered? In relation to the above questions certain age parameters were explored. The issues involved in each of the questions will now be considered.

Various studies have shown that when a young child does not use a numerical strategy to judge the numerosity of a linear array of discrete elements, he tends to use a length strategy based on end points (Gelman, 1969; 1972; Pufall and Shaw, 1972; Siegel, 1974a). Two factors influencing the use of this strategy are the age of the child and set size. Younger children are more apt to use a length strategy than older children; they are more apt to use the strategy on large sets than on small sets. The relation between the endpoint length strategy, age, and set size suggests that this strategy is easier than the numerical strategy. Beilin has argued that the factor which makes length "easier" is its primitive or "immediate" perceptual component. In this it differs from a higher order concept such as number, which is more abstract (Beilin, 1969).

During the development of the length concept, children initially attend to endpoints. However, in stage two, they may attend to aspects of internal segments (Piaget et al., 1960), including numerosity. Supporting evidence for the Piaget's et al. stage two finding comes from the previous experiments in this thesis which show that on small sets some young children will employ some form of numerical strategy when asked to judge length (several children did so on large sets). In Experiment Four, there was evidence that those children who did use a numerical strategy on discrete arrays to judge length were able to use a length strategy based on endpoints to judge continuous length. Thus, while the number strategists had the endpoint strategy available to them they used a numerical strategy to judge length. For these children then, length was not an easier dimension than number. While possibly the absence of a numerical strategy results in children relying on more primitive perceptual attributes, the presence of some form of numerical strategy may result in the primitive attribute being ignored. This problem seems to be conceptual rather than perceptual. Neither the length nor number concepts are, in Piaget's terms, operative.

In the experiments discussed in the preceeding paragraphs most investigators required the child to make verbal judgments of length and number. In the following experiment the author asked children 3-5 to make nonverbal relational discriminations of length and number on discrete arrays of three to four dots. The intention was to determine whether on discrete arrays under conditions which did not depend on the child's understanding words such as "more" and "longer", a child found it easier to make length or number discriminations. It seemed possible that on this type of task, children in the 3-5 age range might find length discriminations easier. However, certain age related effects were anticipated. The younger children, 3-4, might find length discriminations easier than number discriminations. However, the older children (4-5) could find number discriminations easier than length if an increased facility with number made them disregard endpoints.

The second main issue of interest in the experiment had to do with asymmetry in positive and negative dimensional terms. Section three of the introduction reviewed a number of studies which showed that young children's comprehension of "less" (a negative term) lagged behind their comprehension of "more" (a positive term) (Donaldson and Wales, 1970; Klatzky et al., 1973; Palermo, 1973). There is also some evidence that such an asymmetry may exist in the case of other terms such as "longer" - "shorter", "bigger" - "smaller", etc. (E. Clark, 1972; Siegel, 1977). Experiment Four in this thesis provided only weak evidence for a lag in children's comprehension of "less" relative to "more", and no evidence for such a lag in the comprehension of "shorter" relative to "longer". There was, however, a difference in the way children were wrong on the two terms "shorter" and "less". When errors were made on "less" many of the children appeared to be guessing. When errors were made on "shorter" many of the children appeared to be using the numerical strategy incorrectly but consistently.

Klatzky et al. (1973) have argued that the lag in the comprehension of the negative term "less" may reflect a basic cognitive difficulty. They suggested that the usual standard for a comparative judgment is "the small one". To have to evaluate "the small one" requires that the child use an unnatural standard, "the big one". They have presented evidence on a nonverbal task to support that claim for "less". Holland and Palermo have disagreed with Klatzky et al. They have presented evidence that children can learn "less" very rapidly, which argues against a basic cognitive problem. They prefer a frequency or bias interpretation of the "less" phenomena: Children hear the negative term less frequently (1973). However, if either the Klatzky et al. or Palermo arguments are true one would expect to find a lag in non-verbal relational discriminations of "shorter" as well as "less". Piaget's theory, however, would allow a different prediction for "shorter".

According to Piaget's theory very young children's spatial concepts are topologically based. If the child has to compare rows of discrete items, the shorter row, in relation to any longer row of the same number or fewer items, will possess the topological feature of "heaping" (Piaget, 1967). Consequently one might find "shorter" an easier relational discrimination than "longer".

To assess the possible asymmetries on the relational concepts of "more", "longer", "shorter", and "less", the author asked children to make two of the possible four relational discriminations on the nonverbal task. This enabled a refinement of the first question on the relative ease of length and number discriminations, for in light of the arguments on asymmetry, relational discriminations of the positive or negative poles could be different. Different outcomes seemed possible: First, the children might perform in a similar fashion on positive and negative terms on a dimension when the task was nonverbal. This would indicate that the dimensions were perceived in both their positive and negative aspects but that possibly there could be some lag in the attachment of perceptions and strategies to words. Second, the children might make more errors on both "shorter" and "less" relative to their respective positive terms. Third, the children might make fewer errors on "shorter" than on "longer". In relation to age then: Younger children might find any length discrimination easier than any number discrimination, but there are alternate predictions for whether they might find "shorter" easier or more difficult than "longer". Older children might find any number discrimination easier than any length discrimination on discrete arrays. However, there might be no difference in length and number discriminations at this age if numerical strategies do not interfere with endpoint strategies. Possibly "less" might lag behind "more", "longer", and "shorter", if it is a more difficult concept, without "shorter's" advantage on discrete arrays.

The third major concern in this experiment was to explore a number of questions on the relationship between language and thought. In order to assess this relationship the author required that each child do the standard verbal task used in the previous experiments.

The most general question concerned how a child's success on the verbal task related to his success on the nonverbal discrimination task. Some of the many issues in this area were discussed extensively in section three of the introduction, others in section four. The conclusion there was that the evidence supported a "loose" dependency hypothesis. A high degree of success on a quantitative task such as conservation usually signals a high degree of facility with relational terms (Sinclair de Zwart, 1969; Harasym et al., 1971). However, some children succeed on quantitative tasks who fail to comprehend relational terms correctly, and some fail who do seem to understand the terms. It seemed possible that the degree of dependency between the cognitive and verbal domains might vary for the length and number concepts. The argument was made that the length concept has a basic perceptual component which makes discrimination easier on that dimension than on number. If that is so, the dependency between success on verbal and nonverbal tasks requiring length comparisons might be less than on tasks requiring number comparisons. If the more complex concept emerges later, the closer dependency of number strategies and language could reflect either a parallel increase in linguistic and cognitive competence with age, or the reliance of higher order concepts on linguistic mediators.

One of the specific roles proposed for language in thought is the role of mediator. Kendler and Kendler (1969) argued that when a child had a label for a dimension, he found it easier to make two successive discriminations on the same dimension (the reversal shift) than on different dimensions (the nonreversal shift). This theory was discussed in section five. Two factors which relate to performance in the shift paradigm are the age of the child and dimensional preferences.

Prior to the age of six, children seem to find nonreversal shifts relatively easier than reversal shifts; so do white rats. After the age of six, children seem to find reversal shifts relatively easier than nonreversal shifts. This is like the performance of the human adult. The explanation offered for this phenomena is that prior to the age of six, linguistic mediators are not actively available to the child. There are individual differences, however; some children perform like adults earlier than other children. With respect to preference, children find discriminations easier on preferred dimensions (Suchman and Trabasso, 1966).

In this experiment the author wished to determine whether young children found it easier to make a reversal shift (e.g., "more" to "less", or "longer" to "shorter") than a nonreversal shift (e.g., "more" to "longer"), and whether any age or preference factors were operating in the learning of the discriminations. The standard verbal task allowed an estimate of the availability of specific dimensional labels, and of strategy bias, or dimensional preference.

The fourth question was: do linguistic mediators primarily tag specific dimensions, for example, number; or do they primarily tag poles, for example, "big" or "small" in relation to which dimensional strategies are altered. This question related both to the issue of asymmetries and the nature of the linguistic mediator. Baron (1974) has argued that prior to the differentiation of the length and number concepts, the child may possess a concept of "bigness" in relation to which dimensional strategies are altered. Siegel (1977a) has evidence that in both linguistic comprehension and production the child's adeptness with "big" precedes his adeptness with "small". According to Klatzky et al. (1973) judgments at the positive pole are easier than judgments at the negative pole because "the small one" is the natural standard. Given the above arguments and evidence, an outcome which showed that children this age were better at a nonreversal shift then a reversal shift would not require a "white rat" type interpretation, i.e., the children lack active mediators. Rather one might argue that the mediators were possibly of the more general variety such as "big" or "small", and that once a standard for judgment was set, it was easier to stay with that standard than to switch. Also, evidence that a shift from "less" to "more" or "shorter" to "longer" was easier than a shift from "more" to "less" or "longer" to "shorter" would provide some support for Klatzky et al's, (1973) position.

In summary: In order to explore the issues outlined above, the experimenter gave the children in this experiment a nonverbal discrimination task and the standard verbal task. These tasks were given on two separate occasions. In the nonverbal task the children were given the opportunity to learn two of the possible four relational discriminations, "more", "less", "longer", and "shorter". In the verbal task the children had to judge all four terms.

Method

Sixty subjects were tested who ranged in age from 3.1 to 5.3 with a median age of 4.4. There were 29 males and 31 females. The children in the study were attending private or cooperative preschool nurseries and were predominantly of middle-class background. The author tested each child individually on two separate occasions with two tasks: a verbal task and a nonverbal relational discrimination task.

Thirty-one children received the verbal task first, twenty-nine children the discrimination task.

Verbal Task

The verbal task was comparable to that employed in Experiment Four, i.e., the experimenter asked the child to make verbal judgments of "longer", "more", "shorter", "less" on the small set size arrays. The same small set arrays with the same configurations were used as in Experiment Four. There were 12 stimulus cards in all. On six of these the experimenter asked the child questions using "longer" and "more", on the other six, questions about "shorter" and "less". The order was varied: one half the children received "longer" and "more" first; the other half received them second. This was on the same testing day.

Discrimination Task

The discrimination task was nonverbal and required that the child learn two successive discriminations.

The stimuli were comparable to those employed in the verbal task, but, since the discrimination was relational, on the number tasks only configurations 2, 3, and 4, were used; on the length task only configurations 2, 3, and 5. Thirty-six cards made up a set so that each configuration appeared 12 times, no two alike in succession. To balance for number, configuration 5 appeared in both 3 and 4 dot forms. To balance for length, configuration 4 appeared in different lengths corresponding to the smallest possible density of the four dots, i.e., 12 mm. spacing and the largest spread of the three dots. Four sets were employed. The "longer" and "shorter" sets were identical except for the random order. The same was true of the "more" - "less" sets.

The experimenter presented the stimulus cards on a "clown designed to provide feedback, (Figure 5-1). Each card was placed in the clown's hands which were at the child's eye level. Just above the card was a red light designed as one of the clown's buttons. The light was attached to a foot switch and could be flashed when the child made a correct response.

The experimenter told the child that the clown was a happy clown, who showed that he was very happy by lighting up the button. It was his "happy" button. The child could make the clown happy by doing something - by pointing to the good thing (row) on a card.

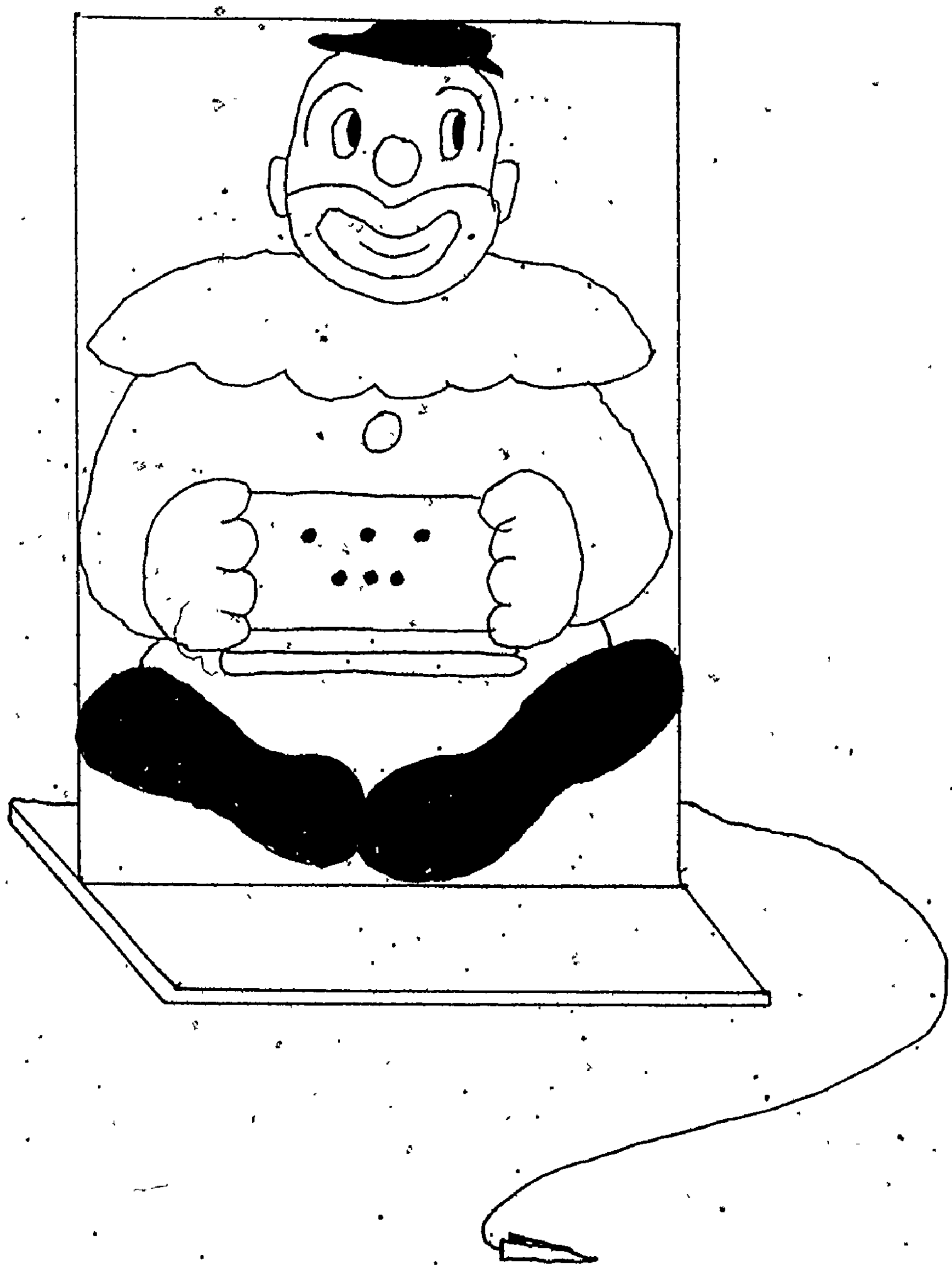


Figure 5-1

Clown used for nonverbal discrimination task.

Each child had a practice run with 5" x 8" cards on which two different animal stickers had been placed in various positions above and below a black line. The experimenter told the child to point to one of the animal stickers, one of them was good. The clown's button would light when the child pointed to the good one. When the child pointed to the animal which the experimenter had arbitrarily designated as correct the clown's light went on and the experimenter said "You are right. That is the good one." If the child was wrong the experimenter said, "You are wrong. That is not the good one. That is the good one." The experimenter then pointed to the correct animal. The feedback procedure was comparable to that used in Experiment Two. It has been demonstrated to be an effective procedure in children's discrimination learning (Bitgood and Jenkins, 1976). Each child was run to a criterion of 9 out of 10 correct responses. This procedure was designed to familiarize the child with the task requirements and also to break any position hypotheses. Very few of the children required more than 10 trials to make this discrimination (N = 2).

Following the practice trials, the experimenter showed the child one of the standard stimulus configurations and told him that now he had to point to the good row. The procedure followed was identical to that for the animal stickers. The child learned either the "longer", "shorter", "more", "less" discrimination first. When a criterion of 9 out of 10 correct in succession was met, or 36 without success, the child was shifted, without notice, to another discrimination, one of the three which he had not had. On the first discrimination 15 children had one of the four possible discriminations. On the second discrimination a group of 15 was subdivided into three groups of five. The experimenter assigned the children randomly to one of the three remaining discriminations. To illustrate: five of the fifteen children who received "longer" on the first discrimination, received "shorter" on the second, five received "more" and five "less". An attempt was made to assign a comparable age range to each group.

The experimenter always introduced a new set of cards for the second discrimination. The same procedures were followed until the child reached nine out of ten correct or 36 without success.

Results

In many of the data analyses only the scores from the first discrimination task were included. The reason for not pooling the child's first and second discrimination scores is that this would have resulted in a mixture of both matched and independent subjects. Such a mixture makes statistical analysis impossible. The second discrimination scores were analyzed independently when it seemed they might validate or clarify results obtained from the first discrimination, or when they were directly of interest, as in the case of the reversal or nonreversal shift.

Prior to the main analyses of the children's performance on the nonverbal discrimination task it was necessary to determine whether there were any order effects related to a child's having had the discrimination or verbal task in the first or second session. The Kruskall-Wallis H test was used to compare the number of trials to criterion on the first discrimination for the 31 children who had the nonverbal discrimination task in the first session, and the 29 children who had it in the second.(Table A5-1, Appendix A, order). The H (X^2_1) = 1.25, $p > .20$ was not significant. Table 5-1 shows the median number of trials to criterion and the mean ranks ($\bar{R}$). A total score of 36 was possible.

Since there were no order effects it was possible to examine the performance of all the children together on the first discrimination (N = 60). To determine whether nonverbal discriminations of length were easier than nonverbal discriminations of number, the number of trials to criterion on the first discrimination for both length terms was compared to the number of trials to criterion on both number terms. The Kruskall Wallis H (X^2_1)= 7.143, $p < .01$ was significant. Length discrimination took fewer trials than number discrimination. Table 5-2 shows that the median trials to criterion on length was 12.5, mean rank 25.02. On number, the median trials to criterion was 36, mean rank 35.98.

The above analysis established that, overall, children found it easier to make relational discriminations on the length dimension than on number. Of some interest in this experiment, however, was whether there were any asymmetries in the four different types of discriminations. Therefore a comparison was made of the number of trials to criterion on the first discrimination for the four types of discriminations, "longer", "shorter", "more", and "less". There were 15 subjects who had each type of discrimination. The Kruskall

Table 5-1

Median number of trials to criterion and mean rank on the first discrimination scores by order of testing session.

	Nonverbal Task	
	First Session (N = 29)	Second Session (N = 31)
median trials	21	14
mean rank	29.32	31.76

Table 5-2

Median number of trials to criterion and mean rank by dimension and discrimination task. (N = 60)

		dimension	
		length (N=30)	number (N=30)
discrimination	First (N=60)		
	median	12.5	36
	$\bar{R}$	25.02	25.98
	Second (N=60)		
	median	26.5	36
	$\bar{R}$	26.73	34.27

Wallis H test yielded an H (X_3^2) = 10.756, $p < .01$. There was, therefore, a significant difference in performance on the four types of relational discrimination.

Table 5-3 shows the median trials to criterion and mean ranks on the first discrimination. With overall significance established it was possible to perform the Protected Rank Sum test to determine which pairs differed significantly. A $z = 1.96$, $p < .05$ was required for significance. The children who had to make the "shorter" discrimination took significantly fewer trials than the children who had to make the "longer", "less", and "more" discriminations, $z = 3.46$, $p < .001$.

The relationship between age and number of trials to criterion on the two dimensions and the four possible types of relational discriminations was of some interest. Therefore, the children were divided into two groups at the median age (4.4). The younger group (children 4.4 and under) had 36 children; the older group had 24. The Kruskall Wallis H test was used to compare the trials to criterion scores of the younger and older groups of children on the length and number dimensions. The H (X_3^2) = 10.430, $p < .001$ was significant.

Table 5-4 presents the median trials to criterion and the mean ranks for the younger and older children on the length and number discriminations. Figure 5-2(a) illustrates the relationships between the two age groups on the two dimensions. A series of Protected Rank Sum tests comparing the different groups gave the following results: The older children took significantly fewer trials to reach criterion

Table 5-3

Median number of trials to criterion and mean rank by type of relational discrimination and order of discrimination task.

Order of discrimination		Type of relational discrimination: more (N=15)	less (N=15)	longer (N=15)	shorter (N=15)
First (N=60)	median	36	36	25	10
	$\bar{R}$	36.93	35.03	31.97	18.06
	$\bar{R}$	35.98		25.02	
Second (N=60)	median	36	36	36	24
	$\bar{R}$	28.66	35.53	28.1	25.37
	$\bar{R}$	32.10		26.74	

Table 5-4

Median number of trials to criterion and mean rank on first discrimination by type of discrimination and age.

Age	Type of Discrimination				
	number		length		
	more	less	longer	shorter	
younger (N=36)					
median	36	36	19	11	26
$\overline{R}$	47.15	39.94	28.94	21.89	33
median	36		13		
$\overline{X}$ R	43.94		25.44		
older (N=24)					
median	10	13	28	10	10
$\overline{R}$	16.5	29.43	36.5	12.33	24
median	11		11		
$\overline{X}$ R	24.04		24.42		

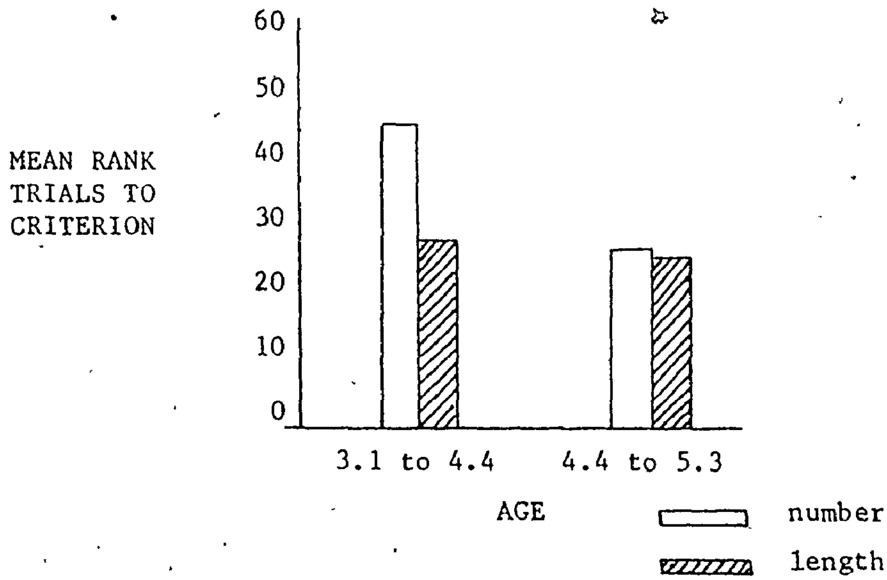


Figure 5-2(a) Mean ranks for trials to criterion measure for younger and older age groups: by dimension (1st discrimination)

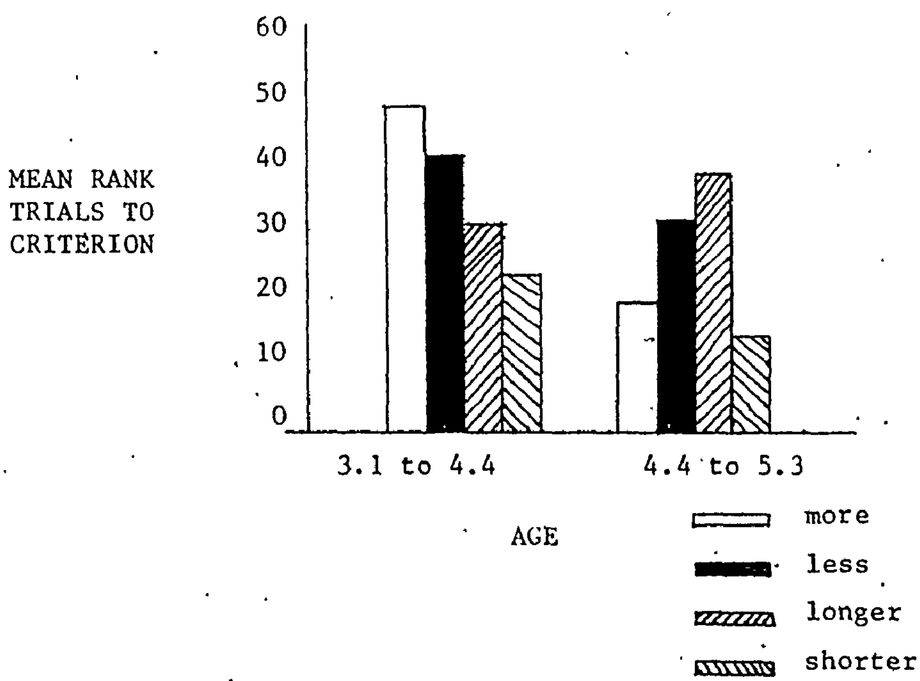


Figure 5-2(b) Mean ranks for trials to criterion measure for younger and older age groups by four types of relational discrimination.

than the younger children. The younger children required significantly fewer trials to criterion on length discriminations than on number. There was no significant difference in performance on the length and number discriminations for the older children. The younger children required significantly more trials to criterion than the older children on number discriminations. There was no difference between the younger and older group on length discriminations. Age changes appear to relate to the improved performance of older children in making nonverbal discriminations of number.

To assess age relationships on the four different types of nonverbal discriminations, the Kruskall Wallis H-test was used to compare the performance of the same younger and older age groups on "more", "less", "longer", and "shorter". The H $(X_3^2) = 24.15$ was significant at $p < .01$. Table 5-4 shows the median number of trials to criterion and the mean rank for each type of discrimination for each age group. Figure 5-3(b) illustrates the relationships.

A series of protected rank sum tests showed that in the younger group children required significantly more trials to criterion on "more" than on "longer" and "shorter". "Less" did not require significantly more trials than "more" nor "longer", but did require significantly more than "shorter". "Longer" and "shorter" did not differ from each other. In other words, in this age group the two length discriminations did not differ from each other in difficulty, neither did the two number discriminations; however, the positive number term "more" differed significantly from both length terms.

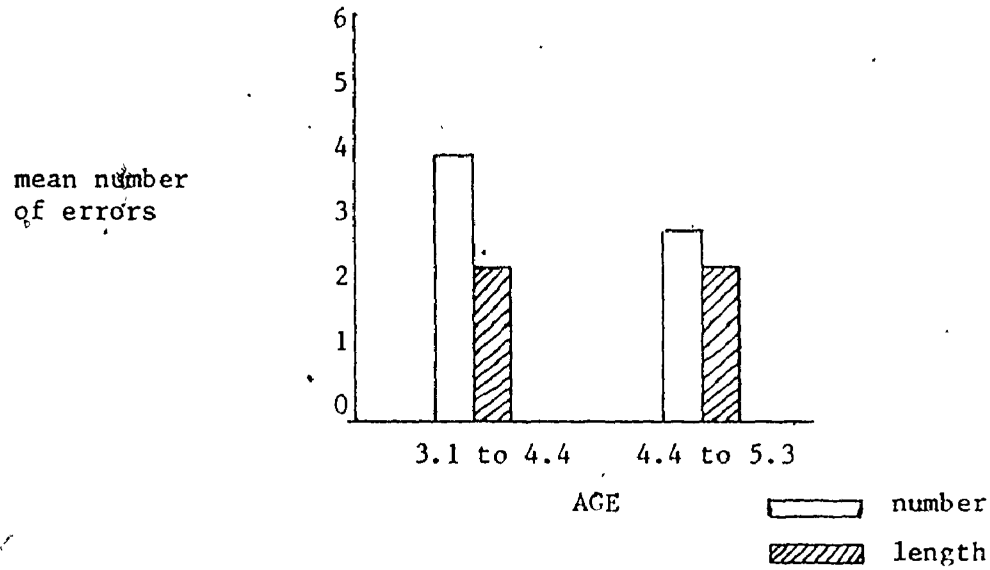


Figure 5-3(a) Mean number of errors on dimensional terms for younger and older age groups

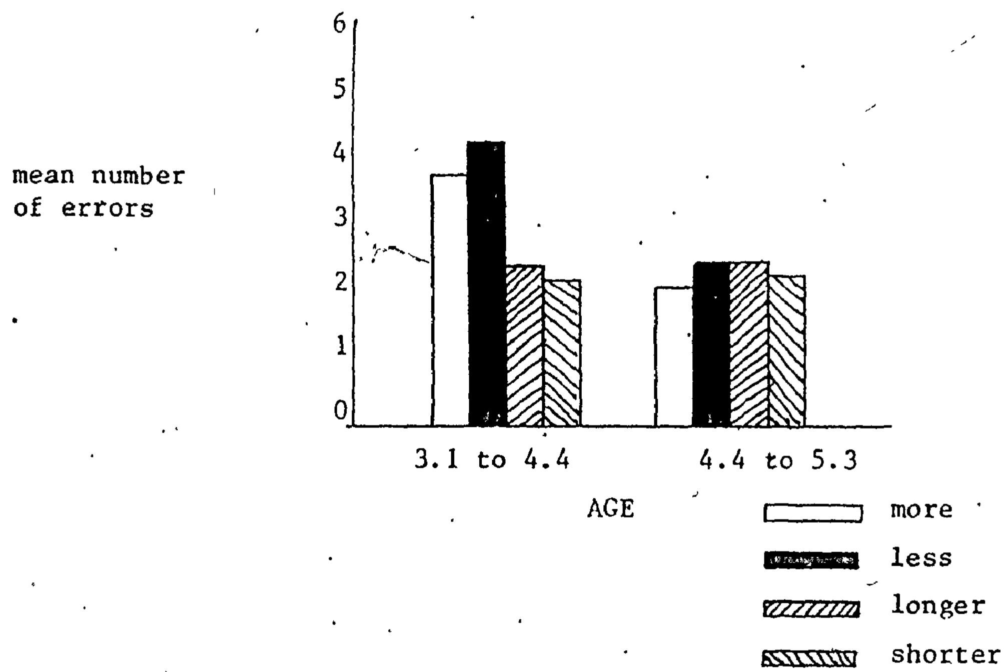


Figure 5-3(b) Mean number of errors on comparative terms for younger and older age groups.

The negative number term only differed significantly from the negative length term. In the older age group "longer" was more difficult than "more" or "shorter" but not than "less". The only term on which there was a significant change with age was "more". The older children required significantly fewer trials to criterion on "more" than the younger children; however, the number of trials to criterion decreased with age for every term except "longer". There appeared to be an increase for this term, although this increase was not significant.

In order to assess the general effects of the child's first discrimination task on his second one, the number of trials to criterion required by each child on the first and second discrimination were compared (Table A 5-1, Appendix A). The Wilcoxon test for the difference between two matched locations yielded a $z = 3.226$, $p < .001$. Table 5-2 shows the median number of trials to criterion and mean ranks for length and number on the second discriminations. The Wilcoxon indicated that the second discrimination was more difficult to learn than the first.

The overall performance of the children on length and number dimensions on the second discrimination was compared through the Kruskall Wallis H test, the H $(X^2_1) = 2.7910$ was not significant. The four terms were also compared. This H $(X^2_3) = 3.133$ was also not significant. While the statistical analyses on the second discrimination were not significant Table 5-3 suggests that again relative to number, a length discrimination was easier.

To determine whether, having learned the first discrimination, it was easier to learn a second discrimination on the same (reversal shift) or a different (nonreversal shift) dimension the performance of those subjects (N=39) who had succeeded on the first discrimination was examined. Those subjects who received the reversal condition (N=13) were compared to those who received the nonreversal condition (N=26) (same or different pole) on a Mann Whitney U-test, $z = 2.065$, $p < .05$. The median trials to criterion of the reversal shift group (median = 13) was significantly less than the median trials to criterion of the nonreversal shift groups (median = 27). The Rank Sum test was then used on the reversal shift data to determine whether there was any difference in the number of trials the children required to make a shift from a negative to a positive pole (N=7), for example, "less" to "more", than from a positive to a negative pole (N=6) for example, "more" to "less". The $z = .71$, was not significant.

Having established that the reversal shift discrimination was easier than the nonreversal shift discrimination, the Mann Whitney U-test was used to make comparisons between the reversal group and the two subgroups which made up the nonreversal shift group. The comparison of the reversal shift group (median = 13, N=13) and the nonreversal shift, new dimension, new pole group (median = 20.5, N=10) did not yield a significant z., $z = 1.80$. The comparison of the differences in the two nonreversal groups, new dimension, new pole (median = 20.5, N=10) versus new dimension, same pole (median = 36, N=16) also did not yield a significant z. The comparison of the reversal shift group (median = 13) and the nonreversal group, new dimension, same pole, (median = 36) gave a significant z, z = 2.149. In other words, from easiest to greatest difficulty in the shift: reversal shift, same dimension, opposite pole; nonreversal shift, new dimension, different pole; nonreversal shift, new dimension, same pole.

To determine whether there was any tendency for younger children to find the nonreversal shift easier than the reversal shift, and older children the reversal shift easier than nonreversal shift, the children in the reversal and nonreversal groups were divided into the younger and older age groups at the median (4.4). The Kruskall Wallis H (X_3^2)= 5.51 was not significant. Both the younger and older children found it easier to make the reversal shift than the nonreversal shift.

Of some interest in this study was the possible relationship between verbal and nonverbal performance. As stated previously there was no evidence for order effects on the discrimination task,. that is, the children's performance on the discrimination task did not appear to benefit or suffer from their having the verbal task in a prior or a subsequent session. The comparable question for language was whether children's performance on the verbal task improved or deteriorated as a result of having had the discrimination task. A t-test for independent samples compared the error scores of the before and after group on the word which was related to the first nonverbal discrimination task (Table A 5-2, Appendix A). The t = 1.48 was not significant. A t-test was also performed on the combined error data for both words related to both discrimination tasks for the before and after groups. The t = .54 was not significant. In addition a Wilcoxon test for matched samples was used to compare the children's performance on words on which they had practice with those on which they did not have practice. It too was not significant. There was, therefore, no evidence for any order effects of session or within subjects effects of practice. Table 5-5 presents the means for both the t-test comparisons. The total number of possible errors for one word was 6, for two words, 12.

To allow for a group comparison of the children's performance on the same comparative terms which they had on the first nonverbal discrimination task, and to evaluate the effects of age, a treatment by levels analysis was performed on the number of errors a child made on the critical term (Table A 5-2, Appendix A). Level was the age of the child in relation to the median (4.4). There were two levels; younger, 3.2 to 4.4 (N=36); older, 4.4 to 5.3 (N=24). These were the same age groups as in the nonverbal relational analyses. Treatments was the number of errors the child made on the comparative term which was the same as the first nonverbal discrimination task which he had. For example, if a child's first nonverbal discrimination was "longer", the score used in the analysis of the verbal task was the number of errors (6 were possible) he made in response to the question "longer". There were four treatment groups then: "longer", "more", "shorter", and "less", with 15 children in each group.

The treatment x level analysis yielded a significant effect of

Table 5-5

Mean number of errors on words related to nonverbal discrimination task.

	Verbal Task	
	First session (N=31)	Second session (N=29)
mean number of errors on term related to first nonverbal discrimination	2.6129	2.690
mean number of errors on both terms related to both nonverbal discriminations	5.586	5.323

age, $F(1,52) = 6.612$, $p < .005$, and an $F(3,52) = 2.78$ for question. The F for question for 3,50 df is 2.79, $p < .05$, and for 3,55 df is 2.78, $p < .05$. Since the df for the $F(3,52)$ was between 3,50 and 3,55 it was taken as significant at $p < .05$. (Source Table B 5-1, Appendix B). The means for the age groups and the terms are presented in Table 5-6 and illustrated in Figure 5-3(a) and (b). A total of 6 errors was possible on each term. The younger children made more errors than the older children. The Duncan's Multiple Range test, $p < .05$, showed that the number of errors the children made in response to questions with "more" and "less" was significantly greater than the number of errors they made in response to questions with "longer" and "shorter". The two number terms did not differ significantly from each other, neither did the two length terms. The reduction in errors with age was attributable to a decrease in errors in response to number questions. There was no change in length questions.

A comparison of the performance on the verbal and nonverbal task via Tables 5-3, 5-4, 5-5,5-6, and Figures 5-2(a) and (b), 5-3(a) and (b) indicates that on both the nonverbal and verbal tasks length judgments or discriminations result in fewer errors than number judgments or number discriminations. Responses to number questions, and nonverbal relational discriminations of number, improved with age. There were several differences in group performance on the verbal and nonverbal task. On the verbal task "shorter" did not result in fewer errors than "longer", as it did on the nonverbal task.

Table 5-6

Total number of errors and means on critical terms according to age.

age	critical term: number — more (N=15)	number — less (N=15)	length — longer (N=15)	length — shorter (N=15)	
younger (N=36)					
total	36	33	20	19	108
N =	10	8	9	9	36
mean	3.60	4.125	2.2	2.11	3.01
	mean 3.86				
older (N=24)					
total	10	15	13	12	50
N =	5	7	6	6	24
mean	2.0	3.20	2.2	2.07	2.08
	2.6				
Total N=15	46	48	33	31	
mean	3.06	3.20	2.2	2.07	
Total N=30	94		64		
mean	3.13		2.13		

Grand Total 158
N = 60
Mean = 2.63

Also, on the verbal task the older children did not make more errors in response to "longer" than to "more" as they did on the nonverbal task.

The above analyses of group performance did not provide any direct measure of the nature of association between a child's performance on the verbal and the nonverbal tasks. Several different analyses were therefore undertaken to attempt to illuminate that relationship: first, an analysis of the relationship between guesser nonguesser classification on the verbal task and performance on the nonverbal task; second, an examination of the association between success on the two nonverbal relational discriminations and the comparable verbal terms; third, an analysis of the relationship between a child's bias as indicated on the verbal task and his performance on the nonverbal task.

As in the previous experiments the children were classified as guessers or nonguessers according to their performance on the six questions asked on the congruent stimuli in response to questions with "longer" and "more" (Table A 5-3, Appendix A). The criterion was set at 4 out of 6 correct responses. The number of guessers according to this criterion was 23, the number of nonguessers, 37. A $X^2_1 = 3.86$, $p < .05$ indicated that there was an association between guesser, nonguesser classification and performance on the first discrimination. The nonguessers were more apt to pass the first discrimination task (28 out of 37); the guessers were as likely to fail as to pass (12 failed, 11 passed).

To assess the relationship between age and guesser-nonguesser category a median split test for age and category was performed. This yielded a $X_1^2 = 25.9666$, $p < .001$. Guessers were very apt to fall below the median age (4.4), whereas nonguessers appeared to be distributed across the entire age range.

The above analyses of guesser-nonguesser provided some information about the relationship between the verbal and nonverbal task. Namely, a child who could make the "same" comparative distinction on a verbal task would probably make a relational discrimination of number or length on a nonverbal task. The child who could not make this distinction on the verbal task was as apt to pass as fail on a nonverbal task. However, a child could be classified a guesser or nonguesser and fail to understand one or both of the linguistic terms which related to his own discrimination tasks. Consequently, the following analyses were carried out to determine whether success on a particular term or terms related to success on that discrimination task.

To determine whether the degree of success on the language task was associated with degree of success on the nonverbal tasks, a complex Chi-square analysis was carried out. The variables were success on the discrimination tasks and the related verbal terms. There were three categories of success on the nonverbal discrimination task: pass both discriminations, only one, or neither. Success on the related comparative term was defined by achieving a score of four out of a possible six correct answers in response to questions on both the congruent and incongruent configurations. There were three categories here also: pass both terms, one term only, neither. The $X_4^2 = 11.195$, $p < .025$ was significant. The contingency coefficient was equal to .39. This result indicated that there was a relationship between the two variables such that degree of success on one task related to the degree of success on the other.

To assess any differences in the nature of the association between linguistic performance and pass-fail on the discrimination task for the length and number dimensions the author carried out separate chi-square analyses for the length and number groups. Thirty children had a number discrimination for their first discrimination. The criterion for success on the language task was again four out of six correct.

A $X_1^2 = 4.999$ for the number group indicated that there was a significant association between success or failure on the number discrimination tasks and success or failure with the comparable term on the verbal task. Fourteen of the thirty children passed the discrimination task, nine passed the language task. The children who passed the language task were also likely to pass the related discrimination (7 out of 9); the children who failed on the language task were also apt to fail on the related discrimination (14 out of 21). Children who passed the discrimination were equally likely to pass or fail the language task (7 passed, 7 failed). Children who failed the related discrimination task were very likely to fail the related language task (14 out of 16). The $X_1^2 = .86$ for the length group was not significant. Twenty-five out of thirty children in this group passed the discrimination, 19 passed the language. Generally, then, performance was better on length, than on number, but there was no language discrimination association for length whereas there was one for number.

The final measure used to assess the relationship between verbal and nonverbal performance was the measure of the child's response bias provided by the language task. For the purposes of this experiment the child's bias was assigned on the basis of his performance on the incongruent configurations in response to the questions "longer" and "more". The child's responses on the incongruent configurations can be classified as either correct, unexplained, or due to the use of a length or number strategy (Table A 5-3, Appendix A). The scoring procedure was outlined in Lawson, Baron and Siegel (1974), Baron, Lawson, and Siegel (1975), and in Experiment Three. A child who made two or three correct judgments of "longer" and at least two errors in response to "more", which could be attributed to the use of a length strategy, was judged to have a length bias. The reverse procedure was used to categorize a child as possessing a number bias. According to these criteria, there were 9 children who had a number bias; 14 who had a length bias, and 37 children who had no apparent bias. The no bias children were not included in the following analyses.

A Rank Sum test was performed on the trials to criterion data from the first discrimination in order to compare the performance of the children who had to learn a discrimination against their bias

(N=11) with the performance of the children who learned a discrimination in the same direction as their bias (12). The z = 1.416 was not significant. The Rank Sum test was also performed on the trials to criterion data on the second discrimination. The z = .66 was also not significant. Table 5-7 presents the medians and mean ranks for both the first and second discriminations. While neither z was significant, the results on both discriminations suggest that a bias toward a dimension observed in a verbal response will have some predictive value for performance on a nonverbal task. Children will either find it somewhat easier to learn a nonverbal discrimination in the direction of their bias, or, they will find it more difficult to learn a discrimination against it.

It seemed possible that the effects of a child's learning a discrimination against, or in keeping with his bias, could be different if the bias was toward length or number. Throughout the preceeding analyses, results had suggested that length, as defined by end points, was an easier discrimination. It might not be as difficult to learn a discrimination on this dimension even if it was against one's bias. On the other hand, number appeared throughout to be a more difficult dimension. Possibly then, it might be relatively more difficult to learn a discrimination on the number dimension particularly if one had a length bias. Consequently, the 23 children were divided into four groups: number bias, (N=9) with and against; length bias (N=14) with and against. Two separate Kruskall-Wallis analyses were done on the trials to criterion on the first and second discriminations.

Table 5-7

Median number of trials to criterion and mean rank for discriminations learned with and against bias (N=23).

discriminations	with bias	against bias
first		
median	21 (N=12)	36 (N=11)
mean R	10.08	14.09
second		
median	27 (N=9)	36 (N=24)
mean R	10.83	12.75

The H (X_3^2) = 6.30, $p < .10$ on the first discrimination was not significant. The H (X_3^2) = 5.06, $p < .20$ on the second discrimination was also not significant. Table 5-8 presents the medians and mean ranks for both the first and second discriminations according to type of bias and type of discrimination. Although the results were not significant they are in the same direction on both discriminations. Children with a number bias seem to perform equally well on length and number discriminations. They seem to perform just as well as the children with a length bias perform on a length discrimination. The poorest performance was given by the children with the length bias who were required to learn a number discrimination. A child who indicates a bias toward number in verbal tasks may have little difficulty with a nonverbal or relational discrimination of length or number. A child who shows a bias toward length in verbal tasks may have difficulty with a nonverbal relational discrimination of number.

Both Piaget's theories of number and length, and findings in the research literature, including those in this experiment, suggest that the use of the end point length strategy to evaluate both spatial and numerical quantities will precede the use of a numerical strategy. Consequently, one might expect to find the children with a length bias relatively younger than the children with a number bias. A Mann Whitney U test was used to compare the ages of the children in the number bias group with those in the length bias group. The U_{9-14} = 25, was significant at the $p < .05$ level (two-tailed). The children in the number bias group were significantly older (median, 4.11, range 3.8 to 5.3) than the children in the length bias group

Table 5-8

Median number of trials to criterion and mean rank for discriminations learned with and against bias. Subjects classified according to type of bias.

discrimination order ↘ / bias → / type →	number (9)		length (14)	
	number	length	number	length
first				
median	11.5	13.00	36	30
$\overline{R}$	9.17	7.33	16.63	11.00
second				
median	17	16	36	28
$\overline{R}$	8.83	8.00	15.19	10.83

(median age 4.2, range 3.5 to 5.2).

Discussion

A number of findings in this experiment lend some support to Beilin's argument that the length concept possesses a more primitive perceptual component than the number concept (Beilin, 1969). First, children in the age range 3-5 found it easier to learn a nonverbal relational discrimination on the length dimension than on the number dimension. Second, on the verbal task children made fewer errors in response to questions employing a length term than a number term. This may indicate that at least some aspects of the referents for length terms are more easily perceptible than the referents for number terms. Third, the bias results from the language task suggest that length discrimination may be unaffected by a child's strategy bias to number, whereas number discrimination may be affected by a child's strategy bias to length. Such a result may indicate either, that length is a more compelling dimension perceptually, at least at a certain stage in development or, that number requires an additional, more difficult operation which children lack at a certain stage in development. For these children (those with a consistent strategy bias toward length) length becomes the only cue for quantity judgments. Finally, the relationships between age and performance on both the verbal and nonverbal task, and age and bias suggest that length (as defined by end points alone) is an easier dimension. Younger children are able to perform as well on length judgments or discriminations as older children and they are more apt to have a length bias than older children.

While the above findings do provide some support for the argument that length possesses a more primitive component than number, that argument must be refined. The refinement is necessary to take into account significant asymmetries in the children's discriminations of positive and negative poles. Over the age range of this experiment "shorter" was an easier discrimination than "longer". While this was the direction of the relationship for these two terms in both age groups, it was in the older group that the difference was significant. There was no evidence of an asymmetry in the positive and negative terms, although the decrease in the number of trials to criterion with age was significant for "more", but not for "less".

Surprising was the ease with which even young children learned the "shorter" discrimination. In light of the fact that the "longer" and "shorter" sets were identical such a finding provides support for Piaget's proposal that young children's spatial concepts are initially topologically based (Piaget, 1967). The "shorter" row which is more "heaped" (or dense) possesses a topological feature which the "longer" row does not. The findings do not support Klatzky et al's (1974) position that judgments of the "small one" are intrinsically more difficult than judgments of the "big one". At least on this type of task there is no evidence for a cognitive deficit argument to explain asymmetries in comparative terms. That does not preclude the possibility that such a deficit might operate in the linguistic domain. Semantically, and in terms of syntactical requirements negative terms are more complex than positive terms (H. Clark, 1970). Such complexity could result in a lag in the attachment of the proper referent or appropriate strategy to the word. If that is the case, the cause of a lag in the comprehension of a negative term may have to be sought in the linguistic context itself.

The only discrimination which appeared to increase in difficulty with age (relative to the other discriminations) was the discrimination of "longer". This may indicate that as children get older they pay less attention to end points. This would make it more difficult for them to make length discriminations. However, the bias results do not suggest that having a strong number bias causes an increase in difficulty in length discriminations. This question requires further study.

The preceding discussion has dealt with the first two questions outlined in the introduction. The third question was concerned with the nature of the association between the nonverbal discrimination task and related language. Various results support the "loose" dependency hypothesis discussed in the introduction. First, the degree of success which a child had on the two discrimination tasks was associated with the degree of success he had on the two verbal tasks. Second, if the child was at least able to make the "same - comparative" distinction (was a nonguesser), or could answer questions employing a particular quantity term reasonably well, he was very apt to learn a related comparative concept in a nonverbal setting. However, the presence of "appropriate language" did not always appear to be essential for success on the discrimination task. While some children answered questions rather badly, they seemed to have nonverbal concepts of "more", "longer" and "shorter" available, for they learned relational discriminations appropriate to these concepts.

In this particular experiment this "loose dependency" may result from certain differences between the verbal and the nonverbal task. The nonverbal task requires only a judgment of inequality, while the verbal task implies the need for both a judgment of equality and inequality by virtue of the form of the question asked, "Are the rows the same length or is one longer?" There is evidence that equality judgments are more difficult than inequality judgments (Beilin, 1969); therefore, one might expect the nonverbal task to be easier than the verbal task for this reason alone. There could be other reasons also. For example, Siegel (1977b) has evidence that young children may do less well on a task with more language cues than on a comparable task with fewer cues. Language cues may confuse young children in some situations.

There were also findings in this experiment that suggested that the nature of the association between the nonverbal discrimination task and related language was different for the length and number concepts, at least in this age range. For length, the association between success and failure on the discrimination task and on the verbal task was not significant. This association was significant for number. The closer association for number may reflect a greater degree of cognitive-linguistic dependence for number. On the other hand it may simply reflect some parallel emergence of cognitive and linguistic skills with age.

The findings on the reversal-nonreversal shift also bear on the language-thought question. The children in this study (all under five) found reversal shifts easier than nonreversal shifts. There was no evidence for age changes. These findings are not in the usual direction of the findings in shift studies. Children in this age group usually perform relatively better on a nonreversal than on a reversal shift. However, the dimensions employed in this study are not strictly parallel to those used in the usual shift study. Normally the dimensions employed are independent, for example, size and colour. In the case of length and number the dimensions are not independent. Also, the change from a length to a number (or number to length) discriminations (nonreversal shift) involved the change of one stimulus card. In the length discrimination all the rows varied in relative length, but one configuration did not vary in number (configuration five). The number set dropped this one stimulus card (conf. 5) which did not vary in number and introduced a new stimulus card (conf. 4) which was the same in length but different in number. This change eliminates the possibility of purely associative learning on the nonreversal shift, and researchers have generally used associative learning to explain the young child's relatively superior performance on the nonreversal shift. (Kendler and Kendler, 1969).

One might argue that the reversal shift children did benefit from possessing a specific mediating term for a specific dimension.

However, at least on the first discrimination task 32 of the children passed the discrimination task who failed on the appropriate verbal term. Eight of the thirteen children in the reversal shift group did not meet the criterion for success on the verbal term but only two of them failed to make the shift. However, given that the verbal task was both relational and absolute, and the discrimination task relational only, it is possible that some of the children who did not pass the criterion did possess a relational understanding of the term, for example, "longer", while not possessing a notion of "same".

In order to study more fully the effect of dimensional preferences on reversal and nonreversal shifts it would probably be best to get more data on just two terms, for example, "longer" and "more". The range of scores on the incongruent stimuli in this study was fairly small. However, the results do suggest that a verbal evaluation of a child's predominant response strategy, if he has one, will help predict his performance on a nonverbal task. If a child is biased toward length, he will perform poorly on number but well on length. If he is biased toward number he will perform well on number but there will be no negative effect on length. It might be recalled that in Experiment One, the child's strategy bias on the verbal task was a good predictor of his performance on a conservation task. The findings in this experiment are in general agreement with those from other discrimination learning experiments (Suchman and Trabasso, 1966) which have demonstrated improved performance on a preferred dimension.

The fourth question raised in this experiment concerned the specific or general nature of the mediator in the shift and the issue of asymmetry. The order of difficulty on the shift: same dimension, opposite pole; opposite dimension, opposite pole; opposite dimension, same pole, suggests that if a child had an appropriate strategy and identified the criterial dimension, he found it easiest to maintain his strategy and switch poles, i.e., "big" - "small", or "small" - "big", when the feedback indicated a new task requirement. Perhaps the order of difficulty reflects the following. If a strategy (either numerosity or end points) is working, stay with it. When the feedback changes, test out the opposite pole on that dimension (the reversal shift). If when the feedback changes and you switch poles, your strategy does not work, stay at the new pole and switch strategy (the nonreversal, different pole, different dimension shift). If when the feedback changes and you switch poles and your strategy does not work, and then you switch strategies and this does not work, switch poles (the nonreversal, different dimension, same pole shift). There was no evidence that it was easier to move from a positive to a negative pole, or a negative to a positive pole.

The implication of this model then is that poles do not operate as dimensions. The "big-small" distinction operates only in relation to a specific dimensional strategy, as the testing element in any quantitative judgment. The conservative aspect of a quantitative judgment for a child who possesses a strategy will be his strategy. This may reflect a developmental order. The first quantitative distinction a child makes is the "big" - "small" distinction, where-after he acquires various specific quantitative strategies. These results support the idea of a specific mediator, for example "number", "length", rather than the idea of a general mediator of the "big-small" variety.

Experiment VI

In Experiments 1 to 5 in this series the experimenter asked the children to make judgments of length and number on rows of dots on cards. These experiments have established that on such a task young children may operate with a strategy bias which depends, at least in part, on set size and the age of the child (Lawson, et al., 1974; Baron et al., 1975). It appears that such strategy biases are: resistant to some irrelevant changes in a set (Experiment 3); attached in some cases to both positive and negative comparative terms (Experiment 4); possibly related in some way to a hierarchy of discriminability (Experiment 5). In the following experiment the author explored with some new tasks three areas of concern which related, at least in part, to the issue of the validity of the findings obtained in the previous experiments.

The first concern in the following experiment was to try to determine whether strategy biases might be found in a task requiring length and number judgments on real objects such as toys. For example, if the experimenter asked children to judge the length and number of two rows of beads, or boxes, or cars, arranged in the same configurations used in the verbal task, might strategy biases appear? In addition might it be possible to demonstrate some systematic relationship between a child's strategy biases on the task with real objects and the standard language task which used cards with rows of dots. The issue might be viewed as one of establishing some ecological validity for those cognitive processes which have been under study here. Recently, both cognitive and developmental psychologists have raised the question of the generalizability of concept tasks which employ highly abstract dimensions (Brooks, 1976; Rosch, 1976).

The second issue arose in part from a concern with ecological validity, and in part from some theoretical aspects of Piaget's theories of length and number. In the real world, children acquire the ability to fit a discrete set of objects such as blocks into an appropriate, conceptually related, object such as a box (for blocks). The child's experience with space and different numbers of objects should teach him that "more" objects require "more" space. "More" can imply "longer". He must also learn (through acquiring number operators) that the extent of a row of objects has nothing to do with the numerical value assigned the set of objects. For example, four dinky cars in a row always make a longer line than three dinky cars in a row (when placed bumper to bumper); however, the spatial extension (length) of a row of Dinky cars never determines the numerosity of that row. In the real world, length can be a reasonably good estimator for number; numbers of objects can also provide a good estimator for length (or space).

The theoretical aspect of the above has to do with the relationship in Piaget's theory between the operations which enable length judgments according to end points, and those which permit unit measurement. The measurement operations require the ability to use an arbitrary, repeated unit. This requires numerical operations. Only when the measurement and end point (order) operations are fully coordinated will the child conserve length quantitatively.[1] Results in Experiment Four showed that the children who used a numerical strategy to judge the length of a row of discrete objects, were quite able to use an end point length strategy to make a judgment of continuous length. Apparently, the end point and numerical strategies were not coordinated in these children. If, however, one could demonstrate that children who were skilful with number (including those who used number to judge length on discrete objects) were better at assigning a row of discrete objects (such as dinky cars) to the correct object of continuous length (such as a trailer truck), one might argue that at some level the measurement and end point operations were coordinated.

The recognition that real world experience teaches that the number of a set of objects is a good estimator for length (if a set is in a row), and the finding that children who used a number strategy to judge the length of rows of discrete objects also used an end point strategy, led the author to expect that children with a number strategy bias (or at least with a number strategy available) might do reasonably well at fitting a row of discrete objects into, or onto, a container of appropriate length. Their "fit" should not be affected by the length cues of the rows of discrete objects. Children with a length bias, or children without a systematic number strategy, might not perform as

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[1] See the discussion of the length theory in Chapter One, section one..

well on a "fit" task, for they might try to fit the longer row of discrete objects (cars) to the longer continuous object (trailer truck) without regard to the number of objects.

The third concern of the author was to attempt to explore young children's understanding of length and number on somewhat different tasks than those used in the previous experiments. A major aspect of this exploration was to be a comparison of the ways in which nonguessers and guessers, as classified on the standard language task, varied in their performance on other kinds of quantitative tasks which required judgments of length and number. It might be recalled that the nonguessers were those children who seemed to have an understanding of the basic terminology in the questions, that is, they were at least able to make the "same" - "comparative" distinction.

The intent of the following experiment, then, was to begin an exploration of the validity of the cognitive, developmental, processes which have been under study in this thesis.

Method

The experimenter gave three different tasks, in three different orders to 36 children who ranged in age from 3.4 to 6.4 with a median age of 4.11. There were 18 male and 18 female subjects. The children attended a parochial elementary school, a private nursery school, or a day care centre. Each child was seen individually.

Task 1: Standard Language Task

Task one was the standard language task used in all the previous experiments. The number of stimulus cards was reduced to six to avoid tiring the child on this one task, since the total time required for all 3 tasks was approximately ½ hour. The six stimulus cards showed 3 congruent and 3 incongruent configurations (Figure 1-1, Expt. 1), one on each card. Configuration one appeared in both a 3 and 4 dot form. The experimenter asked the standard length and number questions about each configuration. At the end of this task the child was given the conservation and length discrimination tests described in Experiments One and Four respectively.

Task II: The Standard Toy and Fit Tasks

On task II the experimenter gave the child a series of sets of individual, identical, play objects such as beads, blocks, cars and swans, arranged in two rows in configurations. These were arranged to correspond to configurations two, three, four, and five of the static configurations found in the Standard Language Task (See Figure 1-1). On each of these configurations she asked the child to judge the number and length of the rows of toys using the same questions as those on the Standard Language Task. This set of questions and the answers to them constituted the Standard Toy Task.

After the experimenter had asked the child about one of these configurations she showed him several play objects, which were conceptually related to the toys in the Standard Toy Task. For example, after the child had judged the rows of beads, she showed the child several strings, which varied in length. She then asked the child to compare the length of the strings. For three of the four configurations of toys, the conceptually related play objects varied only in length. These were string for the beads, boxes for the blocks, transport trucks for the cars. On these objects the children were only asked to judge length. For the swans, the matching items were discrete items—nests; consequently, the experimenter asked the child to judge both length and number.

Finally, the experimenter asked the child to fit each of the rows of discrete objects, judged on the standard toy task, to an appropriate object (in terms of length or number). For example, she asked the child for the fit between each of the rows of beads and the strings. For each row one of the strings gave an exact fit. The child indicated the fit by pointing only, i.e., he did not place the beads on the strings. This part of the task was called the Fit Task.

There was an additional task in which the experimenter asked the child to judge the distance between two sets of two objects (ends of bridges) and the length of two boards (spans of bridges). She then asked the child to fit the appropriate span to the appropriately spaced ends.

In all then there were five subtasks to Task II. These were presented in random order. They are outlined in more detail below and illustrated in Figure 6-1. On the first four subtasks the judgment of a correct fit for both rows of objects required that the child not be misled by length cues, but use number to estimate the amount of space required. On the fifth task the child needed to make a correct "big-small" spatial judgment, on both bridge ends and spans.

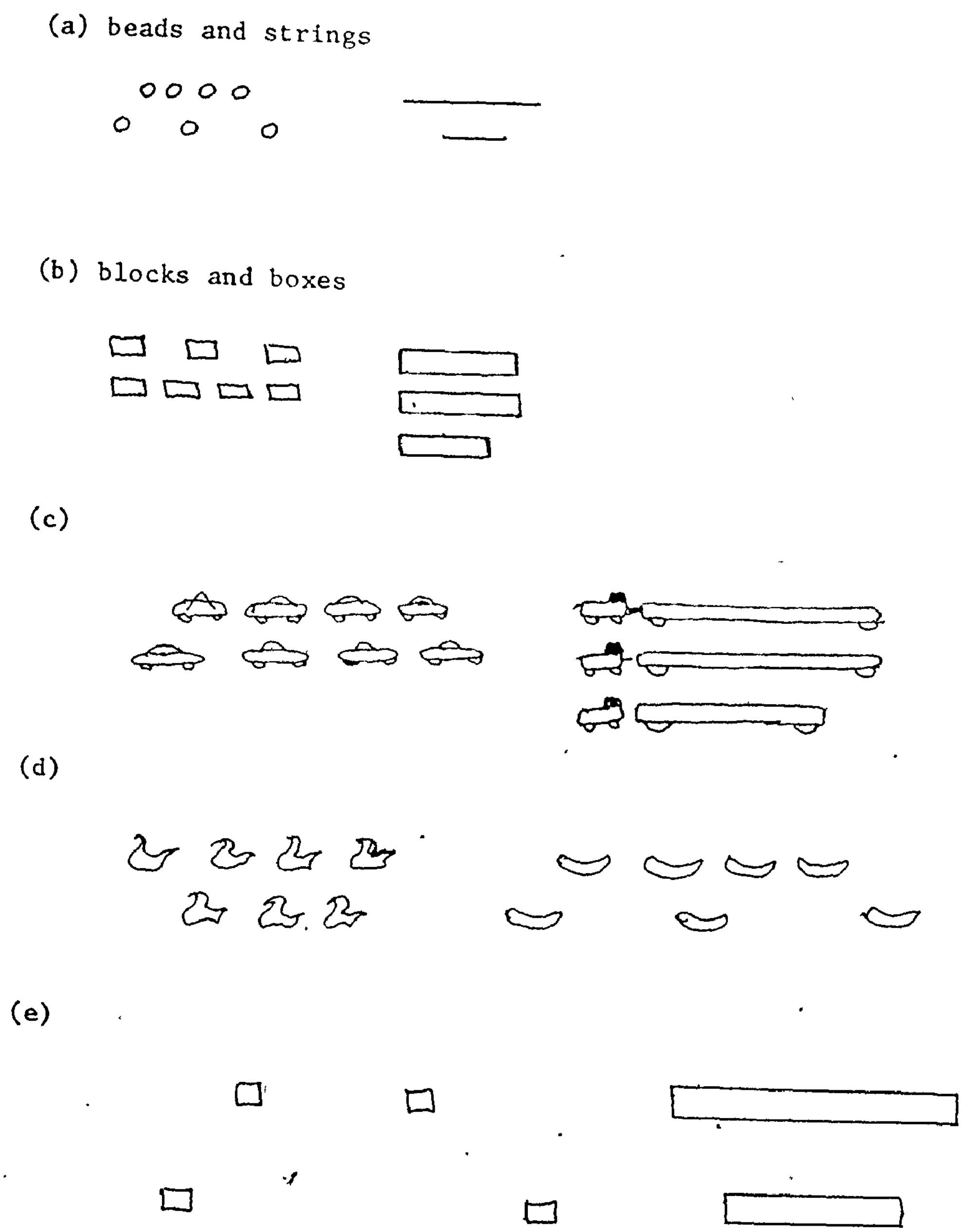


Figure 6-1 Play objects and their arrangements on the Standard Toy and Fit Tasks

Subtask:

(a) The experimenter showed the child two rows of beads, one of four beads, the other of three, arranged as in configuration three. She asked the length and number questions employed in the Standard Language Task. She then showed the child two strings, one short and one long and asked which was longer. Following the child's answer she asked him/her to point to the row of beads which fit on one of the strings, then to point to the row which fit on the other string. The experimenter designated the strings by pointing.

(b) The experimenter asked the child to make length and number judgments on two rows of blocks (4 and 3) arranged as in configuration 4. She then asked the child to make a length judgment on 3 boxes for the blocks: two the same length, one shorter. The question in this case was: "Are the boxes the same length or are some longer?" If the child replied that some were longer the experimenter asked which were longer. She then asked the child to point to the box which one row of blocks fit in, then to point to the box which the other row of blocks fit in. The experimenter designated the rows of blocks by pointing. The three boxes were necessary to allow for the possibility of the child's using a length or number cue.

(c) The experimenter arranged two rows of four cars as in configuration 5. She asked the child to judge both length and number. She then showed the child 3 transport trucks, two of the same length, one shorter. The same procedure was followed as in part (b).

(d) The experimenter presented two rows of swans, one 4 and one 3, in the configuration 2 formation. She asked the child to judge length and number. Then she presented 2 rows of nests in the form of configuration 3 and asked the child to judge length and number. Finally she asked him/her to indicate which rows of swans fit with a row of nests. The experimenter pointed to a row of nests and asked the child to designate the row of swans which fit on it. She pointed to one row of nests, waited for the child's reply, then pointed to the other row of nests.

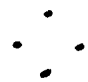
(e) In this instance the experimenter showed the child 4 blocks, one pair spaced more widely apart than the other. She designated these as the ends of bridges and asked the child, "Is one space between the ends of the bridges longer or are they the same length?" She then showed him/her two rectangular blocks of different lengths and asked him/her to judge which was longer. Finally she asked which rectangular block fit on which ends, or alternatively which ends a rectangular block fit on.

Task III: Assorted Tasks for Judgments of Length and Number

Subsection I: Sticks and Pennies

(a) The experimenter showed the child two sticks, one 4 inches (102 mm), one 3 inches (76 mm) long and said, "Here is a stick and here is a stick. I want to tell someone about these sticks. Which is good to say? This stick is longer or this stick is more?" The experimenter pointed to the longer stick while asking the question.

(b) The experimenter showed the child two groups of pennies made up of three and four pennies each, in approximately the following arrangements:

She then said, "Here is a pile of pennies and here is a pile of pennies. Which is good to say: This pile is longer or this pile has more". The experimenter pointed to the more numerous set.

In both (a) and (b) the order of the words varied as did the order of (a) and (b) themselves.

Subsection II: Cars

A child was shown a card like that in Figure 6-2. The card had three rows of cars on it. Row one had four cars which were 25 mm. long, 10 mm. wide, and separated by 10 mm. The total extension was 130 mm. Row two had 3 cars, 25 mm. long, 10 mm. wide, separated by 45 mm. The total extension was 160 mm., i.e., row two was longer than row one. Row three had three cars, 33 mm. long, 10 mm. wide, separated by 15 mm. The total extension was identical to row one, but the cars were longer. In fact the area occupied by the car bodies was equal in these two rows. Thus, rows (1) and (3) were the same length; rows (2) and (3) had the same number; and row 3 had longer cars, but same body area.

There were two parts to subsection II. They were presented in random order.

(a) Production: The experimenter simply showed the child the card and asked the child to tell about it. After the child replied, the experimenter asked if there was anything else to tell. This was continued until the child said he had nothing more to say. His

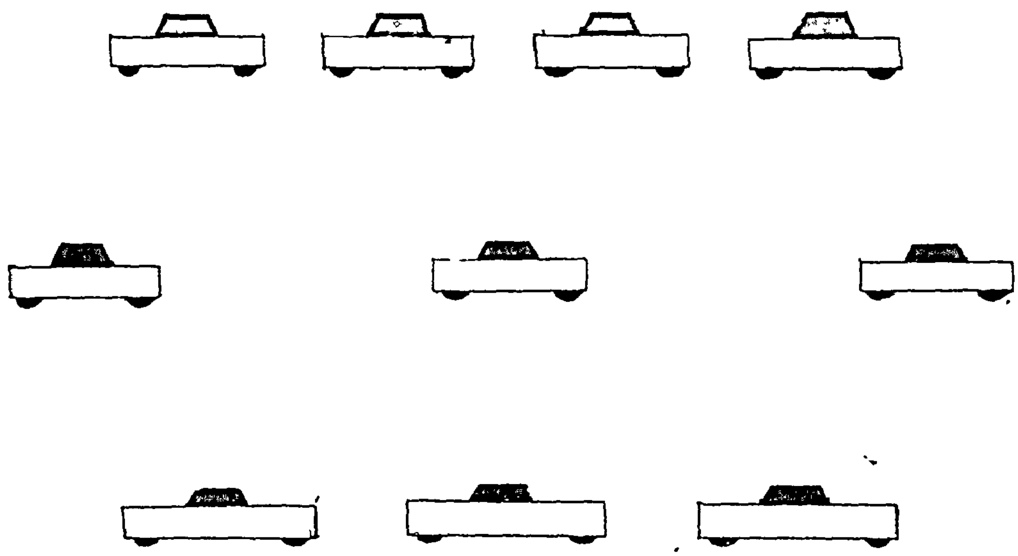


Figure 6-2: Stimulus card used in Cars Task

language was recorded.

(b) Comprehension: In part (b) the experimenter gave the child the following 5 commands in random order.

(1) Show me the row with more cars.

(2) Show me the longer row of cars.

(3) Show me the row with longer cars.

(4) Show me the rows which are the same length.

(5) Show me the rows which have the same number of cars.

These questions were asked in random order. Parts (a) and (b) were presented in random order.

Subsection III: Make Me a Row.

The experimenter gave the child four pennies and asked him/her to make a row of pennies. Following this she gave the child a box containing six pennies and asked the child, in random order, to make: another row the same length; a row which is longer; a row which has more pennies; a row which has the same number of pennies. The child's own standard row was maintained throughout, although it was sometimes necessary for the experimenter to intervene to maintain it.

Results

The children were first classified as guessers and nonguessers on the Standard Language Task according to the procedure employed in the previous experiments. The criterion was set at 5 out of 6 correct responses on the 3 congruent stimuli. This was a slightly more stringent criterion than that used in Experiments Three, Four, and Five. However, it was comparable to that used in Experiment One, (Lawson, Baron, and

Siegel, 1974). In addition, since the author was exploring the development of several new tasks in this experiment she wanted to use a stringent criterion to distinguish guessers from nonguessers.

Eighteen children were classified as guessers, (median age 4.8): these children made an average of 2.8 correct responses on the congruent stimuli. Eighteen children were classified as nonguessers (median age 5.2) (Table A 6-1, Appendix A). These children made an average of 5.8 correct responses on the congruent stimuli. A t-test was performed on the mean number of correct responses made by guessers and nonguessers on the incongruent stimuli, both length and number questions, $t_{34} = 4.1339$, $p < .001$.

Table 6-1 presents the total number of correct responses and means on the congruent and incongruent configurations by subject classification and question. Nonguessers made more correct responses on the incongruent configurations than guessers, 4.17 compared to 2.50. A total of 6 correct responses on the incongruent configurations was possible. Separate sign tests were used to compare the distributions of correct responses on length, and number questions for nonguessers and guessers. The sign test for the nonguessers showed a significant difference in favour of number questions, $z = 2.6046$. The sign test for the guessers showed no difference, $z = 0.0$. Nonguessers made more correct responses on number than length questions. Guessers performed about equally well (or badly) in response to both types of questions

Table 6-2 shows the distribution of errors for guessers and

Table 6-1

Total number of correct responses and means on congruent and incongruent configurations on Standard Language Task by subject classification and question.

		Nonguessers (N=18)		Guessers (N=18)	
		number	length	number	length
Cong.	total	53	52	21	27
	$\overline{X}$	2.94	2.88	1.17	1.5
Incong.	total	44	31	21	24
	$\overline{X}$	2.44	1.72	1.16	1.33
Grand total		97	83	42	51
Grand mean		5.39	4.61	2.33	2.83

		Nonguessers	Guessers
Congruent total		105	48
	$\overline{X}$	5.83	2.67
Incongruent total		75	45
	$\overline{X}$	4.17	2.50

Table 6-2

Distribution of errors on the incongruent configurations on the Standard Language Task by subject classification and question.

Question:	Number		Length		Sum
Error:	Length	Other (O)	Number	Other (O)	
Nonguessers (N=18)	4	3	23	0	30/108
Guessers	19	14	15	15	63/108

nonguessers on the incongruent stimuli. A total of 108 errors (with N=18) was possible, 54 on each question. The majority of errors made by the nonguessers were in response to length questions; all length errors could be attributed to the use of the number strategy. Only one child in the nonguessers' group consistently used a length strategy in response to number questions. The errors of the guessers appear to be evenly distributed over the two dimensions and over the two possible types of errors.

All eight of the children who were classifiable as conservers in this experiment were in the nonguesser category. These children made an average of 1.75 (out of a possible 3) length errors. This contrasted with the performance of the nonconservers in the nonguesser category who only made .80 length errors. The conservers then were twice as likely to overextend the number strategy to the length dimension as the nonconservers.

On the length test given in conjunction with the language task the nonguessers performance was uniformly high, 50/54 correct responses: the guessers performance was not as high, 39/54 correct responses. Eleven of 15 errors were in response to the array which presented two lines of the same length (Table A 6-2, Appendix A). A Cochran's Q. indicated that on the length task the items differed in difficulty. The judgment of the two equal lines (array 2) was much more difficult than the judgment of the other two arrays in which the length of lines differed. A comparable result was obtained in Experiment Four.

The responses of the children on the Standard Toy Task to the length and number questions on the 4 incongruent stimuli (3,4,5, & 3) were scored in the same way as the responses made to the incongruent stimuli on the Standard Language test (Table A 6-3, Appendix A). A t-test comparing the mean number of correct responses made by guessers and nonguessers was significant, $t_{34} = 5.1865$, $p < .001$. Table 6-3 presents the total number of correct responses and means by subject classification and question. Nonguessers made significantly more correct responses than guessers, 6.17 compared to 3.72. A total of 8 correct responses was possible. Separate sign tests were again used to compare the distributions of correct responses on all length and number questions for nonguessers and guessers. Neither sign test showed a significant difference. For the nonguessers, $z = 1.388$, ns., for the guessers, $z = 1.66$. For the nonguessers the direction of difference was the same as on the language task; they made more correct responses on number than length questions. There was a slight tendency for guessers to give more correct responses to length than number questions.

Table 6-4 shows the distribution of errors for guessers and nonguessers. A total of 144 errors was possible, 72 on each question. Nonguessers made relatively more errors on the length questions than on the number questions: (22:11); almost all these errors were due to the use of a number strategy (20:22). Guessers errors were more evenly distributed (42:35), with more errors on number questions. The guessers also made more unexplained errors than the nonguessers (24:5).

An inspection of the pattern of errors on the incongruent

Table 6-3

Total number of correct responses and means on incongruent configurations on the Standard Toy Task. By subject classification and question

	Subject Classification			
	Nonguessers (N=18)		Guessers (N=18)	
	number	length	number	length
Total	61	50	30	37
Mean	3.39	2.78	1.67	2.06
Grand Total	111		67	
Grand Mean	6.17		3.72	

Table 6-4

Distribution of errors on the incongruent configurations: 3, 4, 5, & 3 on the Standard Toy Task. By subject classification and question.

Question	Number		Length		Grand Sum
Type of Error	Length	0 (unexplained)	Number	0 (unexplained)	
Nonguessers	8	3	20	2	33/144
Guessers	29	13	24	11	77/144

configurations on both the Standard Language Test (Table A 6-1, Appendix A) and the Standard Toy Task (Table A 6-2, Appendix A) indicates that whereas a nonguesser tended to make systematic errors on one dimension or the other, a guesser made both systematic and unsystematic errors on both dimensions.

One further analysis was carried out on the first three toy tasks, (a), (b), and (c), to determine whether any of the configurations, 3, 4, and 5, differed in difficulty on length and number questions. Differential difficulty had been shown in Experiment Three using the standard dot cards. Table 6-5 presents the number of correct responses on the three configurations by question and subject classification. A Cochrane's Q on the correct responses to the length question over all subjects indicated that the configurations did differ in difficulty $Q(X_2^2) = 9.49$, $p < .05$. On the number question the Q was also significant, $Q(X_2^2) = 6.19$, $p < .001$. The configuration which produced the least number of correct responses on the length question was configuration 4 (same length, different number) on the number question, configuration 5, (same number, different length). The differences in difficulty appeared to be greatest for the guessers.

A question of major interest in this experiment was whether there was a relationship between an individual child's use of a consistent length or number strategy on the Standard Language Task (incongurent configurations) and on the Toy Task (incongruent configurations). To examine that question, only the data from the nonguessers was analysed, as only they might be said to have consistent quantitative strategies.

Table 6-5

Total number of correct responses on configurations 3, 4, & 5 on the Standard Toy Tasks (a), (b), and (c). By subject classification and question.

	Question					
	Number			Length		
Configuration	3	4	5	3	4	5
Subject Classification						
Nonguessers (N=18)	16	16	13	13	10	14
Guessers (N=18)	10	8	2	10	5	13
Grand Total	26	24	15	23	15	27
Mean	13	12	7.5	11.5	7.5	13.5

The degree of strategy bias for each subject was determined on the two tasks by establishing the proportion of numerical responses (both correct and incorrect as applied to length) to numerical and length responses combined: $\frac{N}{L\&N}$. A value of .50 meant the two strategies were evenly divided, a value below .50 that a length strategy was preferred, a value above .50, a number strategy. Table 6-6 presents the proportions on both tasks.

On the language task 14 out of 18 children showed a number bias. On the Toy Task 10 out of 18 children showed a number bias. Using the proportions from the Language and Toy Tasks, Kendall's tau was calculated to determine the degree of relationship between the ranks for the two sets of proportions, Tau = $\pm$.596, $z = 3.059$, $p < .001$. There was therefore a significant, positive, correlation between the strategy biases on the Language and Toy Tasks. The bias toward number was somewhat stronger on the Language task than on the Toy Task, but the significant positive correlation indicated that an ordered relationship existed between the bias variables on the two tasks.

The next question examined was how well the children judged the fit of one set of play objects to other conceptually related play objects, i.e., beads to string. This was called the Fit Task. There were five subtasks, each required two "fits". To be judged correct on a task the child had to make both fits correctly. The author assumed that two correct fits indicated the use of a number cue. The raw data are found in Table A 6-3, Appendix A. Table 6-7 presents a

Table 6-6

Table of proportions showing strategy biases in Language and Toy Tasks.

Nonguessers

Subject	Language Task	Toy Task
S6	.67	.57
S7	1.00	.75
S9	.00	.00
S13	.50	.50
S14	.67	.50
S15	.67	.63
S16	.67	.29
S18	.50	.38
S19	.67	.25
S24	.67	.50
S26	1.00	1.00
S27	.67	.63
S28	1.00	.88
S29	.50	.75
S31	1.00	1.00
S32	.83	.63
S34	.67	.75
S35	.50	.50

LEGEND

values

.50 - strategies divided

"below" - length strategy probable

"above" - number strategy probable

summary of the number and types of responses made over four fit tasks, a, b, c, and d. A correct response indicated a numerical strategy. An incorrect response could indicate either a length, or an unexplained strategy. An example of a response in the unexplained category was the child judging that both the longer and shorter row of beads fit onto the shorter of the two strings.

Separate binomials were calculated for the number of correct responses for the guessers and nonguessers to determine whether the performance of each of the groups was above chance. The nonguessers made 52 correct responses out of a possible 72. The $z = 3.947$, $p < .001$ indicated that their performance was significantly above chance. The guessers made 34 correct responses out of a possible 72. The $z = 1.086$, NS indicated that their performance was not significantly above chance. A t-test was used to compare the difference in the mean number of correct responses made by the two groups. The $t_{34} = 4.372$, $p < .001$. The nonguessers made significantly more correct responses (mean = 2.89) than the guessers (mean = 1.89).

Thus, the children in the nonguesser category who had shown a high degree of proficiency with, and, indeed, a bias toward number on the Standard Language Task, and a related bias on the Toy Task, performed above chance on the Fit Task. This performance would seem to require some knowledge of the relationship between numbers of objects and the space required by these objects. When these children made an error it was usually due to the use of a length strategy.

Tabl(6-7

Total number of correct and incorrect responses and means over the four fit tasks, a, b, c, and d. By subject classification and type of error.

	Subject Classification	
Responses	Nonguessers (N=18)	Guessers (N=18)
correct	total = 52/72	total = 34/74
	mean = 2.89	mean = 1.89
incorrect	total = 20	total = 36
type of error		
length	total = 17	total = 27
	mean = .94	mean = 1.50
unexplained	total = 3	total = 11
	mean = .17	mean = .61

The children in the guesser category had made equal numbers of correct and incorrect responses on both dimensions on the Language Task and the Toy Task. These children did not perform above chance on the Fit Task. When these children made an error it was also usually due to the use of a length strategy.

The bridge task (task e) required the child to judge the space between bridge ends, the lengths of spans, then fit these together. On this task there was no significant difference in the number of correct responses to the three parts of the task made by the nonguessers and guessers (Table A 6-3, Appendix A). Table 6-8 shows the total number of correct responses made on the three parts of this task. Performance was uniformly high for both guessers and nonguessers. The nonguessers made no errors, 54 correct responses out of 54 possible. The guessers made 47 correct responses. There were only two errors on the Fit Task; these were made by guessers.

Table 6-8 also shows the number of correct responses made on the continuous length questions associated with the Standard Toy Task. These were the responses to the questions to judge the length of the strings, boxes, and trucks. If the responses to these length questions are combined with the responses to the two length questions on the Bridge task, the following mean scores are obtained for the nonguessers and the guessers out of 5 possible: nonguessers, mean = 4.89; guessers, 4.39. A Mann-Whitney U test showed no significant difference between the two groups of children. All the children performed well on the continuous length questions on the Toy Task.

Table 6-8

Total correct responses on the bridge toy task and on the length questions on the string, blocks and trucks (tasks a, b, and c). Means for length questions. Subject classification: nonguessers and guessers.

	Subject Classification	
task	Nonguessers (N=18)	Guessers (N=18)
bridge		
space	18	15
span	18	16
fit	18	16
Total	54	47
string	16	17
boxes	18	17
trucks	18	14
Total	52	48
Grand Total for Five Length Questions	88	79
Mean	4.89	4.39

The remainder of the results are those based on the subtasks of Task III.

Task III

Subsection I: Sticks and Pennies

Table 6-9 presents the proportion of errors made by guessers and nonguessers in response to questions about what was good to say "more" or "longer" when comparing two sticks and two piles of pennies (Table A 6-4, Appendix A). The test for the significance in the difference between the proportions of incorrect responses for the nonguessers and guessers on the sticks was significant at the $p < .001$ level; the test was not significant for the pennies. The children classified as guessers were more apt than nonguessers to say "more" was good in judging the two sticks. Guessers and nonguessers did not differ significantly in their use of the term "longer" in reference to a grouping of pennies.

Subsection 2: Cars Task

Part (a) Production

This was a production task, where the experimenter showed the child a stimulus card with three rows of cars and asked the child to tell about the card. Six subjects refused to respond on this task, consequently there were 16 guessers and 14 nonguessers. The children's responses are presented on Table A 6-5, Appendix A. The total number of responses made by the nonguessers was greater than that made by the guessers, 44 as compared to 32 separate responses. Each subject's first response was classified as quantitative, for example, "bigger"

Table 6-9

Proportion of errors made by nonguessers and guessers in response to questions on sticks and pennies task.

Subject Classification	Sticks	Pennies
Nonguessers	.27	.22
Guessers	.61	.33

"more", "longer", or nonquantitative, for example, "made it". A Chi-square was performed on this data, $X_1^2 = 5.116$, $p < .0225$. There was an association between the production of quantitative terms and nonguesser-guesser classification. Nonguessers were highly likely to comment on some quantitative dimension of the stimulus cards; guessers were as equally likely to comment on nonquantitative as on quantitative aspects.

Part (b) Comprehension

The childrens errors were scored on the five requests on the "Show me" part of the Cars Task. The nature of the errors was assessed, i.e., whether a number error could be attributed to the use of length strategy, or a length error to a number strategy, or an error be classified as unexplained (Table A 6-6, Appendix A). The total number of errors made by the children in response to each "Show me" request are presented in Table 6-10. Neither the nonguessers nor the guessers did well on this task. The nonguessers only made 46 correct responses out of a possible 90, a chance level performance, $z = .00$, $p \leq .50$. The guessers made 30 correct responses out of a possible 90, a below chance performance, $z = -3.396$, $p < .0001$.

Separate Cochrane Q's were calculated for both nonguessers and guessers to determine whether the probability for success was equal across all five requests. The $Q(X_4^2)$ for the nonguessers = 6.62, NS indicated that the probability for success was equal across items . Inspection of the totals on Table 6-10 suggests that for the nonguessers the probability of success was greatest for the two number requests,

Table 6-10

Number of errors made in response to each request on the "Show Me" portion of the Cars Task. Subject classifications: nonguesser, guesser.

		Subject Classification		
question		Nonguessers		Guessers
Show me				
(a) more cars		6	*	12
(b) longer row		10		10
(c) longer cars		11		11
(d) row, same L		11		13
(e) row, same N		6	*	14
* p < .05	Total =	44		60
	Mean =	2.44		3.33

"more" and "same number", and least for all the length requests. The $Q(X_4^2)$ for the guessers = 2.63, p < .10 was also not significant. However, inspection of the totals in this case suggests that for the guessers the probability for success was greatest for the "longer" row request and least for the "same" number request.

Tests for the significance in differences in proportions on each of the requests for the nonguessers and guessers yielded a significant result for the two number requests, p < .05. Nonguessers made significantly fewer errors on both number requests than did the guessers.

The types of errors made by the two groups of subjects in response to the five requests were of interest. On this task a new type of error was possible; a child could attend to the length (or size) of the cars themselves. This might be called a size strategy. It is an addition to the types of errors analyzed in previous tasks. The usual types of errors have been those attributable to the inappropriate use of number, length, or some unknown factor, "unexplained". Table 6-11 shows the distributions of the types of errors made on each of the requests by subject classification.

The nonguessers were most apt to make errors because of an inappropriate use of the number strategy, particularly on the "longer" request. They were least apt to make an error based on the size cue (length of houses). The guessers were most apt to make errors of the "unexplained" variety; all of these occurred in response to questions about "same". They were least apt to make an error because of an

Table 6-11

Types of errors on each of the requests on the Cars Task. By subject classification.

question/error →	Nonguessers (N=18) N	L	S	Ø	Guessers (N=18) N	L	S	Ø
more	–	3	3	–	–	5	7	–
longer row	9	–	1	–	6	–	4	–
longer cars	5	6	–	–	6	5	–	–
same length	5	–	1	5	3	–	2	8
same number	–	1	–	5	–	1	–	13
Total	19	10	5	10	15	11	13	21

N = number strategy
L = length strategy
S = size strategy
Ø = unexplained

inappropriate use of the length strategy. Relative to the nonguessers, the guessers were more likely to make an error based on the size cue.

Subsection 3 "Make Me a Row"

Table 6-12 presents the number of errors made by nonguessers and guessers on each of the items on the "Make Me a Row" Task. (Table A 6-7, Appendix A). The nonguessers made 67 correct responses out of a possible 90 on this task, an above chance performance, $z = 3.02$, $p < .0023$. The guessers made 45 correct responses out of a possible 90; performance in this group of children was at a chance level, $z = .00$, $p < .50$.

Separate Cochrane Q's were calculated for both nonguessers and guessers to determine whether the probability for success was equal across all five items. The $Q(X_4^2)$ for the nonguessers = 2.00, NS. was not significant. The probability for success was equal across items. The $Q(X_4^2)$ for the guessers = 13.52, $p < .001$ was significant indicating that the probability for success was not equal on each of the five items. Inspection of the totals in Table 6-12 suggests that the guesser was most apt to be successful on the "Make me a row" and "Make me a row that is longer" requests, least successful on "more", "same length", and "same number".

The differences in the proprotions of correct responses were tested for the nonguessers and guessers on each of the items. There were no significant differences on the "Make me a row", and "Make me a row which is longer" items. There were significant differences on the other items, ($p < .05$) "same length", "same number", "more".

Table 6-12

Number of errors made in response to eaoh request on the "Make Me a Row Task.

	Subject Classification		
	Nonguessers		Guessers
(1) Make me a row	5		5
(2) Same length	5	*	13
(3) Longer	3		6
(4) More	4	*	10
(5) Same number	4	*	11
Total	21		35
Mean	1.67		1.94

* - significant at $p < .05$

Discussion

The first concern in this experiment was to determine whether quantitative strategy biases could be found in a task requiring length and number judgments of real objects; and, if found, to determine whether a systematic relationship held between these biases and those found on the Standard Language Task. Such strategy biases were found on the Standard Toy Task and they did relate systematically to strategy biases on the Standard Language Task. Apparently the cognitive processes under study in these experiments have not simply been artifacts of a task employing rather abstract dimensions; they have some ecological validity.

On the Standard Toy Task the nonguessers performed well on number questions and showed an overextension of the numerical strategy to length questions. Although fewer children showed a number bias on the Toy Task than on the Language Task the relationship between strategy biases on the two tasks was highly significant. Performance on one task was a very good predictor of performance on the other. Unfortunately for purposes of comparison there were not many children in the nonguesser group who systematically employed the length strategy to judge number. There may have been several reasons for this. First, on the small sets (3-4 dots) number strategists have always been in the majority (Experiments One to Five). Second, relative to the other experiments, a wider age range was used in this experiment in relation to a smaller sample. This reduced the number of younger subjects, and it is amongst the younger subjects in a group of nonguessers (@ 4.0 yrs) that the length strategy usually predominates. On the Toy Task the guessers performed more poorly than the nonguessers. On both the Toy Task and the Standard Language Task their errors were distributed fairly evenly over both dimensions and both types of strategies.

The finding that on the Standard Toy Task different configurations produced different rates of error depending on the question confirmed findings in Experiment Three. The children found it most difficult to judge the number of two rows of toys as the "same", when number was held constant and length varied (configuration five, subtask c). They found it most difficult to judge the length of two rows of toys as the same when length was held constant and number varied (configuration four, subtask b). With real objects also children attend to differences in such abstract dimensions as length and number; their concept of "same" seems somewhat slow in developing and rather easily shaken.

The second concern in this experiment had also to do in part with the matter of ecological validity. This concern was explored in the Fit Task. The nonguessers, the majority of whom were skilful with, and biased toward, number, performed above chance on the Fit Task. Apparently the nonguessers' real world experience taught them that the row with more is longer or takes up more space . These children were able to coordinate sets of discrete objects with other related objects in situations which did not supply one-to-one correspondence cues, and in which attention to length cues would have produced errors. However, while the nonguessers did perform above chance on the Fit Task, their performance was not perfect. Most of these children got three out of four correct. Two factors may account for this less than perfect performance. First, this task, which requires the child to note the relative distance between sets of end points, the relative size of different numbers of items, and then to coordinate his judgments, may be a fairly difficult one for children this age. Second, the less than perfect performance may also reflect the child's knowledge that length cues cannot be completely discounted when making such fits. Under many circumstances the length of a row of discrete items is still a good estimator for the space that those items require.

The guessers were not able to do the Fit Task above a chance level. However, their failure was not simply due to a general inability to follow task requirements. On the bridge subtask, which required spatial coordination, these children performed almost as well as the nonguessers. Their length judgments were also quite good (except judgment of "same length"). Apparently these children's problem was a specific one, having to do with the coordination of sets of discrete toys with either another set of discrete toys, or with objects varying in continuous length. Possibly they were unable to use a number cue systematically. Possibly a failure to understand "same" caused the problem.

The results of the Fit Task also have implications for Piaget's theory of the development of the length concept. On the parts of the Toy Task that required judgments of continuous length the non-guessers' performance was consistently high. These children were apparently quite capable of using end points to evaluate length. On the Standard Language Task, and on the parts of the Toy Task that required judgments of length of a row of discrete items, errors in judging length were due to the use of a number strategy. The children discounted the end points. These findings support those of Experiment Four. They support Piaget's (Piaget et al., 1960) arguments that children in this age group do not have a fully developed concept of length, for they have not fully coordinated the operations which determine the order of end points with the measurement operations which are based on the repetition of an arbitrary unit. However, the nonguessers, who were skilful with number were able to do the Fit task at an above chance level. This result may permit the argument that at some level, perceptual perhaps, the measurement and end point operations are coordinated.

The third concern in this experiment was to explore young children's understanding of length and number on somewhat different tasks than those used in the previous experiments. This was done with particular regard to a comparison of the performance of nonguessers and guessers. In this experiment, as in the previous five, the author classified the children according to their performance on the congruent configurations on the Standard Language Task. The argument was that the children who were able to perform above chance on these configurations were able to make a basic "same-comparative" distinction regardless of strategy biases. They were the nonguessers. The children who were not able to perform above chance on this task were unable to make the "same-comparative" distinction. They were the guessers.

The distinction between nonguessers and guessers seems to be a valid one in the sense of being a good predictor of performance over a variety of tasks. The guessers' performance showed a lack of clear preferences for, or clear rules for application of, either the length or number strategy on discrete arrays. This was apparent on the Standard Language and Toy Tasks. Their judgments of "longer" on a whole object were quite accurate, but they had difficulty with judgments of "same length". They showed problems with judgments of "same" on the "Make Me" and "Show Me" Tasks. They were not very skilful at making fits on the Fit Task. The guessers did not restrict "more" to situations involving discrete sets; on the Sticks and Pennies Task they often said "more" rather than "longer" was good to use in describing the longer of two sticks. "More" apparently had for them a less differentiated meaning than it did for the nonguessers. or perhaps a wider, but less well defined, field of application. Their speech production revealed that they were equally inclined to attend to nonquantitative aspects as to quantitative aspects of a stimulus card presented in an overall context of questions about quantity. None of these children conserved.

The guessers may be of two types: . those who do not understand "same" and who have no numerical or length strategies; those who do not understand "same", but who do have numerical and length strategies available to them. The younger, or less cognitively developed, children may be of the first type.

On the other hand there is evidence that many of the guessers were able to make judgments of "longer" if not of "same length". Their performance on the Make Me a Longer Row Task was as good as that of the nonguessers. This evidence of an earlier emergence of an understanding of length than number is in keeping with the findings of Experiment Five. There is some evidence also that amongst the guessers, number becomes a more salient cue with age (Tables A 6-1, and A 6-7, Appendix A). The guessers who are of the second type respond consistently on configurations where there is a cue for difference, for they may have quantitative strategies available to them for comparative judgments; however, their performance may be limited by the absence of a semantic component which allows judgments of "same".

The nonguessers were consistent in their strategies, as revealed on the Standard Language and Toy Tasks. The majority of them performed very well on number tasks and chiefly showed a number bias when they did make errors on length questions. They seemed to understand the meaning of "more" and "same number" in a wide variety of tasks including the Car and Make Me a Row Tasks. They also restricted the meaning of "more" to situations involving discrete sets (The Sticks and Pennies Task). They used quantitative terms in their descriptions of the Rows of Cars. This probably reflected a sensitivity to the context of the total experiment, a context which called for attention to quantity. The only children in the sample who conserved were in the nonguesser category. In summary, nonguessers were children who apparently were able to make the "same-comparative" distinctions, were sensitive to quantity dimensions and able to apply a quantitative strategy consistently. The errors which they made were usually due to the systematic overextension of a strategy to an inappropriate situation.

There was evidence that there were certain difficulties which both the nonguessers and guessers experienced. Judgments of "same" tended to produce the most errors for both groups of children over a variety of tasks (see, for example, Table A 6-2, Table A 6-5). For the guessers this may reflect an absence of an understanding of "same." This may also be true for the nonguesser; however, it may also be the case that the nonguesser simply lacks confidence in his judgment in the face of perceptible difference in an array.

Both the nonguessers and guessers also experienced difficulty on the "Show Me" portion of the Cars Task. While the nonguessers did better than the guessers, their performance was only at chance. The difficulty of this task for all the children may have been due to the addition of a third quantitative cue, the length or size of the cars. This could have resulted in some confusion.

The specific request to "Show me the row with longer cars " resulted in erroneous responses in 22 of the 36 children, 11 in each of the two groups. The errors could be attributed to both length and number strategies. The children's problem with this request could be due to either semantic or syntactic factors or both. It is possible that children in this age group experience some difficulty in understanding a comparative modifier when it refers to an attribute of an element in a set. It may be that the children could judge one element of one set as longer, or one row of objects with respect to another as longer. However, they may have no understanding of the requirement to make a quantitative judgment of a within group attribute, in the context where the same attribute is relevant to the group as a whole. It is also possible that this outcome is dependent on the structure of the request. For example, "Show me the row which has longer cars" might produce a different result.

Some task needs to be devised to sort out the above possibilities. The required task might involve: assessing children's performance on other comparatives in a similar task, for example "taller"; working with different grammatical constructions; assessing the impact of variations in segment sizes while varying the sizes of rows and the numbers of rows. Such variations might allow one to determine whether the children have yet begun "to make (full) use of the contextual information of syntactic categories (parts of speech) rather than relying strictly on concrete semantic notions (Brown, 1957)". Brown believes that it is just at about this age that children do begin to make full use of contextual information.

The results of the "show me" task have some implications for the interpretation of the findings in the previous experiments. In the Standard Language Task, when a child chooses a row which has more dots, he also chooses a row whose combined elements share a greater area regardless of spatial extension. One could make the argument that the number strategists who picked the row with "more" when judging number and length were attending to area rather than number or length. Two facts should be recalled which bear on the adequacy of such an interpretation. First, if the child is attending to area in such a case, he has attached various aspects of the number strategy to area, i.e., he counts correctly in order to assemble the area. Second, and most important, the children in this experiment who were most skilful in the use of the number strategy, i.e., the nonguessers, were least inclined to refer to the row with longer cars when judging length; rather, they referred to the row with more. The bodies of the cars in each of these two rows when combined yielded equal area (the row with longer cars, and the row with more cars). Therefore, it seems unlikely that area was the basis for the childrens judgments when they chose the row with more as longer. If that were the case more of these children should have chosen the row with longer cars, or on judgments of same length, paired the two rows with equal area. This was not the case. However, the question of the role of area in such a task may warrant more extensive investigation.

Chapter three

Conclusions

This final chapter provides an overview of the relevance of the main research findings presented in Chapter Two to the main issues and general intentions of this thesis outlined at the conclusion of Chapter one.

In all the research presented in Chapter Two the same basic paradigm was used: the author asked young children between 3-7 years, children in Piaget's preoperational period, to make number and length judgments on different configurations of linear, numerical arrays under a variety of task conditions. Using this paradigm, Lawson, et al. (1974), and Baron, et al. (1975) had established that many young children responded to questions about length and number with a consistent strategy bias. Some children used a number strategy to make both number and length judgments; other children used a length strategy to make both length and number judgments. The authors also found the following: conservers in this age group usually had a number bias, indicated by their overextension of the number strategy to the length dimension; a child's bias was frequently manipulable. The experimenter could manipulate the child's bias through changing the set size of the arrays or through training.

The authors inferred from the above findings that the children who demonstrated such strategy biases possessed a superordinate concept of "bigness", but undifferentiated concepts of length and number.

Several different terms and strategies might be attached to a child's superordinate concept, for at least in some cases the children gave evidence of having the strategies appropriate to both number and length concepts available. However, these children apparently lacked a criterion for distinguishing the appropriate situations for the application of a strategy. The individual child's choice of strategy appeared to be determined by: age--the bias toward a numerical strategy increased with age; set size--the bias toward a numerical strategy was found on small set sizes, toward length on large set sizes.

From the Lawson, et al (1974), and Baron et al (1975) experiments, the formal and psychological theories of number, particularly Piaget's, and from related research literature, the author derived six main issues which structured the research questions presented in Chapter Two. These issues are discussed below in light of the major research findings in this thesis.

(1) Various theorists have argued that a child's initial concept of number is multidimensional (Dewey, 1897; Wertheimer, 1912). According to Piaget, children in the preoperational period lack the operations necessary to distinguish number from other quantitative dimensions (Piaget and (Inhelder, 1959). Consequently, one might expect the preoperational child to fail to always distinguish some of the dimensions which are part of his real life experience of number. Experimental evidence existed to show that young children often systematically confused number with length (Gelman, 1972). However, prior to the research carried out in this thesis, little evidence existed to show that young children might confuse length with number, and no evidence for possible developmental

parameters of this type of confusion.

The findings in this thesis support both the notion of an initial multidimensional concept of number and Piaget's contention that children may confuse the strategies appropriate to number and length. Some young children systematically apply number strategies to length as well as length strategies to number (Experiments One to Six). This result was not an artifact produced by using two dimensional arrays. Strategy biases were also found on arrays constructed of toys occupying three dimensional space (Experiment Six).

There was also evidence for developmental changes in the use of the numerical strategy to judge length. On small sets younger children (3-4) were apt to judge number according to a length strategy. On the same set size older children (4 1/2 and up) were apt to judge length according to number (Experiment Three). Apparently as children develop competence in the use of numerical strategies they may apply these strategies to those dimensions such as length which were originally part of their real world experience of number. Such a use of number may result in errors in judging length when the child takes no account of the spatial extension of the rows. However, this use may also facilitate, or at least reflect, the child's understanding of the relationship between numbers of objects and space. This last possibility is supported by the evidence for the skill of the nonguessers on the Fit Task in Experiment Six. Most of these children were biased toward a numerical strategy.

(2) Piaget's theory of the development of the number concept is paralleled by his theory of the development of the length concept. He has proposed that preoperational children do not always distinguish the

situations appropriate to the logical operations, proper to the number concept, from the situations appropriate to the sublogical operations, proper to the length concept (Piaget and Inhelder, 1959). The issue addressed in this thesis was whether a child's strategy bias allowed one to conclude that he only had one set of operations available to him. Baron et al. (1975) had demonstrated that at least for some children variations in the task, such as set size and training, could reveal the availability of another strategy. The findings suggested that while parts of both sets of logical and sublogical operations might be available, the child was biased to respond in certain ways by the task. From Piaget's perspective the child's failure to distinguish the appropriate and inappropriate situations for the two sets of operations is evidence that the operations are not fully developed or coordinated.

Experiments Four, Five, and Six provided additional evidence to support the idea that at least for some preoperational children parts of both sets of operations may be available even though they use a preferred strategy on a particular task. In Experiment Four the children who used a number strategy to make judgments of both "longer" and "shorter" demonstrated that they were quite capable of using an end-point length strategy correctly in judging "longer" and "same length" on pairs of lines of continuous length. In Experiment Five there was evidence that possession of a number bias, as indicated on the Standard Language Task, did not predict any problems in the learning of a length discrimination based on end-points. In Experiment Six there was evidence that the child who used number to judge the length of a row of discrete elements was capable of using an end-point strategy to judge the length of conceptually related objects which varied in continuous length. On the Fit Task these children were also capable of coordinating information about numbers of items and the distribution of these items in space.

However, there were, as well, findings which suggested that for some children some part of, or set of, the operations was not available, or could not be generalized easily to all appropriate situations. Most children in this age group used a length strategy to judge number on large sets (Experiment Three). Some children in the same age group used length to judge number on the small sets. Possibly, numerical operations were simply not present in these children. The children classified as guessers seemed unable to make number judgments (Experiment Six) and found the nonverbal discrimination of small numbers difficult (Experiment Five). Finally, children who showed a length bias on the Standard Language Task had great difficulty with number discriminations. In some children, then, the failure to use a particular strategy may reflect the complete absence of certain operations; in other children such a failure may reflect only a partial absence of, or possibly a lack of coordination of, operations.

(3) According to Piaget's equilibrium model for the development of conservation the probability of a child adopting a particular strategy depends on his current cognitive structure. Prior to attaining full conservation status, i.e., prior to giving evidence of possessing a full and coordinated set of operations, the child's thought is characterized by a tendency to center on, or attend to, single dimensions when making quantitative judgments. The preoperational child is not able to

coordinate information from states and transformations. Until the child reaches the final stage in the development of conservation, transformations of arrays may simply draw the child's attention to a new dimension, i.e., make a new dimension salient. Dimensional saliency may be a major determinant of strategy in the preoperational period and may itself depend on perceptual or cognitive factors. Both the factors of dimensional saliency, and transformation were issues in this thesis with a view to their implications for Piaget's concept of centering and his equilibrium model.

The findings support the idea that saliency may reflect perceptual or cognitive influences, or both. In Experiment Five in the non-verbal discrimination task, children found length as defined by order of end-points a more easily discriminable dimension than number. Consequently, when children find certain operations difficult, perhaps because they are newly acquired, or unpractised, (number operations for example), it is possible that they may respond to a more readily perceptible feature such as end-points. Saliency in this case would reflect the influence of both perceptual and cognitive factors.

Under the task conditions in Experiment Three, the most powerful factor affecting children's judgments was set size. This was true for both the younger and the older children. On large sets length was the salient dimension, on small sets number. The saliency of number on small sets was still high at age seven, as witnessed by those children who used number to judge length. This finding has been independently confirmed by P. Miller and Heller (1976).

The type of configuration also affected children's strategies.

Differences in dimensions were salient features affecting children's judgments. A single difference in a configuration made judgments of "same" difficult. Most of the number errors occurred on the configuration where only length varied and number did not; most of the length errors occurred on the configuration where only number varied and length did not.

The results of Experiment Three indicated that transformations were not necessarily powerful determinants of children's number or length judgments. The children's performance was comparable under both static and transformed conditions. There was no evidence for any age changes with respect to the children's ability to use information from the transformation. Neither was there any indication that the effect of a transformation varied according to the type of configuration.

The above evidence is in agreement with Piaget's contention that centering is a characteristic of the preoperational child's thought. It also lends itself to the argument that examining a child's performance on a single concept may not provide a full picture of the degree to which centering is present. To determine this examination of his performance on a closely related concept may be helpful. For example, those children in Experiment Three who at the age of 6 1/2 to 7 used number to judge length on the small sets (two children did this on the large set) were usually correct on number under both static and transformed conditions. That is, they demonstrated the coordination of operations which supposedly enables decentering on the number concept. However, these children demonstrated centering as a characteristic of their thought by using number to judge length.

Several implications of the above results for Piaget's equilibrium model are the following:

(a) Piaget has stated that initially the child will center on one dimension or another on a chance basis. The findings permit the argument that this might only be true of those children who are totally lacking quantitative strategies (the guessers). Otherwise, the child's tendency to center appears to be under the control of set size and the availability of his estimation strategies.

(b) The importance of observing the effects of a transformation on the development of conservation may be overestimated. The development of the conservation of number may depend on the child attending to different dimensions on different set sizes relating to the level of his estimation skills. He may experience difficulty in his attempts to coordinate judgments on those set sizes which fall in some border-line range. That is, the child may experience cognitive conflict or disequilibrium when he tries to make a number judgment on a set size falling between a small one, where he would attend to number, and a larger one, where he would attend to length. In this thesis such a set might have consisted of four to five dots, although the actual set size where conflict occurred would depend on individual differences in estimation skills.

(4) Both formal and psychological theories of number have assigned a peculiar status to the early numbers in the number series (Beth and Piaget, 1966). Children may have an intuitive grasp of the small numbers and their relationships, or they may actually be able to perform certain operations on small numbers which they cannot perform on large

numbers. Piaget's position on the small numbers was somewhat ambiguous, although he would seem to prefer the idea that they were constructed rather than intuitive. (Beth and Piaget, 1966).

The research findings here are in agreement with those of Gelman (1972), Winer (1974), Lawson et al. (1974) which support the argument that tasks using small number sets may reveal early operational abilities. In Experiment Three, children gave more correct answers to number questions on the small sets than on the large sets, under both the static and transformed conditions. From Experiment Four and Six there was also evidence that a number of children were able to conserve number on the three and four dot arrays.

However, while small numbers may facilitate numerical operations they can apparently hinder length operations for some children; i.e., those children who ignored end-points and used number to determine length. In this category were many of the children who conserved (Experiments Four and Six.) This overextension allows the argument that, while small numbers enable some operations, the numbers are not fully constructed, for the strategies appropriate to number are not fully coordinated with, or differentiated from, those appropriate to length.

(5) In general, psychologists interested in the development of word meaning believe that a child's cognitive status influences his comprehension of language and that changes in word meaning reflect changes in cognitive skills (Sinclair de Zwaart, 1969 ; Siegel, 1977). Children in Piaget's preoperational period (3-7) are of particular interest for students of language, as it is in this period that cognitive and linguistic skills are supposedly converging. Quantitative terms are a major

source of information for the processes involved in the convergence of the linguistic and cognitive systems. There is ample evidence that while the child acquires and coordinates his quantitative strategies, his referents for the quantity terms are not the same as those of adults.

The results of the experiments in this thesis indicate that some children will assign equivalent meanings to two terms such as "more" and "longer" by virtue of employing a single strategy to determine which is "more" and which is "longer". As well the same child can assign both the words a new meaning when the task changes, for example, when the child is required to judge "longer" and "more" on a small set rather than on a large set.

In Experiment Four both positive and negative terms were used in the questions. Here there was evidence that children who used a numerical strategy to make judgments of "longer", also used a numerical strategy to make judgments of "shorter". Noteworthy was the fact that some of these children treated "shorter" as if it meant "less", others simply seemed to guess. There was no evidence that those children who were classified as nonguessers treated negative terms as if they were positive terms.

In these experiments the comparative term "more" appeared to undergo the greatest development or change in meaning to come to most closely approximate the adult term. Evidence from Experiment Six indicated that guessers were prepared to use the term "more" to describe the longer of two sticks, i.e., they did not limit the use of "more" to situations involving discrete elements. The term was treated by the youngest of the nonguessers as if it meant "longer", then correctly by the 4 1/2 year old and older on the small set, and by the 7 year old correctly on the large set.

The term "longer" followed a different course. There was evidence from Experiment Six that "longer" was responded to correctly by both guessers and young nonguessers at least in some contexts, for example, in the Make Me a Row Task. There was also evidence from Experiments Four and Six that with reference to lines of continuous length the term was responded to consistently and correctly by almost all children. However, with the acquisition of a numerical strategy some children treated "longer" as if it meant "more". This persisted in some cases until the children were seven years of age.

In all, the findings support the contention that children's comprehension of quantitative terms changes with changes in quantitative strategies. To the extent that these changes in strategy reflect changes in cognitive status, the development of word meaning can be said to reflect development in cognitive structure. However, the presence of a strategy does not guarantee it will be attached to the appropriate term. Children who used number to assign the meaning of "longer" were able to use end-points, but did not. Some children who used a numerical strategy to judge "shorter" did not use this strategy to judge "less". While the growth of word meaning may reflect the gradual acquisition of quantitative strategies, the attachment of these strategies may reflect not only changes in cognitive structure, but also the frequency of exposure to certain terms such as "less".

(6) Most psychologists interested in the number concept have argued that the study of the factors affecting the use and development of the number

concept can shed some light on the nature of concepts and on the general processes of concept acquisition and cognitive development. In keeping with this tradition the author argued that a comparison of children's performance on both the number and length concepts would allow some reasonable speculation on the following three matters: the appropriate means of evaluating a concept; the relationship between language and thought; the suitability of a model for characterizing quantitative concepts and the course of their development. The results of the experiments as they bear on these three issues will now be considered.

(a) Two findings bear on the question of the criterion for establishing the presence of a concept. These support the argument that the comparison of performance on two concepts can be helpful in considering the appropriateness of a criterion for a concept. Correct answers on a single concept may not themselves be sufficient. First, Baron et al. (1974) made the argument that an important test of the maturity of a child's concept was whether he or she extended the strategies appropriate to that concept to another. Throughout the Six experiments there is evidence for children's overextensions. Even if a child is correct on one concept task, evidence for the immaturity of that concept may come from his errors on another. Second, Piaget has argued that when the child conserves number he gives evidence of the possession of an invariance rule. Such an invariance rule marks the possession of the mature number concept. However, evidence from Experiment Three that children who judged length according to number failed to take account of variance in length allows the argument that variance rules as well as invariance rules may be important markers for the possession of a mature concept.

(b) The findings suggest that the relationship between language and thought might best be described according to a "loose dependency" hypothesis. First, the children who could make the "same-comparative" distinction and who used terms correctly were very likely to be successful on a discrimination task (Experiment Five). However, the relationship between language and success on the discrimination task was much closer for the number dimension than the length dimension. Children who succeeded in successfully answering questions with number terms almost invariably succeeded on a number discrimination; children who failed on the terms almost always failed on the discrimination. There was no significant association for length. Second, in the findings from Experiment Four that some children incorrectly treated "shorter" as if it meant "less", and treated "less" as if they were guessing, there was evidence for the separation of terms and appropriate strategies.

The notion of verbal mediation requires refinement to deal with these findings. Verbal mediators may come to facilitate performance as children get older, but children may well have strategies appropriate to a concept available to them before the verbal mediators are fully present. This is simply another way of saying that words gradually acquire meaning from an individual's experience in the world. The child's experience and understanding of the world will not necessarily bear a one to one relationship with his vocabulary.

(c) Finally, what are the implications of the results for deciding the adequacy of various concept acquisition models such as those of Eve Clark and Jean Piaget? The need to modify the verbal mediation model was discussed above under (b). The findings also suggest that certain assumptions of Eve Clark's component theory of verbal concept acquisition may require modification. In general the findings support Clark's contention that children begin with a higher order, superordinate concept such as "big" which is then refined. However, they do not support Clark's proposal that negative terms are more difficult than positive terms because positive terms permit the use of the natural standard and negative terms do not. Experiment Five demonstrated that in a nonverbal task the negative discrimination "shorter" was significantly easier than the positive discrimination "longer". "Less" was no more difficult than "more". Clark also contended that the children this age first treat negative terms as if they were positive terms. There was no evidence for the positive pole strategy in Experiment Four.

Piaget's theories of number, length, and intellectual development were most useful in determining the specific issues which formed the foundation of this thesis. However, various results of the experiments suggested problems with his theories. These have been discussed throughout Chapter Two. Some of the more significant of these are repeated here. First, Baron et al. (1974) pointed out that Piaget's theory does not allow for a specific prediction of negative transfer across concepts of the type which these authors established through training. Second, there is also evidence throughout the thesis for a wide range of individual differences in strategies and patterns of strategies. This was especially obvious in Experiment Four where the author studied responses to questions employing negative and positive quantity terms. Such a result points to the absence in Piaget's theory of a capacity to predict individual differences as well as invariant sequences. Third, under 6 (a) some difficulties with the conservation criterion for possession of a concept were discussed. For example, a child may conserve number but show that his number concept is immature by using a number strategy to judge length while failing to take account of endpoints. Fourth, there was evidence that children demonstrated centering across concepts and were not always affected by changes in transformation in an irrelevant attribute such as length. These results suggested the need for modifications in the centering concept and in the role assigned transformations in the equilibrium model.

On a more positive note, the following points, highlighted by the research in this thesis, might be considered in speculations on the requirements for an adequate model to characterize quantity concepts and the course of their development. The presence of systematic errors in quantitative strategies was demonstrated by Lawson et al. (1974) and Baron et al. (1975) as well as by Experiments Three, Four, and Six. These systematic errors provide evidence for the presence of some form of stable cognitive structure. The fact that children show different patterns of systematic strategy errors argues for the possibility of individual differences in the order of acquisition of strategies. Prior to the acquisition of the systematic quantitative strategies which result in the systematic errors, a child may very well possess a "stable, cognitive structure"; however, the systematic strategies applied to quantity tasks may not be quantitative strategies. As seen in Experiment Six, the guessers who did not demonstrate the ability to make the "same-comparative" distinction were equally attentive to nonquantitative dimensions of an array as to quantiative dimensions. These childrens' strategies on a quantity task could be systematic but not quantitative.

The emergence of a systematic quantitative strategy may always be marked by its overextension to another quantitative dimension, a result noted in experiments Three and Six. The child may then with time and experience come to restrict his strategies to the appropriate domain. Older children were more apt to be correct than younger children throughout, although age itself was not a determining factor. Experiment Five showed, for example, that nonguessers were present across the age range; guessers, however, tended to be below the median age of the sample.

The findings above suggest that acquisition of quantity concepts would be well characterized by an information processing type model. Such a model allows for the notion of an organism actively engaged with its environment (Bruner, 1956; Piaget, 1952); gradually acquiring the components of a concept. These components may be construed as consisting of habitual plans as Baron has suggested (Baron, 1973). The various plans which make up a concept may be: strategies for discriminating one dimension from another; means for estimating the values along a dimension; the setting of the criteria for the application of the strategies. In the absence of appropriate plan a substitute plan may be implemented. Eventually various plans will come to act as referents for the linguistic system. Such a model allows for stability in cognitive structures, individual differences, active testing of hypotheses, age changes, and a gradual convergence of the cognitive and linguistic system. Such a model also allows one to consider quantity concepts in adults, for it takes account of possible changes in the concepts of number and length as new strategies or plans are learned.

There are many aspects of development of the number and length concepts which require further exploration. From the author's point of view, two of the more interesting questions concern: the possibility raised in Experiment Six that some children might be using an area strategy when they appear to be using number; and the nature of the processes involved in Fit task. It is the author's belief, however, that the research paradigm used throughout the research in this thesis did illuminate some aspects of the above six issues and permitted the realization of the three general intentions of this thesis which were:

(1) to examine some of those factors which influence a child's quantitative judgments on linear, numerical arrays.

(2) to further delineate the nature of the young child's understanding of comparative terms.

(3) to consider some general questions on the nature of concepts and cognitive development.

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Table A3-1

All errors on small set by subject, presentation condition, and grade.

Nursery School   median age 4.8
range 3.7 - 5.4

subject (N=25)	Question: Age	static Number	static Length	transformed Number	transformed Length
3^S1	(3.7)	1	3	2	5
3_S^S2	(3.7)	6	0	4	4
3_S^S3	(3.9)	7	3	6	4
3_S^S4	(3.11)	7	3	4	5
3_S^S5	(3.11)	7	3	5	1
3_S^S6	(4.1)	2	4	2	4
4_S^S1	(4.2)	3	8	2	4
4_S^S2	(4.2)	5	2	4	4
4_S^S3	(4.3)	4	5	3	2
4_S^S4	(4.5)	5	3	6	5
4_S^S5	(4.7)	1	4	0	5
4_S^S6	(4.8)	5	5	4	4
4_S^S7	(4.10)	1	0	0	0
4_S^S8	(4.10)	2	4	0	2
4_S^S9	(4.1)	1	5	1	2
4_S^S10	(4.6)	4	0	5	1
4_S^S11	(4.6)	1	6	1	4
4_S^S12	(4.9)	3	3	4	5
4_S^S13	(5.0)	0	1	0	2
4_S^S14	(5.2)	6	2	5	2
4_S^S15	(5.2)	6	0	6	1
4_S^S16	(5.3)	0	6	0	6
4_S^S17	(5.4)	2	4	2	4
4_S^S18	( )	5	1	6	2
4^S19	( )	0	0	0	2
sum:		84	75	72	80
mean:		3.36	3.00	2.88	3.20

Grand Sum = 311

$\overline{X}$ = 12.44

Appendix A
Expt. 3

Table A3-1

All errors on small set by subject, presentation condition, and grade.

Kindergarten - median age 5 - 6½
range 4.11 - 6.00

subject (N = 18)	Question Age	static Number	static Length	transformed Number	transformed Length
KS_1	(4.11)	3	1	5	1
KS_2	(5.1)	0	1	0	1
KS_3	(5.2)	2	5	1	5
KS_4	(5.2)	4	5	6	6
KS_5	(5.3)	5	2	4	1
KS_6	(5.5)	0	6	1	6
KS_7	(5.6)	4	2	4	3
KS_8	(5.6)	0	0	2	1
KS_9	(5.6)	6	1	6	2
KS_{10}	(5.7)	0	3	0	4
KS_{11}	(5.7)	0	1	0	3
KS_{12}	(5.7)	0	1	0	2
KS_{13}	(5.7)	0	1	0	2
KS_{14}	(5.9)	0	1	0	1
KS_{15}	(5.9)	0	4	0	4
KS_{16}	(5.10)	0	6	0	6
KS_{17}	(5.10)	0	4	0	4
KS_{18}	(6.1)	0	0	1	0
sum:		24	44	30	52
mean:		1.33	2.44	1.67	2.88

Grand Sum = 150

$\overline{X}$ = 8.33

Appendix A
Expt. 3

Table A3-1

All errors on small set by subject, presentation condition and grade.

Grade 1 - median age 6.6
range 6.2 - 7.1

subject (N = 25)	Question: Age	static Number	static Length	transformed Number	transformed Length
IS_1	(6.2)	0	6	0	6
IS_2	(6.2)	0	2	0	0
IS_3	(6.2)	0	1	0	2
IS_4	(6.3)	0	1	0	1
IS_5	(6.5)	0	0	0	0
IS_6	(6.4)	0	6	0	6
IS_7	(6.4)	0	0	0	0
IS_8	(6.5)	1	5	0	6
IS_9	(6.6)	0	5	1	2
IS_{10}	(6.6)	0	6	0	5
IS_{11}	(6.6)	1	0	1	4
IS_{12}	(6.6)	1	3	0	2
IS_{13}	(6.6)	0	0	0	0
IS_{14}	(6.7)	0	1	0	2
IS_{15}	(6.8)	0	2	0	2
IS_{16}	(6.9)	0	3	1	3
IS_{17}	(6.9)	0	3	4	5
IS_{18}	(6.10)	0	3	0	4
IS_{19}	(6.10)	0	6	0	6
IS_{20}	(6.10)	0	2	0	2
IS_{21}	(6.10)	0	2	0	0
IS_{22}	(6.11)	0	2	0	0
IS_{23}	(7.0)	0	2	0	3
IS_{24}	(7.1)	0	5	0	6
IS_{25}	(7.1)	5	2	6	2
sum:		8	68	13	69
mean:		.32	2.72	.52	2.76

Grand Sum = 158

$\overline{X} = 6.32$

Table A3-2

All errors on large set by subject, presentation condition, and grade.

Nursery School - median age 4.10
range 3.7 - 5.1

subject
(N = 27)
N for analyses = 25

Question:	Age	static Number	static Length	transformed Number	transformed Length
$3L_1$	(3.7)	5	6	5	6
$3L_2$	(3.9)	3	0	6	1
$3L_3$	(3.9)	8	7	5	6
$3L_4$	(3.11)	2	7	6	4
$3L_5$	(3.11)	6	0	6	1
$3L_6$	(4.1)	4	1	6	0
$3L_7$	(3.6)	4	0	3	4
$3L_8$	(3.9)	6	3	6	3
$4L_1$	(4.5)	6	2	7	2
$4L_{2*}$	(4.9)	4	2	5	2
$4L_3$	(4.9)	6	3	9	4
$4L_4$	(4.10)	6	0	4	6
$4L_5$	(4.10)	6	2	6	1
$4L_6$	(5.1)	6	3	6	3
$4L_7$	(5.1)	4	2	5	1
$4L_8$	(4.5)	6	0	5	0
$4L_9$	(4.6)	5	5	4	2
$4L_{10}$	(4.6)	0	0	2	3
$4L_{11}$	(4.7)	6	0	6	0
$4L_{12}$	(4.7)	6	0	6	0
$4L_{13}$	(4.9)	6	0	6	0
$4L_{14*}$	(4.9)	6	0	7	2
$4L_{15}$	(4.9)	7	3	4	6
$4L_{16}$	(5.0)	4	1	6	1
$4L_{17}$	(5.1)	7	1	6	3
$3L_9$	( )	6	2	7	3
$3L_{10}$	( )	6	0	4	3
sum:		131	48	136	63
mean:		5.24	1.92	5.44	2.52

Grand Sum = 378

$\overline{X}$ = 15.12

* subjects not included in Anova, total scores, or means.

Table A3-2

All errors on large set by subject, presentation condition, and grade.

Kindergarten - median age 5.5
range 4.11 - 5.9

subject (N = 18)	Question: Age	static Number	static Length	transformed Number	transformed Length
KL1	(4.11)	6	0	5	0
KL2	(4.11)	5	0	6	0
KL3	(5.0)	4	2	4	4
KL4	(5.0)	7	0	6	1
KL5	(5.1)	6	3	4	2
KL6	(5.2)	6	0	6	0
KL7	(5.3)	6	1	5	0
KL8	(5.4)	6	0	6	0
KL9	(5.5)	0	0	0	1
KL10	(5.5)	6	1	8	3
KL11	(5.6)	6	2	6	2
KL12	(5.6)	8	4	6	3
KL13	(5.6)	6	0	6	0
KL14	(5.6)	7	2	8	5
KL15	(5.7)	0	1	1	2
KL16	(5.8)	6	0	6	0
KL17	(5.9)	7	1	6	0
KL18	(5.9)	6	2	4	1
sum:		98	19	93	24
mean:		5.44	1.056	5.17	1.33

Grand Sum = 234

$\overline{X}$ = 13.00

Table 13-2

All errors on large set by subject, presentation condition, and grade.

Grade One - median age 6.6
range 4.1 - 7.0

subject (N = 27) N for analyses = 25		static		transformed	
	Question Age	Number	Length	Number	Length
IL1	(5.1)	2	5	3	4
IL2	(6.0)	6	0	6	0
IL3	(6.0)	5	1	4	1
IL4	(6.0)	4	0	0	2
IL5	(6.0)	6	0	6	0
IL6	(6.3)	1	0	0	0
IL7	(6.3)	7	4	7	4
IL8	(6.3)	1	0	0	0
IL9	(6.4)	3	0	4	1
IL10	(6.4)	1	0	0	0
IL11	(6.5)	5	1	5	0
IL12	(6.5)	1	6	0	6
IL13	(6.5)	1	1	1	0
IL14	(6.6)	3	0	5	0
IL15	(6.6)	2	0	3	0
IL16	(6.7)	1	2	3	0
IL17	(6.7)	0	1	0	0
IL18	(6.7)	0	1	0	0
IL19	(6.7)	5	0	6	0
IL20	(6.7)	4	2	0	1
IL21	(6.9)	1	0	0	0
IL22*	(6.10)	1	1	1	0
IL23	(6.10)	0	0	2	0
IL24	(6.10)	4	5	6	0
IL25	(6.10)	0	0	0	0
IL26*	(6.11)	0	0	0	0
IL27	(7.0)	0	0	0	0
sum:		63	29	61	19
mean:		2.52	1.16	2.44	.76

Grand Sum = 172

$\overline{X} = 6.88$

* subjects not included in Anova, total scores, and means

Table A3-3

Systematic errors on small set by subject, presentation condition, question, and grades.

Nursery School

subject
(N = 25)
N for analyses = 20

Question	static Number	static Length	transformed Number	transformed Length
3S1	1	2	1	5
3S2	3	0	3	2
3S3*	2	3	3	2
3S4*	3	1	1	2
3S5	4	1	3	0
3S6	2	4	2	4
4S1*	1	4	0	1
4S2	4	2	3	3
4S3	2	4	2	1
4S4*	3	2	3	2
4S5	0	2	0	4
4S6	2	2	4	3
4S7	1	0	0	0
4S8	2	3	0	2
4S9	0	4	1	2
4S10	4	0	5	1
4S11	0	6	0	3
4S12*	2	2	2	3
4S13	0	1	0	0
4S14	5	2	5	1
4S15	6	0	6	0
4S16	0	6	0	6
4S17	2	4	1	4
4S18	5	0	5	0
4S19	0	0	0	0
sum:	43	43	41	41
mean:	2.15	2.15	2.05	2.05

Grand Sum = 168

$\overline{X} = 8.4$

* subjects classified as Guessers, not included in Anova, total scores, and means.

Table A3-3

Systematic errors on small set by subject, presentation condition, question and grade.

Kindergarten

Subject (N=18) N=17 for analysis	Static		Transformed	
Question:	Number	Length	Number	Length
KS1	3	1	4	0
KS2	0	0	0	0
KS3	1	5	1	5
KS4*	2	2	3	3
KS5	3	0	3	1
KS6	0	5	1	6
KS7	4	2	2	3
KS8	0	0	2	1
KS9	3	0	2	0
KS10	0	3	0	4
KS11	0	1	0	2
KS12	0	1	0	2
KS13	0	0	0	2
KS14	0	0	0	0
KS15	0	4	0	4
KS16	0	6	0	6
KS17	0	4	0	4
KS18	0	0	1	0
(Sum)	14	32	16	40
(mean)	.82	1.88	1.00	(2.29)

Grand Total = 102

$\overline{X} = 6.00$

* subject classified as guessers, not included in Anvoa, total scores, or means.

Appendix A

Expt. 3

Table A3-3

Systematic errors on small set by subject, presentation condition, question, and grade

Grade 1

Subject (N=25) N=24 for analysis	Static		Transformed	
Question	Number	Length	Number	Length
IS1	0	6	0	6
IS2	0	2	0	0
IS3	0	1	0	1
IS4	0	1	0	0
IS5	0	0	0	0
IS6	0	6	0	6
IS7	0	0	0	0
IS8	1	5	0	6
IS9	0	5	0	2
IS10	0	6	0	5
IS11	1	0	0	2
IS12	0	0	0	2
IS13	0	0	0	0
IS14	0	1	0	0
IS15*	0	2	0	1
IS16	0	3	0	3
IS17	0	1	0	2
IS18	0	3	0	2
IS19	0	6	0	6
IS20	0	2	0	1
IS21	0	2	0	0
IS22	0	2	0	0
IS23	0	0	0	0
IS24	0	5	0	6
IS25	3	1	6	0
Sum	5	58	6	50
Mean	.21	2.42	.25	2.08

Grant Total = 119

$\bar{X}$ = 4.96

*Subjects classified as guessers, not included in Anova, total scores, or means

Appendix A

Expt. 3

Table 3-4

Systematic errors on large set by subject, presentation condition, question, and grade

Nursery School

Subject (N=27) N=24 for analysis

Question	Static Number	Static Length	Transformed Number	Transformed Length
3L1*	4	0	2	2
3L2	2	0	5	0
3L3*	1	0	2	2
3L4	1	4	5	1
3L5	6	0	5	0
3L6	3	1	5	0
3L7	3	0	0	1
3L8	2	0	3	0
4L1	4	0	5	0
4L2	3	2	2	1
4L3*	4	1	3	0
4L4	4	0	4	5
4L5	4	1	4	0
4L6	3	2	5	2
4L7	3	0	4	0
4L8	3	0	3	0
4L9	3	2	4	2
4L10	0	0	0	0
4L11	6	0	6	0
4L12	6	0	6	0
4L13	6	0	6	0
4L14	6	0	6	1
4L15	3	0	2	2
4L16	4	1	5	0
4L17	5	1	3	1
3L9	3	1	4	2
3L10	6	0	4	3
Sum	89	15	96	21
Mean	3.79	.63	4.00	.83

Grand Total = 221

$\bar{X} = 9.208$

Table 3-4

Systematic errors on large set by subject presentation condition, question, and grade

Kindergarten

Subject (N=18)
N for analysis = 15

Question	Static Number	Static Length	Transformed Number	Transformed Length
KL1	6	0	5	0
KL2	5	0	5	0
KL3	3	2	3	2
KL4	6	0	6	0
KL5	5	1	4	2
KL6	6	0	6	0
KL7	6	1	5	0
KL8	6	0	6	0
KL9	0	0	0	0
KL10*	5	0	6	0
KL11	4	0	4	0
KL12*	3	1	3	1
KL13	6	0	6	0
KL14*	3	0	2	1
KL15	0	0	0	0
KL16	6	0	6	0
KL17	6	0	6	0
KL18	5	1	2	0
Sum	70	5	64	4
Mean	4.67	.33	4.27	.27

Grant Total = 143

$\bar{X}$ = 9.53

Appendix A

Expt. 3

Table A3-4

Systematic errors on large set by subject, presentation condition, question, and grade

Grade 1

Subject (N=27)
N=24 for analysis

Question	Static Number	Static Length	Transformed Number	Transformed Length	
IL1*	1	3	1	1	
IL2	6	0	5	0	
IL3	5	1	4	1	
IL4	1	0	0	2	
IL5	6	0	6	0	
IL6	1	0	0	0	
IL7*	4	1	4	1	
IL8	1	0	0	0	
IL9	3	0	4	1	
IL10	1	0	0	0	
IL11	5	1	4	0	
IL12	0	4	0	6	
IL13	0	0	1	0	
IL14	3	0	2	0	
IL15	1	0	2	0	
IL16	0	2	3	0	
IL17	0	0	0	0	
IL18	0	1	0	0	
IL19	5	0	6	0	
IL20	2	1	0	1	
IL21	0	0	0	0	
IL22	0	0	1	0	
IL23	0	0	0	0	
IL24*	2	2	2	0	
IL25	0	0	0	0	
IL26	0	0	2	0	
IL27	0	0	0	0	
Sum	40	10	40	11	Σ101
Mean	1.67	.46	1.67	.46	

Grand Total = 101

$\bar{X}$ = 4.21

*Subjects classified as guessers, not included in Anova, total scores, or means

Table A4-1

All errors on critical terms, "longer" and "more" tested first

Critical Term

Subject	Age	Sex		more	less	longer	shorter
16	3.3	M		9	8	3	7
17	3.8	F		9	7	3	5
18	4.1	F		8	11	7	8
19	4.1	M		9	10	9	9
20	4.3	M		6	5	5	3
21	4.4	F		5	9	6	8
22	4.5	M		2	1	5	10
23	4.6	F		3	6	2	0
24	4.7	F		8	5	2	5
25	4.8	F		7	6	8	8
26	4.9	F		4	6	8	2
27	4.10	M		2	0	2	0
28	5.0	M		1	7	6	7
29	5.0	M		0	1	0	0
30	5.2	F		4	7	6	6
31	5.2	M		3	2	3	5
N=16	Sum		Sum	80	91	75	83
			$\overline{X}$	5.0	5.69	4.69	5.19

Grand Total = 329
$\overline{X}$ for subjects = 20.56
$\overline{X}$ for term 5.14

Appendix A
Expt. 4

Table A4-2

All Errors on Critical terms "shorter" and "less" first

Critical Term

Subject	Age	Sex	more	less	longer	shorter
1	3.9	F	3	7	5	3
2	4.1	M	3	0	6	7
3	4.2	F	10	8	7	9
4	4.3	F	9	11	5	6
5	3.4	M	3	3	6	4
6	4.5	M	3	2	6	7
7	4.5	M	9	8	7	9
8	4.5	F	3	9	5	3
9	3.9	M	12	7	7	5
10	4.9	F	0	3	6	5
11	4.10	F	1	5	6	5
12	4.11	F	4	3	6	4
13	4.11	M	1	7	1	3
14	5.0	M	2	8	0	7
15	5.0	M	1	7	12	10
N=15		Sum	64	88	85	87
		$\overline{X}$	4.27	5.87	5.67	5.8

Grand Total = 324

$\overline{X}$ for subjects = 20.56

$\overline{X}$ for term = 5.40

Table A4-3

Errors on congruent and incongruent configurations. Subjects classified as nonguessers.

Critical Term

	more		longer		less		shorter	
Subject	Cong.	Incong.	Cong.	Incong.	Cong.	Incong.	Cong.	Incong.
1	1	2	1	4	4	3	0	3
2	1	2	0	6	0	0	1	6
5	1	2	0	6	0	3	0	4
20	1	5	3	2	2	3	2	1
6	2	1	0	6	1	1	1	6
8	1	2	2	3	3	6	1	2
23	1	2	1	1	2	4	0	0
10	0	0	0	6	0	3	0	5
11	0	1	0	6	3	2	2	3
27	1	1	0	2	0	0	0	0
12	1	3	1	5	1	2	0	4
13	1	0	1	0	2	5	3	0
28	1	0	0	6	3	4	1	6
29	0	0	0	0	1	0	0	0
14	2	0	0	0	3	5	2	5
31	1	2	0	3	1	1	1	4
N=16	15	23	9	56	26	42	14	49
Incong. $\overline{X}$		1.44		3.50		2.63		3.0
cong $\overline{X}$	.0.94		.56		1.63		.88	

Grand total 2.34

$\overline{X}$ for subjects = 14.63

$\overline{X}$ for term = 3.66

Table A4-4

Errors on congruent and incongruent configurations. Subjects classified as guessers.

Questions	Critical Term							
	more		longer		less		shorter	
Subject	Cong.	Incong.	Cong.	Incong.	Cong.	Incong.	Cong.	Incong.
16	4	5	0	3	5	3	4	3
17	4	5	0	3	2	5	3	2
18	4	4	4	3	5	6	4	6
19	4	5	3	6	5	5	3	6
3	5	5	3	4	5	3	4	5
4	4	5	4	1	5	6	2	4
21	2	3	3	3	3	6	4	4
22	1	1	4	1	1	0	4	6
7	6	3	3	4	4	4	4	5
24	4	4	1	1	5	0	2	3
25	5	2	4	4	4	2	4	4
26	2	2	2	6	2	4	0	2
9	6	6	5	2	2	5	2	3
15	0	1	6	6	3	4	4	6
30	2	2	3	3	3	4	4	2
N=16	53	53	45	50	54	57	48	59
Incong		3.53		3.33		3.8		3.93
Cong	3.53		3.0		3.6		3.2	

Grand Total = 419

$\overline{X}$ for subjects = 27.93

$\overline{X}$ for term = 6.9825

Table A4-5

Age, conservation status, scores on length questions, Nonguessers.

Group	Age	Conservation	Length 1(longer)	Length 2(longer)	Length 3(same)
I					
S13	(4.11)	pass	1	1	1
S14	(5.0)	----	1	1	1
S23	(4.6)	pass	1	1	-
S27	(4.10)	pass	1	1	1
S29	(5.0)	pass	1	1	1
II					
S2	(4.1)	pass	1	1	1
S5	(3.4)	pass	1	1	1
S6	(4.5)	----	1	1	1
S10	(4.9)	pass	1	1	1
S11	(4.10)	pass	1	1	-
S12	(4.11)	pass	1	1	1
S28	(5.0)	pass	1	1	-
III					
S1	(3.9)	----	1	1	1
S8	(4.5)	----	1	1	1
S20	(4.3)	----	1	1	1
S31	(5.2)	----	1	1	1
		10	16	14	13

Table A4-6

Age, conservation status, scores on length questions, Guessers (N=15).

	Age	Conservation	Length 1(longer)	2(longer)	3(same)
16	3.3	----	1	1	-
17	3.8	----	1	1	-
18	4.1	----	1	1	-
19	4.1	----	1	-	-
3	4.2	----	1	1	-
4	4.3	----	1	1	1
21	4.4	----	1	1	-
22	4.5	----	1	1	-
7	4.5	----	1	-	-
24	4.7	----	1	1	1
25	4.8	----	1	1	-
26	4.9	----	1	1	1
9	3.9	----	1	1	1
15	5.0	----	1	1	-
30	5.2	pass	1	1	1
N=15		1	15	13	5

Table A4-7

Total number of intrusions on all terms, nongeussers

Critical term

Subject	r	more	R	less	R	longer	R	shorter	R
1	(11.5)	2	(1.5)	2	(1.5)	3	(3.5)	3	(3.5)
2	(10)	5	(4)	0	(1)	3	(3)	3	(2)
5	(11.5)	3	(2.5)	4	(4)	0	(1)	3	(2.5)
20	(24.0)	6	(3.5)	6	(3.5)	5	(2)	4	(1)
6	(20.0)	4	(3)	7	(4)	2	(1)	3	(2)
8	(22.5)	4	(1)	5	(2.5)	5	(2.5)	6	(4)
23	(15.5)	5	(4)	4	(3)	3	(2)	0	(1)
10	(2.5)	0	(2)	1	(4)	0	(2)	0	(2)
11	(6)	2	(3)	2	(3)	2	(3)	0	(1)
27	(8.5)	3	(4)	2	(3)	1	(1.5)	1	(1.5)
12	(25.6)	7	(3)	4	(2)	9	(4)	2	(1)
13	(22.5)	2	(1)	8	(4)	3	(2)	7	(3)
28	(17.0)	1	(2)	7	(4)	0	(1)	6	(3)
29	(4.0)	0	(2)	2	(4)	0	(2)	0	(2)
14	(6)	2	(3)	4	(4)	0	(1.5)	0	(1.5)
31	(2.5)	1	(4)	0	(2)	0	(2)	0	(2)
N = 16		47	(43.5)	58	(49.5)	36	(34)	36	(33)

R = within subject rank

r = across subjects rank

Table A4-8

Total number of intrusions on all terms, guessers

Critical term

Subject r	more R	less R	longer R	shorter R
16 (18.5)	1 (1)	4 (3)	7 (4)	3 (2)
17 (27)	9 (4)	3 (1)	7 (3)	4 (2)
18 (15.5)	3 (2.5)	5 (4)	3 (2.5)	1 (1)
19 (25.6)	5 (1.5)	6 (3.5)	5 (1.5)	6 (3.5)
3 (13.5)	2 (2.5)	6 (4)	1 (1)	2 (2.5)
4 (29.5)	5 (1)	11 (4)	7 (2.5)	7 (2.5)
21 (8.5)	3 (4)	2 (3)	1 (1.5)	1 (1.5)
22 (21.0)	1 (1)	3 (2)	6 (3)	7 (4)
7 (1)	0 (2.5)	0 (2.5)	0 (2.5)	0 (2.5)
24 (18.5)	2 (2.5)	11 (4)	0 (1)	2 (2.5)
25 (28)	3 (1)	9 (4)	4 (2)	8 (3)
26 (13.5)	4 (3)	5 (4)	1 (1.5)	1 (1.5)
9 (30.0)	12 (4)	11 (3)	6 (1.5)	6 (1.5)
15 (29.5)	2 (1)	4 (2)	12 (3.5)	12 (3.5)
30 (6)	3 (4)	2 (3)	0 (1)	1 (2)
N = 15	55 (35.5)	82 (47)	60 (32)	61 (35.5)

R = within subject rank

r = across subjects rank

Table A5-1

Number of trials to criterion on the nonverbal relational discriminations.

Subject	Age	Sex	Order *	1st discrimination	2nd discrimination
				More	Less
1	3.7	M	—	36	36
2	4.0	F	—	36	29
3	4.4	F	I	36	36
4	4.3	M	I	36	36
5	5.3	M	I	10	10
					Longer
6	3.11	F	I	25	36
7	3.9	M	I	28	23
8	4.4	M	—	36	36
9	4.9	F	—	10	10
10	4.11	F	—	18	36
					Shorter
11	3.1	M	—	36	36
12	4.4	M	I	36	10
13	4.10	F	—	10	18
14	5.2	M	I	12	18
15	3.8	F	I	36	27

* Session one: Discrimination - I
Session one Verbal - ⊢⊣

Table A5-1 (cont'd.)

Number of trials to criterion on the nonverbal relational discriminations.

Subject	Age	Sex	Order	1st discrimination	2nd discrimination
				Less	More
16	3.5	M	—	36	36
17	4.4	F	—	36	36
18	4.11	M	I	36	36
19	5.1	M	—	10	12
20	5.3	F	I	10	11
					Longer
24	3.7	M	I	36	36
25	3.11	F	I	36	36
21	4.2	F	I	10	10
22	4.7	F	—	36	36
27	5.3	M	—	13	36
					Shorter
28	3.8	M	—	36	36
29	3.10	F	I	18	17
30	4.4	M	—	14	36
31	4.9	F	—	36	13
32	5.0	M	I	10	24

Table A5-1 (cont'd.)

Number of trials to criterion on the nonverbal relational discriminations.

Subject	Age	Sex	Order	1st discrimination	2nd discrimination
				Longer	More
33	3.6	F	—	19	36
34	3.11	M	I	32	36
35	4.1	F	—	10	36
36	4.2	M	—	10	27
S_x	5.1	F	I	36	36
					Less
37	3.7	F	—	36	36
38	4.2	M	—	12	28
39	4.6	M	I	36	36
40	4.10	F	I	10	36
41	5.0	M	I	36	36
					Shorter
42	3.7	M	I	25	20
43	4.4	M	I	28	29
44	4.3	F	I	13	10
45	4.7	F	—	28	36
46	4.11	F	—	12	36

Table A5-1 (cont'd.)

Number of trials to criterion.

Subject	Age	Sex	Order	1st discrimination	2nd discrimination
				Shorter	More
47	3.5	M	—	36	36
48	3.2	F	—	10	36
49	3.9	M	I	13	19
50	4.4	M	—	10	15
51	4.10	F	—	10	22
				Shorter	Less
52	3.5	F	I	11	36
53	4.1	M	I	13	36
55	4.3	F	—	13	17
57	4.11	F	—	22	16
58	5.3	M	I	10	36
				Shorter	Longer
59	3.4	F	—	10	13
60	4.3	F	I	11	36
61	4.6	M	—	10	13
63	5.2	M	I	10	12
64	5.1	F	—	9	10

Table A5-2

Total number of errors on critical terms by discrimination groups.

Discrimination Group		more	longer	less	shorter
	S1	3	2	2	2
more	S2	2	3	4	4
-less	S3	4	3	5	1
	S4	3	2	4	2
	S5	1	1	0	1
	S6	4	2	6	5
more	S7	5	4	5	4
-longer	S8	4	4	5	4
	S9	3	2	3	2
	S10	0	1	0	1
	S11	5	5	6	6
more	S12	1	2	4	5
-shorter	S13	2	2	2	3
	S14	4	2	1	1
	S15	5	2	6	4
	S16	4	4	3	4
less	S17	6	6	6	5
-more	S18	3	0	4	4
	S19	0	1	0	0
	S20	2	4	2	1

Table A5-2 (cont'd.)

Total number of errors on critical terms by discrimination groups.

Discrimination		more	longer	less	shorter
	S24	4	2	6	1
less	S25	4	1	3	1
-longer	S21	4	3	4	1
	S22	1	1	3	0
	S27	0	3	1	2
	S28	2	3	5	3
less	S29	5	3	3	3
-shorter	S30	3	3	3	3
	S31	4	2	4	2
	S32	0	2	1	1
	S33	4	1	3	4
longer	S34	3	2	4	6
-more	S35	0	1	0	1
	S36	0	1	0	1
	S_x	2	2	3	2
	S37	4	4	4	4
longer	S38	3	4	4	2
-less	S39	3	2	4	4
	S40	2	2	5	1
	S41	0	2	3	4

Table A5-2 (cont'd.)

Total number of errors on critical terms by discrimination groups.

Discrimination		more	longer	less	shorter
	S42	1	1	2	4
longer	S43	4	3	4	4
-shorter	S44	1	3	1	3
	S45	1	4	5	1
	S46	1	1	1	2
	S47	6	4	5	4
shorter	S48	5	2	4	4
-more	S49	2	3	2	3
	S50	2	1	3	0
	S51	0	1	0	1
	S52	3	2	3	0
shorter	S53	5	3	5	2
-less	S55	0	1	3	1
	S57	1	2	0	3
	S58	3	4	5	2
	S59	3	1	3	2
shorter	S60	0	1	3	3
longer	S61	4	4	4	3
	S63	0	2	2	1
	S64	4	1	5	2
	Total	159	138	175	142
	Mean	2.65	2.30	2.92	2.37

Table A5-3

Responses on congruent and incongruent stimuli - all subjects

Subject	More Congruent	More Incongruent	Longer Congruent	Longer Incongruent
	O	C L O	O	C N O
1	2	2 1 -	1	2 1 -
2	2	3 - -	2	2 1 -
3	2	1 1 1	1	1 2 -
4	1	1 2 -	1	2 1 -
5	-	2 1 -	1	3 - -
6	2	1 1 1	1	2 - 1
7	2	- 3 -	2	1 2 -
8	2	1 2 -	3	2 1 -
9	2	2 - 1	-	1 2 -
10	-	3 - -	-	2 1 -
11	2	- - 3	2	- - 3
12	-	2 - 1	-	1 2 -
13	1	2 - 1	1	2 1 -
14	1	- 3 -	1	2 - 1
15	2	- 2 1	1	2 - 1

C - Correct
L - Length Assimilation Error
N - Number Assimilation Error
O - Unclassifiable Error

Table A5-3 (cont'd.)

Responses on congruent and incongruent stimuli - all subjects

	More		Longer	
Subject	Congruent	Incongruent	Congruent	Incongruent
	O	C L O	O	C N O
16	1	- 3 -	3	2 1 -
17	3	- 1 2	3	- - 3
18	1	1 - 2	-	3 - -
19	-	3 - -	-	2 1 -
20	2	3 - -	2	1 2 -
24	2	1 2 -	2	3 - -
25	1	- 2 1	1	3 - -
21	2	1 2 -	1	1 2 -
22	1	3 - -	-	2 1 1
27	-	3 - -	-	- 3 -
28	1	2 - 1	-	- 2 1
29	3	1 - 2	2	2 - 1
30	1	1 - 2	1	2 1 -
31	2	1 2 -	-	2 - 1
32	-	3 - -	-	1 2 -

Table A5-3 (cont'd.)

Responses on congruent and incongruent stimuli - all subjects

	More		Longer	
Subject	Congruent	Incongruent	Congruent	Incongruent
33	2	1 - 2	-	2 - 1
34	-	- 2 -	2	3 - -
35	2	1 2 -	2	2 1 -
36	3	3 - -	-	2 1 -
x	1	2 1 -	1	2 1 -
37	2	1 1 1	2	1 1 1
38	1	1 1 1	1	- 3 -
39	1	1 2 -	1	2 1 -
40	1	2 1 -	1	2 1 -
41	-	3 - -	1	2 1 -
42	1	3 - -	-	3 - -
43	1	- 3 -	2	2 - 1
44	1	3 - -	-	- 3 1
45	-	2 1 -	-	1 1 1
46	-	2 1 -	-	2 1 -

Table A5-3 (cont'd.)

Responses on congruent and incongruent stimuli - all subjects

	More		Longer	
Subject	Congruent	Incongruent	Congruent	Incongruent
	O	C L O	O	C N O
47	3	- 3 -	1	1 - 2
48	3	1 1 1	2	3 - -
49	-	1 - 2	-	- 3 -
50	-	1 2 -	-	2 1 -
51	-	3 - -	-	2 1 -
52	1	1 - 2	-	1 1 1
53	3	1 - 2	2	2 - 1
55	-	3 - -	-	2 1 -
57	1	3 - -	-	1 2 -
58	2	2 1 -	2	1 2 -
59	1	1 1 1	-	2 - 1
60	-	3 - -	-	2 1 -
61	1	1 1 1	3	2 - 1
63	-	3 - -	1	2 - 1
64	2	1 - 2	-	2 1 -

Table A6-1

Responses on Standard Language Task by subject, configuration, question, and type of error. Age and conservation status indicated.

Legend

C = correct

Ø = unexplained

L = length strategy

N = number strategy

Cr = conserver

		Nonguessers (N=18)										Conservation
		More					Longer					
		Cong.		Incong.			Cong.		Incong.			
Subject	(age)	C	Ø	C	L	Ø	C	Ø	C	N	Ø	
S6	(4.0)	3	–	3	–	–	3	–	2	1	–	
S7	(4.2)	3	–	3	–	–	3	–	–	3	–	
S9	(4.4)	3	–	–	3	–	2	1	3	–	–	
S13	(4.7)	3	–	3	–	–	3	–	3	–	–	
S14	(4.9)	3	–	3	–	–	3	–	2	1	–	
S15	(4.9)	3	–	3	–	–	3	–	2	1	–	Cr
S16	(4.10)	3	–	3	–	–	3	–	2	1	–	
S18	(5.0)	2	1	3	–	–	3	–	3	–	–	
S19	(5.0)	3	–	3	–	–	3	–	2	1	–	Cr
S24	(5.3)	3	–	3	–	–	2	1	2	1	–	Cr
S26	(5.5)	3	–	1	–	2	3	–	–	3	–	Cr
S27	(5.7)	3	–	3	–	–	3	–	2	1	–	
S28	(5.8)	3	–	2	–	1	3	–	–	3	–	Cr
S29	(5.8)	3	–	2	1	–	3	–	2	1	–	
S31	(5.9)	3	–	3	–	–	3	–	–	3	–	Cr
S32	(6.1)	3	–	3	–	–	3	–	1	2	–	Cr
S34	(6.4)	3	–	3	–	–	3	–	2	1	–	Cr
S35	(6.4)	3	–	3	–	–	3	–	3	–	–	Cr

Table A6-1

Responses on Standard Language Task by subject, configuration, question, and type of error. Age and conservation status indicated.

		Guessers (N = 18)											Conservation
		More					Longer						
		Cong.		Incong.			Cong.		Incong.				
Subject	(age)	C	Ø	C	L	Ø	C	Ø	C	N	Ø		
1	(3.0)	1	2	-	-	3	1	2	2	-	1		
2	(3.3)	-	3	-	1	2	1	2	1	-	2		
3	(3.5)	1	2	1	1	1	-	3	1	1	1		
4	(3.7)	1	2	1	2	-	2	1	1	2	-		
5	(4.0)	1	2	-	3	-	3	-	3	-	-		
8	(4.2)	1	2	1	2	-	1	2	1	1	1		
10	(4.4)	3	-	2	1	-	1	2	2	-	1		
11	(4.5)	-	3	-	1	2	2	1	-	1	2		
12	(4.6)	2	1	1	-	2	2	1	-	2	1		
17	(4.10)	1	2	1	1	1	2	1	3	-	-		
20	(5.2)	2	1	2	1	-	2	1	2	1	-		
21	(5.2)	1	2	1	1	1	2	1	3	-	-		
22	(5.2)	1	2	2	-	1	1	2	-	2	1		
23	(5.3)	2	1	1	2	-	2	1	-	1	2		
25	(5.4)	1	2	2	1	-	2	1	-	-	3		
30	(5.9)	2	1	2	1	-	1	2	1	2	-		
33	(6.3)	-	3	2	1	-	1	2	2	1	-		
36	(6.4)	1	2	2	-	1	1	2	2	1	-		

Table A6-2.

Responses on length task given in conjunction with verbal task.

Guessers

Array:	1	2	3
S1	–	1	–
S3	–	1	1
S4	–	1	–
S5	–	–	–
S8	–	1	–
S10	–	1	1
S11	–	1	–
S12	–	–	–
S17	–	–	–
S20	–	1	–
S21	–	–	–
S22	–	1	–
S23	1	–	–
S25	1	1	–
S30	–	–	–
S33	–	1	–
S36	–	1	–
	2	11	2

Nonguessers

Array:	1	2	3
S6	–	–	–
S7	–	1	–
S9	–	–	–
S13	–	–	–
S14	–	–	–
S15	–	–	–
S16	–	–	–
S18	–	–	–
S19	–	–	–
S20	–	1	–
S24	–	–	–
S26	–	–	–
S27	–	–	–
S28	–	–	–
S29	–	–	–
S31	–	1	–
S34	–	–	–
S35	–	1	–
	0	4	0

	Guessers	Nonguessers
Total Errors	15	4
Correct	39	50

Legend

– correct

1 error

Table A6-3

Responses on Standard Toy and Fit Tasks by subject, configuration, question and type of error.

Ø - unexplained
- - correct
N - number strategy
L - Length strategy

Nonguessers

	subtask (a) configuration 3				subtask (b) configuration 4			
	Beads		String	Fit	Blocks		Boxes	Fit
	Length	Number	Length		Length	Number	Length	
S6	0	-	-	-	-	-	-	L
S7	N	-	Ø	-	N	-	-	-
S9	-	Ø	-	-	-	-	-	Ø
S13	-	-	-	-	-	-	-	-
S14	-	-	-	-	-	-	-	L
S15	-	-	-	L	N	-	-	-
S16	-	-	-	-	-	L	-	-
S18	-	-	-	-	-	-	-	L
S19	-	-	-	L	-	-	-	-
S24	-	-	-	-	-	-	-	-
S26	N	Ø	-	L	N	-	-	-
S27	-	-	-	L	N	-	-	-
S28	N	-	-	-	N	-	-	L
S29	-	-	-	-	N	-	-	-
S31	N	-	-	L	N	-	-	-
S32	-	-	Ø	L	-	-	-	-
S34	N	-	-	-	N	-	-	-
S35	-	-	-	L	-	-	-	L

Table A6-3 (cont'd)

Nonguessers

	subtask (c) configuration 5				subtask (d) configurations 2 & 3				
	Cars		Trucks	Fit	Swans		Nests		Fit
	Length	Number	Length		Length	Number	Length	Number	
S6	–	–	–	–	–	–	–	–	–
S7	–	L	–	L	–	–	N	–	–
S9	–	L	Ø	–	–	–	–	L	L
S13	–	–	–	–	–	–	–	–	–
S14	–	–	–	–	–	–	–	–	–
S15	–	–	–	–	–	–	–	–	–
S16	–	Ø	–	Ø	–	–	–	–	–
S18	–	L	–	–	–	–	–	–	–
S19	–	L	–	–	–	–	–	L	Ø
S24	–	–	–	L	–	–	–	–	–
S26	N	–	–	–	–	–	N	–	–
S27	–	–	–	–	–	–	–	–	–
S28	N	–	–	–	–	–	–	–	–
S29	N	–	–	–	–	–	–	–	–
S31	Ø	–	–	–	–	–	N	–	–
S32	–	–	–	L	–	–	N	–	–
S34	–	–	Ø	–	N	–	–	–	–
S35	–	–	–	–	–	–	–	–	L

Table A6-3 (cont'd.)

Guessers

	subtask (a) configuration 3				subtask (b) configuration 4			
	Beads		String	Fit	Blocks		Boxes	Fit
	Length	Number	Length		Length	Number	Length	
S1	–	L	–	L	N	–	–	Ø
S2	Ø	Ø	Ø	L	Ø	Ø	Ø	Ø
S3	N	–	–	L	N	Ø	–	Ø
S4	N	–	–	Ø	N	Ø	–	L
S5	–	–	–	L	–	L	–	L
S8	–	L	–	L	N	Ø	–	–
S10	–	L	–	L	N	–	–	Ø
S11	–	L	–	L	Ø	Ø	–	–
S12	N	–	–	–	–	–	–	–
S17	–	L	–	L	–	L	–	–
S20	–	–	–	–	Ø	–	–	–
S21	Ø	Ø	–	–	–	L	–	Ø
S22	N	–	–	–	N	–	–	–
S23	N	–	–	–	N	–	–	–
S25	–	L	–	L	–	L	–	–
S30	–	–	–	–	N	–	–	–
S33	–	L	–	Ø	Ø	Ø	–	Ø
S36	–	–	–	–	N	–	–	–

Table A6 3 (cont'd)

Guessers

	subtask (c) configuration 5				subtask (d) configurations 2 & 3				
	Cars		Trucks	Fit	Swans		Nests		Fit
	Length	Number	Length		Length	Number	Length	Number	
S1	–	L	–	–	–	–	–	N	L
S2	Ø	Ø	Ø	Ø	Ø	Ø	–	Ø	L
S3	Ø	L	–	–	Ø	Ø	N	L	Ø
S4	Ø	Ø	–	–	–	–	N	–	–
S5	–	L	–	L	–	–	–	L	L
S8	–	L	–	L	–	–	–	L	–
S10	–	L	–	L	–	–	–	L	L
S11	–	L	–	L	–	Ø	N	–	–
S12	–	L	–	L	–	–	N	–	–
S17	N	–	–	–	–	–	–	L	L
S20	–	Ø	Ø	–	–	–	–	–	–
S21	–	L	–	Ø	–	–	N	–	–
S22	–	L	–	L	–	–	N	–	–
S23	–	L	–	L	–	–	N	–	–
S25	Ø	–	Ø	–	Ø	–	Ø	–	–
S30	–	L	–	L	–	–	–	–	Ø
S33	–	L	–	L	–	–	N	–	–
S36	–	Ø	Ø	–	–	–	N	–	–

Table A6-3 (cont'd.)

subtask (e)

	Nonguessers				Guessers		
	space	span	fit		space	span	fit
S6	–	–	–	S1	–	–	Ø
S7	–	–	–	S2	[illegible]	Ø	Ø
S9	–	–	–	S3	–	–	–
S13	–	–	–	S4	–	–	–
S14	–	–	–	S5	–	–	–
S15	–	–	–	S8	–	–	–
S16	–	–	–	S10	–	–	–
S18	–	–	–	S11	Ø	Ø	–
S19	–	–	–	S12	–	–	–
S20	–	–	–	S17	–	–	–
S24	–	–	–	S20	–	–	–
S26	–	–	–	S21	–	–	–
S27	–	–	–	S22	–	–	–
S28	–	–	–	S23	–	–	–
S29	–	–	–	S25	Ø	–	–
S31	–	–	–	S30	–	–	–
S32	–	–	–	S33	–	–	–
S34	–	–	–	S36	–	–	–
S35	–	–	–				

Table A6-4

Responses on Sticks and Pennies Task

	Guessers Sticks	Guessers Pennies		Nonguessers Sticks	Nonguessers Pennies
S1	–	1	S6	–	1
S2	1	–	S7	1	–
S3	–	–	S9	–	–
S4	1	–	S13	–	–
S5	–	–	S14	–	–
S8	1	1	S15	–	–
S10	1	–	S16	–	–
S11	1	–	S18	1	1
S12	1	–	S19	1	–
S17	–	–	S24	–	–
S20	1	–	S26	–	1
S21	–	1	S27	1	–
S22	1	1	S28	–	1
S23	1	–	S29	–	–
S25	–	1	S31	1	–
S30	1	–	S32	–	–
S33	1	–	S34	–	–
S36	–	1	S35	–	–

– indicates a correct choice

1 indicates an error

Table A6-5

Abbreviated forms of subject's responses on production task.

Guessers		Nonguessers	
S1	cars & traffic	S6	-noted number
S2	that one & that one	S7	longer, 3 little
S3	car	S9	longer, 1 & 3 smaller
S4	longer	S13	longer-shorter
S5	lots	S14	-------------------
S8	I don't know	S15	more than, bigger
S10	blocks & wheels	S16	in the middle of a train
S11	a car	S18	longest-because 4
S12	is short	S19	wider; shorter
S17	------------------	S24	longer & longer
S20	(3)bigger (1)more	S26	---------------
S21	(2)longer (1)shorter	S27	car description
S22	more longer	S28	more
S28	made it	S29	bigger
S25	three and four	S31	----------------
S30	smaller	S32	little, more
S33	longer	S34	----------------
S36	-----------------	S35	skinny

Table A6-6

Responses on the "Show Me" portion of the Cars Task.

Nonguessers

Question

	1 "more"	2 "longer"	3 "longer houses"	4 same L	5 same Number
S6	3(S)	1(N)	-	1Ø	Ø
S7	2(L)	1(N)	1(N)	-	-
S9	3(S)	-	2(L)	2&3(N)	2Ø
S13	-	1(N)	2(L)	-	3Ø
S14	-	1(N)	2(L)	-	-
S15	-	1(N)	1(N)	2Ø	-
S16	-	-	1(N)	2Ø	-
S18	-	-	-	(1&2)Ø(S)	Ø
S19	-	-	-	-	-
S24	2(L)	-	2(L)	-	-
S26	-	1(N)	-	2Ø	-
S27	-	-	-	-	(1&3)L
S28	-	1(N)	1(N)	2&3(N)	-
S29	-	-	2(L)	2&3(N)	-
S31	2(L)	1(N)	-	1Ø	3Ø
S32	3(S)	1(N)	1(N)	(2&3)(N)	-
S34	-	3(S)	2(L)	(2&3)(N)	-
S35	-	-	-	-	-

- correct
  number refers to line(s) chosen in error
  letter refers to type of error
N = number
L = length
Ø = unexplained
S = size

Table A6-6 (cont'd.)

Responses on the "Show Me" Task

Guessers

Question

	1 "more"	2 "longer"	3 "longer houses"	4 same L	5 same Number
S1	3(S)	1(N)	-	2∅	3∅
S2	3(S)	3(S)	-	3∅	0
S3	-	-	2(L)	1∅	3∅
S4	3(S)	1(N)	1(N)	1∅	2∅
S5	2(L)	-	2(L)	-	-
S8	3(S)	-	2(L)	1∅	∅
S10	-	-	2(L)	3∅	3∅
S11	-	1(N)	1(N)	(2&3)(N)	3∅
S12	3(S)	3(S)	1(N)	(1&2)(S)	2∅
S17	3(S)	-	-	2∅	1∅
S20	-	3(S)	2(L)	-	-
S21	2(L)	-	-	-	(1&3)(L)
S22	2(L)	1(N)	-	1∅	3∅
S23	2(L)	3(S)	1(N)	-	3∅
S25	-	-	-	(2&3)(N)	1∅
S30	2(L)	-	1(N)	(2&3)(N)	-
S33	3(S)	1(N)	1(N).	(12∅)(S)	all.
S36	-	1(N)	-	-	-

Table A6-7

Responses on the "Make me a Row Task"

Nonguessers

	1	2	3	4	5
	now	same L	longer	more	same N
S6	1	1	–	1	–
S7.	–	–	–	–	–
S9	–	–	–	–	1
S13	–	–	–	–	–
S14	–	–	–	–	–
S15	–	–	–	–	–
S16	–	–	–	–	–
S18	1	1	1	1	1
S19	1	1	–	–	–
S24	–	–	–	–	–
S26	–	–	–	–	–
S27	–	–	–	–	–
S28	1	1	1	1	1
S29	–	–	–	–	–
S31	–	–	–	–	–
S32	1	1	1	1	1
S34	–	–	–	–	–
S35	–	1	–	–	1

Table A6-7

Responses on the "Make me a Row Task"

	Guessers				
	1	2	3	4	5
	now	same L	longer	more	same N
S1	-	1	-	1	1
S2	1	1	1	1	1
S3	-	1	1	1	1
S4	-	1	-	1	1
~~S5~~	-	1	-	1	1
S8	-	1	1	1	-
S10	-	1	1	1	1
S11	-	1	-	-	1
S12	1	1	-	-	-
S17	-	-	-	1	-
S20	-	1	1	1	1
S21	1	1	1	1	1
S22	-	-	-	-	-
S23	1	1	-	-	1
S25	-	-	-	-	-
S30	-	-	-	-	-
S33	1	1	-	-	-
S36	-	-	-	-	1

Legend
- correct
1 incorrect

Table B3-1

Source table for three factor mixed design Anova on all errors. Between factors: set size; within factors: presentation condition and question.

Source	SS	df	MS	F	p
Total	3176.6	543			
Between subjects	1257.4	135			
Set Size	50.05	1	50.05	5.55	<.05
$Error_b$	1207.31	134	9.01		
Within subjects					
transformation	0.811	1	0.81	0.83	
question	91.41	1	91.41	12.03	<.01
set size x transf.	0.05	1	0.04	0.05	
set size x ques.	530.09	1	530.09	69.76	<.001
transf. x ques.	1.34	1	1.34	1.23	
set size x t x qu.	0.01	1	0.01	0.01	
$Error_1$	130.9	134			
$Error_2$	1018.3	134			
$Error_3$	146.4	134			

Table B3-2

Source table for three factor mixed design Anova on all errors. Between factor: set size, within factors: presentation condition and question.

Source	SS	df	MS	F	p
Total	2180.2600	495	4.4046		
Between subjects	625.01				
Set Size	10.74	1	10.7442	2.134	
$Error_b$	614.26	122	5.0349		
Within subjects					
transformation	0.0505	1	0.0505	0.1002	
question	75.0989	1	75.0989	10.44	<.005
set size x transf.	0.1629	1	0.1629	0.3229	
set size x ques.	451.0660	1	451.0660	62.7063	<.001
transf. x ques.	0.0017	1	0.0017	0.0023	
set size x t x s.s.	0.2432	1	0.2432	0.3315	
$Error_1$	61.5366	122	0.5044		
$Error_2$	877.5850	122	7.1933		
$Error_3$	89.5029	122	0.7336		

Table B3-3

Source table for treatment (presentation) by treatment (question) by subjects Anova on subjects classified as length responders

	SS	df	MS	F	p
Presentation	2.7612	1	2.7612	1.3806	
question	798.03	1	798.03	399.0210	<.0001
pres. x ques.	.4580	1	.480	.229	
residual	389.9970	195	2.000		
Total	1544.540	263			

Table B3-4

Source table for treatment (presentation) by treatment (question) by subjects Anova on subjects classified as number responders

	SS	df	MS	F	p
Presentation	0.1454	1	0.1454	.0796	
question	343.3080	1	343.3080	187.8740	<.001
pres. x ques.	.0523	1	.0523	.0286	
residual	230.2440	126	1.8273		
Total	750.5290	171			

Table B3-5

Source table for three factor mixed design Anova on all errors on the small set. Between factor: grade; within factors: presentation and question

Source		df	MS	F	p
Total					
Between subjects	526.747				
Grade	151.9620	2	75.9808	12.721	<.001
$Error_b$	374.7850	63	5.9490		
Within subjects					
transformation	0.4583	1	0.4583	0.4934	
question	88.6704	1	88.6704	11.7266	<.005
grade x transf.	4.7803	2	3.3901	2.5735	
grade x ques.	55.7048	2	27.8524	3.6835	
transf. x ques.	0.8523	1	0.8523	1.1978	
grade x t x g	2.0664	2	1.0332	1.4520	
$Error_1$	58.5115	63	0.9288		
$Error_2$	476.3750	63	7.5615		
$Error_3$	44.8281	63	0.7116		

Table B3-6

Source table for three factor mixed Anova on systematic errors on small set: between factor age ; within presentation condition and question.

Source	SS	df	MS	F	p
Total	899.4960	239	3.7636		
Between subjects					
Grade	43.6583	2	21.8292	5.0256	<.01
$Error_b$	247.5880	57	4.3437		
Within subjects					
presentation	0.1041	1	0.1041	0.2065	
question	78.2041	1	78.2041	10.4263	<.0001
grade x pres.	1.9082	2	0.9541	1.8924	
grade x ques.	43.0083	2	21.5042	2.8670	
pres. x ques.	0.0375	1	0.0375	0.0753	
grade x p x q	0.3247	2	0.1624	0.3260	
$Error_1$	28.7377	57	0.5042		
$Error_2$	427.5380	57	7.50075		
$Error_3$	28.3867	57	0.4980		

Table B3-7

Source table for three factor mixed design Anova on all errors on the large set. Between factor: grade; within factors: presentation and question.

Source	SS	df	MS	F	p
Total					
Between subjects	602.25	65			
Grade	188.576	2	94.3878	14.359	.001
$Error_b$	413.682	63	6.5664		
Within subjects					
presentation	0.2422	1	0.2422	0.2431	
question	546.9700	1	546.9700	89.4935	.001
grade x pres.	4.4839	2	2.420	2.2500	
grade x ques.	414.4844	2	22.2422	3.6392	
pres. x ques.	0.5449	1	0.5449	0.3571	
grade x p x g	2.8174	2	1.4087	0.9232	
$Error_1$	62.7739	63	0.9964		
$Error_2$	385.0460	63	6.1118		
$Error_3$	96.1348	63	1.5260		

Table B3-8

Source table for three factor mixed design Anova on systematic errors on the large set. Between factor: grade; within factors: presentation condition and question.

Source	SS	df	MS	F	p
Total	1180.9300	239	4.9412		
Between subjects					
Grade	83.1082	2	41.5541	11.5369	<.001
$Error_b$	204.8250	57	3.5934		
Within Subjects					
presentation	0.0665	1	0.0665	0.1427	-----
question	453.7500	1	453.7500	97.0586	<.001
grade x pres.	2.3585	2	1.1793	2.5294	-----
grade x ques.	87.7747	2	43.8873	9.3877	<.001
pres. x ques.	0.1499	1	0.1499	0.1582	
grade x p x g	1.8242	2	0.9121	0.9624	
$Error_1$	26.5750	57	0.4662		
$Error_2$	266.4760	57	4.6750		
$Error_3$	54.0244	57	0.9478		

Table B4-1

Source table for two factor mixed design Anova on all errors; between factors; order of testing, within factor; critical term.

Source	SS	df	ms	F	p
Total	480.669	123	-	-	-
Between subjects	157.419	30	-	-	-
Order of testing	9.086	1	9.086	1.78	n.s
Error (b)	148.333	29	5.115	-	
Within Subjects	323.225	93	-	-	-
Critical term	20.859	3	6.953	2.09	n.s.
Term x Order	12.473	3	4.158	1.24	n.s
Error w	289.918	87	3.332		

Table B4-2

Source table for treatments by subjects Anova, all errors on incongruent configurations, 3,4,5. Nonguessers only.

Source	SS	df	MS	F	p
Total	286.43	63	--	-	-
Subjects	79.94	15	--	-	-
Terms	37.81	3	12.61	3.36	< 05
Error	168.68	45	3.75	-	-

Table B4-3

Statistical summary of types of responses of nonguessers on the incongruent configurations

		more			less						longer			shorter					
		C	L	O	C	L	O	P	U	2	C	N	O	C	N	O	P	U	2
Group I (N=5) high performers	Σ	27	0	3	17	2	4	4	3	4	27	2	1	26	3	0	0	1	0
	$\bar{X}$	5.4	.0	.6	3.4	.4	.8	.8	.6	.8	5.4	.4	.2	5.2	.6	0	0	.2	0
Group II (N=7) number strategists	Σ	33	1	8	27	3	5	5	2	3	1	40	1	6	24	1	0	11	2
	$\bar{X}$	4.7	.1	1.1	3.9	.4	.7	.7	.3	.4	.14	5.7	.1	.9	3.4	.1	0	1.6	.3
Group III (N=4) inconsistent strategists	Σ	13	5	6	13	4	3	2	2	5	12	11	1	13	6	5	0	0	3
	$\bar{X}$	3.3	1.3	1.5	3.3	1.0	.7	.5	.5	1.3	3.0	2.8	.3	3.3	1.5	1.3	0	0	.8
	Σ	73	6	17	57	9	12	11	7	12									
	$\bar{X}$	4.6	.4	1.1	3.6	.6	.8	.7	.4	.8	2.5	3.3	.2	2.8	2.1	.7	0	.8	.3

Source Table B4-4

Source Table for Two factor mixed design Anova. Between factor's subject classification: guesser, non-guesser. Repeated measure: Question. Raw data: Intrusions on all configurations.

source	SS	df	ms	F	p
Total	1122.992	123	-	-	-
Between sub.	612.242	30	-	-	-
Condition (classification)	72.942	1	72.942	3.9224	-
Error b	539.3	29	18.596	-	-
Within subjects	510.75	93	5.4919	-	-
Question	42.6611	3	14.22	2.6820	-
Question x classification	6.703	3	2.23	.42	-
Error w	461.338	87	5.302	-	-

Table B5-1

Source table for treatment by level Anova on total number of errors on critical terms. level: children's age; treatment: critical term.

source	SS	df	MS	F	p
Total	133.93	59	-	-	-
age	12.1	1	12.1	6.612	<.005
question	15.27	3	5.09	2.78	<.05
age x question	11.145	3	3.715	2.03	
error	95.41	52	1.83		