# AN INVESTIGATION INTO EXPECTATIONS-DRIVEN BUSINESS CYCLES

# An investigation into expectations-driven business cycles

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# Abstract

In this thesis I explore dimensions through which changes in expectations can serve as a driver of business cycles in a rational expectations setting. Exploiting both the "sunspot" and "news-shock" approaches to expectations-driven business cycles, I use various theoretical models to investigate how changes in expectations may have played a role in macroeconomic events such as the technological revolution of the 1990's and the financial boom and bust of 2003-2008.

In the first chapter, I explore the ability of a model with knowledge capital to generate business cycles driven by expectations of future movement in total factor productivity (TFP). I model knowledge capital as an input into production which is endogenously produced through a learning-by-doing process. When firms receive news of an impending productivity increase, the value of knowledge capital rises, inducing the firm to hire more hours to "invest" in knowledge capital. The rise in the value of knowledge capital immediately raises the value of the firm, causing an appreciation in stock prices. If the expected increase in productivity fails to materialize, the model generates a recession as well as a crash in the stock market.

In the second chapter, I explore the extent to which expectations about

innovations in the financial sector may have contributed to both the boom and bust associated with the "Great Recession". Making a connection between the "boom-years" of easy credit and the crises of 2008, I argue that agents' overly-optimistic expectations of the benefits associated with financial innovation led to a flood of liquidity in the financial sector, lowering interest rate spreads and facilitating the boom in asset prices and economic activity. When the events of 2007-2009 led to a re-evaluation of the effectiveness of these new products, agents revised their expectations regarding the actual efficiency gains available to the financial sector and this led to a withdrawal of liquidity from the financial system, a reversal in credit spreads and asset prices and a bust in real activity. Following the news-shock approach, I model the boom and bust cycle in terms of an expected future fall in the costs of bankruptcy which are eventually not realized. The build up in liquidity and economic activity in expectation of these efficiency gains is then abruptly reversed when agents' hopes are dashed. The model generates counter-cyclical movement in the spread between lending rates and the risk-free rate which is driven purely by expectations, even in the absence of any exogenous movement in bankruptcy costs as well as an endogenous rise and fall in asset prices and leverage.

In the final chapter, I explore the extent to which a "bout of optimism" during a period of technological change such as the 1990's could produce not just a boom in consumption, investment and hours-worked, but also rapid growth in productivity itself. I present a theoretical model where the economy endogenously adopts the technological ideas of a slowly evolving technological frontier, and show that the presence of a "technological gap" between unadopted ideas and current productivity can lead to multiple equilibria and therefore the possibility that changes in beliefs can be self-fulfilling, often referred to as sunspots. In the model these sunspots take the form of beliefs about the value of adopting the new technological ideas, and unleash both a boom in aggregate quantities as well as eventual productivity growth, increasing the value of adoption and self-confirming the beliefs. In this sense, the model provides an alternative interpretation of the empirical news-based results that identify expectational booms that precede growth in TFP. Finally, I demonstrate that the scope for the indeterminacies is a function of the steady-state growth rate of the underlying frontier of technological ideas, and that during times of low growth in ideas or technological stagnation, the potential for indeterminacies and thus belief-driven productivity growth diminishes.

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# Introduction

Despite over a century of research, the source of business cycles is still an extremely contentious issue in academic macroeconomics. Perhaps because of the tendency for tumultuous shifts in asset and credit markets to accompany economic expansions and contractions, many outside of the academic community are sympathetic to the view that aggregate booms and recessions are at least somewhat associated with the beliefs or expectations of economic actors about the macroeconomy. Indeed in the 40 or so years following the Great Depression, this was a prominent view inside academia as well, no doubt in large part due to Keynes' persuasive arguments concerning the role of "animal spirits" in driving the aggregate economy. The emergence of the rational expectations revolution in the 1970's challenged this view however, and with the popularization of real business cycle approach in the early 1980's, a large portion of academic research shifted to studying unanticipated high-frequency shifts in real or monetary factors as a source of business cycles. Yet at least two important developments in the 1990's and 2000's rekindled the interest in expectations in mainstream macroeconomics, this time recast in the discipline of rational expectations: (i) the introduction of models of indeterminate equilibria, whereby expectational "sunspot" shocks act as an independent business cycle impulse, and; (ii) the introduction of models of "news shocks", whereby imperfect signals about future fundamentals can create booms and busts. In this thesis I exploit both of these important developments in the context of three different theoretical models to explore important dimensions through which changes in expectations can serve as a driver of aggregate fluctuations in a rational expectations setting.

While the "sunspot" and "news-shock" approaches are similar in the sense that the business cycles in both are driven by a change in agents' expectations, a significant dimension along which they differ is in the notion of causality between expectations and fundamentals: in the sunspot approach, an exogenous change in expectations can endogenously alter fundamentals, whereas in the news approach, information about a future exogenous change in fundamentals creates a change in expectations. Extending the work of Cass and Shell (1983) and Azariadis (1981) on sunspots and self-fulfilling prophecies, researchers such as Howitt and McAfee (1992), Benhabib and Farmer (1994) and Farmer and Guo (1994) presented a class of non-monetary rational-expectations computable general equilibrium models that could produce business cycle fluctuations with shocks to agents' expectations only. These researchers showed how in models where the production structures yield a multiplicity of dynamic equilibria, a single exogenous expectational shock unrelated to fundamentals - a "sunspot" - could select which of many possible dynamic trajectories an economy follows out of steady-state, producing a business cycle analogous to the "animal spirits" phenomena. Moreover, by virtue of being self-fulfilling, these models are consistent with rational-expectations, since roughly speaking, the economy endogneously produces the behaviour consistent with agents' expectations.

In contrast, in the new-based approach of Beaudry and Portier (2004) and Beaudry and Portier (2007), agents' receive imperfect signals about changes in exogenous future fundamentals, thereby altering their conditional expectation of the future. Since agents know the distribution of both the exogenous shocks to fundamentals as well as that of the news signal however, the models are consistent with rational expectations. Nevertheless, as illustrated by Beaudry and Portier (2004), errors in expectation/forecasting can feature prominently as a source of booms and busts, since the approach allows for an interesting dynamic effect as agents first forecast and then subsequently realize future fundamentals. Beaudry and Portier (2004) showed how a complete boom-bust cycle could result when news about a rise in future productivity first leads to a boom, but then eventual realization that the news was "too optimistic" leads to a bust and recession as agents realize they have over-invested resources and as a result retrench, all in the absence of any actual change in technological fundamentals. Using data that includes stock prices and total factor productivity, Beaudry and Portier (2006) found reduced form evidence for a shock that produces a boom in consumption, investment and hours-worked that precedes growth in total factor productivity (TFP). Moreover, they found that this shock accounts for a material fraction of the variance of measures of aggregate activity at business cycle frequencies. Beaudry and Portier (2006) suggested that one interpretation of this shock is of an expectational news-shock that provides agents with advanced information about changes in TFP in the future.

Both the sunspot and news-based approaches have their advantages depending on the question one intends to study, and in this thesis I exploit both approaches to help understand the extent to which several recent macroeconomic events could be driven by changes in expectations. In particular, the role of errors in forecast/expectations implicit in the news-approach is helpful to understand how we can get both a boom in bust in the 1990's or other periods where technological change was expected to raise TFP, but without necessarily requiring a reduction in TFP during the bust. I consider this application in my first chapter. Similarly, this same connection between forecast/expectations errors and a boom-bust cycle is helpful in trying to understand how the bust of 2007/2008 could have been related to the boom of 2003-2007, in this instance however, driven by a change in expectations about the fundamentals of the financial sector, as opposed to TFP. In this sense, these errors in forecast capture in a relatively simple way the popular notion that the new financial innovations of the day somehow "failed to live up to expectations". I consider this interpretation in my second chapter. Finally, the self-fulfilling nature of the sunspot approach allows one to examine if a change in sentiment alone - rather than a contemporaneous or expected future shift in exogenous technology - could have played a role in not just the boom in consumption, investment and hours-worked during the technological revolution of the 1990's, but also the sudden onset of rapid productivity growth itself from the mid-1990's onwards. I consider this application in my third chapter.

In my first chapter "News and knowledge capital" (with Alok Johri), we investigate a variant of the neoclassical growth model that not only provides for co-movement in response to news shocks, but also helps deal with an asset-pricing issue plaguing researchers. Despite the intuitive appeal of having "booms" driven by the expectation of "good-times ahead", integrating this idea of news about TFP into the standard neoclassical growth model proves to be challenging: when subject to a news-shock about a future increase TFP, consumption booms, yet investment and hours-work fall as the wealth effect associated with the future gain in productivity causes agents to increase not just consumption but leisure also, reducing hours-worked and hence output. As a result, Beaudry and Portier (2006) propose a particular multi-sector variant of the neoclassical growth model that produces a boom in consumption, investment and hours-worked in response to news. Following the work of Beaudry and Portier (2004) and Beaudry and Portier (2007), various researchers such as Jaimovich and Rebelo (2009) and Christiano et al. (2008) presented extensions of the single-sector neoclassical growth model that could produce a co-moving boom in response to news, yet the majority of these variants were unable to simultaneously produce a boom in asset prices, an empirical relation that featured critically in the original empirical identification of news shocks by Beaudry and Portier (2006). In this chapter we present an alternative production structure with a single goods-sector that not only provides for co-movement in response to news about future productivity, but also causes asset prices to rise in response to news. In this sense, the same mechanism that provides for co-movement in response to news also simultaneously implies that stock prices rise immediately in response to news. We model an additional production input called "knowledge capital" which is endogenously produced through a learning-by-doing process that uses hours-worked as an input. When firms receive news of an impending productivity increase, the value of knowledge capital rises, inducing firms to hire more hours to "invest" in developing knowledge capital. The rise in the value of knowledge capital immediately raises the value of firms, causing an appreciation in share prices, in addition to a boom in consumption, investment and hours-worked. If the expected increase in productivity fails to materialize, the model generates a recession as well as a crash in the stock market.

Our interpretation of "news about TFP" in the first chapter is consistent with the majority of the early news shock literature that focused on the role of TFP as a fundamental in the macroeconomy. Yet when we extend this role of fundamentals to other facets of the economy, we can leverage the news-based approach to try and understand how expectations and/or expectational errors can play a role in other aggregate phenomena not associated with TFP. In this regard, in my second chapter "News, Credit Spreads and Default Costs: An expectations-driven interpretation of the recent boom-bust cycle in the U.S." (with Alok Johri), we extend the concept of "fundamentals" beyond that associated with total factor productivity to think about efficiency of the financial sector as a fundamental of the macroeconomy. In doing so, we can then exploit the news-based framework to help understand the role that expectations about the financial sector played in the 2008 financial crisis. Making a connection between the "boom-years" of easy credit and the "great recession" of 2008, we argue that agents' overly-optimistic expectations of the benefits associated with financial innovation led to a flood of liquidity in the financial sector, lowering interest rate spreads and facilitating the boom in asset prices and economic activity. When the events of 2007-2009 led to a re-evaluation of the effectiveness of these new products, agents revised their expectations regarding the actual efficiency gains available to the financial sector and this led to a withdrawal of liquidity from the financial system, a reversal in credit spreads and asset prices and a bust in real activity.

Exploiting the costly-state verification structure of Bernanke, Gertler and Gilchrist (1999), we interpret the efficiency of the financial sector as a function of the loan contract structure that results between lenders and borrowers when the borrower faces a probability of default due to both idiosyncratic and aggregate risk, and the financial intermediary faces monitoring/default costs in the event of loan default. We then assume that this monitoring/bankruptcy cost follows a stochastic process, and study the impact of "news shocks" regarding this process, modeling the boom and bust cycle in terms of an expected future fall in the costs of bankruptcy which are eventually not realized. The build up in liquidity and economic activity in expectation of these efficiency gains is then abruptly reversed when agents' hopes are dashed. The model generates counter-cyclical movement in the spread between lending rates and the risk-free rate which is driven purely by expectations, even in the absence of any exogenous movement in intermediation costs as well as an endogenous rise and fall in asset prices and leverage. Interestingly, because the effect of the shock impacts the wedge between the return to household savings and the marginal product of capital, unlike a TFP shock, there is no direct effect on loosening the resource constraint of the economy. As such, because of the lack of a strong wealth effect that would otherwise depress hours-worked in response to news, the typical mechanisms introduced in the "news shock" literature to induce co-movement in response to TFP shocks are not necessary in this model.

While the news-based approaches above are helpful for understanding the role of expectations in an environment with stochastic shifts in fundamentals, it is also helpful to try and understand the extent to which exogenous changes in expectations can shape fundamentals themselves. In particular, in the third chapter, "From growth to cycles through beliefs", I explore how a sudden bout of "optimism about new technology" during a period of technological change/transition such as the 1990's can trigger both an immediate boom in consumption, investment and hours-worked as well as a delayed and eventual increase in productivity. In doing so, I provide a result broadly consistent with the empirical results of Beaudry and Portier (2006) whereby a boom precedes growth in productivity, yet offering an alternative interpretation of the empirical results in terms of a theoretical model with self-fulfilling properties.

I present a theoretical model where firms endogenously adopt the ideas of an exogenous technological frontier that evolves deterministically and without shocks. I assume the technological ideas are commonly held in the public domain, yet embodied in new capital in the sense that firms must purchase new capital to gain access to the ideas. Firms must then undergo a process of costly-adoption through which they implement the technological ideas into their production process, thereby permanently increasing their productivity of goods production. Importantly, this notion stresses embodiment in the *use of* new investment goods in the sense that using new capital allows firms to transform their production process. This lies in contrast to the notion of embodiment in the *production of* new investment goods, often emphasized in the literature on investment specific technical change such as Hulten (1992) and Greenwood et al. (1997), whereby advancing investment-specific technical change allows those accumulating capital to grow their capital stock faster for a given purchase of investment in units of consumption, without necessarily requiring implementation or adoption.

Physical capital thus plays a very important role in this economy. Unlike the standard neoclassical growth model whereby increases in capital simply increase the scale of production through capital deepening, in this model growth in technological ideas means that increases in capital can alter the blueprint of production itself, increasing the productivity of all factors. As such, periods with an abundance of technological ideas relative to the current state of productivity imply that there are extra returns to capital beyond simply the direct marginal product of capital in production. I show that the presence of a "technological gap" between unadopted technological ideas and current productivity can lead to multiple equilibria and therefore the possibility that changes in beliefs can be self-fulfilling, often referred to as sunspots. These sunspots take the form of beliefs about the value of adopting the new technological ideas, and unleash both a boom in aggregate quantities as well as eventual productivity growth, increasing the value of adoption and self-confirming the beliefs. I demonstrate that the scope for these indeterminacies is a function of the steady-state growth rate of the underlying technological frontier of ideas, and that during times of low growth in ideas - such as those periods outside of technological transition - the potential for indeterminacies disappears. Under this view, technology becomes important for cycles not necessarily because of sudden shifts in the technological frontier, but rather, because it defines a technological regime for the economy such that expectations about its value can produce aggregate fluctuations where in a different regime they could not.

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# Chapter 1

# News and knowledge capital

## 1.1 Introduction

A number of recent studies have attempted to develop models capable of generating expectations-driven business cycles. A key aspect of these cycles is that a boom is created in anticipation of future increases in productivity as opposed to the typical real business cycle model where the boom is driven by an unanticipated contemporaneous rise in productivity. Vector autoregression (VAR) evidence in favour of these cycles is provided in Beaudry and Portier (2004). More recently, Beaudry and Lucke (2009) and Schmitt-Grohe and Uribe (2008) estimate the contribution of anticipated total factor productivity (TFP) shocks along with several other shocks typically used in the business cycle literature and find that anticipated TFP shocks account for a large fraction of the total variation in aggregate series. Despite the possibility that these "news" based shocks play a large role in modern business cycles, there are few models capable of effectively capturing even the most basic empirical features of these cycles. As discussed by Beaudry and Portier (2004), Jaimovich and

Rebelo (2009) and others, the typical business cycle model is unable to deliver booms in which consumption, investment and hours all rise along with output in the periods after the news arrives but before the shock to productivity actually occurs[1].

Beyond co-movement between aggregate quantities, a robust feature of business cycles is co-movement with asset prices. For example, the S&P500 real price index leads GDP by about two quarters: at business cycle frequencies the contemporaneous correlation between the two measures is 0.42 while it is 0.56 two quarters ahead. Moreover, the idea that stock prices respond in advance of the increase in TFP was highlighted in the work of Beaudry and Portier (2006). Intuitively this makes sense: news of impending productivity-increases and the ensuing flow of higher profits should induce an immediate increase in share prices.

The goal of our paper is to deliver co-movement in both aggregate real quantities and asset prices in response to news. This latter feature is an even higher hurdle for most models to cross. In order to get agents to increase investment expenditure on physical capital in advance of the actual rise in TFP, many studies utilize an adjustment cost specification which penalizes changes in the level of investment. An implication of this is that a rise in investment today lowers the value of installed capital. To the extent that the value of firms depends on the installed capital, this effect puts downward pressure on share prices.

[1]This is closely related to the analysis of Barro and King (1984) which showed that consumption and hours-worked will negatively co-move for shocks other than contemporaneous productivity shocks.

In this paper we offer a simple variant of a standard business cycle model that generates the aforementioned co-movement through an intuitive mechanism. The modification is an environment in which agents' actions endogenously create productivity-increasing knowledge through a learning-by-doing (LBD) process. The idea is simple: the news that TFP will rise at a faster than normal pace in the future immediately increases the value of knowledge capital, which we model as an input into the production technology along with labour and physical capital. The rise in the value of knowledge capital creates an incentive for agents to increase the use of labour to accumulate more knowledge capital. The increase in knowledge capital and labour in turn induce investment expenditure on physical capital by raising it's productivity, and the ensuing expansion of output allows both consumption and investment to increase. Since we model firms as storehouses of knowledge capital, the rise in the value of knowledge capital leads to an immediate rise in the value of the firm and therefore its share price. Moreover, since the mechanism in our model resides on the production side of the economy, the model is able to generate an expectational-boom in real quantities and asset prices over a range of preference specifications. In addition, factor prices are procyclical because a rise in knowledge capital raises the marginal productivity of both labour and capital. The absence of adjustment costs prevents unrealistic spikes in interest rates, which are often features of models built to generate co-movement. We think that the introduction of one mechanism that simultaneously delivers all these key features of the data is a strength of the model over others which require one or more modification to the standard growth model to deliver each feature. In light of recent discussions in the literature, it is also interesting to be able to generate expectational-booms in a model without any frictions whatsoever.

The introduction of knowledge capital into the standard one good growth model is, in our opinion, a useful way to try to capture some of the real world complexity surrounding technical change while retaining the simplicity of the original model. While the literature on "news shocks" has focused a lot on the role of wealth effects in explaining why co-movement is hard to achieve, we think part of the problem is that when a productivity shock arrives, it immediately leads to an increase in output without requiring any change in actions on the part of agents. In practice however, the arrival of a new technology itself does nothing to increase output. Considerable resources have to be utilized to re-organize production in the economy including the acquisition of new skills and machines as well as the use of new processes and material and the production of new goods. In a one-good world (as measured by GDP) in which workers perform one identical task (measured by total hours), all these changes are hidden and all that we observe is that aggregate activity goes up in advance of total factor productivity. In other words, the economy needs to reinvent itself to take advantage of the productivity-increases enabled by the new technology, but all we see is a ramping-up of activity. We think that this idea of ramping-up to make the economy conform to the new technology can be captured simply and effectively in terms of investments in knowledge capital. In the model, in response to news about future productivity increases, firms "invest" in building knowledge capital by hiring workers beyond the level dictated by the current marginal product of labour. In doing so, firms sacrifice current profits for anticipated productivity-increases and higher profits in the future. This is similar to the economy devoting resources to re-organizing production activities in order to prepare for a new technology. The by-product nature of the learning-by-doing process also fits well into the one-good, one-task world view of the model. While the mix of tasks performed by workers and the mix of goods produced by firms may change during the expansion phase in order to enable the new technology, all that is recorded at the aggregate level is an increase in output and hours. Similarly in the model, the increase in hours leads simultaneously to more production today and to more knowledge capital which unleashes future increases in productivity.

Thus far, we have described predictions of the model when expected increases in future productivity are realized. The model also has very intriguing implications for situations when agents are disappointed. If the expected productivity shocks fail to materialize, agents find themselves with less wealth than expected and too much physical and knowledge capital relative to the actual state of TFP. This leads to a sharp drop in share prices, induced by a fall in the value of knowledge capital. This “bear market” is accompanied by a recession in which output, investment and hours all fall.

Our work builds on recent business cycle models by Chang et al. (2002) and Cooper and Johri (2002) that incorporate various forms of learning-by-doing into dynamic general equilibrium models and show that they can be an effective propagation mechanism for shocks. While details differ, the two models share the feature that knowledge capital accumulation is a by-product of production activity. Analysis in both papers suggest that models with learning-by-doing improve the ability of the growth model to explain the response of the economy to productivity shocks. Johri (2009) shows similar results for an economy with shocks to monetary policy. Both papers offer aggregate evidence in favour of learning-by-doing and build on an extensive empirical literature which documents the existence of learning effects in all sectors of the economy. Recent studies include Bahk and Gort (1993), Irwin and Klenow (1994), Jarmin (1994), Benkard (2000) and Thornton and Thompson (2001). In this paper we adopt a specification based on that of Chang et al. (2002) where learning occurs as a by-product of past hours-worked. While many other specifications are possible, this one has the advantage of simplicity while still delivering the result [2].

There is a small but growing literature on expectations-driven business cycles. Beaudry and Portier (2004) consider a model with a durable and non-durable good that are produced in two distinct sectors. A complementarity between the two allows both consumption and investment to rise in response to news about a productivity increase in the non-durable goods sector. Jaimovich and Rebelo (2009) propose preferences that reduce or eliminate the strong wealth effect on leisure of an expected future increase in TFP or investment specific technical change. They demonstrate that when combined with capital utilization and adjustment costs to changes in investment, the model produces a strong expectational-boom. Christiano et al. (2008) show that the combination of the same specification of investment adjustment costs

[2] Our work is also related to the ideas of human capital and organizational capital which have been explored in several studies, too numerous to cite.

with habit formation in consumption produces an expectational-boom, however, they find that the model requires an implausible rise in the real interest rate and produces a counterfactual counter-cyclical asset price.[3] They then present a monetary version of the model with nominal wage rigidities and an inflation-targeting monetary authority that creates an expectational boom in real quantities and asset prices without as large a rise in the real interest rate. Den Haan and Kaltenbrunner (2009) present a matching model whereby matching frictions induce firms to post more vacancies in response to news, leading to an increase in employment which allows aggregate consumption and employment to co-move. Dupor and Mehkari (2009) show that a strictly convex frontier between consumption and investment and a high intertemporal elasticity of substitution can also deliver the result. Schmitt-Grohe and Uribe (2008) investigate the role of news shocks in generating economic fluctuations by performing a structural Bayesian estimation on a model featuring habit-formation in consumption and leisure, a flow-specification of investment adjustment costs, and capacity utilization. By allowing for both anticipated (news) and unanticipated components for various shocks, they are able to perform a variance decomposition to determine the relative contribution of anticipated versus unanticipated shocks, and find that anticipated shocks to the permanent and temporary components of TFP account for more than two-thirds of aggregate fluctuations in U.S. postwar quarterly data.

In the remainder of the paper we proceed as follows. In Section 1.2 we discuss an example economy based on Chang et al. (2002). The purpose of

[3]Since we do not use adjustment costs, our model does not suffer from an extreme jump in interest rates.

this example is to show the simplicity and strength of the mechanism built into the knowledge capital economy since it can generate co-movement in consumption, output, hours and investment without any other modification to the one sector growth model. It also illustrates the flexibility of the concept of knowledge capital. Like Chang et al, this section treats knowledge capital as being symmetric with human capital which is accumulated by the worker while Section 3.3 presents a model in which knowledge capital is accumulated by firms. In the former section, payments for knowledge capital go to the worker while in the latter section they lead to operating profits for firms. This feature is crucial for the model to display procyclical stock prices that rise before any changes in TFP. Since we parameterize the model to be consistent with US data and impose constant returns in the production technology, this implies a relatively small contribution of knowledge capital to firm output. As a result, we augment the model with variable capital utilization which magnifies the expectational-boom. We also discuss the impact on our results of changing preferences. The final section concludes.

## 1.2 An example

We begin with a simple example economy based on Chang et al. (2002) that makes clear how the learning-by-doing mechanism allows co-movement of hours, investment and consumption in response to news about a future rise in exogenous total factor productivity. Since this economy is taken more or less directly from Chang et al, we offer very little discussion of the modeling assumptions.[4]

The economy is populated by an infinitely-lived representative household whose preferences are defined over sequences of consumption $C_t$ and leisure $L_t$ with expected lifetime utility defined as

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \{\ln C_t + \chi L_t\}, \tag{1.1}$$

where $\beta$ is the representative household's subjective discount factor and $\chi$ parameterizes the household's relative preference for leisure over consumption.

The representative household operates a production technology that produces output $Y_t$ according to the technology

$$Y_t = A_t \tilde{N}_t^{\alpha} K_t^{1-\alpha}, \tag{1.2}$$

where $A_t$ is the level of an exogenous stationary technology process, $\tilde{N}_t$ is effective labour hours, and $K_t$ is physical capital which accumulates according to

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{1.3}$$

Effective labour is defined as

$$\tilde{N}_t = H_t N_t, \tag{1.4}$$

[4]While Chang et al present a decentralized model, we focus on the associated planner's problem.

where $N_t$ is hours-worked and $H_t$ is the stock of knowledge capital which accumulates according to

$$H_{t+1} = \Psi(H_t, N_t) = H_t^{\gamma} N_t^{1-\gamma}. \tag{1.5}$$

The idea here is that the actual contribution of labour to production is a combination of raw labour hours and knowledge capital which captures information about how best to use the labour input given the state of technology. As discussed in the introduction, the households acquires knowledge capital as a by-product of engaging in production.

Combining (1.2) and (1.4) we get

$$Y_t = A_t F(N_t, K_t, H_t) = A_t (H_t N_t)^{\alpha} K_t^{1-\alpha}. \tag{1.6}$$

The common exogenous total factor productivity process $A_t$ evolves in logs according to the stationary AR(1) process

$$\ln A_t = \rho_A \ln A_{t-1} + \theta_{A,t}, \tag{1.7}$$

where $\rho_a < 1$ and $\theta_{A,t}$ is an exogenous period $t$ innovation which we will define further below.

Each period, the household is endowed with one unit of time that can be allocated between leisure and hours-worked $N_t$ according to

$$N_t + L_t = 1. \tag{1.8}$$

Finally, the economy's resource constraint is given by

$$C_t + I_t = Y_t. \tag{1.9}$$

The planner chooses contingent infinite sequences of $C_t$, $N_t$, $K_{t+1}$ and $H_{t+1}$ to maximize (1.1) subject to (1.3), (1.5), (1.6), (1.8) and (1.9). Making the appropriate substitutions and letting $\lambda_t$ and $\Upsilon_t$ be the period $t$ Lagrange multipliers on (1.9) and (1.5) respectively, the planner's first-order conditions are as follows:

$$u_C(C_t, L_t) = \lambda_t \tag{1.10}$$

$$u_L(C_t, L_t) = \lambda_t A_t F_{Nt} + \Upsilon_t \Psi_{Nt} \tag{1.11}$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \left[ A_{t+1} F_{Kt+1} + 1 - \delta \right] \right\} \tag{1.12}$$

$$\Upsilon_t = \beta E_t \left\{ \lambda_{t+1} A_{t+1} F_{Ht+1} + \Upsilon_{t+1} \Psi_{Ht+1} \right\}. \tag{1.13}$$

where $F_{Nt} = A_t \frac{\partial F(N_t, K_t, H_t)}{\partial N_t}$, $\Psi_{Nt} = \frac{\partial \Psi(H_t, N_t)}{\partial N_t}$ etc. These first-order conditions differ from those of the standard RBC model only by the addition of an additional term in the hours first-order condition (1.11) and an Euler equation for knowledge capital (1.13).

To interpret these two equations, first define $q_{ht} = \frac{\Upsilon_t}{\lambda_t}$ as the value of new knowledge capital in terms of consumption. Applying this definition and substituting out $\lambda_t$, we can re-write the knowledge capital and hours first-order conditions as

$$q_{ht} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ A_{t+1} F_{Ht+1} + q_{ht+1} \Psi_{Ht+1} \right] \right\} \tag{1.14}$$

$$\frac{u_L(C_t, L_t)}{u_C(C_t, L_t)} = A_t F_{Nt} + q_{ht} \Psi_{Nt}. \tag{1.15}$$

Equation (1.14) shows that the value of the marginal unit of knowledge capital in terms of consumption is the stochastically-discounted future lifetime stream of the additional output generated from the additional knowledge capital. Note that the two terms on the right hand side of the equation suggest that additional knowledge not only contributes to output but also raises the marginal effectiveness of each hour in the learning process. Recognizing the connection between hours-worked and the creation of future knowledge capital, the planner does not merely equate the household's marginal rate of substitution of consumption for leisure to the marginal product of labour as would occur in the standard model. Instead, in this model, we see from equation (1.15) that the planner equates the household's marginal rate of substitution to the sum of the marginal product of labour and the value of the additional stock of knowledge generated by an increase in hours-worked today. From the perspective of explaining how the model can generate an increase in hours in response to news about a future productivity shock, it is helpful to note that a change in the value of knowledge capital, $q_{ht}$, will act as a shift factor for the labour supply curve mapped out in hours-productivity space. This shift factor is missing in standard models in the absence of a contemporaneous productivity shock.

While it may appear at first that the by-product nature of the learning process means that knowledge capital (and hence future productivity) can be acquired costlessly, this is not entirely correct. The planner will make considerable "unobserved investments" in knowledge capital. To see this, we can re-write (1.15) as $\frac{u_L(C_t,L_t)}{u_C(C_t,L_t)} - A_t F_{Nt} = q_{ht}\Psi_{Nt}$. The left hand side of this equation shows that the marginal rate of substitution is larger than the marginal product of labour. In other words, the planner is using more labour than is justified by the current payoff in terms of additional output. This additional use of labour is an "investment in the future". The per-hour cost of the investment is on the left hand side of the equation while the per-hour value of the investment is given by the right hand side. We can then define the total unmeasured investment in knowledge capital $\Lambda_t$ as

$$\Lambda_t = \left\{ \frac{u_L(C_t, L_t)}{u_C(C_t, L_t)} - A_t F_{Nt} \right\} N_t. \tag{1.16}$$

### 1.2.1 The impact of news shocks

In this section we explore how news of an impending rise in total factor productivity is received by the economy described above. We contrast this with the response of a similar economy without knowledge capital. Our representation of news shocks is standard and follows Christiano et al. (2008). We provide for news about $A_t$ by defining the innovation $\theta_{A,t}$ in equation (1.7) as

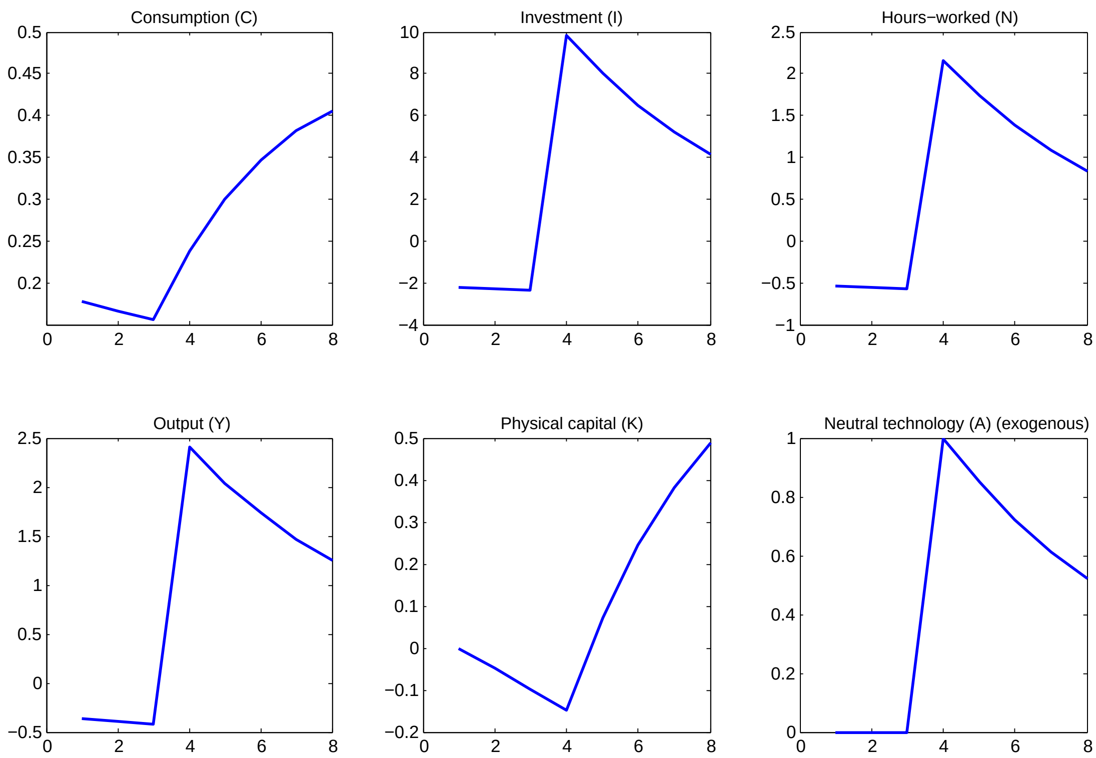
$$\theta_{A,t} = \epsilon^p_{A,t-p} + \varepsilon_{A,t}, \tag{1.17}$$

where $\epsilon^p_{A,t}$ is a news shock that agents receive in period $t$ about the innovation $\theta_{A,t+p}$, and $\varepsilon_{A,t}$ is an unanticipated contemporaneous shock to $\theta_{A,t}$. The news shock $\epsilon^p_{A,t}$ has properties $E\epsilon^p_{A,t} = 0$ and standard deviation $\sigma_{\epsilon^p_A}$, and the contemporaneous shock $\varepsilon_{A,t}$ has properties $E\varepsilon_{A,t} = 0$ and standard deviation $\sigma_{\varepsilon_x}$. The shocks $\epsilon^p_{A,t}$ and $\varepsilon_{A,t}$ are uncorrelated over time and with each other.

Figure 1.1 shows the response of our benchmark standard RBC model to news in period 1 that a temporary but persistent increase in productivity will occur in period 4, represented by $\varepsilon_{At} = 0$ so that $\theta_{At} = \epsilon^p_{A,t-p}$. This and the next figure for the LBD model are based on the news turning out to be correct, ie., productivity actually does rise in period 4.[5] The figure clearly illustrates the difficulties in generating co-movement in response to news shocks. In period 1 consumption rises but investment and hours-worked decrease below

[5]The equivalent baseline RBC model consists of setting $\tilde{N}_t = N_t$ and omitting the constraint (1.5). The common parameterization for both the RBC and LBD model behind these impulse responses is relatively standard: $\beta = 0.99$, $\alpha = 0.67$, $\delta = 0.022$, $\rho_A = 0.85$ and $p = 3$. In the LBD model, we set $\gamma = 0.8$, close to the estimate in Chang et al. A fully calibrated model will be presented in the next section.

Figure 1.1: **Standard RBC** - News shock in period 1 about neutral tech. shock, tech. shock *fully realized* in period 4

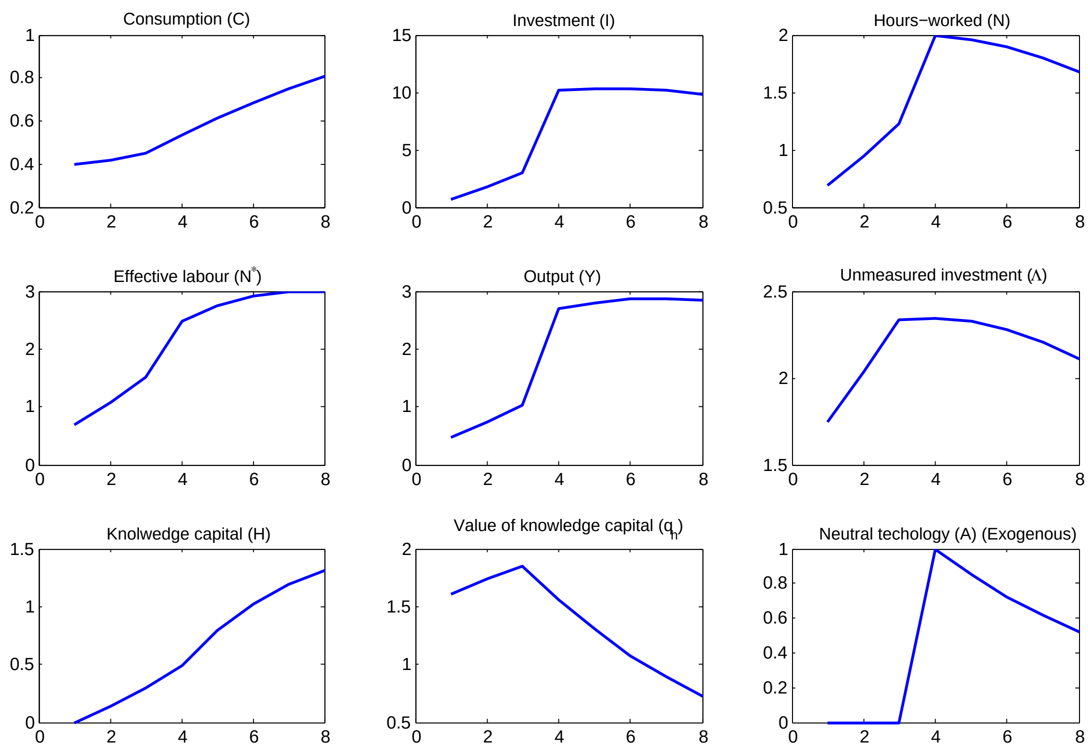



steady state. Thereafter, these three variables slope downwards slightly before reacting positively in the usual manner to the contemporaneous productivity shock in period 4. In response to the news, the wealth effect of the expected increase in future productivity causes households to increase consumption in the initial period, driving down the marginal utility of consumption, and producing a corresponding wealth effect on leisure through the hours first-order condition, causing households to reduce hours-worked. With the capital stock fixed in the initial period, the reduction in hours-worked reduces production, and therefore households "fund" the increase in consumption through a decrease in investment.

Figure 1.2 shows the response of the LBD model to the same news shock. In sharp contrast to the previous figure, hours-worked, investment, consumption and output all increase and slope upwards in response to the news shock. Note in particular the sharp increase in the value of knowledge capital, $q_{ht}$, in response to the news about future productivity increases. This increase in the value of knowledge capital, on its own, induces the agent to increase hours-worked. The aforementioned wealth effect on leisure is still in operation but is trumped by the desire to learn in preparation for the technological change. This can be seen in the plot for unmeasured investment which rises as soon as the news arrives, and peaks in the period before the technology shock actually hits the economy. The increase in hours-worked leads to a rise in output in the current period as well as an increase in knowledge capital in the subsequent period. Anticipating this, agents realize that the productivity of capital will rise in the next period also and therefore are induced to increase investment. The increase in output allows agents to simultaneously satisfy their desire for more consumption and investment.

Why does $q_h$ rise? Firstly, recall that we defined $q_h$ as the ratio of the shadow value of knowledge capital to that of goods, which in this model is also the shadow value of physical capital. The news that TFP will rise in the future causes the shadow value of physical capital to fall but the shadow value of knowledge capital to rise. This discrepancy between the behaviour of the two capital stocks may seem puzzling at first, therefore we discuss them in some detail. It is most convenient to think about the response of these prices

Figure 1.2: **Household knowledge** - News shock in period 1 about neutral tech. shock, tech. shock *fully realized* in period 4




in terms of demand and supply of the resources used to create the two capital stocks. On the demand side, the situation is symmetric. The planner realizes that the marginal products of physical and knowledge capital will both rise with TFP in period four, thus more of each input will be desired at that point. The supply side is, however, dramatically different in period four because of the technological environment of the model. The productivity shock implies that there will be additional goods available for use in consumption and for the creation of physical capital, even if the planner makes no changes at all. Thus there is a large increase in the supply of goods which drives its price down. In contrast, the technology does not expand the supply of knowledge capital in the economy, nor the primary input into knowledge capital, hours-worked. Rather, due to the wealth effect, the demand for leisure will increase, squeezing the availability of market hours needed for the creation of knowledge capital. This increase in the cost of creating knowledge capital leads to a rise in its shadow price. All together, this implies that the shadow value of goods falls while that of knowledge capital rises, both of which lead to a rise in q, the consumption value of knowledge capital. The form of the accumulation technology for knowledge capital determines how far forward in time the initial rise in $q_{ht}$ occurs. The curvature in the functional form and the presence of constant returns in labour and knowledge capital all play a quantitative role in this regard. Diminishing returns to hours in the creation of knowledge encourages the planner to spread out the "investment" period. Furthermore, the presence of knowledge capital on the righthand side of the equation implies additional knowledge raises the marginal return to each hour in terms of knowledge created.

While the results of this section illustrate clearly the manner in which learning-by-doing is able to generate expectations driven business cycles, we think important characteristics of expectational booms cannot be explained by it. The most important of these is the co-movement of firm equity share prices. Often discussion of booms in the media do not distinguish between increases in the value of financial assets and in real quantities that macroeconomists tend to focus on. To an extent this could be because both tend to rise together in these boom periods. Beaudry and Portier (2006) show that for the US, "news", as captured by innovations in their VAR results, lead to immediate increases in stock market values which are subsequently followed by increases in TFP. It would appear that stocks rise in anticipation of future increases in profits due to the increase in TFP. Our knowledge capital model has similar features. News about impending increases in TFP leads to an increase in the value of knowledge capital. If knowledge capital were accumulated by firms, this rise in value would also raise the value of the firms themselves. Share prices would rise in anticipation of the extra profits to be generated in the future. Interestingly, this suggests that the learning mechanism can simultaneously explain not only the co-movement in real quantities like hours-worked and investment, but also the increase in asset values.

This concept of firm value as a function of firm-specific knowledge is consistent with the idea of firm value in the organizational capital literature where organizational capital is typically viewed as an unobserved input into production. For example, Prescott and Visscher (1980) refer to information accumulation within the firm as an explanation for the firm's existence. This information affects its production possibilities set, and thus acts as an asset for the firm which gives it value. Our interpretation of this value is similar: knowledge capital is productivity-enhancing, allowing firms to produce additional output for given levels of labour and capital without having to pay out additional rents in the future, creating a stream of profits which provide value to the firm. It differs from many models of organizational capital such as Atkeson and Kehoe (2005) where the evolution of organizational capital is exogenous and not controlled by the firm. Moreover, our interpretation of knowledge capital as an asset owned by the firm is consistent with that idea of Rosen (1972), who in reference to this type of knowledge writes that "specific knowledge is vested 'in the firm'. Then the asset is transferable by selling the firm, whose price, net of physical capital value, is in fact the market value of its specific capital".

## 1.3 An economy with firm-specific capital

We now present our full model where knowledge is accumulated by firms as opposed to by workers as in the example economy. This will imply that firms will increase labour demand in response to the news, as opposed to workers increasing labour supply [6].

The economy consists of a continuum of identical infinitely-lived households on a unit measure, and a single competitive firm. Since we will impose constant returns to scale in production, it is convenient to assume that production occurs at a single representative firm that nonetheless behaves competitively and takes factor prices as given. This assumption has the advantage of suppressing notation associated with shares belonging to different firms. In general, we use lowercase variables to represent individual household quantities and economy-wide prices, and uppercase variables to represent firm quantities. For notational simplicity, we assume that households own the stock of physical capital and sell capital services to the firm. In addition to markets for labour services, capital services and goods, we assume the existence of a stock market

[6] It is entirely likely that both mechanisms are present in the data, but we explore only the former for clarity.

where households can buy and trade equity shares in the firm that represent claims to the firm's future profits.

### 1.3.1 Household

The household side of the model is relatively standard so we discuss it briefly. An individual household has preferences defined over sequences of consumption $c_t$ and leisure $l_t$ with expected lifetime utility defined as

$$\mathcal{U} = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{1.18}$$

where $\beta$ is the household's subjective discount factor.

Each period, the household supplies hours-worked $n_t$ for wage $w_t$ and capital services $\tilde{k}_t$ for price $r_t$. In addition, it receives dividend income $d_t$ for each unit of its outstanding holdings of firm equity $z_t$. For convenience, we normalize the firm's outstanding number of shares to unity, and thus the household trades fractions of the firm's single equity share. The household allocates its earnings between consumption, investment in physical capital and equity shares. The household's period $t$ budget constraint is given by

$$c_t + i_t + v_t z_{t+1} = w_t n_t + r_t \tilde{k}_t + [v_t + d_t] z_t, \tag{1.19}$$

where $c_t$ is consumption, $i_t$ investment in physical capital and $v_t$ the price of equity. Capital services are defined as

$$\tilde{k}_t = u_t k_t, \tag{1.20}$$

where $k_t$ is the household's stock of physical capital and $u_t$ is the utilization rate of that capital. The household's physical capital evolves according to

$$k_{t+1} = [1 - \delta(u_t)]k_t + i_t \tag{1.21}$$

where the depreciation function $\delta(\cdot)$ satisfies the conditions $\delta'(\cdot) > 0$, $\delta''(\cdot) \geq 0$.

The household's problem is to choose sequences $c_t$, $n_t$, $u_t$, $k_{t+1}$ and $z_{t+1}$ to maximize (2.1) subject to (2.2), (1.20) and (1.21), yielding the standard first-order conditions

$$u_c(c_t, l_t) = \lambda_t \tag{1.22}$$

$$u_l(c_t, l_t) = \lambda_t w_t \tag{1.23}$$

$$\delta'(u_t) = r_t \tag{1.24}$$

$$\lambda_t = \beta E_t \left\{\lambda_{t+1} \left[r_{t+1} u_{t+1} + 1 - \delta(u_{t+1})\right]\right\} \tag{1.25}$$

$$v_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ v_{t+1} + d_{t+1} \right] \right\} = \sum_{s=1}^{\infty} E_t \left\{ \beta^s \frac{\lambda_{t+s}}{\lambda_t} d_{t+s} \right\}. \tag{1.26}$$

where it is clear from (1.26) that as usual the price of the firm's share $v_t$ will equal the stochastically-discounted lifetime stream of the firm's dividends beginning in period $t+1$.

### 1.3.2 Firm

The firm produces output according to

$$Y_t = A_t F(N_t, K_t, H_t) = A_t N_t^{\alpha} \tilde{K}_t^{\theta} H_t^{\varepsilon}, \tag{1.27}$$

where $A_t$ is aggregate exogenous neutral productivity defined as in (1.7) and $H_t$ is the firm's stock of firm-specific knowledge capital, and where we restrict $\alpha + \theta + \varepsilon = 1$ to impose constant returns to labour, capital services and knowledge capital in production.

The firm's knowledge capital evolves as in our example economy as

$$H_{t+1} = \Psi(H_t, N_t) = H_t^{\gamma} N_t^{1-\gamma}. \tag{1.28}$$

Each period, the firm pays out a dividend $D_t$ to shareholders defined as

$$D_t = Y_t - w_t N_t - r_t \tilde{K}_t. \tag{1.29}$$

Since the firm accumulates firm-specific knowledge capital through an internal learning-by-doing process, it faces the dynamic problem of choosing sequences of $N_t$, $\tilde{K}_t$ and $H_{t+1}$ to maximize current and expected future lifetime dividends

$$V_t = D_t + E_t \sum_{s=1}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \{D_{t+s}\} = D_t + \bar{V}_t \tag{1.30}$$

subject to (1.27), (1.28), and (1.29), where the term $\beta^s \frac{\lambda_{t+s}}{\lambda_t}$ is the household's stochastic discount rate for period $t+s$, and where we have defined $\bar{V}_t = E_t \sum_{s=1}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \{D_{t+s}\}$ as the end-of-period discounted value of the firm's future lifetime stream of profits. Letting $q_t$ be the Lagrange multiplier associated with (1.28), and making the appropriate substitutions, the firm's first-order conditions are then

$$w_t - A_t F_{Nt} = q_t \Psi_{Nt} \tag{1.31}$$

$$r_t = A_t F_{\tilde{K}t} \tag{1.32}$$

$$q_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ A_{t+1} F_{Ht+1} + q_{t+1} \Psi_{Ht+1} \right] \right\}. \tag{1.33}$$

Analogous to our example economy, the firm's knowledge capital first order condition (1.33) shows that the value, in terms of profits, of an additional unit of firm-specific knowledge is the stochastically discounted future lifetime stream of additional output created by that additional knowledge. As such, the firm's hours first-order condition now shows that in determining its optimal use of labour, the firm considers both the direct marginal productivity of that labour in current production plus the value in terms of profits of the additional future lifetime output brought about by increasing its stock of firm-specific knowledge from hiring more labour hours today.

Note that once the firm has created the extra unit of knowledge capital, its contribution to additional output each period $t+s$ thereafter, as given by $A_{t+s}F_{Ht+s}$ in (1.33), represents a stream of profits for the firm over the life of the knowledge capital. This occurs because once it is created, it is held costlessly by the firm. To see this, combine (1.29), (1.31), (1.32) and (1.33) along with the specific functional forms of $F(\cdot)$ and $\Psi(\cdot)$ to give

$$q_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ \frac{D_{t+1}}{H_{t+1}} + q_{t+1} \frac{H_{t+2}}{H_{t+1}} \right] \right\}, \tag{1.34}$$

which shows that the marginal value of an additional unit of knowledge capital is the additional profit created by the extra unit (which happens to also equal the average profit per unit) plus the value of future units of knowledge capital made possible.

Recall from the previous section that the right hand side of (1.31) may be thought of as unmeasured investment by the firm in knowledge capital per hour. The extent to which this term influences the firm's labour decision will depend on the current value of knowledge capital, $q_t$. When $q_t > 0$, the firm will wish to use labour at a level in excess of that of a standard neoclassical firm. From the lens of the standard firm's problem (where firms hire labour up to the point where the marginal product of labour equals the wage rate), it appears as if the firm wishes to hire "too much" labour because the marginal product is below the current wage rate. In fact, the firm is investing in knowledge capital by trading off lower current profit for higher future profit. This investment in knowledge capital responds to $q_t$, the value of knowledge, and shows up as "unmeasured investment" since the investment is embedded in wage payments. Like the example economy, if news about future changes in TFP increases $q_t$, the firm will respond by attempting to hire more labour. This will, in turn, raise the value of the firm and it's shares. It is easy to check that changes in the value of the firm are in fact equal to the total value of unmeasured investment. We show in the Appendix that the end-of-period $t$ value of the firm and therefore price of equity can be expressed as

$$\bar{V}_t = q_t H_{t+1}, \tag{1.35}$$

which shows that the value of the firm is determined by the total value of its existing stock of knowledge, obtained as a product of the marginal value of firm-specific knowledge and the stock of firm-specific knowledge.[7]

[7]By assuming that the value of the firm derives solely from its knowledge capital we do not mean to suggest that physical capital plays no role. However, it is convenient to let households accumulate physical capital and focus the analysis of the firm's problem on the novel mechanism. In any case, the role played by capital and variation in the price of capital on firm values is well documented and understood.

### 1.3.3 Equilibrium

Equilibrium in this economy is defined by contingent infinite sequences of $c_t$, $n_t$, $\tilde{k}_t$, $u_t$, $k_{t+1}$, $z_{t+1}$ for each household, $N_t$, $\tilde{K}_t$, $H_{t+1}$, $Y_t$ for the firm, aggregate states $\int_0^1 H_t di = \mathbb{H}_t$, $\int_0^1 k_t di = \mathbb{K}_t$, and prices $w_t = w(\mathbb{H}_t, \mathbb{K}_t, A_t)$, $r_t = r(\mathbb{H}_t, \mathbb{K}_t, A_t)$ and $v_t = v(\mathbb{H}_t, \mathbb{K}_t, A_t)$ that satisfy the following conditions: $(i)$ the allocations solve each household's problem taking prices as given; $(ii)$ the allocations solve the firm's problem taking prices as given; $(iii)$ the equity market clears, $\int_0^1 z_t di = 1$; $(iv)$ the labour market clears, $\int_0^1 n_t di = N_t$; $(v)$ the capital services market clears, $\int_0^1 \tilde{k}_t di = \tilde{K}_t$, and; (vi) the aggregate resource constraint holds, $C_t + I_t = Y_t$, where $\int_0^1 c_t di = C_t$ and $\int_0^1 i_t di = I_t$. Finally, we note that (1.26) and (1.30) imply that $v_t = V_t$.

### 1.3.4 Solution method and parameterization

In order to solve the model it is convenient to work with the associated central planner's version given in the Appendix. We solve the model by linearizing the model equations around the steady-state and then use the singular linear difference system reduction method of King and Watson (2002). We assign values to the parameters of the model using typical values established in the literature, and later provide sensitivity analysis to discuss the dependence of the results on these values.

First, we set the share of time allocated to the market in steady-state $N_{SS}$ to 0.2, and the household's subjective discount factor $\beta$ to 0.99. For the knowledge capital parameters, we start by choosing $\varepsilon = 0.15$, which is approximately the midpoint of the range of 0.08-0.26 estimated in Cooper and

Johri (2002). This value is equivalent to a learning rate of around 10%, which is half of that typically estimated in the learning literature. This is also the value of the contribution of organizational capital used in Atkeson and Kehoe (2005). For the knowledge capital accumulation equation we pick $\gamma = 0.8$, which is close to the value estimated by Chang et al. (2002). As we will show later, we find that the model results are quite robust to variations in $\gamma$ over the range of values estimated in the literature such as Cooper and Johri (2002) and Johri and Letendre (2007).

Next, we choose the remaining parameters in the production technology and the capital depreciation rate. We require that the model deliver a steady-state labour share, $S_N$, of approximately 0.67, which in the model is given by

$$S_n = \alpha + \left( \frac{(1-\gamma)}{\xi + (1-\gamma)} \right) \varepsilon, \tag{1.36}$$

where $\xi = 1/\beta - 1$.

This yields a value of 0.53 for $\alpha$, and, with constant returns to $N_t$, $\tilde{K}_t$ and $H_t$ in the production function, a value of 0.32 for $\theta$. Next, we determine the capital depreciation rate such that the model delivers a capital-output ratio of 10, yielding a value of 0.022 for $\delta$.

The parameterization of the learning-by-doing technology has implications for steady state profit. We show in the Appendix that with constant returns in both $F(\cdot)$ and $\Psi(\cdot)$, the share of profit is very small but positive and is given by

$$\frac{D}{Y} = \left(\frac{\xi}{\xi + (1-\gamma)}\right)\varepsilon, \tag{1.37}$$

where $\xi = 1/\beta - 1$ is the household's subjective discount rate, and $\gamma$ is the parameter in the accumulation equation for knowledge capital.

We note that the above expression (1.37) for $\frac{D}{Y}$ assumes that the household accumulates physical capital, and thus this profit share represents the steady state contribution of knowledge capital to profit net of physical capital. For the above parameterization, $\left(\frac{\xi}{\xi+(1-\gamma)}\right) \approx 0.048$ yielding $\frac{d}{y} \approx 0.007$.

We set the elasticity of the marginal capital depreciation function $\epsilon_u = \frac{\delta''(u)}{\delta'(u)}u$ to 0.15, which is within the range of values considered by King and Rebelo (2000) and the same value as that used by Jaimovich and Rebelo (2009).

For the exogenous technology shock process that includes news shocks, we set the persistence to $\rho_A = 0.85$, which is in the middle of the values of 0.83 and 0.89 estimated by Christiano et al. (2008) and Schmitt-Grohe and Uribe (2008) respectively. Following the literature, we set $p = 3$, implying that in each period agents receive news about total factor productivity 3 periods in the future.

Since the learning-by-doing mechanism in this model is primarily a production-side mechanism, we explore the impact of three different forms of preferences on our results. These are:

1. Standard indivisible labour preferences separable in consumption and

leisure with specification

$$u(c_t, l_t) = \frac{s_t^{1-\sigma}}{1-\sigma} + \chi \frac{(l_t)^{1-\nu}}{1-\nu}, \tag{1.38}$$

with $\sigma = 1$ and $\nu = 0$, therefore implying log consumption and linear leisure per Hansen's (Hansen, 1985) indivisible labour model.

2. Indivisible labour preferences not separable in consumption and leisure of the form used by King and Rebelo (2000) in their application of Rogerson and Wright's (Rogerson and Wright, 1988) generalization of indivisible labour to nonseparable preferences. These preferences still fall within the general class of "KPR preferences" described in King et al. (1988). With these preferences, the stand-in representative agent has the preference specification

$$u(c_t, l_t) = \frac{1}{1-\sigma} \left\{ c_t^{1-\sigma} \upsilon^*(l_t)^{1-\sigma} - 1 \right\} \tag{1.39}$$

where $\upsilon^*(l) = \left[ \left(\frac{1-l_t}{H}\right) \upsilon_1^{\frac{1-\sigma}{\sigma}} + \left(1 - \frac{1-l_t}{H}\right) \upsilon_2^{\frac{1-\sigma}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}}$, and where $H$ is the fixed shift length, and $\upsilon_1$ and $\upsilon_2$ are constants representing the leisure component of utility of the underlying employed group (who work $H$ hours) and unemployed group (who work zero hours) respectively. We set $\sigma = 2$ in this case. [8]

3. "JR preferences" of the form proposed by Jaimovich and Rebelo (2009),

[8] We note that for $\sigma = 1$, the linearized form of these nonseparable indivisible labour preferences is equivalent to the linearized form of the standard separable indivisible labour preferences that we consider in the first preference case.

Table 1.1: Firm knowledge capital interpretation - calibration

| $\sigma$ | $\nu$ | $\zeta$ | $\beta$ | NSS | $\alpha$ | $\theta$ | $\varepsilon$ | $\delta(u)$ | $\epsilon_u$ | $\rho_A$ | $p$ | $\gamma$ | $\eta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *KPR separable indivisible labour preferences* | | | | | | | | | | | | | |
| 1 | 0 | n/a | 0.99 | 0.2 | 0.53 | 0.32 | 0.15 | 0.022 | 0.15 | 0.85 | 3 | 0.8 | 0.2 |
| *KPR nonseparable indivisible labour preferences* | | | | | | | | | | | | | |
| 2 | n/a | n/a | 0.99 | 0.2 | 0.53 | 0.32 | 0.15 | 0.022 | 0.15 | 0.85 | 3 | 0.8 | 0.2 |
| *JR preferences* | | | | | | | | | | | | | |
| 1 | 1.16 | 0.01 | 0.99 | 0.2 | 0.53 | 0.32 | 0.15 | 0.022 | 0.15 | 0.85 | 3 | 0.8 | 0.2 |

with specification [9]

$$u(c_t, l_t, x_t) = \frac{(c_t - \psi(1-l_t)^{\nu} x_t)^{1-\sigma} - 1}{1-\sigma}, \tag{1.40}$$

$$x_t = c_t^{\zeta} x_{t-1}^{1-\zeta} \tag{1.41}$$

and where we set $\sigma = 1$, $\nu = 1.16$ and $\zeta = 0.01$ based on Schmitt-Grohe and Uribe's (Schmitt-Grohe and Uribe (2008)) estimation of these preferences. As detailed in Jaimovich and Rebelo (2009), the $\zeta$ parameter has the effect of parameterizing the wealth effect to leisure and nests both "GHH preferences" ($\zeta = 0$) proposed by Greenwood et al. (1988) and "KPR preferences", with lower $\zeta$ implying a lower wealth effect to leisure.

[9] The inclusion of the term $X_t$ introduces another state variable into our system ($X_{t-1}$) and another first-order condition for $X_t$ to the household problem. See Jaimovich and Rebelo (2009) for a complete discussion of these preferences.

Figure 1.3: **Firm-specific knowledge: separable indivis lbr pref's -** News shock in period 1 about neutral tech. shock in period 4, tech. shock *fully realized* in period 4

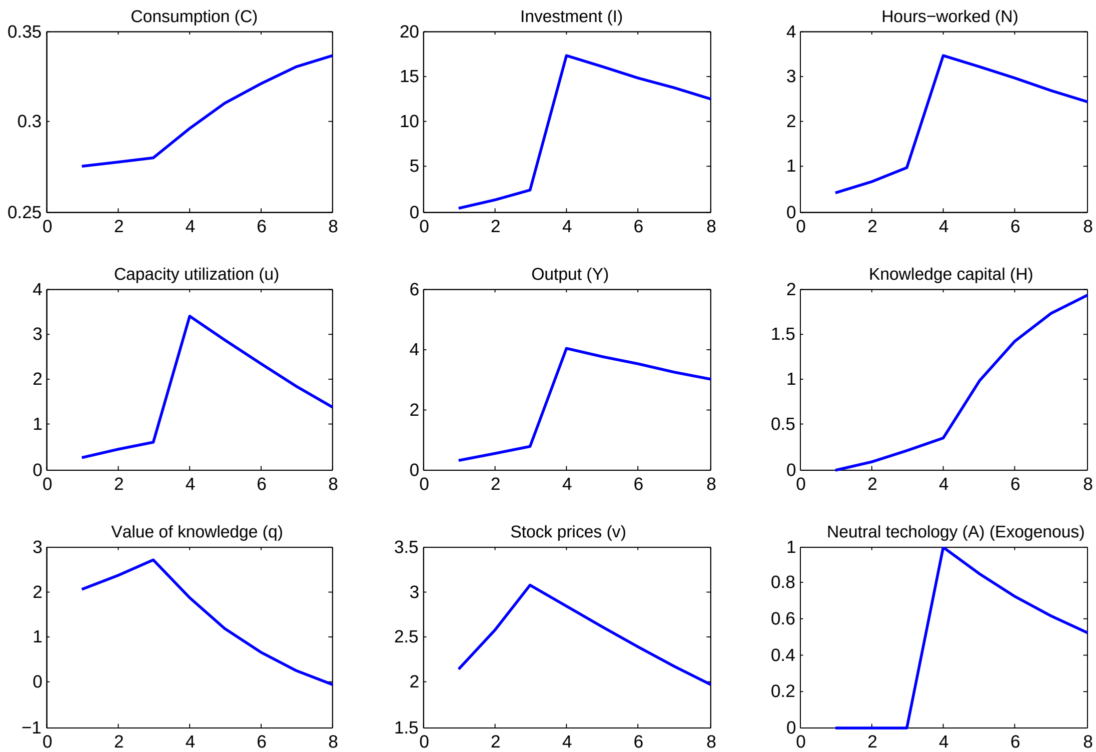


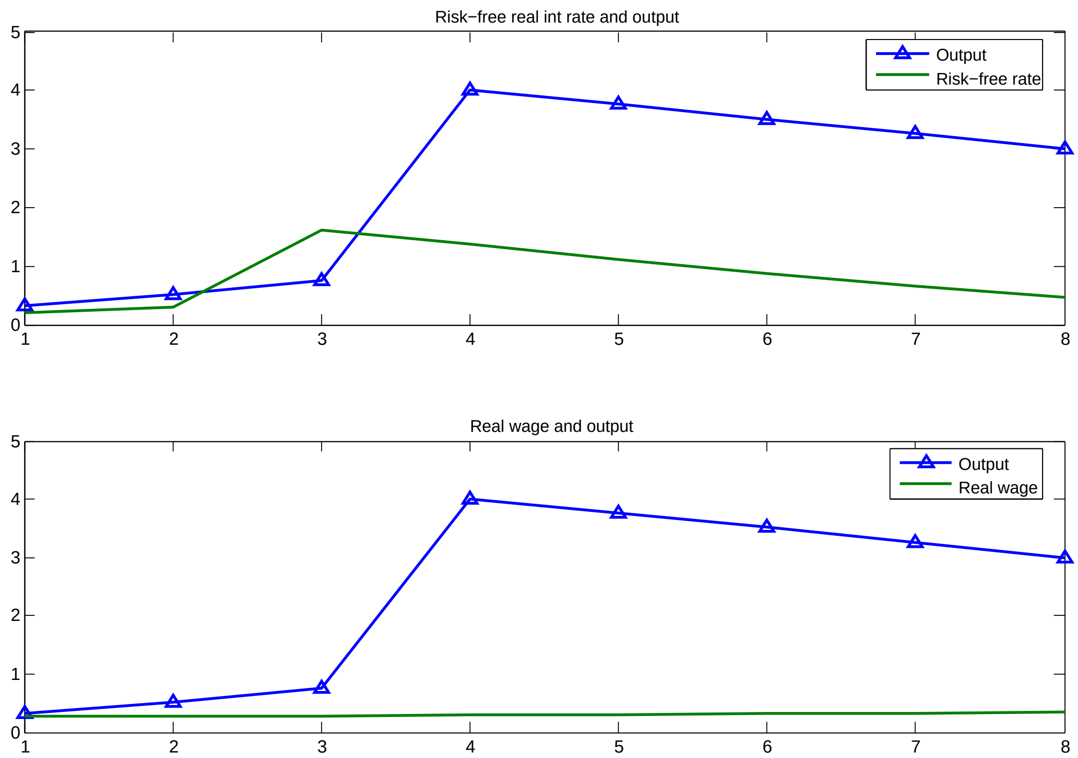

Table 1.1 summarizes our parameterization of the model.

## 1.4 Results

We begin this section with a discussion of how the economy described above reacts to news of a 1% increase in TFP in period 4, and then an eventual realization of that shock in period 4. As discussed above, we present results for three different preferences.

Figure 1.3 shows the response of the economy using separable preferences.

Figure 1.4: **Factor prices: separable indivis lbr pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *fully realized* in period 4




As expected from the results of Section 1.2, consumption, investment, hours-worked and output all rise above steady state levels immediately. Upon receipt of the news, the rise in the value of knowledge capital $q$ shifts out the firm's labour demand as the firm realizes that it needs to invest in accumulating more knowledge capital by increasing hours, creating a corresponding increase in demand for capital services, and raising the overall level of production. More interestingly, stock prices jump up by over 2 percent upon arrival of the news, and then continue to rise, peaking in the period before the technology shock actually raises TFP. [10] This pattern of share prices reacting well in advance of any movements in productivity is reminiscent of the discussions in Beaudry and Portier (Beaudry and Portier (2006)). The rise in the value of the firm results both from a rise in the value of knowledge capital, $q$, which rises two percent and stays above steady state for several periods, and the increase in knowledge capital itself, which begins to rise in the period after the news arrives.

Figure 1.4 shows the response of factor prices. Both wages and interest rates rise along with output but their response is more muted than that of output.

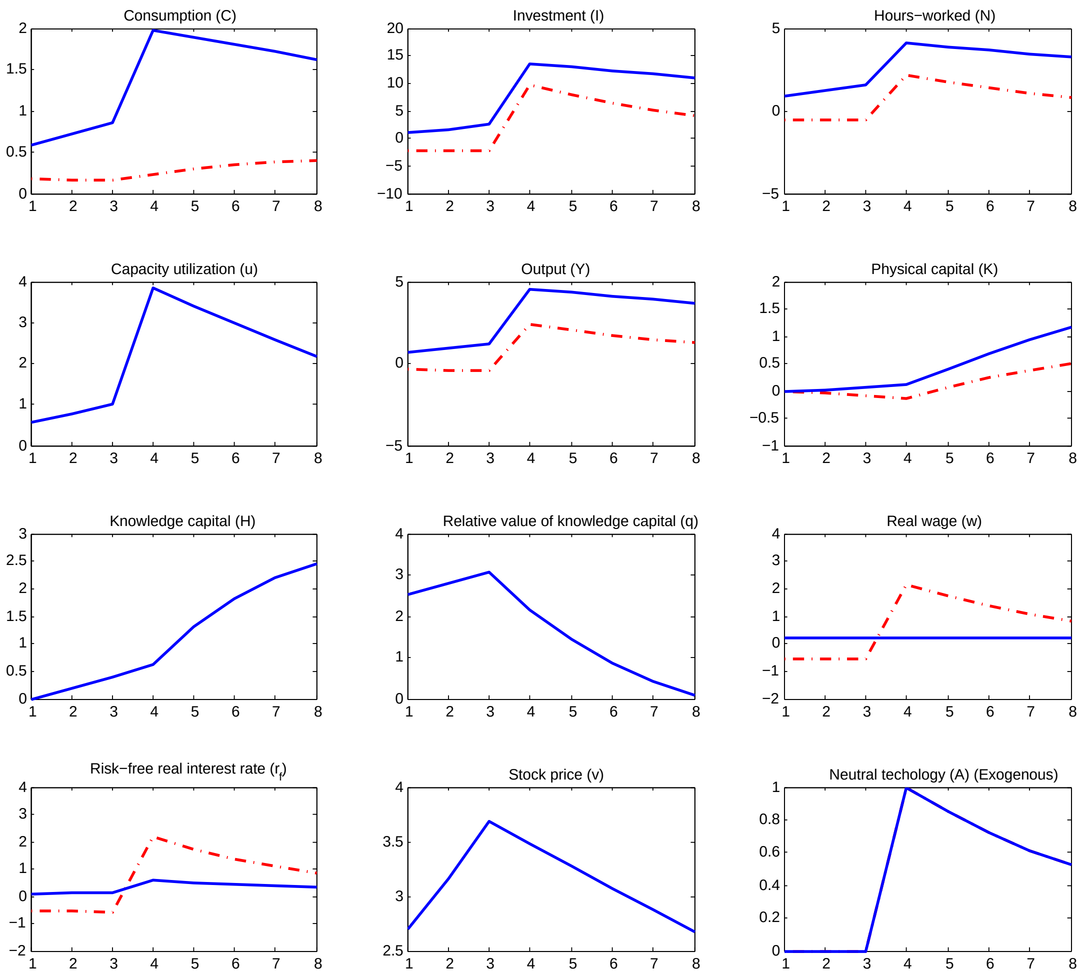
### 1.4.1 The effect of varying preferences

We can influence the response of consumption and investment by altering preferences. Below we consider two possibilities. In Figure 1.5 we show the case of indivisible labour with nonseparability in consumption and leisure. Figure 1.6 shows the response of preferences based on Jaimovich and Rebelo (2009). As both figures show, consumption now rises more than in the case of separable preferences, both in response to arrival of the news, and when the TFP shock actually hits. In both cases investment responds less aggressively.

For the nonseparable indivisible labour preferences, with $\sigma > 1$, the marginal utility of consumption is increasing in hours-worked, making consumption track closer to hours-worked. As discussed by King and Rebelo (2000), when $\sigma > 1$, the combination of nonseparability of consumption and leisure and in-

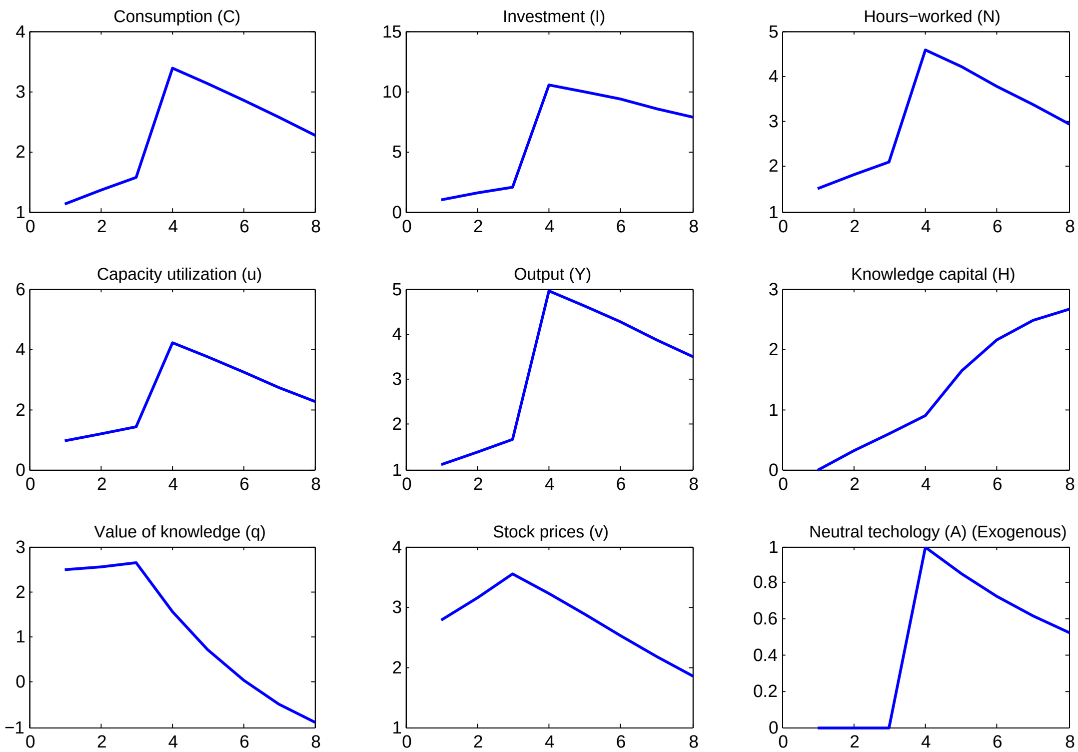
[10] The model is also capable of generating expectational-booms in response to investment specific technology shocks. Plots for these are available from the authors.

Figure 1.5: **Firm-specific knowledge: nonseparable indivis lbr pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *fully realized* in period 4




divisibility in hours-worked imply that the consumption of the employed group will exceed that of the unemployed group. An increase in total hours, which occurs along the extensive employment margin, represents an increase in the number of individuals moving from unemployment to employment. Since the employed enjoy higher consumption levels, total consumption responds more

Figure 1.6: **Firm-specific knowledge: JR pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *fully realized* in period 4

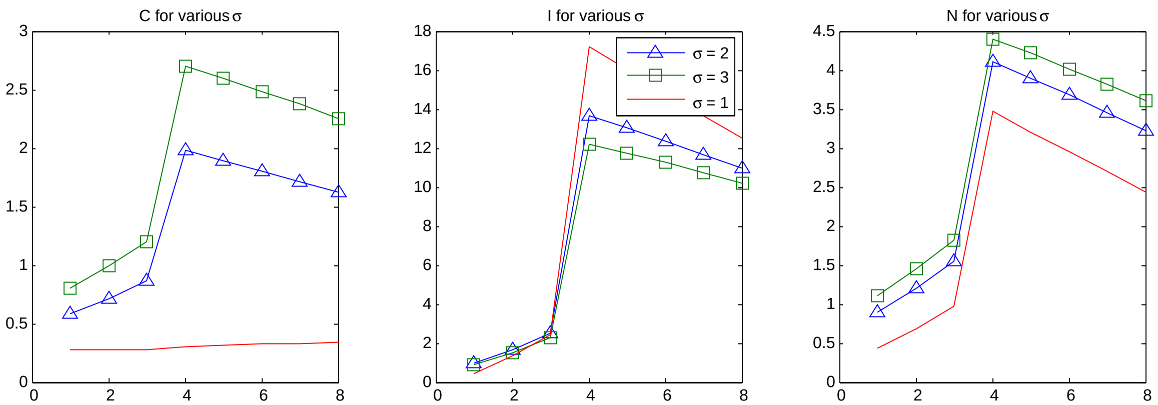



than in the separable case. Figure 1.7 shows the impact of changing sigma on the response of consumption, investment and hours as $\sigma$ varies from 1 to 3.[11] The sharp increase in the response of both variables as $\sigma$ increases is clearly visible.

The nonseparablility built into JR preferences also boosts the response of consumption by making the marginal utility of consumption depend on labour. These preferences, however, also offer the benefit of being able to parameterize

[11]Recall that for $\sigma = 1$, the linearized form of the generalized nonseparable indivisible labour preferences is equivalent to the linearized form of the standard separable indivisible labour preferences with log consumption that we consider in the first preference case.

Figure 1.7: **Effect of varying $\sigma$: nonseparable indivis lbr pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *fully realized* in period 4



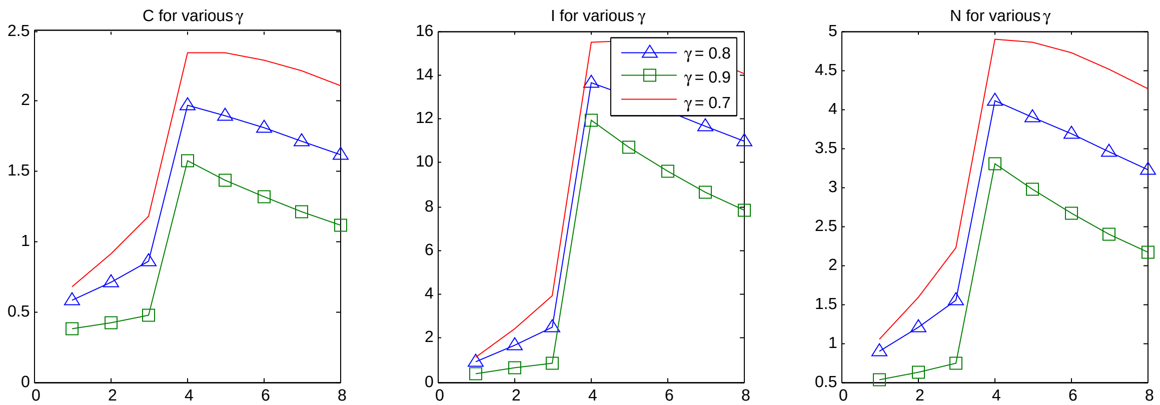

the wealth effect on leisure. Given the choice of $\zeta = 0.01$, the wealth effect on leisure is so small that it does not counteract the shift in labour demand caused by the jump in $q$, and thus the response of hours is greater than in the previous cases.

Having discussed the impact of changing preferences on the results, for the remainder of our results we will focus on the nonseparable case where $\sigma = 2$.

### 1.4.2 Effect of varying $\gamma$

Next we turn to the impact of varying $\gamma$. In Section 1.3.4 we stated that our chosen value of $\gamma = 0.8$ represented a typical value from the literature. Figure 1.8 shows the effects on consumption, investment and hours of varying $\gamma$. Clearly the model continues to display co-movement over this range of $\gamma$, however, the increased curvature from using a lower $\gamma$ strengthens the response of hours and therefore the other variables over this range.

Figure 1.8: **Effect of varying $\gamma$: nonseparable indivis lbr pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock *fully realized* in period 4




### 1.4.3 The role of capital utilization

Due to the assumption of constant returns to all factors in production, the contribution of knowledge capital to output is much smaller than was the case in the human capital example economy discussed earlier. As a result, with $K$ and $H$ fixed in the initial period, without capital utilization, the increase in hours-worked alone cannot raise output sufficiently to finance both an increase in consumption and investment in period 1. Adding capital utilization to the model allows capital services to expand along with labour and therefore increase the responsiveness of output. The optimal determination of utilization can be seen by combining the household's first-order condition for utilization (1.24) with the firm's first-order condition for capital services (1.32), and imposing equilibrium to give

$$\delta'(u_t)K_t = A_tF_{ut}. \tag{1.42}$$

Note that unlike models that include both utilization and intertemporal adjustment costs to capital or investment, here there is no direct intertemporal link to the optimal level of utilization (such as through changes in the relative price of investment or capital which would alter the cost of adjusting utilization). Utilization simply responds to changes in its marginal product through the variation of the other factors of production, or changes to the stock of capital. Thus the role of capacity utilization in the model is to simply amplify the boom.

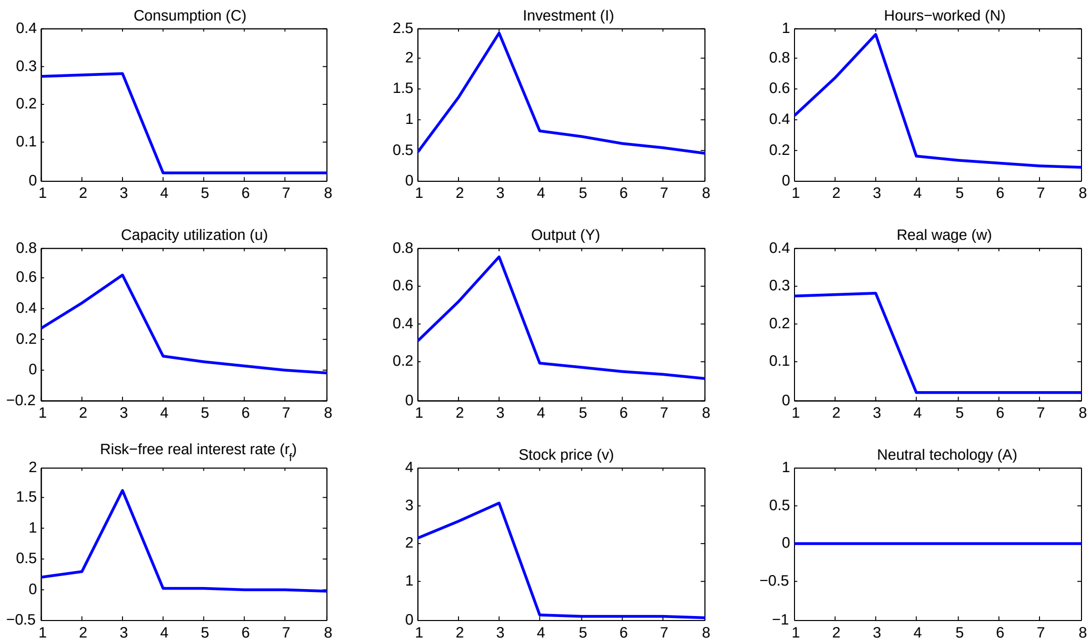
However, while capacity utilization acts as a magnification device for the boom, it cannot deliver one in the absence of knowledge capital. Absent knowledge capital, hours would fall upon receipt of the news which would reduce the marginal productivity of varying capital utilization and this would, in turn (1.42) induce a reduction in utilization which would further magnify the contraction of output [12].

### 1.4.4 A Boom-Bust Episode

Thus far, we have considered the artificial situation in which the expected increases in productivity are fully realized. In reality, agents' forecasts about future fundamentals will be imperfect, and expectations will be continuously revised. A question investigated at length by Beaudry and Portier (2004) and Christiano et al. (2008) asks whether "boom-bust" behaviour in aggregate quantities and asset prices could result when a news shock turns out be to ex-post "too optimistic" and expectations are as a consequence revised down-

[12] Impulse response plots for this case are available from the authors.

Figure 1.9: **A boom-bust episode: nonseparable indivis lbr pref's** - News shock in period 1 about neutral tech. shock in period 4, tech. shock ***not*** *realized* in period 4




wards. In this section we briefly consider the extreme situation in this regard where agents receive news of an expected future increase in TFP that turns out be to fully unrealized.

Figure 1.9 shows the response of our model economy to news of a 1% increase in TFP in period 4 and then no eventual realization of that shock in period 4. Since agents proceed as if the shock will be realized, the response of the model in periods 1 to 3 is identical to the previous case: firms and households ramp-up investment in physical and knowledge capital, hours, output, factor prices and stock prices rise along with consumption. When the shock fails to materialize in period 4 however, agents revise their expectations downward and a recession ensues. Agents realize that the excess capacity of the economy needs to be worked off and the value of knowledge capital plummets. This leads to a sharp revision in the value of firms and a "correction in the stock market". While stock prices fall quickly, the response of other variables is more gradual. The last panel of the figure shows that this boom-bust in asset prices and quantities occurs in the absence of any exogenous technical change. More importantly, consistent with discussions in Beaudry and Portier (2004), the model generates both a crash in stock prices and overall recession without any true technological regress or variation in monetary policy.

### 1.4.5 Robustness

In this section we explore the model's sensitivity to key parameter values with regards to its ability to generate an expectations-driven business cycle. We maintain a strict definition of this type of cycle that in response to news, $C$, $I$ and $N$ must be at or above steady-state in period 1, and slope upwards beginning either in period 1 or thereafter. Table 1.2 shows the results of our robustness check for the three different preference specifications. Following the approach of Jaimovich and Rebelo (2009), we vary only 1 parameter from each baseline paramaterization and report the range of the parameter over which the model can still exhibit an expectations-driven business cycle.

It is clear that the parameter ranges are most limited for the KPR separable preferences, especially for the parameters $\epsilon_u$ and $\nu$ that provide for the high-

Table 1.2: Robustness

| *Preferences* | $\varepsilon$ [a] | $\epsilon_u$ | $\gamma$ [b] | $\nu$ | $\zeta$ |
|---|---|---|---|---|---|
| KPR separable | 0.12-0.54 | 0.01-0.18 | 0.57-0.87 | -0.67-0.54 | n/a |
| KPR nonseparable | 0.06-0.34 | 0.01-0.31 | 0.29-0.94 | n/a | n/a |
| JR | 0.01-0.25 | 0.01-0.71 | 0-0.98 [c] | 0.83-1.59 | 0-0.15 |

[a] Due to CRS in $F(\cdot)$, a change to $\varepsilon$ implies a change to $\alpha$ and/or $\theta$. For this exercise we keep $\theta$ constant (through constant $\delta$) and vary $\alpha$. This alters the labour share slightly but by no more than 0.02- for all parameter ranges in this column.

[b] A change in $\gamma$ affects the labour share, but the change is less than 0.02 unless otherwise indicated.

[c] Labour share varies 0.68-0.63 over this range

substitution response. Allowing for nonseparability significantly expands the parameter range, especially by lowering the required $\varepsilon$. We note that without adjustment costs to investment or capital, a typical parameterization for the first two KPR preference specifications that "fails" to exhibit an expectations-drive business cycle under this definition is often characterized by $C$ and $N$ that are above steady state and sloping upwards in accordance with our definition, but an $I$ that begins below steady state - in some cases less than 0.01% below steady state - yet still sloping upwards and often above steady-state in periods 2 or 3, and giving the overall impression of a "boom". While we could likely remedy this behaviour and expand our robustness set by adding real frictions to the model, for the purposes of illustrating our primary mechanism we abstain from additional features and disqualify any parameterization that causes either $C$, $I$ or $N$ to move even slightly below steady state.

Allowing for JR preferences expands the parameter range even further, especially for $\varepsilon$, $\epsilon_u$ and $\nu$ because the absence of a strong wealth effect to leisure eliminates the need for as elastic a response of output. Even though JR preferences allow for very low learning rates as captured by $\varepsilon < 0.05$, this leads to small increases in variables above their steady state values. For the parameter $\zeta$, we note that above 0.15, while $C$, $I$ and $N$ rise above steady state, $N$ and/or $I$ begin to slope downwards following their initial rise and thus we exclude values above 0.15 given our definition.

Across all the preference specifications it is clear that our model requires a high labour supply elasticity. Even with JR preferences with no wealth effect, our range of $\nu = 0.83 - 1.59$ is low compared to the range of robustness for the equivalent parameter found by Jaimovich and Rebelo (2009) in their model. This is not surprising, however, given that the critical mechanism in our model requires a substantial increase in labour supply in response to the shift in labour demand induced by the news.

## 1.5 Conclusion

In this paper we highlight the role of knowledge capital in enabling the existence of expectations driven cycles. We present a model in which firms accumulate knowledge capital as a function of hours-worked at the firm. Since the learning process is internalized by firms, their demand for labour exceeds that implied by equating the wage rate to the marginal product of labour. This occurs because firms take into account not only the current increase in output but also the value of the additional knowledge capital generated by the marginal hour of work hired. This latter effect operates as a “shift” factor for a labour demand curve drawn in wage and hours space and is key to enabling an expectations driven cycle. When news of future increases in technology arrive, the value of the firm’s knowledge capital rises. This induces the firm to hire more labour at any given wage rate and results in increased production. The subsequent increase in knowledge capital also induces the accumulation of physical capital in anticipation of higher productivity next period. Meanwhile households wish to consume more in anticipation of higher income when the new technology eventually arrives. The increase in hours allows output to rise enough for both consumption and investment to co-move. When expectations about future productivity increases are not realized, the model generates a complete boom-bust cycle.

We note that unlike most models of expectations driven cycles, the rise in investment occurs in the absence of any investment adjustment costs or any other frictions. The ability of the model to generate these cycles in the absence of adjustment costs on changes in investment, a feature that is often built into business cycle models, is worth emphasizing. These costs often have unpleasant implications for factor prices as well as for firm values. As discussed by Christiano et al. (2008) and Jaimovich and Rebelo (2009) they imply that the price of capital will fall and interest rates spike upwards in an unrealistic way. Since the model does not have adjustment costs, it does not rely on changes in the price of capital to raise firm values. Rather, the key mechanism is the rise in the value of knowledge capital and its accumulation that raises the value of the firm. This leads to an appreciation in the price of equity shares. Evidence suggests that the boom in stock prices leads increases in total factor productivity. We show our model is consistent with this lead-lag relationship. Moreover the very mechanism that generates the expectational-boom also leads to a rise in asset values.

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## 1.6 Appendix

### 1.6.1 Central Planner's problem

The representative household has preferences defined over sequences of consumption $C_t$ and leisure $L_t$ with expected lifetime utility defined as

$$\mathcal{U} = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t), \tag{1.43}$$

where $\beta$ is the representative household's subjective discount factor and the period utility $u(C_t, L_t)$ function falls within the standard general class of preferences detailed in King et al. (1988).

The representative household operates a production technology that produces output $Y_t$ according to the technology

$$Y_t = A_t F(N_t, K_t, H_t) = A_t N_t^{\alpha} \tilde{K}_t^{\theta} H_t^{\varepsilon}, \tag{1.44}$$

where $A_t$ is the level of an exogenous stationary technology process, $N_t$ is hours-worked, $\tilde{K}_t$ is capital services and $H_t$ is the stock of knowledge capital.

Capital services are defined as

$$\tilde{K}_t = u_t K_t, \tag{1.45}$$

where $K_t$ is the stock of physical capital, and $u_t$ is the utilization rate of that capital. Physical capital evolves according to

$$K_{t+1} = [1 - \delta(u_t)]K_t + I_t \tag{1.46}$$

where $I_t$ is investment, and where the depreciation function $\delta(\cdot)$ satisfies the conditions $\delta'(\cdot) > 0$, $\delta''(\cdot) \geq 0$.

The common exogenous total factor productivity process $A_t$ evolves in logs according to the stationary AR(1) process

$$\ln A_t = \rho_A \ln A_{t-1} + \epsilon^p_{A,t-p} + \varepsilon_{A,t}, \tag{1.47}$$

where $\rho_a < 1$, $\epsilon^p_{At}$ is a news shock that agents receive in period $t$ about the innovation $\theta_{At+p}$, and $\varepsilon_{A,t}$ is an unanticipated contemporaneous shock.

The stock of knowledge capital $H_t$ evolves according to

$$H_{t+1} = \Psi(H_t, N_t) = H_t^\gamma N_t^{1-\gamma}. \tag{1.48}$$

Each period, the representative household is endowed with one unit of time that can be allocated between leisure and hours-worked $N_t$ according to

$$N_t + L_t = 1. \tag{1.49}$$

Finally, the economy's resource constraint is given by

$$C_t + I_t = Y_t. \tag{1.50}$$

Combining the above equations, the planner's consolidated resource constraint is

$$C_t + K_{t+1} - [1 - \delta(u_t)]K_t = A_t N_t^\alpha (u_t K_t)^\theta H_t^\varepsilon. \tag{1.51}$$

The central planner chooses contingent infinite sequences of $C_t$, $N_t$, $u_t$, $K_{t+1}$ and $H_{t+1}$ to maximize (1.43) subject to equations (1.48) and (1.51).

Letting $\Upsilon_t$ and $\lambda_t$ be the period Lagrange multipliers on (1.48) and (1.51)) respectively, the planner's first-order conditions are as follows:

$$u_C(C_t, L_t) = \lambda_t \tag{1.52}$$

$$U_L(C_t, L_t) = \lambda_t A_t F_{Nt} + \Upsilon_t \Psi_{Nt} \tag{1.53}$$

$$\delta'(u_t)K_t = A_t F_{ut} \tag{1.54}$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} \left[ A_{t+1} F_{Kt+1} + 1 - \delta(u_{t+1}) \right] \right\} \tag{1.55}$$

$$\Upsilon_t = \beta E_t \left\{ \lambda_{t+1} A_{t+1} F_{Ht+1} + \Upsilon_{t+1} \Psi_{Ht+1} \right\} \tag{1.56}$$

Define $q_{ht} = \frac{\Upsilon_t}{\lambda_t}$ as the value of new knowledge capital in terms of consumption. Applying this definition and substituting out $\lambda_t$, we can re-write the knowledge capital and hours first-order conditions as

$$q_{ht} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ A_{t+1} F_{Ht+1} + q_{ht+1} \Psi_{Ht+1} \right] \right\} \tag{1.57}$$

$$\frac{u_L(C_t, L_t)}{u_C(C_t, L_t)} = A_t F_{Nt} + q_{ht} \Psi_{Nt} \tag{1.58}$$

### 1.6.2 The value of the firm

In this section we investigate steady-state firm profits and the time $t$ value of the firm. As in the main body of the paper, lower case variables indicate individual agent quantities or economy-wide prices, and upper case variables indicate aggregate quantities.

#### Steady-state firm profits

First, we re-write the firm's two technologies $F(\cdot)$ and $\Psi(\cdot)$ without imposing any particular returns to scale as

$$Y_t = A_t F(N_t, \tilde{K}_t, H_t) = N_t^{\alpha} (\tilde{K}_t)^{\theta} H_t^{\varepsilon} \tag{1.59}$$

and

$$\Psi(H_t, N_t, \tilde{K}_t) = H_t^{\gamma} N_t^{\eta}. \tag{1.60}$$

Since $H_{t+1} = H_t^{\gamma} N_t^{\eta}$, in steady state $H = N^{\frac{\eta}{1-\gamma}}$ and thus

$$Y = AN^{\alpha + \frac{\eta\varepsilon}{1-\gamma}} \tilde{K}^{\theta} = A\tilde{F}(N, \tilde{K}), \tag{1.61}$$

so that in steady state the firm's production function can be expressed as a function of just labour and capital services. From (1.61) we can see that for a given $\alpha$, $\theta$ and $\gamma$, we can impose constant returns to labour and capital services in steady-state such that $\alpha + \theta + \frac{\eta\varepsilon}{1-\gamma} = 1$ by either: (i) $\eta < 1-\gamma$ (DRS in $\Psi(\cdot)$) and $\varepsilon > 1-\alpha-\theta$ (IRS in $F(\cdot)$), or (ii) $\eta > 1-\gamma$ (IRS in $\Psi(\cdot)$) and $\varepsilon < 1-\alpha-\theta$ (DRS in $F(\cdot)$), or (iii) $\eta = 1-\gamma$ (CRS in $\Psi(\cdot)$) and $\varepsilon = 1-\alpha-\theta$ (CRS in $F(\cdot)$. In this paper we impose case (iii), and as such in steady-state,

$$Y = AN^{1-\theta}\tilde{K}^{\theta}., \tag{1.62}$$

where we explicitly note that despite the presence of knowledge capital as an input into production, in steady state, the production of $Y$ displays CRS to just labour and physical capital services. We can see how this relates to firm profits in steady-state by expressing $D = Y - wN - r\tilde{K}$ as a share of output as

$$\frac{D}{Y} = 1 - \frac{wN}{Y} - \frac{r\tilde{K}}{Y} = 1 - S_N - S_{\tilde{K}}, \tag{1.63}$$

where $S_N$ is the steady-state labour share and $S_{\tilde{K}}$ is the steady-state capital services share. Applying (1.59) and (1.60) to the firm's first-order conditions gives h and

$$S_{\tilde{K}} = \theta, \tag{1.64}$$

so that

$$\frac{D}{Y} = 1 - \alpha - \theta - \left(\frac{\eta}{1/\beta - \gamma}\right)\varepsilon = 1 - [\alpha + \left(\frac{\eta}{1/\beta - \gamma}\right)] - \theta, \tag{1.65}$$

or in our case with $\eta = 1-\gamma$ and $\varepsilon = 1-\alpha-\theta$

$$\frac{D}{Y} = \left(\frac{1/\beta - 1}{1/\beta - \gamma}\right)\varepsilon = \left(\frac{\xi}{\xi + (1-\gamma}\right)[1-\alpha-\theta], \tag{1.66}$$

where $\xi = 1/\beta - 1$ is the household-owner's subjective discount rate. From (1.65) and (1.66) it is evident that the steady-state profit share will be affected not only by the steady-state returns to scale to $N$ and $\tilde{K}$ as implied in (1.61), but also the household-owner's subjective rate of time discount $\beta$. With $\beta < 1$, the share of profits is slightly positive, even though $F(\cdot)$ exhibits constant returns to $N$ and $\tilde{K}$.

### Period $t$ dynamic value of firm

Having established the firm's steady-state profits, we now obtain an expression for the dynamic period $t$ value of the firm. We follow an approach similar to that used by Jaimovich and Rebelo (Jaimovich and Rebelo (2009)) to value the firm using a recursive formulation of the firm's problem.

First, we note that the stochastic process (1.47) can be represented by the first-order system

$$\ln A_t = \rho_A \ln A_{t-1} + \Omega_t^p + \varepsilon_{A,t} \tag{1.67}$$
$$\Omega_t^p = \Omega_{t-1}^{p-1} \tag{1.68}$$
$$\Omega_t^{p-1} = \Omega_{t-1}^{p-2} \tag{1.69}$$
$$\vdots \tag{1.70}$$
$$\Omega_t^{p-p+1} = \Omega_t^{p-p} = \Omega_{t-1}^{0} \tag{1.71}$$
$$\Omega_t^0 = \epsilon_t. \tag{1.72}$$

Defining the $p \times 1$ vector $\Omega_t = \begin{bmatrix} \Omega_t^P \\ \Omega_t^{P-1} \\ \vdots \\ \Omega_t^0 \end{bmatrix}$, the system can then be represented compactly using matrix notation by

$$\ln A_t = \rho_A \ln A_{t-1} + \Omega_t^p + \varepsilon_{A,t} \tag{1.73}$$
$$\Omega_t = B\Omega_{t-1} + G\epsilon_t \tag{1.74}$$

where $B$ is a $p \times p$ matrix with all zeroes except for ones on the first diagonal above the main diagonal, and $G$ is a $p \times 1$ vector with all zeroes except for a one in the final row. Using this notation, the agent's exogenous states are then $A_t$ and $\Omega_t$, the former representing the current state of technology and the latter the agent's current information set of news shocks relevant for forecasting the future state of technology.

We can then re-formulate the firm's problem (1.30) recursively as

$$V(H, A, \Omega) = \max_{N, \tilde{K}', H'} \left\{ \lambda [AF(N, \tilde{K}, H) - wN - r\tilde{K}] + \beta EV(H', A', \Omega') \right\} \tag{1.75}$$

subject to

$$H' = \Psi(H, N) \tag{1.76}$$

and where the firm takes as given from the aggregate

$$\mathbb{K}' = \mathbb{K}(\mathbb{H}, \mathbb{K}, A, \Omega) \tag{1.77}$$
$$\mathbb{H}' = \mathbb{H}(\mathbb{H}, \mathbb{K}, A, \Omega) \tag{1.78}$$
$$\ln A_t = \rho_A \ln A_{t-1} + \Omega_t^p + \varepsilon_{A,t} \tag{1.79}$$
$$\Omega_t = B\Omega_{t-1} + G\epsilon_t \tag{1.80}$$
$$\lambda' = \lambda(A, H, K, \Omega) \tag{1.81}$$
$$w = w(\mathbb{H}, \mathbb{K}, A, \Omega) \tag{1.82}$$
$$r = r(\mathbb{H}, \mathbb{K}, A, \Omega) \tag{1.83}$$

Note that we have defined $V(H, A, \Omega)$ in terms of the household-owners' utility as given by $\lambda$, so that in terms of the notation used in (1.30) in the main text, $V_t = \frac{V(H,A,\Omega)}{\lambda}$. Letting $\Upsilon$ be the Lagrange multiplier on (1.76), we write (1.75) as

$$V(H, A, \Omega) = \max_{N, \tilde{K}', H'} \left\{ \lambda[AF(N, \tilde{K}, H) - wN - r\tilde{K}] + \Upsilon[\Psi(H, N) - H'] + \beta EV(H', A', \Omega') \right\}. \tag{1.84}$$

Solving the maximization on the right-hand side gives

$$w = AF_N(N, \tilde{K}, H) + \frac{\Upsilon}{\lambda}\Psi_n(h, n) \tag{1.85}$$
$$r = AF_{\tilde{K}}(N, \tilde{K}, H) \tag{1.86}$$
$$\Upsilon = \beta EV_1(H', A', \Omega'). \tag{1.87}$$

Now define $\bar{V}(H, A, \Omega) = \beta EV(H', A', \Omega')$ as the end-of-period value of the firm, which is related to (1.30) in the main text by $\bar{V}_t = \frac{\bar{V}(H,A,\Omega)}{\lambda}$, and to the price $v_t$ of a share of the firm's equity through the household's Euler equation for equity (1.26) as $v_t = \bar{V}_t = \frac{\bar{V}(H,A,\Omega)}{\lambda}$.

Since it can be shown that $\bar{V}(H, A, \Omega)$ is homogenous of degree 1 in $H$ (which we prove below in 1.6.2 below), we can write

$$\bar{V}(H, A, \Omega) = \beta EV_1(H', A, \Omega)H'. \tag{1.88}$$

Substituting the firm's $H'$ first-order condition (1.87) into (1.88) then gives

$$\bar{V}(A,H) = \Upsilon h' \tag{1.89}$$

as the end-of-period value of the firm in terms of the household-owners' utility. Using the notation in the main text, this then gives

$$\bar{V}_t = v_t = \frac{\Upsilon_t}{\lambda_t} H_{t+1}. \tag{1.90}$$

Or, defining $q_t = \frac{\Upsilon_t}{\lambda_t}$,

$$\bar{V}_t = v_t = q_t H_{t+1}, \tag{1.91}$$

so that the period $t$ price $v_t$ of the firm's equity share is the product of the value of knowledge capital in terms of consumption today and next period's stock of knowledge capital.

It only remains to establish that $\bar{V}(H,A,\Omega)$ is homogenous of degree 1 in $H$, which we do in the following section.

**Degree 1 homogeneity of $\bar{V}(H,A,\Omega)$**

The firm's recursive problem (1.75) in our model has the unique property that both the return-function $\lambda[AF(N,\tilde{K},H) - wN - r\tilde{K}]$ and the constraint $H' = \Psi(H,N)$ are homogeneous of degree 1 (hod 1) in $N$, $\tilde{K}$ and $H$. In what follows we will show that these properties then imply that $V(H,A,\Omega)$ and thus $\beta EV(H',A',\Omega')$ are hod 1 in $H$.

First, defining any contingent sequence $\{H_t\}_{t=1}^{\infty}$ as a plan, let

$$\Pi(H_0,A_0,\Omega_0) = \{\{H_t\}_{t=1}^{\infty} : H_{t+1} = \Psi(H_t, N^*(H_t,A_t,\Omega_t)), t = 0,1,...\} \tag{1.92}$$

be the set of plans that are feasible from $(H_0,A_0,\Omega_0)$ in that they satisfy the initial conditions $H_0$, $A_0$, and $\Omega_0$ and the constraint $H_{t+1} = \Psi(H_t, N^*(H_t,A_t,\Omega_t))$, where $N_t = N^*(N_t,A_t,\Omega_t)$ is the policy function for $N_t$, and $\underline{N}$ a typical feasible plan in $\Pi(N_0,A_0,\Omega_0)$. Then let

$$u(\underline{H}) = \sum_{t=0}^{\infty} \left\{ \lambda[AF(N^*(H_t,A_t,\Omega_t), \tilde{K}^*(H_t,A_t,\Omega_t), H_t) - wN^*(H_t,A_t,\Omega_t) - r\tilde{K}^*(H_t,A_t,\Omega_t)] \right\} \tag{1.93}$$

be the discounted sum of values of the return function over some feasible plan $\underline{H}$, where $\tilde{K}_t = \tilde{K}^*(H_t,A_t,\Omega_t)$ is the policy function for $\tilde{K}_t$. The maximum value function is then defined as $V^*(H_0, A_0, \Omega_0) = \max_{\underline{H} \in \Pi(H_0, A_0, \Omega_0)} u(\underline{H})$. By the hod 1 of our return function,

$$\begin{aligned}
&\lambda[AF(N^*(\theta N_t, A_t, \Omega_t), \tilde{K}^*(\theta H_t, A_t, \Omega_t), \theta H_t) - wN^*(\theta H_t, A_t, \Omega_t) - r\tilde{K}^*(\theta H_t, A_t, \Omega_t)] \\
&\quad = \theta\lambda[AF(N^*(H_t, A_t, \Omega_t), \tilde{K}^*(H_t, A_t, \Omega_t), H_t) - wN^*(H_t, A_t, \Omega_t) - r\tilde{K}^*(H_t, A_t, \Omega_t)]
\end{aligned} \tag{1.94}$$

for some value $\theta$, and therefore

$$\begin{aligned}
u(\theta\underline{H}) = \theta \sum_{t=0}^{\infty} \Big\{ \lambda[AF(N^*(H_t, A_t, \Omega_t), \tilde{K}^*(H_t, A_t, \Omega_t), H_t) - wN^*(H_t, A_t, \Omega_t) - r\tilde{K}^*(H_t, A_t, \Omega_t)] \Big\} \\
= \theta u(\underline{H}).
\end{aligned} \tag{1.95}$$

Similarly, by the hod 1 or our constraint, $\Psi(\theta H_t, N^*(\theta H_t, A_t, \Omega_t)) = \theta\Psi(H_t, N^*(H_t, A_t, \Omega_t))$, and therefore

$$\underline{H} \in \Pi(H_0, A_0, \Omega_0) \Leftrightarrow \theta\underline{H} \in \Pi(\theta H_0, A_0, \Omega_0). \tag{1.96}$$

As a result,

$$\begin{aligned}
V^*(\theta H_0, A_0, \Omega_0) &= \max_{\theta\underline{H} \in \Pi(\theta H_0, A_0, \Omega_0)} u(\theta\underline{H}) \\
&= \max_{\underline{H} \in \Pi(H_0, A_0, \Omega_0)} \theta u(\underline{H}) \\
&= \theta V^*(H_0, A_0, \Omega_0),
\end{aligned} \tag{1.97}$$

which implies that for any given state $(H, A, \Omega)$ , $V(\theta H, A, \Omega) = \theta V(H, A, \Omega)$, and therefore $\beta EV(\theta H', A', \Omega') = \theta\beta EV(H', A', \Omega')$, and thus $\bar{V}(H, A\Omega)$ is homogenous of degree 1.

# Chapter 2

# News, Credit Spreads and Default Costs: An expectations-driven interpretation of the recent boom-bust cycle in the U.S.

## 2.1 Introduction

> Many of the financial institutions and instruments caught up in the crisis are part of the centuries old phenomenon of financial innovation. The new instruments, often devised to avoid regulation, are then proved to be successful or not by the test of financial stress such as we have been recently encountering (Bordo, 2007).

In this paper we explore the role of changes in expectations about future increases in the fundamentals of the financial sector in inducing boom-bust cycles in macroeconomic aggregates. Most DSGE models of the recent financial crisis and recession begin with various financial shocks such as shocks to collateral values or credit constraints in order to connect fluctuations in the financial sector to real economic activity[1]. Moreover, this literature typically stresses the role of contemporaneous shocks arriving in 2007/2008 and pays relatively little attention to the years leading up to the crisis. In contrast, we show that changes in expectations about the costs associated with defaults can lead to movements in credit spreads, asset prices, leverage, aggregate net-worth and total quantity of credit as well as aggregate macro quantities to simultaneously explain both the "great recession" period as well as the boom that preceded it[2].

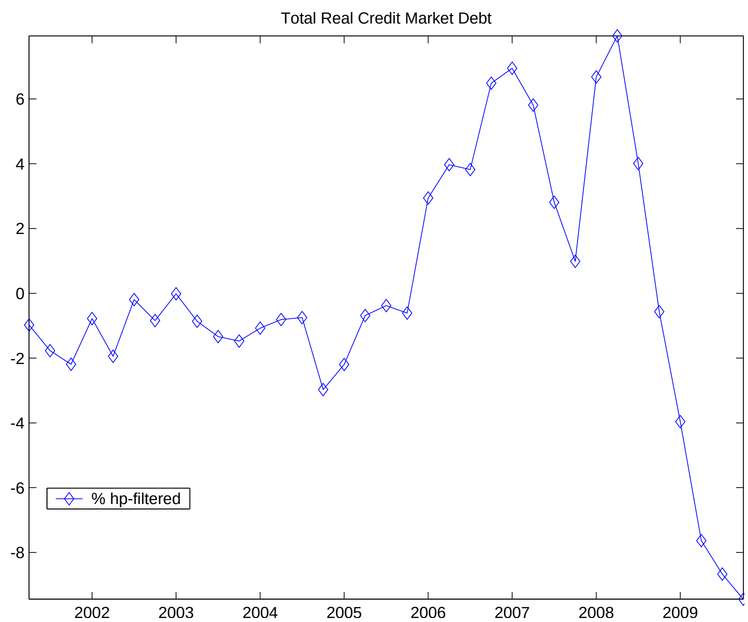
Before discussing the model further, it is useful to consider some of the boom-bust patterns exhibited by the recent U.S. data. The years leading up to the "great recession" were a time of rapid innovation in the financial industry. This period also saw a fall in interest rate spreads, and a boom in liquidity that accompanied the boom in real activity, especially investment. We wish to explore the possibility that these were related phenomena. The linkages are easy to see: the emergence and rapid adoption of new financial products and practices could have led agents to expect a fall in the overall costs of intermediation which in turn engendered the flood of liquidity in the financial sector, lowered interest rate spreads and facilitated the boom in economic activity. When the events of 2007-2009 led to a re-evaluation of the effectiveness of these new products, agents revised downwards their initial ex-

[1]See Gertler and Karadi (2011), Gertler and Kiyotaki (2010), Christiano, Motto and Rostagno (2010), Gilchrist, Ortiz and Zakrajsek (2009), Jermann and Quadrini (2009), Nolan and Thoenissen (2009).

[2]Beaudry and Lahiri (2009) has a similar interest in linking the crisis to the preceding period. Unlike us they focus on the lack of productive investment opportunities available at that time which induce liquidity in the system.

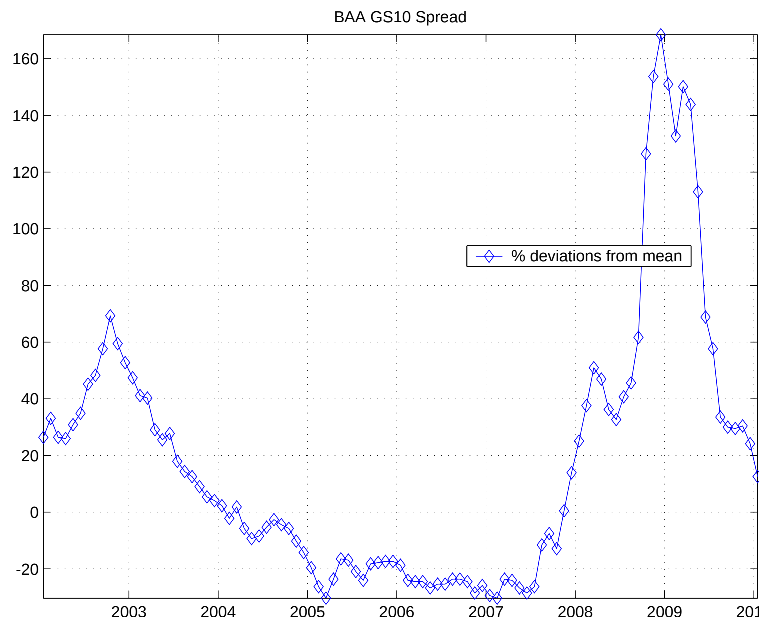
pectations regarding the actual efficiency gains, and this led to a withdrawal of liquidity from the financial system, a reversal in interest rates and a bust in real activity. Figures 2.1 and 2.2 display this boom-bust cycle in credit and interest rates for the US economy. Figure 1 displays the rapid rise in the total level of real credit relative to its long run trend and the subsequent pronounced bust that followed. Figure 2.2 displays the behaviour of the spread between the yield on BAA bonds and the ten year treasury bond over the same period. As is clear from the graph, the spread fell roughly 25 percent below mean levels and then rose to well over 100 percent during the crisis.

Figure 2.1: Total Real Credit Market Debt




While the role of technical progress and innovation in goods production has been central to business cycle models in the last three decades, innovation in the financial sector has not received the same attention in the business cy-

Figure 2.2: BAA-GS10 Spread




cle literature even though it has been widely discussed in the financial press. The decade leading up to the financial crisis was especially a time of rapid innovation in the financial sector. Particularly important to the crisis was the development of new debt instruments such as residential and commercial mortgage-backed securities, collateralized debt obligations (CDOs), collateralized loan obligations, asset-backed commercial paper (ABCP), structured investment vehicles and the widespread use of credit default swaps to insure against default. A brief look at two instruments elucidates this point. For example, the total amount of asset backed commercial paper doubled from around 600 billion in January 2001 to over 1.2 trillion in mid 2007. A similarly rapid expansion took place with credit default swaps. According to the international swaps and derivatives association (ISDA Market Survey 2010), the market for CDS rose from about 900 billion in 2001 to 62 trillion in 2007.

In the context of our macroeconomic analysis, we abstract from the fact that assets come in many different risk levels and that different investors have varying tolerance for risk. Instead, following the macroeconomic literature on agency costs, we model intermediaries as agents that originate a portfolio of loans, some of which will be defaulted upon. The intermediary expects to lose a fraction of the value of these loans due to various bankruptcy and monitoring costs. As part of our abstraction, we view the emergence of new financial products in terms of these costs: the ability to move risky loans off one's balance sheet or the ability to buy insurance against a default event using a CDS may be interpreted as lowering the expected losses associated with any default episode. The rapid development and adoption of these new financial products suggests the possibility that expectations of future losses associated with defaults may have fallen too much relative to the actual efficiency gains that these products could deliver. The optimism of lenders regarding the costs associated with defaults creates expectations that the return on deposits with financial intermediaries will rise which in turn leads to a large increase in the amount of funds in the financial system.

To this end, we model bankruptcy/monitoring costs as a stochastic process, and interpret shocks to this process as stochastic variation in financial innovation [3]. While it abstracts from the complexity of credit markets, we believe it is useful to think of innovation in the financial sector in terms of a

[3]Levin et al. (2004) present a partial-equilibrium representation of a costly-state verification problem similar to that in Bernanke et al. (1999), using a time-varying cost of monitoring parameter in order to quantify the time-series properties of this parameter over a panel of 900 firms from 1997Q1 to 2003Q3.

random efficiency parameter in parallel to the way we think of technical change in product markets in terms of TFP shocks [4]. We interpret the period leading up to the crisis as being one in which this parameter governing bankruptcy costs was low or expected to be below its steady state level. Furthermore, we interpret the widespread scrutiny of the financial sector and their products that began in 2007 in terms of revision of expectations - that in the future this parameter would be much higher. By interpreting this phenomena in terms of a change in expectations about future fundamentals - in this case the fundamentals of the financial intermediation process - we exploit the ideas of the recent "news shock" business cycle literature that investigates the role of changes in agents' expectations about future total factor productivity (TFP) in producing business cycle fluctuations [5]. We find the case of an *unfulfilled* news shock especially instructive [6]. In our exercise, agents receive news that this cost parameter will fall at some future date, but this news turns out to be false when the date finally arrives. In anticipation of the cost savings, agents flood the system with liquidity, which creates an investment boom, higher production and employment, a fall in credit spreads and a rise in asset prices. When the news turns out to be false, a bust ensues with falling investment and

[4]This distinguishes us from a number of other studies of the financial crisis that focus on shocks to the system that increase aggregate risk. We do not deny that it may be useful to think of shocks in the financial sector in terms of an increase in risk but think it is useful to study different aspects of the crisis in order to gain a full understanding of what happened.

[5]See Beaudry and Portier (2004), Beaudry and Portier (2006), Christiano, Ilut, Motto and Rostagno (2008), Jaimovich and Rebelo (2009), Gunn and Johri (2011a), Den Haan and Kaltenbrunner (2009), Schmitt-Grohe and Uribe (2009), Khan and Tsoukalas (2009), and Dupor and Mehkari (2009).

[6]Beaudry and Portier (2004) illustrate a special case where agents first receive news about a future increase in TFP and then subsequently find out that these expectations were "overoptimistic", in the sense that the expected change in TFP fundamentals is not realized ex-post and therefore unfulfilled.

employment along with a sudden rise in credit spreads and fall in asset prices. Interestingly the entire boom and bust occur without any actual change in the cost parameter.

We use a simple and stylized model of financial intermediaries to capture the essence of the above observations and embed this into a relatively straightforward real dynamic general equilibrium business cycle model.[7] A financial intermediary issue debt instruments to households and uses the proceeds to make loans to entrepreneurs to finance purchases of new capital. A zero profit condition ties the interest paid to lenders to the interest rate charged to borrowers but the two are not equal because financial intermediaries must cover the costs associated with those firms that are unable to repay their loans. A fall in these costs allows the spread between rates charged to borrowers and lenders to fall. Interestingly, spreads decline in the boom and rise in the bust even in the absence of any actual movement in the cost parameter. This occurs because net-worth rises endogenously during the boom phase and falls during bust independent of changes in the exogenous bankruptcy costs, improving the overall return recovered from defaulted loans during the boom, and worsening it during the bust. As a result, the financial intermediary can charge a lower interest rate relative to the safe-rate on the loan portfolio during the boom phase as compared to the bust. In this sense, the model generates fluctuations in spreads that are purely driven by changes in expectations.

[7] The role of the Federal Reserve and monetary policy before and during the crisis has been the subject of much debate and research. We deliberately choose to work with a real model in order to keep attention focused on the issues at hand. In this context we also note our focus on interest rate spreads as opposed to the level of short term interest rates that may be more under the control of monetary authorities.

In the next section we present our model. Section 3 discusses how we parameterize the linearized model. Section 4 presents both an illustration of the optimal contract, and simulation results for the response of the model to contemporaneous shocks, and fulfilled and unfulfilled news shocks. Section 4 concludes.

## 2.2 Model

Our model embeds the financial accelerator mechanism of Bernanke, Gertler and Gilchrist (1999) into an otherwise standard real business cycle model. Since we will not study the role of monetary policy in this paper, we omit the New Keynesian elements present in that model. The model economy consists of a representative infinitely-lived stand-in household, one each of a single goods-producer, capital-producer and financial intermediary who all nonetheless act competitively, as well as a unit measure of risk-neutral entrepreneurs. The household owns the goods-producer and capital-producer as well as the financial intermediary. The goods-producer produces output with labour and capital, paying wages to households and renting capital from entrepreneurs. Entrepreneurs purchase capital from the capital producer, financing their capital with their own wealth as well as from loans from the financial intermediary. The entrepreneurs' capital returns are subject to idiosyncratic shocks observable to the entrepreneurs but not the financial intermediary, and thus the lending arrangement between the financial intermediary and a given entrepreneur involves agency costs. The financial intermediary finances its loans to entrepreneurs by issuing risk-free securities to households. The capital-producer creates new capital by purchasing output from the goods market and combining it with existing capital.

In addition to markets for labour and goods, we assume the existence of a market for household deposits (financial securities), a market for intermediated loans, and a market for capital goods.

### 2.2.1 Household

The representative stand-in household has preferences defined over sequences of consumption $C_t$ and hours-worked $N_t$ with expected lifetime utility defined as

$$\mathcal{U} = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \tag{2.1}$$

where $\beta$ is the household's subjective discount factor and the period utility function $U(C_t, N_t)$ follows the class of preferences described in King, Plosser and Rebelo (1988).

The household enters into each period with total financial securities $A_t$, earning the riskless gross rate of return $R_t^d$ on its financial securities, receiving the wage rate $w_t$ for supplying hours $N_t$ to the goods-producing firms, and receiving a share of profits from the capital-producers, goods-producers and financial intermediary, denoted collectively as $\Pi_t$. At the end of the period, the household chooses its consumption $C_t$ and its holdings of financial securities $A_{t+1}$ to deposit with the financial intermediary.

The period $t$ household's budget constraint is given by

$$C_t + A_{t+1} = R_t^d A_t + w_t N_t + \Pi_t, \tag{2.2}$$

where the interest rate $R_t^d$ is determined in the previous period.

The household's problem is to choose sequences $C_t$, $N_t$, and $A_{t+1}$ to maximize (2.1) subject to (2.2), yielding the respective first-order conditions

$$u_C(C_t, N_t) = \lambda_t \tag{2.3}$$

$$-u_N(C_t, N_t) = \lambda_t w_t \tag{2.4}$$

$$\lambda_t = \beta R_{t+1}^d E_t \left\{ \lambda_{t+1} \right\}, \tag{2.5}$$

where $\lambda_t$ refers to the Lagrange multiplier on (2.2).

### 2.2.2 Goods-producer

The goods-producing firm produces output $Y_t$ according to a constant returns to scale technology given by

$$Y_t = \tilde{N}_t^{\alpha} K_t^{1-\alpha}, \tag{2.6}$$

where $\tilde{N}_t$ is total hours-worked, and $K_t$ is physical capital rented from entrepreneurs at the rental rate $r_t$. Hours-worked is a composite of both household and entrepreneurial labour, such that

$$\tilde{N}_t = N_t^{\Omega}(N_t^e)^{1-\Omega} \tag{2.7}$$

where household labour $N_t$ is acquired at wage-rate $w_t$ and entrepreneurial labour $N_t^e$ is acquired at wage-rate $w_t^e$.

The firm sells its output in the goods market where it is used as consumption by households or as additions to the capital stock by capital-producers. Each period the firm chooses $N_t$, $N_t^e$ and $K_t$ to maximize its profits $\Pi_t^g = Y_t - w_t N_t - w_t^e N_t^e - r_t K_t$, yielding the first-order conditions

$$w_t = \Omega\alpha\frac{Y_t}{N_t} \tag{2.8}$$

$$w_t^e = (1-\Omega)\alpha\frac{Y_t}{N_t^e} \tag{2.9}$$

$$r_t = (1-\alpha)\frac{Y_t}{K_t}. \tag{2.10}$$

### 2.2.3 Financial Intermediary

At the end of each period $t$ the financial intermediary makes a portfolio of loans to the measure of entrepreneurs, with $B_{t+1}(i)$ denoting the loan to the $i$th entrepreneur, funding this portfolio of loans by issuing securities, $A_{t+1}$, to the household that promise a riskless gross return, $R^d_{t+1}$. For simplicity, the financial intermediary issues no equity and has no other sources of funds. As such, in order for the competitive financial intermediary to guarantee the risk-free return on its household securities each period, it must generate a total return on its loan portfolio in each aggregate contingency to just cover its opportunity cost of funds on the household securities. Although loans to entrepreneurs are subject to both idiosyncratic and aggregate risk, by virtue of the entrepreneurs being risk neutral, as in Bernanke et al. (1999) we can assume that each entrepreneur is willing to bear all the aggregate risk on its loan and thus make state-contingent loan payments that ensure that in each aggregate state of the world the financial intermediary achieves an expected return (where the expectation is over the idiosyncratic returns of the entrepreneur) equal to the intermediary's opportunity cost of funds. This leaves the financial intermediary with only the idiosyncratic risk associated with individual loans, which it can diversify away by virtue of holding a large loan portfolio. As such, in each aggregate state in period $t$, the financial intermediary's budget constraint is

$$\xi_t = R^d_t A_t, \tag{2.11}$$

where $\xi_t$ is the intermediary's return on its entire loan portfolio after idiosyncratic uncertainty has been realized, and where $R_t^d$ and $A_t$ are predetermined. We will now first discuss the entrepreneurial technological environment before detailing the financial contracts between the financial intermediary and the entrepreneurs.

### 2.2.4 Entrepreneurs

Risk-neutral entrepreneurs accumulate physical capital. At the beginning of each period, entrepreneurs rent their capital $K_t(i)$ to the goods-producer at rental rate $r_t$. At the end of the period, they sell their existing capital to the capital-producer at price $q_t$, and then immediately buy back, at price $q_t^n$, their desired level of capital, $K_{t+1}(i)$, to hold into next period. Entrepreneurs finance these capital purchases with their own end-of-period net-worth, $X_{t+1}(i)$, and new loans from the financial intermediary $B_{t+1}(i)$, such that their financing satisfies

$$q_t^n K_{t+1}(i) = X_{t+1}(i) + B_{t+1}(i). \tag{2.12}$$

Entrepreneur $i$'s return to capital is subject to both idiosyncratic and aggregate risk, such that its ex-post return to holding capital from $t$ to $t+1$ is given by

$$R_{t+1}^k(i) = \omega_{t+1}(i) R_{t+1}^k \tag{2.13}$$

where $\omega(i)$ is a random variable providing an idiosyncratic component to entrepreneur $i$'s return, and where

$$R^k_{t+1} = \frac{r_{t+1} + q_{t+1}}{q^n_t}. \tag{2.14}$$

is the ex-post return on capital averaged across all entrepreneurs. The market prices $r_t$, $q_t$ and $q^n_t$ and thus $R^k_{t+1}$ are functions of the aggregate state of the economy. The random variable $\omega$ is i.i.d across firms and time, has cumulative distribution function $F(\omega)$, and is normalized so that $E\omega = 1$.

To prevent entrepreneurs from self-financing and eliminating the need for external finance in the long run, we assume as in Bernanke et al. (1999) that each entrepreneur faces a constant probability, $\gamma$, of surviving into the next period. When entrepreneurs die (and at no other time), they consumer their entrepreneurial equity.

Finally, entrepreneurs supply a unit time endowment inelastically to the good-producers at wage-rate $w^e_t$.

### 2.2.5 Agency problem and debt-contract

As in Bernanke et al. (1999), we assume that the financial intermediary can observe the average return to capital $R^k_t$ but not an entrepreneur's idiosyncratic component $\omega_t(i)$, unless it pays a a monitoring cost. As such, as illustrated by Townsend (1979), the parties can adopt a financial contract that minimizes the expected agency costs, in the form of risky-debt where the monitoring costs are incurred only in states where the an entrepreneur fails to make promised debt payments. Under this structure, as discussed by Williamson (1987) and

Bernanke et al. (1999), the monitoring costs can be interpreted as "default costs" or "bankruptcy costs". We assume that these default costs are a fraction, $\theta_t$, of the entrepreneur's gross payout, $\omega_t(i)R_t^k q_{t-1}K_k(i)$,, however, unlike Bernanke et al. (1999), we follow Levin et al. (2004) in assuming that $\theta_t$ is time varying and follows a stochastic process, the properties of which we will describe below. Moreover, we assume that $\theta_t$ is an exogenous aggregate state that is common to all entrepreneurs, and which is observable by all agents in the economy.

The specific timing of a typical entrepreneur's choices and the contract are as follows: at the end of period t, the entrepreneur chooses its capital expenditures, $q_t^n K_{t+1}(i)$ and associated level of borrowing, $B_{t+1}(i)$, with knowledge of neither the aggregate state in period $t+1$ nor the idiosyncratic realization of $\omega$ in period $t+1$, $\omega_{t+1}(i)$. Conditional on these choices, the terms of the contract between the financial intermediary and the entrepreneur specify a contractual non-default state-contingent gross interest rate, $R_{t+1}^l$ that ensures that in each aggregate state of the world, the financial intermediary achieves an expected return equal to the its opportunity cost of funds. In the event that the entrepreneur's idiosyncractic returns are insufficient to cover its contracted debt payments, the entrepreneur defaults and goes bankrupt, handing over all remaining gross returns to the financial intermediary, leaving the gross returns less default costs to the financial intermediary. Note that given the state-contingent contract structure, the loan rate $R_t^l(i)$ will adjust in period t to reflect the ex-post realization of the aggregate state in $t$.

Carlstrom and Fuerst (1997) and Bernanke et al. (1999) show that such a contract can be represented with a cut-off value $\bar{\omega}_t(i)$ defined as

$$\bar{\omega}_{t+1}(i)R^k_{t+1}q_tK_{t+1}(i) = R^l_{t+1}(i)B_{t+1}(i), \tag{2.15}$$

where if the entrepreneur's realization exceeds the threshold such that $\omega_{t+1}(i) \geq \bar{\omega}_{t+1}(i)$, the entrepreneur pays the financial intermediary the contracted amount $R^l_{t+1}(i)B_{t+1}(i)$, keeping the amount $\omega_{t+1}(i)R^k_{t+1}(i)q_tK_{t+1}(i) - R^l_{t+1}B_{t+1}$, and if $\omega_{t+1}(i) < \bar{\omega_{t+1}}(i)$, the entrepreneur defaults, receives nothing, and the financial intermediary receives $(1-\theta_t)\omega_{t+1}(i)R^k_{t+1}(i)q_tK_{t+1}(i)$. As with $R^l_t(i)$, $\bar{\omega}_t(i)$ adjusts to reflect the aggregate ex-post realizations of the aggregate state in period $t$.

Given these contract details, we can write the financial intermediary's expected return on a given loan contract in a given aggregate contingency in period $t+1$ as

$$\xi_{t+1}(i) = [1-F(\bar{\omega}_{t+1})]R^l_{t+1}(i)B_{t+1}(i)+(1-\theta_{t+1})\int_0^{\bar{\omega}(i)} \omega_{t+1}(i)R^k_{t+1}(i)q_tK_{t+1}(i)dF(\omega) \tag{2.16}$$

Substituting in (2.15), we can write (2.16) in terms of the cut-off $\bar{\omega}$ as

$$\xi(\bar{\omega}_{t+1}(i),\theta_{t+1}) = \left[[1-F(\bar{\omega}_{t+1})]\bar{\omega}_{t+1}(i) + (1-\theta_{t+1})\int_0^{\bar{\omega}(i)} \omega_{t+1}(i)dF(\omega)\right] R^k_{t+1}(i)q_tK_{t+1}(i). \tag{2.17}$$

Defining the financial intermediary's expected share of gross returns $\Gamma(\bar{\omega}_t)$

as

$$\Gamma(\bar{\omega}_t) = [1 - F(\bar{\omega}_{t+1})]\bar{\omega}_{t+1}(i) + \int_0^{\bar{\omega}(i)} \omega_{t+1}(i)dF(\omega), \tag{2.18}$$

and defining $G(\bar{\omega}_t)$ as

$$G(\bar{\omega}_t) = \int_0^{\bar{\omega}} \omega dF(\omega) \tag{2.19}$$

we can re-write the financial intermediary's expected return on a given loan contract in a given aggregate contingency as

$$\xi_{t+1}(\bar{\omega}_{t+1}(i), \theta_{t+1}) = [\Gamma(\bar{\omega}_{t+1}) - \theta_{t+1}G(\bar{\omega}_{t+1})]\, R^k_{t+1}(i)q_t K_{t+1}(i) \tag{2.20}$$

where the terms in square brackets represent the financial intermediary's share of profits net of default costs. The requirement that the financial intermediary earn an expected return in every aggregate contingency equal to its opportunity cost of funds,

$$\xi_{t+1}(\bar{\omega}_{t+1}(i), \theta_{t+1}) = R_{t+1}B_{t+1}(i) \tag{2.21}$$

then serves as a restriction to define a menu of contracts over loan quantity and cut-off value for the entrepreneur. Substituting in (2.12) and (2.20) we can then write this as

$$[\Gamma(\bar{\omega}_{t+1}) - \theta_{t+1}G(\bar{\omega}_{t+1})]\, R^k_{t+1}(i)q_t K_{t+1}(i) = R^d_{t+1}\left(q^n_t K_{t+1}(i) - X_{t+1}(i)\right) \tag{2.22}$$

which for a given level of net-worth $X_{t+1}(i)$ defines a menu of contracts relating the entrepreneur's choice of $K_{t+1}(i)$ to the cut-off $\bar{\omega}_{t+1}(i)$.

### 2.2.6 Entrepreneur's contract problem

The entrepreneur's expected gross return, conditional on the ex-post realization of the aggregate state but before the resolution of idiosyncratic risk, is given by

$$V^k_{t+1}(i) = \int_{\bar{\omega}_{t+1}}^{\infty} \omega_{t+1}(i) R^k_{t+1}(i) q_t K_{t+1}(i) dF(\omega_{t+1}) - R^l_{t+1}(i) B_{t+1}(i). \tag{2.23}$$

Substituting in the definitions above yields

$$V^k_{t+1}(i) = \left[1 - \Gamma(\bar{\omega}_{t+1}(i))\right] R^k_{t+1} q_t K_{t+1}(i). \tag{2.24}$$

where $1 - \Gamma(\bar{\omega}_{t+1})$ is the entrepreneur's expected share of gross returns.

For a given level of net-worth $X_{t+1}(i)$, the entrepreneur's optimal contacting problem is then

$$max_{K_{t+1}(i), \bar{\omega}_{t+1}(i)} E_t\{V^k_{t+1}(i)\} \tag{2.25}$$

subject to the condition that the financial intermediary's expected return on the contract equal its opportunity cost of its borrowing, equation (2.22). Letting $\lambda_{t+1}(i)$ be the ex-post value of the Lagrange multiplier conditional on realization of the aggregate state, the first-order conditions are then

$$\Gamma'(\bar{\omega}_{t+1}) - \lambda_{t+1}\left[\Gamma'(\bar{\omega}_{t+1}) - \theta_{t+1}G'(\bar{\omega}_{t+1})\right] = 0 \tag{2.26}$$

$$E_t\left\{\left[1 - \Gamma(\bar{\omega}_{t+1})\right]\frac{R^k_{t+1}}{R^d_{t+1}} + \lambda_{t+1}\left(\left[\Gamma(\bar{\omega}_{t+1}) - \theta_{t+1}G(\bar{\omega}_{t+1})\right]\frac{R^k_{t+1}}{R^d_{t+1}} - 1\right)\right\} = 0 \tag{2.27}$$

$$\left[\Gamma(\bar{\omega}_{t+1}) - \theta_{t+1}G(\bar{\omega}_{t+1})\right]R^k_{t+1}(i)q_tK_{t+1}(i) - R^d_{t+1}\left(q^n_tK_{t+1}(i) - X_{t+1}(i)\right) = 0 \tag{2.28}$$

where (2.26) and (2.28) hold in each contingency, but (2.27) holds only in expectation.

### 2.2.7 Capital-producer

The competitive capital-goods producer operates a within-period technology that combines existing capital with new goods to create new installed capital. At the end of each period it purchases existing capital $K^k_t$ from entrepreneurs at price $q_t$, combining it with investment $I_t$ purchased from the goods market to yield new capital stock $K^{nk}_t$, which it sells back to entrepreneurs in the same period at price $q^n_t$. The capital-producer faces adjustment costs in the creation of new capital, and incurs depreciation in the process, so that

$$K^{nk}_t = (1-\delta)K^k_t + \Phi(\frac{I_t}{K^k_t})K^k_t. \tag{2.29}$$

The capital-goods producer chooses $K_t^{nk}$, $K_t^k$ and $I_t$ to maximize profits $\Pi_t^k = q_t^n K_t^{nk} - q_t K_t^k - I_t$. Substituting (2.29) into this expression gives $\Pi_t^k = q_t^n(1-\delta)K_t^k + q_t^n \Phi(\frac{I_t}{K_t^k})K_t^k - q_t K_t^k - I_t$. The producer's optimal choices of $I_t$ and $K_t^k$ then leads to,

$$q_t^n = \frac{1}{\Phi'(\frac{I_t}{K_t^k})}. \tag{2.30}$$

$$q_t = q_t^n \left[(1-\delta) + \Phi(\frac{I_t}{K_t^k})\right] - \frac{I_t}{K_t}. \tag{2.31}$$

### 2.2.8 Stochastic process $\theta_t$

The default cost process, $\theta_t$ evolves according to the stationary AR(1) process

$$\ln \theta_t = \rho \ln \theta_{t-1} + \mu_t, \tag{2.32}$$

where $\rho < 1$ and $\mu_t$ is an exogenous period $t$ innovation which we will define further below. Note that shocks to $\theta$ will cause the spread between interest rates charged to borrowers and paid to lenders to vary over time so that the financial intermediary's zero-profit condition is satisfied.[8]

[8]This is reminiscent of the risk-premium shocks used in Amano and Shukayev (2009) which induce exogenous movements in the spread between risk-free and risky assets. Note that in our model, the spread between the loan rate and the risk-free rate is actually endogenous. Indeed as discussed in the results section, movements in this latter spread can be induced, purely by changes in agents' expectations.

**News shocks**

We want to explore the possibility that agents react to information about changes in the financial sector in advance of the actual occurrence of these shocks. This fits in with the idea of expectations driven cycles in the news shock literature. Our representation of news shocks is standard and follows **?**. We provide for news about $\theta_t$ by defining the innovation $\mu_t$ in equation (2.32) as

$$\mu_t = \epsilon^p_{t-p} + \varepsilon_t, \tag{2.33}$$

where $\epsilon^p_{t-p}$ is a news shock that agents receive in period $t-p$ about the innovation $\mu_t$, and $\varepsilon_t$ is an unanticipated contemporaneous shock to $\mu_t$. The news shock $\epsilon^p_t$ has properties $E\epsilon^p_t = 0$ and standard deviation $\sigma_{\epsilon^p}$, and the contemporaneous shock $\varepsilon_t$ has properties $E\varepsilon_t = 0$ and standard deviation $\sigma_{\varepsilon_x}$. The shocks $\epsilon^p_t$ and $\varepsilon_{A,t}$ are uncorrelated over time and with each other.

### 2.2.9 Equilibrium

Equilibrium in this economy is defined by a contingent sequence of decision rules $C_t$, $C^e_t$, $N_t$, $N^e_t$, $I_t$, $A_{t+1}(i)\forall i$, $K_{t+1}$, $B_{t+1}(i)\forall i$, $\bar{\omega}_{t+1}$, $K^{nk}_t$, $K^k_t$, prices $w_t$, $w^e_t$, $r_t$,$R^d_{t+1}$, $R^l_{t+1}$, $R^k_t$, $q_t$, $q^n_t$, that satisfy the following conditions: $(i)$ the allocations solve the households', goods-producers', financial intermediary's, entrepreneurs' and capital producers' problems, taking prices as given, $(ii)$ all markets clear, $(iii)$ the resource constraint $C_t + C^e_t + q^n_t\Phi(\frac{I_t}{K_t}K_t) + \theta_t G(\bar{\omega}_t)q^n_{t-1}R^k_t K_t = Y_t$ holds, where $\int_0^1 K_{t+1}(i) = K_{t+1}$, $\int_0^1 B_{t+1}(i) = B_{t+1}$,

$\int_0^1 X_{t+1}(i) = X_{t+1}$, $\int_0^1 C^e_{t+1}(i) = C^e_{t+1}$, and where all entrepreneurs choose the same cut-off such that $\bar{\omega}_{t+1}(i) = \bar{\omega}_{t+1}(j) = \bar{\omega}_{t+1}$.

Equilibrium in the capital goods market implies that $K^{nk}_t = K_{t+1}$ and $K^k_t = K_t$, and equilibrium in the securities market implies that $A_t = B_t$.

In equilibrium the financial intermediary's return on its entire loan portfolio just covers its opportunity cost of funds, implying that its budget constraint holds in every aggregate contingency and after idiosycratic uncertainty is resolved as

$$[\Gamma(\bar{\omega}_{t+1}) - \theta_{t+1} G(\bar{\omega}_{t+1})]\, R^k_{t+1}(i) q_t K_{t+1}(i) = R^d_{t+1} A_{t+1}. \tag{2.34}$$

Aggregate net-worth evolves as the accumulated gross returns of surviving entrepreneurs plus their labour income. Letting $V_t$ be aggregate gross entrepreneurial returns, we can compute it as the average gross idiosyncratic returns,

$$V_t = [1 - \Gamma(\bar{\omega}_t)]\, R^k_t q^n_{t-1} K_t, \tag{2.35}$$

which after making substitutions yields

$$V_t = R^k_t q^n_{t-1} K_t - \left[R^d_t B_t + \theta_t G(\bar{\omega}_t) R^k_t q^n_{t-1} K_t\right], \tag{2.36}$$

so that aggregate net-worth evolves as

$$X_{t+1} = \gamma V_t + w^e_t. \tag{2.37}$$

Finally, entrepreneurial consumption $C_t^e$ is equal to the aggregated gross return of dying entrepreneurs,

$$C_t^e = (1-\gamma)V_t. \tag{2.38}$$

For reference later in discussion of our results, we also define the equilibrium real risk-free net interest rate as $r_t^f = \frac{1}{E_t\beta\frac{\lambda 1t+1}{\lambda_{1t}}} - 1$, the credit spread as $R_t^l - R_t^d$, and leverage as $L_t = \frac{q_t^n K_{t+1}}{X_{t+1}}$.

## 2.3 Parameterization

In this section we present an illustrative calibration that we will use in the next section for our simulation analysis. We assign values to parameters using typical values established in the literature, or where there is a lack of precedent, we choose the parameters to match relevant steady state quantities in the model economy with analogous quantities in the data. For parameters relevant to the financial contract, we follow closely the calibration of Bernanke et al. (1999). Finally, we solve the model by using standard methods to linearize the non-linear system about the unique steady state.

Beginning with the parameters common to standard real-business cycle models, we set the household's subjective discount factor $\beta$ to 0.99, implying a net annualized risk-free interest rate of 4.1%, and implying a quarterly gross return on household financial assets $R^d = (1 + r^f)^{0.25} = 1.0101$.

On the production side, we set labour's share in production, $\alpha = .67$, and the depreciation of physical capital, $\delta$ to 0.025.

For the capital adjustment cost, our solution method requires that we need only specify the elasticity of the price of capital with respect to the investment capital ratio. We follow Bernanke et al. (1999) in setting this to 0.25. In the next section we report results for a version of the model without adjustment costs.

We use preferences of the form used by King and Rebelo (2000) where the stand-in representative agent has the preference specification

$$u(C_t, N_t) = \frac{1}{1-\sigma}\left\{C_t^{1-\sigma}\upsilon^*(N_t)^{1-\sigma} - 1\right\}, \tag{2.39}$$

where $\upsilon^*(N_t) = \left[\left(\frac{N_t}{H}\right)\upsilon_1^{\frac{1-\sigma}{\sigma}} + \left(1 - \frac{N_t}{H}\right)\upsilon_2^{\frac{1-\sigma}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}}$, where $H$ is the fixed shift length, and $\upsilon_1$ and $\upsilon_2$ are constants representing the leisure component of utility of the underlying employed group (who work $H$ hours) and unemployed group (who work zero hours) respectively. For $\sigma > 1$ these preferences are not separable in consumption and leisure, and for $\sigma = 1$ they reduce down to standard separable indivisible labour preferences with log-consumption and linear leisure. We report results for both cases in the next section. We set the fraction of the population working on average, $f^w$ to 0.6, and the average household's share of time allocated to market work $N_{ss}$ to 0.3. In our impulse-response analysis of the non-separable case, we set $\sigma = 2$, which is within the range studies reported by King and Rebelo. This then yields a ratio of consumption of those employed to consumption of those unemployed of 2.26. For our simulations involving separable preferences, we set $\sigma = 1$. We also explored other non-separable preferences which give similar results and these are available from the authors upon request.

For the parameters associated with the financial contract and the entrepreneur, we follow Bernanke et al. (1999) in setting these parameters such that in steady state, the external finance spread, $R^k - R^d$, equals 0.005 quarterly, leverage, $K/X$, is approximately 2, and the fraction of entrepreneurs defaulting each quarter is 0.076. As such, we set the quarterly survival rate of entrepreneurs to 0.9795, the variance of $log\bar{\omega}$ to 0.0727, and steady-state fraction of gross returns lost in default, $\theta$, to 0.115.

For our illustrations we choose default cost shocks that cause the credit-spread to fall by the same order of magnitude as seen in the US over the period preceding the financial crisis. Depending on which assets are used for this calculation, the spread decreased from 25 percent to over 120 percent between 2002 and 2007 [9]. Since these exercises are meant to be a quantitative illustration of the mechanisms in our model, we arbitrarily choose a shock that reduces default costs by 50 percent. The $\theta$ process has an AR(1) co-efficient of .9722 which we estimated from the spread between the BAA corporate bond yield and the ten year government bond yield measured as percent deviations from the mean value of the spread[10].

For reference later, we refer to the model with non-separable preferences ($\sigma = 2$) and adjustment costs as the "full-model", and the version of the model with separable preferences ($\sigma = 1$) and no adjustment costs as the "baseline"

[9]Details of these calculations are available from the authors.

[10]These series were obtained from FRED and the annualized rates were converted to quarterly frequency to be consistent with the model

model.

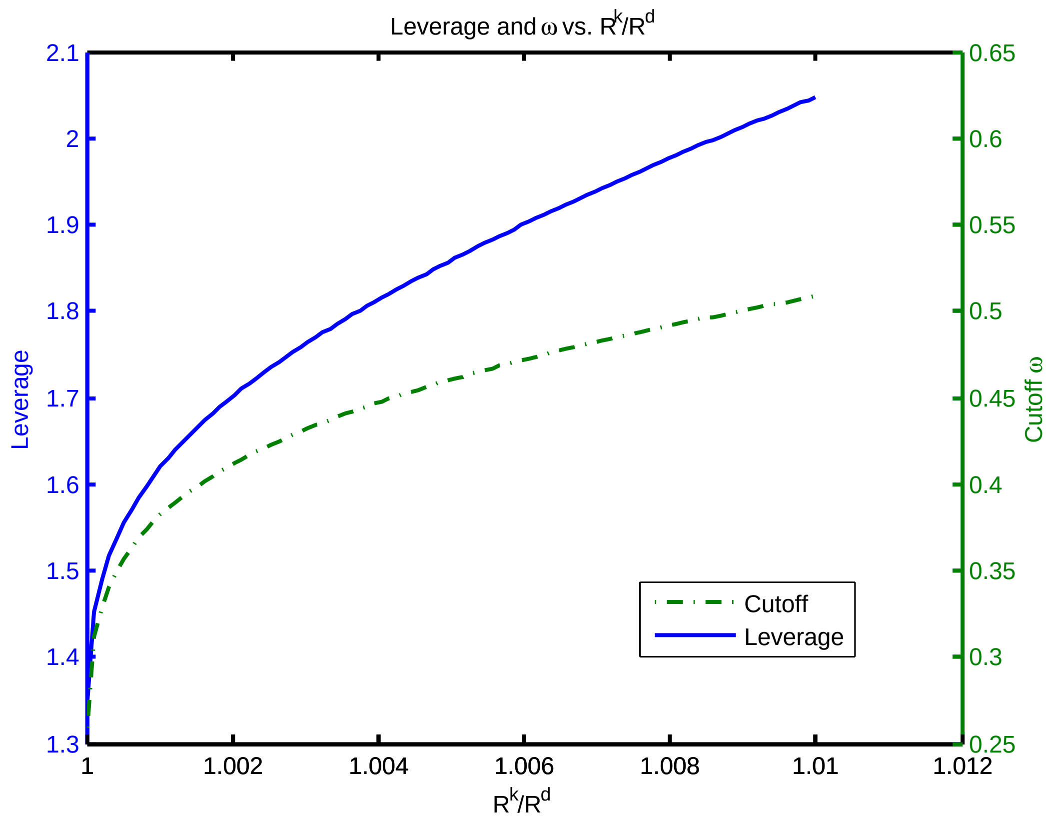
## 2.4 Results

### 2.4.1 Sensitivity of contract to $\frac{R^k_{t+1}}{R^d_{t+1}}$ and $\theta_t$

Recall from our earlier discussion that the optimal contract at the end of period $t$ is defined by the pair $(\bar{\omega}_{t+1}, \text{Ł}_{t+1})$, where $\bar{\omega}_{t+1}$ is a list of cut-off productivities specifying the state-contingent cut-off productivity level associated with each aggregate contingency $(\frac{R^k_{t+1}}{R^d_{t+1}}, \theta_{t+1})$ in $t+1$, and $L_{t+1}$ is the end-of-period leverage, predetermined relative to all $t+1$ contingencies, as a function of the aggregate state in period $t$. We will often refer to this pair as "the contract" in our discussions below. Before we discuss how our model responds to both contemporaneous and news shocks it will be helpful to understand how the contract itself responds to expected changes in either $\frac{R^k}{R^d}$ or in $\theta$ while holding the other constant. This is potentially useful because a shock to $\theta$ will have a direct impact on the contract as well as an indirect impact via the general equilibrium movements in $\frac{R^k}{R^d}$. Figure 2.3 shows the results of this exercise, holding $\theta$ constant at its calibrated steady-state value while varying $\frac{R^k}{R^d}$ around its steady-state value. Similarly, Figure 2.4 shows the results holding $\frac{R^k}{R^d}$ constant at its calibrated steady-state value and varying $\theta$ around its steady-state value. As discussed in Bernanke et al. (1999), Figure 2.3 shows that both leverage and $\bar{\omega}_{t+1}$ rise along with an expected increase in $\frac{R^k}{R^d}$. In contrast, Figure 2.4 shows that an increase in the cost-of-default, $\theta_t$, is associated with a fall in leverage and in $\bar{\omega}_{t+1}$. To understand this latter result, first note that there is an inefficient loss of surplus in the event of bankruptcy: the contract allocates the surplus generated by the project between the entrepreneur and the lender but a fraction of this surplus is lost to both parties in the event of default. The lender's zero-profit condition implies that higher leverage is accompanied by a higher interest rate on loans, which in turn implies a higher probability of default. A higher cost-of-default, $\theta_t$, forces the financial intermediary to cover the lower return on defaulted loans with a higher interest rate for each level of leverage. Faced with this new menu of leverage and interest rates, an entrepreneur will prefer to choose a lower combination of leverage and interest rates which in turn implies a lower probability of default, implying a lower cut-off productivity. This has the effect of reducing the dead-weight losses associated with default when default costs are rising.

With an eye on our discussion of impulse responses below, we note that the effects of varying $\frac{R^k}{R^d}$ and $\theta$ discussed above provide two opposing forces operating on leverage when $\theta$ is shocked. As we will see, the general equilibrium impact of a fall in $\frac{R^k}{R^d}$ on leverage can partially offset and sometimes overturn (such as when adjustment costs are present) the partial-equilibrium contract tendency for leverage to rise in response to a fall in $\theta$. These results serve to emphasize the idea that shocks to the cost-of-default parameter act as a time varying wedge in the relationship between leverage and external finance premium discussed in Bernanke et al. (1999).

Figure 2.3: Optimal contract: leverage (L) & $\bar{\omega}$ vs. $\frac{R^k}{R^d}$: $\theta$ constant

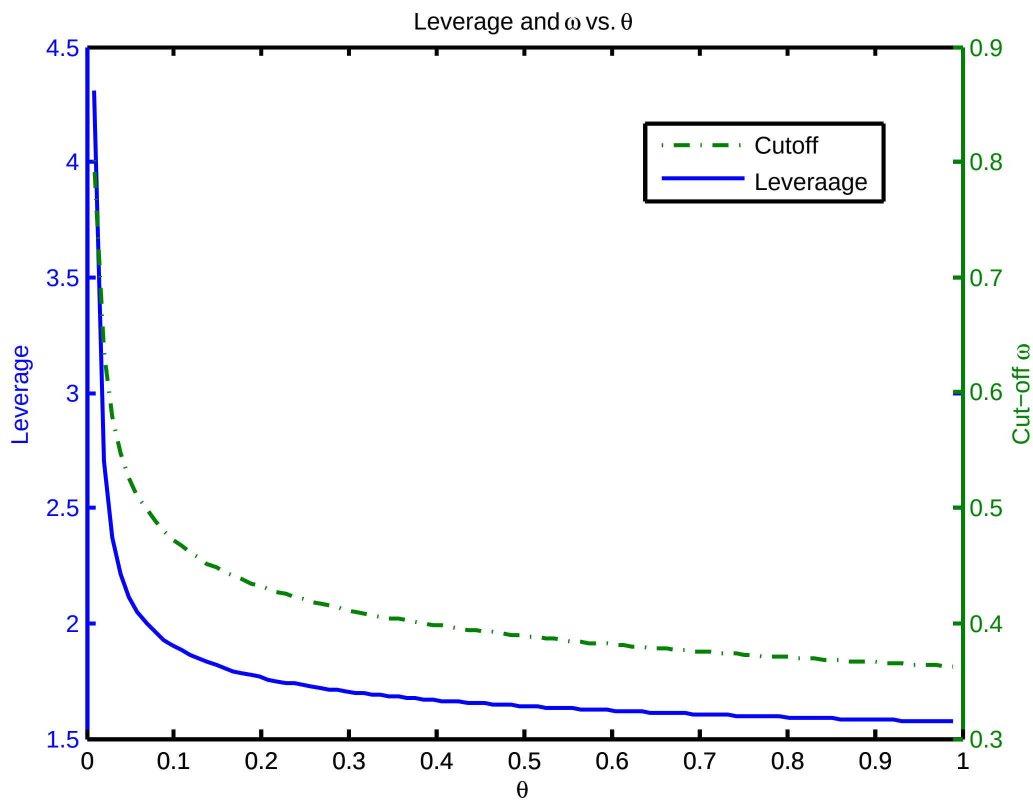



### 2.4.2 Impulse response to cost of default shocks

In this section we use a linearized and parameterized version of the model economy to illustrate how a fall in the default cost, $\theta_t$, can lead to a large boom in economic activity and a fall in the credit spread. We begin by first exploring the response of the model to a contemporaneous shock that reduces default costs before subsequently moving on to explore the response to news shocks about future reductions in default costs.

Figure 2.5 illustrates the response of the model when $\theta_t$ unexpectedly falls

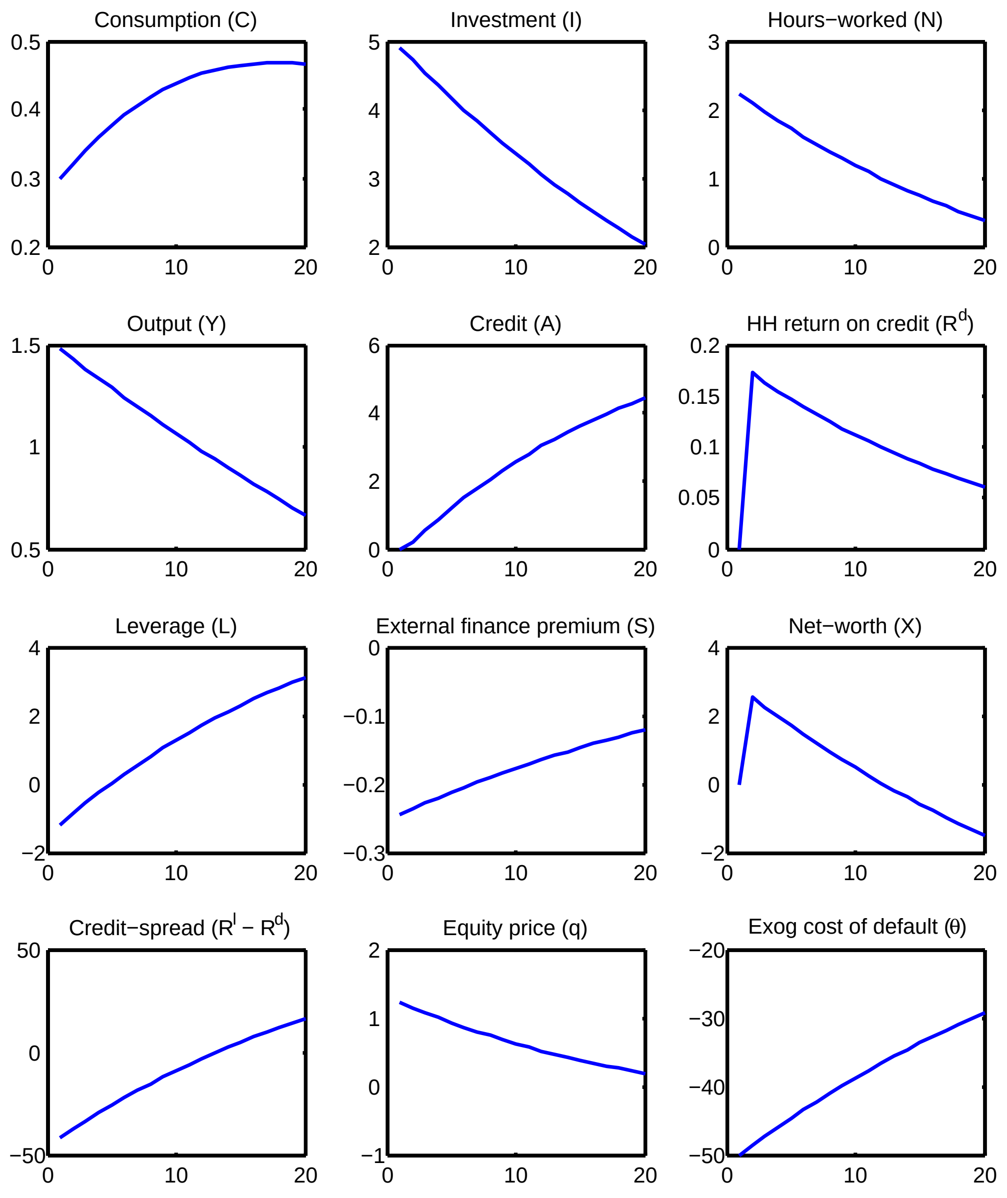
Figure 2.4: Optimal contract: leverage (L) & $\bar{\omega}$ vs. $\theta$: $\frac{R^k}{R^d}$ constant




fifty percent below it's steady state value for our "full model" with adjustment costs and non-separable preferences. The shock creates an immediate boom in consumption, investment, output and hours-worked, with the largest impact in the period of the shock and a persistent decline towards steady state. The boom in real activity is accompanied by a rise in credit, the price of capital and net-worth while the credit spread and external finance premium fall on impact.

How should we understand this response? It is useful to separate the impact

Figure 2.5: Contemporaneous reduction in default cost $\theta$: Full model




period from the rest of the response since the shock is unexpected. Firstly, in the impact period, certain aspects of the contract are already determined from the end of the last period: the amount borrowed by the entrepreneur (conditional on last period net-worth) and therefore leverage. Since $\bar{\omega}_t$ is state contingent, it is free to move in the impact period. The fall in $\theta_t$ in period 1 implies that for a given $\frac{R_t^k}{R_t^d}$, $\bar{\omega}_t$ must change to satisfy the zero profit condition of the financial intermediary. Since the reduction in $\theta_t$ means that the financial intermediary recovers a larger fraction of the value of defaulting loans in period 1, it can earn less on its portfolio of non-defaulting loans and still satisfy the zero-profit condition. As a result, $\bar{\omega}$ falls in the impact period. This increases entrepreneurs' share of profits that period, and increases their net-worth in period 1 slightly. Net worth is further raised by the rise in the price of capital due to the presence of adjustment costs.

Secondly, beyond the impact effect, since the $\theta_t$ process is persistent, agents will now expect default costs to be below steady state in the following periods. Since agents must choose the amount of new capital and new borrowing for period 2 at the end of period 1, unlike the impact effect of $\theta_t$, this expected change in $\theta_{t+1}$ now impacts the optimal contract. In the previous section we showed that an expected reduction in $\theta$ implies an increase in leverage and an associated increase in $\bar{\omega}$ for a given external finance premium, $\frac{R^k}{R^d}$. The fall in the external finance premium is enough in this instance to overturn this effect, so that leverage $L_2$ in fact falls. Nonetheless, the increase in net worth is sufficient to create an increased demand by entrepreneurs for new capital $K_2$ which is also associated with an increased level of borrowing through the financial intermediary. The extra funds are raised by selling additional financial securities, $A_2$, to the household, which is willing to supply the additional savings only at a higher rate of return, $R_2^d$. Note that while both $R^k$ and $R^d$ rise, the spread between them falls due to the lower default costs.[11] The increased demand for capital is accompanied by an increase in investment by capital producers in the impact period in anticipation of the extra demand. The presence of adjustment costs drives up the marginal cost of producing new capital, thus driving up its price, $q_t$. This yields a capital gain for entrepreneurs which increases the gross return on capital $R^k$, driving up end-of-period net-worth. The extra investment raises the demand for goods in the impact period which is met by an increased supply from goods producing firms. The additional goods can only be produced by hiring more labor since the current stock of capital is fixed in the impact period. This hiring occurs because the rise in $R_2^d$ leads to a rise in the household's marginal utility of income in period 1 relative to that of the future which shifts its labour supply outwards, resulting in an increase in hours worked in the initial period. The additional wage income allows both consumption and investment in financial securities to rise. Finally, combining the equilibrium zero-profit condition (2.34) with the cut-off definition (2.15), we end up with an expression that relates the credit spread to $\theta$ and $\bar{\omega}$,

$$\frac{R_t^l}{R_t^d} = \frac{\bar{\omega}_t}{\Gamma(\bar{\omega}_t) - \theta_t G(\bar{\omega}_t)}. \tag{2.40}$$

On its own, a fall in $\theta$ induces a fall in the credit spread while a fall in $\bar{\omega}$

[11]The external finance premium would shrink to zero if the default/monitoring costs were zero. As discussed by Gale and Hellwig (1985) and Levin et al. (2004), a costly-state verification problem with a zero-cost of monitoring is essentially equivalent to the case of symmetric information.

induces a further decrease in the impact period. As discussed above, in the following periods, $\bar{\omega}$ rises while $\theta$ remains below steady state, thus the two exert opposing forces on the credit spread which slowly rises.

The above effects are repeated in the future periods while $\theta_t$ remains below steady-state, albeit at a gradually dampening rate as the demand for new physical capital falls with the increasing $\theta$. This can be seen clearly in the figure where investment is at its maximum in the initial period when $\theta_t$ is at its minimum.

We now discuss the role of capital adjustment costs and non-separable preferences ($\sigma > 1$) to our results. As might be expected, the main impact of capital-adjustment costs is on investment and the price of capital (which is fixed in their absence). Without adjustment costs, aggregate macro variables and especially investment rise to higher levels in the impact period but the qualitative behaviour of these variables is very similar to those shown in Figure 2.5. Since the price of capital does not rise, net worth responds less than shown here. The additional demand for capital is financed by additional leverage so the optimal contract picks out higher levels of $\bar{\omega}$ relative to the adjustment cost case to satisfy the zero-profit condition. While not shown here, these impulse response plots are available in an appendix.

Turning to the role of preferences, since the shock drives a wedge between the return on capital, $R^k$ and $R^d$, with standard preferences separable in consumption and leisure the shock causes negative co-movement between consumption and investment as the household forgoes consumption in order to increase savings. For the particular preferences we use in this paper, this case occurs when $\sigma = 1$ which yields preferences with log-consumption and linear leisure. For $\sigma > 1$ however, the preferences are non-separable in consumption and leisure, and imply that the consumption of the underlying employed households exceeds that of the underlying unemployed household members. As economic activity increases above steady state, the proportion of unemployed agents falls so that aggregate consumption rises along with total hours. The net effect is thus to make the marginal utility of consumption increasing in hours-worked, causing consumption to track closer to that of hours. As such, the primary purpose of these preferences parameterized under $\sigma > 1$ is to get consumption to co-move with total hours. Impulse responses for the case of $\sigma = 1$, are also available in an appendix. With the exception of consumption which falls below steady state, the qualitative movement of all other variables is very similar to those shown here. For completeness, in the appendix we also show impulse responses to the contemporaneous shock to $\theta_t$ for the baseline model (separable preferences and no adjustment costs).

### 2.4.3 News shocks

We now explore how the model economy responds to a change in expectations about the future costs of default. As before, we will focus on a fall in $\theta$ of the same magnitude, the difference being that agents will now anticipate the shock and respond optimally to it as soon as the news arrives. In all cases discussed in this section, the news is received in period 1 about a shock that occurs after two years (period 8). We begin our analysis with news shocks that are fulfilled, ie., the news turns out to be correct. Since we have already discussed the impact of a decline in $\theta$ in the previous section, we will focus our analysis on the response of the model economy to the periods before the shock is actually realized. As before, our "full model" includes both adjustment costs and non-separable preferences. We begin with a discussion of this model and then we show separately the impact of removing adjustment costs and of working with separable preferences corresponding to our "baseline model". As we will see, neither are crucial for generating news based booms, however they help improve the model response in some ways. Following this, we show the response of the"full mode" to news that turns out to be false, and discuss how this case may offer some insight into the events of the recent financial crises in the United States.

Before beginning the formal analysis of the impulse responses, it is useful to contrast a default cost shock with a more familiar total factor productivity (TFP) shock. When TFP rises in the future, the economy will have a relative abundance of goods in the future whereas a fall in default costs has no such direct effect on loosening the resource constraint of the economy. Indeed to the extent that the demand for capital increases in anticipation of a fall in default costs, the shadow value of goods will rise which will raise the marginal utility of consumption and, holding other things constant, shift labor supply out in a manner reminiscent of our discussion in the previous section. This is the opposite of what happens with a TFP shock. [12][13].

[12] An implication of this is that the typical mechanisms introduced in the"news shock" literature to induce co-movement in response to TFP shocks are not needed for default cost shocks.

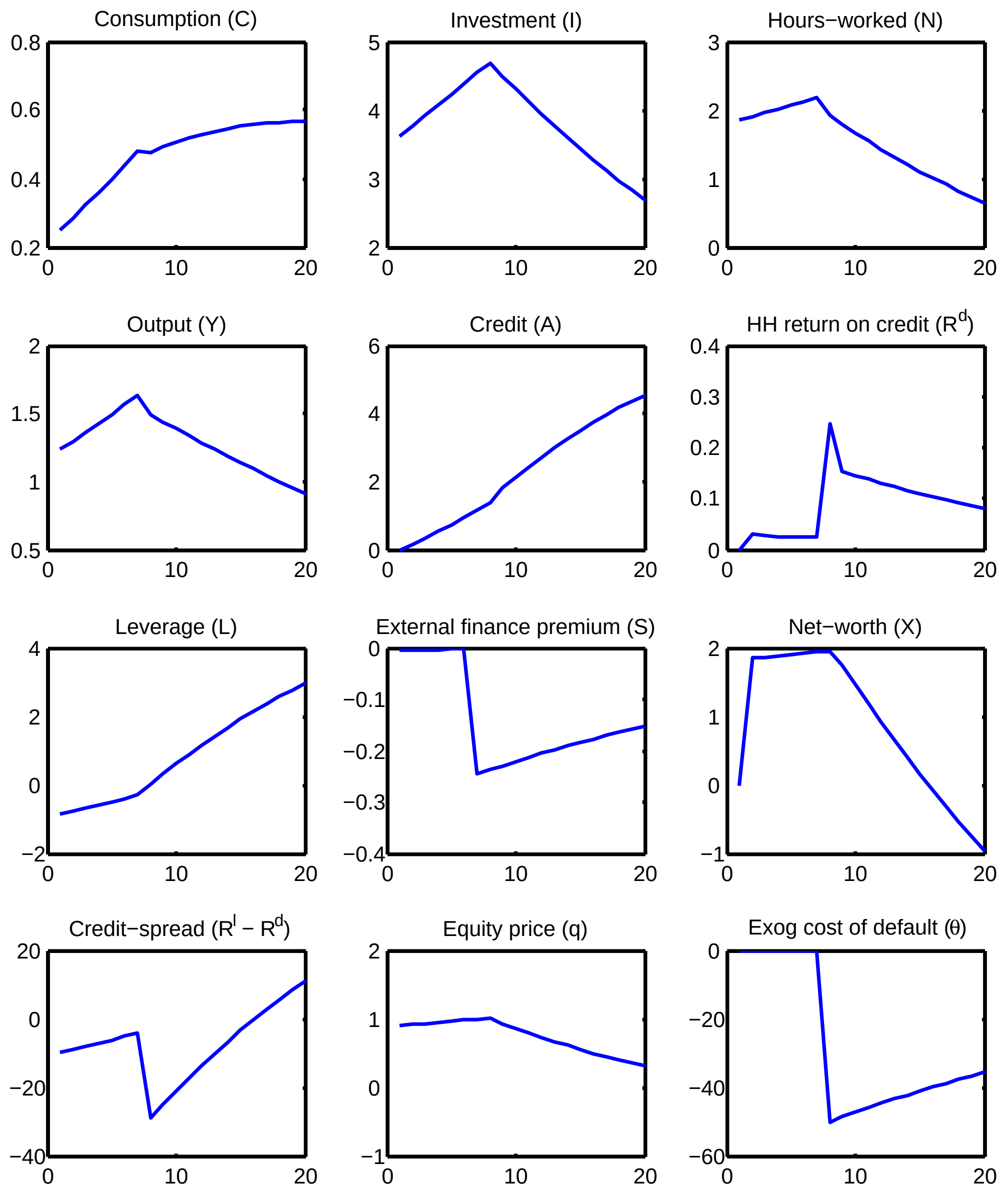
[13] In Gunn and Johri (2011b) we demonstrate how including a particular form of portfolio adjustment costs can facilitate a news-boom in a model where firms are constrained to borrow investment and/or their wage bill prior to production.

### A fulfilled news-shock

Figure 2.6 illustrates the response of our full model to the news that default costs will fall after two years and the news turns out to be correct. As can be seen from the figure, the news creates an immediate expansion in economic activity in the impact period. Investment, hours, output and consumption jump up in the first period and rise until a peak is reached in the period that (or one period before) the actual reduction in $\theta$ arrives. During this period, the household increases its investment in the securities of the financial intermediary, which in turn lends more to entrepreneurs who use the extra credit to purchase additional capital goods. The rise in investment and production of goods is accompanied by an increase in total hours worked, consumption and a rise in the price of capital, the net worth of entrepreneurs as well as a fall in the credit spread.

Why does the model economy boom from periods 1 to 7, prior to any actual change in $\theta$? To clearly illustrate this mechanism, it is helpful to remove the effect of adjustment costs and non-separable preferences. Figure 2.13 illustrates the response of our baseline model with no adjustment costs and separable preferences ($\sigma = 1$) to the news that intermediation costs will fall after two years and the news turns out to be correct. To describe how the model produces a news-driven boom when agents receives news in period 1, we will work backwards from period 7 when agents expect that in period 8 the actual shock will reduce $\theta$ below its steady state level. Looking forward one period to period 8 when they expect a lower $\theta$, entrepreneurs will choose to borrow more to finance new capital purchases for any given level of net worth

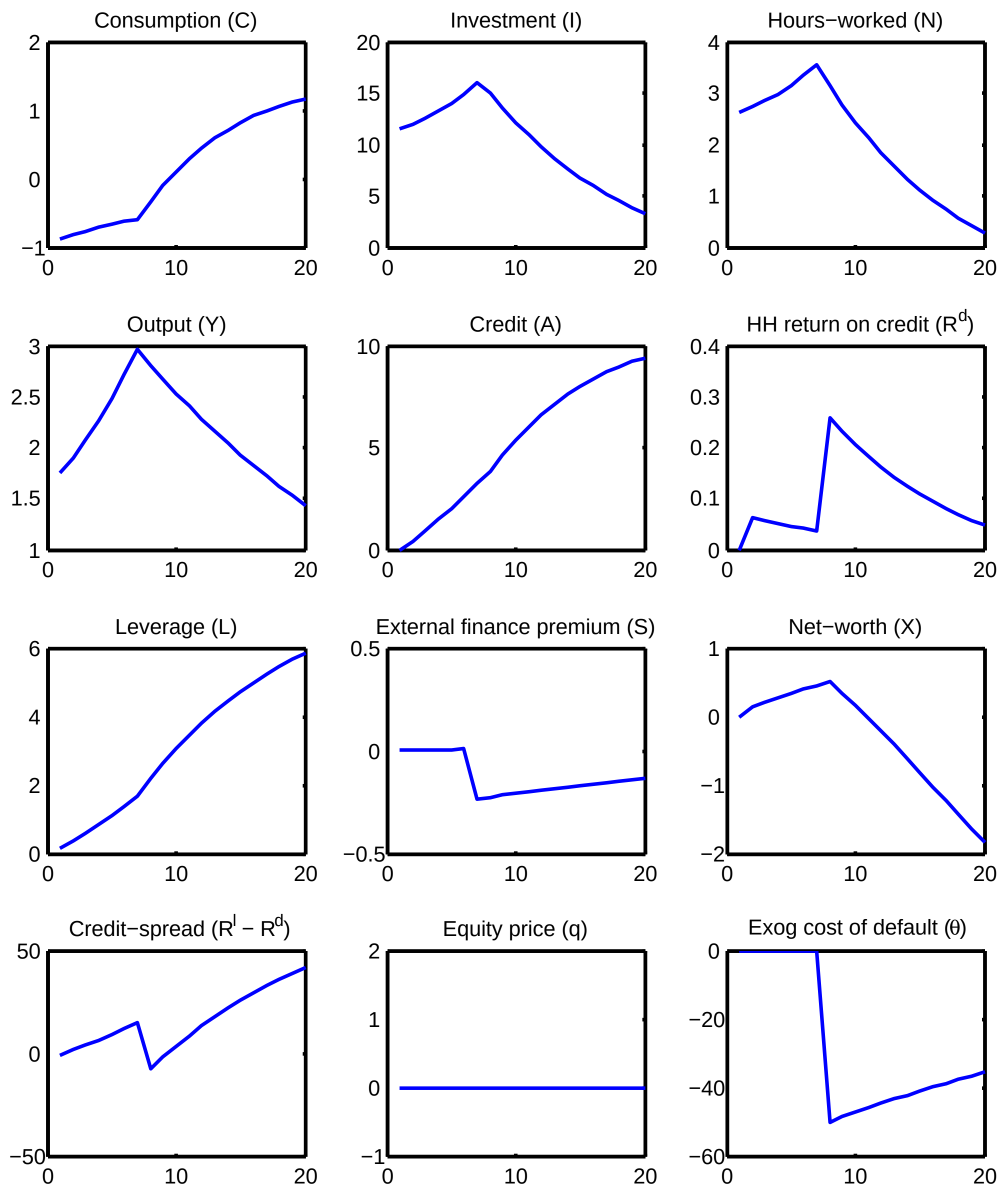
Figure 2.6: News about reduction in default cost $\theta$ in period 8 - reduction in default cost **realized** in period 8: Full-model



while still satisfying the financial intermediaries zero profit condition, leading to an increase in demand for new capital and new loans in period 7. From consumption smoothing motives, we know that the household will accept a big change in its marginal utility of consumption only if its reward for saving jumps up sufficiently. This effect can be seen in the jump in the return on the household's assets, $R^d$, held between period 7 and 8 which induces the household to trade off its period 7 consumption to fund the increase in demand for loans. Critically, this combination of a high expected marginal utility of consumption in period 8 and the high real interest rate in period 7 imply that the household's marginal utility of consumption in period 7 will also be high. This as a result creates an expansionary effect on the household's labour supply in period 7 as the household desires to raise its work effect while its marginal utility of consumption is high.

The general equilibrium consequences of the increase in labor supply on one hand and the increase in demand for capital on the other, lead to an increase in hours-worked in period 7 at good-producing firms and a corresponding rise in investment and output. The additional labor input raises the marginal product of capital and in turn the return on capital, $R^k$. The rationale for why these variables should also rise above steady state in period 6 and prior can be similarly worked out. Working backwards from period 7, entrepreneurs in period 6 anticipate that $R^k$ will be high in period 7 and this induces them to demand additional capital for next period as well as additional loans, given net worth. At the same time, the household expects its marginal utility of consumption to be high in period 7, and is willing to postpone consumption

Figure 2.7: News about reduction in default cost $\theta$ in period 8 - reduction in default cost **realized** in period 8 - Baseline model: no adjustment costs, sep pref's



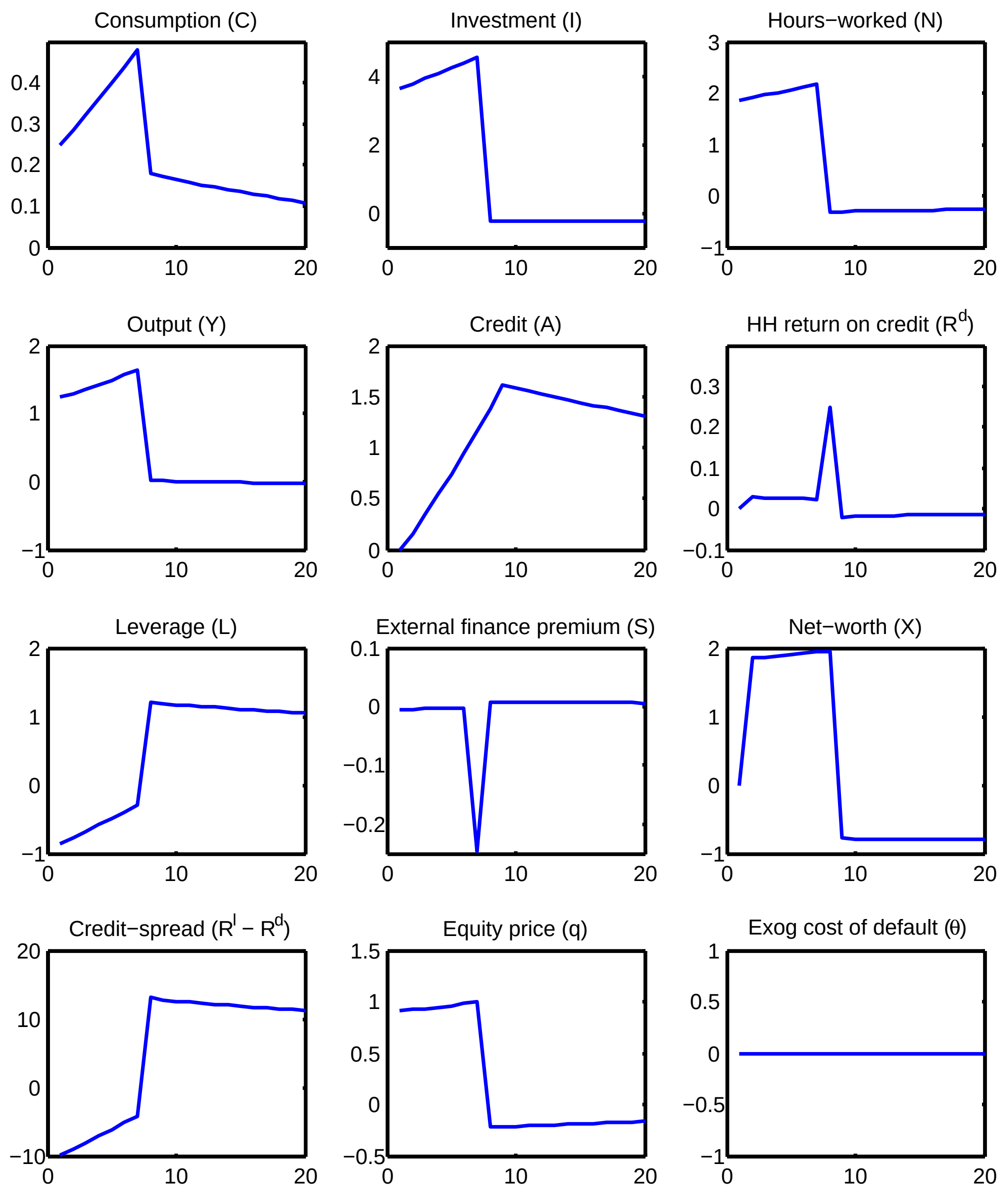
to provide the additional loans in return for a higher $R^d$, pushing up the household's real interest rate between period 6 and 7. Once again, the rise in the marginal utility of consumption pushes out labor supply in period 6 thus generating a boom in loans, hours, investment and output as previously discussed. This effect then continues backwards in each period until period 1 when the household first receives the news.

While this baseline model delivers booms in response to good news, a couple of the predictions of the baseline model need improvement. First, the model predicts that loan rates to entrepreneurs relative to the safe rate rise during the boom phase before shocks are actually realized. This occurs because the extra leverage taken on by entrepreneurs implies higher losses for the financial intermediary on defaulting loans in an environment where borrowing costs, $R^d$ are above steady state. The zero-profit condition of the intermediary is restored by raising loan rates. As a result, the credit spread is above steady state levels before $\theta$ actually falls. The addition of adjustment costs fixes this problem. With adjustment costs present, the price of capital rises immediately upon the agents receiving news, driving up the entrepreneurs' net worth, and improving their balance sheets. As a result the financial intermediary can charge a lower interest rate, $R^l$, relative to its opportunity cost, $R^d$, thus lowering credit spreads in the periods prior to the expected shock actually hitting. Thus, the model endogenously generates a countercyclical credit-spread and procyclical asset prices, even in advance of any actual changes in the cost of default parameter, $\theta$. Other variables such as consumption, hours, investment and total credit behave similarly to the model without adjustment costs. Second, as discussed above, consumption, while on a rising path, is below steady state levels over this period. Adding in non-separable preferences, gives us the"full model", which as discussed earlier delivers co-movement in consumption and hours. It is worth noting in this context that while consumption rises, the marginal utility of consumption is still above steady state so that our analysis above remains relevant to this case.

**An unfulfilled news-shock: interpreting the 2003-2008 episode as an expectations-driven boom bust cycle**

In this exercise, we explore the role of expectations more fully by studying the case where the news of future efficiency gains turn out to be completely false in that the gains never materialize. This situation is depicted in Figure 2.8. The response of the economy in the first seven periods is exactly the same as for the case where the news turns out to be true. Agents arrive into period 8 expecting to observe a large and persistent fall in default costs but these expectations turn out to be completely false. In fact, $\theta$ remains at the steady state level and agents must reverse their steps. This reversal leads to a sharp bust in economic activity, as total hours-worked and output fall below steady state levels. The sudden bust is especially evident in investment which goes from being over 4 percent above steady state to below steady state levels. While consumption falls, the movement is relatively muted. The bust in quantities is accompanied with a rapid change in prices: the price of capital falls in period 8 and pulls down the net worth of entrepreneurs. As a result there is a sharp increase in the credit spread from being roughly 4 percent below steady state to over 13 percent above steady state, reflecting the sudden deterioration in the quality of the entrepreneurs' balance sheets. Overall Figure 2.8 is an illustration of a complete boom-bust cycle which is driven entirely by expectations of future intermediation efficiency gains that are never realized. The bust lasts for a number of years and the economy is still operating below steady state levels roughly ten years or forty quarters after the news is first received. We find this scenario particularly interesting because a change in expectations is the only source of a large and persistent endogenous movement in the credit spread and in the price of capital without any underlying movement in the actual default-cost parameter. In the introduction we showed plots of interest rate spreads and credit just before and during the recession. Figures 1 and 2 showed that the fall in spreads was accompanied by an expansion of credit in the economy in the period before the financial crisis. This was followed by the crisis period during which spreads spiked sharply and credit plummeted. This inverse relationship between spreads and credit is also delivered by the model as can be seen in Figure 2.8.

Figure 2.8: News about reduction in default cost $\theta$ in period 8 - reduction in default cost **unrealized** in period 8 - Full model



## 2.5 Conclusions

In this paper we build a business cycle model with costly-state-verification in which changes in agents expectations about a future fall in default/monitoring costs can lead to an immediate expansion in liquidity in the financial system, a fall in credit spreads, a rise in asset values and net worth as well as a boom in economic activity, all of which precedes any actual change in these intermediation costs. Likewise, expected increases in costs would lead to a credit contraction, higher spreads and a fall in economic activity. We go on to show that an expectations driven boom in production and credit can subsequently be followed by a bust if the expectations turn out to be false. Consistent with the model, the negative co-movement of credit spreads on the one hand and total credit and economic activity on the other was part of the recent boom-bust cycle in the U.S.

We argue that the years preceding the financial crisis were a period of rapid technological change in the financial sector when a number of new financial products as well as practices were introduced. Given the novelty of many of these innovations and speed of adoption, it is likely that agents had very high expectations of the financial efficiencies resulting from these developments. The events of 2007 then led to a sharp downward revision in the expected efficacy of these products. At the same time, concerns regarding the stability of the financial system may have also contributed to the expectation that intermediation costs would be much higher going forward, than previously expected. Our model attempts to provide a stylized economy that can help understand the consequences of these changes in expectations about intermediation costs. Intermediation efficiency is incorporated into the financial sector using an exogenous process on the fraction of the value of a project that is lost when default occurs.

The events of 2007-2009 have cast a spotlight on the financial sector and revealed a complex set of phenomena that contributed to the worst recession in the post-war era. We have tried to contribute to our overall understanding of what happened in this period by focusing on one possible source of the great expansion in liquidity that preceded the recession and its eventual decline. Our explanation of this liquidity boom has focused on overoptimistic expectations of efficiency gains in the financial sector in the context of innovation whereas much of the discussions in the financial press have focused on the effect of low policy rates such as the Federal Funds Rate. Interestingly, our model generates declines in interest rate spreads during the boom phase followed by sharp increases in these spreads once the bust begins even in the absence of any monetary authority in the model. We note that many spreads in this period were far less under the control of the monetary authority and sometimes moved against the prevailing direction of the policy rate which suggests that they may have been susceptible to changes in expectations. Developing a monetary version of the model which can incorporate the behaviour of the fed so that predictions can be made about the level of interest rates would be an interesting avenue for future work.

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## 2.6 Appendix

Figure 2.9: Contemporaneous reduction in default cost $\theta$ - No adjustment costs, non-sep pref's

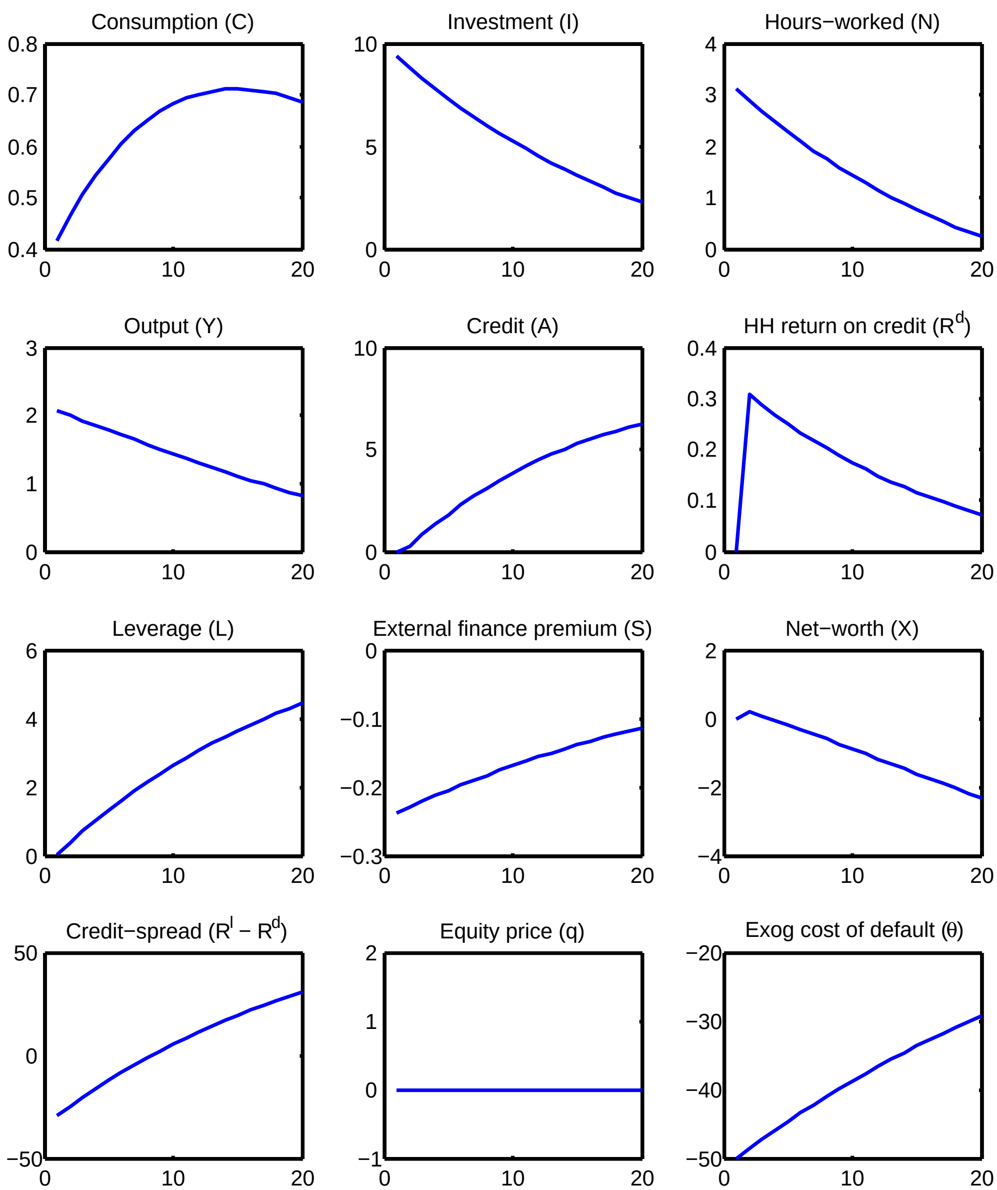


Figure 2.10: Contemporaneous reduction in default cost $\theta$ - Adjustment costs, sep pref's

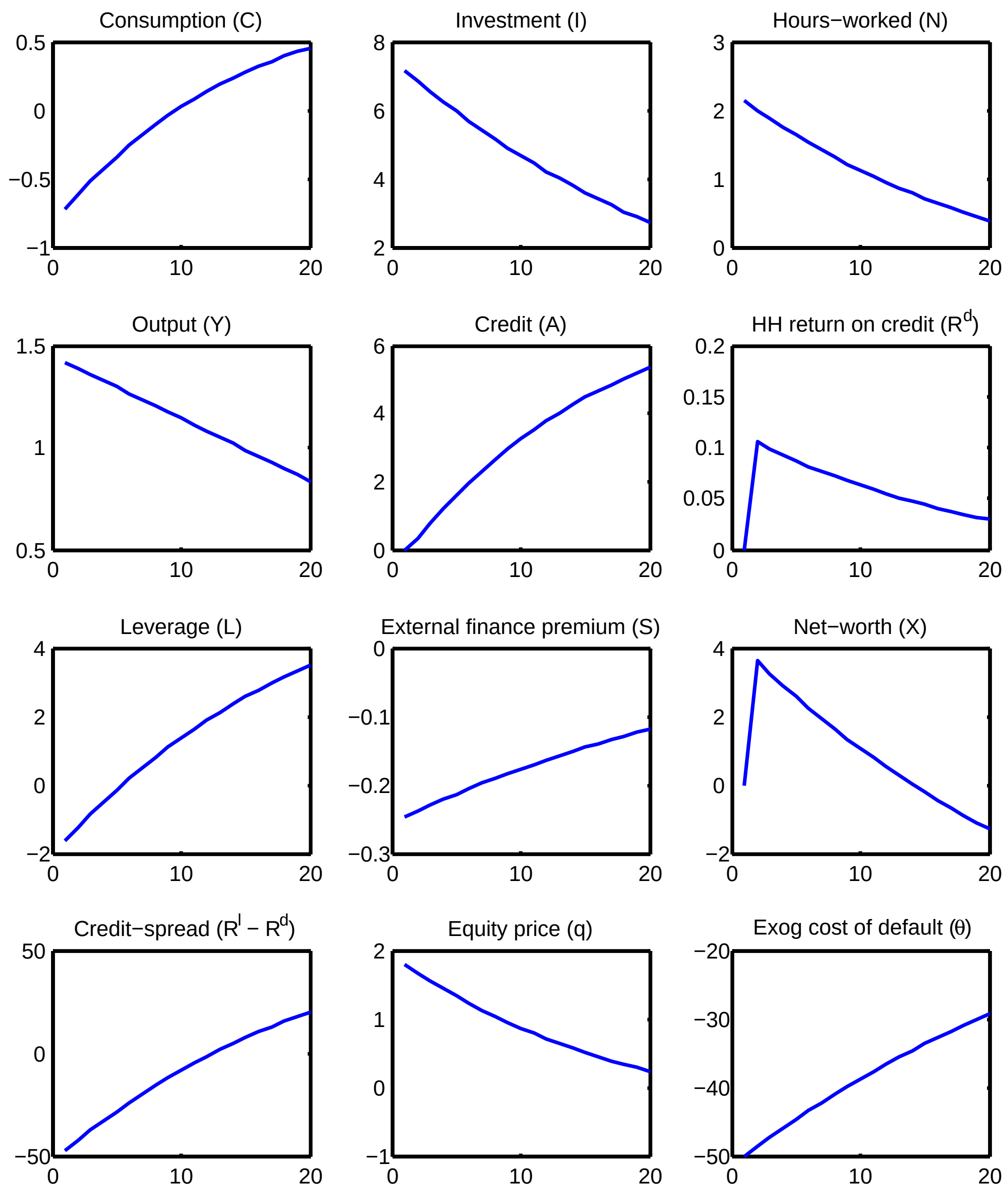


Figure 2.11: Contemporaneous reduction in default cost $\theta$ - Baseline model: no adjustment costs, sep pref's

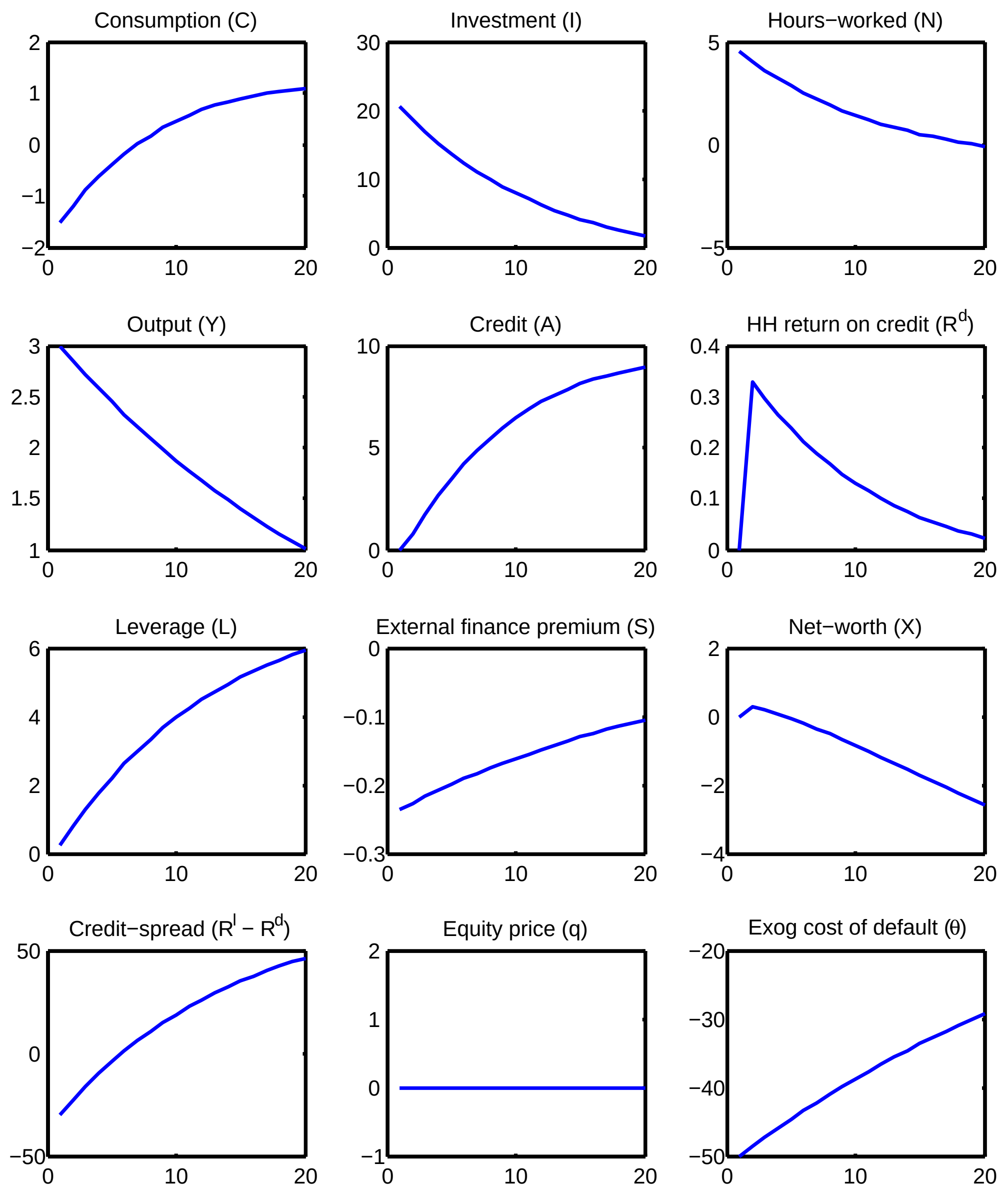


Figure 2.12: News about reduction in default cost $\theta$ in period 8 - reduction in default cost **realized** in period 8 - No adjustment costs, non-sep pref's

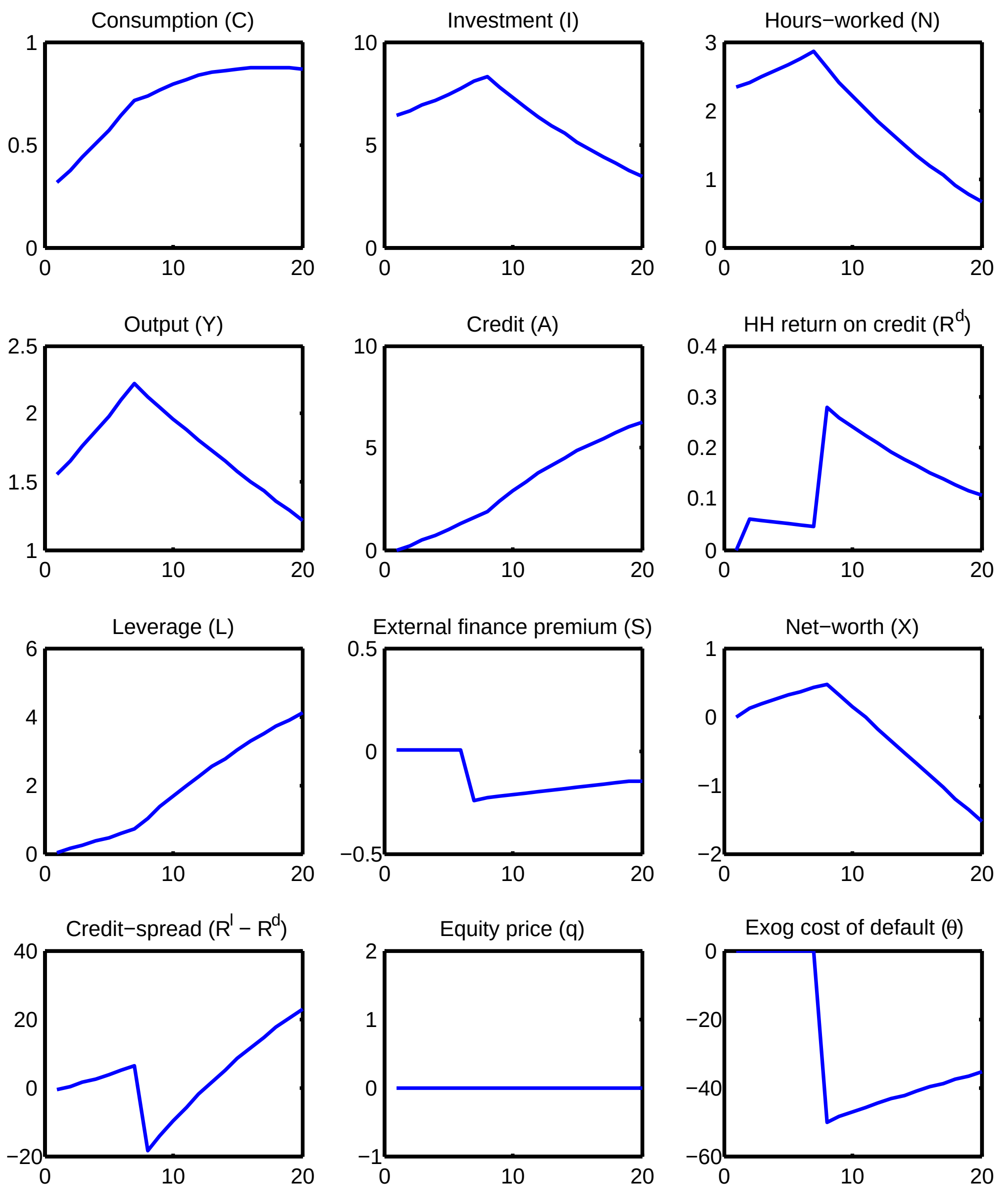


Figure 2.13: News about reduction in default cost $\theta$ in period 8 - reduction in default cost **realized** in period 8 - Adjustment costs, sep pref's

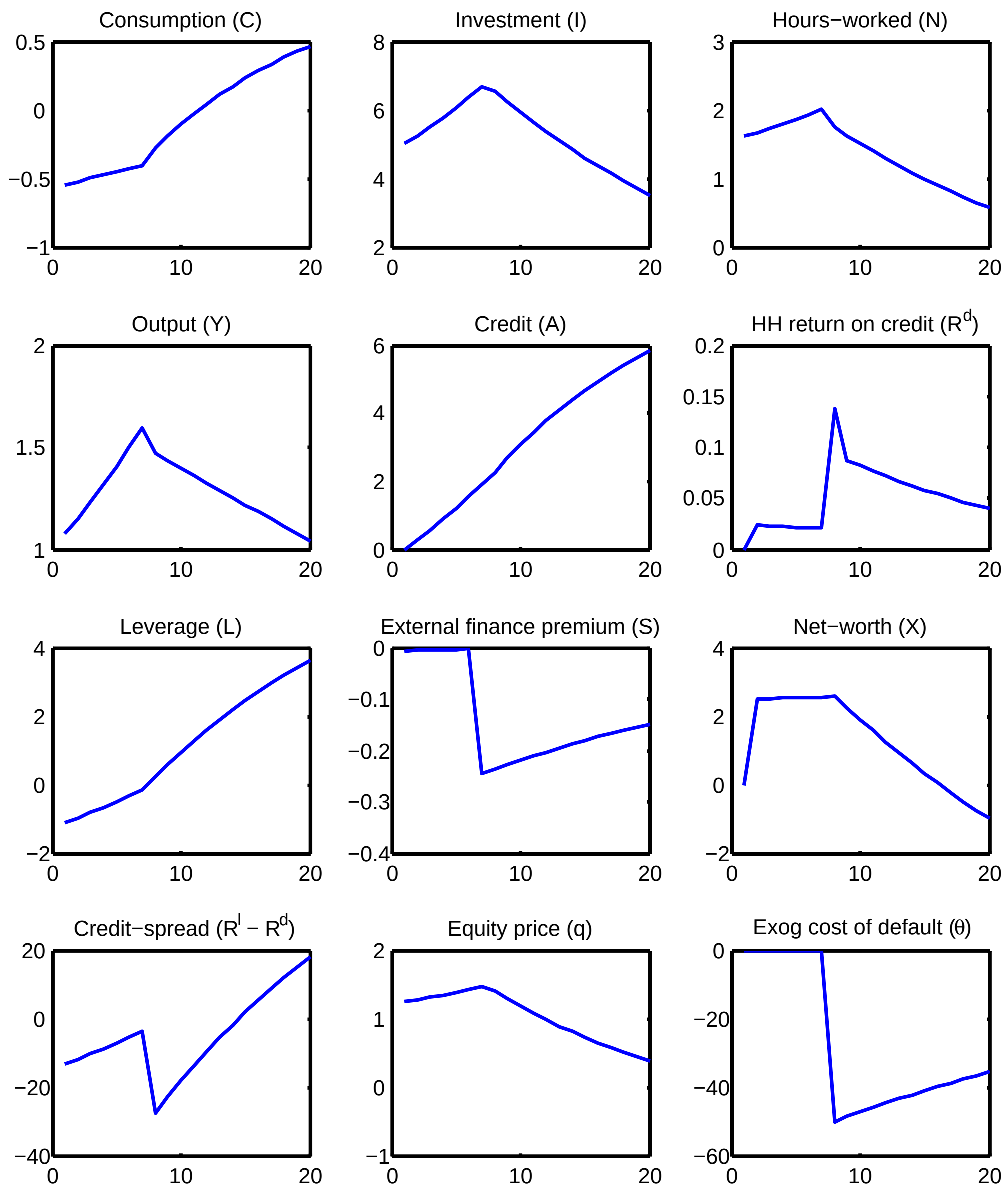
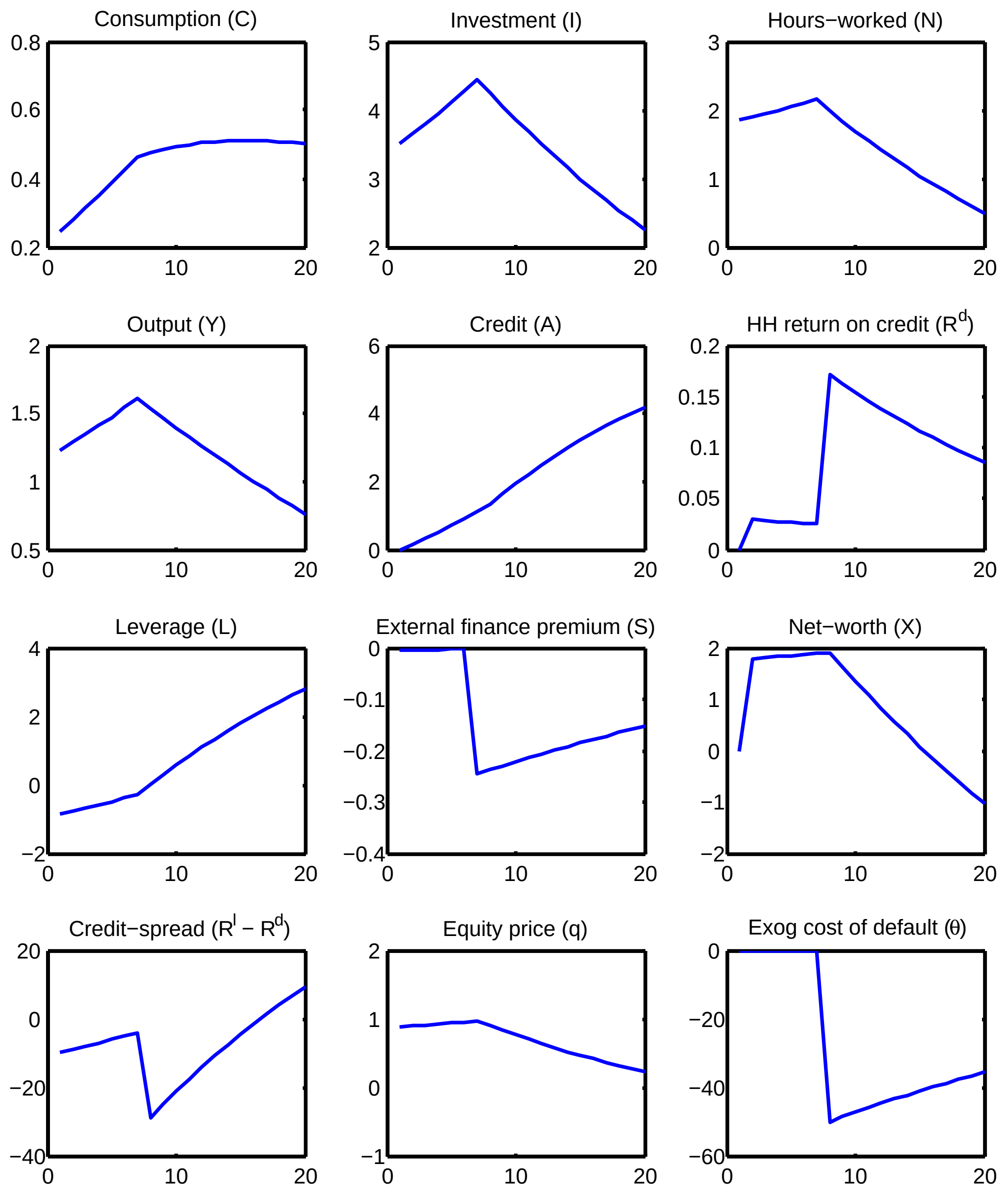


Figure 2.14: News about reduction in default cost $\theta$ in period 8 - reduction in default cost **realized** in period 8 - Cost paid to household



# Chapter 3

# From growth to cycles through beliefs

## 3.1 Introduction

Can technology be important for aggregate fluctuations even if technology *shocks* are not? Much of the debate about the extent to which fluctuations are related to technology is framed in the context of theoretical models which involve sudden shocks to innovation or technology - either unanticipated or anticipated - and empirical methods that seek to identify these shocks. Yet accounts such as that of David (1990) of the larger periods that spawned the booms that many would argue to be the most likely related to technology - such as the information technology (IT) boom of the 1990's - reveal slow transformational shifts in the role of technology in the economy well before the boom periods. In the 1990's case in particular, we see a culmination of a technological revolution that began 20 years prior in the 1970's. Does viewing a boom embedded in a technological transition simply as a stochastic shift of the technological frontier unconnected to the processes driving the transition give us a complete picture of the forces driving the boom? Or rather, are these booms somehow related to the lower-frequency transformational change that preceded them?

In this paper I argue that an important feature of technological change is to define a particular *technological regime* that may influence the high and medium frequency dynamics of the economic system independent of any shocks to technology itself. I present a theoretical model where a frontier of "technological ideas" evolves gradually and without shocks, yet where the economy endogenously adopts these ideas into production at higher frequencies based on agents' self-fulfilling beliefs about the value of adoption. This frenzy of adoption then leads to a boom in aggregate quantities followed by an eventual increase in productivity growth. As a result, the rate of realization of the benefits provided by the technological frontier is a function of changes in beliefs about the value of technology as opposed to sudden shocks to technology itself as in many models in the literature. Yet the underlying nature of the frontier is critical to determining the possibility that changes in beliefs can be self-fulfilling and thus produce an adoption boom. The self-fulfilling beliefs in the model take the form of stationary sunspot equilibria associated with indeterminacies due to the presence of a "technological gap" between unadopted ideas and current productivity, and I demonstrate that the scope for these indeterminacies is a function of the steady-state growth rate of the frontier. When growth in ideas is high and a sufficient gap exists between ideas and productivity, indeterminacies can exist, whereas during times of low growth in ideas, the potential for indeterminacies diminishes and thus reduces the dimension of the state-space of possible shocks. Under this view, technology becomes important for cycles not necessarily because it produces sudden shifts in the frontier, but rather, because it defines a technological regime for the economy whereby beliefs about its value can produce aggregate fluctuations where in a different regime they could not. To the extent then that a technological transition such as the IT revolution of the 1970's-1990's spawns an extended period of growth in transformational ideas, the transition thus defines a technological regime which creates the possibility that beliefs about the technology can be self-fulfilling, providing for the existence of a belief driven boom. Thus the model is consistent with the view that a boom such as the 1990's may have resulted from sudden optimism of the benefits of the transformational ideas that had gradually evolved out of the preceding years of the IT revolution.

This "animal spirits" or self-fulfilling view of cycles is of course not new. The ideas themselves grew popular from the writing of Keynes, and through the 1980's and 1990's researchers such as Howitt and McAfee (1992), Benhabib and Farmer (1994) and Farmer and Guo (1994) formalized them into rational expectations models of aggregate fluctuations driven by beliefs. Yet the vast majority of these models exploit externalities or other structural features of the economy that are arguably structurally-ever-present in goods production or its connected markets, whether during a technological transition or not, and in this regard could be argued to describe "normal" business cycles as a result of random fluctuations of beliefs. Was the boom of the 1990's simply a result of a random wave of optimism unconnected to the IT revolution, or was there something different about this period that catalyzed this response? Was there something different about the boom of the 1990's compared to that from 2001-2007? I argue that from the perspective of models of aggregate fluctuations, a period such as the 1990's is in fact different because the preceding technological transition introduced structural features that *allowed* beliefs to become important in a way that they would not during a "normal" cycle. One important implication of this is that technology need not *always* be important for cycles: just as a technological transition can define a regime with a high-growth in technological ideas that enables a boom, a period where the distance between the frontier of new ideas and those ideas already in practice is low - resulting from either a stagnation in ideas growth or a previous adoption boom which exhausted further gains - could limit the role of technology in a belief-driven boom.

I present a model economy where firms increase their productivity by implementing the technological ideas from an exogenous technological frontier. These technological ideas are in the public domain and freely available to all firms, yet they are embodied in new capital in the sense that firms must purchase new capital to gain access to the ideas. After purchasing capital, firms then undergo a process of costly-adoption through which they learn how to tailor and apply these general ideas to their specific production processes, thereby permanently increasing their productivity of goods production. Importantly, this notion of embodiment stresses the impact of technological ideas in the *use of* new investment goods in the sense that using new capital allows firms to transform and reorganize their production process. This lies in contrast to the notion of embodiment in the *production of* new investment goods, often emphasized in the literature on investment specific technical change such as Hulten (1992) and Greenwood et al. (1997), whereby advancing investment-specific technical change allows those accumulating capital to grow their capital stock faster for a given purchase of investment in units of consumption, without necessarily requirement implementation or adoption.

Physical capital as a result plays an extremely critical role in this economy beyond its standard role in goods production by acting as a conduit of technological ideas and knowledge. Unlike the standard neoclassical growth model whereby increases in capital simply increase the scale of production through capital deepening, in this model growth in technological ideas means that increases in capital can also alter the blueprint of goods production itself, increasing the productivity of all factors. As such, during periods with an abundance of technological ideas relative to the current state of productivity, there are potentially high returns to marginal investment.

Both this critical role for physical capital as well as the delayed realization of productivity benefits is motivated by a host of studies following the boom of the 1990's that link information technology capital with eventual delayed productivity gains. For example, Basu and Fernald (2008) study a data-set of 40 industries in the U.S. over the period of 1986 to 2004 and find that TFP gains after mid-1990's were broad-based across industries, located primarily in information and communications technology (ICT) capital-*using* rather than *producing* industries, and that industry TFP accelerations in 2000's were positively correlated with industry ICT capital growth in mid-1990's.

To give some concrete substance to what I am referring to as "technological ideas" it is helpful to look at a specific example. One such candidate is the concept of supply chain management which was popularized in the 1990's during the IT revolution and involved the process by which firms plan and manage the flow of goods through the various stages of their businesses, from procurement to production to distribution. Most observers generally acknowledge that the IT revolution began many years prior in the mid-1970's, and over the next ten to fifteen years the primary impact of this revolution within incumbent firms in the economy was to automate or replace many individual tasks within those firms. Eventually as the late 1980's approached however, this gave way to a system of transformational ideas about how to reorganize processes within firms, between firms, and between firms and customers to exploit the connectivity, visibility and analytics provided by the the new hardware and software technologies. Importantly, these ideas were not the result of any one innovation by a firm or sector, but rather were well known and in the public domain in business schools, consultancies and within those in strategic capacities in the businesses themselves by early 1990's when the firms began to implement them, evolving slowly over time through the various efforts of those who interacted and studied the newly emerging technologies. Finally, while these ideas may have existed in the public domain, in order to implement them firms first had to purchase the necessary hardware and software infrastructure to interact with their existing capital stock.

To model the endogenous adoption process, I use a costly-adoption specification similar to Greenwood and Yorukoglu (1997), Comin and Hobijn (2007), and Nelson and Phelps (1966) that models the movement of productivity in practice towards some limit defined by the technological frontier, with the distance between productivity in practice and that of the frontier being commonly referred to as the "technological gap". I then extend this concept to an environment of capital-specific learning where the effective frontier faced by an individual firm is endogenously-controlled by that firm as a function of its history of purchases of physical capital. This reliance on investment in new physical capital to push a firm's effective frontier forward is a critical ingredient in producing indeterminacies in the model. Starting from some arbitrary equilibrium path, a conjectured belief by the firm about an increase in return to adoption immediately drives up the value of increasing the effective frontier that it faces in adoption, which being a function of investment in physical capital means that during adoption physical capital provides an additional return to the firm beyond its direct impact in goods production. Combined with the process of costly adoption of the firm's effective frontier this pursuit of investment then interacts with the labour market resulting in shifts in both labour demand and supply that increase the return to all the firm's accumulated factor inputs. Yet since all firms ultimately confront a given frontier of established technological ideas within a given technological paradigm, with each additional unit of productivity created firms know that they are closing in on the technological limit, and thus through time firms value new adoption less and less, imposing the necessary stability on the system to return it to the balanced growth path. Thus the presence of the "technological gap" is critical to the dynamics of the model economy, since despite the high returns to investment in adoption, the exogenous technological frontier means that the economy overall faces a dynamic diseconomy of scale as it "closes the gap" on the theoretical frontier, invoking an effect similar to that illustrated by Howitt and McAfee (1988), who show how counteracting multiple externalities can lead to locally stable equilibria in systems with multiple equilibria. At the core of this idea in my model is the interplay between medium to high-frequency efforts of the economy to move towards its theoretical frontier in response to changes in beliefs, and the low-frequency phenomena that move the frontier forward and thus constrain the economy's short-run expansionary efforts.

The remainder of the paper is structured as follows. In Section 3.2 I discuss how my approach relates to other works in the literature not discussed above. In Section 3.3 I describe the technological environment, outline the model, and define the equilibrium and balanced growth path. In Section 3.4 I investigate the role of indeterminacies in the model, illustrating the dependence of the scope for indeterminacies on the underlying growth regime, and examining the response of the model economy to sunspot shocks. Section 3.5 concludes.

## 3.2 Relation to existing literature

In this section I give a brief explanation of how the ideas and elements in this paper relate to other research in the literature.

In attempting to make a link between endogenous adoption and aggregate fluctuations, I am proceeding in the spirit of Comin et al. (2009), who model an endogenous adoption process in response to stochastic shifts in the technology frontier. Unlike Comin et al however, I attempt to investigate the link between endogenous adoption and aggregate fluctuations in the absence of any sudden shocks to the technology frontier, and in an environment where the only form of uncertainty is changes in beliefs about the value of adoption.

By considering the relation between adoption and beliefs, I draw on ideas similar to those in implementation models such as Shleifer (1986), Francois and Lloyd-Ellis (2003) and Francois and Lloyd-Ellis (2008), whereby firms must choose to implement a new technology to realize its productivity benefits. While these papers show how clustering of firm-level implementations can lead to endogenous cycles in an environment where profits related to the innovation are short-lived, I focus on the expanding role of physical capital to enable adoption, as well as the dynamic interaction of a slowly evolving technology and the constraint it places on an economy that adopts to it at varying rates driven by beliefs. Moreover, while Schleifer investigates the role of adoption during "normal" cycles where the benefits of innovation occur immediately after adoption, I focus on sub-periods where technological change alters the dynamics of the economy and produces delayed realization of benefits.

In drawing a connection between growth and cycles, I focus on a different aspect than that emphasized by researchers working with endogenous growth models, such as King and Rebelo (1986), Comin and Gertler (2006) and Comin (2009). These papers integrate growth and cycles based on a core of an endogenous growth model whereby temporary shocks unrelated to technology can have a permanent effect on output through the innovative activities of the firms, impacting the aggregate stock of "innovations" that ultimately lead to permanent growth. In contrast, in my model there is no endogenous growth; the aggregate stock of ideas evolves smoothly and unconnected to the business cycle, yet beliefs about the value of the technology can affect the *rate* that the economy realizes these benefits. Moreover, I focus on how a change in technological regime would cause the economy to respond differently during for example the 1970's versus the mid-1990's, as opposed to investigating the role of technology or innovation in "all" cycles. Additionally, in proposing an alternative connection between growth and medium-frequency fluctuations, I take advantage of the work of Comin and Gertler (2006) and Comin (2009) that presents evidence linking high and low-frequency fluctuations. I highlight, however, a direction of causation that works in reverse: whereas they describe how low-persistence shocks that produce business cycles can then lead to medium frequency fluctuations, in my model the emergence of certain growth regimes provides for the possibility of belief shocks that produce an immediate short-run burst of activity that eventually settles into a persistent response well into the medium-term frequencies.

In the sense that the role of beliefs relies on the underlying fundamentals of technology, my work is also related to the "news shock" idea spearheaded in a recent literature by Beaudry and Portier (2004), and further investigated by Christiano, Ilut, Motto and Rostagno (2008), Jaimovich and Rebelo (2009), Comin et al. (2009), Gunn and Johri (2011), and Dupor and Mehkari Dupor and Mehkari (2010) [1]. Under this idea, agents act on information they receive about expected changes in future technological fundamentals. This contrasts

[1] See also, Schmitt-Groh and Uribe (2009), Khan and Tsoukalas (2009) and Barsky and Sims (2010).

with my model where there are no sudden shocks in the frontier. Furthermore, I am modeling self-fulfilling beliefs whereby the economy must endogenously create the growth that confirms its expectations, rather than having the realized change in productivity be independent of the actions of the agents, as in most of the models in the news literature with the exception of Comin et al. (2009), and to some extent Gunn and Johri (2011). Nevertheless, the ideas from the news shock literature and the concept I highlight in this paper are complementary. I focus on the dynamics of the economy conditional on a certain underlying growth rate of ideas, and don't consider the dynamics of the transition between regimes of low or high growth in ideas. One could argue that the ideas of the news shock literature become especially important at the interface of transitions between growth regimes, as agents in the economy begin to learn about the increased growth potential and new stock of innovations of the new technological regime. In this regard, both the modeling strategies of Gunn and Gunn and Johri (2011) and Comin et al. (2009) are especially complementary to this model. In a similar vein, my model is complementary to the ideas of Aghion and Howitt (1998) who provide an endogenous rationale of why a boom may arise from a slow growth process, but nevertheless share a similar focus with the news literature in implying an increase in future productivity at a growth threshold.

Finally, I follow a long line of researchers including Benhabib and Farmer (1994), Benhabib and Nishimura (1998), Bennett and Farmer (2000) and Farmer and Guo (1994) who exploit variants of the neoclassical growth model to investigate the connection between stationary sunspot equilibria and aggregate fluctuations, and also build upon other research that examines the impact of externalities in the neoclassical growth model such as Baxter and King (1991) and Cooper and Johri (1997). Extending these approaches, I focus on how structural change created by changing growth regimes changes the potential for indeterminacies. Moreover, I also introduce a new modeling mechanism whereby the mechanisms driving indeterminacies depend on the growth rate of the frontier, and therefore may exist only during certain technological regimes.

## 3.3 Model

The economy consists of an infinitely-lived representative household, a single final goods firm that nonetheless acts competitively, and a continuum of monopolistically competitive intermediate goods firms on a unit measure, each ith firm producing a differentiated good. Intermediate goods firms own both their physical and intangible capital stocks, financing their expenditures through shares sold to households.

### 3.3.1 Final goods firm

The final goods producer purchases intermediate goods $y_t(i)$ from intermediate goods firms and combines these goods into a single final good $y_t$ according to the technology

$$Y_t = \left( \int_0^1 y_t(i)^\nu di \right)^{\frac{1}{\nu}}, \tag{3.1}$$

where $\nu \in (0, 1)$ determines the elasticity of substitution between the intermediate goods. The producer then sells the final good into the final goods market to be used as consumption for households or investment for intermediate goods firms. Each period the producer chooses its demand for each intermediate good $y_t(i)$ by maximizing its profits given by

$$Y_t - \int_0^1 P_t(i) y_t(i) di, \tag{3.2}$$

where $P_t(i)$ is the relative price of the ith intermediate good $y_t(i)$ in terms of the final good $y_t$. The resulting optimality condition then yields a demand function for the ith good as

$$y_t(i) = P_t(i)^{\frac{1}{\nu - 1}} Y_t. \tag{3.3}$$

### 3.3.2 Intermediate goods firms

Each ith intermediate goods firm produces differentiated output $y_t(i)$ with an associated *firm-specific productivity* $h_t(i)$. Firms increase their productivity by adopting freely-available technological ideas $\Psi_t$ into their production process, where $\Psi_t$ represents the technological frontier of ideas about how to organize production related to new physical capital. Since these ideas pertain to recent physical capital, the quantity of ideas that the firm can implement into production will be some function of its current and past investments in physical capital. A firm's *potential productivity* $j_t(i)$ thus represents the quantity of ideas from the frontier $\Psi_t$ that the firm can adopt at time $t$ based on its investment history, with the property that the more a firm invests in physical capital, the more ideas from the frontier $\Psi_t$ the firm will be able to implement. Firms then convert their stock of potential productivity $j_t(i)$ into firm-specific productivity $h_t(i)$ through a costly learning and adoption process through which they learn to tailor the ideas to their specific production process.

Firms produce their output $y_t(i)$ according to the production function

$$y_t(i) = h_t(i)^{\epsilon}(X_t n_{yt}(i))^{\alpha}\tilde{k}_t(i)^{\theta} \tag{3.4}$$

where $h_t(i)$ is firm-specific productivity, $X_t$ is exogenous labour-augmenting technical change, $n_{yt}(i)$ is labour allocated to goods production and $\tilde{k}_t(i)$ is physical capital services [2]. Capital services $\tilde{k_t(i)}$ is defined as $\tilde{k}_t(i) = u_t(i)k_t(i)$, where $u_t(i)$ is the utilization rate of the stock of physical capital $k_t(i)$.

Firms accumulate physical capital according to

$$k_{t+1}(i) = [1 - \delta(u_t)]k_t(i) + i_t(i) \tag{3.5}$$

where $i_t(i)$ is investment in units of the final good, and the function $\delta(\cdot)$ is a standard time-varying cost of utilization as a convex function of the utilization rate, with properties $\delta'(\cdot) > 0$, $\delta''(\cdot) > 0$.

Firms grow their firm-specific productivity $h_t(i)$ through a process of costly learning and adoption during which they allocate labour to tailor and implement the ideas reflected in their potential productivity $j_t(i)$ into their spe-

[2]Labour-augmenting technical change $X_t$ is necessary for calibration to summarize the effect of other contributions to growth beyond the ideas frontier $\Psi_t$, but is not neccessary for the problem

cific production process. One salient feature of technological implementation/adoption processes is that it takes *time* to implement a technology. The literature has posited various reasons for this, but here I follow Comin and Hobijn (2007), Greenwood and Yorukoglu (1997), and Nelson and Phelps (1966) in simply specifying the adoption process as a partial-adjustment equation of the form

$$h_{t+1}(i) = h_t(i) + h_t(i)\Big[1 - \frac{h_t(i)}{j_{t+1}(i)}\Big]\Phi\Big(n_{ht}(i)\Big), \tag{3.6}$$

where $\Phi\Big(n_{ht}(i)\Big) = \tau_0 n_{ht}(i)^{\eta}$, $\tau_0$ is a constant, $0 < \eta \leq 1$, and $n_{ht}$ is labour that the firm allocates to the adoption process [3].

Equation (3.6) has the property that if the realizations of $\Phi\Big(n_{ht}(i)\Big) < 1$, $h_t(i)$ is bounded above by $j_{t+1}(i)$, imposing a type of "limit to learning" on the firm whereby the firm cannot increase its productivity $h_{t+1}(i)$ beyond its productivity potential $j_{t+1}(i)$. In contrast with Greenwood and Yorukoglu (1997), Comin and Hobijn (2007) and Nelson and Phelps (1966) where the bound of adoption is tied to the overall technological frontier outside of the

[3] One way to motivate this equation is to make a probabilistic interpretation of implementation based on approaches by Howitt and Mayer-Foulkes (2005) and Comin and Gertler (2006). For example, a firm that attempts to increase its productivity from the $h_t(i)$ to the potential $j_{t+1}(i)$ through implementation is successful with probability $\omega_t(i)$, which is an endogenous function of the firm's choices. As such, the firm's expected productivity next period is $E_t(i)h_{t+1}(i) = \omega_t(i)j_{t+1}(i) + (1-\omega_t(i))h_t$. The probability $\omega_t(i)$ that the firm is successful is an increasing function of the resources the firm directs towards adoption, and a decreasing function of the relative distance between the firm's current productivity and its potential productivity, $\frac{h_t(i)}{j_{t+1}(i)}$, such that $\omega_t(i) = \Phi\Big(n_{ht}(i)\Big)\frac{h_t(i)}{j_{t+1}(i)}$. Substituting this definition of $\omega_t(i)$ into the expression for expected productivity next period and re-arranging then yields $E_t(i)h_{t+1}(i) = h_t(i) + h_t(i)\Big[1 - \frac{h_t(i)}{j_{t+1}(i)}\Big]\Phi\Big(n_{ht}(i)\Big)$, giving the specification in (3.6) above in expectation. Comin and Gertler (2006) and Comin et al. (2009) then use the assumption that once a technology is in use, all firms have it to facilitate aggregation without expectation.

control of the firm, here the bound $j_{t+1}(i)$ is endogenously controlled by the firm as a function of its history of investment in physical capital, acting as an "effective frontier" facing the firm. On the balanced growth path when growth in ideas is positive, $h_t(i)$ will converge to a constant gap between $h_t(i)$ and $j_t(i)$ and thus a constant gap between $h_t(i)$ and $\Psi_t$. In the special case of a "technologically stagnant" era where there is no growth in ideas, $h_t(i)$ will converge to $\Psi_t$ such that the gap is zero.

A firm's potential productivity $j_t(i)$ evolves as some function of its current and past investment history relative to the state of technological ideas $\Psi_t$ during which these investments in physical capital are made, such that $j_{t+1}(i) = J(j_t(i), i_t(i), \Psi_t)$ . To ease notation, it is helpful to define $x_t = \frac{j_t(i)}{\Psi_{t-1}}$ to represent the current *fraction* of ideas contained in the frontier $\Psi$ that the firm *can* implement based on its current history of investment, and thus rewrite the function $J(\cdot)$ as $x_{t+1}(i) = X(x_t(i), i_t(i), \Psi_t)$.

The function $X(\cdot)$ essentially enforces the requirement that since the ideas $\Psi_t$ relate to physical capital, the firm must invest in order to implement these ideas. Thus the function is essentially a property of the *technology* of production. In reality, the ability to implement ideas that depend on the characteristics of physical capital will be some complicated function of the history of investment purchases, to the extent that capital from different periods is not alike. For example, a firm in the 1990's that wished to implement supply chain management ideas required not only the physical infrastructure such as vehicles and warehouses necessary to operate its business, but also the necessary information technology hardware and software that enables the supply chain planning and optimization. To the extent that latter (software) requirements represent an expansion in the variety of capital goods through time relative the former (computers and before that warehouses), the function $X(\cdot)$ would require new purchases of capital to complement an accumulated stock of older capital as a prerequisite for implementing the supply chain management ideas.

To captures these feature most simply, I assume a simple log-linear specification of $X(\cdot)$ similar to that in Stadler (1990) to describe the evolution of $x_t$ through time,

$$x_{t+1}(i) = \Gamma_t x_t(i)^{\phi} i_t(i)^{\rho}, \tag{3.7}$$

where $\Gamma_t = \Gamma(J_t, I_t, \Psi_{t-1})$ is a scale-factor necessary for balanced growth that I will define later, and where $0 \leq \phi < 1$ , $0 \leq \rho < 1$ [4]. Equation (3.7) simply expresses the evolution of the fraction of ideas that the firm can implement next period as a function of that fraction this period and new investment this period.

To understand this equation, it is helpful to first consider the extreme condition where $\phi = \rho = 0$: in this case the firm need not do anything to implement $\Psi_t$; it spills over simply as an external effect independent of the firms current and past investment. For $\rho > 0$ however, the rate that the firm's potential productivity $j_t(i)$ grows relative to the frontier of ideas $\Psi_t$ depends

[4] The results of the model hold under other specifications for the function $x_{t+1}(i) = X(x_t(i), i_t(i), \Psi_t)$ . One such specification, symmetrical to that used for firm-specific productivity $h_t(i)$, is $x_{t+1}(i) = x_t(i)/g_t^{\Psi} + x_t(i)/g_t^{\Psi}\left[1 - x_t(i)/g_t^{\Psi}\right]\kappa_0 \frac{i_t(i)}{k_t(i)}$, where $\kappa_0$ is a constant, $g_t^{\Psi}$ is the growth rate of $\Psi$, and where the firm internalizes all the variables indexed with i. As will become clear later however, the current specification in equation (3.7) offers the advantage of economizing the aggregate state vector under a symmetric equilibrum.

on its new investment in physical capital. A value of $\phi > 0$ along with $\rho > 0$ makes $j_{t+1}(i)$ a function of both its current and past investment, such that a firm that accumulates a larger stock of new capital early on can expect higher growth in potential productivity. This essentially allows investment now to increase the future effectiveness of investment in growing $j$, implying a backwards compatibility of future ideas to past investment such that a firm can exploit some portion of future growth in ideas $\Psi_t$ based on past investment.

Finally, firms purchase total labour $n_t(i)$ at wage $w_t$, and allocate it between goods production and adoption according to

$$n_{y_t}(i) + n_{ht}(i) = n_t(i). \tag{3.8}$$

Note that despite the fact that $\Psi_t$ is freely available, it differs from the concept of TFP in a neoclassical growth model or a model with external effects in goods production in that the need to purchase physical capital in order to implement the free ideas requires that a firm *internalize* these ideas as additional benefits that may come with purchasing new capital. This has important consequences then for income distribution since it implies that in order to increase productivity some time in the future, a firm may be willing to invest more in physical capital in the present then is justified by that extra capital's direct returns in production in the present. As such, either decreasing returns to scale to physical capital and labour or a positive markup over marginal cost associated with imperfect competition are necessary to allow the firm to operate in such an environment without driving profits negative.

### Intermediate goods firms' problem

Defining $S_t = (\Psi_t, Z_t, K_t, J_t, H_t)$ as a vector of aggregate state variables beyond the control of the firm, and $s_t(i) = \Big(k_t(i), j_t(i), h_t(i)\Big)$ as the corresponding vector of state variables under the control of the firm, each intermediate goods producer solves the recursive problem

(3.9)

$$V(S_t, s_t(i)) = max_{n_y(i), n_h(i), i_t(i), u_t(i), s'(i)} \left\{ d_t(i) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} V\Big(S_{t+1}, s_{t+1}(i)\Big) \right\}$$

subject to (3.5), (3.7) and (3.6), where $d_t(i) = P_t(i)y_t(i) - w_t(S_t)n_t(i) - i_t(i) = Y_t^{1-\nu} y_t(i)^{\nu} - w_t(S_t)n_t(i) - i_t(i)$ is the firm's dividend, and $\beta E_t \frac{\lambda_{t+1}}{\lambda_t}$ is the household owner's stochastic discount factor, yielding the optimal policy rules $k'(i) = k(S_t, s_t(i))$, $j'(i) = j(S_t, s_t(i))$, $h'(i) = h(S_t, s_t(i))$, $n_y(i) = n_y(S_t, s_t(i))$, $n_h(i) = n_h(S_t, s_t(i))$, $i(i) = i(S_t, s_t(i))$ and $u(i) = u(S_t, s_t(i))$. Letting $q_{kt}(i)$, $q_{jt}(i)$ and $q_{ht}(i)$ be the Lagrange multipliers on firm i's physical capital ($k$), firm-specific productivity potential ($j$) and firm-specific productivity ($h$) accumulation equations respectively, the firm's first-order conditions are as follows:

$$w_t = \nu\alpha \frac{P_t(i)y_t(i)}{n_{yt}(i)} \tag{3.10}$$

$$w_t = q_{ht}(i)h_t(i)\Big[1 - \frac{h_t(i)}{j_{t+1}(i)}\Big]\Phi'\Big(n_{ht}(i)\Big) \tag{3.11}$$

$$1 = q_{kt}(i) + q_{jt}(i)\rho\frac{j_{t+1}(i)}{i_t(i)} \tag{3.12}$$

$$q_{kt}(i)\delta'(u_t(i))k_t(i) = \nu\theta\frac{P_t(i)y_t(i)}{u_t(i)} \tag{3.13}$$

$$q_{jt}(i) = q_{ht}(i)\left(\frac{h_t(i)}{j_{t+1}(i)}\right)^2\Phi(n_{ht}(i)) + E_t\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}q_{jt+1}(i)\phi\frac{j_{t+2}(i)}{j_{t+1}(i)}\right\}, \tag{3.14}$$

$$q_{kt}(i) = E_t\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}\left(\nu\theta\frac{P_{t+1}(i)y_{t+1}(i)}{k_{t+1}(i)} + q_{kt+1}(i)\left[1-\delta(u_{t+1}(i))\right]\right)\right\} \tag{3.15}$$

(3.16)
$$q_{ht}(i) = E_t\left\{\beta\frac{\lambda_{t+1}}{\lambda_t}\left(\nu\epsilon\frac{P_{t+1}(i)y_{t+1}(i)}{h_{t+1}(i)} + q_{ht+1}(i)\Big[1+\left(1-\frac{2h_{t+1}(i)}{j_{t+2}(i)}\right)\Phi\Big(n_{ht+1}(i)\Big)\Big]\right)\right\}.$$

Equations (3.10) and (3.11) are the firm's y-hours and h-hours first-order conditions respectively, and show that the firm allocates labour between goods production and adoption to equalize the marginal products of labour in each use. Note in (3.11) that the gap $[1-\frac{h_t(i)}{j_{t+1}(i)}\Big]$ increases the technical effectiveness of hours in adoption, whereas the shadow value $q_{ht}(i)$ expresses the marginal value of adoption in terms of the firm's output. Combining these two equations along with the firm's hours allocation constraint (3.8) yields an expression for the firm's total labour demand,

$$w_t = \frac{1}{n_t(i)}\left(\nu\alpha y_t(i) + q_{ht}(i)h_t(i)\Big[1 - \frac{h_t(i)}{j_{t+1}(i)}\Big]\eta\Phi\Big(n_{ht}(i)\Big)\right) \tag{3.17}$$

where it is clear that both the shadow value $q_{ht}$ and the gap $[1 - \frac{h_t}{j_{t+1}}]$ act as shift-factors for the firm's labour demand curve in wage-hours space. Given the dependence of the gap $[1 - \frac{h_t(i)}{j_{t+1}(i)}\Big]$ on the existence of growth in aggregate $J_t$ due to positive trend growth in $\Psi_t$, the magnitude of this shift factor will be a function of the underlying technological environment. At the extreme in a technologically stagnant era this gap will go to zero, such that the second term in (3.17) drops out and and the firm's labour demand collapses to that of the standard one-sector neoclassical model.

The firm's investment first-order condition (3.12) shows that in determining investment, the firm considers both its benefit in adding to the physical capital stock as well as its potential contribution in developing a productivity potential that will eventually lead to productivity improvements, illustrating the by-product nature of investment in this economy. The firm values each of these two effects according to their respective shadow prices.

The firm's utilization first-order condition (3.13) shows simply that in choosing $u$, the firm equates the marginal product of $u$ in goods production to the marginal cost of adjusting utilization, where the marginal cost reflects both physical depreciation of physical capital, and the relative value of physical capital in terms of the consumption good.

Equation (3.14) describes the firm's optimal choice of its potential productivity next period, $j_{t+1}$. Being a necessary input into producing firm-specific productivity, $h_t$, the first term on the right shows how the value of potential productivity $j_t$ is a function of the value of new firm-specific knowledge. The final term captures the contribution of the additional $j$ in raising future $j$.

Both the physical capital and firm-specific productivity first-order conditions (3.15) and (3.16) relate the respective forward-looking shadow prices to the sum of the stochastically-discounted future marginal products of each of these factors in goods production.

It is important to note that the impact of the technology related to adoption exists only while the technological gap is positive. When $\Psi$ growth and hence $J$ growth is high, the firm can invest in new capital, increasing it's potential productivity, $j_t$, and thus maintaining a positive gap $\left[1 - \frac{h_t(i)}{j_{t+1}(i)}\right]$ such that it can transfer labour out of goods production into adoption to increase productivity. Yet during periods of low-growth this effect is diminished, and during technological stagnation it is eliminated: without growth in $\Psi_t$, no amount of investment can increase the firm's potential productivity $j_t$, and thus labour transferred out of goods production into adoption is useless.

### 3.3.3 Household

The household side of the model is standard so I discuss it briefly. The representative household has preferences defined over sequences of consumption $C_t$

and leisure $L_t$ with expected lifetime utility defined as

$$\mathcal{U} = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \tag{3.18}$$

where $\beta$ is the household's subjective discount factor, and the period utility function $u(C_t, L_t) = \frac{1}{1-\sigma}\{[C_t v(L_t)]^{1-\sigma} - 1\}$ is of the class of preferences described in King et al. (1988). The household's budget constraint is given by

$$C_t + \int_0^1 v_t(i) A_{t+1}(i) di = w_t N_t + \int_0^1 [d_t(i) + v_t(i)] A_t(i) di, \tag{3.19}$$

where $v_t(i) = v_i(S_t)$ is the price of firm i's share, $A_t(i)$ the household's holdings of shares of firm i, $w_t = w(S_t)$ is the wage, $N_t$ is hours-worked, and $S_t$ is a vector of aggregate state variables.

Finally, each period, the household is endowed with one unit of time that it allocates between leisure and hours-worked according to

$$N_t + L_t = 1. \tag{3.20}$$

The household solves the recursive problem $V(A_t, S) = max_{C,N,L,A'}\{u(C_t, L_t) + \beta E_t V(A_{t+1}, S_{t+1})\}$ subject to (3.19) and (3.20), where $A_t$ represents a vector of the portfolio of firms' shares, yielding the policy rules $A' = A(A, S)$, $C' = C(A, S)$ and $L_t = L(A, S)$. Letting $\lambda_t$ by the Lagrange multiplier on

(3.19), the firm's first-order conditions are

$$u_c(C_t, L_t) = \lambda_t \tag{3.21}$$

$$u_l(C_t, L_t) = \lambda_t w_t \tag{3.22}$$

$$v_t(i) = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[ d_{t+1}(i) + v_{t+1}(i) \right] \right\} \quad \forall i. \tag{3.23}$$

### 3.3.4 Aggregate technology

To approximate what in reality may be a gradual rate of change of publicly-available technological ideas $\Psi_t$, I assume that $\Psi_t$ grows deterministically as

$$\Psi_t = \Psi_{t-1} g_{\Psi}^{era}, \tag{3.24}$$

where the growth rate $g_\Psi$ is relatively constant over a "technological era" but over the long run will change and thus represent a structural shift in the economy, represented by the index "era" [5] [6]. For example, Fernald (2007) detects evidence of a break in the trend of average labour productivity growth such that average growth during from the 1950's to early 1970's, and after the mid-1990's was much higher than it was in between these periods. In the context of my model, I interpret these high-growth regimes as periods of structural change that increase the deterministic growth rate of the frontier from its value $g_{\Psi}^{L} \approx 1$ in the low-growth period to some value $g_{\Psi}^{H} > 1$ in the high-growth period, as an approximation to what in reality may be a gradual acceleration of the technological frontier. As such, in this model, a period such as the 1990's that yielded transformational technical change over a relatively short ten-year period would be represented by a structural shift in the growth rate parameter $g_{\Psi}$ at some point prior to this period. It is important to note however that my argument in the paper does not rely on the *timing* of this structural change. For example, I am not implying that belief-driven fluctuations occur simultaneously with this structural change. Rather, the *existence* of a period of sufficiently positive growth in ideas simply opens up the possibility that a belief-driven fluctuation may occur.

In order to properly calibrate the model to average long-run growth rates over high and low growth periods, I also include deterministic labour-augmenting

[5] Alternatively one could model $\Psi_t$ with a stochastic trend where its growth rate follows a highly-persistent process subject to low-variance shocks, such that $\Psi_t$ evolves gradually over time. As will become apparent later however, I use a linear approximation in my analysis, and thus what becomes most important for the model dynamics I consider is only the steady-state drift portion of the growth rate, even though in reality the non-linear dynamics may be important.

[6] It would be relatively straightforward to endogenize the growth rate of the theoretical frontier $\Psi_t$ using the various mechanisms in the endogenous growth literature, however it would complicate the model without necessarily further illuminating my central point.

technical change $X_t$ in the model to represent "other" sources of growth, where $X_t$ follows the process

$$X_t = X_{t-1} g_X \tag{3.25}$$

and where the growth rate $g_X$ is constant over "high" and "low" technological eras.

### 3.3.5 Equilibrium

I define the aggregate quantities $K_t = \int_0^1 k_t(i)di$, $J_t = \int_0^1 j_t(i)di$, $H_t = \int_0^1 h_t(i)di$, $N_t^f = \int_0^1 n_t(i)di$, $I_t = \int_0^1 i_t(i)di$ and $A_t^f = \int_0^1 a_t(i)di$ associated with the intermediate goods producers. The market clearing conditions in the model economy for the labour market, stock market and goods market as then as follows:

$$N_t = N_t^f \tag{3.26}$$

$$Y_t = C_t + I_t \tag{3.27}$$

$$A_t^f = 1 = A_t, \tag{3.28}$$

where the left-hand and right-hand sides of each of the equalities are the supply and demand sides respectively in the markets for each quantity.

A rational expectations equilibrium for this economy is then a collection of policies for households $a' = a(S_t)$, $n = n(S_t, a_t)$, $l = l(S_t, a_t)$, policies for intermediate goods firms $k'(i) = k(S_t, s_t(i)$, $j'(i) = j(S_t, s_t(i))$, $h'(i) = h(S_t, s_t(i))$, $n_y(i) = n_y(S_t, s_t(i))$, $n_h(i) = n_h(S_t, s_t(i))$, $i(i) = i(S_t, s_t(i))$ for $i \in [0, 1]$, policies for the final goods producer $y = y(S_t)$, $y(i) = y(i)(S_t)$, price systems $w(S_t)$, $v_i(S_t) \quad \forall i$, and aggregate laws of motion $K = K(S)$, $J = J(S)$, and $H = H(S)$, such that: (i) households solve their problem (ii) intermediate goods producers solve their problems; (iii) the final goods producer solves its problem; (iv) the markets in equations (3.26), (3.27) and (3.28) clear, and; (v) a fixed point such that the individual firm's policy rules confirm the aggregate laws of motion.

I consider a symmetric equilibrium where $p_t(i) = p_t$, $y_t(i) = y_t$, $n_{yt}(i) = n_{yt}$, $n_{ht}(i) = n_{ht}$, $n_t(i) = n_t$, $k_t(i) = k_t$, $j_t(i) = j_t$, $h_t(i) = h_t$ and $i_t(i) = i_t$. Substituting into the definitions of the aggregate quantities associated with the intermediate firms then implies that $k_t = K_t$, $j_t = J_t$, $h_t = H_t$, $n_{yt} = N_{yt}$, $n_{ht} = N_{ht}$, $n_= N_t$, $i_t = I_t$ and $u_t = u_t$.

I define the scale factor $\Gamma_t = (\frac{J_t}{\Psi_{t-1}})^{1-\phi}/I_t^{\rho}$ so that under a symmetrical equilibrium, the intermediate goods firms's $x$ accumulation equation (3.7) reduces to $\frac{J_{t+1}}{J_t} = \frac{\Psi_t}{\Psi_{t-1}}$, such that $J$ grows at the same rate as $\Psi$. As such, under the resulting transformed stationary system, we can effectively remove $J_t$ from the equilibrium system, reducing the endogenous states down to the stationary forms of $K_t$ and $H_t$. Nevertheless, despite there being no endogenous movement in $J_t$, the conditions imposed under the intermediate goods firm's $j_t(i)$ accumulation equation (3.7) continue to influence the firm's behaviour through its first-order conditions [7].

Substituting $y_t(i) = y_t$ into the final goods aggregate technology (3.1) yields the condition $y_t = Y_t$. Recognizing that under perfect competition the final goods firm's profits will be zero then implies that $p_t(i) = p_t = 1$. Finally, substituting $y_t = Y_t$, $h_t = H_t$, $n_{yt} = N_{yt}$ and $k_t = K_t$ into the ith intermediate goods firm's production function (3.4) yields the aggregate production function

$$Y_t = H_t^{\epsilon}(X_t N_{yt})^{\alpha} \tilde{K}_t^{\theta}. \tag{3.29}$$

In a symmetrical equilibrium, all firm's shadow prices of $k$, $h$ and $j$ will be equivalent. To represent this in equilibrium system I redefine these internal prices in aggregate in terms of household utility as $\mu_t = q_{kt}\lambda_t$, $\Upsilon_t = q_{ht}\lambda_t$ and $\zeta_t = q_{jt}\lambda_t$. The resulting equilibrium dynamic system is represented by the following system of equations:

$$Y_t = H_t^{\epsilon}(X_t N_{yt})^{\alpha}(u_t K_t)^{\theta} \tag{3.30}$$

[7] The model results are not dependent on limiting the endogenous role of $J_t$ in this manner. As I indicated in an earlier footnote, the results of the model hold for other specifications of $x_{t+1}(i) = X(x_t(i), i_t(i), \Psi_t)$ , and under the alternate specification $J_t$ is an endogenous state variable in both the non-stationary and stationary system.

$$N_{y_t} + N_{ht} = N_t. \tag{3.31}$$

$$K_{t+1} = (1 - \delta_k)K_t + Z_t I_t \tag{3.32}$$

$$H_{t+1} = H_t + H_t[1 - \frac{H_t}{J_{t+1}}]\tau N_{ht}^{\eta} \tag{3.33}$$

$$w_t = \nu\alpha\frac{Y_t}{N_{yt}} \tag{3.34}$$

$$w_t = \frac{\Upsilon_t}{\lambda_t} H_t[1 - \frac{H_t}{J_{t+1}}]\eta\tau N_{ht}^{\eta-1} \tag{3.35}$$

$$\lambda_t = \mu_t + \zeta_t \rho\frac{J_{t+1}}{I_t} \tag{3.36}$$

$$\frac{\mu_t}{\lambda_t}\delta'(u_t)K_t = \nu\theta\frac{Y_t}{u_t} \tag{3.37}$$

$$\mu_t = \beta E_t\left\{\lambda_{t+1}\nu\ \theta\frac{Y_{t+1}}{K_{t+1}} + \mu_{t+1}\left(1-\delta_k\right)\right\} \tag{3.38}$$

$$\Upsilon_t = \beta E_t\left\{\lambda_{t+1}\nu\epsilon\frac{Y_{t+1}}{H_{t+1}} + \Upsilon_{t+1}\left[1+\left(1-2\frac{H_{t+1}}{J_{t+2}}\right)\tau N_{ht+1}^{\eta}\right]\right\} \tag{3.39}$$

$$\zeta_t = \Upsilon_t\left(\frac{H_t}{J_{t+1}}\right)^2\tau N_{ht}^{\eta} + \beta E_t\zeta_{t+1}\phi\frac{J_{t+2}}{J_{t+1}}. \tag{3.40}$$

$$\frac{J_{t+1}}{J_t} = \frac{\Psi_t}{\Psi_{t-1}} \tag{3.41}$$

$$C_t^{-\sigma}\upsilon(L_t)^{1-\sigma} = \lambda_t \tag{3.42}$$

$$C_t^{1-\sigma}\upsilon(L_t)^{-\sigma}\upsilon'(L_t) = \lambda_t w_t \tag{3.43}$$

$$C_t + I_t = Y_t. \tag{3.44}$$

It is important to realize that when there is no growth in $\Psi$, the system essentially reduces down to the neoclassical growth model where $H_t$ is fixed through time. This can be seen in the above system by substituting $\Psi_{t+1} = \Psi_t = J_{t+1} = J_t = H_t = H_{t+1}$ into the above system.

For reference later in the discussion of the results, I also defined equilibrium observed total factor productivity (TFP) as

$$TFP_t = Y_t - \alpha N_t - (1-\alpha)u_t - (1-\alpha)K_t \tag{3.45}$$

according to the standard definition that uses total labour (as opposed to $N_{yt}$), assumes constant returns to labour and physical capital, and as well controls for the variable contribution of capacity utilization.

### 3.3.6 Balanced growth path and steady state

I define a balanced growth path for this economy whereby $N_t$, $N_{yt}$, $N_{ht}$ and $u_t$ are constant, and the other endogenous variables inherit trends as some function of the trend in $X_t$ and $\Psi_t$. The equilibrium system implies that C, I, Y, D, w, v and K contain trend $X_t^Y = \Psi_t^{\frac{\theta}{1-\theta}} X_t^{\frac{\alpha}{1-\theta}}$, $\lambda_t$ contains trend $1/X_t^{\sigma}$, $J_t$ contains trend $X_t^J = \Psi_t$ and $H_t$ contains trend $X_t^H = \Psi_t$. On the balanced growth path, the growth rates are then $g^y = \frac{X_{t+1}^Y}{X_t^Y}$, and $g^{\Psi} = \frac{\Psi_{t+1}}{\Psi_t}$ and $g^x = \frac{X_{t+1}}{X_t}$ for all t.

I then perform the following transformation such that each resulting variable is stationary on the balanced growth path: $\tilde{C}_t = \frac{C_t}{X_t^Y}$, $\tilde{I}_t = \frac{I_t}{X_t^Y}$, $\tilde{Y}_t = \frac{Y_t}{X_t^Y}$, ...etc., $\tilde{K}_t = \frac{K_t}{X_{t-1}^K}$, $\tilde{J}_t = \frac{J_t}{X_{t-1}^J}$, $\tilde{H}_t = \frac{H_t}{X_{t-1}^H}$, $\tilde{\lambda}_t = \lambda_t {X_t^Y}^{\sigma}$, $\tilde{\mu}_t = \mu_t \frac{X_t^K}{{X_t^Y}^{1-\sigma}}$, $\tilde{\zeta}_t = \zeta_t \frac{X_t^J}{{X_t^Y}^{1-\sigma}}$, and $\tilde{\Upsilon}_t = \Upsilon_t \frac{X_t^H}{{X_t^Y}^{1-\sigma}}$. Under this transformation, the stationary system now has just two endogenous state variables, $\tilde{K}_t$ and $\tilde{H}_t$. Finally, the resulting stationary system contains a unique non-stochastic steady state.

## 3.4 Examining the role of self-fulfilling beliefs

In this section I explore the properties of the model under parameterizations that produce indeterminacies such that sunspot expectation shocks can produce fluctuations in the absence of any shocks to technology. I first describe my solution method and baseline parameterization for a "high-growth" period based on quarterly data, and then investigate how the potential for indeterminacies varies with the underlying growth rate of $\Psi$. Finally I illustrate the impulse response of the model economy to the sunspot shocks.

### 3.4.1 Solution method

I first linearize the model around the non-stochastic state state, resulting in a first-order linear system of the form

$$E_t \mathcal{S}_{t+1} = A\mathcal{S}_t + B\epsilon_t \tag{3.46}$$

where $\mathcal{S}_t = [\hat{k}_t, \hat{h}_t, \hat{\mu}_t, \hat{\Upsilon}_t, \hat{\zeta}_t,]'$, and $\epsilon_t = [0,0,0,0,0]'$ such that there is no external source of uncertainty. Hats above variables denote %-deviations from steady state.

The linear system (3.46) contains two predetermined endogenous states ($k$,$h$) and three forward-looking non-predetermined co-states ($\mu$,$\Upsilon$,$\zeta$). The system will exhibit saddle-path stability if the number of eigenvalues of the matrix A outside of the unit circle is equal to the number of forward-looking non-predetermined variables, and will display indeterminacy if the number of eigenvalues of A lying outside the unit circle is less than the number of forward-looking non-predetermined variables.

To analyze the response of the system to intrinsic uncertainty, I follow the approach of Farmer (1999) and replace the expectations of a variable with the variable less the expectational error, so that now in this case (3.46) re-writes as,

$$\mathcal{S}_{t+1} = A\mathcal{S}_t + B\varepsilon_t \tag{3.47}$$

where $\varepsilon_t$ is now defined as $\varepsilon_t = [0,0,w_t^{\mu}, w_t^{\Upsilon}, w_t^{\zeta}]'$, where $w_t^{\mu} = E_t\mu_{t+1} - \mu_{t+1}$, $w_t^{\Upsilon} = E_t\Upsilon_{t+1} - \Upsilon_{t+1}$ and $w_t^{\zeta} = E_t\zeta_{t+1} - \zeta_{t+1}$ are the one-step ahead forecast errors on the Lagrange multipliers. Note also that by definition the expectational error of a predetermined variable is zero yielding the two zeros in $\varepsilon_t$.

For the parameterizations that I consider that yield indeterminacy, the matrix $A$ has one less root outside the unit-circle than forward-looking variables, leaving two forward-looking variables with unstable roots. Thus under indeterminacy we can interpret the expectational errors above as iid sunspot shocks. I then diagonalize the system and iterate out the two remaining unstable roots as in a saddle-path solution, yielding a restriction on (3.47) that relates the two unstable forward-looking variables to the stable variables. Similar to the multi-sector model of Benhabib and Nishimura (1998), the three sunspot shocks cannot be chosen independently since there is a joint restriction imposed on them from iterating out the two unstable roots.

After solving out the unstable roots, the system reduces down to

$$\tilde{\mathcal{S}}_{t+1} = \tilde{\mathcal{A}}\tilde{\mathcal{S}}_t + \tilde{\mathcal{B}}\tilde{\varepsilon}_t \tag{3.48}$$

where now $\tilde{\mathcal{S}}_t = [\hat{k}_t, \hat{h}_t, \hat{\Upsilon}_t]'$, and $\tilde{\varepsilon}_t = [0, 0, e_t^{\Upsilon}]'$ and where $e_t^{\Upsilon}$ is an iid sunspot shock to $\Upsilon$, the value of $H$. Note now that all the roots of $\tilde{\mathcal{A}}$ are inside the unit circle, and the system is a Markovian stable process such that any value of $e_t^{\Upsilon}$ will set the system on a stable path that eventually returns to steady state.

These sunspot expectational shocks involve the Lagrange multipliers of $K$, $J$, and $H$ which by definition measure the marginal value of these predetermined states. Thus in the context of the model one can interpret an expectational shock to $\Upsilon$ as a self-fulfilling belief by the agents about the value adoption since a marginal change in $H$ constitutes a marginal adoption by the firm of the technology of the frontier.

### 3.4.2 Parameterization

In this section I detail an illustrative calibration for a "high-growth" period that features positive growth in the frontier of technological ideas. Where possible I assign values to parameters using restrictions on the model steady-state with values established in the literature.

I approximate what may in reality be a gradual increase in the rate of ideas-growth and associated dynamic expansion of the technological gap as a structural break in the steady-state growth rate of the parameter $\Psi$, creating an additional source of growth beyond the "other factors" contained within the labour-augmenting growth factor $X_t$. In the data this break in $\Psi$ would *eventually* show up as a structural break in measured average productivity as firms adopt and develop firm-specific productivity. Thus I exploit existing empirical analysis by others of structural breaks in average labour productivity in post-war U.S. data to calibrate the magnitude of the growth rate of $\Psi$ [8]. It is important to note however that my model implies that the *timing* of the structural break in $\Psi$ need not necessarily coincide with an observed structural break in average labour productivity, since in the theoretical model there is an implementation and adoption phase that delays productivity gains. Moreover, since the rate of change of adoption and therefore realized productivity is a function of the exogenous beliefs of agents, even if the gap is large, unless firms are optimistic about its value, they may not pursue adoption with enough fervour to produce concentrated productivity growth.

Fernald (2007) finds break-dates in average labour productivity in the

[8] See Fernald (2007) and Kahn and Rich (2007).

early-1970s and the mid-1990's, separating data productivity series into 'high" periods (from the 1950's to the early 1970's and after mid-1990's) and "low" periods (from 1970's to mid-1990's) with growth rates of approximately 3.25% and 1.5% respectively. For the purposes of illustration, I assume that the other source of growth in the model - labour augmenting growth - is constant throughout the entire post-war period, and then that growth in technological ideas makes up the difference between "high" and "low" growth regimes with growth rates of 2.5% and 1.5% respectively [9], such that I interpret the "high" regimes as periods where the "technological gap" opens up. Given these break-dates identified by Fernald, my interpretation of breaks related to the gap is consistent with the empirical evidence of Cummins and Violante (2002), who estimate a Nelson-Phelps "technological gap" style adoption model, and find that the gap increased from the mid-1950's to the early 1970's, and from the mid-1990's until the turn of the century.

Since during the "low" regime I assume that $\Psi$ is near-stagnant, during this regime the growth rate of output $g_y$ is related to the growth rate of labour-augmenting technical change $g_x$ by $g_y^{low} = g_x^{\frac{\alpha}{1-\theta}}$, which given the quarterly growth rate of output $g_y^{low} = 1.015^{.25}$, determines $g_x$. During the "high" regime, the growth rate of output is related to both sources of growth by $g_y^{high} = g_\Psi^{\frac{1}{\alpha}} g_x$, yielding $g_\Psi = \left(\frac{g_y^{high}}{g_X}\right)^\alpha$, which given the quarterly growth rate of output $g_y^{high} = 1.025^{.25}$, determines $g^\Psi$. Both of these expressions are de-

[9] There is much evidence that the rate of growth of investment-specific technological change accelerated in the 1990's, such as that of Cummins and Violante (2002). Incorporating investment-specific technical change (ISTC) into the model and allowing for a structural break in ISTC for the "high" regime however does not materially impact my results, and therefore I neglect it for simplicity

pendent on a parameterization for $\alpha$ and $\theta$ which I will determine below.

In the model, the overall labour share in output $S_N$ is a function of labour in both the Y and H uses, such that

$$S_N = S_{N_y} + S_{N_h}, \tag{3.49}$$

where $S_{N_y} = \nu\alpha$ and $S_{N_h}$ are the Y and H labour shares in output respectively. (2010) calibrate the cost of $R\&D$ using a result from Corrado et al. (2009) that investment in R&D in the US corporate investment sector is approximately 5.7% of corporate income, which is analogous to $S_{N_h}$ in this model. Using a value of $S_{N_h} = 0.057$ from Comin and Hobijn, and setting the overall labour share $S_N$ to a 0.70 during the "high" period yields a labour share in goods production of $S_{N_y} = (0.7 - 0.057) = 0.643$. During the "low" period, since there is no growth in $\Psi$, $S_{N_h}$ will then approach zero, implying that the overall labour share in the economy decreases from 0.7 to 0.643. Note however that the labour share in goods production $S_{N_y}$ remains constant at 0.643 over both the "high" and 'low" growth periods.

To parameterize the shares of factors in goods production I first assume that the firm-specific productivity $H$ acts as a capital-augmenting growth factor, implying that $\epsilon = \theta$. While this suggests possible increasing returns to $N_y$, $K$ and $H$ in goods production, it should not be directly interpreted as an indicator of the short-run point-in-time returns to scale of the production of $Y_t$ as in models with contemporaneous externalities such as Baxter and King (1991) or Benhabib and Farmer (1994), since in this model $H_t$ is not a function of contemporaneous (internal or external) goods-labour and/or physical capital. Instead, $H_t$ in this context acts like a dynamic complementarity that changes marginal cost over time, in the vein of that estimated by Cooper and Johri (1997). Moreover, the impact of this dynamic complementary may only be temporary, since only during regimes with positive growth in the ideas frontier does $H$ grow; during regimes where there is no growth in $\Psi$, $H$ is constant and the production function will act as a standard production function with constant or decreasing returns to labour and capital. Nevertheless it is important to note that the results of the model are not dependent on the assumption of $\alpha + \theta + \epsilon > 1$. An alternative parameterization featuring constant returns to $N_y$, $K$ and $H$ such that $\alpha + \theta + \epsilon = 1$ still yields indeterminacies, however, it is not clear that this parameterization would make economic sense in this model since it would imply significant decreasing returns in production during times of no growth in $\Psi$ and thus $H$ is fixed.

Next to specify the curvature $\nu$ on the final goods aggregator and the degree of returns to scale to $N_y$ and $K$ in intermediate goods production I use values similar to Atkeson and Kehoe (2007) who model technological transition featuring a intermediate goods production function with inputs of intangible capital, labour and physical capital and with decreasing returns to labour and physical capital. I set $\nu = 0.95$, implying a markup of 5.3%, and $\alpha + \theta = 0.95$. In comparison, Atkeson and Kehoe use 0.9 and 0.95 for the analogous quantities, the former of which implies a markup of 11% in their model. As Atkeson and Kehoe discuss, their markup of 11% is consistent with evidence in Basu and Fernald (1995) and others, and decreasing returns of

0.95 is consistent with a wide range of empirical work that finds estimates in the rage of 0.9 to 1. Using these values then yields $\alpha = S_{N_y}/\nu = 0.643/0.95 = 0.6768$, and $\theta = \epsilon = 0.2732$.

Having determined $\alpha$, we can now determine $g_x$ and $g_\Psi$ using the expressions from earlier, yielding $g_x = 1.0040$ and $g^\Psi = 1.0065$. Given these values, we can then determine the parameters in the H-accumulation equation using the expression for $S_{N_h}$, $S_{N_h} = \nu\theta\eta\frac{g^\Psi - 1}{\frac{g^\Psi}{\beta g y^{1-\sigma}} - (1+[1-2\frac{H}{Jg^\Psi}]\Phi(N_h))}$, where in steady-state $\frac{H}{Hg^\Psi} = \frac{1-\Phi(N_h)-g^\Psi}{\Phi(N_h)}$. Since the "gap" nature of the H-accumulation equation provides for significant implicit decreasing returns to scale as the firm closes its gap, for the baseline parameterization I assume no curvature on $N_h$, setting $\eta = 1$, noting that providing for curvature on $N_h$ such as letting $\eta = \alpha$ (to equate the labour intensity in the two uses of labour) does not dramatically impact the results, given a constant calibration for $S_{N_h}$.

For the convex cost of capacity utilization, my solution method requires that I need only specify the elasticity of marginal depreciation to utilization, $\epsilon_u = \frac{\delta''(u)}{\delta'(u)}u$, which I set to 0.56 based on the estimates of Burnside and Eichenbaum (1996), and the steady state value of depreciation $\delta(u_{ss}) = \delta_k$, which I set to the standard value of 0.025.

To promote comovement, I use preferences not separable in consumption and leisure of the form used by (King and Rebelo (2000)) where the the stand-in representative agent has the preference specification

$$u(C_t, L_t) = \frac{1}{1-\sigma}\left\{C_t^{1-\sigma}v(L_t)^{1-\sigma} - 1\right\} \tag{3.50}$$

where $\upsilon(L_t) = \left[\left(\frac{1-L_t}{H}\right)\upsilon_1^{*\frac{1-\sigma}{\sigma}} + \left(1 - \frac{(1-L_t)}{H}\right)\upsilon_2^{*\frac{1-\sigma}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}}$, and where $H$ is the fixed shift length, and $\upsilon_1^*$ and $\upsilon_2^*$ are constants representing the leisure component of utility of the underlying employed group (who work $H$ hours) and unemployed group (who work zero hours) respectively. Basu and Kimball (2002) empirically investigate the general class of King et al. (1988) preferences not additively separable in consumption and leisure and find estimates of the *labour held constant elasticity of intertemporal substitution in consumption* of 0.5-0.67 during the sample period 1982 to 1999, larger than the near-zero values of the intertemporal elasticity of consumption estimated by Hall (1988) that assumed no non-separabilities in consumption and leisure. During the sample period 1949 to 1982 they estimated this quantity to be not significantly different from zero, in line with the results of Hall (1988). Thus to represent both these periods, I choose a value of the labour held constant elasticity of intertemporal substitution in consumption of 0.25, which in this model is equal to $1/\sigma$, implying $\sigma = 4$, in the range of the value of $\sigma = 3$ used by King and Rebelo (2000) in an illustration of these preferences. I then set the average household's share of time allocated to market work $N_{ss}$ to 0.3, and the average household's subjective discount factor $\beta$ to 0.9934.

The remaining parameters $\phi$ and $\rho$ in the firm's j-accumulation equation control the dependency of adoption of physical capital, and have a very significant impact on the region of indeterminacy in addition to impacting the steady-state K-Y ratio and therefore equilibrium profit share. The steady state

K-Y ratio is given by the expression

$$\frac{k}{y} = \frac{\nu\theta}{1/[\beta g^{y\,1-\sigma} - (1-\delta_k)/g^k]} + \left(\frac{\tilde{\zeta}}{\tilde{\Upsilon}}\right)\left(\frac{\tilde{\Upsilon}h}{\tilde{\lambda}y}\right)\rho\frac{1/h}{i/k} \tag{3.51}$$

where $\frac{\tilde{\zeta}}{\tilde{\Upsilon}} = \frac{\left(h/g^h\right)^2}{1-\beta g^{y\,1-\sigma}\phi}$, $\frac{\tilde{\Upsilon}h}{\tilde{\lambda}y} = \frac{\nu\theta}{1/[\beta g^{y\,1-\sigma} - (1+(1-h/g^h)\Phi(N_h))/g^h}$ and $\frac{i}{k} = 1-(1-\delta_k)/g^k$. Note that in (3.51), the first term on the right-hand side is the standard expression for the K-Y ratio based on the contribution of K to the production of Y. The second term however reflects the additional contributions to the steady-state capital stock as a result of the firms' internalizing the additional benefit of purchasing physical capital to grow productivity, beyond that of the marginal production of capital in goods production. All else equal, this second term is increasing in both $\phi$ and $\rho$. Since the equilibrium profit (or dividend) share is given by $\frac{D}{Y} = 1 - S_N - \frac{I}{K}\frac{K}{Y}$, we can pin down a combination of $\phi$ and $\rho$ based on a plausible steady-state profit share through the effect on the K-Y ratio for a given $S_N$. In this model, the profit share is related to the important quantity $\frac{wN}{C}$ by $\frac{wN}{C} = \frac{S_N}{1-\frac{I}{Y}} = \frac{S_N}{S_N+\frac{D}{Y}}$ . The quantity $\frac{wN}{C}$ is important because it is readily observable in the data, as discussed at length by Farmer and Ohanian (1999) and Basu and Kimball (2002), and moreover provides a link to the non-separable preference specification in this model through the relation $\frac{\upsilon'(L)}{\upsilon(L)}L = \frac{wN}{C}$. Farmer and Ohanian estimate this quantity to be 0.97 over the period 1929 to 1988, which in this model thus implies a steady state profit share of $\frac{D}{Y} = S_N\frac{(1-\frac{wN}{C})}{\frac{wN}{C}} = 0.0216$. For illustration, I report the impulse-response simulations for two different combinations of these two parameters: ($\phi$, $\rho$) = (0.8,0.85) and ($\phi$, $\rho$) = (0.9,0.45), yielding profit shares in the "high" growth period of 1.9% and 2.5% respectively.

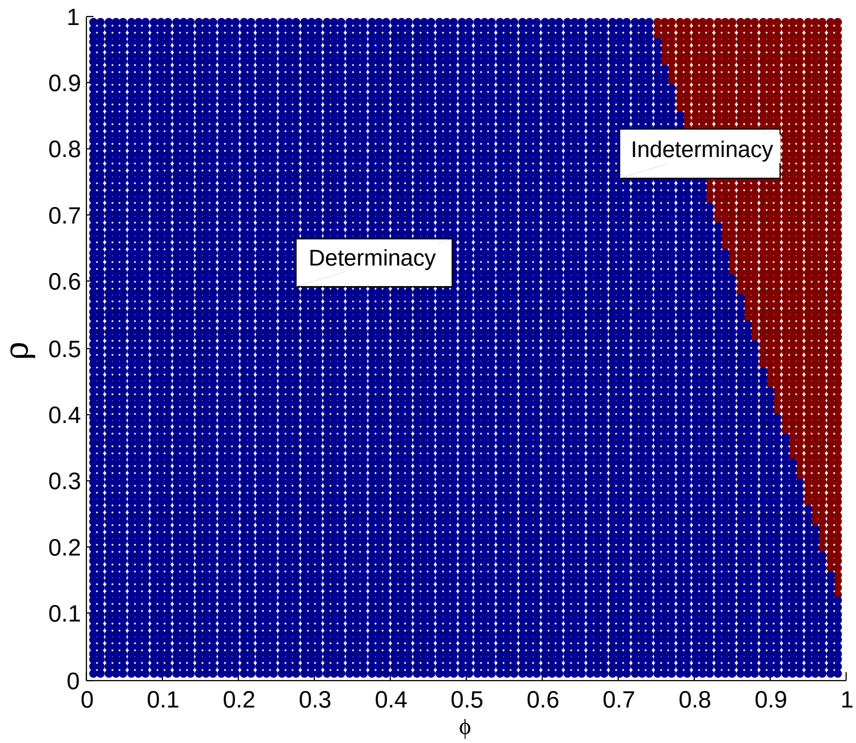
### 3.4.3 Characterization of indeterminacy

In this section I numerically characterize indeterminacy in the model in terms of the parameters $\phi$ and $\rho$, as well as the dependence of the scope for indeterminacy on the growth rate of the ideas frontier $g^{\Psi}$.

#### Dependence of indeterminacies on $\phi$ and $\rho$

Using the baseline parameterization I solve the model for each combination of $\phi$ and $\rho$ on a 100x100 grid ranging from 0 to 1 for each of these parameters, determining the stability properties of the system for each combination. Figure 3.1 shows the results of this exercise. Recall that $\rho$ captures the extent to which a given firm must itself invest in new capital contemporaneously to exploit the new ideas, and $\phi$ the extent to which a firm's past purchases of new capital allow it to exploit new ideas for a give level of new investment. Interestingly, the results from the figure imply then that each firm needs to "do something" purposefully to drive indeterminacy, in the form of actively purchasing the new capital which allows it to exploit the new ideas, rather than simply just receiving a costless spillover externality independent of its own investment actions. Indeed at the extreme case discussed earlier where $\phi = 0$ and $\rho$ - whereby the firm can adopt new ideas without investing - the system is completely determinant. The role of physical capital as an enabler of ideas in this economy is thus a critical ingredient for generating indeterminacies.

Figure 3.1: Dependency of indeterminacy region on $\Phi$ and $\rho$ for baseline parameterization



**Dependence of indeterminacies on steady-state growth rate of the ideas frontier, $g^\Psi$**

I now attempt to characterize the relation between the scope for indeterminacies in the model economy and the underlying steady state growth rate of $\Psi$, and thus by doing so suggest that the potential for adoption booms fueled by self-fulfilling "animal spirits" is dependent upon the growth rate of technological ideas.

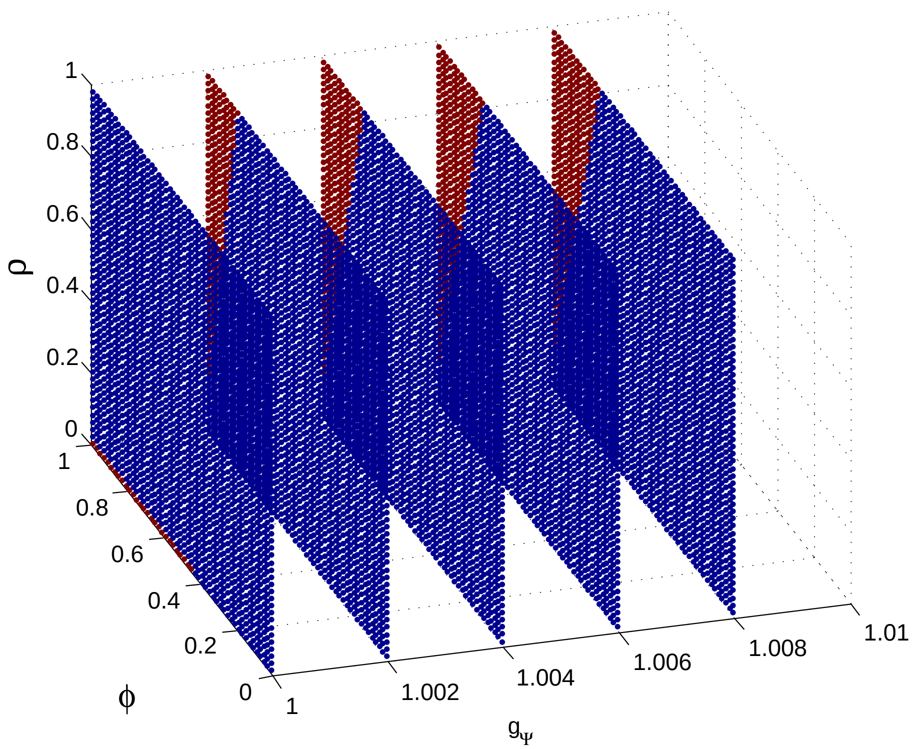
Starting with the baseline parameterization, I vary the value of the growth rate parameter $g^\Psi$, keeping constant the remainder of the baseline parameters, and for each different growth rate $g^\Psi$ I solve the model for each combination of $\phi$ and $\rho$ on a 100x100 grid as in the previous exercise, determining the properties of the system for each combination of $g^\Psi$, $\phi$ and $\rho$.

Figure 3.2 shows the results of this exercise, plotting the resulting properties of determinacy as a function of combinations of $\phi$ and $\rho$ for 5 different vertical "slices" of $g^\Psi$ (ie each slice essentially repeats the exercise of Figure 3.1 for a different $g^\Psi$).

Importantly, note that as the underlying growth rate of the embodied frontier $g^\Psi$ decreases, the scope for indeterminacy decreases, to the point where it disappears as the parameterization approaches very low growth rates and the limit with technological stagnation of $g^\Psi = 1$, where the system is completely saddle-path stable for all combinations of $\phi$ and $\rho$.

Recall that the "size" of the steady-state technological gap $(\Psi - H)$ on the balanced growth path varies positively with the underlying growth rate of $\Psi$. Intuitively, as the model approaches very low growth rates, the gap becomes

Figure 3.2: Dependency of region of indeterminacy on growth rate of ideas frontier $g_\Psi$



small enough such that given a set of beliefs about the value of the $K$ and $H$, the additional benefit provided by investment in physical capital as an enabler of knowledge diminishes as the falling gap reduces the technical effectiveness of labour in adoption.

### 3.4.4 Response to iid sunspot shocks

Figure 3.3 shows impulse response functions relative to trend of the model economy with $(\phi,\, \rho) = (0.9.0.45)$ to a 1% iid sunspot shock on $\Upsilon_t$, the Lagrange multiplier on $J$, interpreted as a belief shock about the value of adoption. Firstly, note immediately in period 1 that $\Upsilon_t$ rises by the amount of the shock, reflecting the change in value from the belief shock. These optimistic beliefs about the value of adoption then lead to an increase in demand for the two primary inputs into adoption - investment in physical capital and labour allocated to adoption - increasing the rate of adoption of technological ideas and as a consequence producing an immediate jump in aggregate consumption, investment, hours-worked in total and in Y-hours and H-hours, as well as a drop in the real wage. Following the initial adoption frenzy, both firm-specific productivity $H$ and TFP increase gradually with a lag. Note that the initial boom also leads to an initial drop in TFP as a result of firms reducing their allocation of total labour in goods production and increasing their labour allocation in non-production adoption activities. Both the initial drop in TFP when investment surges and the eventual delayed increase in TFP is consistent with the empirical results of Basu and Fernald (2008) who in addition to finding a positive correlation with lagged ICT investment also find a negative correlation of TFP with contemporaneous investment. The drop in the real wage prior to the eventual delayed increase in TFP is also consistent with the findings of McGrattan and Prescott (2009) in their study of the boom of the 1990s. Finally, note that after the initial short-run dynamics of consumption, investment and hours in the first several quarters, these variables display significant persistence, staying above trend well into the range generally associated with medium-frequency fluctuations, driven by the slow and delayed increase in $H$ and $K$. This property is consistent with a potential link between high-frequency fluctuations associated with business cycles and medium-frequency fluctuations per the "medium-term" cycles phenomena discussed by researchers such as Comin and Gertler (2006).

It is important to understand that the impulse responses only show movement relative to trend, and that the existence of the technological gap is a key driver of the dynamics of the model economy. Inspecting the aggregate $H$ accumulation equation (3.33) shows that as long $\frac{H_t}{J_{t+1}}\Phi(N_{ht}) < 1$, the value of $H$ in levels that *includes* trend cannot decrease. Although relative to the trend growth the dynamics of $H$ is temporary, in the non-detrended economy these movements represent *permanent* increases in $H$. While this increase represent gains that the economy would have *eventually realized anyway* under the counterfactual situation *without* a belief shock, the effects of the belief shock work to produce a concentrated period whereby the economy "captures" these productivity gains at a faster rate that it would have in the absence of a change in beliefs. Moreover, recalling that on the balanced growth path the economy converges to a "constant gap" between $H$ and $\Psi$, these temporary bursts of activity represent the economy temporarily "narrowing" the gap smaller than the value that is consistent with balanced growth. As a result, eventually the forces that push the economy back to its balanced growth equilibrium decrease the endogenous rate of $H$ accumulation while the economy "waits" for the slowly growing $\Psi$ frontier to "catch up" such that the technological gap widens and again returns back to constant value consistent with the balanced growth path. Thus realized growth slowdowns naturally follow realized growth spurts in this economy, as the endogenous forces of adoption interact with the constraints of the slowly moving theoretical frontier.

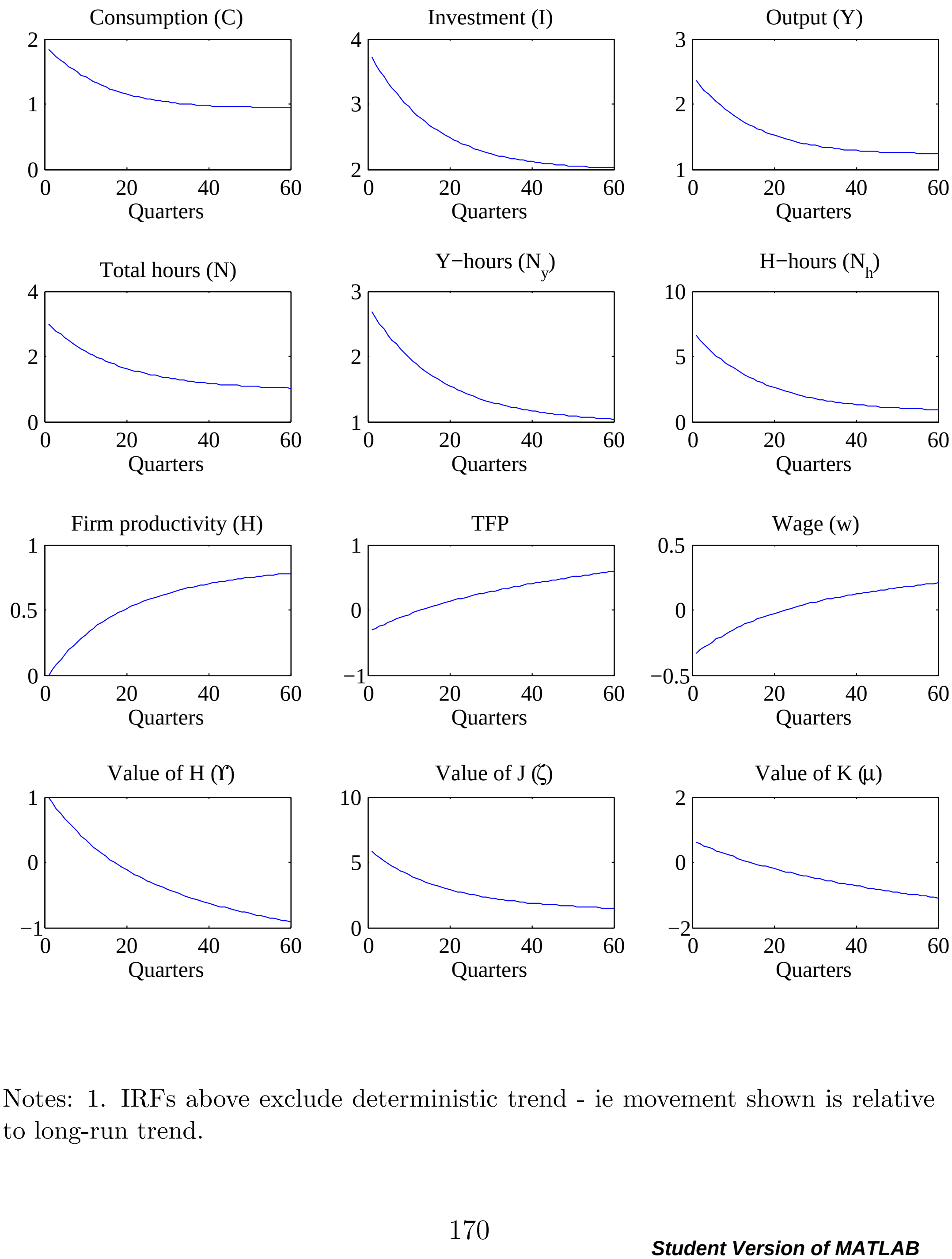
What produces the self-fulling effect in this economy? From the perspective of the firm, its beliefs about an increase in the value of firm-specific productivity $\Upsilon$ and thus the returns to H leads to a desire to increase H, which in turn leads to an increase in demand for the two primary inputs into $H$: firm-specific productivity potential $J$, and H-hours $N_h$. This is evidenced by the effect of the sudden rise $\Upsilon$ in both the firm's $j_{t+1}$ first-order condition and h-hours first-order condition, which for convenience I re-state, this time using the multipliers in terms of household utility,

$$\zeta_t(i) = \Upsilon_t(i)\left(\frac{h_t(i)}{j_{t+1}(i)}\right)^2 \Phi(n_{ht}(i)) + E_t\left\{\zeta_{t+1}(i)\phi\frac{j_{t+2}(i)}{j_{t+1}(i)}\right\} \tag{3.52}$$

$$w_t = \frac{\Upsilon_t(i)}{\lambda_t}(i)h_t(i)\left[1 - \frac{h_t(i)}{j_{t+1}(i)}\right]\Phi'\Big(n_{ht}(i)\Big). \tag{3.53}$$

I will consider the effects in these two first-order conditions in turn.

Figure 3.3: Response to iid sunspot shock about value of $H$, relative to trend: $\phi = 0.9; \rho = 0.45$




Notes: 1. IRFs above exclude deterministic trend - ie movement shown is relative to long-run trend.

First, recalling that the firms adoption process is bounded by its potential productivity $j$, there are high returns to the firm to increasing the upper bound of productivity, and therefore the increase in $\Upsilon$ immediately leads to a large increase in the value of potential knowledge $\zeta$. This effect can be seen in the $j_{t+1}$ first-order condition (3.52), where all else equal, this rise in $\Upsilon$ in (3.52) causes the value of potential knowledge $\zeta$ to rise, essentially working through a relative price margin similar to an effect described in Benhabib and Nishimura (1998). For a given value of physical capital $\mu$ - which reflects the future expected returns to physical capital in goods production through the $k_{t+1}$ first-order condition (3.15) - this rise in the value of potential knowledge $\zeta$ thus causes the firm to increase investment to reduce the marginal production of investment in potential knowledge, $\rho\frac{j_{t+1}}{i_t}$ through the investment first-order condition,

$$\lambda_t = \mu_t(i) + \zeta_t(i)\rho\frac{j_{t+1}(i)}{i_t(i)}, \tag{3.54}$$

reflecting the fact that the firm must purchase new capital to increase its productivity potential.

Since the additional investment must come at the expense of consumption and the household wishes to smooth consumption over time, the marginal utility of consumption and thus $\lambda_t$ increase both now and in the future, driving up the real interest rate and increasing the return to $K$ and $H$ in future periods, again through a similar relative price effect, thus contributing to the rise in $\mu_t$ and partially supporting the conjectured belief about $\Upsilon_t$. Moreover, this rise in $\lambda_t$ both now and in the future also has the effect of keeping $\zeta_t$ high in future periods through the effect of the investment first-order condition in future periods, since all else equal an increase in investment would tend to lower $\mu_t$ in future periods. This has the important result of amplifying the initial increase in $\zeta_t$ since from the firm's $j_{t+1}$ first-order condition (3.52) the value of potential productivity in the present also depends on the value of $j$ to future $j$-growth. This effect is proportional to the parameter $\phi$ however, and small values of $\phi$ in effect act like a large depreciation of $j$ and can thus limit the rise in $\zeta$. Consistent with the numerical evaluation of indeterminacy earlier, this effect underscores the necessity of relative high values of $\phi$ for indeterminacy [10].

Now turning to the labour channel, the effect of the initial rise $\Upsilon$ in the H-hours first-order condition (3.53) increases the firm's demand for labour in adoption, $n_h$. Since there is no shift in the productivity of goods production in the initial period however, the marginal product of $N_h$ in goods production doesn't shift initially and therefore the firm's total labour demand shifts out in an attempt to satisfy the increase in $N_h$ though additional labour [11]. In tandem with this shift in labour demand, the high cost of current consumption caused by the investment opportunities produces an increased willingness of the household to substitute out of current leisure, creating a shift in labour supply and lowering the real wage in the initial period. The net effect in the

[10] Note that endogenous growth in $J$ would also serve to amplify the increase in $\zeta$, but since under the current specification for $J$-growth $J$ does not vary independent of $\Psi$ in equilibrium, this margin is shut down under this specification.

[11] Again under an alternate specification, growth in $J$ independent of $\Psi$ would increases the size of the gap $\left[1 - \frac{h_t(i)}{j_{t+1}(i)}\right]$, further increasing the demand for $N_h$, but this margin is shut down under the current specification.

labour market is a rise in total hours, an effect which is amplified by capacity utilization through the impact of the increase in $N_h$ on the marginal product of utilization. This labour market effect then continues into subsequent periods through the propagation effects discussed earlier, and is further amplified by the gradual rise in $K$ and $H$ which increase labour demand further.

As a result, both the relative price effects through investment and rapid expansion in labour allow the future marginal products of $K$ and $H$ to rise despite "investment" in these factors rising also, thus confirming the original conjectured beliefs.

Importantly however, the sink-dynamics of the stationary sunspot equilibrium require not just a self-reinforcing return, but also a channel of stability that pulls the system back to steady-state and keeps it off the explosive path. The effect of the technological gap in this model provides a critical role in this regard. To see this, it is helpful re-state the firm's $h_{t+1}$ first-order condition,

(3.55)

$$\Upsilon_t(i) = \beta E_t \left\{ \lambda_{t+1} \nu \theta \frac{P_{t+1}(i) y_{t+1}(i)}{h_{t+1}(i)} + \Upsilon_{t+1}(i) \Big[ 1 + \left( 1 - \frac{2h_{t+1}(i)}{j_{t+2}(i)} \right) \Phi\Big( n_{ht+1}(i) \Big) \Big] \right\}.$$

Note that the last term $\left(1 - \frac{2h_{t+1}(i)}{j_{t+2}(i)}\right)$ varies through time as a result of changes in the technological gap. As the firm grows its productivity $h$ over time, it narrows the gap between current productivity and the technological frontier, thereby reducing future growth in productivity. Dynamically narrowing the gap thereby gradually reduces the benefit of adoption, reducing $\Upsilon_t$ over time. Both the combined effect of the narrowing gap and the reduction in $\Upsilon_t$ over time then reduces the amount of labour the firm allocates to adoption, and the reduction in $\Upsilon_t$ over time reduces the value of new investment, gradually pulling the system back to steady state.

Since firms must invest in new capital to reap the productivity gains, the channels through which a belief shock in this economy impacts aggregate quantities shares similarities with a broader class of "investment shock" models that affect the marginal efficiency of investment, including those describing investment specific technical change, credit and capital installation shocks, such as in Greenwood et al. (1988), Fisher (2006), Primiceri et al. (2006) and Justiniano et al. (2010). These models all describe a variation of a shock that drives a wedge into the household's Euler equation, making current consumption expensive as the household seeks to increase investment. While not shown in Figure 3.3, in this model the effects working through the investment first-order condition produce an initial drop in the relative price of physical capital in terms of consumption, $q_t^k = \frac{\mu_t}{\lambda_t}$, in response to the belief shock. Thus the model produces endogenous movement in $q_t^k$ which is consistent with findings in this literature regarding countercyclical movements in the relative price of installed capital as a result of changes in the marginal efficiency of investment an/or investment specific technical change. This literature also typically finds that these shocks imply negative co-movement between consumption and investment in versions of the models that stay close to the neoclassical core. Without preferences with non-separabilities in consumption and leisure, this model would suffer the same co-movement issues. With non-separable preferences however, the marginal utility of consumption is increasing in hours worked, and therefore the rise in labour through the interactions in the labour market cause a similar rise in consumption.

It is important to note that capacity utilization plays subordinate role in this economy, and is not key to driving indeterminacy [12]. To see the effect of utilization, we can re-state the firm's utilization first-order condition as

$$\frac{\mu_t(i)}{\lambda_t(i)}\delta'(u_t(i))k_t(i) = \nu\theta\frac{P_t(i)y_t(i)}{u_t(i)}. \tag{3.56}$$

Both the expansion of labour that increases the marginal product of utilization, and the drop in $\frac{\mu_t}{\lambda_t}$ that reduces the cost of utilization increase the rate of utilization in response to the belief shock, amplifying the expansion of output in the early periods.

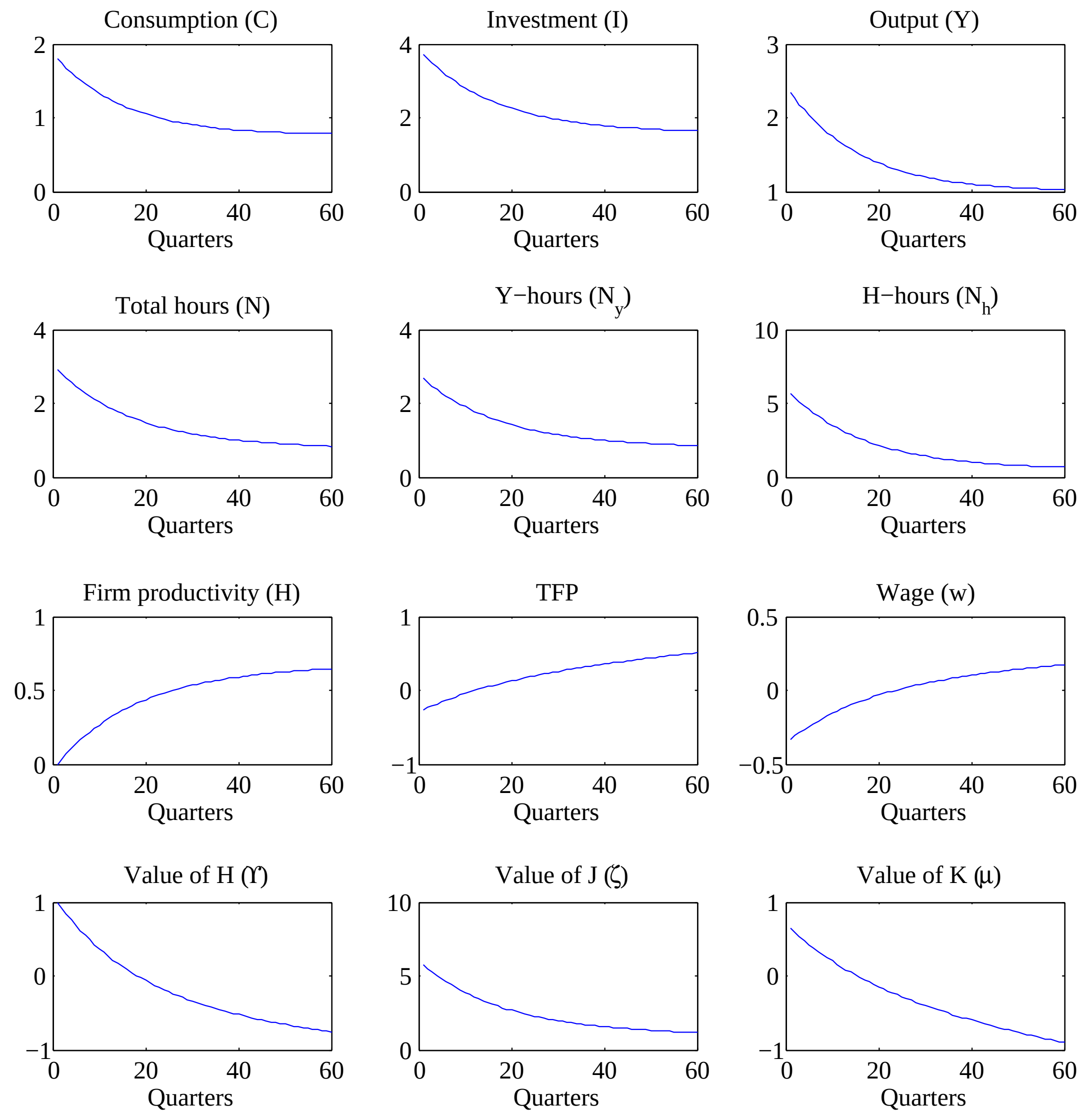
Figure 3.4 shows the response of the model economy to the same shock as Figure 3.3, this time for $(\phi,\, \rho) = (0.8{,}0.85)$. As is clear from the graph, the response of the model economy for this combination of $(\phi,\, \rho)$ is very similar that that in Figure 3.3.

## 3.5 Conclusion

In this paper I argue that the technological frontier need not undergo sudden shifts to influence the dynamics of the economy in the short and medium run. I present a theoretical model where the technological frontier moves slowly and without shocks, yet where the economy adopts to this frontier endogenously

[12] Variants of the model with either extremely high costs of utilization or no utilization at all produce very similar regions of indeterminacy.

Figure 3.4: Response to iid sunspot shock about value of $H$, relative to trend: $\phi = 0.8; \rho = 0.85$




Notes: 1. IRFs above exclude deterministic trend - ie movement shown is relative to long-run trend.

at higher frequencies based on agents' beliefs about the actions of others. As a result, the rate of realization of the benefits provided by the technological frontier is independent of shocks to that frontier. Yet the underlying growth rate of the frontier is critical to determining the possibility that expectations can in fact influence the dynamics through its impact on indeterminacies. Thus I am ultimately providing an argument of how technology may be important not through technology shocks, but because it establishes a technological regime for the economy that either enables or inhibits expectations to play a role.

This result has a number of interesting implications. First, the argument highlights the need to properly account for structural change, not just in empirical methods, but also in theoretical models that often form the basis for the emphasis in these methods. Empirical researchers have certainly attempted to control for the slower-moving forces that change state of the system through structural change, and in fact recent work by Comin et al. (2009) and Fernald (2007) shows that correctly controlling for the long (yet stationary) cycles in hours-worked that appear in the data is critical to obtaining an unbiased assessment of the response of hours-worked to neutral technology shocks. Yet the argument I am making suggests that obtaining a full account of the impact on technology and aggregate fluctuations may go beyond that just associated with identifying shocks that have a long-run impact on productivity. In particular, in my model structural change from changing growth regimes allows for the influence of belief shocks, thereby not only altering the conditional response of variables to other shocks, but also changing the state space of the shocks themselves. Moreover, since in the model long-run growth is deterministic, these belief shocks *do not* alter the permanent long-run level of labour-productivity; they just change the *rate* that the economy realizes these fundamental changes. As such, empirically-identified shocks that are deemed unrelated to technology may in be fact be related to beliefs about the technological regime, masquerading as “normal” temporary disturbances.

Second, the technological regime change that I highlight has the potential to create not just a role for unobserved beliefs to influence dynamics, but also the potential to influence the response to other shocks, such as monetary policy or credit, such that the structural change itself becomes at least as important as the exogenous shock. This idea of structural change in technology runs parallel with the discussions of causality related to the response of regime change in monetary policy and its reaction to exogenous shocks. Cochrane (1994) suggests that the answer to the question “What exogenous shocks account for business cycle fluctuations” has “more limited applications than is usually recognized”, and goes on to propose an example where oil price shocks have a small effect on the economy, yet trigger a severe response of monetary policymakers that produces a recession. Did the oil shock cause the recession? In the context of my model, this property supports the intuitive notion that the response of the economy to a given shock depends on the state of the system - in this case the technological state - and implies that the dynamic effect of a given shock may not necessarily be stable over time, a result that has strong implications for empirical identifications.

Third, while I am modeling endogenous adoption, I am not modeling endogenous growth, and the dependence of the endogenous adoption on the technological state means that adoption and R&D activities will be different during "normal times" versus technological transition. This property thus allows the model to break any stable link between R&D effort and productivity realization, a property held by many endogenous growth models linked to the business cycles, freeing it from the criticism of Jones (1995) of weak evidence between R&D effort and productivity.

Fourth, the model contains the implication that "bad" shocks such as contractionary credit shocks unrelated to technology in the midst of a technological era don't permanently impact the level of output following the recession; they only "delay" the eventual realization of the benefit of the slowly evolving technological frontier. This contrasts with models where the aggregate growth in aggregate ideas that gives rise to technological change is connected to high-frequency business cycle forces, and where recessionary forces therefore reduce the rate of ideas generated, permanently reducing the level of output. Econometric evidence provided by Baxter and King (1991) suggests that recessions don't permanently impact the level of output. Moreover, a corollary of this in my model is that the rate of growth following non-technological recessions *may* increase not just due to increasing the utilization of resources that were under-utilized during the recession, but also because the economy moves further below its technological potential, increasing the upside benefit of closing the technological gap [13]. In this sense, whereas a belief-driven boom would allow the economy to "pull-forward" technological benefits, a contraction would cause it to "push back" technological benefits.

[13] I say "may" because it of course depends upon whether the contraction is embedded in a technological transition.

Finally, since the additional returns to capital that drive indeterminacies may exist only temporarily, it poses implications for empirical methods that seek to evaluate the plausibility of models with indeterminacies by determining whether industry data exhibit the degree of returns and scale and externalities used by the sunspot literature. While a particular growth regime may in the aggregate last upwards of 10-15 years such as in the 1990's, in a given industry the effects may be more concentrated in time. Furthermore, the returns to scale may not be as evident in the variation of output as they are in eventual productivity increases resulting from purchases of "new era" capital. This proposition is particularly interesting in light of the empirical evidence found by Basu and Fernald (2008) between industry capital use and eventual productivity increases. As such, the model underscores the importance of controlling for structural change related to capital transitions in industry-level regressions seeking to determine plausible degrees of returns to scale.

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# Conclusion

Economists and non-economists alike have long held that the beliefs of economic agents play a role in driving the business cycle. While the rational-expectations revolution of the 1970's shifted the emphasis away from the role of beliefs as a driver of business cycles in mainstream macroeconomics, the emergence of the "sunspot" and "news-shock" approaches in the 1990's and 2000's reinvigorated this research area within the macroeconomic mainstream. In this thesis I have used both of these developments as a means for exploring the role of changes in expectation in important macroeconomic events.

In the first chapter, I show how a model augmented with an additional production input call knowledge capital acquired through learning-by-doing can generate a boom in both macroeconomic quantities as well as asset prices in response to news that total factor productivity will rise in the future. This alternative production structure offers an attractive modeling mechanism for achieving results that are consistent with the empirical news identifications in the sense that the same mechanism that provides for comovement in quantities also causes asset prices to rise. Moreover, the mechanism offers intuitive appeal in the sense that it implies that the firms in the economy must "do something" to prepare and acquire knowledge to allow them to respond optimally to the expected TFP. In the context of a period of rapid technological change such as the 1990's in particular, this is especially appealing, to the extent that it alludes to the notion that a period of significant technological transformation such as the 1990's requires the economy to transform itself to accommodate the technological change. Through the lens of this model, the information firms received about future changes in information technology compelled them to undergo a period of rapid re-organization and knowledge acquisition, the pursuit of which led to a rapid boom in macroeconomic quantities in advance of realizing the technological gains.

In the second chapter, I show how extending the news-based approach to examining fundamentals in the financial sector can help shed some light on the role that unfulfilled expectations may have played in the boom-bust of 2003-2008. Using a financial accelerator modeling structure, I examine the potential for financial innovation to impact the expected cost of bankruptcy associated with originating loans. I then use the news-based framework to illustrate how boom-bust cycles in macroeconomic and financial quantities and prices could result if agents had high expectations of the innovations that were subsequently crushed. In contrast to many existing approaches in the DSGE literature that rely on contemporaneous shocks such as a decrease in the price of housing or a decrease in the quality of capital, this result is consistent with the view that seeds of the crisis were sewed well before 2007/2008 in the boom years of high-expectation about financial innovation.

Finally, in the third chapter, I show how sudden shocks to expectations themselves can trigger a change in future fundamentals, in contrast to the news-based approach whereby expectations change in response to signals about changes in future fundamentals. I show that in response to a positive sunspot expectational shock, aggregate macroeconomic quantities boom immediately, and precede a gradual and permanent rise in productivity. In doing so, I provide an alternative interpretation of the empirical news-based results that identifies expectational booms that precede growth in TFP. In this context, a positive shock to expectations represents a period of “optimism”, and in this regard I am fundamentally exploring how enthusiasm and optimism about the benefits of a new technology can end up shaping those benefits.