BEHAVIOUR OF REINFORCED CONCRETE JOINTS

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by

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A Thesis

Submitted to the Faculty of Graduate Studies

in Partial Fulfillment of the Requirements

for the Degree

Doctor of Philosophy

McMaster University

April, 1978

To My Dear Wife SANAA

whose

Patience, Understanding, and Assistance

are deeply appreciated.

DOCTOR OF PHILOSOPHY (1978) McMASTER UNIVERSITY

(Civil Engineering) Hamilton, Ontario

TITLE: Behaviour of Reinforced Concrete Joints

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NUMBER OF PAGES: xvii, 308

ACKNOWLEDGEMENTS

I wish to express my sincere gratitude to Dr. R.G. Drysdale for his guidance and interest during the course of this study. It has been a privilege and a pleasure to work under his supervision.

I am also indebted to the members of my supervisory committee, Dr. A.C. Heidebrecht, Dr. J. Morrison, Dr. G.AE. Oravas, and Dr. H. Robinson for their valuable suggestions.

I would like to take this opportunity to thank McMaster University, and the National Research Council of Canada for financial support, and Dr. J.J. Emery for lending his computer subroutines.

Last but not the least, I must mention my deepest appreciation to my parents, without whose help and encouragement this thesis would have been difficult to complete.

ABSTRACT

No rigorous explanation exists for the behaviour of reinforced concrete joints. The lack of understanding the complex interaction may account for continued use of inadequate joint details particularly for knee joints subjected to opening moments. The object of this research was to provide a better understanding of joint behaviour through a combined experimental and analytical investigation.

In the experimental program, 6 specimens with different types of joint details were tested under opening moment. These details reflected recent recommendations by other investigators and some adaptations of their ideas. The results served as evidence of the ability to produce effective and simple details as well as serving as a basis for evaluating the reliability of the analytical model.

A method of analysis has been developed using the finite element method to model the behaviour of reinforced concrete including the interactions between concrete and steel in the form of bond and dowel forces. One of the principle features of this method of analysis is the built in ability to trace the propagation, location and orientation of cracking. This feature was tested by comparing predicted behaviour of beams failing in shear and flexure with test results. Very good agreement with the well documented phenomena of various

modes of cracking and failure provided evidence of the validity of the analytical method.

The analyses of the joints reproduced cracking patterns and sequences as well as capacities which are quite close to the test results. The analytical information helped to identify the primary causes of failure or premature failure and thereby provided a more rational basis for suggesting alternate joint details which will behave much better than inadequate details which are currently recommended.

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CHAPTER 1

INTRODUCTION

1.1 Foreword

This thesis deals with a field of concrete research on which relatively little previous work has been done. The topic of this investigation is the behaviour of the joint between reinforced concrete beams and columns or between other intersecting members.

The advantages of modern design theories including the ultimate strength design method may be negated to a certain extent by the apparent lack of information on joint behaviour and joint detailing. In fact, it has been shown (50, 58, 74, 91, 97) that in many cases the capacity of members cannot be utilized because of inadequate capacity of joints. The lack of adequate information about the interaction of beams and columns at their intersections is reflected by the omission of this topic in textbooks (96) and by the rather noncommittal recommendations contained in Section 7.11 of ACI 318-71, "Building Code Requirements for Reinforced Concrete" (69).

Recently ACI-ASCE Committee 352 published "Recommendations for Design of Beam-Column Joints in Monolithic Reinforced Concrete Structures" (104). However, these recommendations are restricted to joints other than knee joints which is the particular subject of this dissertation. Considering the possible range

of variables, the inapplicability of the traditional analytical methods, and the complexity of behaviour, it is not surprising that no comprehensive evaluation of this problem is available.

Many parameters may influence the behaviour of reinforced concrete joints. The effects of altering some of these parameters are not known. Some important parameters to be considered are:

(1) details of the steel reinforcing bars used in the joint region,

(2) properties of the materials used,

(3) physical dimensions of the intersecting members,

(4) steel percentage used for both the intersecting members and the joint region,

(5) anchorage length provided for reinforcing bars in the joint region,

(6) number and size of stirrups used, if any, and their shape.

The large number of parameters involved and the inherent complexity of a theoretical solution have so far precluded a thorough study of the effects of the above parameters.

1.2 Definitions

The definitions listed below are provided to avoid the need for repeating lengthy descriptions or explanations in the text.

Diagonal Bar is a reinforcing bar which is placed diagonally as close to the inside corner of the joint as the cover will allow (see Fig. 1.1). The cross-sectional area of this type of reinforcement will be denoted as A_{sd}.

Diagonal Stirrups are stirrups placed diagonally to join the inside and outside corners of the joint, as shown in Fig. 1.1. Their cross-sectional area is denoted as A_{st}.

Joint, unless otherwise stated, the word, "Joint" will be used in place of "knee or corner beam-column connection subjected to positive movement which tends to open the angle between the intersecting members".

Joint Efficiency is the ratio between an experimentally or analytically obtained capacity and the theoretical flexural capacity of the larger of the two intersecting members.

Joint Element is an element which joins steel and concrete elements at their interface in the finite element analysis and which represents bond-slip and dowel action. The same joint element may be called a bond and/or a dowel element (linkage).

Positive or Negative Moments are bending moments which tends to open or close the angle at the joint respectively.

Theoretical Capacity is the flexural capacity calculated using statics and the Whitney stress block (96) representation for concrete in compression.

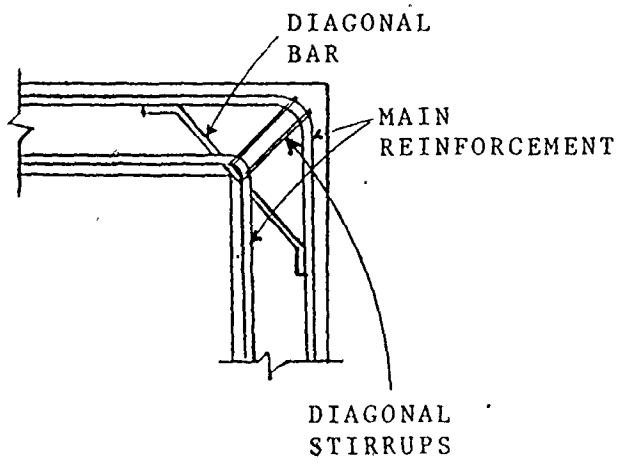


Fig. 1.1 DEFINITION OF REINFORCING STEEL TYPES IN JOINTS

1.3 Purpose and Scope

The primary objective of this investigation was to contribute to the knowledge of the behaviour of reinforced concrete joints. In a more restrictive sense the major area of investigation was concerned with the behaviour of joints subjected to positive moment. It was decided that both experimental and analytical studies would be undertaken. The experimental program would provide personal experience with testing joints and would help to gain a physical appreciation of their behaviour as well as providing well documented data for comparison with results from the analytical study. Because of the range of the range of the parameters involved, development of an analytical method was necessary in order to compliment the experimental program and provide a means for extending the study in the future.

1.3.1 Scope of the Experimental Program

An extensive experimental program was not within the scope of this investigation. Other investigators (74,97) who have done extensive testing with relatively little theoretical interpretation have not been able to provide a thorough explanation of the behaviour of joints. Therefore, the major emphasis of the experimental part of this study was to provide reliable data which could be used to confirm the adequacy of the analytical model. However, an important consideration was also to try to develop a simple and more efficient detail which could improve current practice.

The test specimen, loading conditions, method of testing and range of variables were chosen so that the test results would be

as informative in themselves as possible, and the number of tests would be minimized while still providing a reasonable basis for comparison with the analytical results. It was preferred that the specimen dimensions be as large as was practical in order to reduce doubts concerning size effects when relating test values to actual structures. In addition, fabrication tolerances would be less significant for larger overall dimensions.

1.3.2 Scope of the Analytical Investigation

The main objectives of the analytical investigation was to develop an analytical model to predict the behaviour of reinforced concrete joints. (However, it was anticipated that this would be a useful tool for studying other structural elements.) This analytical model should take into account the important aspects which affect the behaviour of reinforced concrete structures.

A review of the major factors involved in the behaviour of reinforced concrete will help to identify difficulties associated with developing such a method of analysis. Ngo and Scordelis (39) indicated that some of these difficulties are as follows:

1. Non-homogeneity of concrete and its non-linear stress-strain relationship.
2. Intricate interaction of steel and concrete relative to bond-slip characteristics and dowel action.
3. Time-dependent deformation of concrete.
4. Cracking and crack propagation with accompanying changes in stiffness.
5. Aggregate interlock and dowel effects varying with crack propagation.
6. Limited knowledge of the response of concrete

subjected to biaxial and triaxial states of stress.

Earlier studies of these aspects and the experience of others (68,99), indicated that conventional methods of analysis would not adequately accommodate inclusion of these aspects. That is, use of plane section behaviour with assumed elastic or elastic-plastic materials would not be acceptable for developing an envisioned analytical procedure which could more adequately incorporate the influences of the above effects. Thus, it was concluded that the nature of reinforced concrete, coupled with the complexities of the joint behaviour itself, required adaptation and development of more sophisticated analytical methods to facilitate a more detailed and accurate investigation.

The development of high speed digital computers and the formulation of the finite element method permit the development of an efficient analytical model which can result in a more realistic evaluation of the behaviour of reinforced concrete structures.

It was intended that the development of this analytical tool would contribute to the knowledge of joints and that this tool would also help in the development of rules for the design of efficient joints. If was though that particular emphasis should be placed on the modelling of cracking and crack propagation as a means of evaluating effects of different joint details on the behaviour of joints.

1.4 Outline and Resume of the Thesis

To help the reader follow the organization of the material in this thesis, the following outline is presented.

Chapter 2 contains a review of selected literature covering the following:

1. Testing and detailing of reinforced concrete joints.
2. Development and use of the finite element method.
3. Load-deformation characteristics for materials used in connection with the analytical method.

Chapter 3 contains a description of the test specimen and the test set-up. Also the basic material properties which are required in the analysis are reported and discussed. The experimental results of the six joints tested are reported in Chapter 4 and general observations on the behaviour and relative merits of different joint details are made. In Chapter 5, the description of the method of analysis and the development of the computer program are given. To provide an alternate method for evaluating the reliability of the computer program, analytical results for simply supported reinforced concrete beams are presented and compared with test results in Chapter 6. An evaluation of these results including the effects of variations of different parameters were discussed because of the interesting nature of the findings. The analytical results for joints are presented and compared with the experimental data in Chapter 7. In all these chapters pertinent observations and conclusions are made as they become apparent. Finally in Chapter 8, a brief summary of the results of the experimental and analytical work and the final conclusions are presented along with some recommendations for future research.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

As discussed in Chapter 1, study of the problem under consideration requires a basic understanding of behaviour of reinforced concrete members and the properties of the materials. These aspects will be discussed in the chapters where they are used. This chapter will be devoted to a review of the pertinent literature dealing with the particular problems focused on in this thesis.

The literature review is organized into three major groupings, each dealing with information related to specific aspects of this investigation. These are:

a. Details of tests on Beam-Column joints.

b. Theoretical Modelling of reinforced concrete structures using the finite element method.

c. Description of Force-Displacement relationships for use in the finite element analysis of reinforced concrete.

Other literature which is relevant but more commonly used is not reviewed here but is referenced throughout the text. Some of the references reported in the following sections were

not directly available to the author. Summaries or translations of these were found in other publications. Reference to the original publication is included in the review for the sake of completeness. These references are identified with square brackets in the Bibliography and the publications in which they were summarized are shown at the end of each reference.

2.2 Current and Previous Joint Detailing Practice:

In this section a selection of the most frequently used reinforcing details for corner joints subjected to positive bending are reported. (1, 2, 3, 4, 6, 19, 20, 52, 63, 74, 91, 97, 100, 103). Some of these details as well as some others were investigated by the author.

In 1964, Robinson (20) suggested the reinforcements detail shown in Fig. 2.1 for piers, abutments for bridges.

Boughton proposed the detail shown in Fig. 2.2 for square water tanks in the design aid, "Reinforced Concrete Detailers' Manual", (52) London 1969. He also suggests the detail in Fig. 2.3, for corners in frames subjected to positive moment.

For corners subjected to positive moment the report, "Standard Method of Detailing Reinforced Concrete", by the Concrete Society in conjunction with the Institution of Structural Engineers in London, 1970 (63), provides the following recommendations for corner reinforcing details

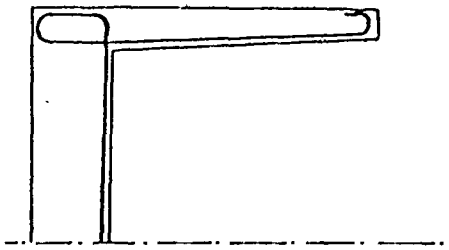


Fig 2.1 DETAIL PROPOSED BY ROBINSON (20)

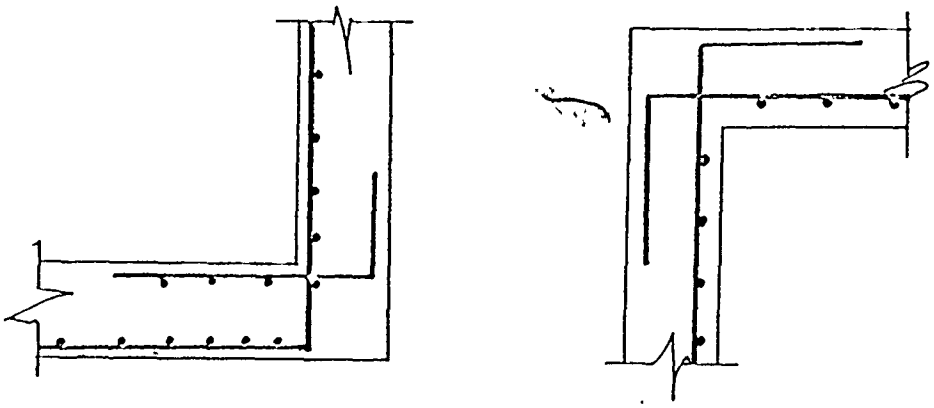


Fig. 2.2 DETAIL PROPOSED BY BOUGHTON (52)
FOR TANKS.

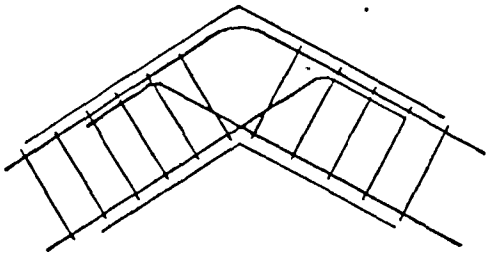


Fig. 2.3 DETAIL PROPOSED BY BOUGHTON (52)
FOR FRAMES.

as indicated in Fig. 2.4: "Looped bars as shown in figure (a) should be avoided. Better arrangements consist of two straight bars with right-angle bends sufficient to provide proper anchorage (figure (b)) or two hairpin bars (figure (c))".

The simple corner detail shown in Fig. 2.5 was recommended in 1974 by ACI Committee 315, "Manual of Standard Practice for Detailing Reinforced Concrete Structures" (100).

In 1976, the ACI-ASCE Joint Committee 352 report "Recommendations for Design of Beam-Column Joints in Monolithic Reinforced Concrete Structures" (104) provided tentative recommendations for proportioning and detailing of beam-column joints in monolithic reinforced concrete structures. Several limitations were imposed on the recommendations. The following paragraph, probably represents the most noteworthy restriction relative to this research investigation: "It should be noted that shear provisions cited above may not be applicable to joints in which only two members meet, such as knee joints or where the column above the joint is considerably smaller than the one below. All data cited above were obtained from tests of joints with columns above and below the core. Special problems often arise with corner or knee joints. Knee and corner joints require special reinforcement to prevent failures produced by bursting or spalling stresses in the joint. Results of tests of corner and knee joints are reported in the literature, and methods for reinforcing such joints are presented therein."

Looped bars as shown in figure (a) should be avoided. Better arrangements consist of two straight bars with right-angle bends sufficient to provide proper anchorage (figure (b)) or two hairpin bars (figure (c)).

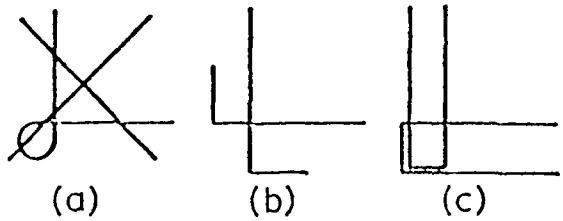
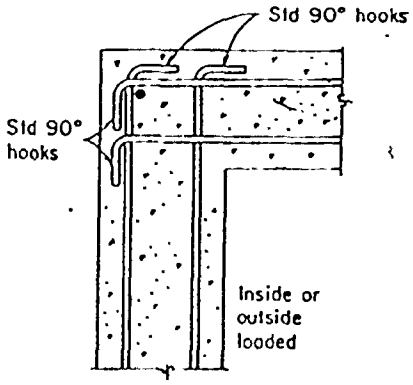


Fig. 2.4. DETAILS SUGGESTED BY THE INSTITUTION OF STRUCTURAL ENGINEERS IN LONDON.
(Reproduced from Reference 63)



Typical corner detail

Fig. 2.5. DETAIL RECOMMENDED BY ACI 315-74.
(Reproduced from Reference 100)

The above certainly indicates the difficulty in understanding and detailing corner joints. In addition it should be mentioned here that the above statement that methods for reinforcing are available should be qualified to indicate that not all of the suggestions are adequate.

2.3 Experimental Research on Joints.

In this section both early (pre 1945) and more recent research on corner joints are summarized. Special attention is given to joints subjected to positive bending moment.

2.3.1 Joints subjected to Negative Bending Moment:

In 1934 (1) and more recently (4, 19, 74, 91, 97), it has been shown that when a corner is subjected to a negative bending moment, detailing presents no major difficulties. It was shown (91, 97) that when the tension reinforcement is detailed, as is commonly done, so that it passes around the outside of the corner joint, no serious loss of efficiency should be anticipated. One reason for premature failures in corner joints subjected to negative bending moment is the possibility of a splitting failure under the bend of the reinforcing bar (Fig. 1.I). Experimental investigations (1, 2, 3) showed that the ability to develop higher steel stress prior to splitting failure in the concrete increased with the radius of curvature of the bend, but not in direct proportion. However, it was shown (65) that no such splitting failure should be expected if the current codes' (63, 69)

recommendations regulating the bend radi for reinforcing bars are observed.

This brief review was included to indicate that there are no major problems with these joints under negative moments.

2.3.2 Early Research on Corner Joints Subjected to Positive Bending Moment:

Pioneer studies of corners subjected to positive bending moment dealt mainly with looped patterns of reinforcement. Use of loops was regarded as the most obvious way of placing the reinforcement in a corner. Wästlund (1, 2, 3) systematically investigated the capacity of concrete to resist bearing stresses caused by such reinforcing loops. This large number of tests demonstrated that understanding the behaviour of these corners was not a simple problem. In order to try to determine the possible influences of the various factors, a number of test series were carried out in which the concrete quality, the radius of curvature of the loop, the width of the concrete specimen perpendicular to the plane of the loop and the model scale was systematically varied. Fig. 2.6 illustrates the type of angle beam specimen which was used in this investigation. Concrete having a cube strength of 190 kg/cm^2 (2700 psi) and plain bars were used in most of the tests. Steel stresses at failure were reported to be from 2,000 to 3,000 kg/cm^2 (28.5 to 42.5 ksi). These values are appreciably below the strengths of the presently used steel and concrete. Another essential difference is that the currently used deformed bars have different bond characteristics than the plain bars which were used for

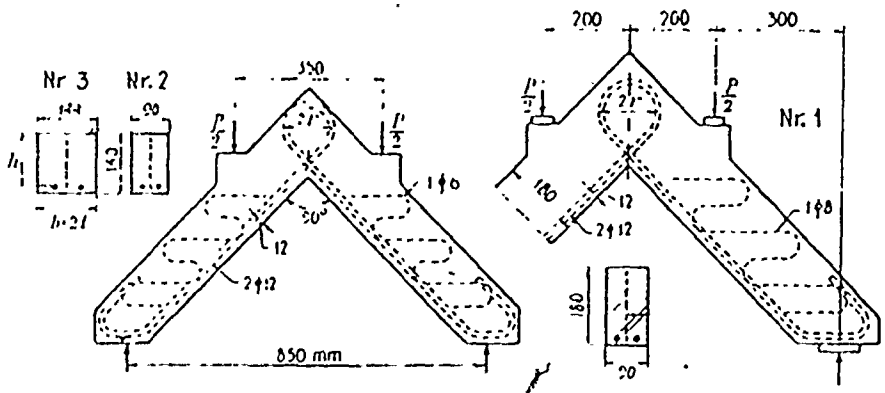


Fig. 2-6. WÄSTLUND TEST SET-UP[1]
(Reproduced from Reference 97)

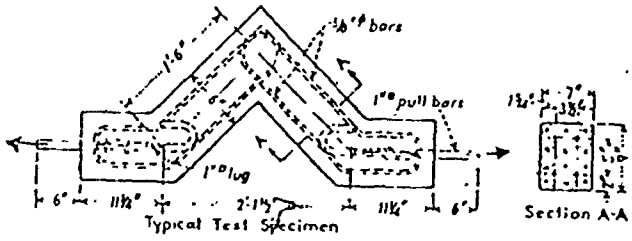


Fig. 2-7 . TEST ARRANGEMENT BY GUMENSKY[4]
(Reproduced from Reference 97)

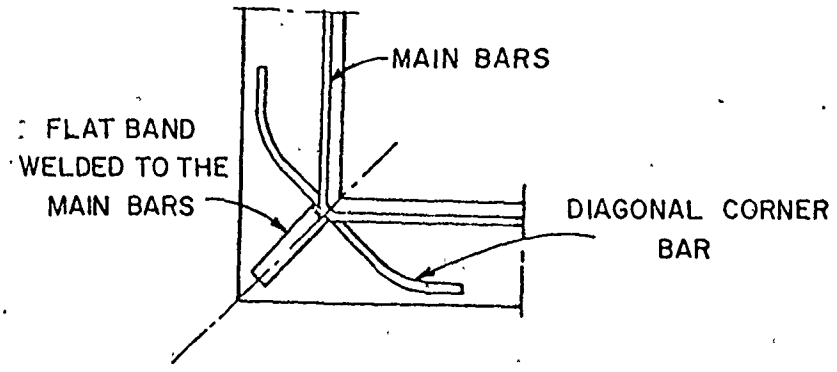


Fig. 2-8. DETAIL SUGGESTED BY POSEY AND KOFOID.
(Reproduced from Reference 6)

these tests. As a result of Wästlund's experimental work, some empirical equations were derived to obtain the steel stresses on failure of the concrete. However, the applicability of these equations to current conditions is limited.

Gumensky, (4) in the U.S.A. carried out tests in 1939 on concrete corners in tension. He used the simple test specimen and loading arrangement sketched in Fig. 2.7. Five reinforcing patterns were compared. It was found advantageous to use reinforcing loops in combination with a cross bar in a fillet. In 1943, Posey and Kofoid (6), carried out tests on concrete corners with a test method similar to that of Gumensky (4). They developed the corner detail reproduced in Fig. 2.8. It consisted of main bars which were bent very tightly around the inner corner and which were supplemented by diagonal corner bars, and flat bands welded to the main reinforcement. They found this detail to be superior to using loops. Although the detail looks different the same general idea was developed in the 1970's by Mayfield, et. al. (74, 91).

The pioneering work reported in this section was of a great value. The problems and difficulties in detailing joints subjected to positive moment were pointed out. However, none of the details developed 100% efficiency even with the low concrete and steel strengths used at the time of these investigations.

2.3.3 Recent Research on Corner Joints Subjected to Positive Bending Moment:

Relatively few researchers have reported studies on the particular subject of this thesis, reinforced concrete joints. The main purpose here is to gather the available information on corner joints subjected to positive moment. This information is included to provide a broader background for understanding of the problem. In this section an overall review of the subject is presented. Then special concentration on corner joints subjected to positive bending moment follows to assist in understanding how these results influenced the course of this study.

Although little information on the actual behaviour of joints in buildings was available, Hanson and Connor (41) in 1967 commented that, "Where unsatisfactory performance during earthquake shocks is observed, it can usually be attributed to quality of materials or construction, or to poor reinforcement details in the structural joints." They considered that unsatisfactory details of joint reinforcement were not limited to seismic design, but that good design practice requires that joints be designed to be as strong as the adjoining members.

In 1968, Beaufait and Williams (50), in a preliminary investigation on the behaviour of reinforced concrete frames, also considered reinforcing details which were later (65) described as "classic" detailing errors. Beaufait and Williams tested a series of pin-supported portal frames subjected to

either a repeated, reversible, horizontal sway force, or a single horizontal sway force, at the level of the beam. The objectives of their preliminary project were to study the effects of the loading on the ultimate strength of the concrete frame, and to investigate the influence of different patterns of reinforcement at the corner joints. The three corner details tested are shown in Fig. 2.9. Their results showed that the distribution of the reinforcement at the joint influenced the response of the structure. Detail (a) resulted in a rather sudden collapse of the frame due to the lack of continuity of the reinforcement. Detail (b) exhibited more ductility than (a). Detail (c) had the highest load capacity of those frames which were loaded to failure by a single horizontal force. It had less than 39% efficiency and failed due to the tendency of the tension bars to straighten out. The details tested were not examples of good design. However, the tests clearly indicated what kind of corner details should be avoided. Beaufait and Williams concluded that "the placement of reinforcement at the joints is critical and the manner in which continuity of the reinforcement is developed at the joints can and does have a very definite influence on the behaviour of the structure."

Swann (58, 66), in 1969 and 1970, found that some detailing arrangements in current practice were not as strong as other, less conventional joint details. The object of his preliminary investigation was to obtain data on details in common use and to develop more efficient ones.

Specimens representing the right-angle bend of a

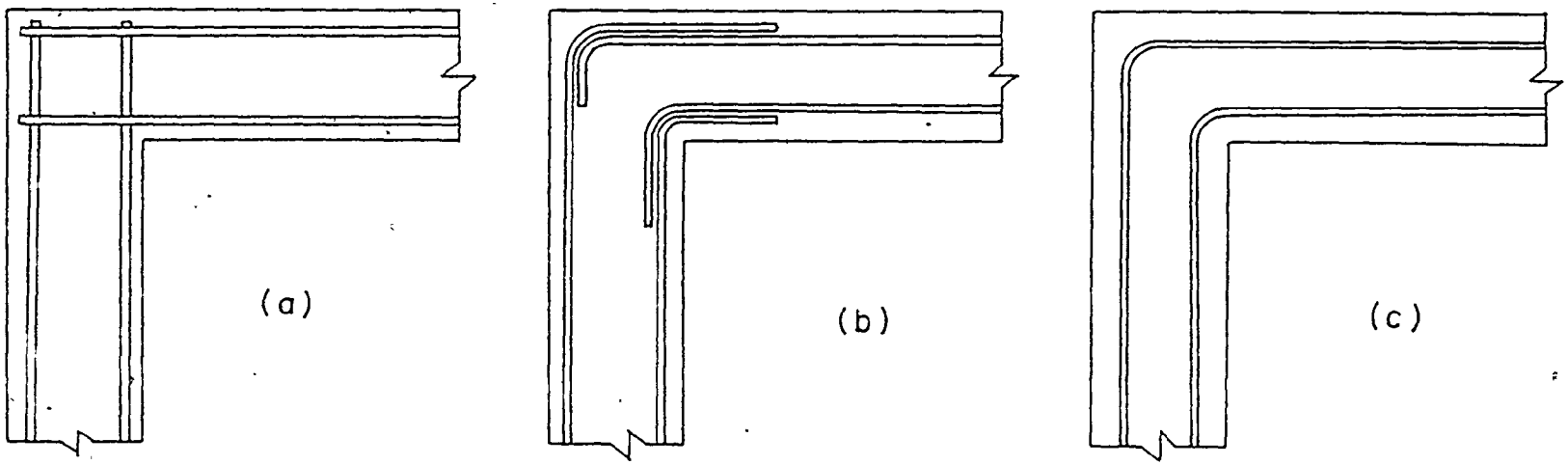


Fig.2.9 DETAILS OF CORNER CONNECTION-REINFORCEMENT TESTED BY BEAUFAIT AND WILLIAMS[50]

portal frame were tested with both positive and negative bending moment. The detailing arrangements shown in Fig. 2.10, range, as Swann indicated from the simplest possible arrangements, Details (1) and (2), through details having varying degrees of popularity, Details (3), (4), (5), (6) and (7), to innovations specifically designed to resist the anticipated forces such as Details (8), (9) and (10).

For positive bending moments, Swann found that Details (9) and (10) performed significantly better than the others. In cases where the bending moment tended to close the right angle, bearing failure of the concrete occurred at the bends of the main reinforcement. Swann's specimens had at least 3% tension reinforcement ratios. This may in part explain why some of the low joint efficiencies were obtained. The work by Swann pointed out deficiencies in detailing practice but did not result in any new detailing proposals.

In 1970 Mayfield et al. (74, 91) started an extensive experimental program at the University of Nottingham, England. Devoted mainly to investigating the effects of corner detailing on lightweight concrete, the program was carried out in two stages. In the first stage 48 reinforced lightweight concrete corners were tested. The effects of 12 types of reinforcement details on ultimate strength, stiffness and cracking were studied. The test results were reported jointly by Mayfield, Kong, Bennison and Davies (74). The test arrangement and dimensions of the test specimen as well as the 12 details

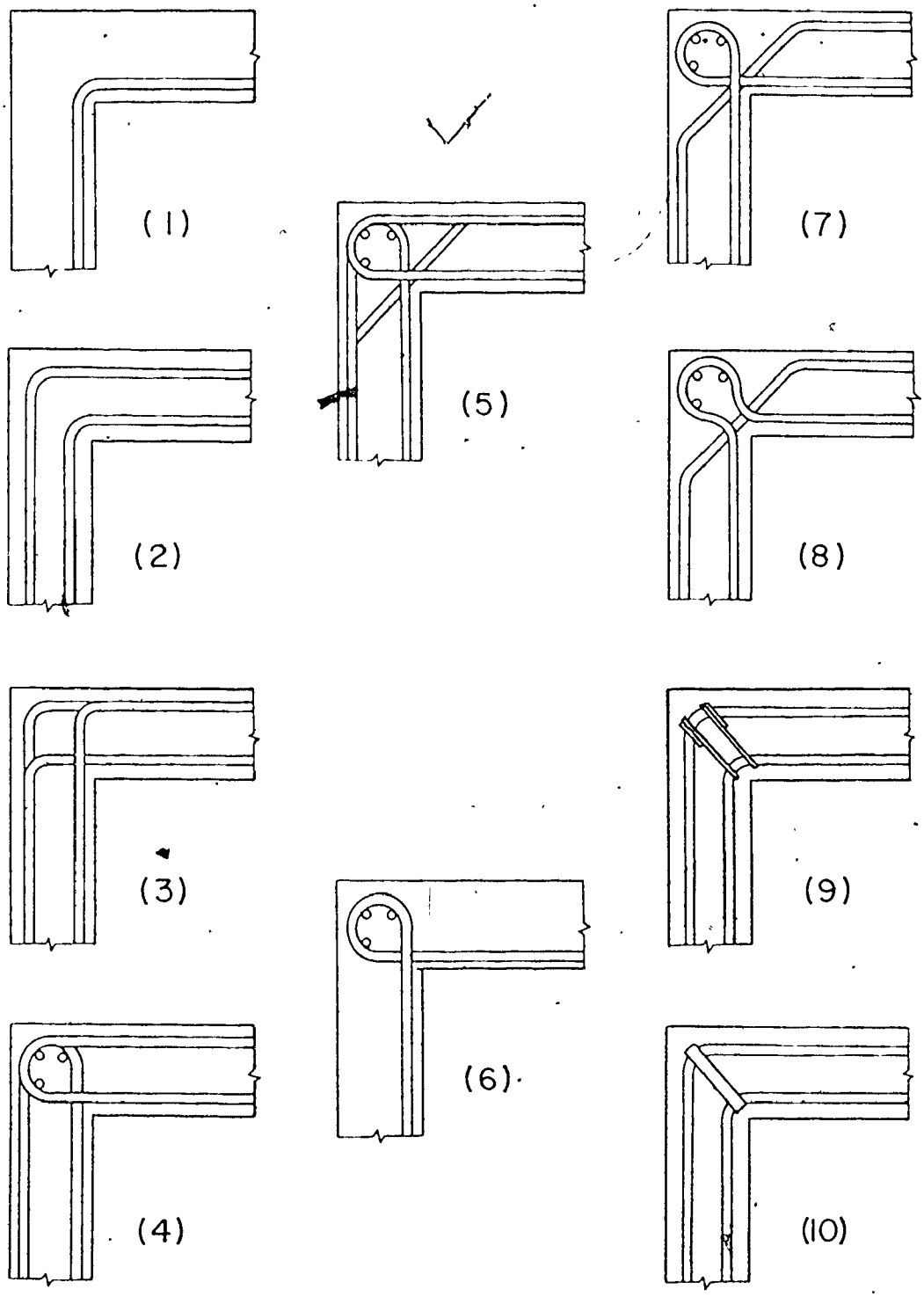


Fig. 2.10 DETAILS OF CORNER CONNECTION REINFORCEMENT TESTED BY SWANN[58]

of corner reinforcement are reproduced in Figs. 2.11 and 2.12. The details were tested for both negative and positive bending moments. They concluded that no major difficulties when a negative moment was applied should be expected. However, when positive moment was applied, the best detail was found to be Detail (6) shown in Fig. 2.12. Its efficiency was only 75% of the capacity of the intersecting members. It was also concluded that some corner details (1 & 2) commonly used in practice were not as efficient as other less conventional (Detail 6), but simpler details. Tables, 2.1, 2.2 and 2.3, reproduced from reference (74), summarize the details of reinforcement sizes and concrete strengths, test results for negative and for positive bending moments respectively. These results constitute a major source of data and for positive moment indicate efficiencies ranging from 17% to 75%.

Having identified the problems involved with corners subjected to positive bending moment a second stage was reported by the same team (91) in 1972. Fifty-four reinforced lightweight concrete corners subjected to positive bending moment were tested. The effects of 28 different types of reinforcing details (Fig. 2.13) on ultimate flexural strength, stiffness, and cracking were studied.

Tables 2.4 and 2.5 were reproduced from reference (91) to show the details of reinforcement sizes and concrete strengths, and the test results respectively. It was found that the use of two sets of mutually perpendicular diagonal reinforcement was a promising method for developing flexural strength, reducing crack widths and producing reasonably

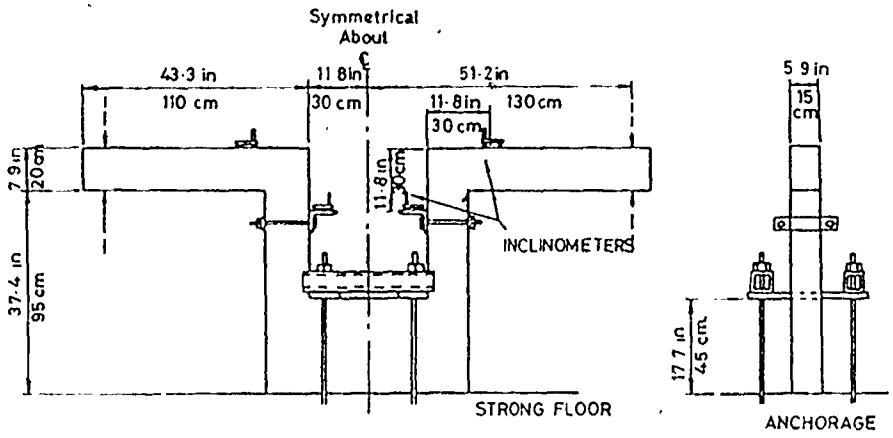


Fig.2.11. Test arrangement and dimensions of test specimens

Reproduced from Reference 74

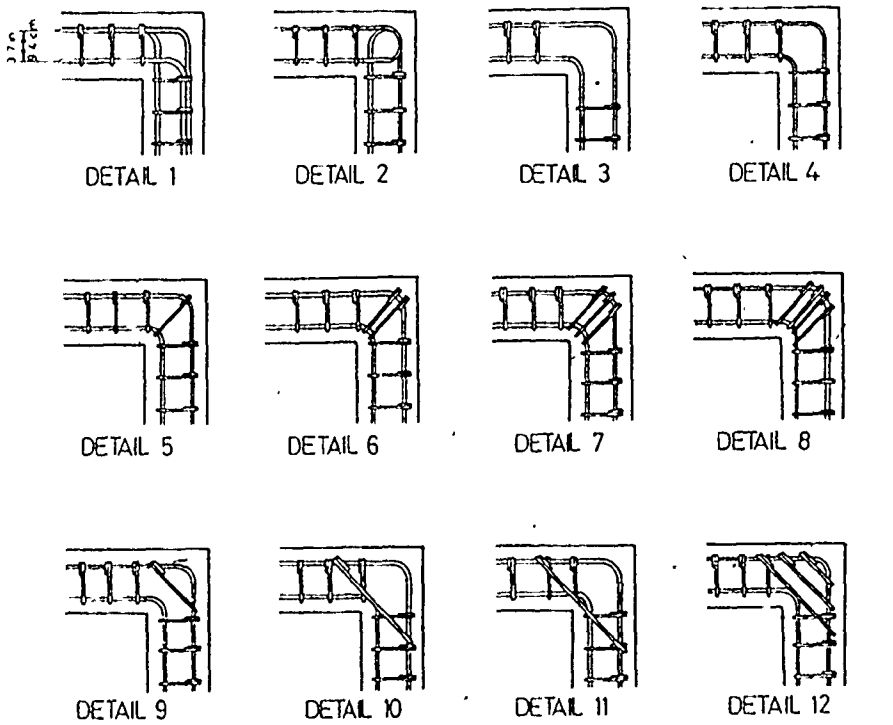


Fig.2.12. Details of corner reinforcement (The longitudinal steel extends to the end of the flexural member in all details, and is terminated by a standard hook)

Reproduced from Reference 74

Table 2.1 (Reproduced from Reference 74)

DETAILS OF REINFORCEMENT SIZES AND CONCRETE STRENGTHS

Detail Reference No	Corner Reference No*	Diameters of: Main bars in. (cm)	Diagonal stirrups, in. (cm)	Other stirrups, in. (cm)	Concrete Cylinder Strength, psi (kgf/cm²)
1	1-1 , 1-2	0.47 (1.2)	None	0.24 (0.6)	3000 (210)
	1-3 , 1-4	0.47 (1.2)	None	0.24 (0.6)	
2	2-1 ; 2-2	0.47 (1.2)	None	0.24 (0.6)	2700 (190)
	2-3 ; 2-4	0.47 (1.2)	None	0.24 (0.6)	
3	3-1 ; 3-2	0.47 (1.2)	None	0.24 (0.6)	3400 (240)
4	4A-1 ;4A-2	0.47 (1.2)	None	0.24 (0.6)	3100 (220)
	4B	0.47 (1.2)	None	0.32 (0.8)	2400 (170)
	4C	0.47 (1.2)	None	0.47 (1.2)	
5	5A-1 ,5A-2	0.47 (1.2)	0.47 (1.2)	0.24 (0.6)	3500 (245)
	5A-3 ,5A-4	0.47 (1.2)	0.47 (1.2)	0.24 (0.6)	2900 (205)
	5B-1 ,5B-2	0.47 (1.2)	0.24 (0.6)	0.24 (0.6)	2850 (200)
	5B-3 ,5B-4	0.47 (1.2)	0.24 (0.6)	0.24 (0.6)	3300 (230)
	5C-1†,5C-2†	0.47 (1.2)	0.47 (1.2)	0.24 (0.6)	2800 (195)
	5D-1†,5D-2†	0.47 (1.2)	0.24 (0.6)	0.24 (0.6)	2500 (175)
	5E-1 ,5E-2	0.63 (1.6)	0.24 (0.6)	0.24 (0.6)	3500 (245)
	5F-1 ,5F-2	0.32 (0.8)	0.24 (0.6)	0.24 (0.6)	3200 (225)
6	6A-1 ;6A-2	0.47 (1.2)	0.24 (0.6)	0.24 (0.6)	2900 (205)
	6B-1 ,6B-2	0.47 (1.2)	0.32 (0.8)	0.24 (0.6)	2950 (210)
	6C-1 ,6C-2	0.47 (1.2)	0.47 (1.2)	0.24 (0.6)	2850 (200)
7	7-1 , 7-2	0.47 (1.2)	0.24 (0.6)	0.24 (0.6)	2500 (185)
8	8-1 , 8-2	0.47 (1.2)	0.24 (0.6)	0.24 (0.6)	3000 (210)
9	9-1 , 9-2	0.47 (1.2)	0.24 (0.6)	0.24 (0.6)	2900 (185)
10	10A-1 ,10A-2	0.47 (1.2)	0.24 (0.6)	0.24 (0.6)	2650 (185)
	10B	0.32 (0.8)	0.24 (0.6)	0.24 (0.6)	2500 (175)
11	11	0.32 (0.8)	0.24 (0.6)	0.24 (0.6)	2500 (175)
12	12-1 ; 12-2	0.47 (1.2)	0.24 (0.6)	0.24 (0.6)	2800 (200)

*Notation for corner reference number. The figure before the hyphen indicates the type of corner detail (Fig 3), and the figure after the hyphen indicates the specimen number. Thus 3-2 represents Detail 3—Corner Specimen 2. Where different bar sizes were used for the same type of corner detail, distinguishing letters, A, B, C etc. are used.
†Diagonal stirrups in these corners were deliberately misplaced by 15 deg to observe the effect of inaccurate bar placing.

Table 2.2 (Reproduced from Reference 74)

TEST RESULTS (LOAD-CLOSING CORNERS)

Corner Reference No†	M_c'/M_c	At 40 percent M_c: Rotation, radians x 10^{-4}	Crack widths: Average, mm	Maximum, mm	At 80 percent M_c: Rotation, radians x 10^{-4}	Crack widths: Average, mm	Maximum, mm
1-3	1.25	2.1	0.1	0.1	7.1	0.2	0.2
1-4	1.27	1.5	NV*	NV*	4.3	0.2	0.2
2-3	0.84	3.4	0.2	0.2	11.7	0.6	1.1
2-4	1.17	3.2	0.2	0.25	13.8	0.8	1.2
3-3	1.27	2.6	0.2	0.2	9.3	0.4	0.65
4A-3	1.36	2.5	0.2	0.2	8.2	0.4	0.5
5A-3	1.17	3.7	0.1	0.1	10.0	0.2	0.25
5A-4	1.25	2.6	0.1	0.1	8.7	0.2	0.25
5B-3	1.25	3.1	0.1	0.15	8.7	0.3	0.4
5B-4	1.29	4.0	0.1	0.15	9.9	0.3	0.4
5C-2	1.30	2.5	0.15	0.25	7.3	0.4	0.5
5D-2	1.34	2.9	0.1	0.1	7.3	0.25	0.5
5E-2	0.89	4.5	0.2	0.3	13.0	0.45	0.5
5F-2	1.45	2.7	NV*	0.2	7.4	0.4	0.4
12-2	1.22	1.9	0.2	0.15	7.0	0.4	0.5

*NV = none visible.
†Corner notation as in Table 1

Table 2.3 (Reproduced from Reference 74)

TEST RESULTS (LOAD-OPENING CORNERS)

Corner Reference No†	M_c'/M_c	At 20 percent M_c: Rotation, radians x 10^{-4}	Crack widths: Average, mm	Maximum, mm	At 40 percent M_c: Rotation, radians x 10^{-4}	Crack widths: Average, mm	Maximum, mm
1-1	0.19	F*	F	F	F	F	F
1-2	0.19	F	F	F	F	F	F
2-1	0.44	0.8	NV‡	NV	NF‡	NF	NF
2-2	0.43	0.2	NV	NV	6.6	0.2	1.4
3-1	0.16	F	F	F	F	F	F
4A-1	0.20	1.3	0.1	0.2	F	F	F
4B	0.22	2.4	0.2	0.2	F	F	F
4C	0.24	1.8	0.1	0.2	F	F	F
5A-1	0.53	1.3	0.1	0.1	8.1	0.4	0.7
5A-2	0.43	5.1	NV	NV	11.7	0.5	0.8
5B-1	0.41	1.7	0.1	0.2	NF	NF	NF
5B-2	0.46	4.0	0.1	0.1	13.0	0.6	1.0
5C-1	0.48	1.1	0.1	0.1	9.2	0.7	1.0
5D-1	0.43	1.9	0.1	0.2	NF	NF	NF
5D-2	0.25	2.8	0.2	0.4	F	F	F
5F-1	0.65	2.8	NV	NV	4.8	0.2	0.8
6A-1	0.48	0.8	0.1	0.1	1.2	0.8	1.4
6A-2	0.48	0.9	0.1	0.1	7.1	0.5	1.0
6B-1	0.90	0.7	NV	NV	4.0	0.2	0.8
6B-2	0.83	0.7	NV	NV	3.8	0.2	0.4
6C-1	0.60	1.2	0.1	0.1	8.1	0.4	0.8
6C-2	0.75	0.6	NV	NV	3.0	0.2	0.4
7-1	0.58	0.7	NV	NV	4.3	0.3	0.8
7-2	0.60	0.5	NV	NV	2.1	0.1	0.2
8-1	0.30	0.8	0.1	0.1	5.8	0.2	0.6
8-2	0.45	0.8	NV	NV	5.4	0.4	0.8
9-1	0.28	0.7	NV	NV	F	F	F
9-2	0.21	2.2	0.2	0.2	F	F	F
10A-1	0.24	1.7	NV	NV	F	F	F
10A-2	0.22	3.6	NV	NV	F	F	F
10B	0.66	0.2	NV	NV	10.0	NV	NV
11	0.73	0.2	NV	NV	0.5	NV	NV
12-2	0.37	0.5	NV	NV	F	F	F

*F = Failed. ‡NV = None visible. ‡NF = Near failure
†Corner notation as in Table 1

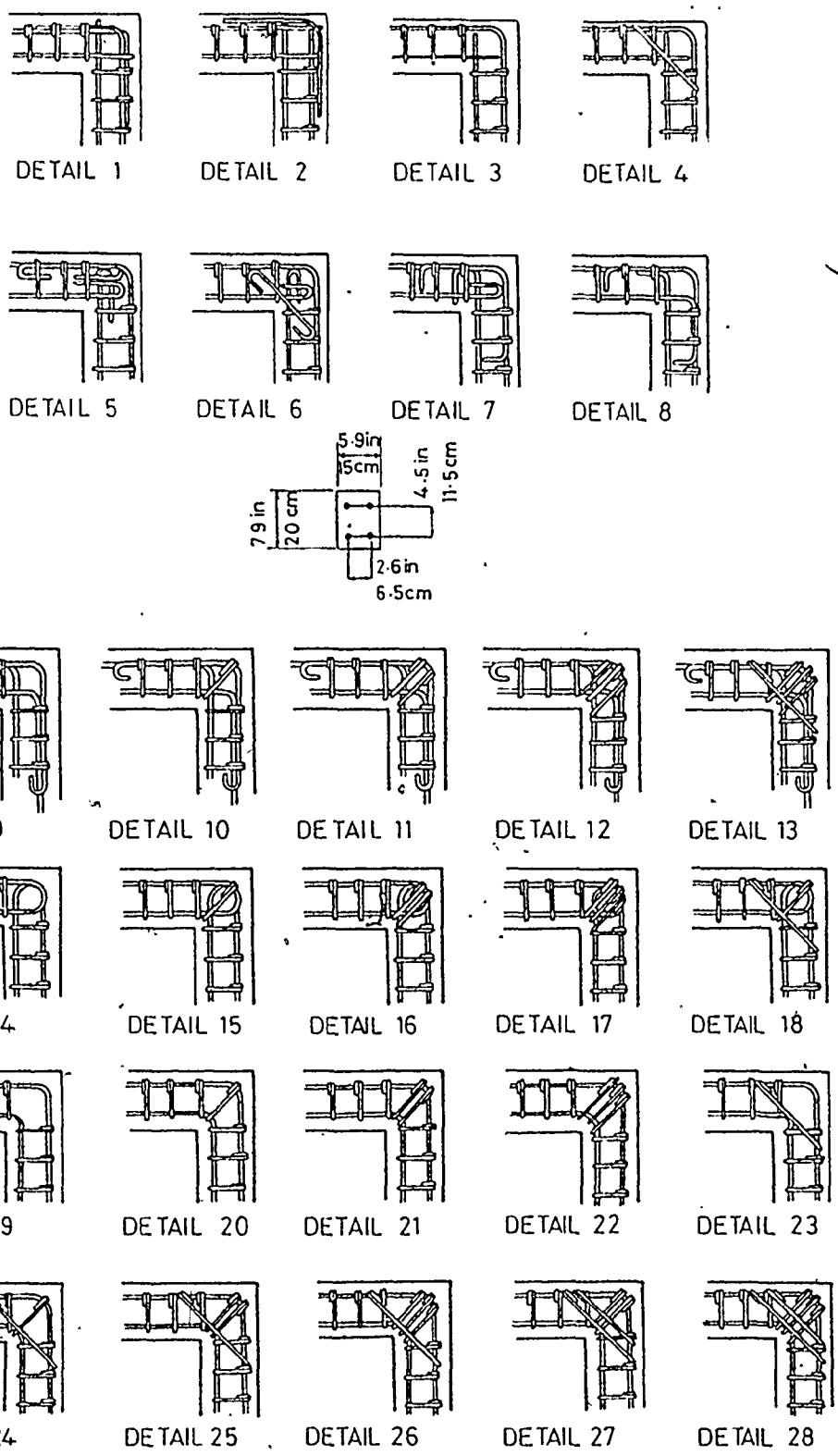


Fig. 2.13. Details of corner reinforcement (main longitudinal bars extend to end of flexural member and are terminated by a standard hook)

Reproduced from Reference 91

ductile behaviour. Of all the details tested, Detail 26 gave the best overall performance. This was the only detail to exceed 100% corner efficiency. It is apparent however, that this detail is only good for low percentages of reinforcement. The efficiency ranged from 99% to 130% for a tension steel percentage of 0.52. When the percentage of tension steel was increased to 1.34, the efficiency dropped dramatically down to 38%.

In 1972, Balint and Taylor (85) in England made the following recommendation: "The provision of a splay adds greatly to the strength of opening flexural corners and should be provided whenever possible". They suggested that the detail reproduced in Fig. 2.14 may be used for tensile steel reinforcement ratios up to 1%. The reinforcement detail consists of hair pins and splay bars. Balint and Taylor (85) and also Nilsson (97) concluded that sufficient corner strength can always be obtained by using a haunch.

In 1973, Nilsson documented in a comprehensive report (97) published by the National Swedish Building Research the results and conclusions of a research program in which he tested 78 corner and joint specimens tested from 1965 to 1973.

The following causes of failure in corner joints were discussed:

1. Diagonal tension failure.

Table 2.4 (Reproduced from Reference 91)

DETAILS OF REINFORCEMENT SIZES AND CONCRETE STRENGTHS

Detail reference no	Corner reference no*	Diameters in (cm) Main bars	Diagonal reinforcement Type (i)	Diagonal reinforcement Type (ii)	Concrete cylinder strength psi (kg/cm²)
1	1-1 , 1-2	0.25(1.0)	None	None	2500 (195)
2	2-1 , 2-2	0.25(1.0)	None	None	2730 (195)
3	3	0.25(1.0)	None	None	2534 (250)
4	4	0.25(1.0)	None	0.77(1.0)	3550 (250)
5	5-1 , 5-2	0.25(1.0)	None	None	3180 (270)
6	6-1 , 6-2	0.25(1.0)	None	0.25(1.0)	2180 (150)
7	7-1 , 7-2	0.25(1.0)	None	None	2700 (223)
8	8-1 , 8-2	0.25(1.0)	None	None	2200 (200)
9	9	0.25(1.0)	None	None	2500 (175)
10	10	0.25(1.0)	0.24(0.6)	None	2580 (175)
11	11	0.25(1.0)	0.24(0.6)	None	2600 (183)
12	12	0.25(1.0)	0.24(0.6)	None	2404 (183)
13	13A	0.25(1.0)	0.24(0.6)	0.24(0.6)	2300 (175)
	13B	0.25(1.0)	0.24(0.6)	0.25(1.0)	2500 (175)
14	14	0.25(1.0)	None	None	2330 (166)
15	15	0.25(1.0)	0.24(0.6)	None	2300 (160)
16	16	0.25(1.0)	0.24(0.6)	None	2150 (150)
17	17	0.25(1.0)	0.24(0.6)	None	2134 (150)
18	18A	0.25(1.0)	0.24(0.6)	0.25(1.0)	2500 (175)
	18B	0.25(1.0)	0.24(0.6)	0.25(1.0)	2500 (175)
19	19-1 , 19-2	0.25(1.0)	None	None	2730 (195)
20	20-1 , 20-2	0.25(1.0)	0.24(0.6)	None	2730 (195)
21	21A 1 , 21A 2	0.25(1.0)	0.24(0.6)	None	2400 (168)
	21B	0.25(1.0)	0.24(0.6)	None	1900 (135)
22	22A-1 , 22A-2	0.25(1.0)	0.24(0.6)	None	2700 (190)
	22B	0.25(1.0)	0.24(0.6)	None	2900 (203)
	22C	0.47(1.2)	0.32(0.8)	None	2300 (160)
23	23	0.47(1.2)	None	0.24(0.6)	2300 (180)
24	24A	0.25(1.0)	0.24(0.6)	0.24(0.6)	2700 (275)
	24B	0.47(1.2)	0.24(0.6)	0.24(0.6)	2500 (175)
25	25A	0.25(1.0)	0.24(0.6)	0.24(0.6)	2500 (223)
	25B	0.25(1.0)	0.24(0.6)	0.25(1.0)	2800 (200)
	25C	0.47(1.2)	0.24(0.6)	0.24(0.6)	2500 (175)
	25D	0.25(1.0)	0.24(0.6)	0.25(1.0)	2700 (190)
	25E	0.25(1.0)	0.24(0.6)	0.25(1.0)	2100 (148)
26	26A	0.25(1.0)	0.24(0.6)	0.24(0.6)	2700 (155)
	26B-1 , 26B-2	0.25(1.0)	0.24(0.6)	0.25(1.0)	2500 (175)
	26C	0.25(1.0)	0.24(0.6)	0.25(1.0)	2300 (205)
	26D	0.25(1.0)	0.25(1.0)	0.25(1.0)	2434 (200)
	26E	0.25(1.0)	0.24(0.6)	0.24(0.6)	2526 (175)
	26F	0.42(1.0)	0.25(0.8)	0.25(1.0)	2300 (175)
27	27	0.25(1.0)	0.24(0.6)	0.25(0.8)	2800 (200)
28	28	0.25(1.0)	0.25(0.8)	0.24(0.6)	2500 (200)

*Notation for corner reference number. The figure before the hyphen indicates the type of corner detail (Fig 1) and the figure after the hyphen indicates the corner specimen number e.g. 5-2 represents Detail 5 — Corner Specimen 2. Where different bar sizes were used for the same type of corner detail distinguishing letters A, B, C etc are used.

†Diagonal reinforcement Type (i) is along the line from the inside reentrant angle to the outside of the corner as in Detail 26. Diagonal reinforcement Type (ii) is perpendicular to Type (i) as in Detail 23.

Table 2.5 (Reproduced from Reference 91)

TEST RESULTS (LOAD OPENING CORNER)

Corner* reference No	M_c / M_r	At 30 percent M_r: Net rotation radians $\theta_c \times 10^3$ †	At 30 percent M_r: Maximum crack widths, mm	At 60 percent M_r: Net rotation radians $\times 10^3$ †	At 60 percent M_r: Maximum crack widths, mm	At 80 percent M_r: Net rotation radians $\times 10^3$ †	At 80 percent M_r: Maximum crack widths mm
1-1	0.42	−0.11	NV‡	0.41	0.1	F§	F
1-2	0.29	0.04	NV	3?	F	F	F
2-1	0.45	−0.23	NV	0.13	0.12	F	F
2-2	0.49	−0.09	NV	0.61	0.1	F	F
3	0.43	−0.4	NV	0.21	0.1	F	F
4	0.30	−0.36	NV	−0.82	0.1	F	F
5-1	0.61	−0.20	NV	0.5	0.1	NF§	NF
5-2	0.61	−0.41	NV	0.24	0.1	NF	NF
6-1	0.82	−0.06	NV	−0.15	0.1	0.75	0.2
6-2	0.85	−0.7	NV	0.32	0.5	0.98	0.1
7-1	0.54	−0.23	NV	0.5	0.2	F	F
7-2	0.61	−0.04	NV	0.43	0.1	NF	NF
8-1	0.45	−0.06	NV	0.32	0.1	F	F
8-2	0.53	−0.07	NV	−0.83	0.1	F	F
9	0.52	−0.13	NV	0.4	0.1	F	F
10	0.62	−0.23	NV	0.4	0.1	NF	NF
11	0.52	0.43	NV	1.34	0.15	4.04	0.5
12	0.92	−0.06	NV	0.96	0.1	3.18	0.35
13A	0.95	0.06	NV	0.72	0.15	3.4	0.4
13B	0.95	−0.29	NV	−0.22	0.15	0.89	0.3
14	0.72	−0.06	NV	0.5	0.2	1.27	0.3
15	0.88	−0.33	NV	0.23	0.1	1.22	0.2
16	0.83	0.19	NV	0.5	0.1	1.27	0.3
17	0.75	0.41	NV	0.14	0.2	1.88	0.4
18A	0.94	−0.44	NV	0.5	0.1	2.04	0.25
18B	1.02	−0.5	NV	−0.23	0.1	0.24	0.15
19-1	0.42	−0.28	NV	NF	NF	F	F
19-2	0.31	−0.19	NV	2.30	0.2	F	F
20-1	0.58	0.04	NV	3.98	0.2	F	F
20-2	0.80	0.24	NV	3.14	0.2	F	F
21A-1	0.62	−0.23	NV	1.86	0.2	4.0	1.1
21A 2	0.61	0.01	NV	0.47	0.1	NF	NF
21B	0.77	−0.09	NV	0.96	0.1	3.14	0.6
22A-1	0.42	−0.13	NV	1.4	0.15	NF	NF
22A-2	0.53	−0.25	NV	1.18	0.15	NF	NF
22B	0.66	−0.33	NV	0.9	0.2	1.62	0.8
22C	0.85		0.1		0.8		1.8
23	0.45		0.86		1.5	F	F
24A	0.81	0.13	NV	0.6	0.1	2.36	0.5
24B	0.35		NV		0.1	F	F
25A	0.86	−0.32	NV	0.48	0.1	1.83	0.25
25B	0.91	−0.18	NV	−0.17	0.06	1.58	0.2
25C	0.68		NV		0.2	NF	NF
25D	0.82	−0.29	NV	0.2	0.06	3.42	0.38
25E	0.98	−0.19	NV	0.1	0.1	1.9	0.2
26A	1.06	0.22	NV	0.15	0.06	0.47	0.2
26B-1	1.26	0.83	NV	0.18	0.1	1.1	0.25
26B-2	1.05	0.06	NV	0.32	0.1	2.04	0.2
26C	0.59	0.15	NV	0.48	0.06	1.73	0.2
26D	1.20	−0.19	NV	−0.1	NV	0.52	0.1
26E	0.29	0.81	0.1	F	F	F	F
26F	0.27	0.19	0.1	F	F	F	F
27	0.98	−0.13	NV	0.22	NV	0.72	0.1
28	0.81	−0.07	NV	0.5	1.4	1.17	0.3

* Corner notation as in Table 1
† NV = None visible
‡ F = Failed
§ NF = Near failure

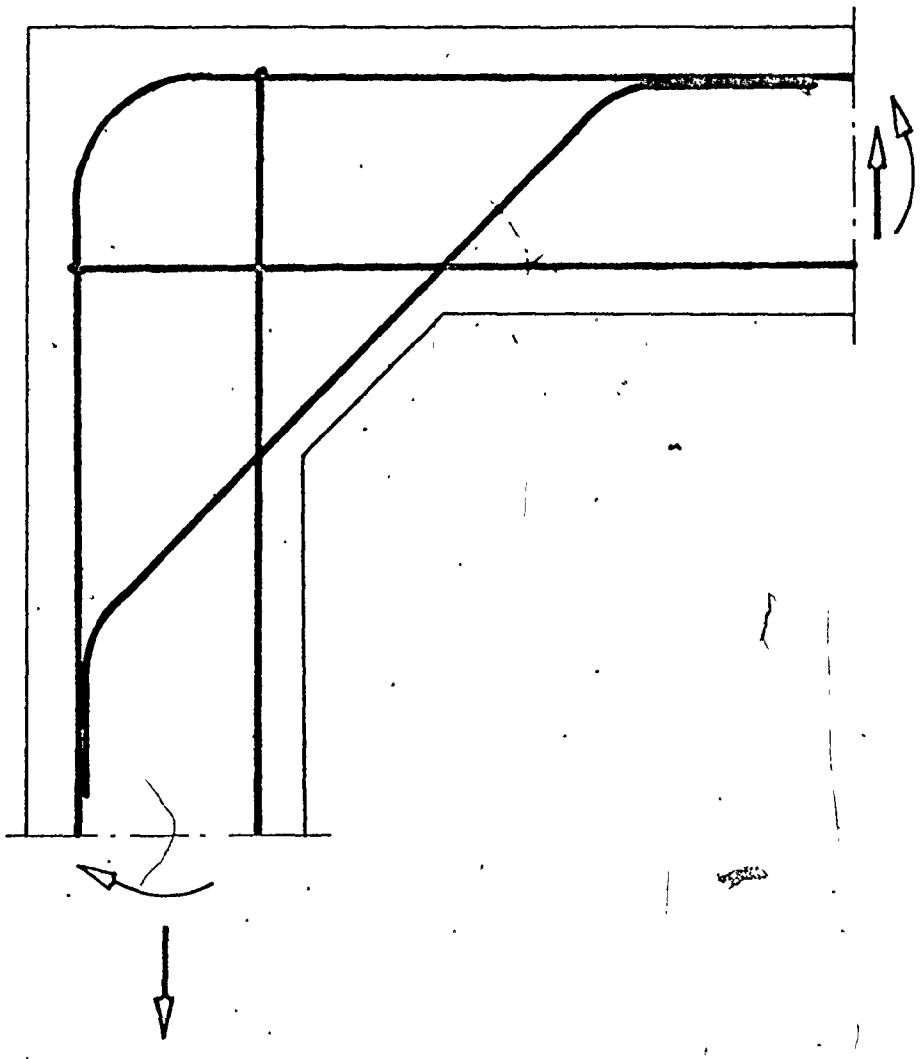


Fig. 2.14 CORNER DETAIL SUGGESTED BY BALINT AND TAYLOR[85].

2. Splitting failure.

3. Failure due primarily to yielding of the reinforcement.

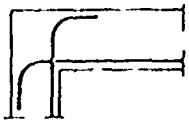
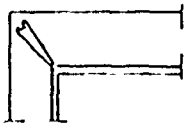
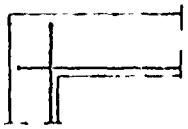
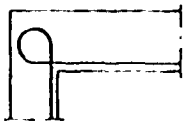
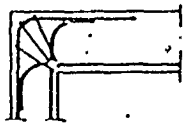
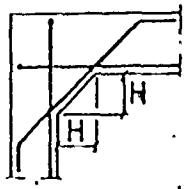
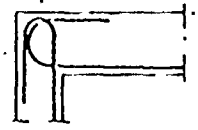
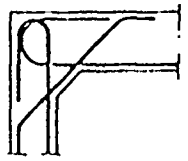
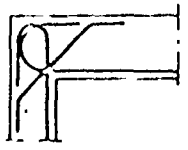
4. Anchorage failure.

5. Failure due primarily to crushing of concrete.

Of these causes of failure, special attention was devoted to diagonal tension failure. Simple expressions based on truss analogies were derived for the analysis of this type of failure. The emphasis of the experimental work centred on corners subjected to positive moment (tension on the inside). Some solutions for reinforcing such joints were offered in his document (97). Other types of joints with similar problems, such as corners in retaining walls, T-joints, and X-joints were also studied by Nilsson.

Proposals were put forward (97) for simple and functional reinforcement layouts for corners and some other common types of joints in reinforced concrete structures subjected to bending. Nilsson restricted the percentage of reinforcement in his details to certain amounts depending on the steel grade. However, he recognized the fact that much is still to be done in this field in order to more clearly understand the behaviour of reinforced concrete beam-column connections. Nilsson's work appears to be very thorough and his design recommendations are very beneficial. Document D7: 1973 (97), along with his other reports and publications listed in reference (97) can be of great help as detailing

Table 2.6 (Reproduced from Reference 97)
Nilsson's Joint Details and Test Results

Specimen number	Haunch size H cm	Concrete strength σ_c kgf/cm^2	σ_{sp} kgf/cm^2	Failure moment M_{ut} kgfm	Calculated ultimate mom. M_{uc} kgfm	$\frac{M_{ut}}{M_{uc}}$ %	Inside corner crack width at $M_{uc}/1.8$ mm
 U 21		339	27.3	990	3135	32	Failed
 U 27		277	21.1	1840	2990	61	0.60
 U 15		289	25.6	2227	3290	68	0.70
 U 12		335	31.2	2474	3220	77	0.27
 U 28		272	20.5	2540	3185	79	0.26
 UV 1	15	305	23.7	3160			
UV 2	5	345	25.2	3120			
 U 24		398	25.9	2804	3240	87	0.34
 UV 3	10	318	24.1	3712			0.08
UV 4	5	277	22.3	3505			0.06
 UV 5		335	26.2	3629	3180	114	0.11
UV 6		292	24.3	3505	3040	115	0.13
UV 7		339	18.4	3773	3070	123	0.13

references. In Table 2.6, his test results are reproduced (97). Details UV5, UV6, and UV7 were found to be the most efficient.

Although the behaviour of beam-column connections under cyclic loading is beyond the scope of this investigation, a list of references is included in the Bibliography to assist future investigators in the subject.

Example of valuable work on T-joints subjected to reversable loading were conducted at the University of Toronto by Uzumeri et. al. (101) and at the University of Canterbury in New Zealand, by Park and Paulay, (95, 103). Among other variables, different placement patterns of tension steel and of stirrups were studied. Both studies contain efficient detailing examples and contributed to a more complete understanding of the subject.

2.4 Development of the Finite Element Method in the Analysis of Reinforced Concrete Members:

2.4.1 General Background

Significant improvements and developments of the methods of reinforced concrete design were achieved as a result of continued research over the years. Design theories were proposed as early as 1850 with the application of reinforced concrete slab construction which appeared in France and England. (10, 14).

Working stress and ultimate strength design methods gained varying degrees of acceptance between 1900 and 1970. These two methods have been derived from

extensive experimental and analytical research work by many investigators. From this vast body of research data, building codes have adopted these two simplified methods which were proved to be adequate for a large proportion of the problems encountered in practice. However, the simplifying assumptions associated with these methods are not adequate for some of the complex interaction in reinforced concrete such as those outlined in Section 1.3.2. Designers are often confronted with special cases where conventional methods are ineffective. A common design solution where cost is low, is to make sections larger and/or more heavily reinforced to eliminate any possibility of failure. In problems where failure can lead to catastrophic results, such as loss of life, damage to costly equipment etc., load tests on prototypes and models must be carried out even if they are expensive and time consuming.

Researchers also need more sophisticated methods of analysis to be able to describe more accurately the behaviour of reinforced concrete and thereby be in a position to suggest better design formulae for the building codes.

The development of high speed digital computers and the formulation of the finite element method has

provided a means for the development of analytical models which can lead to a realistic evaluation of the behaviour of reinforced concrete. As a result it is possible to focus on the state of internal stresses and displacements in a manner which seems necessary if the detailed behaviour of reinforced concrete is to be properly investigated (39).

Formulation of a sophisticated form of such an analysis can be a difficult task not only in terms of the numerical solution of the non-linear problem arising from material properties but also in terms of the definition of realistic relationships governing material and structural behaviour. Considering that prediction of the behaviour of a single homgeneous isotropic material is commonly based on approximations such as idealized elasto-plastic stress-strain properties and empirical yield criteria, it may be easy to understand why the modelling of the behaviour of such a complex composite material as reinforced concrete has been subjected to many approximations and assumptions.

Existing experimental evidence and the corresponding

empirical or semi-empirical relationships for each individual material (steel and concrete) offer reasonable guidelines for modelling of their behaviour. However, as will be discussed later, interaction of concrete and steel has not been studied sufficiently to have the same degree of confidence. Interactions between the two materials such as bond-slip behaviour and dowel action, and also aggregate interlock between two concrete interfaces are important aspects which require considerably more study.

In this investigation what is judged to be the most reliable data (as documented in Sections 2.5 and 3.2) on each subject is used. To minimize the inaccuracies involved in the analysis, material properties were kept as close as possible to those from which the above relationships were developed.

2.4.2 Development of the Finite Element Method

The gradual trend toward representation of a continuum by an aggregation of discrete elements seems to have resulted from recognition that this idealization is more amenable to a numerical solution. The resulting structure is analyzed as a highly indeterminate structure using either the force or the displacement approach. The difference between these appraoches lies in the selection of the unknowns to be initially solved and in the variation in the matrix quantities associated with their formulation.

The basic concept of the displacement method is that any structural member may be considered as an assemblage of individual elements interconnected at a finite number of nodal points. By placing certain constraints on displacements, conformity is assured between elements. It is the finite nature of the connectivity which makes it possible to formulate simultaneous algebraic equations and which distinguishes the method from continuum mechanics.

Displacement methods have been shown (90) to be more versatile than force methods since their formulation is simpler for a majority of structural analysis problems. One of the earliest representations of a continuum by discrete elements appears to have originated in the early 1940's with Hrennikoff (5) and McHenry (7). Besides inaccuracies due to the bar element chosen, the method gave rise to very large arrays of equations which could not easily be solved due to the lack of electronic digital computers.

Matrix formulations and the use of electronic digital computers developed rapidly in the air-craft industry. Further development of the finite element method from approximately 1955 to 1965 was reported mostly in the journals of aeronautical societies.

Turner and Clough (15) in 1956 introduced a two-dimensional plate element in the analysis of air-craft structure. Rectangular and triangular plane element stiffness derivations for plane stress condition were presented in 1960 by Clough (17).

With the increased availability of faster computers with larger core storage capacities, and because of its apparent potential, the finite element method has undergone a very rapid development in the last decade. New families of 2 and 3 dimensional elements have been developed to solve linear and non-linear problems in plates, shells, arches and solids and hundreds of papers have been published. Special conferences entirely devoted to the subject have been held, such as those held at Wright-Patterson Air Force Base (25, 42), in Japan, (55) at Nashville, (57), and in Montreal (89). Development of such elements and methods have been summarized in books published by Zienkiewicz (37), Prezemienieki (43), Holand and Bell (59), and Desai and Abel (80). Books dealing with the matrix formulation have also been published by numerous authors among whom are Gere and Weaver (26), Rubinstein (31), and Meek (70).

2.4.3. Analysis of Reinforced Concrete Using the Finite Element Method:

Comparatively few publications in the ever growing reference list on finite element method have been devoted to the analysis of reinforced concrete. One of the first applications of the Finite Element Method to concrete was reported by Clough, Wilson, Sims and Rhodes (21, 23) who studied the behaviour of a gravity dam in which deep cracking had been observed. The crack was represented in the finite element

system by separation of elements along the crack line where independent nodal points were assumed for the elements on each side of the crack. Concrete was assumed to behave elastically under all applied loads.

Zienkiewicz and Cheung (37) described a simple way to simulate crack formation in low tensile capacity materials such as rocks and concrete by changing the overstressed element into an element with a zero stiffness in the direction perpendicular to the crack. Ngo and Scordelis (39) introduced in 1967 special elements to model the interaction between steel and concrete. In their study, simple beams with tensile reinforcement only were represented by triangular finite elements and the concrete-steel interaction was idealized by springs of zero length having constant stiffnesses for both bond-slip and dowel force-dowel displacement relationships. A number of beams with various predefined crack patterns were analyzed to evaluate concrete and steel stresses, bond stresses and deflections. The influence of decreasing stiffness due to various crack lengths was studied. Both steel and concrete were assumed to exhibit linear elastic behaviour.

Nilson (51) extended these studies by defining a non-linear bond-slip relationship and by introducing non-linear behaviour of concrete in the analysis of tension specimens which were axially or eccentrically loaded. Incremental loading procedures were used to follow the non-linearities of the materials. The concrete elements were considered to have failed when the principal tensile stress reached the modulus of

rupture. Execution of the program was then halted and the cracked element was replaced by two smaller elements with new node numbers on each side of the crack. A completely new topology was thus defined and this new structure was reloaded from zero load until another element cracked.

Franklin (60) extended the previous work by developing nonlinear analysis capable of redistributing stresses after cracking by iterating at each load increment. Triangular, rectangular, quadrilateral and rod elements were used with and without bond links. It was observed that the bond phenomenon was important for modelling beams to predict failure loads.

In 1970, Ngo, Franklin and Scordelis (68) formulated a more refined model of a reinforced concrete beam in which they introduced aggregate interlock linkages along, predefined diagonal tension cracks of various lengths. Linear behaviour of materials was retained. Beams with and without stirrups were analyzed. One of the significant results was that they determined the horizontal and vertical spring forces simulating bond and dowel forces which have no counter-part in experimental data.

A limit analysis of reinforced concrete slabs using a mixed finite element model with both force and displacement variables was reported by Backlund (79). Mufti et. al. (72), Spokowski (88), and Houde (93) using non-linear material

properties and bond-slip relations derived by Nilson (51) and Houde (93), developed computer programs with incremental loading procedures which delete the cracked element stiffness and redistribute the strain energy to the adjacent uncracked elements. Loov (94) in 1972 directed particular attention towards problems associated with cracking. Initiation and propagation of cracks were considered to occur through elements rather than being restricted to following the element boundaries.

2.5 The Force-Displacement Relationships for the Finite Element Analysis of Reinforced Concrete

In this section the state-of-the art for the development of the following three important force-displacement relationships are discussed:

1. Bond-slip relationship
2. Dowel Action
3. Aggregate Interlock.

The stress-strain relationships for concrete and steel are discussed in the next chapter where data from the experimental program are also presented.

2.5.1 The Bond-Slip Relationship

Bond stress is the unit shearing force parallel to the axis of the reinforcing bar and on the steel-concrete interface. The bond stresses transfer the force from the steel bar to the surrounding concrete and vice versa.

Bishara (8), Brice (9), Mains (11), Bresler and Bertero (30), Perry and Thompson (35) and more recently, Nilson (48, 78, 92), and Houde (93) have studied the bond-slip problem.

The work of the first four researchers was pioneering work concentrating on the techniques to measure bond and slip. No specific bond-slip relationships were developed.

In 1966 Bresler and Bertero (30) tested concentrically reinforced specimens having one number 9 bar. They measured the end slip versus the bond stress. They reported results on both the bond stress distribution and the measured end slips.

Nilson (48) used Bresler and Bertero's results to derive a tentative local bond stress-slip relationship. He calculated the bond stress distribution along the length of the bar from the values of the steel stresses determined at closely spaced locations (1-2 inches). Slip between concrete and steel was defined as the difference between the steel displacement and the concrete displacement at particular locations, since it was difficult to measure the actual value of slip experimentally. Although there was considerable scatter, Nilson fitted the results of the experiments into the following third degree polynomial.

$$\dot{U} = 3.606 \times 10^{6} d - 5.356 \times 10^{9} d^{2} + 1.986 \times 10^{12} d^{3} \quad \text{.....(2.1)}$$

where U is the nominal bond stress (psi) and d is the local bond slip (inch).

More recently, Nilson (78, 92) measured the steel strain along the length of an embedded reinforcing bar to provide the basis for calculation of bond stress variation. He measured the internal strains in the concrete close to the interface at a number of locations along the embedded length. By numerical integration of both the steel strains and the concrete strains, displacement and slip were determined. Nilson used the method he described to investigate the bond stress versus slip relations for a No. 8 standard deformed reinforcing bar in two types of specimens, using concrete strengths in the range of 4000 psi.

He proposed a bilinear bond stress-slip relation as follows

$$U = 3100(1.43S + 1.50)\sqrt{f'_c}\, d \quad \text{.....(2.2)}$$

and

$$U_u \leq (1.43S + 1.5)\sqrt{f'_c} \quad \text{.....(2.3)}$$

where,

S = distance from the loaded end face of specimens tested (and by analogy, the distance from a crack face in typical members).

U = bond stress (psi)

U_u = peak bond stress (psi)

d = slip

f'_c = cylinder strength of concrete (psi)

Nilson found that both the peak local bond stress and the bond stress-slip modulus vary with distance from the loaded end of the specimens tested. He gave representative equations for peak bond stress and for bond stress as a function of slip. However, he claimed no generality for his results because the data were available only for a limited range of variables. He reported that some of the variables that may have an effect on the relationship are size, type, number and spacing of bars, concrete strength, type of specimen, and type and duration of loading.

In 1974 Haude and Mirza (99) tested two types of specimens; tension and anchorage (Beam-ends). Sixty two concentric tension tests were conducted on specimens with varying cross-sections and reinforced with a single steel bar (either No. 4, 6, or 8). They measured the steel strain and the end slip in twelve of the concentric tension tests with No. 8 bars and in six of the twenty-four beam-end tests. Concrete strength was varied from 3,000 to 7,000 psi. They found that the average crack spacing in the tension specimen to be equal to three times the concrete cover thickness. They proposed that the slip can be expressed by the following empirical expression,

$$d = k f_s^{1.1} (A_c/A_s)^{1/3} \quad \text{.....(2.4)}$$

where,

f_s = the applied steel stress at the face (ksi)

A_s, A_c = area of steel bar and concrete in tension respectively

k = experimental constant (in/ksi) with an average value of 0.19

Houde and Mirza found that the bond forces are dependent on the embedment length, the concrete cross-section and the concrete strength.

For the series of tests with No. 8 bars, local bond stresses at known points along the bar were calculated at various load levels using the bar forces. Slip was evaluated from eqn. (2.4). All results were then normalized to a concrete strength of 5,000 psi using the factor $(f'_c / 5{,}000)^{1/2}$. For slip up to the point where the peak bond force is reached the following fourth degree polynomial was proposed to relate the local bond stress U and the local bond slip d,

$$U = 1.95E + 06d - 2.35E + 09d^2 + 1.39E + 12d^3 - .33E + 15d^4 \quad \text{.....(2.5)}$$

The initial bond-slip modulus (dU/dd) was seen to be close to 2.0×10^6 psi. A similar value is obtainable from Nilson's bilinear relationship, equation (2.2), when f'_c = 5000 psi and S = 5.33 inch. Figure (2.15) shows the curve obtained from Houde and Mirza's (93) mathematical expression versus his test results.

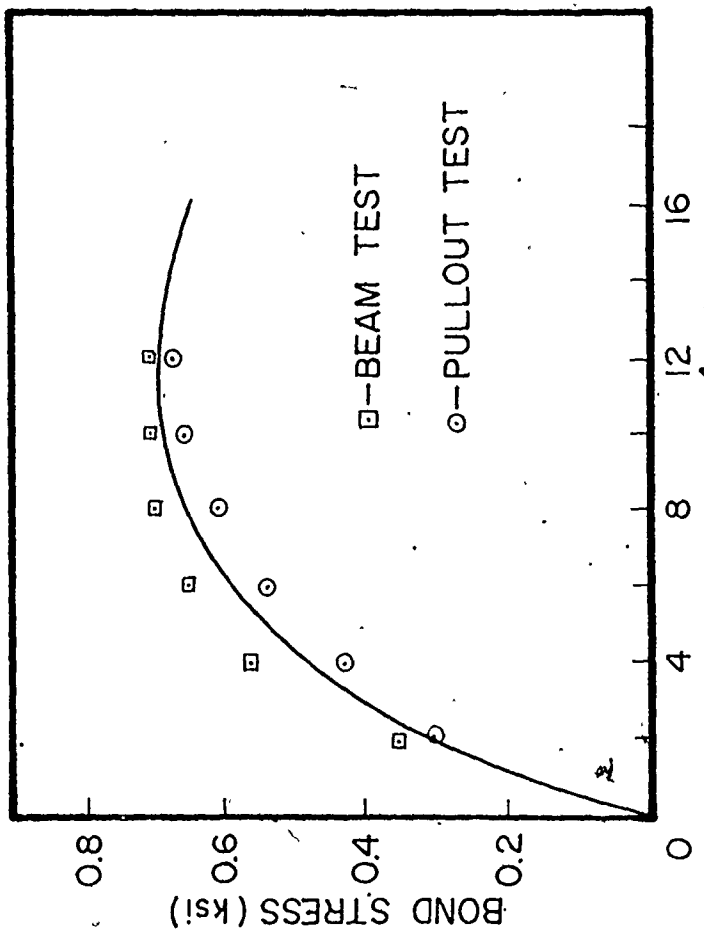


Fig. 2.15 UNIT BOND STRESS vs. UNIT SLIP
REPROCED FROM REFERENCE(93)

Although Nilson found the peak bond stress to be proportional to the distance from the crack face, Houde and Mirza did not observe such a relationship in their investigation.

In this study, Houde and Mirza's equations are used. It was thought that adding the distance to the crack face as a variable which effects the bond-slip relationship would add to the complexity of the numerical solution without either theoretical justification or strong experimental evidence.

It seems evident that the current knowledge of the bond stress-slip relationship is still very limited. Some of the important factors that may affect this relationship have yet to be studied. Some of these variables were reported by Nilson (92) as mentioned earlier.

2.5.2 Dowel Action

Dowel action describes a mechanism by which shear is transferred across cracks in reinforced concrete members. In addition to longitudinal forces, the reinforcement is required to carry dowel forces which are perpendicular to its axis.

Krefeld and Thurston (33, 34), Fenwick and Paulay (49), Taylor (44, 64) and Houde (93) studied the dowel phenomenon.

Krefeld and Thurston (33, 34) tested divided beams which were joined by reinforcing bars of different sizes. Beam width, concrete cover and distance of the preformed crack from

the support were varied. They proposed a force-displacement relationship which takes all of the above mentioned variables in addition to the concrete strength into consideration. Their work is valuable. However, the inclusion of the distance from the crack to the beam support as a parameter restricts the use of their equations to beam applications.

Fenwick and Paulay (49) reported that effects of embedment length, number,diameter, arrangement and bond properties of bars, shear displacement across the crack and the tensile stress in reinforcing bars, are most important. Using direct shear testing models, they studied the effect of embedment length in addition to the parameters studied by Krefeld and Thurston (33, 34). However, the steel reinforcements were not subjected to tension in these experiments.

Taylor (44, 64) adopted the same specimens as developed by Krefeld and Thurston. He studied the effects of concrete strength, number and layout of reinforcing bars, shear span, cover, and beam width. He did not study the effect of bar diameters. Taylor suggested a force-displacement relationship which contained a large constant value term which made his equation insensitive to changes in net beam width and in tensile strength of concrete which he had found to be the most important factors associated with this phenomena.

Houde (93) recently developed equations for the dowel cracking loads and the dowel load-displacement relationship which take beam net width and concrete strength into account.

He developed the equations using his own test results and test data from others (44, 49, 64). This formed part of an extensive program to derive force-displacement relationships to be used in finite element analyses of reinforced concrete members. All his test results for dowel action were normalized to a common concrete strength level of 4,300 psi using the multiplier $(f_c'/4300)^{1/2}$. The following equations which he proposed are used in this investigation to describe the dowel phenomena.

$$V_d = 2{,}000\ D_f\ \Delta_{crack} \quad \text{.....(2.6)}$$

where $$D_f = 40\ b_n (f_c')^{1/3} \text{ (lb.)} \quad \text{.....(2.7)}$$

V_d = dowel load (lbs)

Δ_{crack} = dowel displacement (in)

$= 5 \times 10^{-4}$ in.

D_f = dowel cracking load (lbs)

b_n = net width of beam (in)

therefore

$$V_{d_{crack}} = 40\ b_n (f_c')^{1/3} \quad \text{.....(2.8)}$$

2.5.3 Aggregate Interlock:

Aggregate interlock describes another mechanism for transferring shear across cracks. When concrete cracks, the surfaces of cracks are usually rough and irregular. Relative displacement along the crack is necessary to cause shear transfer by aggregate interlock.

Aggregate interlock was studied by Nowlen (47) Fenwick (49), Gergely (53), Loeber (61) Taylor (67) and Houde (93).

Nowlen (47) concluded from tests under dynamic loading that maximum aggregate size is important especially for wide cracks. He did not give any specific values or equations to represent his conclusions.

Gergely (53) concluded from his direct shear tests that the contribution of aggregate interlock in the shear capacity of a cracked member can be as high as 40%.

Fenwick (49) using direct shear tests, developed a shear stress-shear displacement equation which takes crack width and concrete compressive strength into account. The expression developed by Fenwick gives results essentially equivalent to the ones obtainable from the equations derived recently by Houde (93).

Loeber (61) reported that the effect of aggregate size is not important.

Taylor (67) using beam-end type and direct shear tests found that concrete strength, aggregate type and displacement ratio (or inclination of crack) have sizeable effects on the phenomenon. He also implied that his equations could only be used in the analysis of beams similar to the limited number he tested, and did not claim the generality of his equation.

Houde (93) tested 32 specimens similar to Fenwick's where he studied the effects of crack width, concrete strengths,

and maximum aggregate size. The shear displacements were always measured along a 45 degree crack line. Houde developed with a relatively simple equation that relates shear stress and displacement per unit crack length with crack width and concrete strength as variables.

The equation developed by Houde (93) to determine the shear stress transmitted by aggregate interlock action is expressed as follows:

$$fa_i = 57\ (1/c)^{3/2}\ (f'_c/5{,}000)^{1/2} \Delta_s\ (psi) \ldots\ldots (2.9)$$

where,

fa_i is the shearing stress (psi)

c is the crack width (inch)

Δ_s is the shear displacement along the crack (inch/inch)

This expression gives results essentially equivalent to the ones derived by Fenwick from tests on specimens similar to those of Houde.

The study of the effects of aggregate size greater than 3/4" or less than 3/8", the effects of the actual problem of progressive crack opening has not been found in the literature. Due to computer core storage limitations and to the fact that aggregate interlock is only effective for a limited range of structural elements for which the shear crack widths maintain fairly small values during the

loading to failure , this phenomena was not included in the present analysis.

2.6 Summary

In this chapter, the literature review was organized into three major groupings, each dealing with information related to specific aspects of this investigation. These were:

a. Details of tests of Beam-Column joints.

b. Theoretical modelling of reinforced concrete structures using the finite element method.

c. Description of Force-Displacement relationships for use in the finite element analysis of reinforced concrete.

Other literature which is relevant but more commonly used was not reviewed in this Chapter, but is referenced throughout the text.

CHAPTER 3

MATERIAL PROPERTIES, TEST SPECIMEN, AND EQUIPMENT AND INSTRUMENTATION

3.1 Introduction

This chapter provides information about the materials used in the experimental test program. Included are the experimental stress-strain results for the concrete which was used to fabricate the joint test specimens described later. The correlation of these results with a general stress-strain equation is also presented. In addition the stress-strain relationship for the reinforcing steel used in the joint test specimens is described. This data and its mathematical representation provide necessary information for the numerical analyses of the joint test specimens which are presented in Chapter 7.

This chapter also provides explanations for the choice of test specimen shape and contains information on details of the specimens such as dimensions, tolerances, and fabrication methods. In addition the instrumentation and measurements to be taken and the test set-up and equipment are described and discussed. The joint test procedure is included in Chapter 4, with the joint test results.

3.2 Material Properties

Only one predesigned (36) concrete mix was used in this testing program. Many variables and the corresponding uncertainties pertaining to these were eliminated by using a standard mix. This mix was used in most research on concrete structures done in McMaster University. Such properties of concrete as the stress-strain relationship, increased strength with age, creep, and shrinkage were readily available. The use of this predesigned mix necessitated checking the accuracy of the predetermined stress-strain relationship. Tests to determine this relationship were performed for each specimen.

3.2.1 Concrete Mix and Batching Procedure

Table 3.1 lists the proportions by weight of the concrete mix as well as the weights required for each batch. The weight of the aggregate was for the air-dried condition. Concrete slump and aggregate types are also included, in Table 3.1.

Three batches of concrete were required to complete the casting of each test specimen. One concrete pour consisted of the following group of specimens:

a) One rectangular frame (described in Section 3.3).

b) 6 standard 6 inch diameter by 12 inch long cylinders.

Immediately preceding mixing of the first batch, a "butter" batch of one quarter the regular batch was mixed to condition the nine cubic feet horizontal drum mixer. This

concrete was then thrown out. Subsequent batches were mixed in rapid succession to avoid drying out of the mixer between batches. Each batch was allowed to mix for five minutes after the last of the water had been added. The complete mixing and concrete placing operation took about 40 minutes.

The horizontally cast frame and the standard cylinders were poured in three equal layers corresponding to the three concrete batches. Each layer was then placed and vibrated as soon as it was removed from the mixer. The concrete was internally vibrated using a 1 1/4 inch diameter poker type vibrator. For the top layer, the forms were filled so that the concrete was about one half of an inch over the top of the form after vibrating. The excess was then trowelled off level with the top of the form. It was believed that by following this procedure any tendency for the formation of a weaker top surface layer of concrete associated with migration of water due to vibration would be avoided.

Component	Percent By Weight	Weight in pounds per Batch
Portland Cement Type 1	14.0	127.37
Water	9.1	82.55
Fine aggregate (Washed pit run sand, fineness Modulus = 2.51)	46.6	423.61
Coarse Aggregate (3/8" max. size crushed limestone)	30.3	275.28
TOTAL	100.00	908.81
Slump for a standard 12 inch slump cone = 2.25 inch Volume per batch = 6.0 cubic feet (approximately)		

Table 3.1 Concrete Mix Data

3.2.2 Curing of the Concrete

About five hours after placing, when the concrete had begun to harden, wet burlap was placed over the specimens and kept moist. The curing continued to the age of 21 days at which time the specimens were moved to the structural testing floor to facilitate instrumentation and other preparations for testing. After 28 days from casting, the testing was begun.

3.2.3 Concrete Stress-Strain Relationship

The concrete cylinders were cast to provide individual and collective strength and stress-strain information for the specimens. At least 4 cylinders were tested at 28 days to determine the concrete strength and the stress-strain curve. Two or more of each were instrumented to provide the stress-strain information. The cylinders were instrumented with two sets of gauge points placed on opposite sides of the cylinder. These gauge points were centred on the concrete cylinders with an eight inch gauge length. A Demec Strain Indicator, which is a demountable mechanical strain indicator, was used to measure the strains. This instrument was calibrated into divisions of 0.00001 inches per inch and could be read accurately to one division as determined by repeated readings.

The cylinders were capped with a molten sulphur compound and were tested in a 300 kip capacity hydraulic "Tinius Olson" testing machine. Readings were taken at 10 kip intervals up to about 80 percent of the cylinder strength. After

exceeding 80% of the cylinder strength, strain measurements were taken on one side of the cylinder without stopping loading until material failure was imminent. Comparison of the strengths of cylinders used for the stress-strain measurements with companion cylinders tested only to determine strength indicated that there was no apparent loss of strength due to the incremental loading required to take strain measurements.

The average of the strains from the cylinder tests were used for plotting the stress-strain curve. Although it was not possible to obtain reliable strain readings at strains beyond the ultimate strength of the cylinders (too much energy release), the measurements taken indicated a gradual decrease of stress up to about 0.0025 inches/inch. This data agreed reasonably well with Saenz's (24) equation. This equation for the concrete stress-strain relationship has been used in other studies (51, 88, 93) and is accepted as a good generalization for a wide range of concrete strengths. Therefore, it was adopted to facilitate use of the analytical model for concrete properties other than those used in the experimental program.

The concrete stress-strain curve was used to determine values for the moduli of elasticities used in the numerical analysis. The curve represented by Saenz's (24) equation was plotted in Fig. 3.1, along with the actual curve obtained experimentally. Saenz's equation (24) is

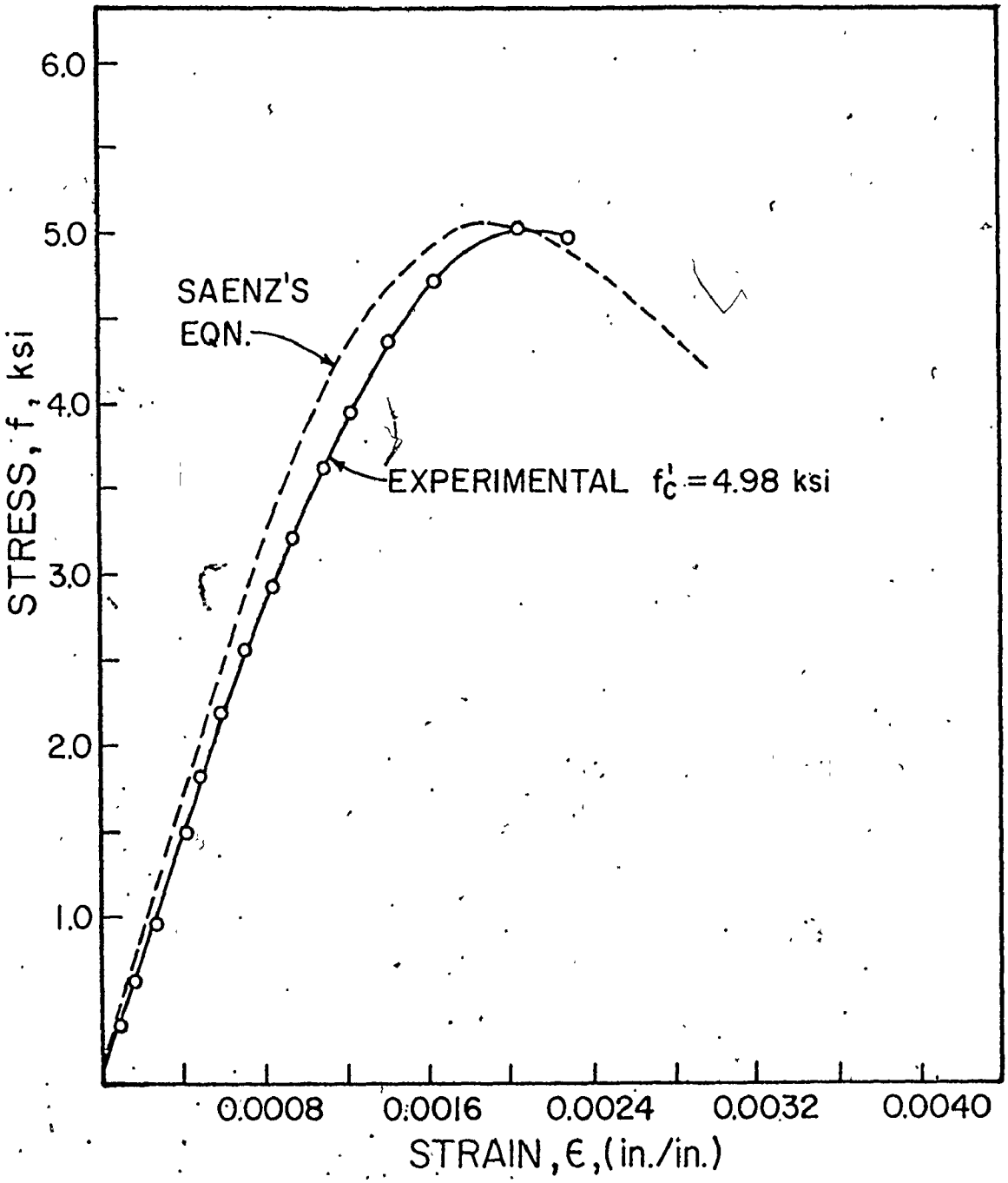


Fig. 3.1 TYPICAL EXPERIMENTAL CONCRETE STRESS-STRAIN DIAGRAM AS COMPARED WITH SAENZ'S EQUATION.[24]

$$f = \frac{E_c \varepsilon}{1 + (R + R_E - 2)\frac{\varepsilon}{\varepsilon_0} - (2R-1)\left(\frac{\varepsilon}{\varepsilon_0}\right)^2 + R\left(\frac{\varepsilon}{\varepsilon_0}\right)^3} \quad \text{.....(3.1)}$$

where

$$R = \frac{R_E (R_f - 1)}{(R_\varepsilon - 1)^2} - \frac{1}{R_\varepsilon} \quad \text{.....(3.2)}$$

$$R_E = \frac{E_c}{E_0} \quad \text{.....(3.3)}$$

$$R_f = \frac{f'_c}{f_f} \quad \text{.....(3.4)}$$

$$R_\varepsilon = \frac{\varepsilon_f}{\varepsilon_0} \quad \text{.....(3.5)}$$

and where

f = concrete stress

ε = concrete strain

ε_0 = concrete strain corresponding to maximum stress

f'_c = maximum concrete stress

ε_f = maximum concrete strain at failure

f_f = concrete stress at maximum strain

E_c = initial tangent modulus of Elasticity

$E_o = \frac{f'_c}{\varepsilon_0}$

From equation (3.1) both the tangent modulus of elasticity, E_t,

and the secant modulus, E_s, have been derived as follows:

$$E_t = \frac{df}{d\varepsilon} = \frac{E_c\left(1+C_1\left(\frac{\varepsilon}{\varepsilon_0}\right)^2-2C_2\left(\frac{\varepsilon}{\varepsilon_0}\right)^3\right)}{\left(1+C_3\left(\frac{\varepsilon}{\varepsilon_0}\right)-C_1\left(\frac{\varepsilon}{\varepsilon_0}\right)^2+C_2\left(\frac{\varepsilon}{\varepsilon_0}\right)^3\right)^2} \qquad (3.6)$$

$$E_s = \frac{f}{\varepsilon} = \frac{E_c}{1 + C_3\,\frac{\varepsilon}{\varepsilon_0} - C_1\left(\frac{\varepsilon}{\varepsilon_0}\right)^2 + C_2\left(\frac{\varepsilon}{\varepsilon_0}\right)^3} \qquad (3.7)$$

where,

$$C_1 = 2R - 1$$

$$C_2 = R$$

$$C_3 = R + R_E - 2$$

Saenz suggested the following values for ε_0, ε_f, f_f and E_c from the analysis of test results of Desayi and Krishnan (11).

$$\varepsilon_0 = 10^{-5}(f_c')^{.25}[31.5-(f_c')^{.25}] \text{ in./in., } f_c' \text{ in psi} \qquad .....(3.8)$$

$$E_c = 10^5\,(f_c')^{.5}/[1.0+0.006(f_c')^{.5}] \text{ psi, with } f_c' \text{ in psi} \qquad .....(3.9)$$

The values of ε_f and f_f suggested by Saenz to be used in his equation are as follows:

f_c' (psi)	ε_f (in./in.)	f_f (psi)
1275	.0035	1060
3005	.0030	2700
4440	.0025	4190
7180	.0025	6800

Since all of the concrete strengths used in this investigation were in the vicinity of 5000 psi, ε_f was set at .0025 and f_f was approximated to 0.94 f_c' which is fairly close to the actual suggested values for f_c' over 4440 psi. The values of E_c from eqn. (3.9) were found to be higher than those reported elsewhere (61, 86) for concrete in this strength range. The ACI (61) equation for E_c was found to agree better with the experimental values obtained in this investigation.

$$E_c = 33(w)^{1.5} (f_c')^{1/2} \quad \text{.....(3.10)}$$

where, w = 145 pcf for normal weight concrete.

This substitution for equation (3.9) is also appropriate for various concrete strengths. Although the validity of equation (3.9) is questioned, it does not affect the general validity of Saenz's equation (3.1). Therefore equation (3.10) was used in the analysis in place of Saenz's equation (3.9). The curve shown in Fig. 3.1 which represents Saenz's equation was plotted using the value of E_c obtained using equation (3.10).

The validity of using the stress-strain relationships derived from a concentric compression test in eccentric compression analysis has been questioned by numerous researchers. Rüsh (16) and Struman et.al. (29) reported that strain gradient retards cracking and stated explicitly that much higher stress-strain peaks can be expected in eccentric compression. As much as 70% higher stress and 50% higher strain peaks were suggested by Struman et.al. (29). Many other researchers, Clark et.al. (40) and Hognestad et.al. (13)) illustrated more conclusively that the strain gradient has minimal effect on the stress value but tends to increase the maximum strain attained before cracking occurs. The effect of the strain gradient, in addition to crack retardation, is to introduce a different strain rate in the various fibres. Assuming a plane section to remain plane, at least at a certain distance from the beam supports, each fiber is deformed at a different rate proportional to their distance from the neutral axis. The increased deformability of concrete with time (creep) is well known to be a function of the level of sustained stress and may explain some of the differences already noted (40).

The maximum strains at failure suggested by Saenz (24) were obtained from uniaxial (concentric) compression tests. Although these values were incorporated in the representation of the stress-strain relationship, they were not used as the limiting compressive strain which determines the criteria of failure for concrete in compression. This limiting strain was set at

0.003 in./in. for the range of concrete strengths used in the test program. This value is higher than that suggested by Saenz by 20% as proposed by Lui et.al. (86,87). Further discussion on the criteria of failure of concrete in compression is included in Chapter 6.

3.2.4 Concrete In Tension

The tensile strength of concrete is usually neglected in reinforced concrete structural design. This slightly conservative practice has been adopted because of the relative weakness of concrete in tension as compared with compressive strength. The purpose for which the finite element analysis in this investigation is intended, requires recognition of the importance of including the tension properties of concrete. The extent of the propogation of cracking in a reinforced concrete member depends largely on the tensile strength of concrete.

The level of the tensile strength obtained by testing is dependent on the type of the test and on the specimen size. Examples of tensile strengths reported in the literature (32, 93, 98) were $4.5\sqrt{f_c'}$, $7.0\sqrt{f_c'}$ and $9.6\sqrt{f_c'}$ for axial tensile tests, splitting (Brazilian) test, and flexure tests, respectively.

Although, there is some indication in the literature that some small amounts of yielding may occur prior to failure (98), the stress-strain relationship of concrete in tension can be represented by a straight line without serious loss of accuracy (32). A modulus of elasticity in tension equal to the

initial tangent modulus of concrete in compression is used in this investigation up to the value of the ultimate tensile strength.

3.2.5 Reinforcing Steel

High grade deformed reinforcing steel was used in the experimental part of this investigation. The minimum nominal yield stress of 60 ksi specified by the manufacturer was always obtained. Unfortunately the flat yield plateau was not evident in the stress-strain curve for the main steel reinforcement. The transition from elastic behaviour to the strain hardening region occured between 65 and 72 ksi. A typical ultimate tensile strength of over 115 ksi was obtained from the average of several test specimens. Relevant details of these test results which were required for the numerical analyses are reported in Chapter 4. Figure 3.2 shows a typical stress-strain curve for the number 6 bars used to reinforce the joint specimens.

Twenty-two inch test lengths were randomly cut from some of the bars to determine the stress-strain properties of the reinforcement. A 120 kip capacity hydraulically operated "Tinius-Olson" Testing Machine was used to perform the tests. Reference holes for the Demec Strain Indicator were drilled on opposite sides of the test lengths and located on the bar deformations. Strain measurements were taken at 2500 lb. increments and at approximately 500 lb. increments after yielding.

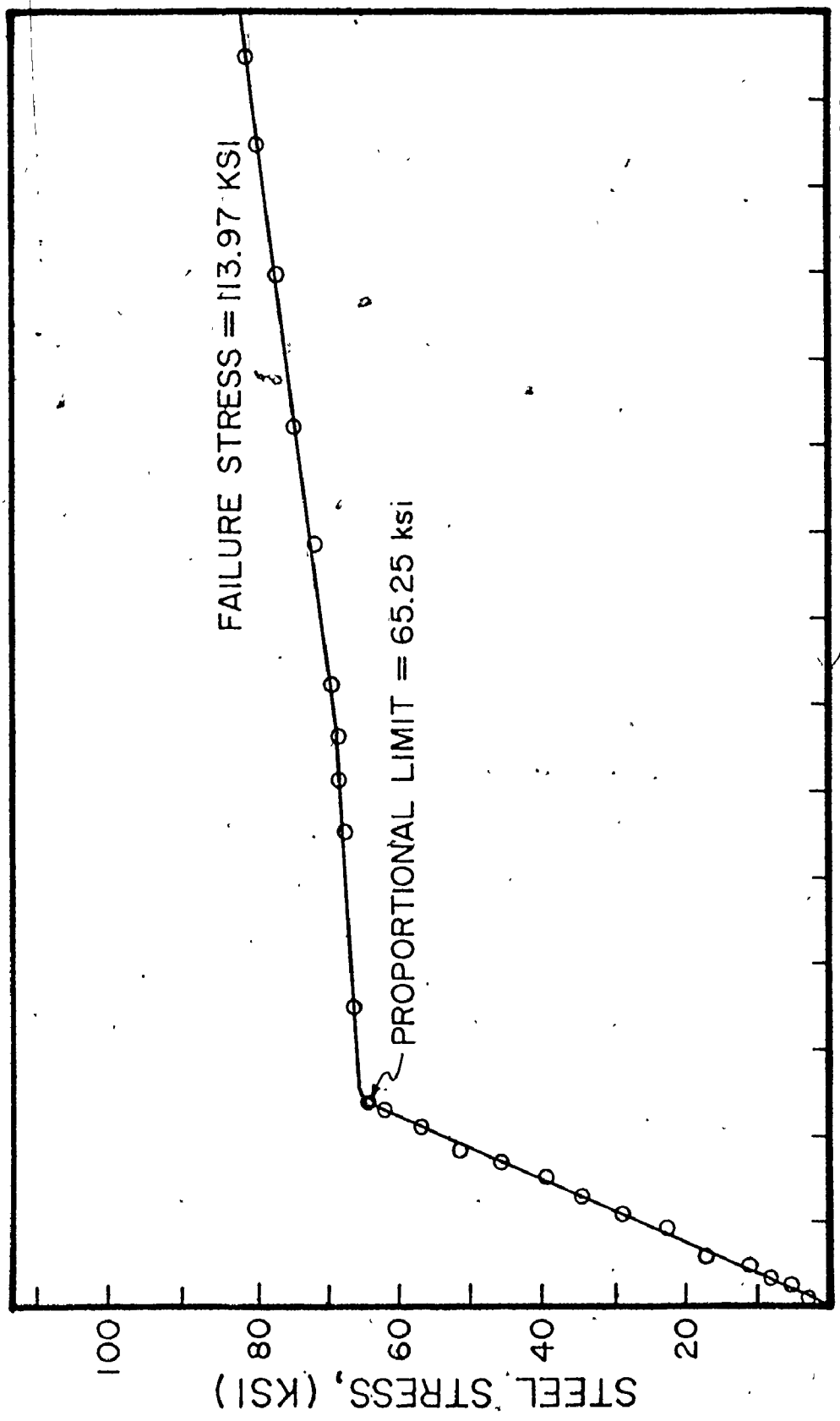


Fig. 3.2 TYPICAL AXIAL STRESS–STRAIN DIAGRAM FOR TENSION TESTS OF No.6 BAR SPECIMENS.

The actual average areas of the bars were found by weighing the samples and measuring their lengths. It was felt that the extra complexities of trying to mathematically model the ill-defined yield region of the stress-strain curve for the steel reinforcement was not justified. Therefore, a well defined yield point followed by a short yield plateau leading to a strain hardening region as shown in Fig. 3.3 was adopted in the analysis.

3.3 Development of the Shape of the Test Specimen

A major consideration in the design of the joint test specimen was that it should be large enough to incorporate a wide range of reinforcing details and to be representative of dimensions encountered in current practice, but also small enough to facilitate handling and testing. A typical type of specimen, which was considered to fulfil these requirements, consisted of two members meeting at right angle as shown in Fig. 3.4.a and used by Swann (58, 66). The simplicity of the design was offset by the many envisaged disadvantages. The 'L' shape, with a centre of gravity situated outside the members, would cause lifting and handling problems. In addition, a sophisticated base fixing attachment would be necessary to provide the required stability for testing.

Figure 3.4.b shows the shape of the specimen used by Nilsson (97) which appeared to produce a design with few handling or stability problems. However, the joint not being

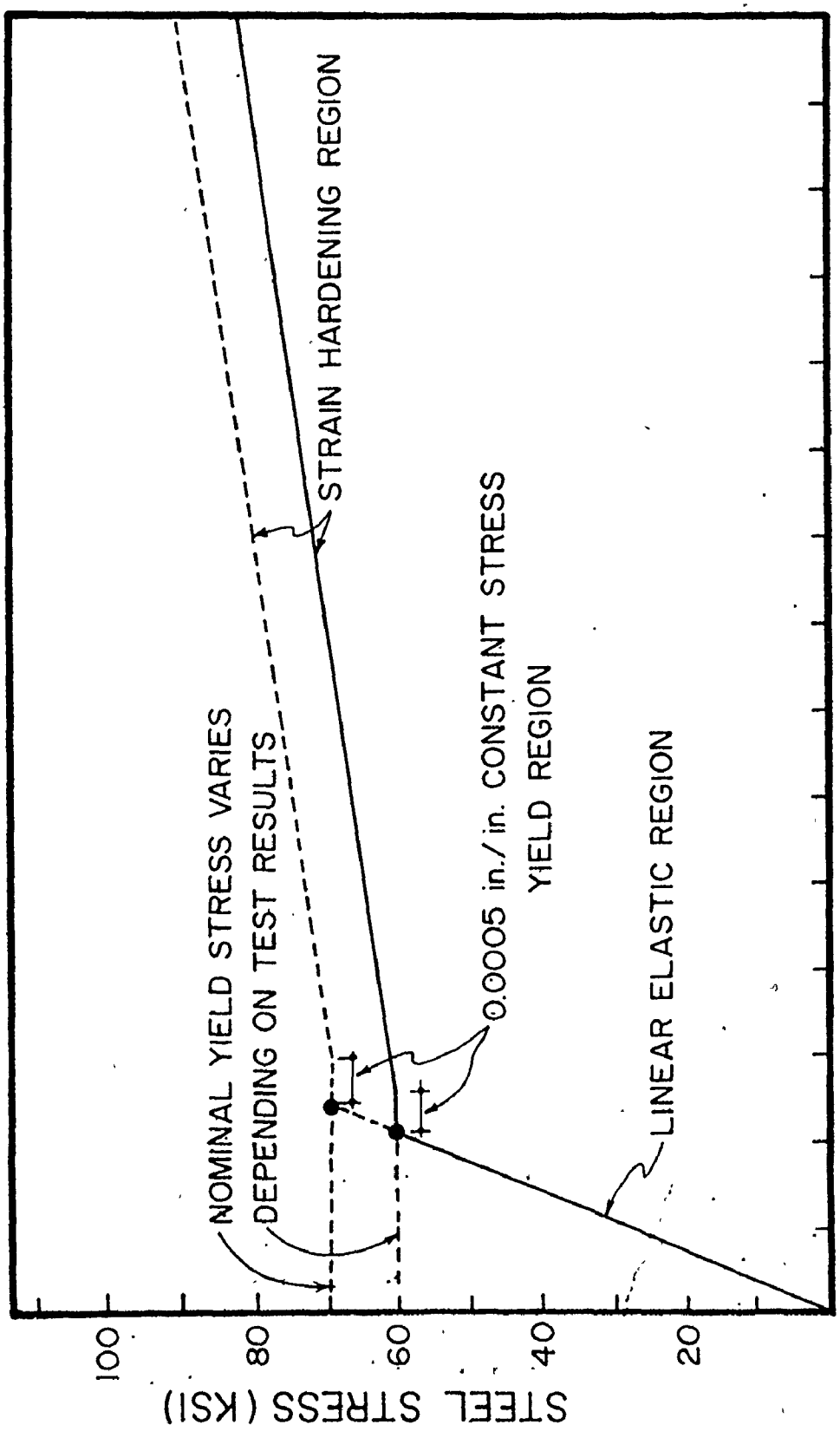


Fig. 3.3 IDEALIZED STRESS–STRAIN CURVE FOR STEEL REINFORCING BARS.

tested still requires restraint of some form if each joint is to be tested individually. As an alternative it was thought that a specimen with a large base member, arranged as shown in Fig. 3.4.c would give stability and enable the load to be applied either from above or below. However, this arrangement would yield only one test joint per specimen.

Mayfield et.al. (91) combined two specimens of the type shown in Fig. 3.4.c to form a specimen similar to the one shown in Fig. 3.4.d. The excess weight and bulk of this specimen and the relatively complex reinforcing required if smaller dimensions were used led to the modified shape developed for this investigation.

Figure 3.4.e shows the shape of the test specimen adopted for this experimental program. It was a relatively simple matter to fix the test specimen to the laboratory floor in a way which would ensure the independent testing of each corner joint.

The dimensions chosen were considered to meet the demands previously outlined. Detailed dimensions on a schematic drawing of the joint specimen and a photograph for the test setup are included in Section 3.4.2.

3.4 Preparations and Equipment for Joint Tests:

This section briefly describes the procedures and equipment used to prepare the joints for testing. In addition, the test equipment and measuring devices are described.

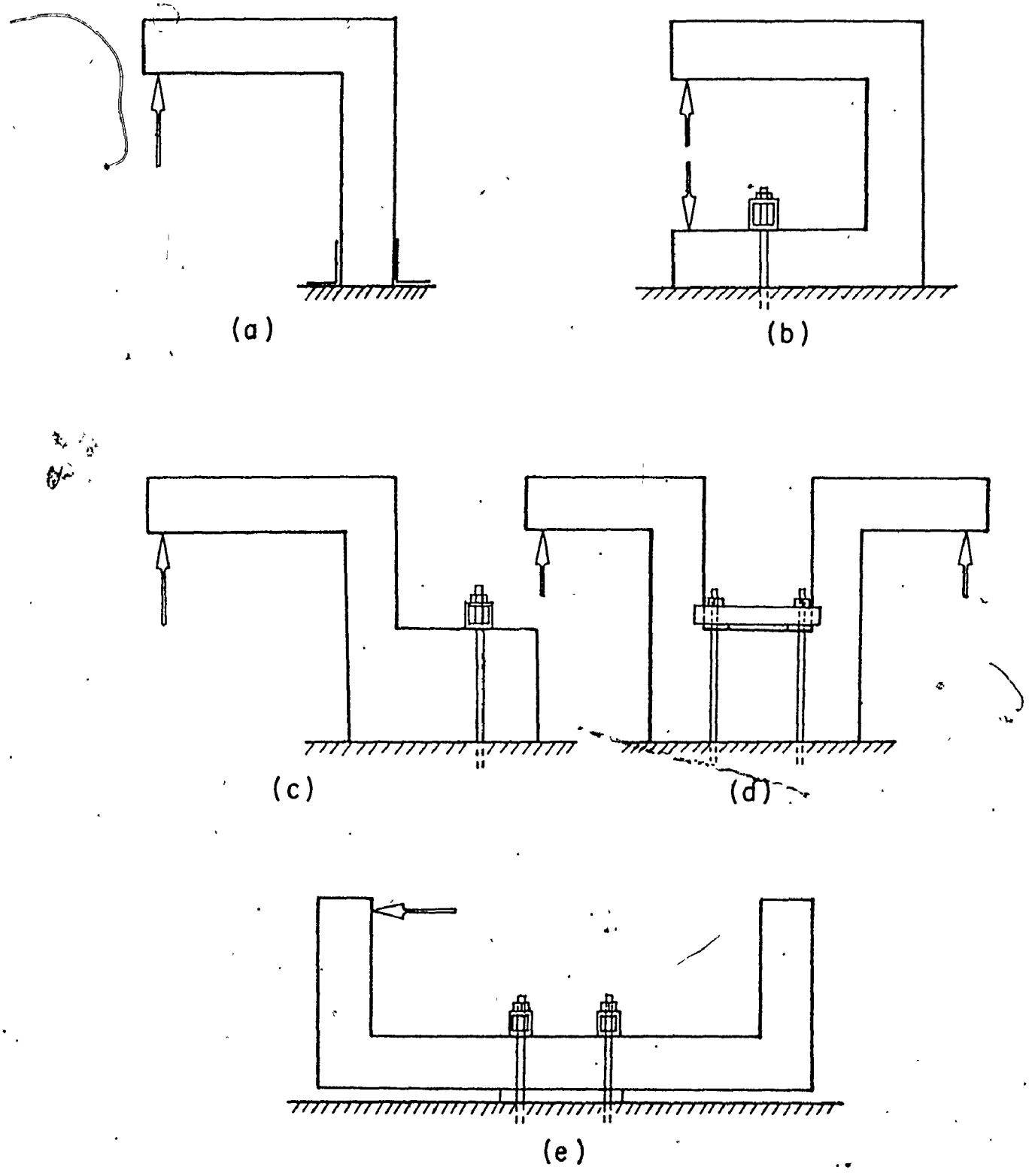


Fig. 3.4 SKETCHES OF POSSIBLE TYPES OF TEST SPECIMENS.

3.4.1 Forms and Steel Reinforcing Cages:

As was mentioned in Section 3.2.1, one specimen having two joints and six cylinders were cast at one time. Steel forms available from previous concrete research in the Applied Dynamics Laboratory were used to fabricate the test specimens. The use of the steel forms permitted fabrication to close dimensional tolerances (approximately ± 1/16 in.)

The reinforcing steel cages for the joints were made by tying the bars to 1/4 inch diameter stirrups spaced at 6 inch intervals. These stirrups were accurately produced on a specially designed bending device. The reinforcement was firmly wired into each corner of the stirrups. After each cage was completed, the spacing of the reinforcing steel was rechecked. If necessary, adjustments were made by prying apart with a lever system, and re-wiring the bars to the ties. Small steel seats were used to correctly position and hold the cage in place in the form. These seats were kept out of the joint region to avoid any complicating effects in this critical region.

3.4.2 Preparation of the Specimens for Testing

After curing of the concrete, the specimens were moved to the test floor where they were prepared for testing. The following steps were required:

(a) Fixing the Specimen to the Test Floor:

The frame was accurately positioned between the two columns which had been prestressed to the floor. Figures 3.5 and 3.6, show two 8 in. by 8 in. hollow structural sections (C and D) 3' - 0" apart which were used to fix the specimen to the floor. The specimen was kept six inches above the floor level by resting it on a 6 in. by 9 in. H.S.S. (E). The length of the elevating H.S.S. was made 3' - 8" to match the outside dimension between the other two H.S.S.'s which were placed on the top as shown in Figs. 3.5 and 3.6. Using four, 2 1/2 inch diameter bolts the 2 top H.S.S.s were then tightened to the specimen. The amount of tightening was determined so that the bearing stress in the concrete under the H.S.S.s was approximately 500 psi.

(b) Installation of Loading Equipment:

Two hydraulic jacks (20 and 50 kip) capacity were installed as shown in Figs. 3.5 and 3.6. The top jack was mounted on a sliding plate and was continuously lowered during loading at amounts equal to the vertical deflections measured by the dial gauge designated with the letter A in Fig. 3.5. The sliding plate was bolted to a channel which spanned between the two columns and was in turn bolted to each of them with six 1 inch diameter bolts as shown in Fig. 3.6. A roller bearing plate was mounted on the frame opposite to the loading cell, using two 3 inch angles joined by two 1/4 inch diameter threaded rods which were tightened around the vertical member of the specimen, to fix the angles in place. The bearing plate

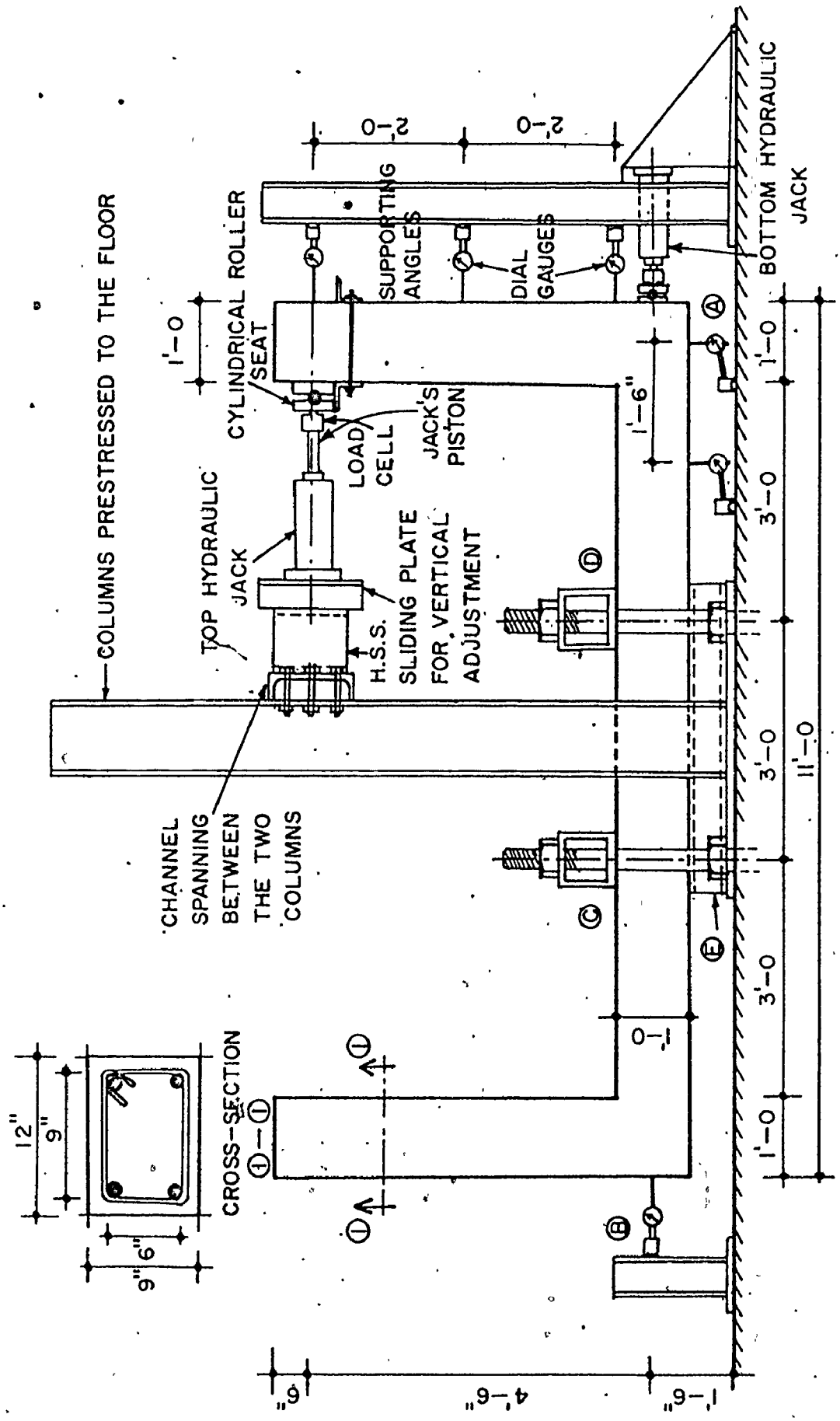


Fig. 3.5 SCHEMATIC DRAWING OF THE JOINT SPECIMEN AND TEST ARRANGEMENT.

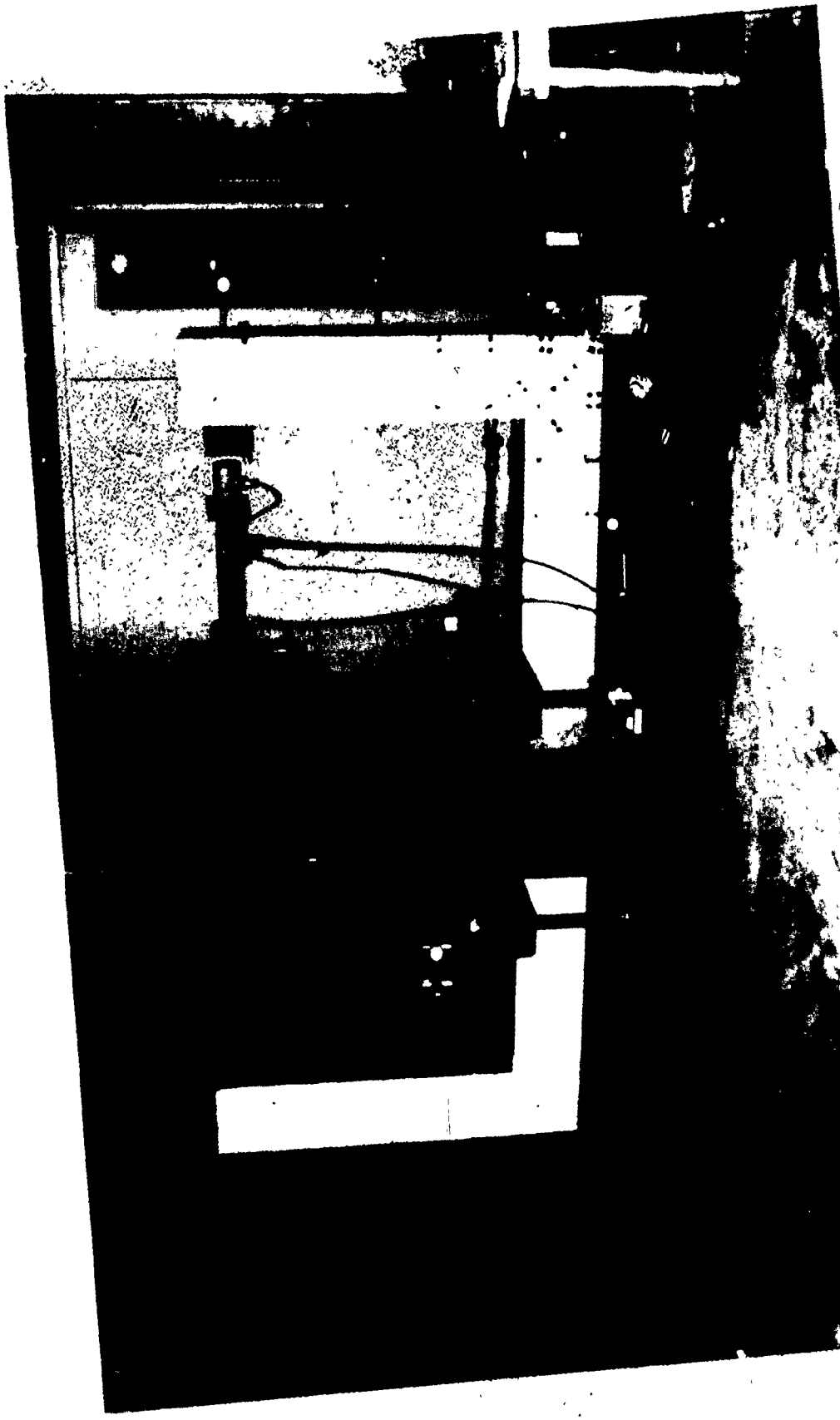


FIG 36 PHOTOGRAPH SHOWING SPECIMEN PREPARED FOR TESTING

was then simply rested on the projecting leg of the angle.

The bottom jack was mounted directly on a channel reinforced with three brackets. This arrangement was bolted to the floor using two, 2 1/2" diameter bolts. A roller bearing was then placed against the frame and was used to transfer the jack's force from the load cell to the frame.

Calibrated load cells were used to accurately control the applied loads. These were placed between the end of the piston and the bearing plate for both locations. During testing the load levels for both jacks were always maintained at the same value. This created a pure bending moment region in the horizontal member of the specimen. (The increase or decrease of the load level in the bottom jack to apply axial compression or tension on the horizontal member to simulate columns in actual structures is possible with this same arrangement.). However, to eliminate variation of this parameter, the testing program was confined to cases with pure bending moment in the horizontal member.

(c) Instrumentation for Deflection and Strain Measurements:

For each test, six dial gauges were positioned at the locations shown in Fig. 3.5. The precision with which the dial gauges could be read was one half of a division which represents 1/1000 of an inch. The three gauges mounted opposite the vertical member were installed to facilitate reading the horizontal deflection of the vertical member. The two dial

gauges placed underneath the horizontal member were installed for vertical deflection measurements. All gauge readings were then used to calculate joint rotations. The gauge which was positioned at the joint not being tested (designated by letter B in Fig. 3.5, was provided to detect any sliding of the specimen. Since the loads applied by both top and bottom jacks were intended to be kept very close to being balanced no sliding was expected.

The Demec Strain Indicator, which is a demountable mechanical strain gauge, referred to in section 3.2.3, was also used for measuring the joint strains.

Demec points were installed as can be seen on the right hand side of the specimen shown in Fig. 3.6. The points were spaced at 8 inches, some of which were overlapped to get several readings at distances less than 8 inches apart. Two rows of Demec points were installed in the compression zone and one row was used at the level of the tension reinforcement. In the joint region several additional points were used to measure the strains at 45 and 135 degrees from the horizontal.

3.5 Summary

In this chapter, the basic material properties for the concrete and the steel used in the fabrication of the test specimens, the choice of the shape of the specimen, and the preparations and equipment along with description of the test setup for joint testing were discussed. Care was taken in the fabrication and testing of the joints and other test specimens

in order to reduce the influence of test variability. This was especially important because of the limited number of test specimens planned for this investigation. Additional aspects related to the testing procedure will be discussed in Chapter 4.

CHAPTER 4

MCMASTER UNIVERSITY JOINT TEST RESULTS

4.1 Introduction

The fabrication and instrumentation of the test joints, the materials used and their properties, and the joint test equipment were described in Chapter 3. The details of the joints tested, the test procedure and the basic test results are presented in this chapter. The test results are compared with results from the numerical analysis method in Chapter 7. Based simply on observations from the tests some preliminary observations regarding joint detailing were possible and are presented at the end of this chapter.

4.2 Requirements for Satisfactory Details:

At present there is no recognized procedure for the design of reinforced concrete joints and there are no comprehensive detailing standards to be complied with other than those associated with the general design of the intersecting members. A designer normally arranges the steel around the corner according to the existing practice of his company, or may be guided by the various publications of technical bodies, or by available detailing manuals which in fact do not specifically address the problem of joints. Some of the more popular corner details were discussed in Chapter 2. An

illustration of the speculative nature of this area of design is the fact that some of the details preferred in the past are now specifically described as details to be avoided (20, 63). It is apparent from Fig. 2.4 that the reinforcement was not specifically designed to resist the anticipated forces. Indeed, little distinction was made between corners with applied load tending to open the angle and those with the applied load tending to close it. It appears that ease of construction was considered to be of overriding importance.

One of the aims of this investigation was to establish experimentally joint details which would perform satisfactorily when positive bending moment is applied.

In order for a reinforcing layout to be considered satisfactory, reinforcement should be placed in such a way that the joint meets certain fundamental requirements. These are summarized as follows:

1. The joint should be capable of developing a moment at least as large as the maximum calculated failure moment in the intersecting members. Flexural failure may in this case occur outside the joint, which, then does not limit the load bearing capacity of the remainder of the structure.

2. If the first requirement is not met, then the joint should satisfy the ductility necessary for redistribution of forces in the structure without brittle failure of the joints.

3. Cracks preferentially occur on the inside of joints subjected to tension. The widths of corner cracks should therefore be limited to an acceptable value under service loads.

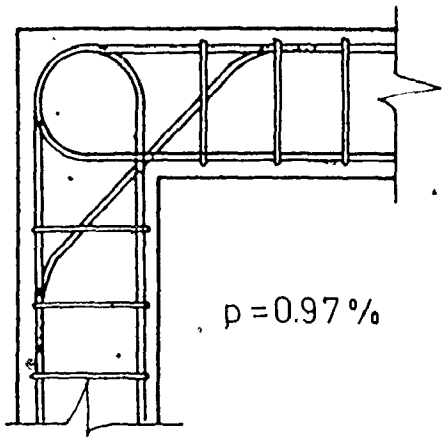
4. The reinforcement should be easy to fabricate and to fix in place. One measure of the ultimate worth of this investigation will be its influence on design practice where practical considerations and simplicity in construction are important requirements.

The experimental contribution towards finding this satisfactory layout consisted of tests on six corner joints comprising four different details. Two of these were reproduced arrangements reported to be satisfactory by Mayfield et. al. (74,91) and Nilsson (97). Three of the four details tested were considered satisfactory, (two of these were developed by the author). One of the details was considered to be superior to the others. Two more tests were performed on this same detail; one to confirm the original behaviour and the other to determine the influence of increasing the ratio of steel reinforcement on the behaviour of the joint.

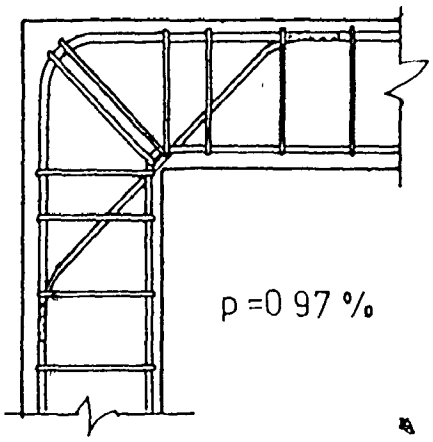
4.3 Choice of Different Details to be Tested

4.3.1 General

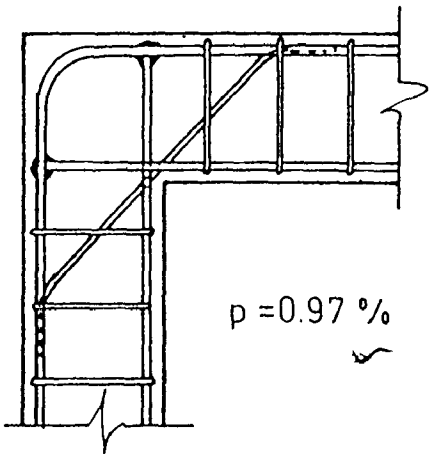
The corner details studied in this investigation are shown in Fig. 4.1. As was described in Chapter 3, two joint specimens were cast at the same time. Initially, the



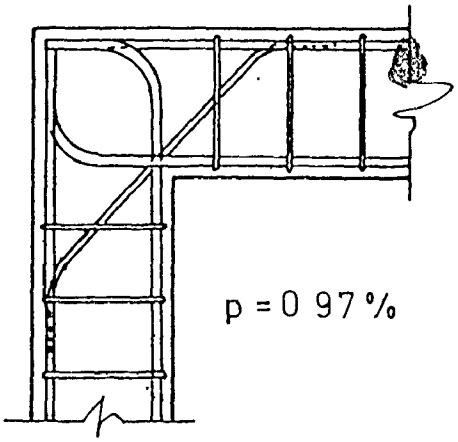
(a) DETAIL (1)
(from Reference 97)



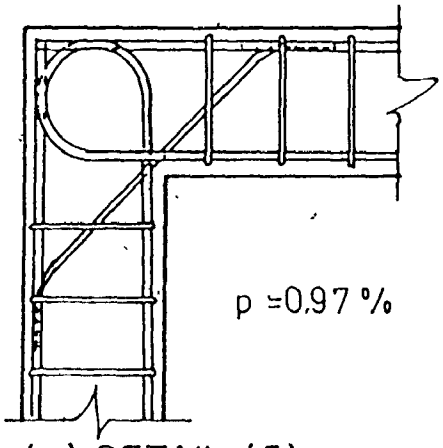
(b) DETAIL (2)
(from Reference 91)



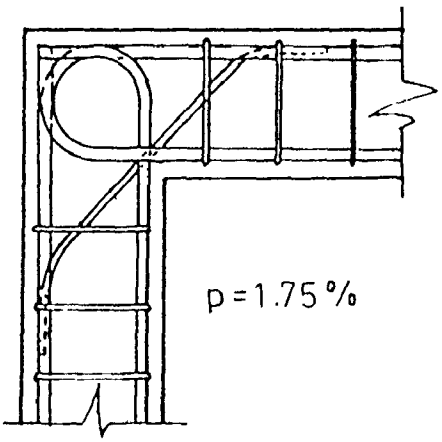
(c) DETAIL (3)



(d) DETAIL (4)



(e) DETAIL (5)



(f) DETAIL (6)

Fig. 4.1 STEEL REINFORCEMENT DETAILS USED IN THE TEST PROGRAM.

experimental program studied two corner joint details which were reported in the literature (91, 97) as being highly efficient details. Some progress toward the development of new details was then made using a series of four tests. The reasons for the choice of each of the details is discussed in this section.

In the experimental part of this study, details which had been shown to be inadequate in other experimental programs (74, 91, 97), were avoided. It is relevant to mention again that some of these inadequate details are widely used and are recommended in some manuals (63, 100) for reinforced concrete detailing practice. Some of these details were shown in Fig. 2.1 to 2.5.

4.3.2 Descriptions of the Joint Details Tested:

4.3.2.1 Detail (1):

Detail (1) was proposed as the result of one of the most comprehensive experimental programs on the subject of joint detailing. The detail developed by Nilsson (97) in Sweden, as shown in Fig. 4.1a, is governed by the following recommendations. "The main reinforcement from the inside tension face is to be extended out into the joint, from where it is to return into the compression zone of the same cross-section to the same level as the inclined reinforcement. The main reinforcement is to be laid as near to the face as the cover specified in the standards will allow. The main

reinforcement is to be designed on the basis of the forces and moments in the intersecting members. The sections are to be regarded as singly reinforced and the calculations can be carried out by elementary bending theory, the effect of the inclined reinforcement being ignored. The spacing of the bars should be the same at both sections. The inclined reinforcements is to have an area approximately 50% of that of the largest main reinforcement. The spacing of the inclined bars should be the same as for the main reinforcement in the intersecting members."

This detail is now adopted by the building authorities in Sweden and thought of as a very efficient detail.

The upper limit on the reinforcement percentage was set at 1.2% for 57 ksi (Grade Ks 40) steel and 0.8% for 85 ksi (Grade Ks 60) steel where these are all based on using around 4500 psi concrete.

In the current investigation, the amount of steel chosen for the main reinforcement was 2 No. 6 bars having an area of 0.884 $in.^2$ or p = 0.97%. The inclined bars were 2 No. 4 bars having an area = 0.393 $in.^2$, or 44.4% of the area of the main reinforcement. The quality of the high strength steel and concrete were discussed in the previous Chapter.

The results of testing this detail are discussed in Section 4.5.

4.3.2.2 Detail (2):

Detail (2) was the second detail tested in this investigation which was developed by other researchers. The detail shown in Fig. 4.1. (b), proposed by Mayfield et.al. (91) was found to be the most efficient detail as a result of testing over one hundred joints. The joints tested by Mayfield et.al. were reported in Chapter 2 and the details were reproduced in Fig. 2.12 and 2.13. They adopted an obvious method of using stirrups to strengthen some of the simple yet deficient reinforcement details.

As shown in Fig. 4.1.b the main tension reinforcement had to be bent very tightly around the inner corner. It was found very difficult to get adequate concrete cover in the corner when the bending radii rules specified by the current codes were observed. Mayfield et.al. did not recommend any specific amount of steel for stirrups. Instead they concluded that a suitable combination of stirrups and inclined reinforcement according to Details 13, 18, 26, in Fig. 2.13 provides a promising method for reducing crack widths and achieving reasonable ductility, and that this detail will be capable of producing corner efficiencies exceeding 100%.

One shortcoming of using stirrups in corners is that the practical use of such a layout would likely be restricted to beam-column joints. For joint corners in retaining walls for example, stirrups could be difficult to place.

In this investigation Detail (2), was very similar to detail (26) by Mayfield et.al. The only change was that the (inclined) diagonal stirrups were replaced by 2No. 4 bars, the area of which was 0.393 in.^{2}. The main tension reinforcement was maintained at 2 No. 6 bars as for Detail (1) (p = 0.97%).

Two No. 3 two-branch stirrups having an area of 0.44 in.^{2} (or stirrup to main tensile reinforcement ratio of 0.5) were used to join the tension and compression reinforcement along the line from the inside to the outside of the corner. It was anticipated that the stirrup area might be low. However, it was used as an initial guideline for the behaviour of this detail. Increasing the amount of stirrups in the corner would have made the placing of concrete such a difficult process that incomplete compaction in the joint region would have been quite possible. Discussion of the effect of increasing the stirrup area on the behaviour of the corner joint is included in Chapter 7.

4.3.2.3 Detail (3):

Detail (3) was the first of a series of four details which were tested in an attempt to develop an efficient and simple detail. A few difficulties were encountered in fabricating the steel for Detail (1). The exact spacing required between tension and compression reinforcement was not easy to obtain. Bending steel 180^o was found to be impractical. Also fitting beam and column stirrups was rather difficult.

The area of stirrups which would be required using Detail (2) to obtain reasonable efficiency was anticipated to be large. This would be particularly true considering the crowding of the steel in the joint region.

It was thought that Detail (3), shown in Fig. 4.1.c could be a step towards simplifying the joint detail without sacrificing the desired efficiency.

From review of other tests (97) it appeared to be of prime importance to confine the core of the joint region to avoid diagonal tension failures, hence the detail was designed as follows:

1. The tension reinforcement was composed of 2 No. 6 straight bars in each of the two intersecting members. These were extended to meet and were welded to the compression reinforcement near the outer faces.

2. The compression reinforcement consisted of two No. 6 bars bent at 90 degrees as shown in Figure 4.1.c.

3. The diagonal (inclined) reinforcement of 2 No. 4 bars were also added. This type of reinforcement was common to all the tests performed in this investigation and as it is discussed in Chapter 7, was found to be necessary.

Detail (3) was simple to fabricate. However, the availability of welding equipment is essential.

4.3.2.4 Detail (4) :

Detail (4), shown in Fig. 4.1.d was developed to avoid either welding the reinforcing steel or bending the

steel more than 90°, or using stirrups. The tension reinforcement consists of 2 No. 6 bars in each intersecting member, each bent at 90 degrees. The steel is placed in the cage so that the end of the 90° hook reaches the outer corner of the joint (with the appropriate cover). The addition of concrete stubs to one or both of the intersecting members would be beneficial for providing end anchorage especially for larger steel sizes or smaller member dimensions (95). Compression steel was simply 2 No. 6 straight bars stopped at the outer corner of the joint. Obviously this arrangement is not adequate for loads tending to close the joint. Inclined diagonal bars of the same size for the three previous details were added.

4.3.2.5 Detail (5) :

Detail (5) was made essentially the same as detail (4). The main purpose was to confirm the results obtained from the test on detail (4). One change made to accommodate the possibility of load reversals was that compression steel was made continuous using 90° bent bar as shown in Fig. 4.1.e. This detail was also suggested by Park and Paulay, for joints with large interesting members, in their text (103) which was published after this experimental program was conducted.

4.3.2.6 Detail (6) :

The effect of higher percentage of reinforcement on the behaviour of the joint was examined by testing Detail (6). The layout was exactly the same as that for Detail (5), with only the amount of reinforcement changed. Two No. 8 bars were used for tension reinforcement in each member with of compression ste (p = p' = 1.75%). The

inclined diagonal bars were made of 2 No. 6 bars, with an area ratio of 56%, of the area of the main tension reinforcement.

4.4 Test Procedure

The testing rig described in Chapter 3, was used to load the test specimens to failure. The test procedures for Details (1), (2), (3), & (4) were identical, but different from those for Details (5) and (6) which were subjected to 4 cycles of load reversals before loading to failure. Test procedures for the two groups are described in this section.

4.4.1 Incrementally Increasing Load Tests

In this group of tests the two jacks (top and bottom) shown in Fig. 3.5 were activated simultaneously. The jacks were both in compression and were acting in opposite directions. The top jack pushed the vertical member to open the corner. This resulted in a constant shearing force and linearly changing bending moment on the vertical member. The bottom jack applied the same amount of force as the top one to eliminate the tension from the horizontal member which is, therefore subjected to pure bending moment only.

Initial readings were taken, then the loads were applied in increments. After each 1.0 kip increment, dial gauge readings and Demec readings were taken to measure deflection and strains. Also the cracks were traced using a magnifying flashlight and were marked to show the crack propagation at the different load levels. The 1.0 kip increments were decreased to.5 kip increments near the failure loads.The load levels for both

jacks were continuously monitored and adjusted to maintain constant loads during the process of taking measurements.

After all the measurements were taken, the load was increased to the next load level and the whole procedure was repeated. This process was continued until failure of the specimen.

Strain measurements were not taken after the joint region was cracked or spalled in a way that measurements would no longer represent strains.

Dial gauges placed under the specimen to facilitate the calculations of joint rotations were removed when the joint was judged to be quite near failure. This precautionary measure was taken to avoid damage to the gauges.

Details of test results are discussed in the following sections of this chapter.

4.4.2 Test Procedure for Joints Subjected to Load Reversals, Details (5) and (6) :

After the completion of the first four tests it was decided that the next two specimens would be subjected to load reversals before the joint was loaded to failure. It was thought that this type of information would provide a worthwhile additional aspect for the evaluation of the adequacy of the detail. In addition to evaluating behaviour for bending in both directions, it could provide some indication of the joints' capabilities for withstanding cyclic effects.

Since cyclic loading was not within the original scope of this investigation and was included only to provide an indication of overall suitability of this detail, the test procedure was designed so that damage to the specimen during cyclic loading would be minimal. The precaution taken was to limit the inelastic deformations and thereby minimize damage, unless it happened that a defective detail was used which exhibits unfavourable characteristics.

Discussions with Tso* and Paulay** helped to clarify the author's ideas on this matter. It was decided that an arbitrary deflection be chosen and the joint be opened then closed to get a specified deflection in both directions. This procedure was to be repeated for four cycles, then the specimen was to be loaded to failure with the load opening the joint.

The criterion by which the joint would be judged to be adequate was to maintain at least 85% of the original load after 4 cycles of loading to the specified deflection, and to maintain its ultimate capacity when finally loaded to failure.

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* Private communication with W.K. Tso, Professor & Chairman Department of Civil Engineering, McMaster University, Hamilton Ontario, Canada.

** Private communication with T. Paulay, Professor, Department of Civil Engineering, University of Canterbury, New Zealand.

An arbitrary deflection of 1.00 inch was chosen for both joints (5) and (6). This amount of deflection, which represented nearly four times the measured deflection for Detail (5) at the theoretical working load level, was chosen so that the main reinforcement would be in the vicinity of yielding. This precaution was taken to avoid any possibility of severe damage during load reversals. This aspect was of great concern to the author because of the limited number of tests which were planned for this investigation.

When the top jack was pushing to open the joint, the test procedure was the same as that discussed in the previous section for Details(1), (2), (3) and (4). However, when the top jack was pulling to close the joint, the bottom jack was not activated thereby allowing a compressive force equal to the load applied by the top jack to be transferred to the horizontal member. Strain measurements were not taken after the first cycle of loading was completed.

4.5 Test Results

4.5.1 General Behaviour

The test procedure and arrangements of steel reinforcement were discussed in the previous sections. In this section and the following sections the test results and some preliminary observations are presented.

The load-deflection curves for the six joints are shown in Fig. 4.2. Load-deflection or moment-rotation curves are considered to be reliable bases for comparisons of

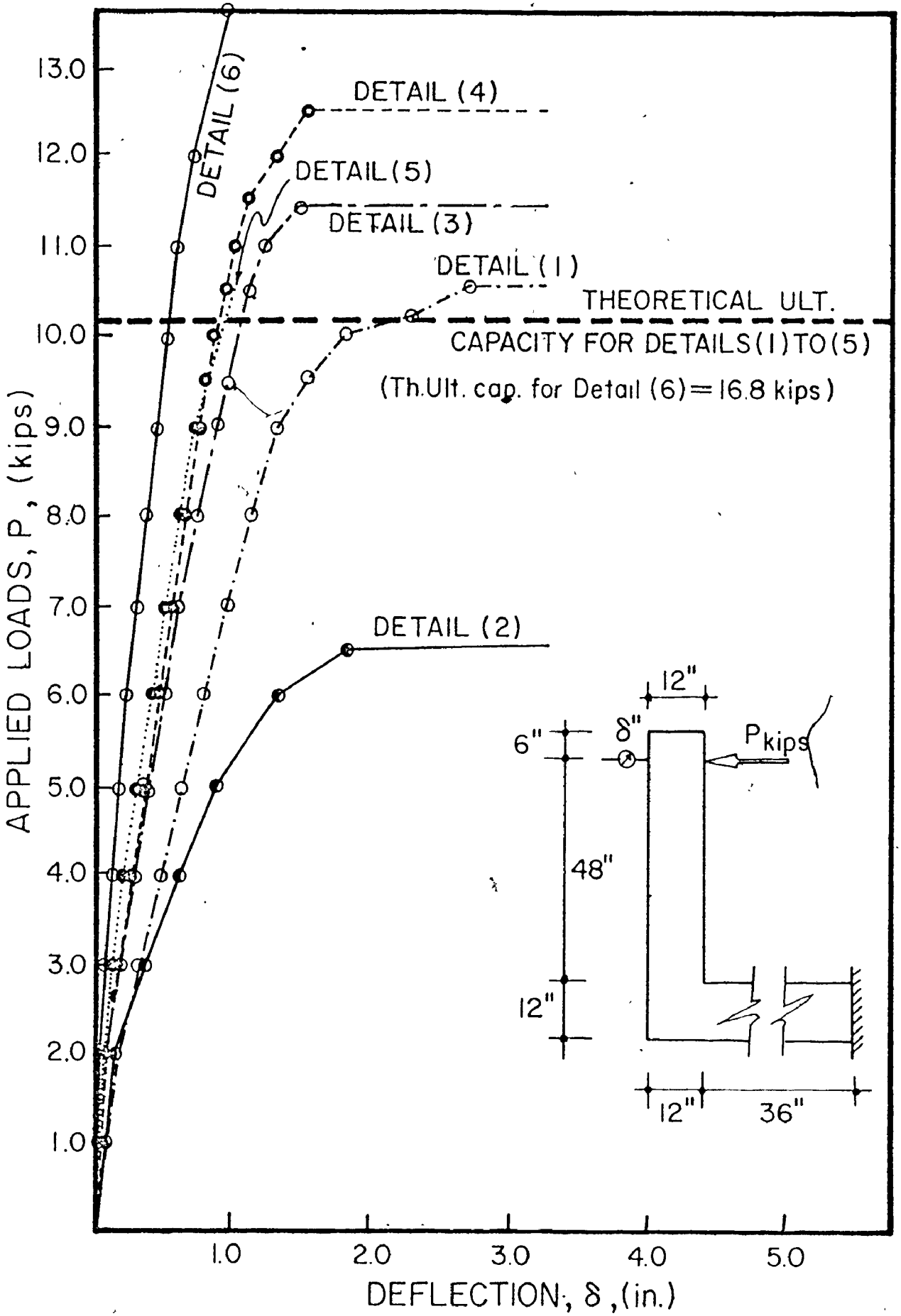


Fig 4.2 FCTION 'S TESTED T DETA

behaviour of structures since the basic test measurements are used.

From Figure 4.2, it can be seen that the results are divided into four distinct behaviour groups. Detail (2) has a very low load capacity and comparatively unfavourable load-deflection characteristics. Detail (1) which has an intermediate load capacity also has an improved load-deflection characteristic. Details (3), (4) and (5) have capacities which are higher than Detail (1) and the load-deflection curves are quite consistent. Detail (6) was the only joint tested with a different percentage of reinforcement. A direct comparison between the capacity of this joint and of the others is not valid. However, the joint capacity was close to the calculated capacity of the intersecting members according to the ultimate strength theory using the Whitney Stress Block (96).

Figure 4.3 shows the moment-rotation curves for all the details. The same general characteristics as for the load-deflection curves were observed except that the behaviour of Detail (1) was closer to that of Detail (3), (4) and (5).

The load-deflection curves for Details (5), and (6) shown in Fig. 4.2 are for the opening part of first loading cycle only. The full load-deflection curves for all cycles of loading for these details are shown in Figs. 4.4 and 4.5 respectively.

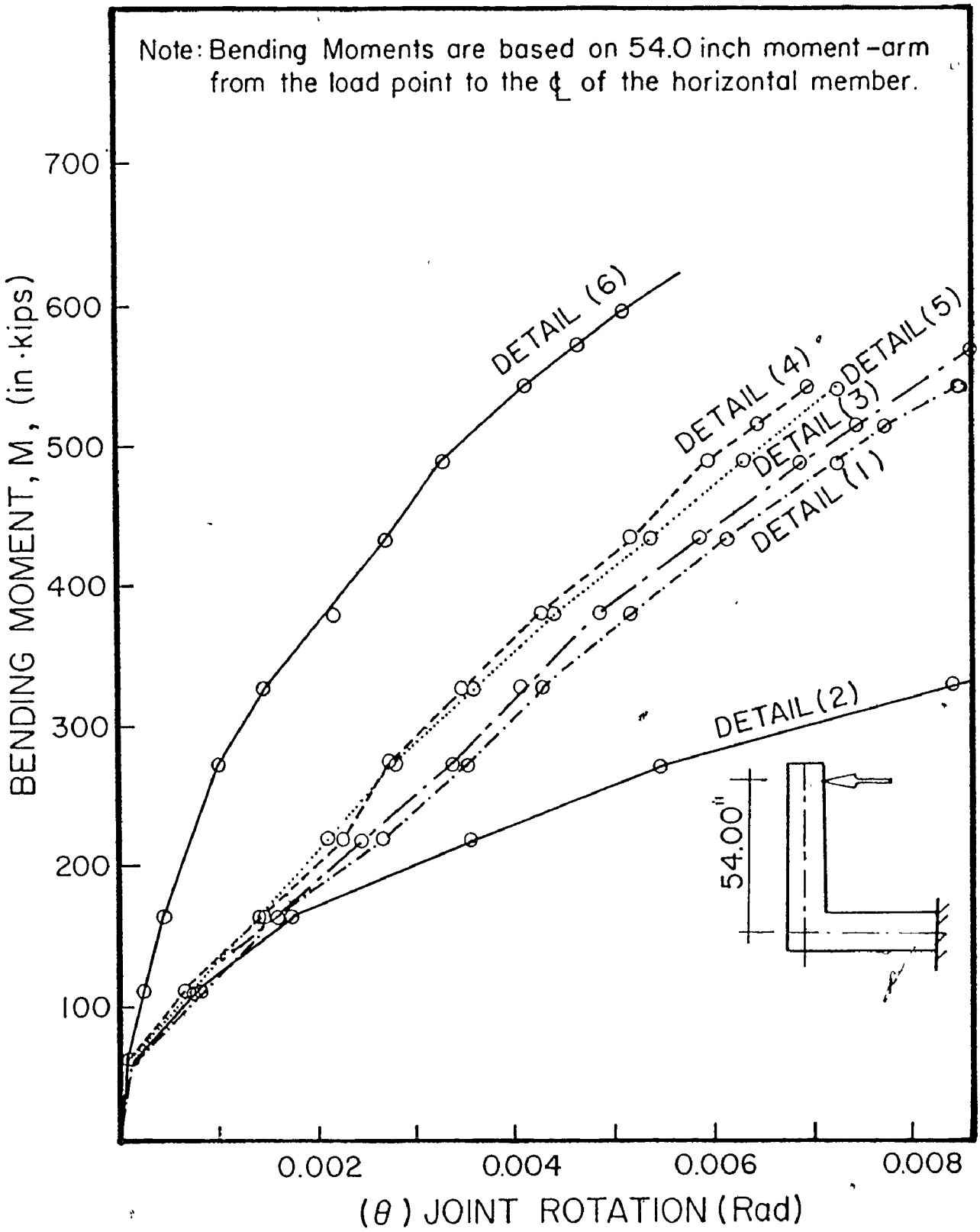


Fig. 4.3 MOMENT–ROTATION RESULTS FOR TESTED
T DETAILS

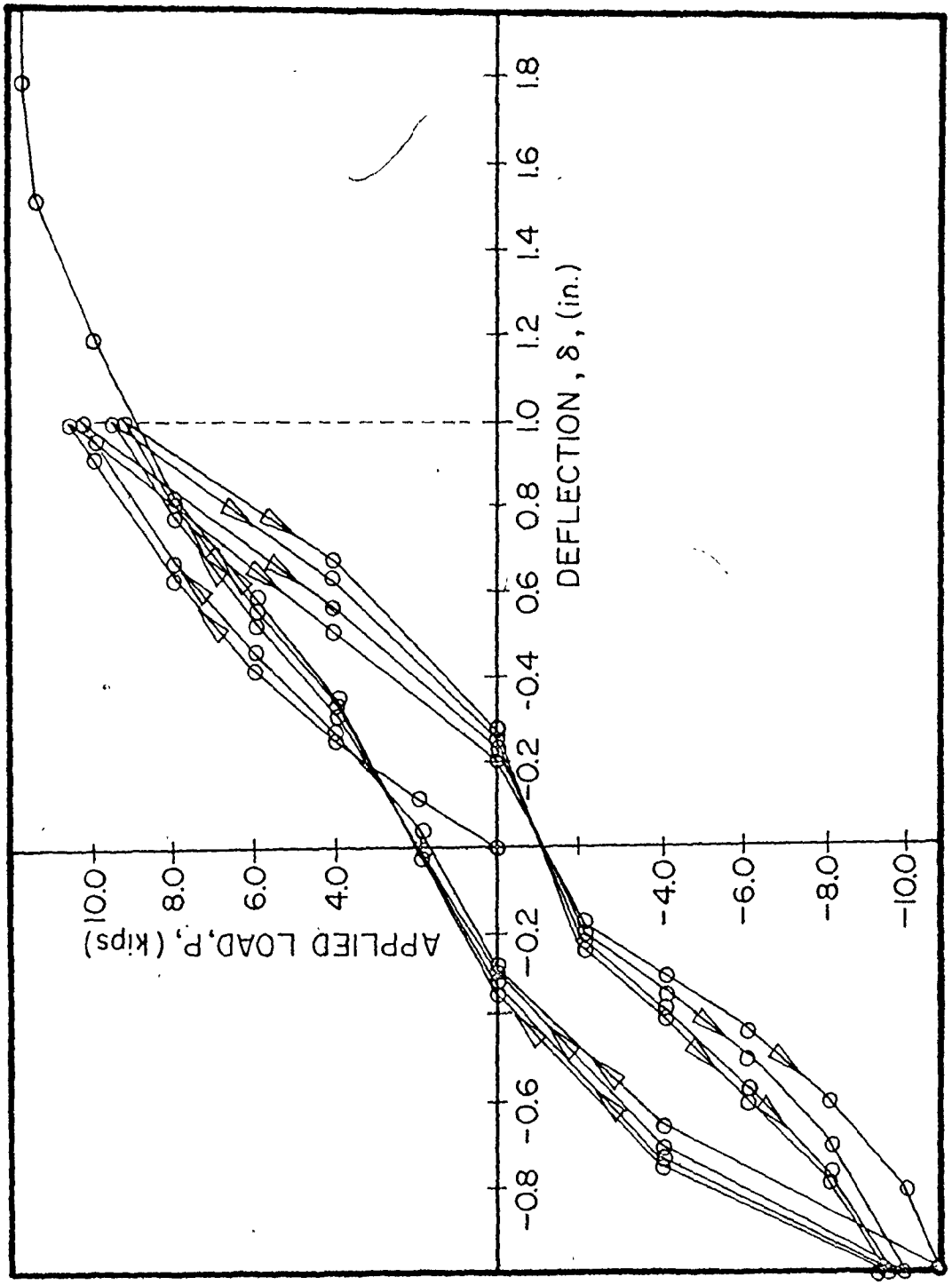


Fig. 4.4 LOAD-DEFLECTION RESULTS FOR 4 CYCLES OF LOADING, DETAIL (5)

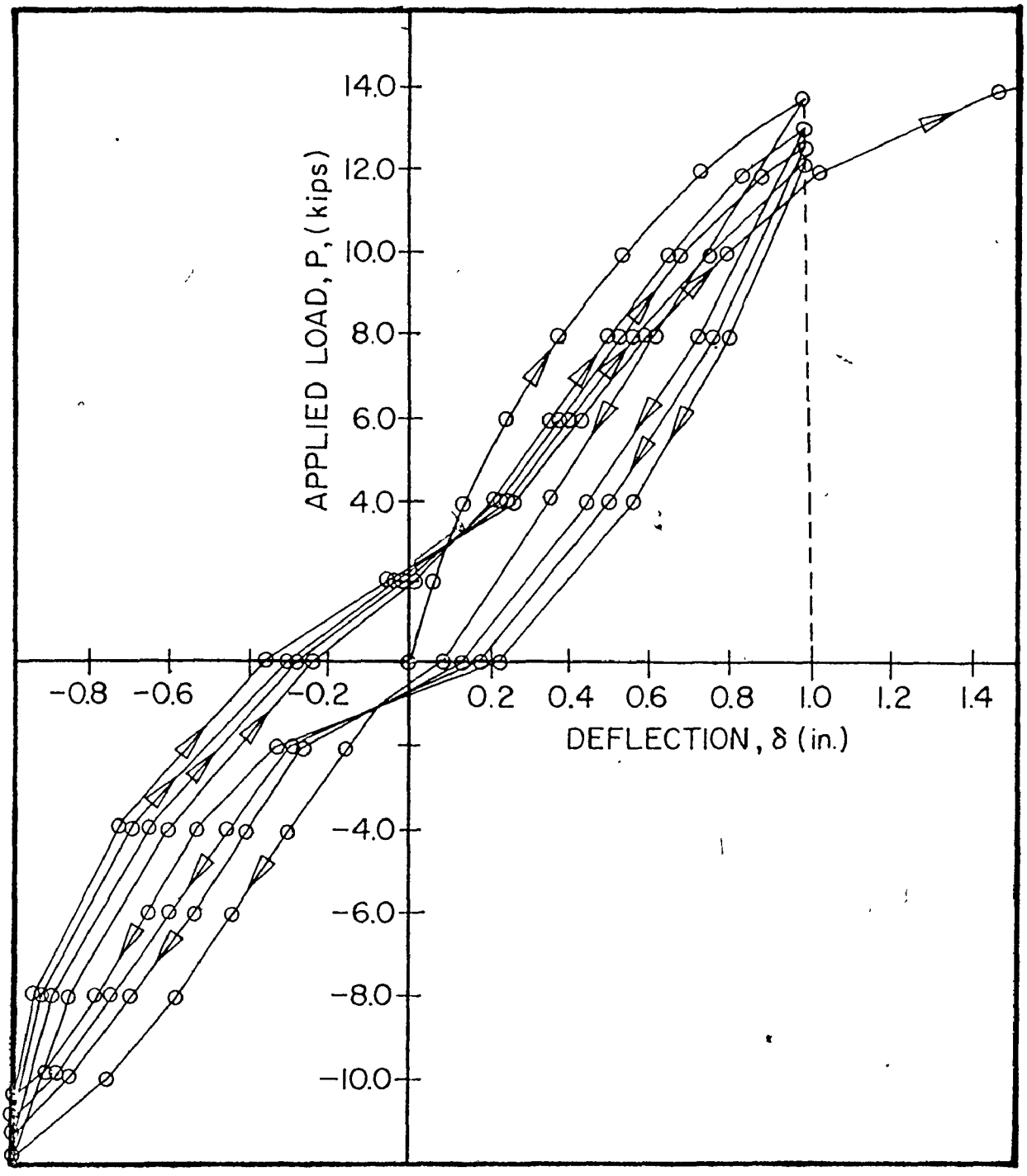


Fig. 4.5 LOAD–DEFLECTION RESULTS FOR 4 CYCLES OF LOADING, DETAIL (6)

4.5.2 Cracking Patterns

The cracking patterns at failure for Details (1) to (4) and after the first cycle of loading for Details (5) and (6) are presented in Figures 4.6 through 4.11. These drawings are scaled reductions of tracings made on the tested specimens.

The numbers shown beside the cracks represent the load levels at which the cracks had propagated to the points where the numbers are written.

From Figures 4.6 to 4.11 it is clear that the flexural cracks in the two intersecting members are not identical for all the tested specimens having equal amounts of steel reinforcement. However, crack spacing and crack propagation are fairly consistent.

Crack widths were not measured in this investigation but visual observations didn't show any great differences between different specimens for regions outside of the joint. However, major differences for cracking in the joint region between some of the details were observed.

Table 4.1 shows the measured crack spacings outside the joint region for each test specimen. Average, minimum and maximum crack spacing are given for every specimen. The mean and standard deviation of all the crack spacings of the specimens with equal amounts of steel reinforcement are also given.

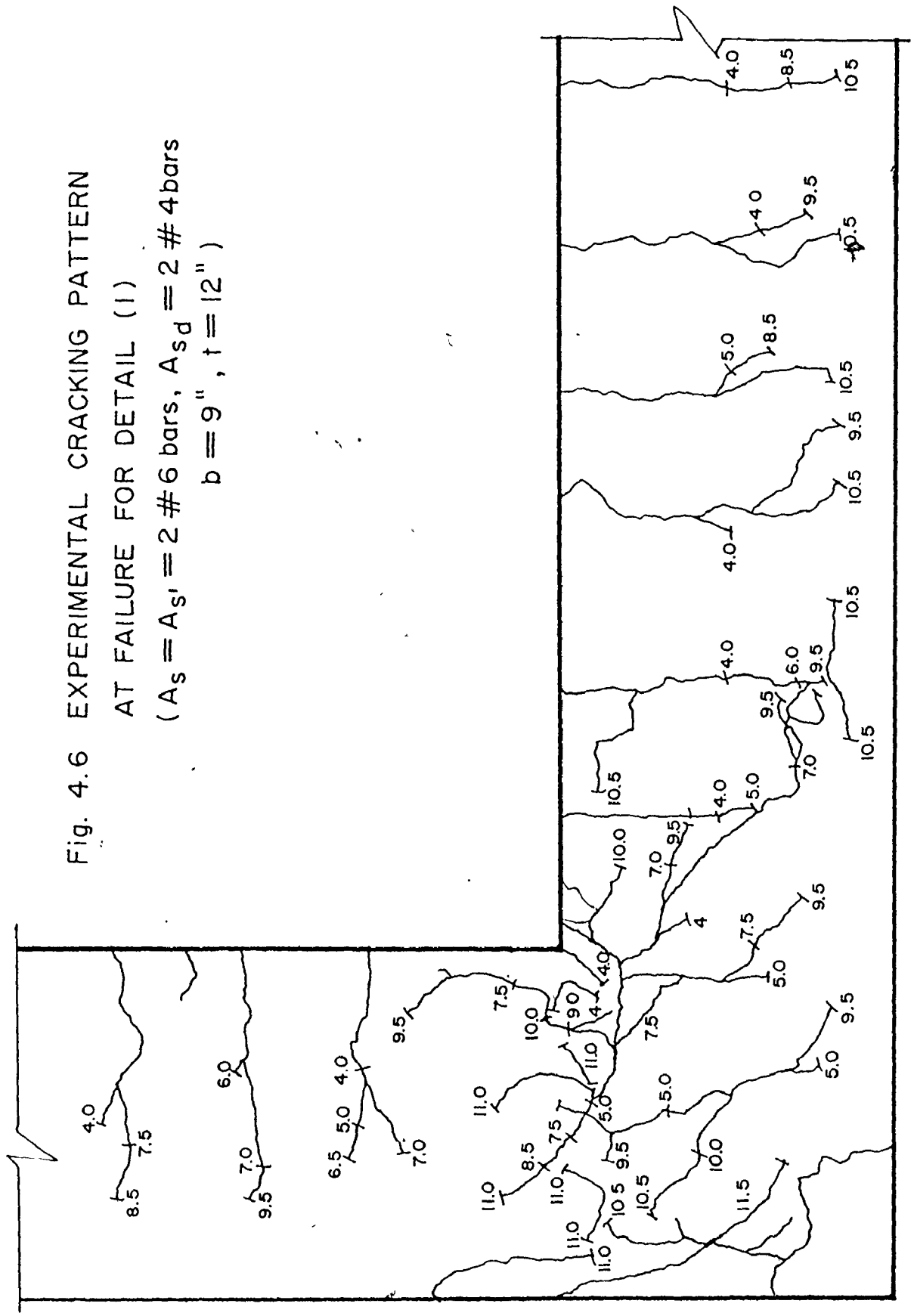


Fig. 4.6 EXPERIMENTAL CRACKING PATTERN AT FAILURE FOR DETAIL (1) ($A_s = A_{s'} = 2\,\#6$ bars, $A_{sd} = 2\,\#4$ bars, $b = 9''$, $t = 12''$)

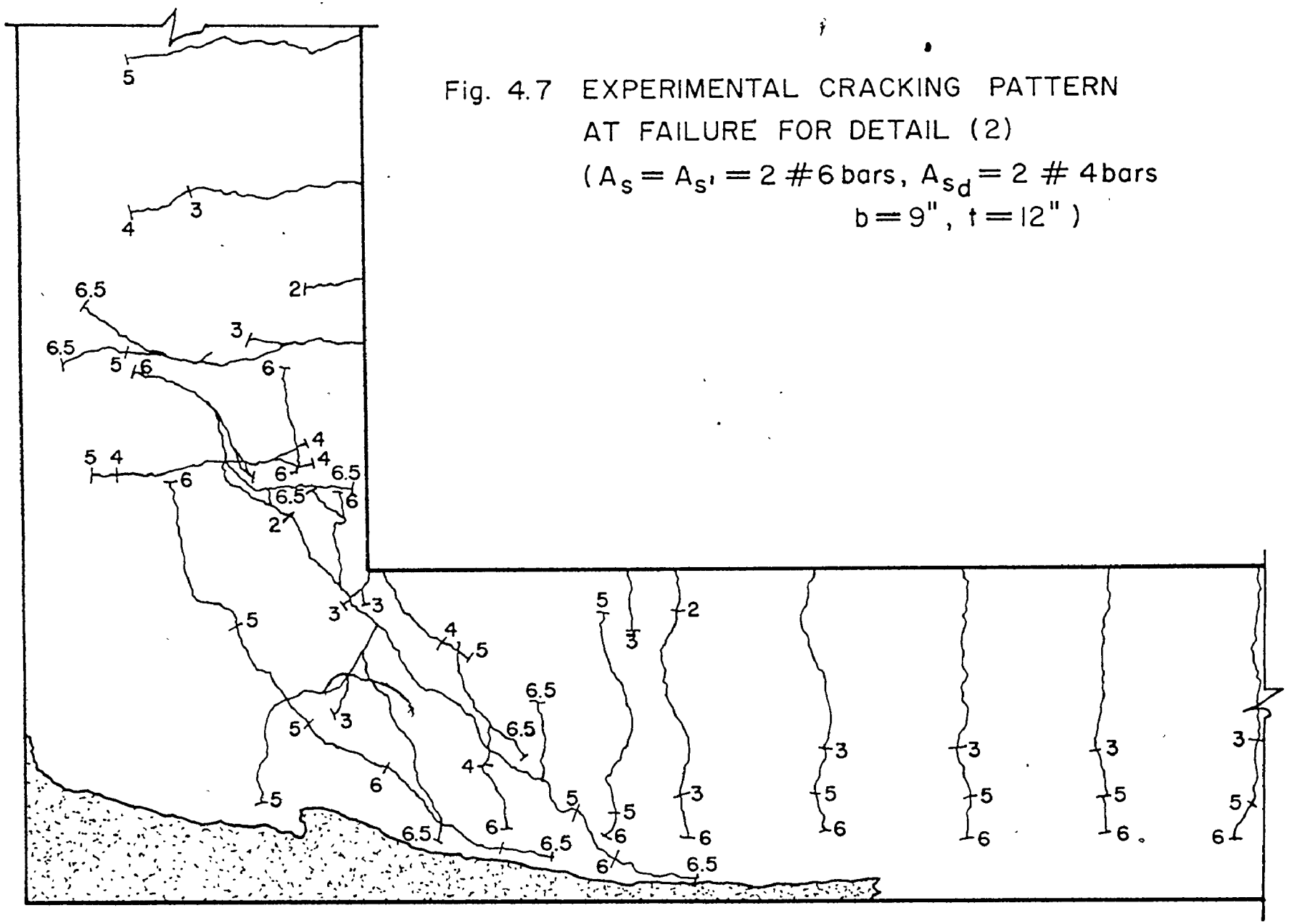


Fig. 4.7 EXPERIMENTAL CRACKING PATTERN AT FAILURE FOR DETAIL (2)
($A_s = A_{s'} = 2\ \#6$ bars, $A_{sd} = 2\ \#4$ bars
$b = 9"$, $t = 12"$)

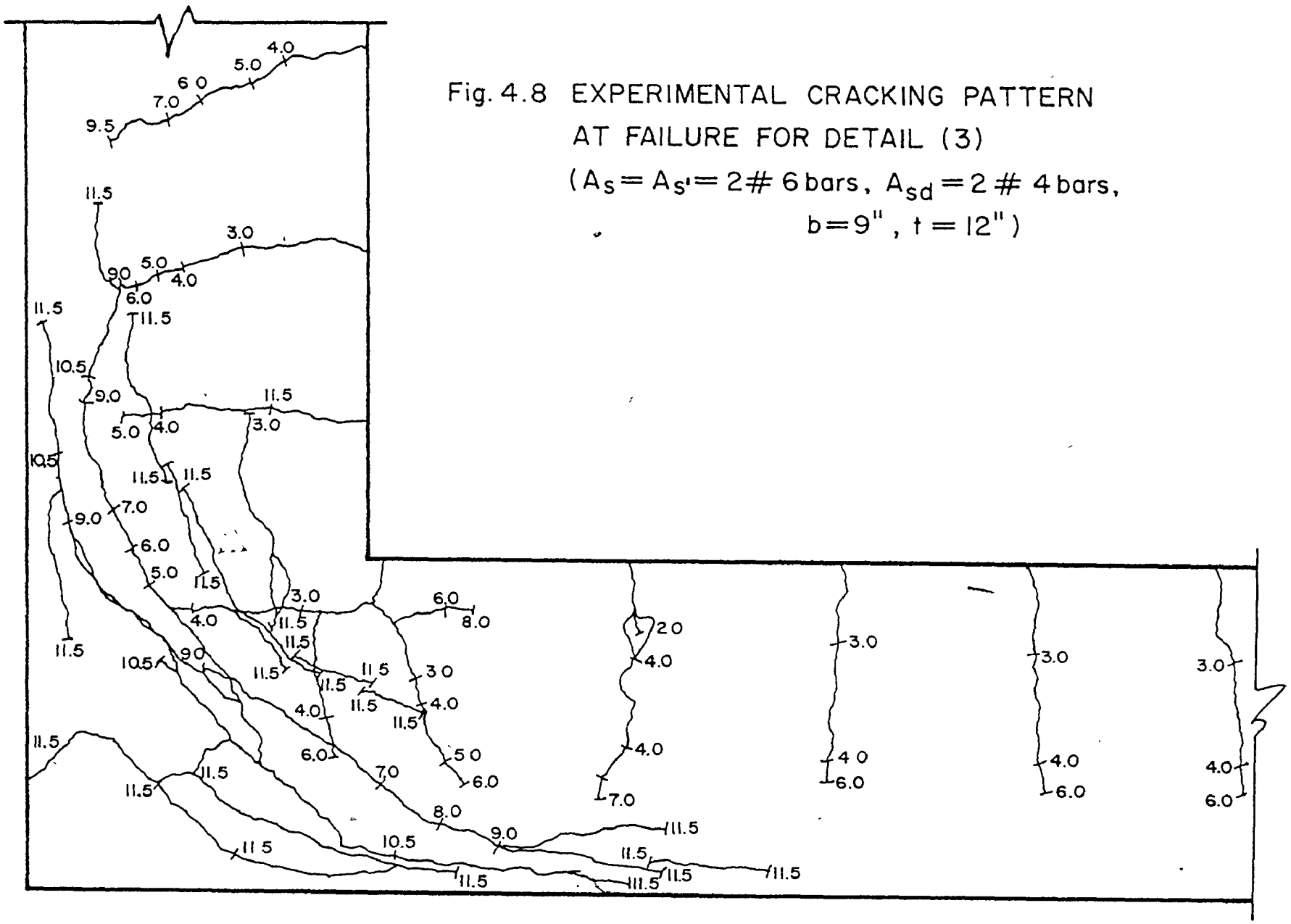


Fig. 4.8 EXPERIMENTAL CRACKING PATTERN AT FAILURE FOR DETAIL (3)
($A_s = A_{s'} = 2\#6$ bars, $A_{sd} = 2\#4$ bars, $b = 9''$, $t = 12''$)

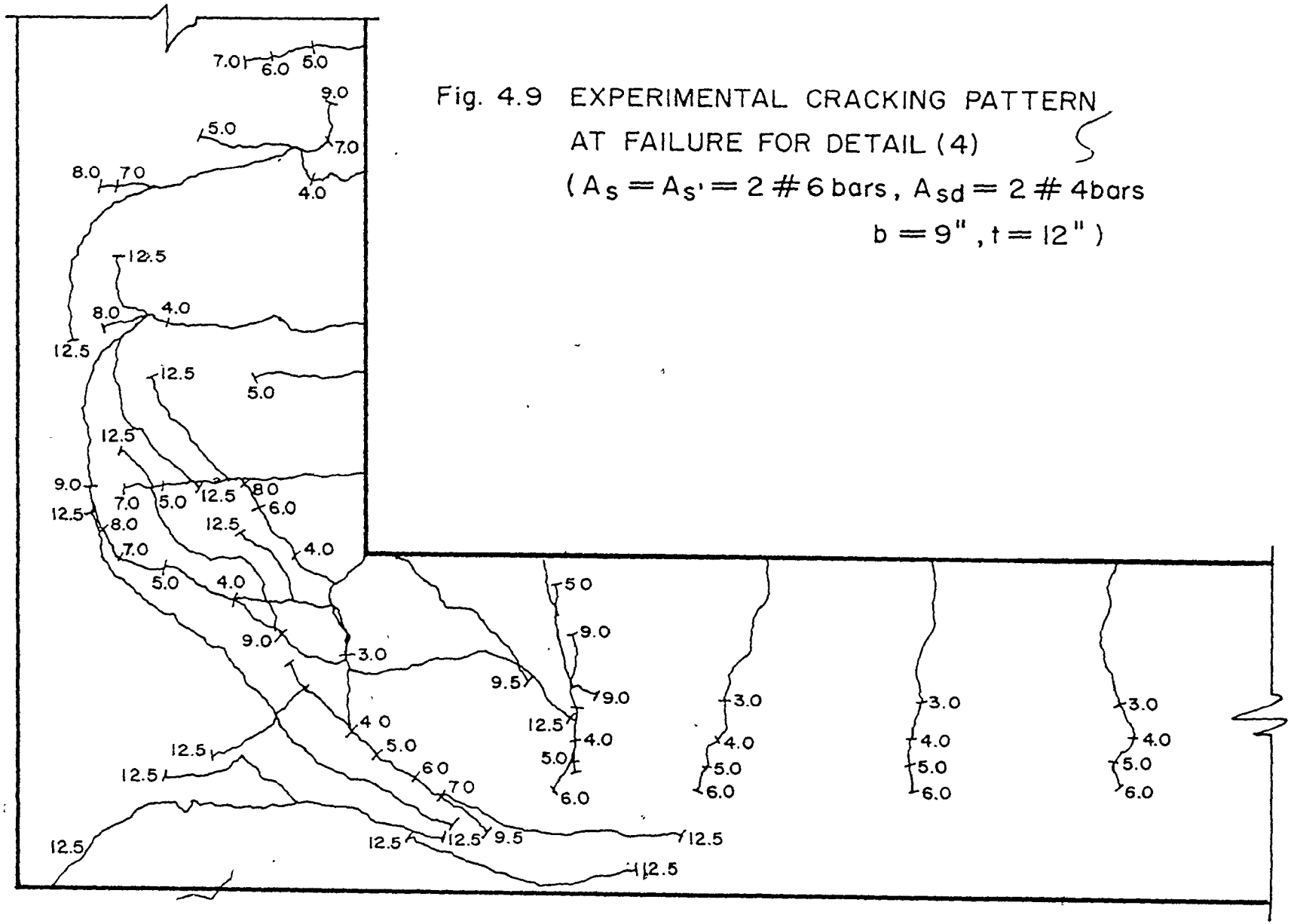


Fig. 4.9 EXPERIMENTAL CRACKING PATTERN AT FAILURE FOR DETAIL (4)
($A_s = A_{s'} = 2\#6$ bars, $A_{sd} = 2\#4$ bars
$b = 9"$, $t = 12"$)

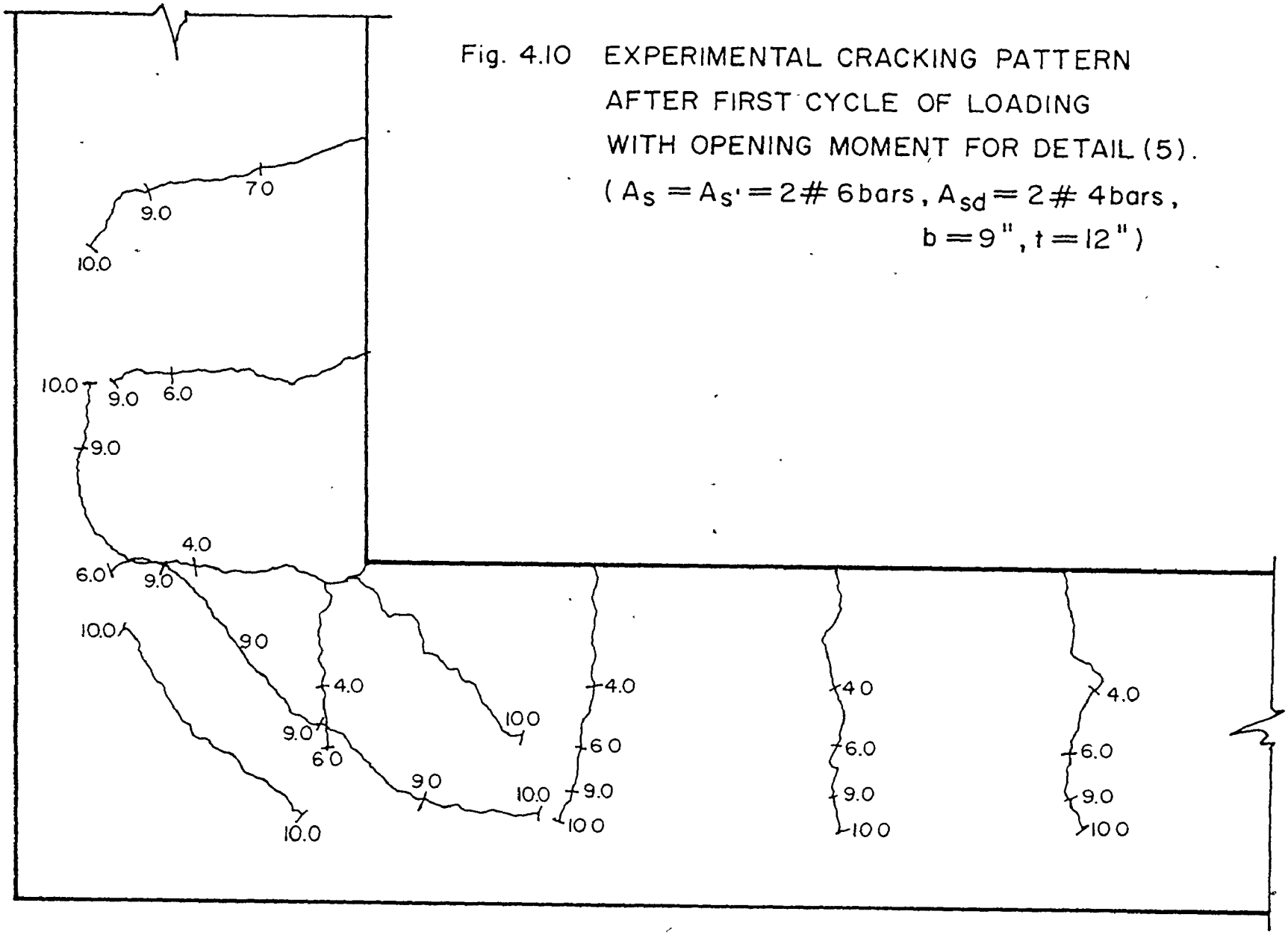


Fig. 4.10 EXPERIMENTAL CRACKING PATTERN AFTER FIRST CYCLE OF LOADING WITH OPENING MOMENT FOR DETAIL (5). ($A_s = A_{s'} = 2\#6$ bars, $A_{sd} = 2\#4$ bars, $b = 9"$, $t = 12"$)

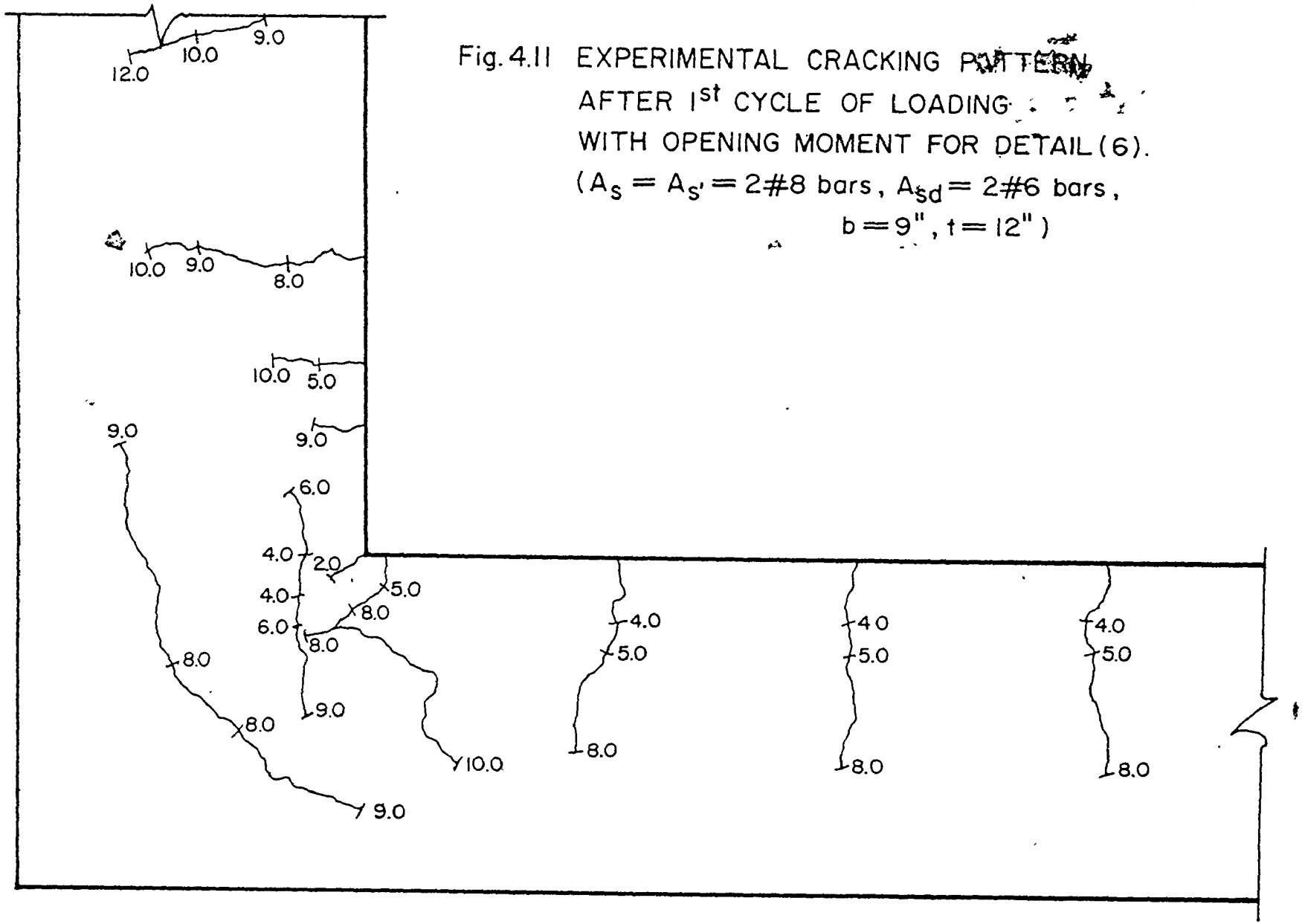


Fig. 4.11 EXPERIMENTAL CRACKING PATTERN AFTER 1st CYCLE OF LOADING WITH OPENING MOMENT FOR DETAIL (6). ($A_s = A_{s'} = 2\#8$ bars, $A_{sd} = 2\#6$ bars, $b = 9"$, $t = 12"$)

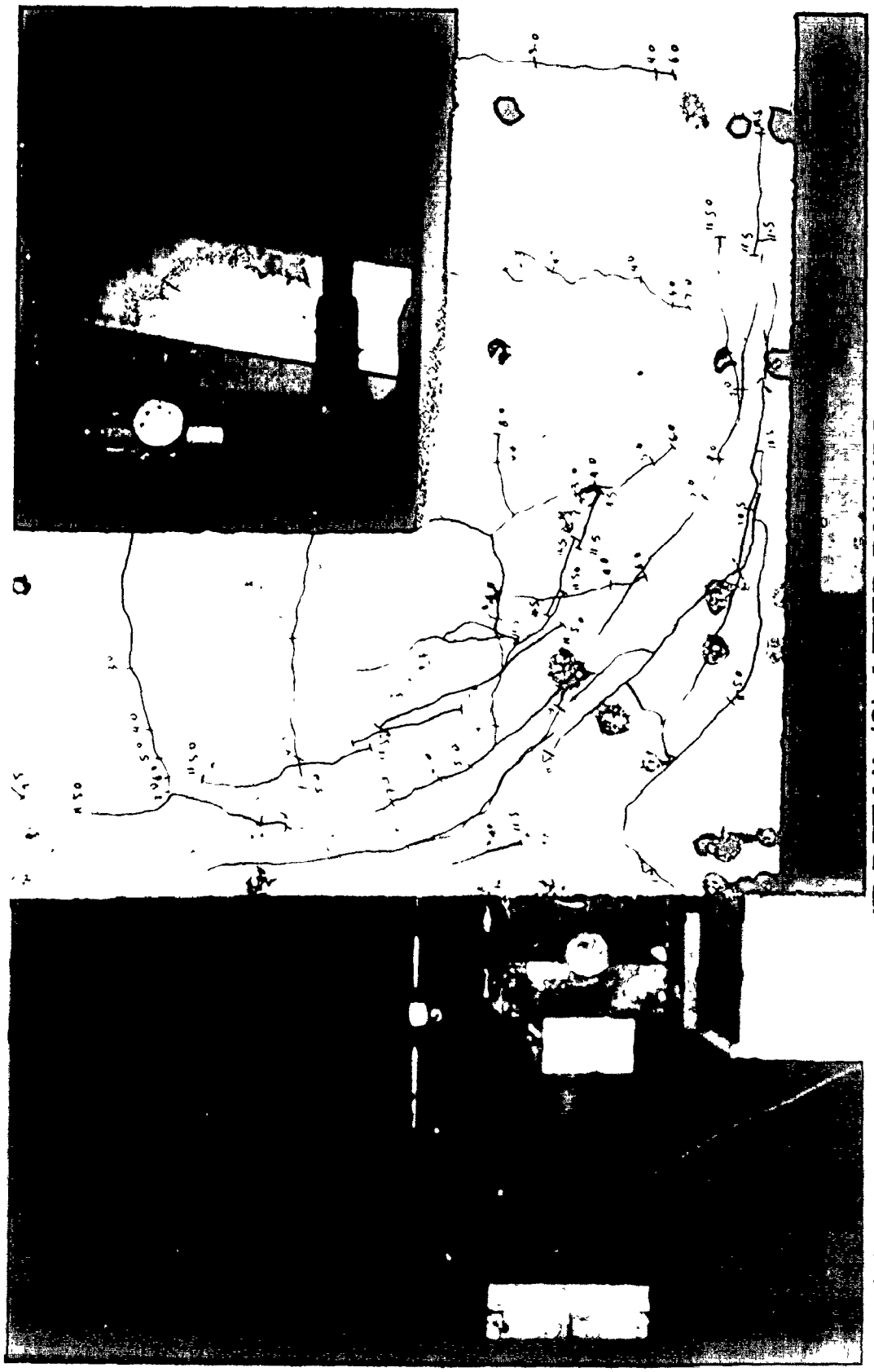
Table 4.1 Measured Crack Spacing Outside the Joint Region.

Crack Spacing (inches)			
Joint Detail	Average	Maximum	Minimum
(1)	4.9	6.0	3.6
(2)	5.1	5.8	3.6
(3)	7.15	8.8	6.6
(4)	6.27	6.4	5.6
(5)	7.8	8.4	7.6
(6)	7.67	9.5	6.2
MEAN* = 6.24"		Standard Deviation* = 1.26	
*Excluding detail (6) (different reinforcement ratio)			

Figures 4.12 to 4.15 show photographs for Details (3), (4), (5) and (6) at failure respectively.

Large differences in the cracking patterns at the joints regions of the different specimens can be observed from Figures 4.6 to 4.11. Diagonal cracking which is considered to be the main concern in the joint regions varies considerably. It is generally acceptable that the more the diagonal crack propogation is delayed, the stronger the joint will be (74, 97). Three types of cracks in the joint region have been identified and are illustrated in Figure 4.16. These are:

1. Flexural Cracks designated as type (a)



VT DETAIL (3) AFTER FAILURE

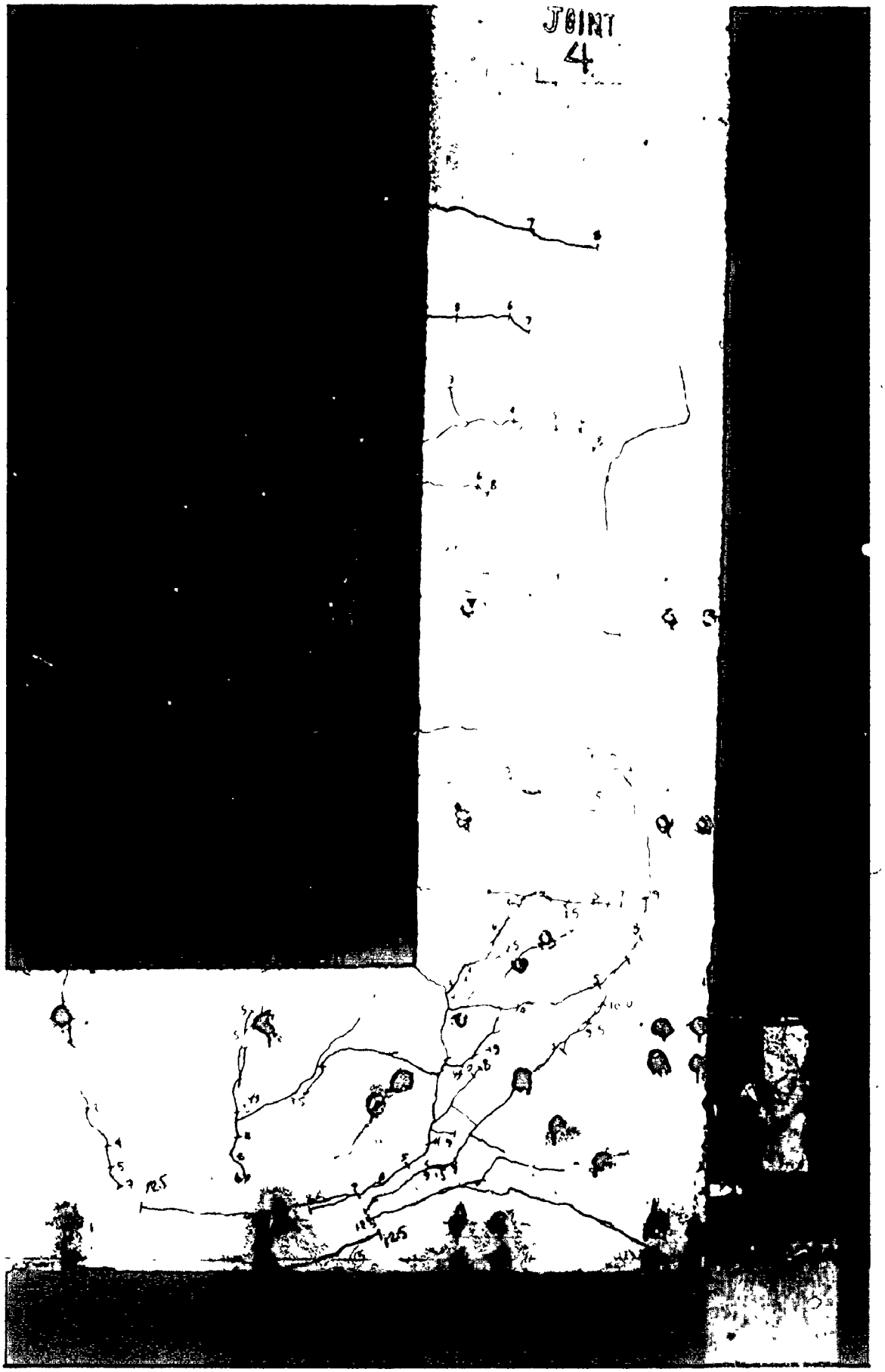


FIG. 4.13 JOINT DETAIL (4) AFTER FAILURE

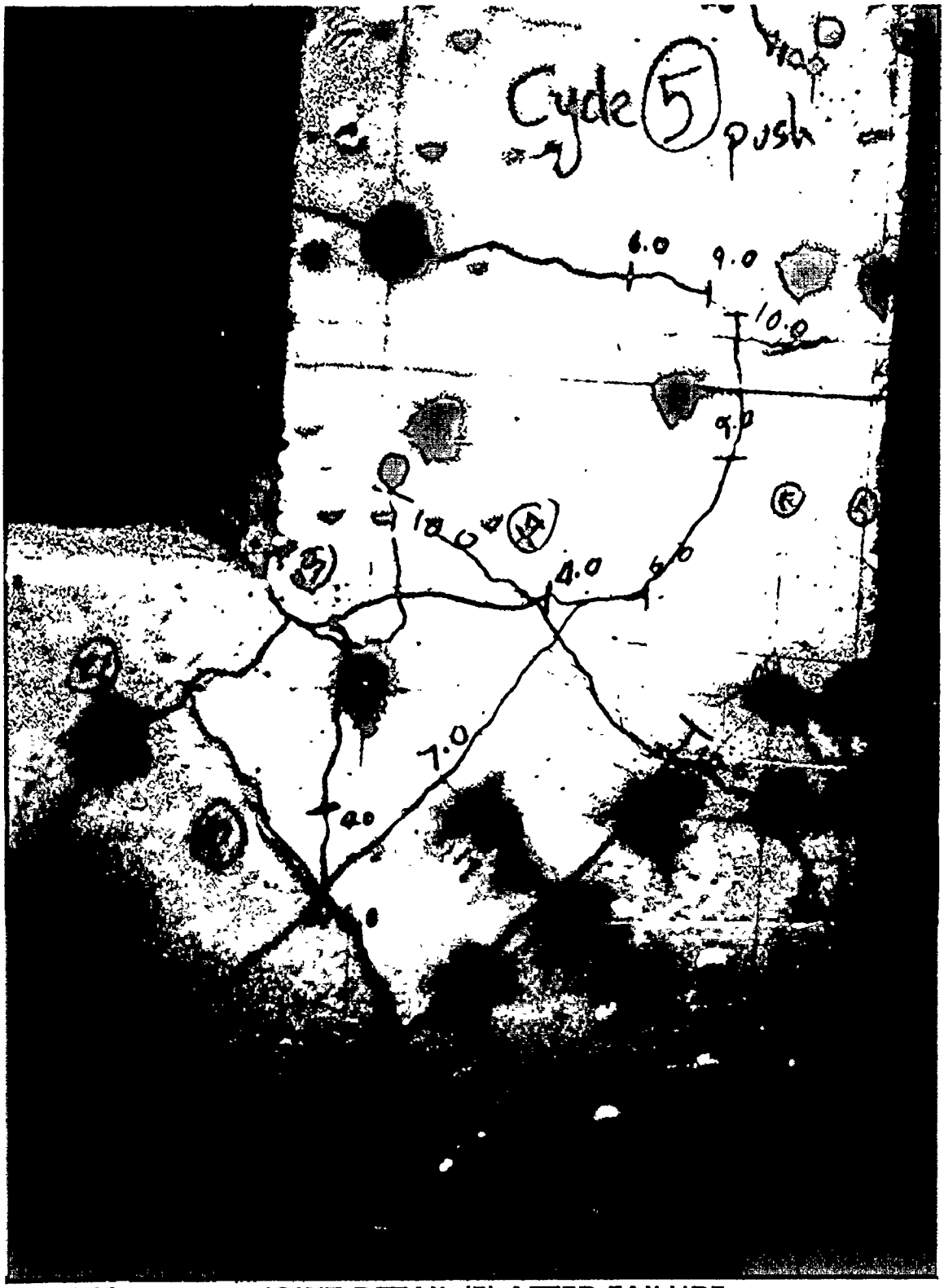


FIG. 4.14 JOINT DETAIL (5) AFTER FAILURE

JOINT DETAIL (6) AFTER FAILURE

(2) Corner tension cracks designated as type (b)

(3) Diagonal tension cracks designated as type (c)

Crack type (b) is inevitable in any corner joint subjected to moment tending to open the angle. Diagonal reinforcement was provided in all the specimens to limit the width and the propagation of this type of crack.

Crack type (a) can be seen at the joint region in all the specimens except for Detail (2) where premature yielding of the stirrups diagonally joining the compression and tension reinforcement (as shown in Fig. 4.1.b) caused the cracks to propagate diagonally in both directions instead of being controlled vertically and horizontally.

The formation of type (c) cracks signifies the main differences between one joint detail and another. Also cracks of type (a) will usually propagate diagonally as designated by type (d) in Fig. 4.16. These cracks may eventually cause a diagonal tension failure of the joint if stirrups were not provided at these locations in the two intersecting members. Type (d) cracks shall be regarded as different from type (c) although their directions are usually the same. They form at different locations and they may form even in the presence of positive confinement for the joint core.

The approximate load levels at which cracks of types (a), (b), (c) and (d) were first observed expressed as percentages of the calculated ultimate load capacities of the intersecting members using the Whitney Stress Block (96) are presented

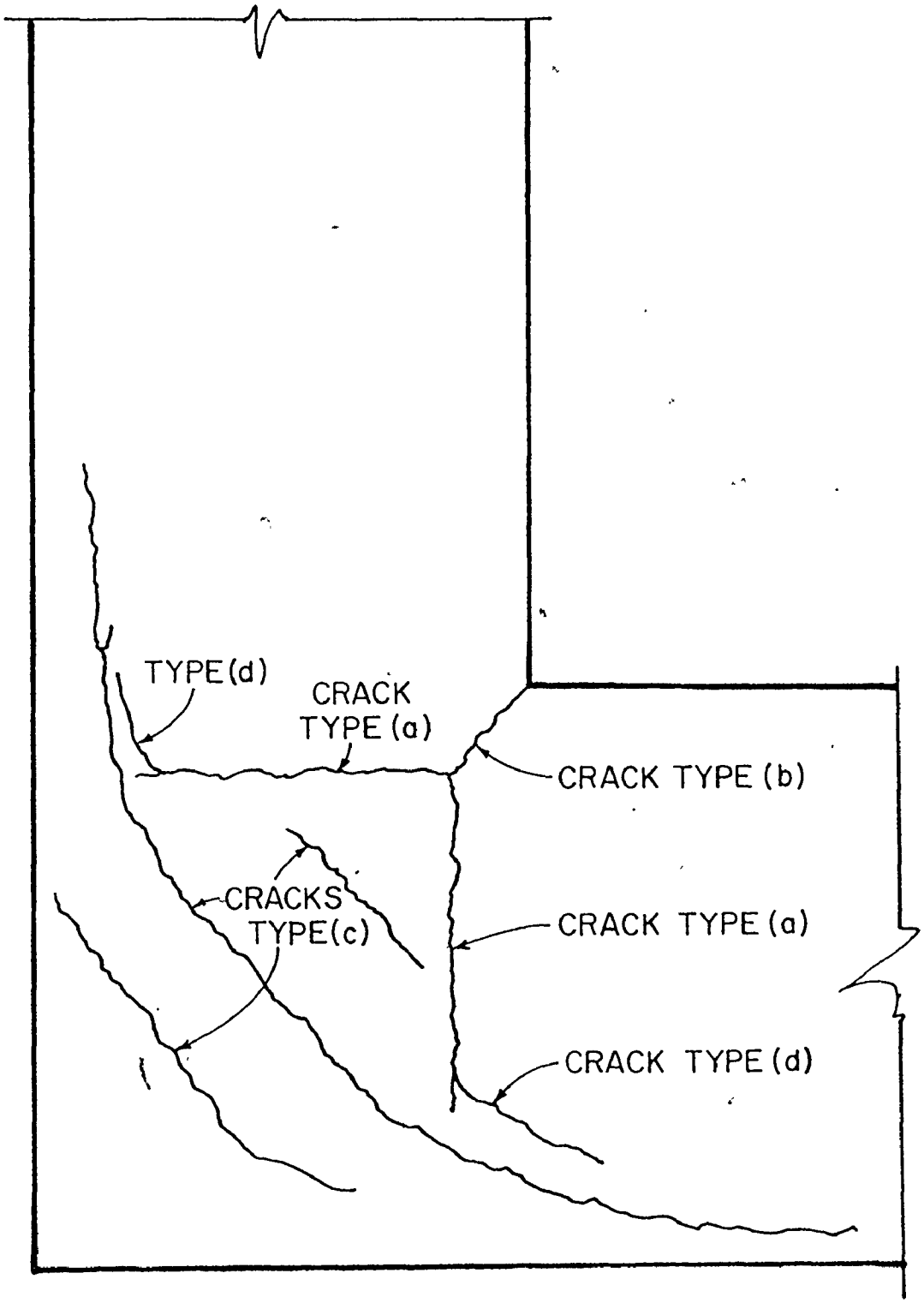


Fig. 4.16 POSSIBLE CRACK TYPES IN JOINT REGION.

in Table 4.2. The experimental ultimate capacity as a ratio of the calculated capacity is also shown in the same table.

Table 4.2 SUMMARY OF JOINT TEST RESULTS

Joint Detail Number	f'_c ksi	Steel Prop. limit f_y, ksi	% Load at crack occurrence / calculated ultimate load — CRACK TYPE (a)	(b)	(c)	(d)	Ult. cap. kips	exp. ult.cap / Cal. ult. cap %
(1)	4.98	69	50	40	50	75	10.5	104%
(2)	4.98	69	-	30	20	-	6.5	64%
(3)	5.11	68	30	30	70	50	11.5	114%
(4)	5.11	68	40	30	90	50	12.5	124%
(5)	5.36	71	40	30	70	90	11.6	115%
(6)	5.36	71	24	12	50	50	14.75	88%

NOTE:

The capacities of Details (1) to (4) are the capacities from one cycle load test while these for (5) and (6) are the capacities after 4 cycles of load reversals as previously discussed.

4.6 Observations

In this chapter the test results for the joints were presented and discussed. Further discussions are contained in Chapter 7 where comparisons between experimental and analytical results are made. Discussions of the mode of failure of each specimen is also presented in Chapter 7.

From considerations of just the test results the following observations can be made:

(1) Detail (2) for the quantity of diagonal stirrups used in the test has the least capacity and the lowest overall performance of all the details tested. In Chapter 7 the effect of increasing the amount of diagonal reinforcement shall be discussed.

(2) Details (1) and (3) performed satisfactorily from a capacity and displacement point of view. However, their fabrication required more effort than was required, for Details (4), (5) and (6). The practicality of welding is an important requirement for the fabrication of Detail (3).

(3) Details (4), (5) and (6) were relatively simple to fabricate and generally have a capacity in excess of that required by the design of their intersecting members. The exception is Detail (6) which had a slightly lower capacity than that required. This however, may have been due to the deterioration of the joint during the cyclic loading. This observation and other factors such as size and anchorage of

reinforcement are discussed in Chapter 7.

It is thought, therefore, that a reinforcement layout as shown in Details (4), (5) and (6) may be superior to other widely used details.

The test program studies only a very limited range of variables and, therefore, additional evidence is required to confirm the applicability of any of these details. However, a full experimental study of this nature is beyond the scope of this investigation. Instead, subsequent chapters describe the analytical approach which was developed not only to try to help explain the performace of established joint details but to complement experimental work in the development of other joint details.

CHAPTER 5

ANALYTICAL METHOD AND DESCRIPTION OF THE COMPUTER PROGRAM

5.1 Introduction

To develop a better understanding of the behaviour of reinforced concrete joints is the primary objective for this research. The use of inadequate joint details may be due to the lack of rigorous explanation for the complex interaction found in joint behaviour. Experimental research can be of great value to assist in the process of developing better joint details. However, development of an analytical method based on rational modelling of concrete and steel and their interactive behaviour is helpful in two ways. It will not only complement the experimental results by facilitating a better understanding of the behaviour of an experimentally developed joint but should also be of great assistance in actually developing satisfactorily performing joints through understanding and analytical testing of their complex behaviour.

This chapter contains the descriptions of the computer program developed by the author to analyse the behaviour of reinforced concrete members. The particular objective for developing this program was to model reinforced concrete beam-column connections and simple beams. The program is applicable only to cases with monotonically increasing loads and a major

feature is the ability to introduce and trace crack growth. Joint elements (linkage elements at steel and concrete interface) are also introduced to permit the modelling of bond-slip and of dowel action. The feature of permitting cracks to develop is very necessary for a proper analysis of reinforced concrete members of the types intended in this investigation. However, this introduces computational complexities such as iterations required for establishing crack stability and the necessity to continuously alter the stiffnesses of the finite elements to allow for this as well as other non-linearities involved in the analysis.

5.2 Idealizations Introduced in the Finite Element Method of Analysis

Some idealizations of the concrete and steel configurations and the nature of their interconnectivity are required since the analysis is limited to being finite and two dimensional.

A plane stress analysis of a reinforced concrete member using the finite element method is possible assuming that

1. Steel reinforcing bars are uniformly distributed across the member.
2. Loads are applied in a plane parallel to the length and depth directions.
3. The length of the member is relatively large as compared to its cross sectional dimensions.

In some instances, to be valid, analysis must account for the

three dimensional nature of the stress and strain fields. For example, this is important in the anchorage zone of a prestressed concrete beam or in beams with large widths and a single reinforcing bar.

The finite element idealization of members reinforced with "m" bars of diameter "ϕ", distributed over the width "b" of the member is shown in Fig. 5.1. For analytical purposes it is convenient to replace each round bar with an equivalent rectangular bar of the same cross sectional area. The dimension of the equivalent bar in the depth direction is kept the same as "ϕ" to maintain realistic cross sectional geometry which is especially important with complex arrangements of reinforcement. The width of the idealized steel element is modified to

$$S = m \frac{\pi \phi}{4} \qquad \text{.....(5.1)}$$

The modified width of concrete elements at the steel level becomes

$$b_{net} = b - S \qquad \text{.....(5.2)}$$

where b = total member width.

The finite element idealization of a continuum is made by subdividing this continuum into a finite member of elements interconnected at a number of discrete nodes. The elements can have various shapes such as triangles, rectangles or quadrilaterals. The shape and the number of elements used in an analysis are usually chosen by the user to suit various contours and to obtain sufficient precision consistent with a reasonable number of degrees of freedom. Concrete and steel elements are

idealized by using constant stress triangles in this investigation. Triangular elements best suit the complicated reinforcing steel arrangements used in this study. Due to the large number of elements required for the tracing of crack growth in concrete elements, the use of more than two degrees of freedom per node was not possible with the available computer facilities.

The complex interaction between concrete and steel at their interface is idealized by special discrete elements called, hereinafter, "joint elements". These joint elements were developed by Goodman et.al. (46), to permit deformations in the direcitons parallel and perpendicular to their lengths. These special linkages have no thickness, therefore, they eliminate the introduction of a non-realistic concrete or steel configuration. Section 5.3.3 is devoted to the description of this type of element.

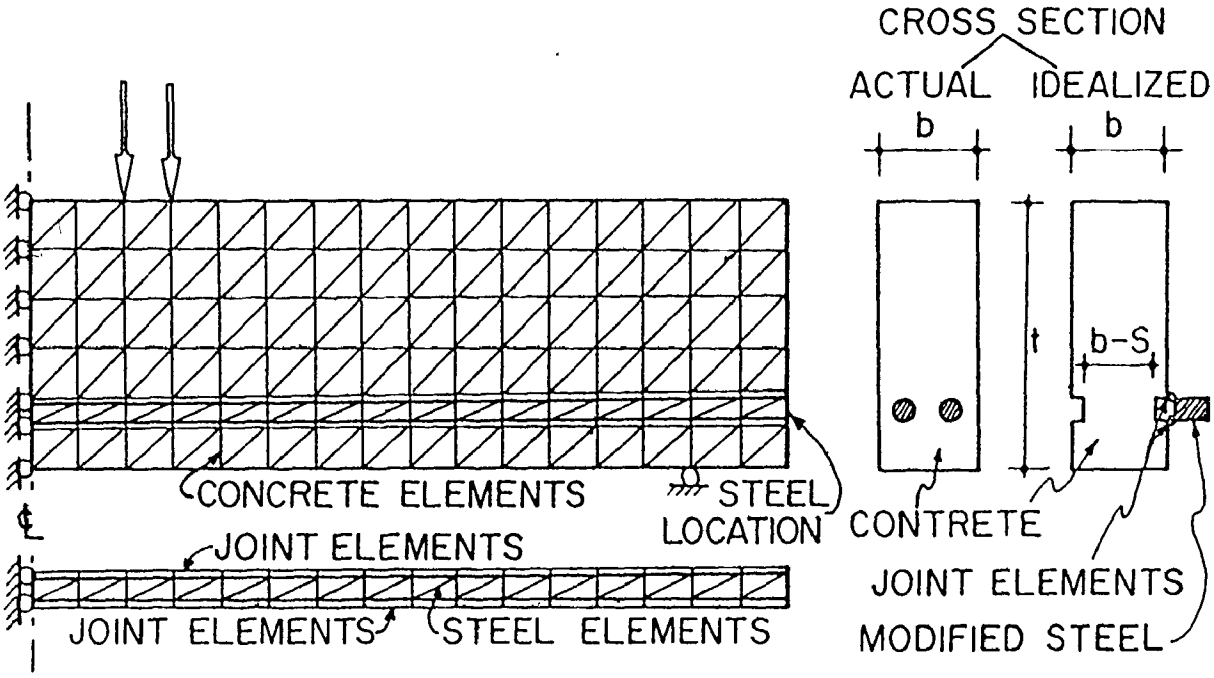


Fig. 5.1 FINITE ELEMENT IDEALIZATION OF A SIMPLE BEAM.

5.3 Mathematical Formulation

5.3.1 Concrete and Steel Element

The finite nature of the connectivity between the elements makes possible the formulation of simultaneous algebraic equations relating the nodal forces to the nodal displacements in each element. These equations are equal in number to the number of degrees of freedom chosen. In the present analysis of plane stress cases, two degrees of freedom per node are incorporated. In matrix notation the equations are as follows:

$$\{P_e\} = [k_e]\{\delta_e\} \quad \text{.....(5.3)}$$

where,

$\{P_e\}$ = nodal force vector

$\{\delta_e\}$ = nodal displacement vector

$[k_e]$ = element stiffness matrix

The description of the derivation of the stiffness characteristics of plane stress elements used in this investigation can be found in many finite element method text books. (37, 59, 80).

The element stiffness matrix

$$[k] = t\,[B]^T [D]\,[B]\,dA \quad \text{.....(5.4)}$$

where, $[B]$ = strain matrix

$[D]$ = elasticity matrix

The strain matrix [B] relates the strain vector $\{\varepsilon\}$ to the nodal displacement vector $\{\delta_e\}$ hence,

$$\{\varepsilon\} = [B]\{\delta_e\} \quad \text{.....(5.5)}$$

where,

$$[B] = \{ \frac{\partial u}{\partial x} \qquad \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \} \tag{5.6}$$

and u and v are the displacement functions in the form of polynomials with the number of coefficients equal to the number of the degrees of freedom chosen for the element. A closed form solution of the integrals in the stiffness matrix can readily be obtained for constant or linear strain triangles as well as for rectangular elements.

In the case of a general quadrilateral element, Zienkiewicz (37), reported that "It will, however, be evident on inspection that integration of the various expressions required for the formulation of the stiffness and strain matrices will present difficulties which are almost insurmountable if closed form expressions are desired. This is due to the inverse of the Jacobian matrix, in which polynomials occur in the denominator of various expressions. It seems essential, therefore, to apply numerical integration procedures. Application of Gauss' rule or other similar techniques will permit numerical evaluation of the appropriate integrals. Computer programs, however, become longer now, and it is perhaps problematical whether the improved accuracy would not be better achieved by the use of a finer subdivision in the simpler formulation of triangular elements."

The elasticity matrix [D] relating stresses to strains should incorporate the biaxial stress state which is often present in reinforced concrete members. Because concrete does not behave elastically, the values of the elastic moduli included

in the elasticity matrix are actually variable quantities depending on the strain level. Thus this nonlinearity can be incorporated in an inelastic analysis when an incremental loading technique is used. The orthotropic elasticity matrix can be written (37) as:

$$[D] = \begin{bmatrix} \frac{E_x}{1-\nu_x \nu_y} & \frac{E_x \nu_y}{1-\nu_x \nu_y} & 0 \\ \frac{E_y \nu_x}{1-\nu_x \nu_y} & \frac{E_y}{1-\nu_x \nu_y} & 0 \\ 0 & 0 & G \end{bmatrix} \qquad (5.7)$$

where E_x and E_y = elastic moduli in orthogonal x and y directions.

ν_x and ν_y = Poisson's ratio in the x and y directions

G = Shear modulus of elasticity

In this investigation ν_x and ν_y were set equal and constant throughout the analysis. ($\nu_{concrete} = 0.2$, $\nu_{steel} = 0.3$)

In equation 5.7, the increase in effective stiffness in either direction in the presence of normal compressive stress is due only to the Poisson's ratio effect. With Poisson's ratio for concrete assumed to be 0.20, and ratio of principal stresses equal to 1.0, a stiffness increase for 20% higher than the uniaxial stiffness was calculated by Liu et.al. (86). They observed experimentally that in the presence of biaxial compression, a stiffness increase greater than that which can be

explained by Poisson's effect alone, existed, They also concluded (87) that the main cause of this supplementary stiffness is the confinement of potential microcracking in the presence of biaxial compression. The supplementary stiffness was estimated to be 15% for a ratio of principal compressive stresses equal to 1.0. To account for both Poisson's effect and microcracking confinement, Liu et.al. (86), proposed, in uncoupled form, the following constitutive relationship

$$\{\sigma\} = \begin{bmatrix} E_{xb} & 0 & 0 \\ 0 & E_{yb} & 0 \\ 0 & 0 & G_b \end{bmatrix} \{\varepsilon\} \qquad (5.8)$$

where E_{xb} and E_{yb} are the effective modulii in directions x and y, respectively, obtained from their proposed strss-strain relationship for concrete submitted to biaxial compression. G_b is the shear modulus for biaxially-stressed concrete. Good agreement was reported between the proposed biaxial stress-strain relationship and the experimental data of Kupfer et.al. (56) who noted that the strength increase for equal compressive stresses in the two principal directions is approximately 16% above the uniaxial strength. No experimental data were found (86) to check the shear modulus, G_b. For biaxial tension and tension compression cases, uniaxial stress-strain relations were found to be adequate, (87).

In the present investigation, the elasticity matrix equation (5:7) is used with the assumption that $E_x = E_y$. The value of the Modulus of Elasticity $E = E_x = E_y$ is found from the stress-strain equation of concrete in uniaxial compression.

The minimum principal strain in an element under biaxial stresses is used to find the modulus of elasticity of the element. The modulus of elasticity of the element in compression is continuously changed according to the non-linear stress-strain relationship of concrete in uniaxial compression.

A constant modulus of elasticity equal to the initial tangent of the stress-strain curve in axial compression is incorporated in the analysis for elements in tension until the failure criterion in tension is met.

5.3.2 Cracked Concrete Elements

Concrete elements in tension will crack when they meet their failure criterion. In this investigation a concrete element in tension is considered to crack when the maximum principal stress exceeds the modulus of rupture of concrete. Exact modelling of cracking is difficult because of the different problems associated with crack formation.

To accurately simulate cracks in elements new nodes should be introduced in the cracked element and this element should then be subdivided into new elements. This technique has great complexities because of the creation of new irregular shapes of elements, the solution of which are much more complex than triangular or rectangular elements. The introduction of new nodes may also cause problems with the computer core storage. Increasing the number of degrees of freedom of the structure shall also result in a much higher cost per run.

An alternative to subdividing the element into two new elements which may have irregular shapes is to separate the nodes of existing elements at their interface. However, this

solution, besides confining crack paths to element boundaries, creates the same problem with increasing the number of nodes and consequently increasing the number of degrees of freedom incorporated in the analysis as discussed above.

Another alternative, which may replace dividing the cracked elements or separating the nodes, is to reduce the stiffness, of the element in the direction normal to the crack according to the technique reported by Zienkiewicz (37). This technique which was developed for stratified materials is incorporated in the analysis. This assumption, although, it may not be the most realistic, was thought to be a definite improvement over the relatively crude process which have been used by many other investigators by totally deleting the element from the global matrix when cracked.

One of the complex phenomena created upon cracking of concrete is the aggregate interlock which was described in Chapter 2. Simulation of aggregate interlock can be done using one of the two following techniques:

1. Introduction of new nodes at cracks can be used to transfer shear across the crack. This can be accomplished by joining the corresponding concrete nodes with joint elements which have stiffnesses representing the aggregate interlock action. This, however, complicates the computer programming to a great extent considering the unrestricted growth of cracks which is incorporated in this analysis. The predefinition of cracks to overcome this difficulty nullifies the significance

of this technique for tracing growth of cracks. The increase in the required computer core storage associated with any method which involved an increase in the number of degrees of freedom normally made that method impractical.

2. The use of shear retention techniques have been recently developed and used by some investigators (105) to overcome the problems of core storage when other methods of modelling aggregate interlock were used. In this method, of shear retention, the shear modulus, G, in cracked elements is reduced to a smaller value. Since it is known that aggregate interlock depends on the crack width, this shear retention factor should be a function of crack width. However, it was found (105) that in many cases a constant value of reduced G gave better results. The reliablity of this technique and the necessary factors required for its use have not yet been developed.

In this investigation the inclusion of the aggregate interlock phenomena was neglected. This was thought to be an appropriate assumption, because aggregate interlock may only be effective in beams with a limited range of shear span to depth (a/d) ratios. The inclusion of the effects of aggregate interlock phenomenon in the present analysis was not possible for joints because of computer core storage limitations. To study the effects of including shear retention factors, to simulate aggregate interlock, on the behaviour of a joint is beyond the scope of this study.

The reduced stiffness of a cracked element $[k_c]$ and the stresses and strains in this element are found as follows:

For a cracked element as shown in Fig. 5.2, the following assumptions are made:

1. The element has no stiffness in the direction normal to the crack (i.e. $E'_y \rightarrow 0$)
2. Poisson's effect is neglected in the cracked element (i.e. $\nu \rightarrow 0$)
3. The shear stiffness of a cracked element is significantly reduced (i.e. $G \rightarrow 0$)

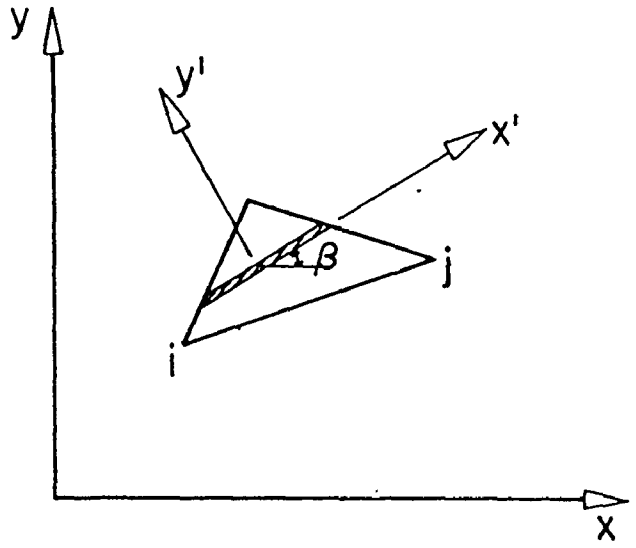


Fig. 5.2 IDEALIZATION OF A CRACKED CONCRETE ELEMENT.

Then, where [D'] is the elasticity matrix in local coordinates (x', y'),

$$[D'] = \frac{1}{1-\nu^2}\begin{bmatrix} E_x' & \nu E_x' & 0 \\ \nu E_y' & E_y' & 0 \\ 0 & 0 & G(1-\nu^2) \end{bmatrix} \quad \ldots\ldots(5.9)$$

Therefore, for the cracked element, [D'] reduces to

$$[D'] = \begin{bmatrix} E_x' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \ldots\ldots(5.10)$$

and if $[D] = [T][D'][T]^T$ .....(5.11)

where

[D] = elasticity matrix --global coordinates.

[T] = strain transformation matrix.

$$[T] = \begin{bmatrix} \cos^2\beta & \sin^2\beta & -2\sin\beta\cos\beta \\ \sin^2\beta & \cos^2\beta & 2\sin\beta\cos\beta \\ \sin\beta\cos\beta & -\sin\beta\cos\beta & \cos^2\beta-\sin^2\beta \end{bmatrix} \quad \ldots\ldots(5.12)$$

where,

β = angle of inclination of the crack with the global x axis.

then, $[k_c]$ the stiffness matrix of the cracked element becomes

$$[k_c] = [B]^T [D] [B] t \quad \ldots\ldots(5.13)$$

or

$$[k_c] = [B]^T [T] [D] [T]^T [B] t.\Delta \quad \ldots\ldots(5.14)$$

where,

t = thickness of the element

Δ = area of the element

and for plane stress elements the strain matrix [B] becomes:

$$[B] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \qquad \text{.....(5.15)}$$

to find the stress in the cracked element

$$\{\sigma\} = \{\varepsilon\}^T [D] \qquad \text{.....(5.16)}$$

$$\{\sigma\} = \{\varepsilon\}^T [T] [D'] [T]^T$$

where $\{\sigma\}$, $\{\varepsilon\}$ are the stress and strain vectors respectively.

5.3.3. Stirrup Elements

The idealization and modelling of stirrups to accurately describe their effect on the behaviour of a reinforced concrete structure is relatively difficult to achieve using a two dimensional analysis. Conventional stirrups, either closed or U-shaped, confine a portion of the concrete and, where closely spaced, increase the ductility of the member and its rotational capacity (18). However, this was not of great significance for the problems analyzed in this investigation, since the few stirrups provided in the joints tested were not closely spaced. Furthermore, the analyzed beams had no web reinforcement.

During the development of the computer program it was evident that a three dimensional analysis was impossible for practical considerations associated with the available

computer facilities (core storage and computational time),

In this investigation segments of stirrups are represented by four-degree of freedom bar elements, Fig. 5.3, described by a stiffness matrix:

$$[k]_{st} = \frac{E_s A_{st}}{L_s} \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta\sin\theta & -\sin^2\theta \\ \cos^2\theta & \cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & \sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \quad (5.17)$$

where,

θ = angle of inclination of local x-axis to global x-axis

E_s = Modulus of Elasticity of stirrup steel

A_{st} = Cross sectional area of stirrup

L_s = Length of the stirrup

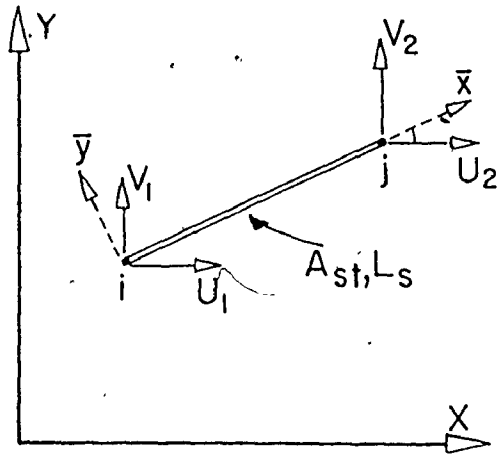


Fig. 5.3 IDEALIZATION OF STIRRUP ELEMENTS.

These stirrup elements can be connected between any two concrete or steel nodes directly or with the use of joint elements, depending upon whether or not relative slip displacement is expected.

If large steel forces and large relative displacements between the steel and concrete are anticipated, joint elements should be introduced. If the stirrup is located so that no large deformations or end slip are expected the stirrups can be connected directly to steel nodes. This procedure will tend to minimize computer core storage; which is usually a problem with a finite element analysis of the nature intended in this investigation.

5.3.4 Joint Elements

Mathematical representations of bond-slip, aggregate interlock and dowel action relationships are not easy to produce. An idealization was introduced in 1967 by Ngo and Scordelis (39) using four-degree of freedom spring linkages. These links connected corresponding nodes on concrete and steel elements at their interface to simulate bond-slip and dowel action, and they were used to connect corresponding nodes of concrete on either side along a crack to simulate aggregate interlock. This idealization has been used by Ngo and Scordelis (68), Nilson (48) Spolowski (88), Houde and Mirza (99) and others to predict the behaviour of reinforced concrete structures. The only drawback of this technique is that it only insures compatibility conditions at the nodes. This concept was broadened in 1968 by Goodman et.al.(46) when they introduced joint elements in an application to jointed rock masses.

The joint element as shown in Fig. 5.4, has a finite length "L" but zero thickness as the nodal pairs (1,4) and (2,3) initially have identical coordinates. In this figure, the joint elements is in a local coordinate system with the x-axis along

the length, and the origin taken at the centroid of the joint element.

The relative displacement vector of the top and bottom of the joint element is given by

$$W = \begin{Bmatrix} W_s^{top} - W_s^{bottom} \\ W_n^{top} - W_n^{bottom} \end{Bmatrix} \qquad \text{.....(5.18)}$$

where W_s^{top} and W_s^{bottom} are the displacements in the tangential direction of the top and bottom parts of the joint element, respectively, and W_n^{top} and W_n^{bottom} are the displacements in the normal direction of the top and bottom parts of the joint element, respectively. This relative displacement vector is related to the external applied forces by

$$\{P\} = [K]_{jt} \{W\} \qquad \text{.....(5.19)}$$

where

$\{P\}$ is defined as

$$\{p\} = \begin{Bmatrix} P_s \\ P_n \end{Bmatrix} \qquad \text{.....(5.20)}$$

where P_s and P_n are the forces per unit length in the tangential and normal directions respectively.

$[K]_{jt}$ is defined as

$$[K]_{jt} = \begin{bmatrix} k_s & 0 \\ 0 & k_n \end{bmatrix} \qquad \text{.....(5.21)}$$

where K_n is the normal force per unit length, required to produce a unit deformation in the direction normal to the joint, and similarly K_s applies to the direction tangent to the joint.

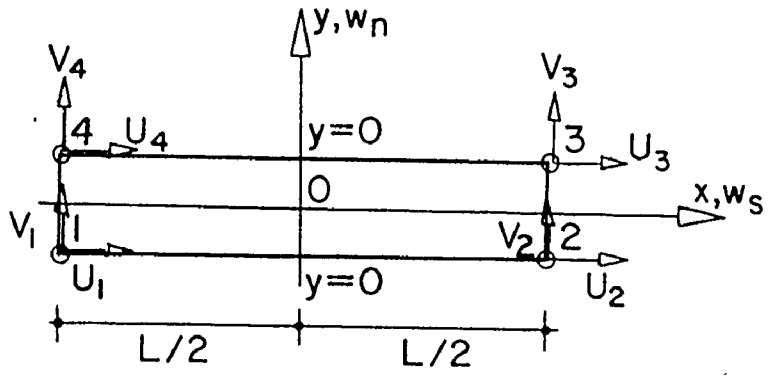


Fig. 5.4 JOINT ELEMENT REPRESENTATION(46)

In terms of the local coordinates the energy stored in a joint element due to applied forces per unit length acting through the displacements is:

$$\phi = 1/2 \int_{L/2}^{L/2} \{w\}^T [P]\, dx \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5.22)$$

If the displacements {w} were replaced by the nodal point displacements {U} through a linear interpolation formula the energy is:

$$\phi = 1/2 \; L \{U\}^T [K] \{U\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (5.23)$$

where [K] is the joint element stiffness per unit length. Development of [K] is given in full detail in reference (46).

The structural (global) stiffness matrix for the entire system of elements and joints is then assembled by

properly adding corresponding terms of elements contributing stiffness at each nodal point.

Once the solution is obtained, the joint forces (or stresses) are calculated from the nodal displacements. Derivation of the joint stresses in terms of displacements is also given in the reference (46).

5.4 Program Description

5.4.1 General

A computer program utilizing the finite element technique has been developed to study the behaviour of reinforced concrete members. The program was of a general nature regarding the shapes and types of structures it can handle. However, it was written mainly to analyze simple reinforced concrete beams and joints and has not been tested for other types of elements.

In the course of developing the program, numerous problems were encountered. Some of these problems were related to the nature of the analysis and the expected inaccuracies accompanying a discrete solution to a problem of a continuous nature. The other type of problems was due to the limited computer core storage available and to finally trying to reduce the cost per run to a feasible amount. The nature of some of the problems and how they were tackled shall be discussed in Section 5.4.4. This might be helpful to future investigators who are working in the same field.

5.4.2 Function of the Main Program and the Subroutines:

The program was written to run on the CDC-6400 computer available at the McMaster University Computing Centre. Due to core storage limitations (124 K maximum) it was necessary to convert the program to run also on the IBM-370 computer, available at McMaster University for administrative use. The IBM-370 has a core storage capacity of 700K, however, double precision is necessary for any problem where solution of ill-conditioned matrices is expected. The introduction of joint elements and the imposition of cracks in concrete elements usually results in an ill-conditioned matrix which requires double precision. The CDC-6400 computers' single precision was almost equivalent to double precision on the IBM-370.

All of the simple R.C. beam runs were made on the CDC-6400 Computer because it was more accessible than the IBM-370. Also beam runs could be handled with reasonable accuracy using the available CDC-6400 core storage capacity. Analysis of joints were all made on the IBM-370 computer.

The Program consists of a main Routine (MAIN) and 17 subroutines. The organization of the main program, of the subroutines and the auxiliary storage files is shown in Fig. 5.5. The functions of the subroutines are to formulate the "Global Stiffness Matrix of the Structure", to solve for the displacements, strains, and stresses corresponding to the applied loads, to check the failure criteria of the different elements and modify their stiffness accordingly, and to store information regarding the cracking history.

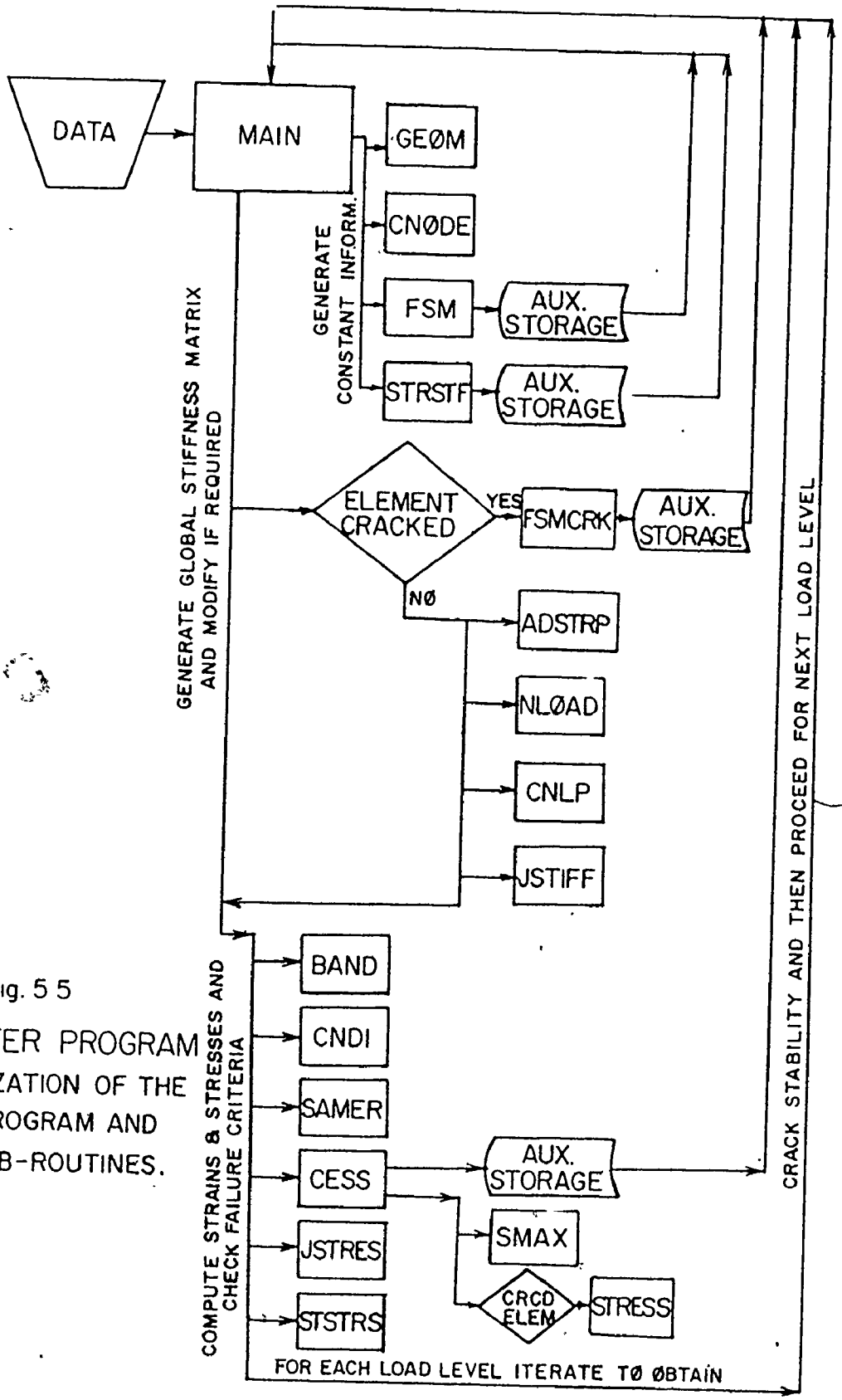


Fig. 5 5

COMPUTER PROGRAM ORGANIZATION OF THE MAIN PROGRAM AND THE SUB-ROUTINES.

Appendix A contains a detailed description of the function of each subroutine and a listing of the computer program.

5.4.3 Program Logic and Operation

One of the interesting features of the program is its ability to predict crack patterns. During the development, the general trend of the crack patterns predicted by the analysis agreed in a very encouraging fashion with crack patterns from experimental work. Therefore, the logic of the program was changed from the usual application of preselected load increments on the structure to a logic which allows the analysis to predict crack growth in a more realistic manner.

To more accurately predict crack patterns in reinforced concrete structures, it was necessary to let only one element crack at a time. The element in the structure which is stressed the most is the only element that should be allowed to crack at any particular iteration for crack stability. Even when there is more than one element in the structure which meets the failure criterion, the most highly stressed one is the only one that should be allowed to crack. Modification of the stiffness of the cracked element is then performed according to the direction of the crack (normal to the maximum principal tensile stress) and re-entered in the global stiffness matrix. This technique allows the redistribution of stresses to take place before any other element is allowed to crack. This results, in most cases, in relieving

some of the surrounding elements which were overstressed and prevents the occurrence of an unnecessary crack. The topology of the structure will then be the same as that of an actual structure with a crack in the process of propagation. Although this technique greatly increased the computer time required , it was found in earlier trials that allowing several overstressed elements to crack at one time created discrepancies (especially for cracking patterns) which were not justified for the sake of the reduced cost of running the program.

After crack stability is achieved for a certain load level (i.e. no elements are overstressed for this particular load level), the next load increment is not imposed on the solution but rather is automatically determined within the program based on the level of stress in the next most highly stressed element which did not exceed the failure criteria. However, if this increment is larger than a predetermined limiting size of increments, it is then set at the smaller value to minimize divergence resulting from the nonlinear material properties used in this analysis.

During the iterations for crack stability, all concrete elements, steel elements and joint elements are constantly modified to include the effects of the nonlinear relationships of concrete in compression, bond-slip and of the strain hardening of steel. Convergence was normally obtained fairly readily and in most cases no further iterations for material properties were required. However, divergence occurs in some elements which are near failure because of the extra sensitivity due to more pronounced nonlinear

material behaviour.

The convergence to a correct solution despite all of the involved nonlinearities can be achieved by iterating around material properties until the errors are within acceptable limits. This is not included in the present analysis, however, because of the extra cost involved in running the program and since this slight divergence was only noticed quite near failure and in relatively few elements.

Although the expression "increment" was used in the above discussion, in this analysis the total load is applied on the structure rather than the easier to employ but less accurate method of superimposing incremental effects. The possibility of employing an incremental technique or a mixed procedure utilizing a combination of the incremental and total load schemes, although not used in this investigation, was also incorporated in the computer program for future studies.

The reliability and suitability of the program has been documented by comparison with some of the available experimental data. This is reported and discussed in the following chapters.

Although the title of this thesis is "Behaviour of Reinforced Concrete Joints", the program's accuracy was tested and developed first by using simple reinforced concrete beams. There has been much more experimental work done in this area and researchers are more familiar with their general behaviour so that visual comparison and simplified mathematical calculations can more easily reveal discrepancies in the analysis than would be the case for joints. Beam

results are included in Chapter 6, whereas the results for joints are introduced in Chapter 7.

5.4.4 Some Problems Associated with Finite Element Analysis of Reinforced Concrete

a. Element Size

"There is no doubt that the solution to plane elasticity problems is, in the limit of subdivision, an exact solution. Indeed at any stage of a finite subdivision it is an approximate solution.

The total strain energy obtained during any stage of approximation will be below the true strain energy of the exact solution. In practice it will mean that the displacements, and hence also the strains and stresses, will be underestimated by the approximation, in its general picture. However, this is not necessarily true at every point of the continuum individually; hence the value of such a bound in practice is not great" (37).

What is important to the engineer is to know the order of accuracy achievable for typical problems using a certain fineness, of element subdivision. In any particular case the error can be assessed by comparison with known, exact solutions or by a study of the convergence, using two or more stages of subdivision.

In a stress field which varies either linearly or nonlinearly, the basic assumption of constancy of stress within elements means that the solution will be approximate

only. In the case of a beam subjected to constant bending moment with a fairly coarse subdivision of elements, the axial stress given by the element straddles the exact values and, in fact, if the constant stress values are associated with centroids of the elements the best fit line, Fig. 5.6, reproduced from reference (37) should be close to the exact stresses. The stresses perpendicular to the axis of the

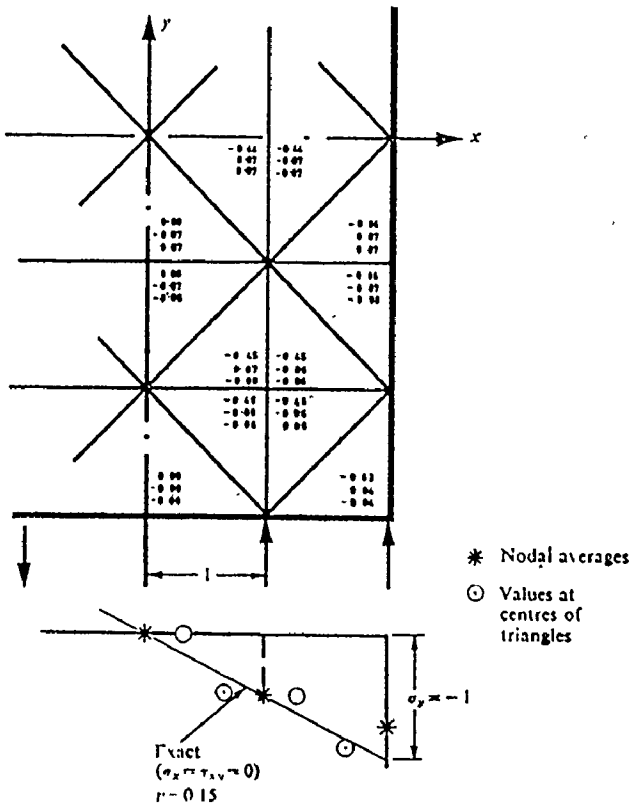


Fig. 5.6 Pure bending of a beam solved by a coarse subdivision into elements of triangular shape. (Values of σ_y, σ_x, and τ_{xy} listed in that order)

(Reproduced From Reference 37)

member and the shear stress components also differ from the exact values, which are simply zero. However, it is noted that they will oscillate by equal and in most cases small

amounts around the exact values. At internal nodes, if the average of stresses of surrounding elements is taken, it will be found that the exact stresses are very closely represented. The average at external faces however, is not so close. Figure 5.6 shows that σ_y obtained from the nodal averages is very close to the exact solution at the internal nodes while it is underestimated at the external face. Weighting of the averages near the faces of the structure can be used as a further refinement.

Representation of the stresses by nodal averages is a definite improvement of the approximation. However, it is preferable to use a finer subdivision wherever possible and when the accuracy requires it.

In this investigation reasonably fine subdivisions were always used whenever the core storage permitted. When the computer core storage capacity limited the desired fineness, averaging of the stresses at the nodes was applied. When this averaging technique is employed the deflections of the analyzed structure are usually considered a reliable indication for the degree of precision achieved in the cases where the accuracy of the calculated strains and stresses cannot be verified.

It is worthwhile noting again that,because of the lower bound nature of the solution obtained from the finite element method of analysis, for a coarser mesh, the underestimation of the computed deflections, strains and stresses

will increase.

b. Efficient Use of Core Storage:

Minimizing the band width of the global stiffness matrix is necessary for finite element analyses using the "Choleski" technique in solving the banded matrix. As mentioned before, accuracy increases with fineness of the element mesh. The band width reduction or minimization becomes then a matter of great importance to save on the required computer core storage. To get the minimum possible band width for a structure such as the corner joints, (i.e. members with a sudden bend) it was found very useful to number the nodes in diagonal or fan-type fashion rather than using a vertical or horizontal numbering scheme.

c. Data Generation

To generate the large amount of data required to drive the program, especially for joints, development of automatic and semi-automatic data generators, data checkers, and routines for optimization of the band width were found to be quite helpful.

5.5 Additional Information

The cost of running the program was found to be relatively reasonable for beams (i.e. between 8 and 20 CP minutes execution time on the CDC-6400 computer). However analysing corner or other types of joints which require much larger core storage and a larger number of nodes and elements was expensive. The amount of data available from these runs,

however, justified the higher cost in a satisfactory way. (The approximate average time on the IBM-370 computer was around 100 CP minutes for execution). In terms of time, information obtained and actual cost, this compares favourably with experimental work.

The current rapid development in the speed and storage capacities of modern computers should make this analytical tool even more economically feasible in the near future.

In the next two chapters the analytical results are presented analysed and compared with experimental data. Final conclusions are presented in Chapter 8. A complete listing of the computer program is included in Appendix A.

CHAPTER 6

APPLICATION OF THE FINITE ELEMENT METHOD TO SIMPLE REINFORCED CONCRETE BEAMS

6.1 Introduction

A great deal of work has been done on simply supported reinforced concrete beams by numerous investigators (28, 33, 38, 44, 49, 64, 67). Simple beams were thought of as a reliable means of developing the proposed method of analysis. Their well documented behaviour allowed the performance of visual and rough mathematical checks which were easier to formulate and more readily revealed any discrepancies in the analysis than would be the case for joints. They were also much more economical to analyze using the computer. All the same programming features for joints were also required for beams, therefore no major programming modification were required to analyze the joints. This is not to say that the results of the beam analyses are not useful in themselves. In fact as will be shown, the results reproduce cracking behaviours and failures in a fashion which documents observed phenomena.

In this chapter comparisons with results for beams tested by Leonhardt and Walther (28), are made. Load-deflection, ultimate capacity, type of failure, stress distributions,

and crack patterns from the analyses are presented and where possible compared with the test results.

Five beams were analyzed and are discussed in this chapter. The choice of beams from Leonhardt and Walther's (28) test data was made so that the reliability of the method of analysis could be assessed and so that the sensitivity to the following variables could be tested.

1. The effects of different a/d (shear span/depth) ratios.
2. The effects of bond-slip characteristics.
3. The effects of different material properties.
4. The effects of cracking on the overall behaviour of simple reinforced concrete beams.

In the next chapter, the joint test results are compared with the analytical results obtained from the developed computer program.

6.2 Test Data and Description of the Analyzed Beams

In 1962, Leonhardt and Walther (28), tested a series of beams to investigate the influence of the moment versus shear relationship. This relationship has been established as a very important criterion for the shear carrying capacity, for beams with point loads and uniform loads. All the beams were rectangular with the same 19 cm by 32 cm (7.48 in x 12.6 in) cross section and were reinforced with two 26 mm diameter (1.024 in) Bst IIIb steel bars in the longitudinal direction. To avoid anchorage failure rather long cantilevered beam

ends with stirrups as reinforcing were employed, although stirrups were not used in the main parts of the beams. Figure 6.1.a shows a sketch of the test set-up and the dimensions used.

In the case of beams with point loads, the a/d ratio was changed so that the distance between the symmetric load points and the supports was different. The distance "a" between the two point loads was chosen to be small and to be the same for most of the beams. Thus different span lengths resulted and also different slenderness ratio for the beams. The left load distribution plate (3.0 inch wide) was made smaller than the right one (5.0 inch wide) to find out if different local pressures had any influence. No influence was found and hence the size of the loading plate was neglected in this investigation and all of the comparisons are made for a constant point loading pattern. No beams with uniform loads were considered in this investigation.

The beams were kept moist and tested without any time for drying to avoid shrinkage influences. The material properties, exact dimensions and the type of steel reinforcing bars used for the five analysed beams are included in Table 6.1. The concrete was proportionately sandrich (60% from 0-7 mm (9/32 in.) with a cement content of 290 kg/m^3 (490 lbs/yd^3) cement type 475 and a water-cement ratio = 0.68.

Deflections were measured in the middle of the beam and crack patterns were recorded at every loading step. The widths of cracks were not measured for the beams analysed here.

Beam	l in.	a in.	a' in.	t in.	d in.	b in.	a/d	cube strength psi	f'_c* psi	Steel type
4	66.93	26.38	14.17	12.6	10.6	7.48	2.5	5050	4290	Rippentor stahl Bst IIIb
7/1	122.00	53.15	15.70	12.6	10.6	7.48	5.0	5290	4500	
9/1	228.35	74.4	79.55	12.6	10.75	7.48	6.92	5430	4620	
EA1	78.74	29.5	19.74	12.6	10.6	7.48	2.78	3570	2860	Rippentor-stahl Blankerstah st 37 k
EB1	78.74	29.5	19.74	12.6	10.6	7.48	2.78	3570	2860	

Table 6.1 Dimensions and Material Properties.

*f_c' values are calculated as 80% of the cube strength for concrete strengths under 5,000 psi and 85% of the cube strength for concrete strengths higher than 5,000 psi. The above values were arbitrarily chosen with guidance from the tables and equations developed by Evans and L'Hermite both referenced in the text, "Properties of Concrete" by Neville, (98).

To find the influence of the bond of the longitidunal reinforcing on the shear carrying capacity for rectangular beams without shear reinforcing, Leonhardt and Walther (28) tested four beams under two point loading with the same span lengths, cross-section areas, concrete strengths and reinforcing ratio, where only the quality of the bond of the longitudinal reinforcing, was changed. The beams contained no shear reinforcing, but beam ends protruded far over the supports to avoid anchorage failures. The anchorage zone did not have any stirrups. The quality of the bond was changed either by changing the number and diameter of the reinforcing bar or by varying the condition of the surface of the reinforcement. It is known that bond is better for smaller diameter reinforcement because the ratio between circumferential area and cross-sectional area becomes greater.

Beam EA1 was reinforced with 2, 24 mm (.945 in) diameter and 1, 6 mm. (.236 in.) diameter bars having ribbed surfaces (Rippentorstahl BST 1116). Beam EB1 was reinforced with 2, 25 mm (.984 in.) diameter cold drawn machine steel (St 37 k). This steel is round and has a very smooth mirror like surface. In order to anchor the smooth rods adequately at their ends ribbed pieces of rod were welded on. The a/d ratio for all of these tests for influence of bond was 2.78.

The concrete used had a normal composition with 251 kg/m^3 (423 lbs/yd^3) of Pz 375 cement with a water-cement ratio of 0.75. The cube strength had an average value of

252 kg/cm^2 (3584 psi). The tensile strength under bending averaged 41.4 kg/cm^2 (589 psi). The stress-strain curves for both types of steel used in specimens EA1 & EB1 did not show any noticeable difference in the range of steel stresses which coincided with shear failure. With the exception of beams EA1 and EB1, the cracking patterns for the beams analyzed in this chapter were clear but the numbers on the photographs were not readable even with the use of a magnifying glass. Therefore, it was not possible to compare the sequence of crack occurrence and crack propagation with the predicted behaviour for any beams but EA1 and EB1. However, the crack patterns are compared for these other beams.

The stress-strain diagram for concrete was not included in Leonhardt and Walther results, however, the information provided was adequate for the use of Saenz's (24) equations which are employed in this investigation.

The stress-strain curves for the reinforcing steel used in the beam tests are shown in Fig.6.1.b. However, an idealized elastic-plastic stress-strain relationship was employed in the analysis, since post-yielding did not have any influence on the behaviour of the tested beams.

For all deformed bars, (ribbed bars) used in the beam tests, the bond-stress slip characteristics have been assumed to be represented by the equations developed by Houde and Mirza (99) which were described earlier in Chapter 2. Also the dowel action equations described in Chapter 2

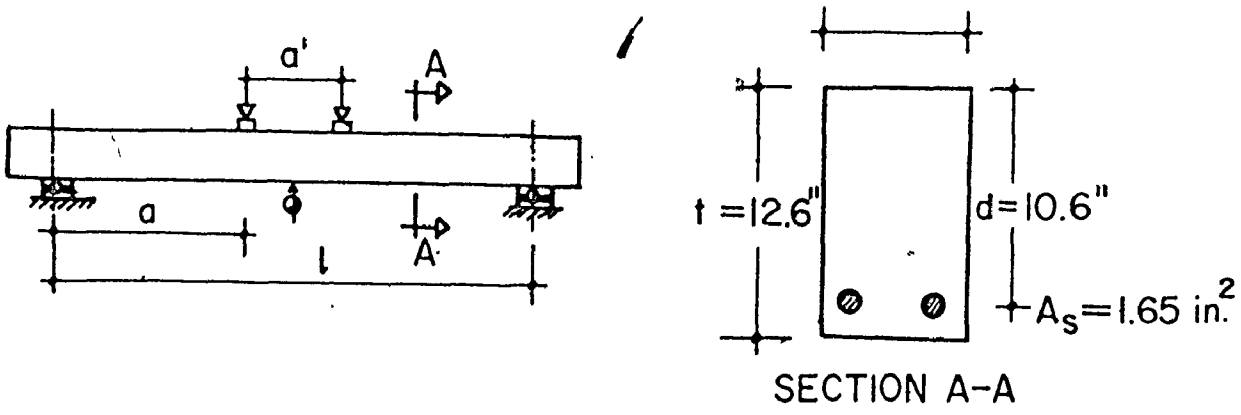


Fig. 6.1a DIMENSIONS OF THE ANALYSED BEAMS.

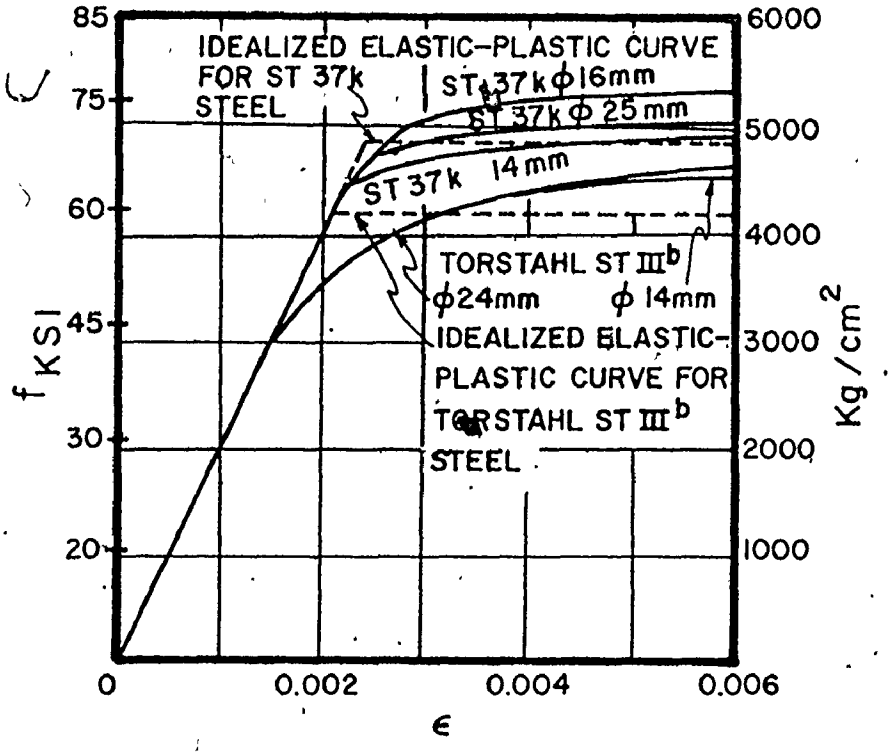


FIG. 6.1b STRESS-STRAIN DIAGRAM FOR STEEL USED IN LEONHARDT AND WALTHERS' BEAMS REPRODUCED FROM REFERENCE (28)

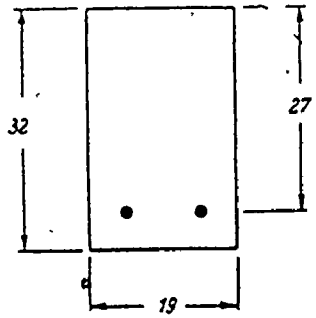
are used in the analyses.

6.3 Summary of Test Results

In this section the test results (28) for all of the beams tested with point loads are summarized, even though only five of these beams were analysed in this chapter. The pictures shown in Fig. 6.2 were reporduced from reference (28) and show the crack patterns after failure for beams with different a/d ratios. For the shortest beam, beam (1), the diagonal crack is steep because of the stress components, F_y, between load and the support allow the directions of the principal tensile stresses to become flatter than 45°. For beams (2) and (3), the shear cracks which led to failure, extended under the load into the zone of pure bending because the concrete compressive strength in bending is increased directly under the load because of the local confinement due to the load. Although the concrete actually crushed in the zone of pure bending, this must be considered a shear failure because of the depth of the compression zone for bending was decreased much more by the shear crack than by the flexural cracks.

For beams (1) to (4), the shear cracks developed beyond the flexural cracks only at relatively high loads. In beams (5) to (8) shear cracks sometimes developed out of the bending cracks and at other times they were new cracks which formed at an angle right across existing flexural cracks. In the top region they were very flat, due to the presence of the high longitudinal compressive stresses.

FIG. 6.2



Crack and failure pictures of beams with points loads (the numbers indicate the load in tons for which a crack penetrated up to the indicated location). The Beams 4 to 6 were taped at the destroyed side after failure load was reached and loaded again up to failure at the other side.

REPRODUCED FROM REFERENCE (28)

Bez.	l	a	M/Q · h
–	m	m	–
1	0,90	0,27	1,0
2	1,15	0,40	1,5
3	1,45	0,54	2,0
4	1,70	0,67	2,5
5	1,95	0,81	3,0
6	2,35	1,10	4,0
7/1	3,10	1,35	5,0
8/1	3,60	1,62	6,0
10/1	4,70	2,16	8,0
9/1	5,80	1,89	7,0

In general as shear cracks open at higher loads a greater part of the shear force is transferred through the lower reinforcing bars. Because of this the concrete cracks and splits along or below the reinforcing with the crack possibly extending up to the supports. Extension of cracking of this type is generally a near failure or post failure phenomena.

The location of failure is always close to the loading point, where $\frac{M}{Vd}$ reaches its maximum value. Where,

M = external moment applied to beam

V = Vertical total shear due to external loading

d = distance between the centroidal axes of the steel bars and the top concrete face.

For $\frac{M}{Vd} \geq 7$, the beam pairs (9) and (10) were reported to have failed in bending and in this case because of crushing of the concrete before the yield stress of the reinforcing steel was reached.

For beams (1) to (6) the critical cracks propogated gradually over several load increments until finally the compression zone was destroyed. For the beam pairs (7) and (8), the sudden failure was typified by rapid development of a flat shear crack. The lack of warning of shear failure was more pronounced in the case of point loads far from the support compared to loads close to the support (smaller a/d ratios).

Nonetheless, they found that less reinforcing was required to avoid shear failures in slender beams than for shorter beams where the ratio of shear capacity to flexural capacity is lower when no shear reinforcing is used.

Flexural and shear stress distributions along the beams were not included in the test results. However, some discussion of analytical flexural and shear stress distributions will be presented in the following section for beam (7) even though no corresponding experimental data was provided.

6.4 Failure Criteria for Beams:

6.4.1 General

The various modes of failure for simple R.C. beams include cracking or crushing of concrete under complex states of stress, as for example, when diagonal tension cracking, shear compression failure, or splitting occurs. Other modes of failure also include yielding of reinforcing steel, breakdown of bond or end anchorage, breakdown of aggregate interlock, and breakdown of dowel action.

All of the above mentioned failure modes and the failure criterion adopted for each mode are included in the present analysis either explicitely or implicitly, with the exception of aggregate interlock. Although, relatively easy to incorporate in the analysis, the inclusion of aggregate interlock creates computer core storage difficulties and requires a special node numbering scheme which was not practical with the available computing facilities.

6.4.2 Criteria of Failure for Concrete in Compression:

The development of a failure criterion for concrete subjected to combined states of stress is a complex subject. Although, the formulation of such a criterion has received considerable attention in the past, this did not result in the development of a well established or widely recognized single failure criterion.

It is not intended here to attempt a review of the different theories for concrete failure criteria. The subject is documented in several publications (12, 54, 56, 87) and many others not included in the Bibliography.

Some of the existing theories are:

1. The maximum strain theory.
2. The maximum stress theory (Rankine).
3. The maximum shear (internal friction) theory by Coulomb.
4. Mohr's failure enevelope theory.
5. Cowan failure envelope theory.
6. Modified Cowan theory by Zia.

The maximum strain theory, although it may not be the most accurate, was successfully used by several investigators (39, 51, 68, 89) using the finite element method of analysis to study the behaviour of reinforced concrete structures.

The maximum strain theory states that failure occurs when the maximum compressive strain in a concrete element reaches a limiting strain (this strain was set at 0.003 as

described in Chapter 3). It was felt that this theory can provide a rational basis for the study of the behaviour of reinforced concrete members of the nature intended in this investigation. This theory can be changed within the computer program to any other theory with no major effort.

6.4.3 Failure Criteria for Concrete in Tension:

For concrete in tension a crack is introduced when the maximum tensile stress in an element exceeds a predetermined tensile strength. The tensile strength for all concretes used in the analysis of beams was taken as the modulus of rupture represented by the following expression recommended by the Comité European du Beton

$$f_t = 9.5\ (f'_c)^{1/2} \qquad \text{.....(6.1)}$$

The above expression is in better agreement with the values reported by Leonhardt and Walther (28) than the following expression suggested at the University of Illinois:

$$f_t = \frac{3{,}000}{4 + \frac{12{,}000}{f'_c}} \qquad \text{.....(6.2)}$$

Both expressions are reported and referenced in the text book "Properties of Concrete" by Neville (98). As was mentioned in Chapter 3, although there is some indication in the literature that some small amount of yielding may occur prior to failure (98), the stress-strain relationship of concrete in tension can be represented by a straight line without serious loss of

accuracy (32). A modulus of elasticity in tension equal to the initial tangent modulus of concrete in compression is used in this investigation up to the value of the ultimate tensile strength.

The direction of a crack in an element is the direction perpendicular to that computed for the maximum principal tensile stress exceeding the ultimate strength of concrete in tension.

Although it was noticed in the early stages of developing the computer program, that some cracked elements tended to close at loads near failure, this feature was not included in the final analyses used in this thesis. When this feature was included in the development of the analysis it caused cracking instability, in some instances, in those elements (i.e. cracks opening and closing several times) because of the difficulty involved in determining their new moduli of elasticity after crack closure.

6.4.4 Failure Criteria for Bond Elements:

Bond elements (Joint elements) joining the steel and concrete elements at their interface behave non-linearly according to the equation developed by Houde and Mirza (99) which was discussed in Chapter 2. Equation (2.4.5) best represents the bond-slip relationship before the peak bond stress value is reached (93). The bond-slip behaviour past the peak point, shown in Fig. 2.14, was reported (93) to be dependent on the distance from the end face of loading

(representing crack location). For points at some distance from the end face (3 to 4 inches and more) the bond stress was almost constant for all slip values. For smaller distances, bond stresses decreased progressively with increasing slips. This loss of bond transfer capacity near the end faces was explained by Houde (93) to be a result of the splitting cracks observed. Since the normal cracks spacing for the particular concrete cover and the type of reinforcing used in this analysis ranged between 3.0 to 6.0 inches,the use of a progressive decrease of bond stresses with increasing slips is more appropriate, than a constant stress for all slips.

Houde and Mirza's equation (2.4.5) was examined for slips beyond the peak bond stress value and it was found that nearly half of the peak bond stress value was obtained at a slip = 0.002 inches. Houde and Mirza did not suggest a specific maximum slip in their study. However, the values suggested by Nilson (78, 92) ranged between 0.0008 and 0.003 inch for different distances from the crack face.

In this investigation the failure of a bond linkage (joint element failure in the direction parallel to its length) is considered to occur when a slip value of .002 inch is reached.

When several bond elements fail, the reinforcement bar usually depends on the bond elements near its ends to provide end anchorage. If these latter elements also fail the whole beam will very likely fail immediately due to bond

breakdown.

6.4.5 Failure Criteria for Dowel Elements:

Two failure criteria were considered for joint elements representing dowel action. These were for failure in tension or failure in compression. When shear is transferred across a crack through the steel bars the dowel force in the joint elements on the top at the L.H.S. of the beam are in tension as shown in Fig. 6.3, and the bottom elements are in compression.

When the tensile stress in the elements reaches the direct adhesion strength between steel and concrete the two surfaces separate. However, the compressive forces at the bottom can still transfer the shear force from the right to the left of the crack until finally a concrete tension crack forms along the reinforcement, as a result of the tensile stresses caused by the pressure on the concrete cover. Therefore, the failure of the joint elements in tension is usually of no great significance whereas the elements in compression cause tension cracks in neighbouring concrete element. Hence the breakdown of dowel action is implicitly determined from the failure of the concrete elements along the steel bar and not explicitly from the failure of the joint elements.

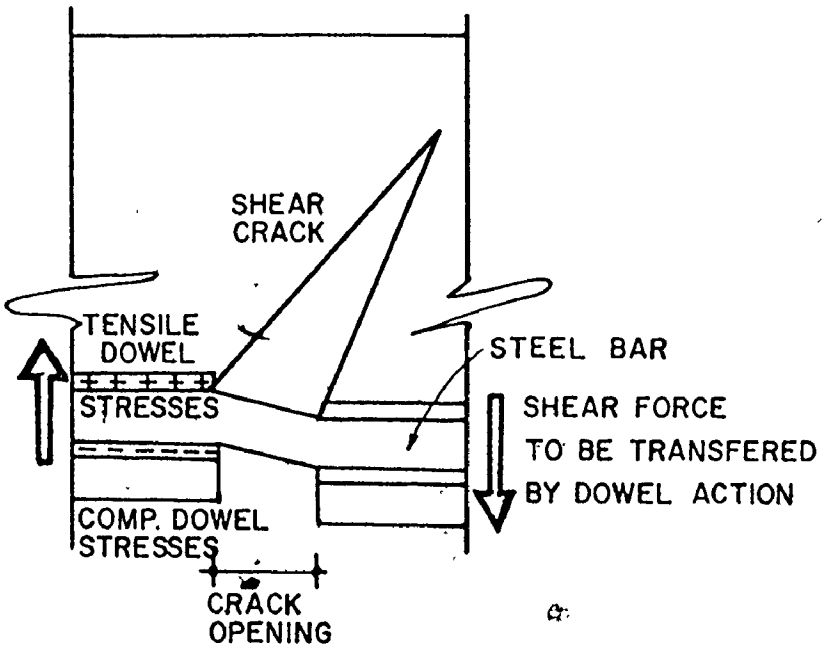


Fig. 6.3 DOWEL STRESSES IN BEAMS.

The stiffnesses which represent dowel action of the joint elements were discussed in Chapters 2 and 5.

The tension failure criterion of the joint elements employed in the analysis was:

$$f_{t_j} = f_t \qquad \text{.....(6.3)}$$

f_{t_j} = the maximum tensile strength of the joint elements normal to their axes.

f_t = modulus of rupture of concrete as defined in equation (6.1)

6.4.6 Definition of Modes of Failure for Beams:

The possible modes of failure for a beam can be described as follows:

1. Flexural failure occurs when the beam attains its full flexural capacity.

This type of failure is either caused by excessive yielding of steel reinforcement followed by crushing of the concrete, or by compression failure of the fully effective compression zone (i.e.not reduced by any type of cracking other than the flexural cracks). For the case where steel yields first, the failure is accompanied by large deflections.

2. Shear failure can be a reason for the beam not attaining its full flexural capacity as defined in (1). Shear failures can be subdivided into two main divisions. These are diagonal tension failure and shear-compression failure. In the first case, the failure crack develops from a normal flexural crack and travels very rapidly towards the load point. At the same time, a splitting crack usually develops along the line of the steel reinforcement and propogates towards the support. In the second case, the crack is straighter and steeper and develops also fairly rapidly as the beam splits into halves. Fenwick (49) describes these two kinds of behaviour as "beam action" in the former case and "arch action"

in the latter. The beam capacity is reduced by the formation of either one of the two types of shear cracks. Diagonal tension failures are usually sudden with very few or no signs of failure. Differentiation between diagonal failure and shear compression failure is not an easy task using a numerical analysis of the type proposed in this investigation. However, shear-compression failure and diagonal tension failure were detected analytically for beams (4) and (7/1) respectively as shall be discussed later in this chapter.

3. Bond or end anchorage failures are not as difficult to detect by the analysis as they sometimes are from test observations. As discussed previously in section 6.4.4, end anchorage or bond failure is identified once all the bond elements fail due to excessive slippage. This includes the elements provided for end anchorage, if any, and the beam is not able to carry any more load. This, of course, is true only if the beam did not fail in the previous steps for some other reason such as flexural or shear failures and exhibit apparent bond failure as a post-failure phenomenon. Interpretation requires examination of the sequence of occurrence of failures of different types of elements.

4. Dowel failure is usually considered to be another type of shear failure which is caused by the extension of a horizontal crack along the reinforcing steel from the point of an existing shear crack toward the support. Failure of this type may also be easily detected using the proposed analysis after studying the sequence of events and when the picture of the predicted crack pattern is completed.

5. The aggregate interlock type of failure which is another form of shear failure was not included in the analysis of the beams for the reasons mentioned in section 6.4.1. The exclusion of aggregate interlock phenomena from the analysis may affect the results. However, varied opinion in this matter especially considering beams with wide shear cracks supported the decision not to try to include this effect in the analysis. It was thought that these effects would not have any sizeable influence on failure loads for the problems considered in this study. The decision not to include it was also based on practical limitations of computer storage capacity as discussed previously.

6.5 Comparison of Experimental and Analytical Results

6.5.1 General:

In this section the results obtained using the analytical model described in Chapter 5 are presented and compared with the results of the tests conducted by Leonhardt and Walther (28), which were summarized earlier in this chapter.

Crack patterns, load-deflection curves and ultimate capacities are compared. Comments, explanations of results and presentations of some stress distributions are also included in the following sections of this chapter.

6.5.2 General Observations on Load-Deflection Results:

The experimental and analytical load-deflection curves are presented and discussed in this section. Figures 6.4 to 6.7 show the load-deflection results from the analysis and tests for beams (7/1), (9/1), EA1, and EB1 respectively. The experimental load-deflection curve for beam (4) was not available for comparison. As can be noted, the computer analysis generally tends to slightly underestimate the deflections. This tendency is natural because the analysis is a lower bound solution which tends to underestimate deformation values for any given load when the finite element mesh is not fine enough. As was discussed in Chapter 5, the degree of fineness of the mesh is the main factor that determines the cost of running the program. However, it was considered that the degree of accuracy obtained is acceptable from the engineering point of view and for the purposes of this investigation.

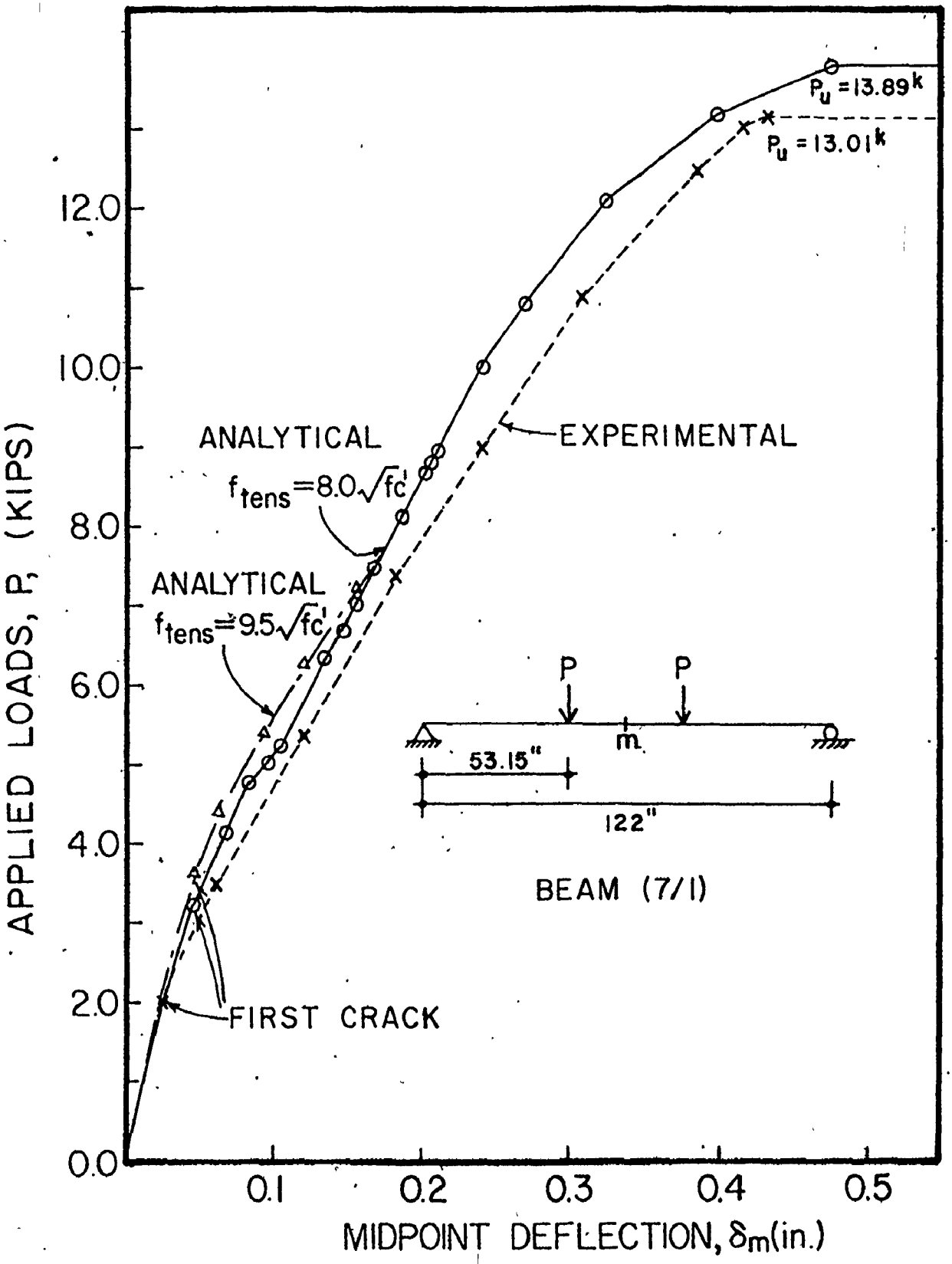


Fig. 6.4 LOAD–DEFLECTION RESULTS FOR BEAM (7/1).
a/d = 5.0

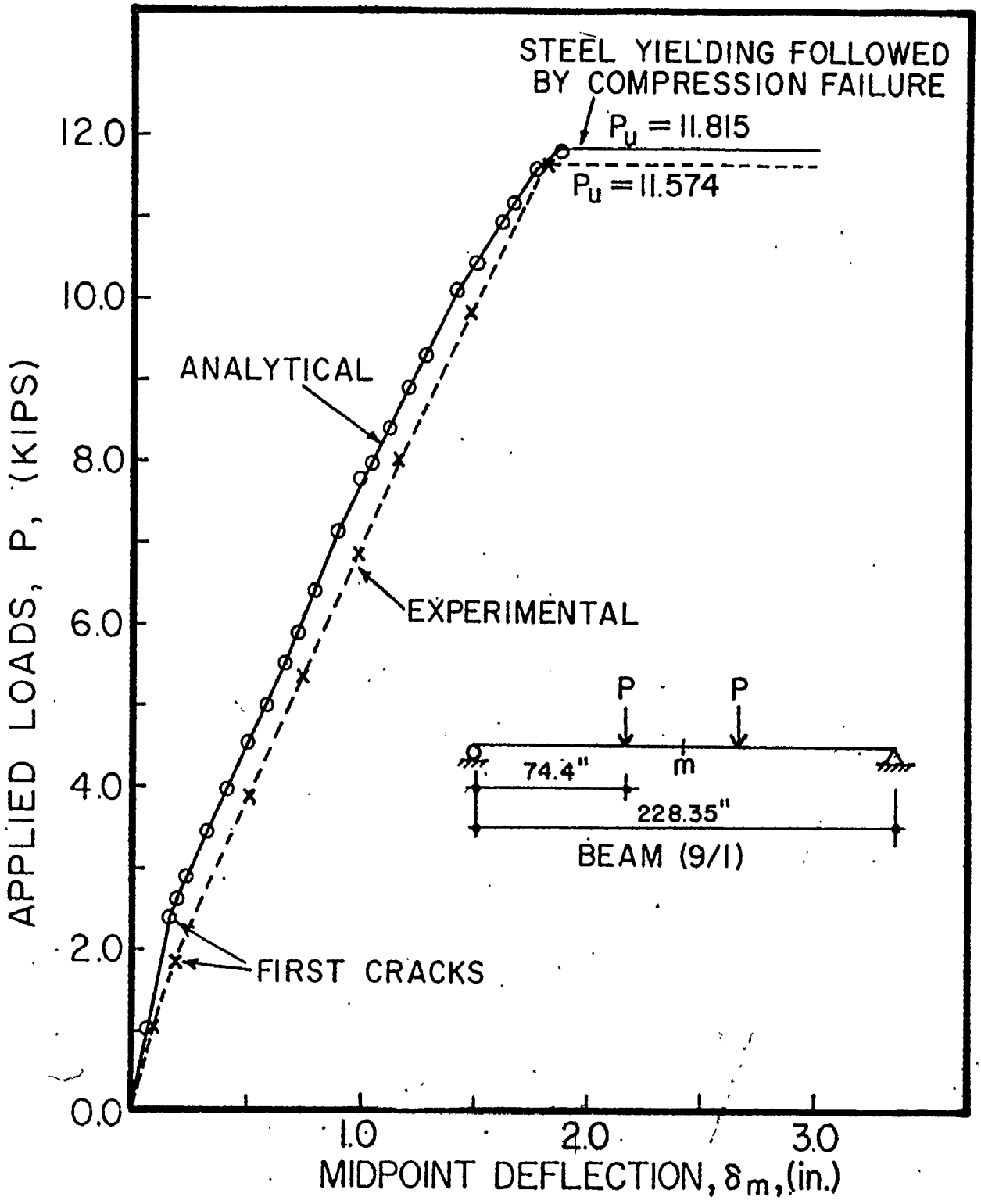


Fig. 6.5 LOAD–DEFLECTION RESULTS FOR BEAM (9/1)
$a/d = 7.0$

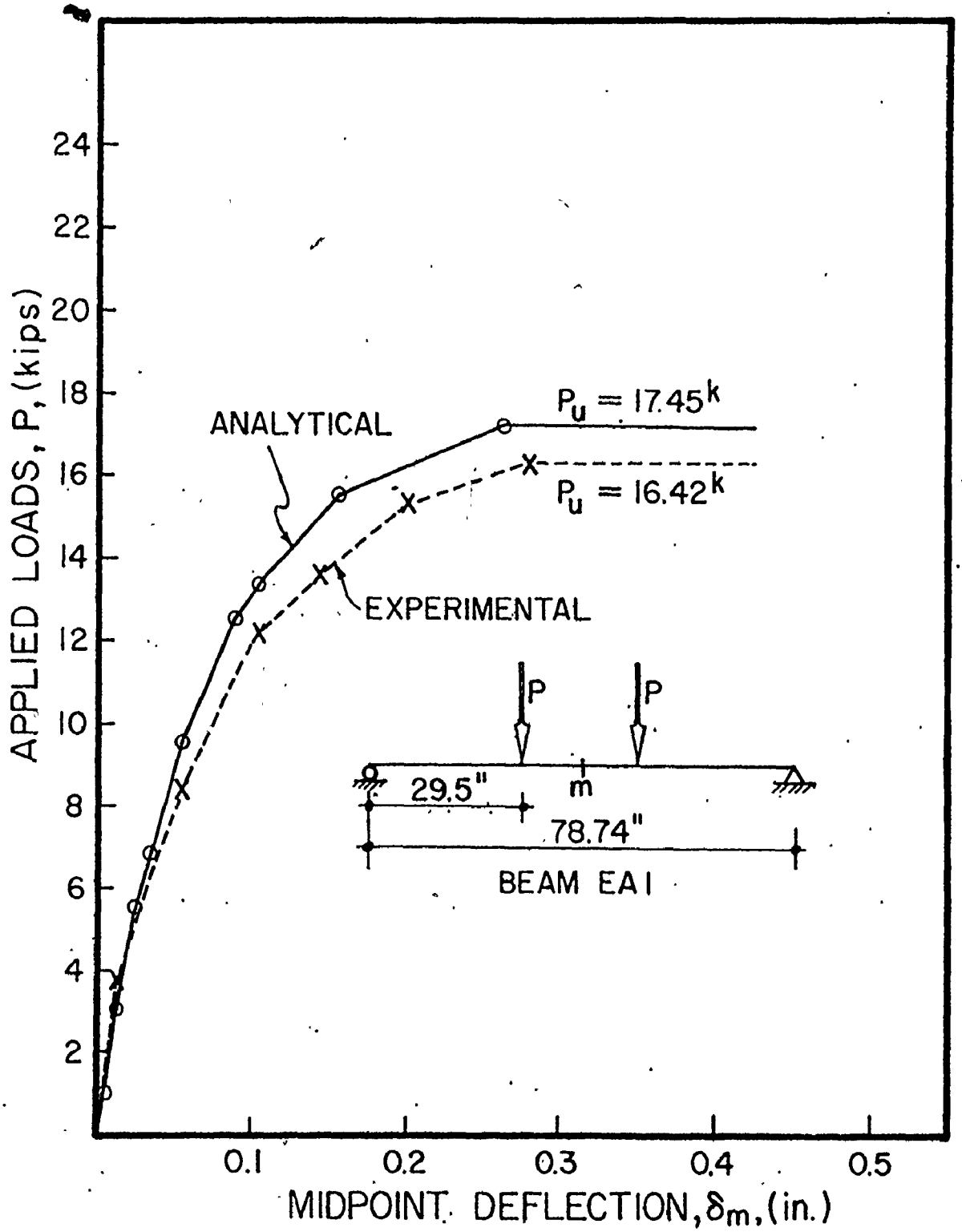


Fig. 6.6 LOAD DEFLECTION RESULTS FOR BEAM EA I
NORMAL BOND – $a/d = 2.78$

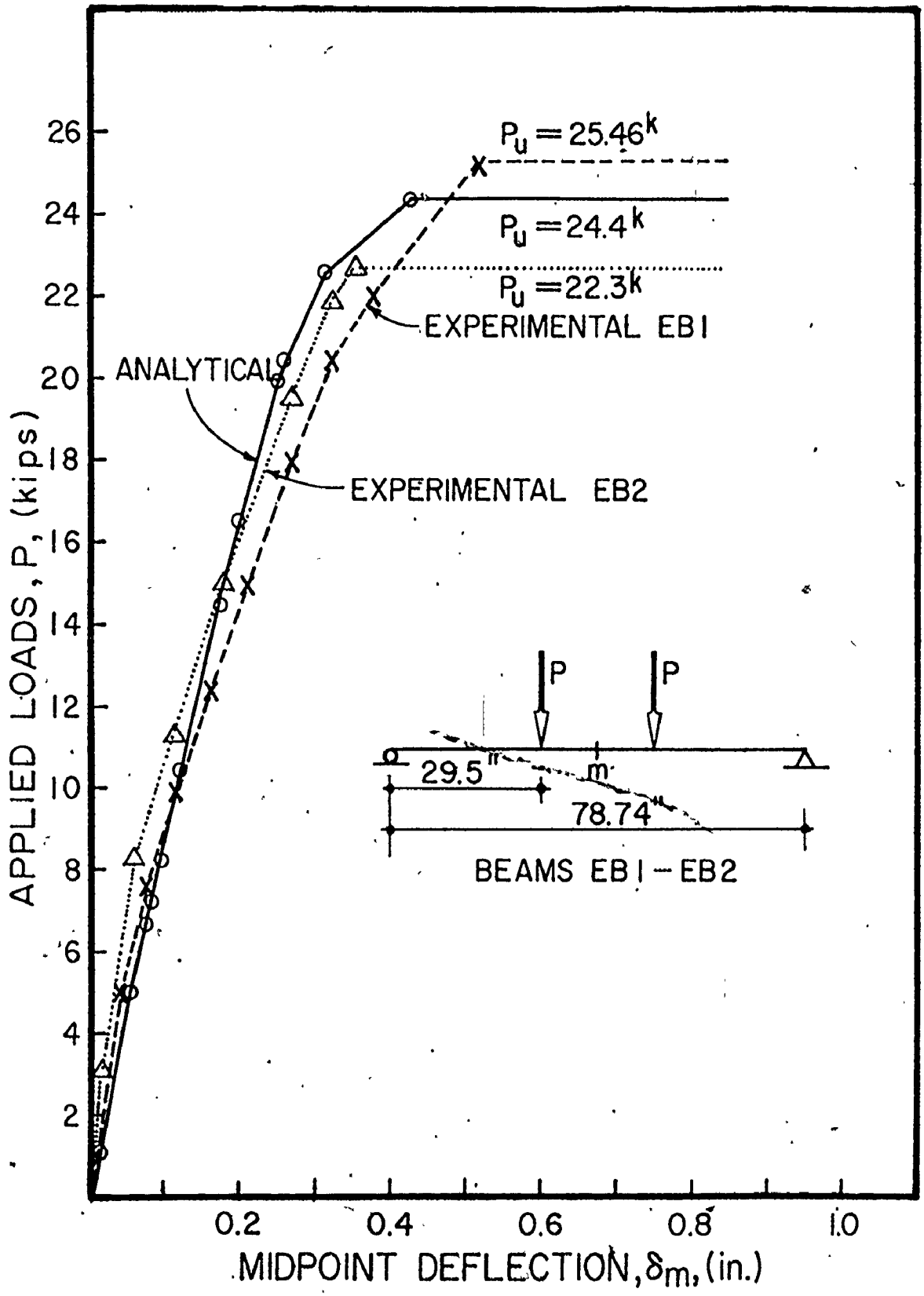


Fig. 6.7 LOAD DEFLECTION RESULTS FOR BEAMS EB1–EB2
POOR BOND, a/d = 2.78

In Figure 6.4, two curves from the analysis are compared with the experimental curve. One analysis was done using a modulus of rupture, f_t, of $9.5\sqrt{f'_c}$ and the other using $8.0\sqrt{f'_c}$. This comparison permitted the study of the sensitivity of the analysis to a change in the value of the modulus of rupture which determines the failure criterion for concrete in tension. As can be seen the analytical load-deflection curves were slightly apart in the first loading stages, and the load at first cracking was proportionately smaller for the smaller value of f_t. However, when the height of flexural cracking stabilized at a load near 7.0 kips, the two deflection curves coincided.

In conclusion, a generally close agreement between the analyses and the test results was obtained using only a moderately fine finite element mesh.

6.5.3 Cracking Patterns and Failure Modes:

In this section predicted crack patterns from the proposed method of analysis are compared with the crack patterns taken from pictures of test specimens in Reference 28. The crack patterns shown in Fig. 6.2 were scaled up in Figs. 6.8 to 6.12 to facilitate comparison with predicted cracks. Figures 6.8 to 6.12 are for beams (4), (7/1), (9/1), EA1 and EB1 respectively. The sequence of test crack occurrence is only shown for beams EA1 and EB1 since for the remainder, this information was not readable. The results for each beam are discussed separately in more detail below.

6.5.3.1 Beam (4)

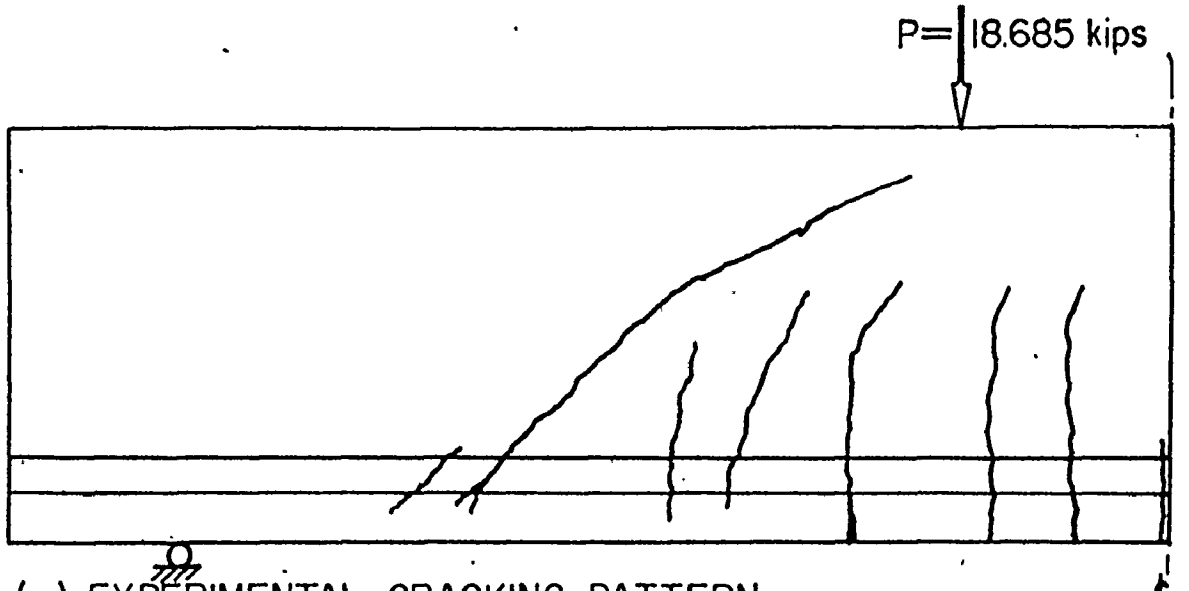
The experimental crack patterns for the left and right hand sides of the beam shown in Figures 6.8a and 6.8b respectively. The predicted crack pattern is superimposed on the finite element mesh used as shown in Fig. 6.8c. The failure loads are shown for both the test and the analysis. These loads and the first cracking loads are included in Table 6.2.

The two sides of the tested beam are shown to illustrate the degree of variation of cracking patterns and crack locations for supposedly identical conditions.

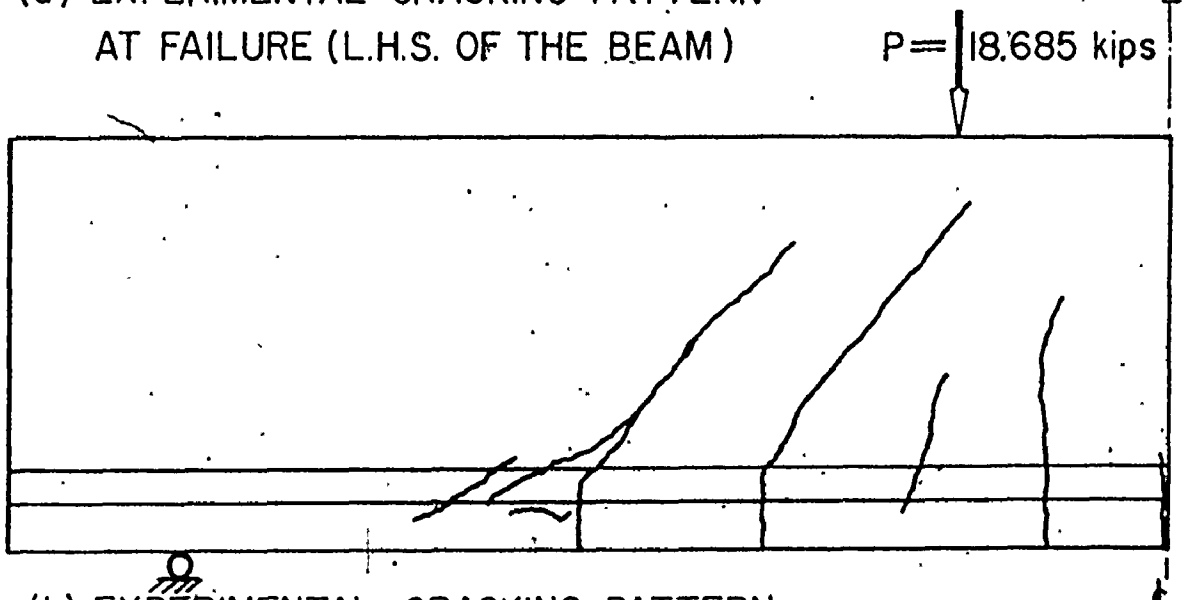
The numbers shown in Fig. 6.8c indicate the sequence of the crack occurrence predicted by the analysis. The load levels corresponding to these numbers were listed in Table 6.3 for all beams.

As can be seen from Fig. 6.8c not all of the elements in the constant moment region are cracked even at failure and the effect of the bond-slip phenomenon on the spacing and sequence of the cracking can be noticed. More detailed discussion on this effect is included for Beam (9/1).

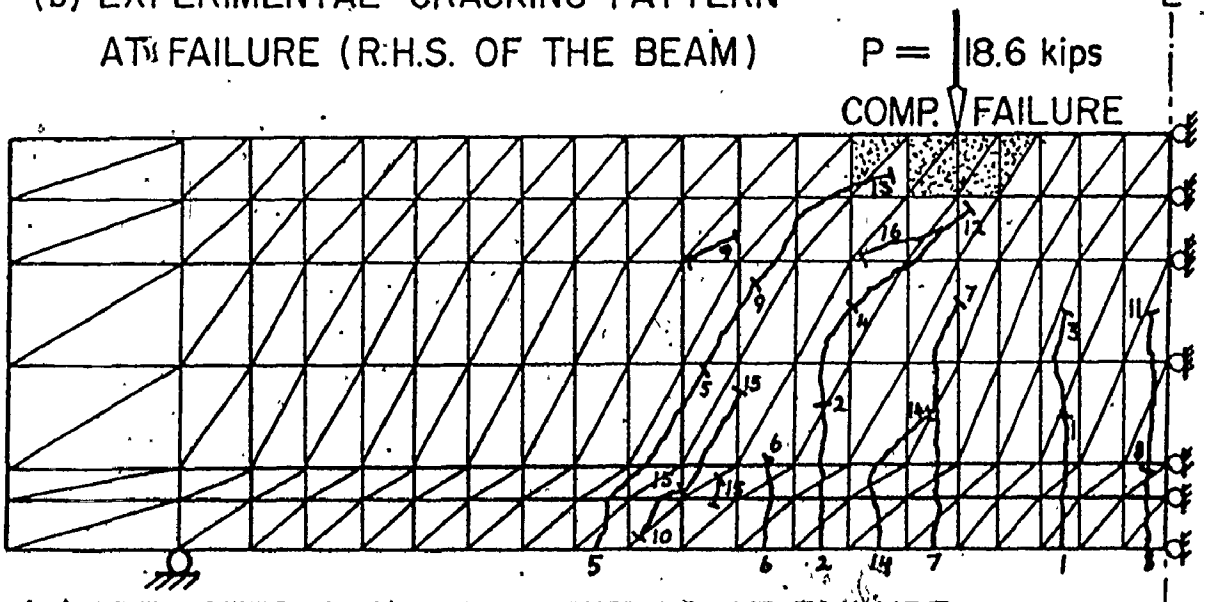
Shear-compression failure occurred when the crack closest to the support (which was the fifth crack to occur) had propagated during load step 13, and substantially reduced the comparison zone near the load location. The second major crack identified by load steps 2, 4, and 12 may also have had some influence on a reduced compression zone. When shear



(a) EXPERIMENTAL CRACKING PATTERN AT FAILURE (L.H.S. OF THE BEAM)



(b) EXPERIMENTAL CRACKING PATTERN AT FAILURE (R.H.S. OF THE BEAM)



(c) PREDICTED CRACKING PATTERN AT FAILURE.

Fig 6.8 CK PATT ILTS FOR M (a 2.5)

Table 6.2 Beam Cracking and Failure Loads.

BEAM	First Cracking Load			Ultimate Failure Load		
	Experimental (kips)	Analytical (kips)	Analysis / Experiment	Experimental (kips)	Analytical (kips)	Analysis / Experiment
4	-	5.64	-	18.685	18.60	0.99
7/1	-	2.89	-	13.01	13.887	1.067
9/1	-	2.44	-	11.574	11.815	1.021
EA1	5.12	5.42	1.062	16.42	17.45	1.059
EB1	4.80	5.02	1.046	25.46	24.4	0.96

Load Step No.	Beam (4)	Beam (7/1)	Beam (9/1)	Beam EA1	Beam EB1	Load Step No.	Beam (9/1)
1	5640	2886	2440	5423	5020	30	8101
2	7510	3443	2493	6837	5253	31	8165
3	8960	4332	2536	9491	6662	32	8211
4	9430	4433	2809	10041	7137	33	8312
5	12470	4887	3465	10226	8069	34	8440
6	12492	5560	3507	12223	10225	35	8689
7	13475	5855	3714	12583	14394	36	8825
8	14528	5864	3728	13517	16506	37	9187
9	15370	6163	3885	15777	19957	38	9270
10	15381	6428	4001	15984	20460	39	9299
11	15695	6703	4099	16782	22703	40	9665
12	18196	6962	4149	17451	24400	41	9712
13	18600	6988	4194			42	9976
14	18600	7906	4278			43	10056
15	18600	8525	4439			44	10097
16	18600	9625	4523			45	10151
17		9637	4778			46	10309
18		10344	5033			47	10480
19		11083	5160			48	10695
20		11162	5309			49	10877
21		12038	5508			50	10949
22		12973	5826			51	11105
23		13887	6397			52	11263
24		13887	6545			53	11349
25		13887	7114			54	11407
26		13887	7357			55	11449
27		13887	7738			56	11572
28			7789			57	11689
29			8035			58	11815

Table 6.3 Loads in lbs. Corresponding to load step numbers

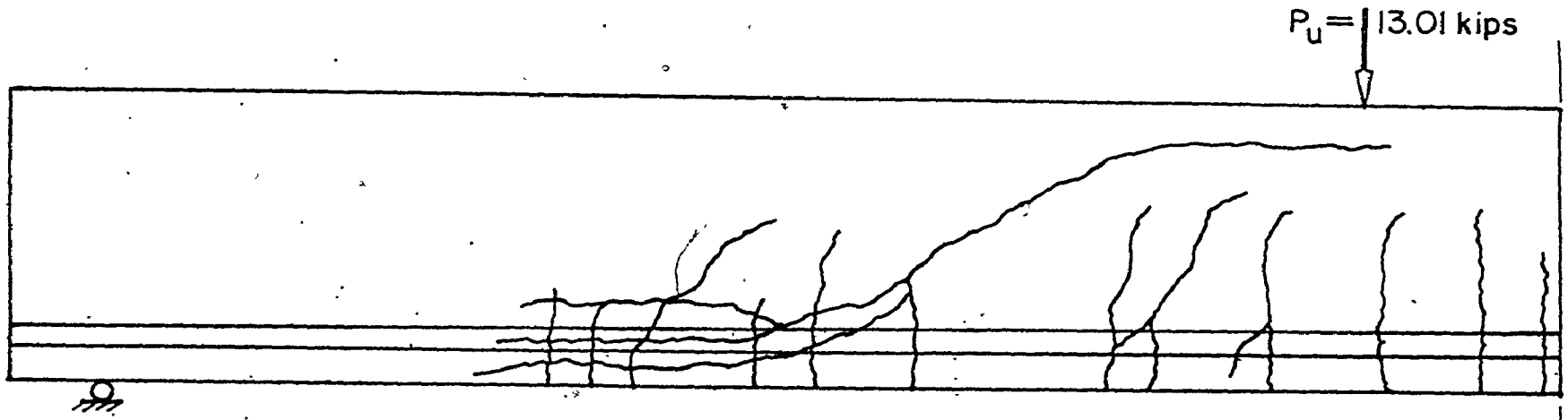
compression failure occurred, post-failure cracks 14-14, 15-15 and 12-16 developed. No bond break down or anchorage failure occurred because proper end anchorage was provided.

There was very close agreement between the test and the analysis results for the cracking patterns (spacing, height, shape and location) and the failure loads. Also, the mode of failure reported from the test (shear-compression failure) was predicted by the analysis.

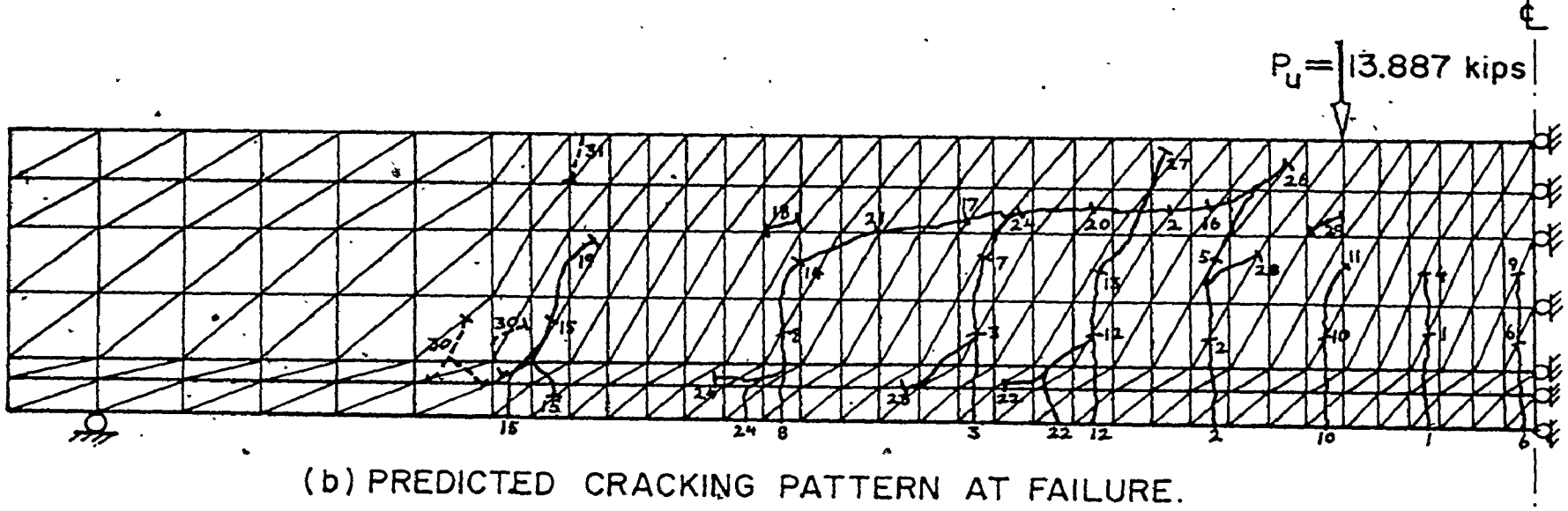
(6.5.3.2) Beam (7/1)

Beam (7/1) was chosen to be analyzed in this thesis because ~~its~~ a/d ratio (5.0) was in the middle range for diagonal tension failure. More detailed analytical results for this beam than for the others are included for this reason. Detailed drawings showing crack propagation at different loading steps are included, along with stresses in concrete and steel reinforcement. Also, the bond stresses along the reinforcement for different load levels are presented and discussed in the next section.

A sudden diagonal tension failure of beam (7/1) was reported by Leonhardt and Walther (28). It was significant to see the same type of distinct crack propagation and failure detected by the analysis. As can be seen in Figure 6.9 the crack associated with failure is very flat compared with the corresponding cracks shown in Fig. 6.8 for beam (4). This flat crack was initiated from the flexural-shear crack, 8-14-21 and several other cracks which opened near the top of



(a) EXPERIMTAL CRACKING PATTERN AT FAILURE.



(b) PREDICTED CRACKING PATTERN AT FAILURE.

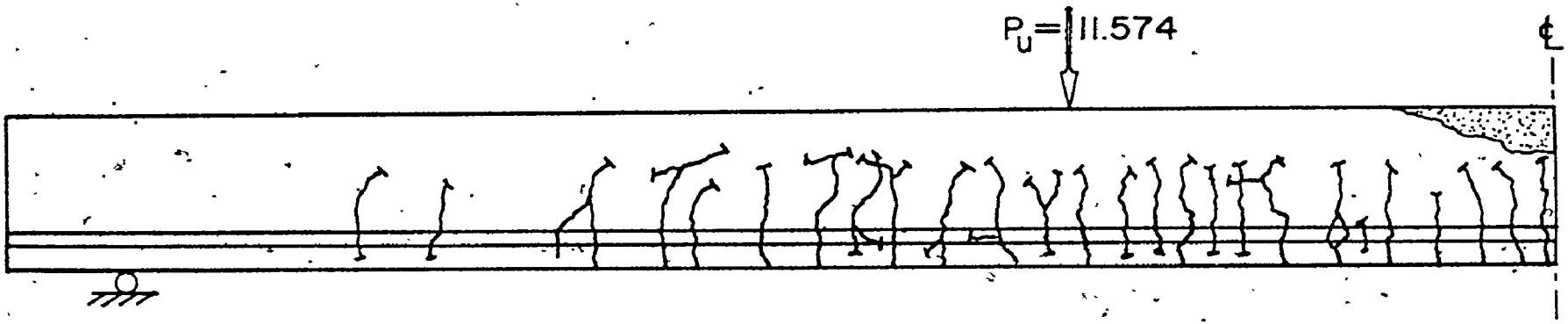
Fig. 6.9 CRACKING PATTERN RESULTS FOR BEAM 7/1 (a/d = 5.0)

existing flexural cracks at very flat angles and at different stages of loading. All of these small flat cracks, designated by 16, 17 and 20, were joined by crack 8-14-21 at load step 26 to form a diagonal tension crack which extended to near the loading point and which caused the beam to fail by crushing of the compression zone around the loading point. The same observation regarding the cracking pattern discussed for beam (4) apply here.

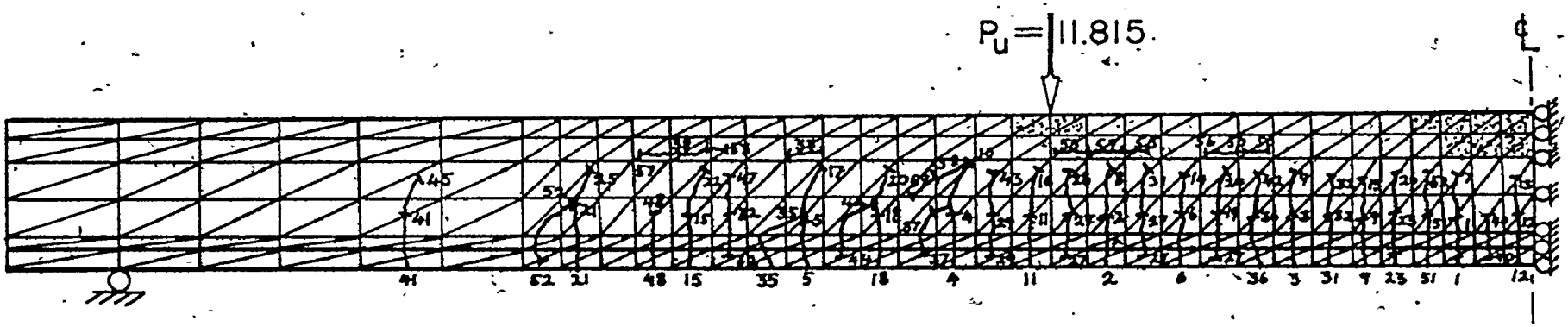
It is also very interesting to observe that dowel cracks have occurred in the shear span at loads near failure, steps (22, 23, and 24) and after initial failure. The excessive post-failure dowel cracking which cannot be prevented in tests was not predicted by the analysis. The analysis was stopped at a preselected limiting deflection before the rest of the dowel cracks along the reinforcement had the chance to form.

6.5.3.3 Beam (9/1)

Experimental results have shown that Beam (9/1) with an a/d ratio near 7, should fail in flexure for normal loading conditions and normal bond-slip characteristics. The reported test failure load of 11.57 kips is very close to the theoretical ultimate flexural capacity of the beam. Flexural failure can be identified from observation of the cracking pattern shown in Fig. 6.10 at failure of the beam. Although some short flat cracks appear near the top of some of the flexural cracks in both the shear span and the constant moment region, no diagonal cracks can be seen in the crack pattern



(a) EXPERIMENTAL CRACKING PATTERN AT FAILURE.



(b) PREDICTED CRACKING PATTERN AT FAILURE.

Fig. 6.10 CRACKING PATTERN RESULTS FOR BEAM 9/1 (a/d = 6.92)

at failure. Yielding of the steel followed by crushing of the concrete in compression was predicted from the analysis. This does not exactly agree with the test result, where concrete in compression crushed prior to yielding of the steel. This disagreement may be partly due to the closeness of the steel amount used compared to the balanced steel of the section, and partly due to the idealization of the stress-strain relationship by an elastic-plastic curve . (Percentage of steel was 2.2, and balanced steel is 2.3%).

As can be seen from Fig. 6.10b almost every row of elements is cracked. This is due to the coarse mesh used for this beam which was necessary because it was fairly long. The size of the elements is almost equal to the crack spacing found in the experimental work and reproduced in the analysis of other beams using a finer element mesh. The cracks did not occur simultaneously and the sequence of crack occurrence followed the theory of cracking (27). Cracking would usually begin with large spacing. The the distance between two cracks, (usually called primary cracks) would normally be halved by another crack which is usually termed a secondary crack. The secondary crack may or may not propogate fully depending on its location relative to all other cracks. This process of halving the crack spacing continues until a typical minimum crack spacing for the particular geometry is reached.

Although a fairly coarse element mesh size was used as compared to the other beams analysed, this did not affect the degree of accuracy of the analysis to any appreciable

extent as can be judged from comparisons of ultimate load, the load-deflection results, and the cracking patterns.

6.5.3.4 Beams EA1 & EB1

Beams EA1 & EB1 were analysed to demonstrate the effect of the bond-slip relationship on the behaviour of simply supported R.C. beams. Beams EA1 & EB1 have the same a/d ratio of 2.78. In this range of a/d ratios, reduction of capacity from flexural strength due to shear type failure is most marked. Beam EA1 had deformed reinforcing bars with normal bond while Beam EB1 had all the same physical qualities of beam EA1 except that the steel bars were very smooth and had a mirror like surface. Therefore, the bond would be expected to be very poor.

Figure 6.6 shows the load-deflection curves for beam EA1 obtained from the experiment and the computer analysis. Also, the ultimate capacities of the beam is indicated for both. The predicted ultimate load is 6.3% higher than the experimental value. The agreement between the analytical and experimental results can be considered quite acceptable.

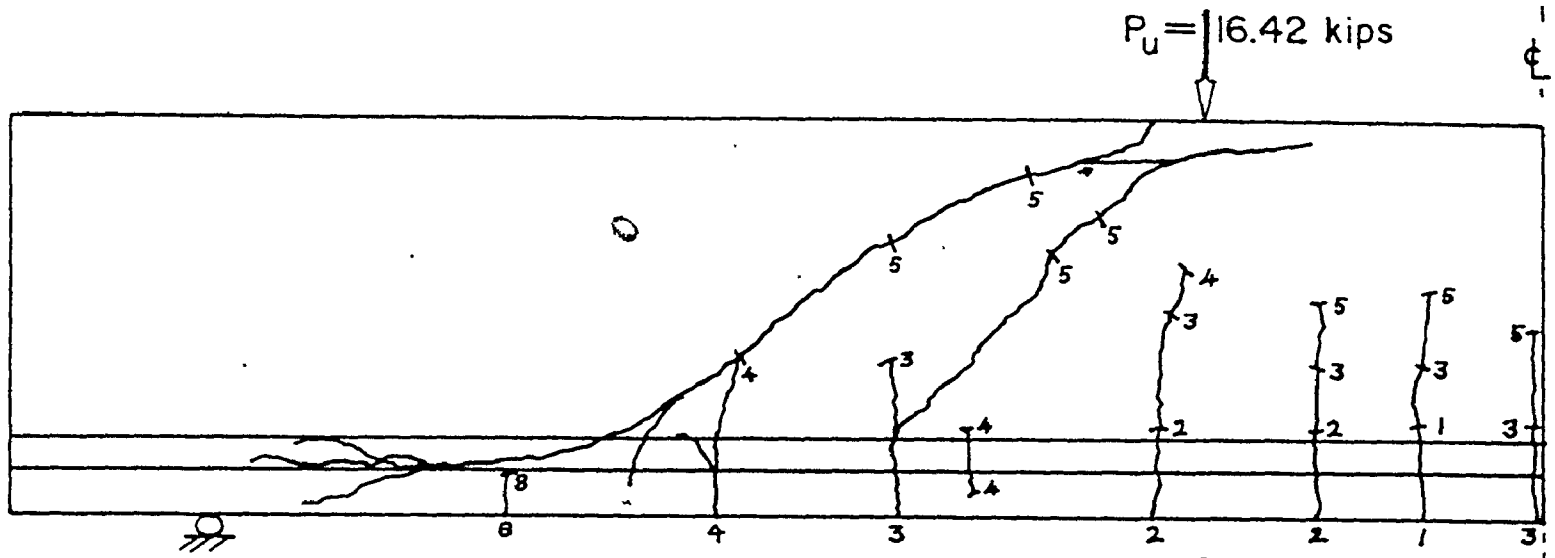
The analytical and experimental load-deflection curves for beam EB1 are shown in Fig. 6.7 along with the corresponding ultimate load values. The experimental load deflection curve for beam EB2 which is another beam similar to beam EB1 except that it had slightly better bond, is also shown. This slightly better bond was obtained by using smaller but still

smooth bars, having the same total cross-sectional area as for beam EB1. Small values for stiffness were assigned to the bond elements in the computer program where zero stiffness would cause an error in the stiffness matrix solution. No specific values for bond-slip were given for these beams. However, the adhesion between the bars and the concrete can provide some bond resistance. Three arbitrary values of 1%, 5% and 10% of the normal bond stiffness were analyzed. The 10% value gave the most realistic result. Therefore it was used in the analysis with the same failure criterion as that for normal bond.

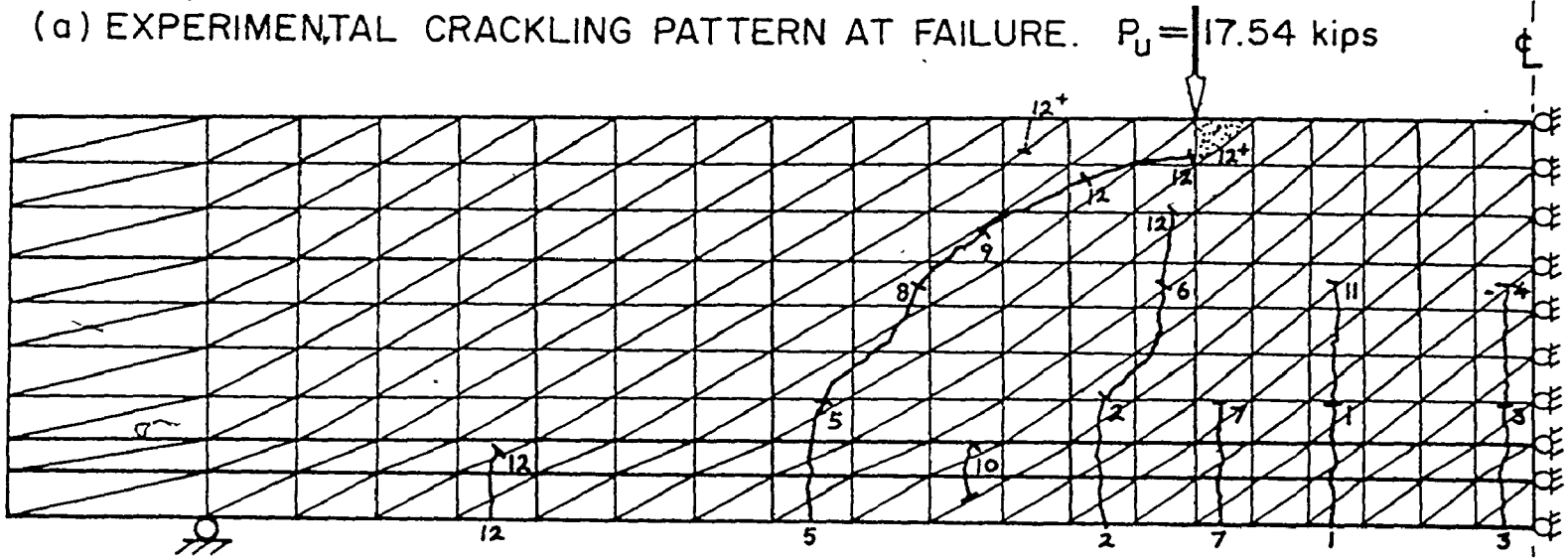
The analytical load-deflection curve lies very close to both the EB1 and EB2 experimental curves, however, the predicted ultimate load was 4.2% lower than the experimental load for beam EB1 & 9.8% higher than the experimental load for beam EB2. This discrepancy may indicate that the bond-slip values used in the computer program did not accurately represent the actual characteristics of either beam EB1 or beam EB2.

Comparison of the crack patterns on Figures 6.11 and 6.12 shows the dramatic difference in behaviour due to different bond-slip characteristics for otherwise identical beams.

Figure 6.11a shows the experimental cracking pattern at failure for beam EA1, while Fig. 6.11b shows the analytically predicted cracking pattern for the same beam. The crack occurrence sequence is shown for both the test and the analysis.

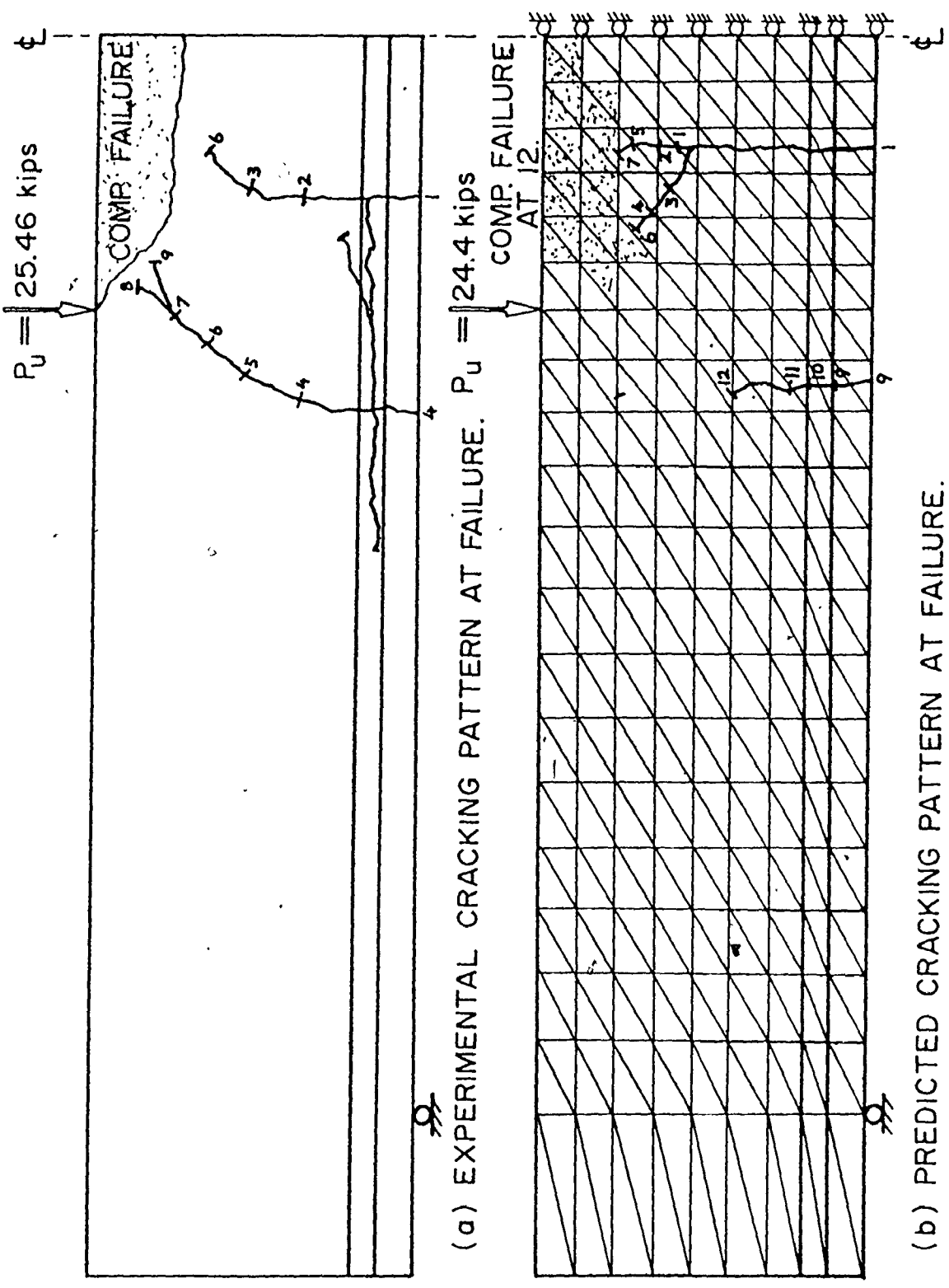


(a) EXPERIMENTAL CRACKLING PATTERN AT FAILURE. P_u = 17.54 kips



(b) PREDICTED CRACKING PATTERN AT FAILURE.

Fig. 6.11 CRACKING PATTERN RESULTS FOR BEAM EA I (NORMAL BOND, a/d=2.78)



(a) EXPERIMENTAL CRACKING PATTERN AT FAILURE.

(b) PREDICTED CRACKING PATTERN AT FAILURE.

Fig. 6.12 CRACKING PATTERN RESULTS FOR BEAM EB1 (POOR BOND, a/d=2.78)

It can be seen that the agreement between crack sequences, crack locations (crack spacing) and the general crack pattern at failure is quite close. The same agreement can be observed for beam EB1 from Figs. 6.12a and 6.12b. Only two cracks occurred in both the experimental and predicted cracking patterns For this latter beam, failure occurred, experimentally and analytically due to compressive crushing of the concrete at loads just below the theoretical flexural capacity of the beams. The difference between the modes of failure for beams EA1 and beam EA2 can be easily visualized from the predicted cracking patterns. Beam EA1 failed in a shear-compression mode whereas beam EB1 failed in a flexural-compression mode.

From the experimenal and the analytical ultimate failure loads for beams EA1 and EB1, it can be concluded that, contrary to the general notion of stronger being better, stronger bond causes premature shear-compression or diagonal tension failures in beams with a/d ratios in this range. Also failure loads have been shown to be quite near their flexural capacity when reinforced with low bond bars provided that they are adequately anchored at their ends. However, the widths of cracks will be much larger than those for better bonded reinforcement. Larger crack widths occur due to the concentration of all the reinforcing elongations at relatively fewer crack locations when bond is poor.

6.5.4 Detailed behaviour of Beam (7/1) at different loading Steps:

Figures 6.13 to 6.35 show the extent of cracking at different loading steps as predicted by the analysis. Also, the longitudinal and shear stresses in the concrete are shown for selected sections at several load steps. The longitudinal steel stresses are shown for all load steps and bond stresses along the bar are shown at several intervals upto near failure. [Please note: The stresses shown in these figures are drawn to different scales for each loading step. Shear stresses have their own scale which is shown for every figure. The steel stresses for the elements along the bar length are joined in the figures to show continuous stress curves for the longitudinal steel reinforcing bars. However, bond stresses are shown as discrete values. When bond is broken, (i.e. when a joint element fails in the direction parallel to the bar axis) an X mark is made to indicate the failure of this particular joint element].

This series of figures illustrates how the flexural cracks initiated and propagated until they stabilized in height around the 8,000 lbs. load level. At the load level of 9,625 lbs. flat cracks occurring immediately above some of the existing flexural cracks started to appear and flexural cracks started to branch at the bottom forming another crack. Failure occurred suddenly, at the 13,887 lbs. load level, when the flat cracks were all joined with an inclined crack in the shear span as shown with the dotted line, in Figure 6.35.

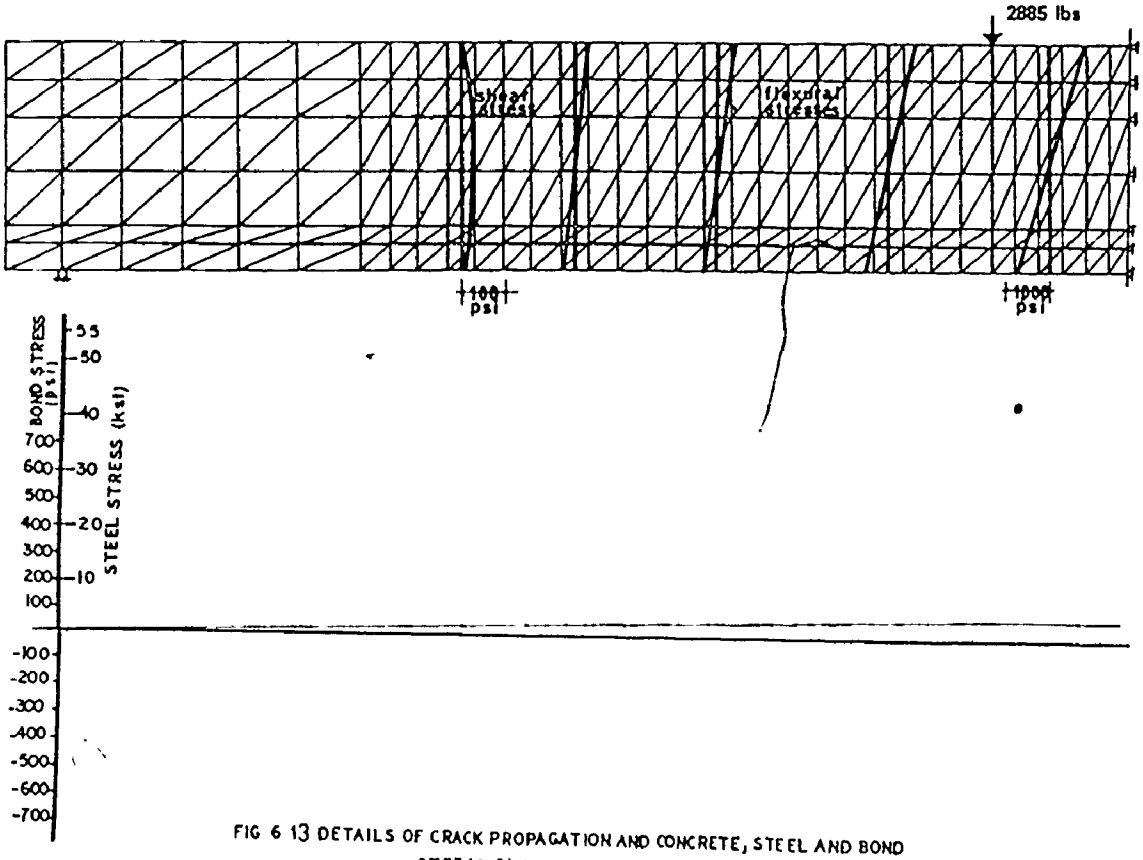


FIG 6 13 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

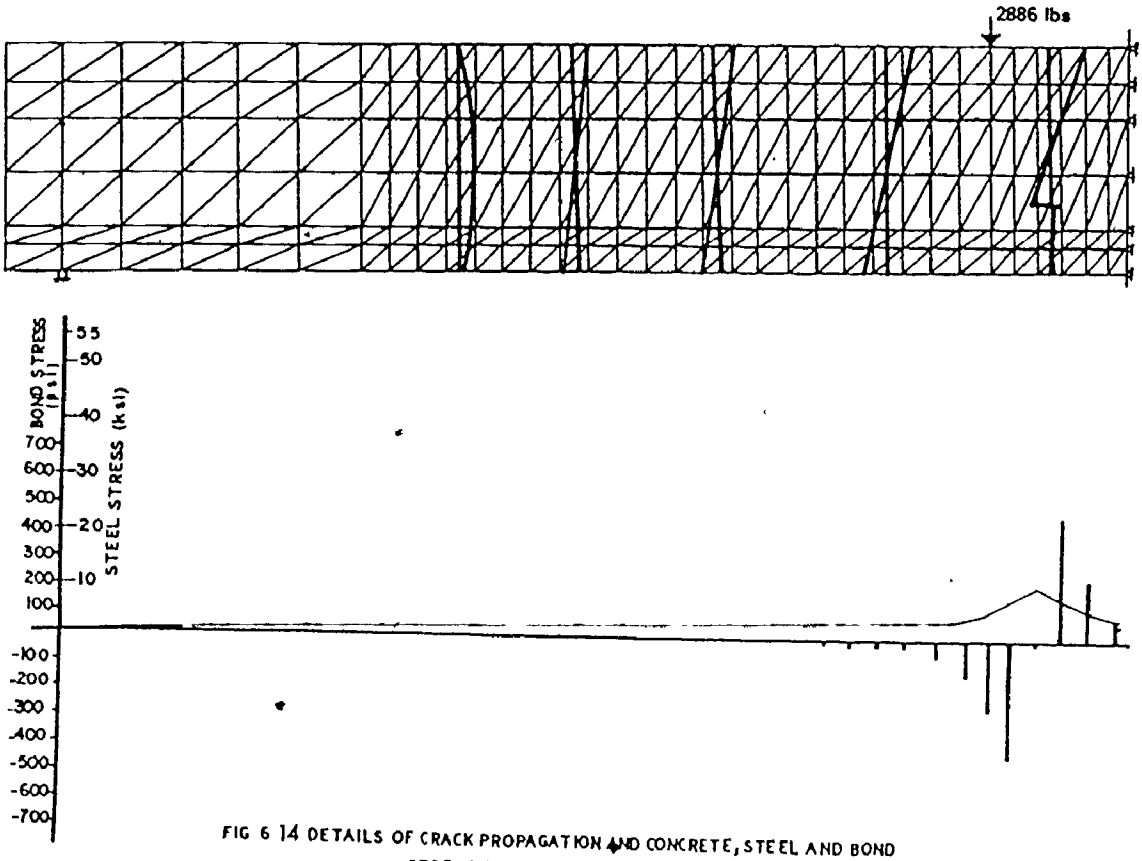


FIG 6 14 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

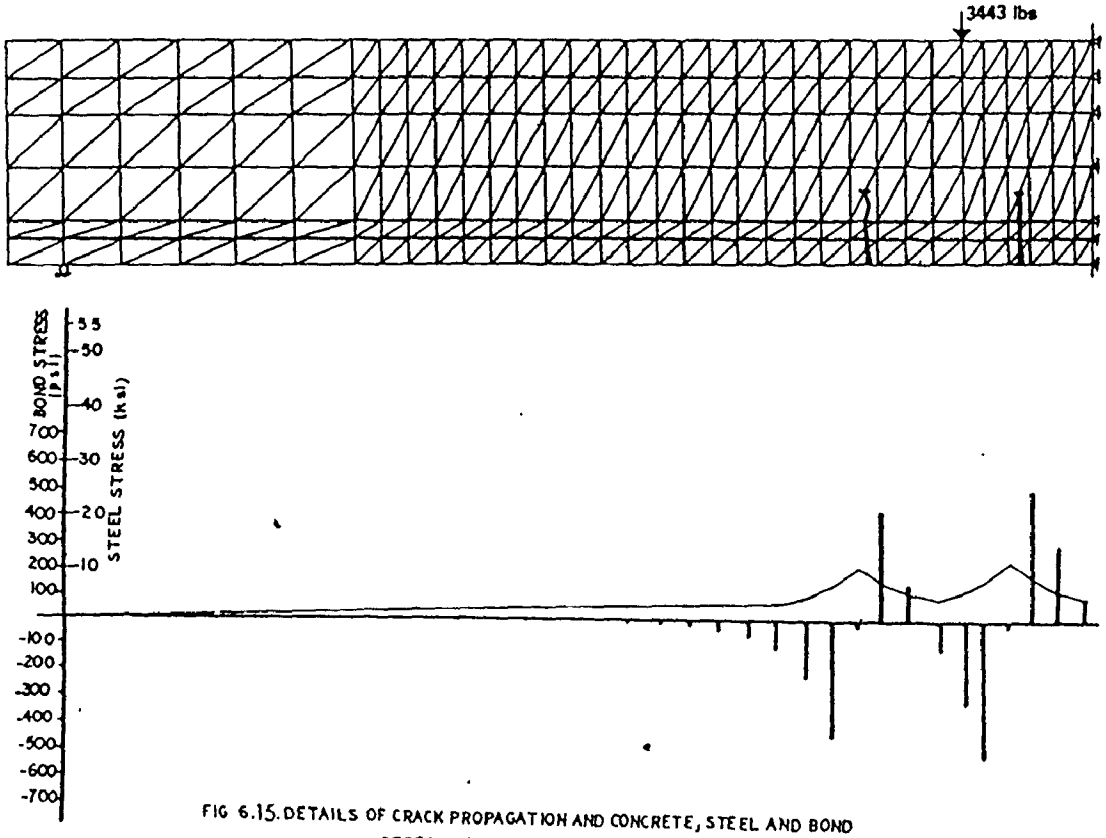


FIG 6.15. DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM (7/1)

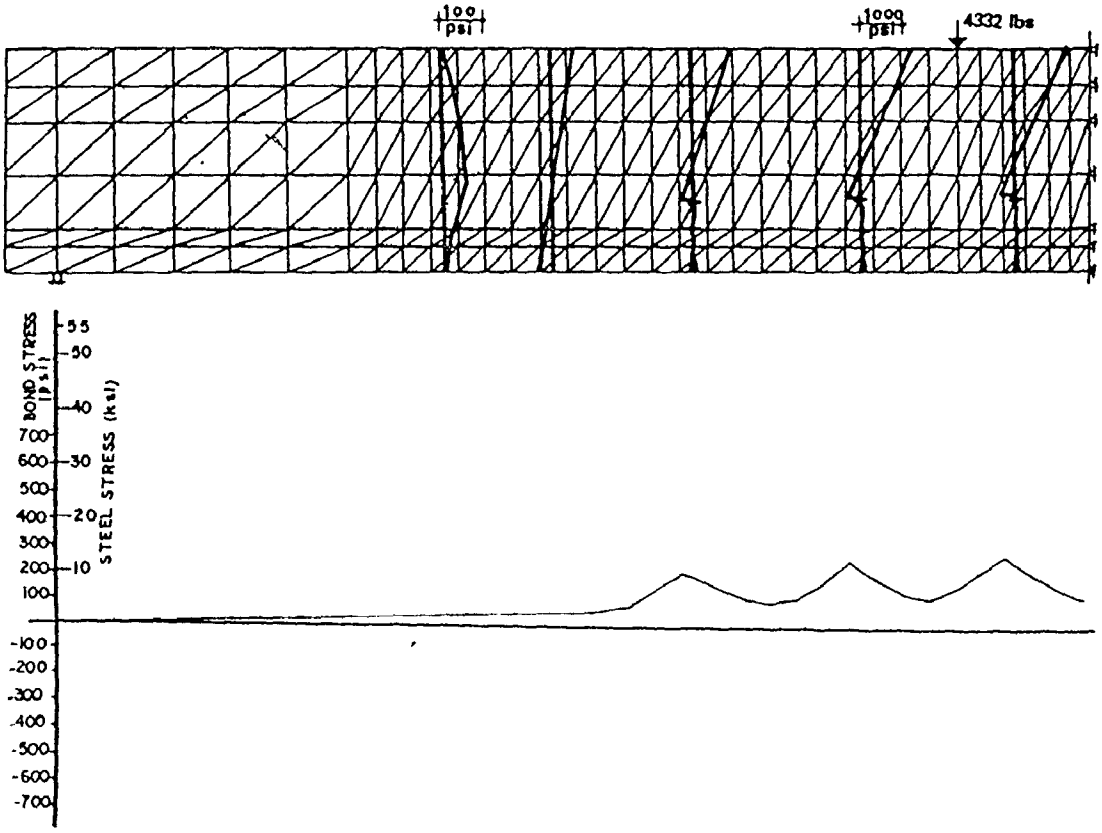


FIG 6 16 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM (7/1)

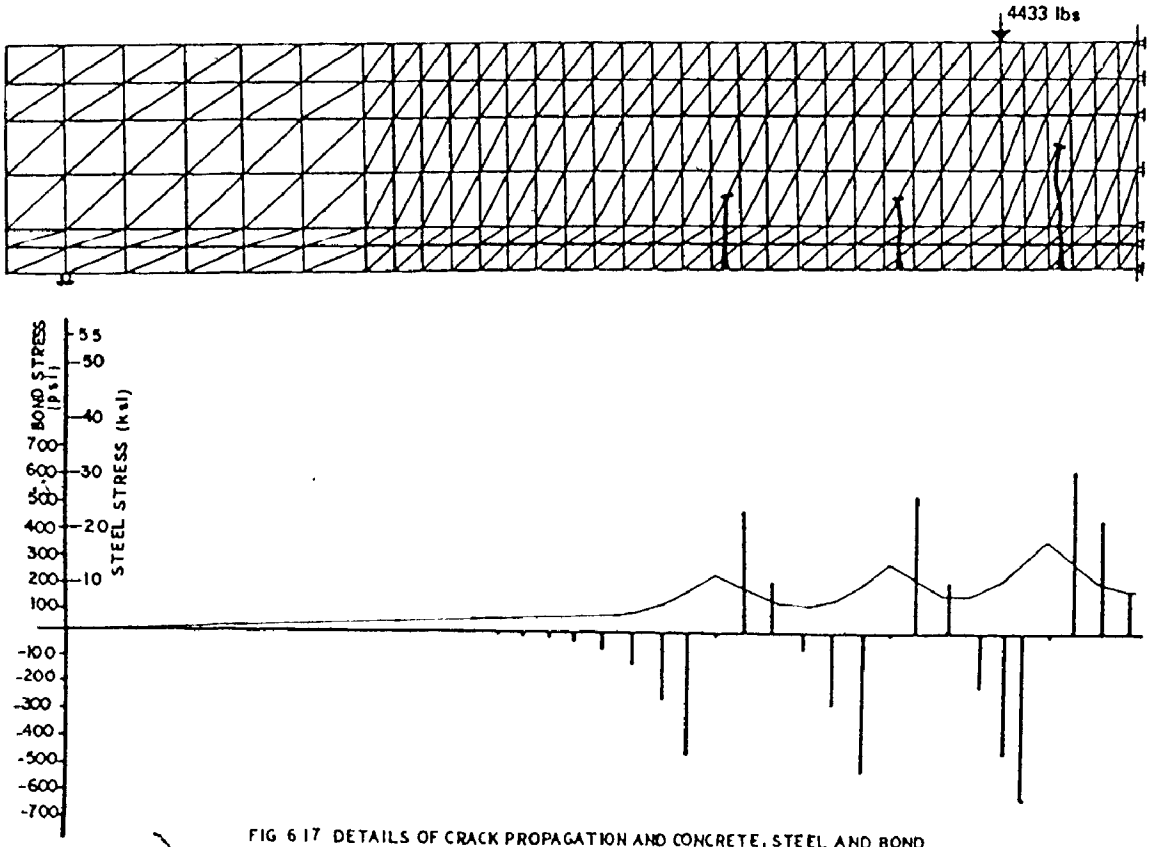


FIG 6 17 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

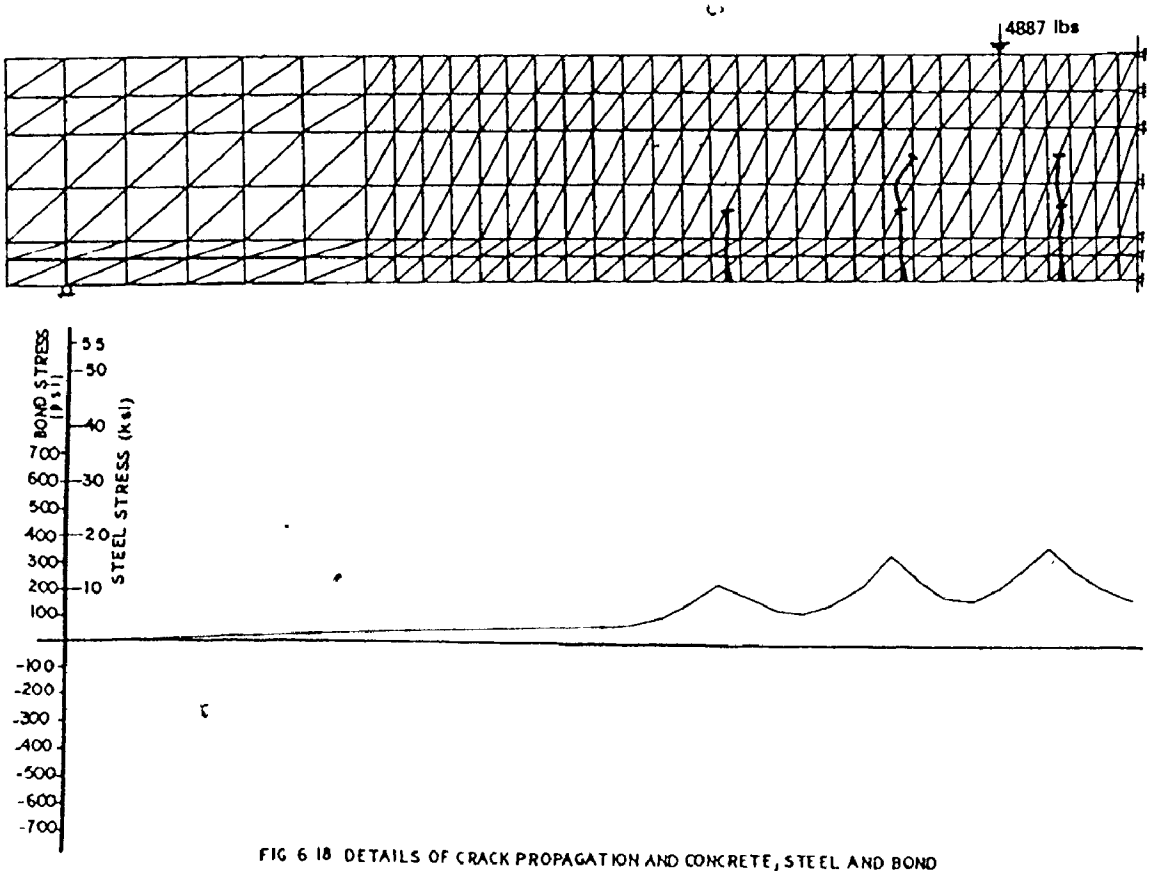


FIG 6 18 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

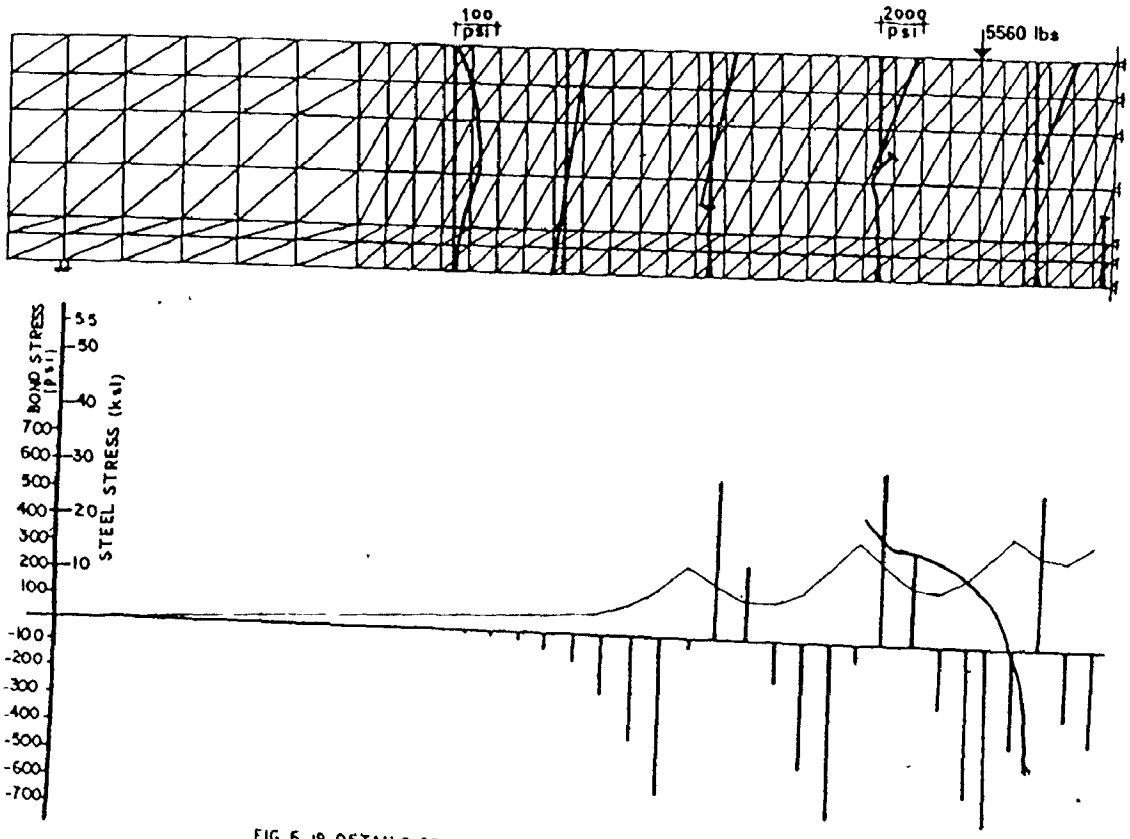


FIG 6 19. DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

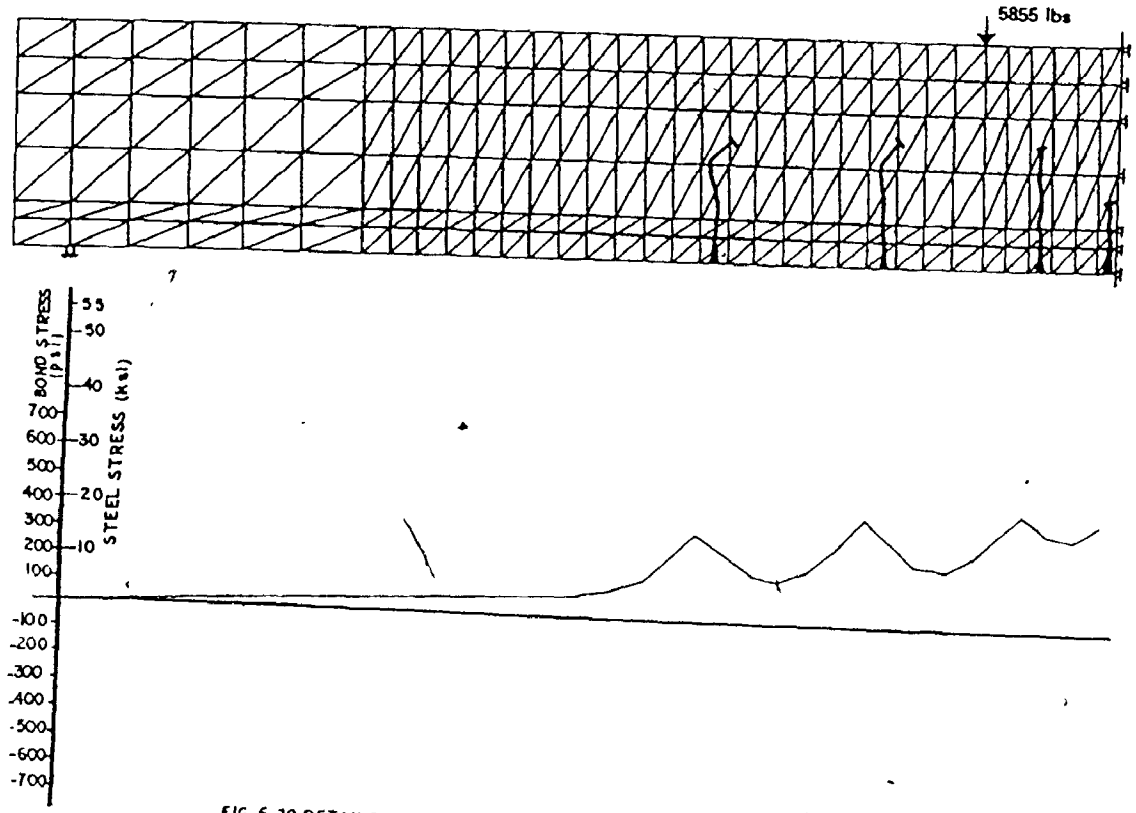


FIG 6 20 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

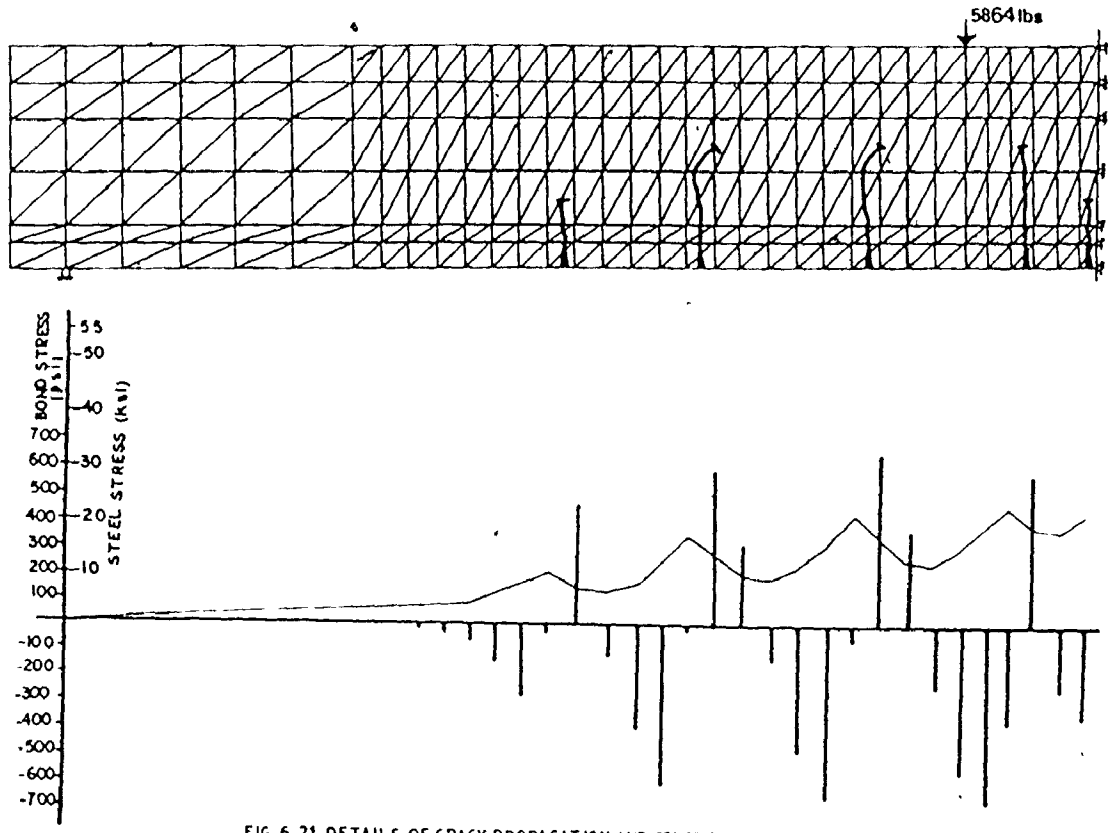


FIG 6 21 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

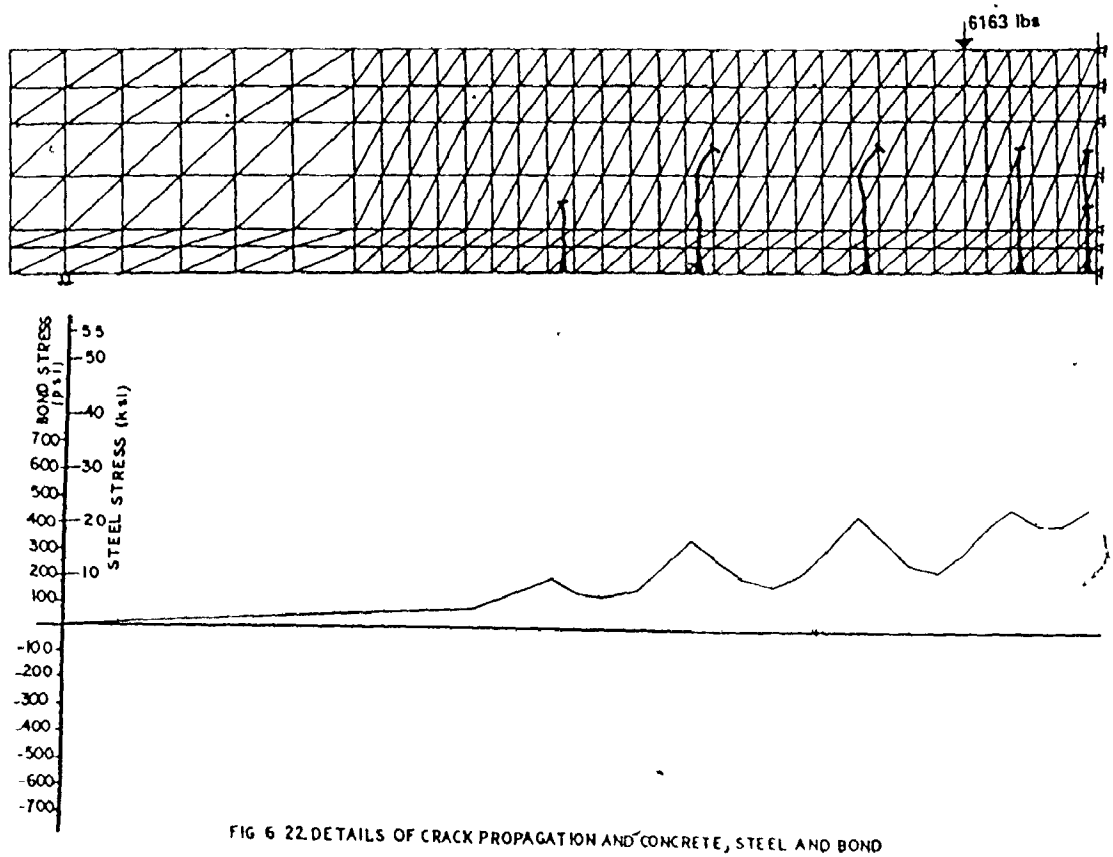


FIG 6 22 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

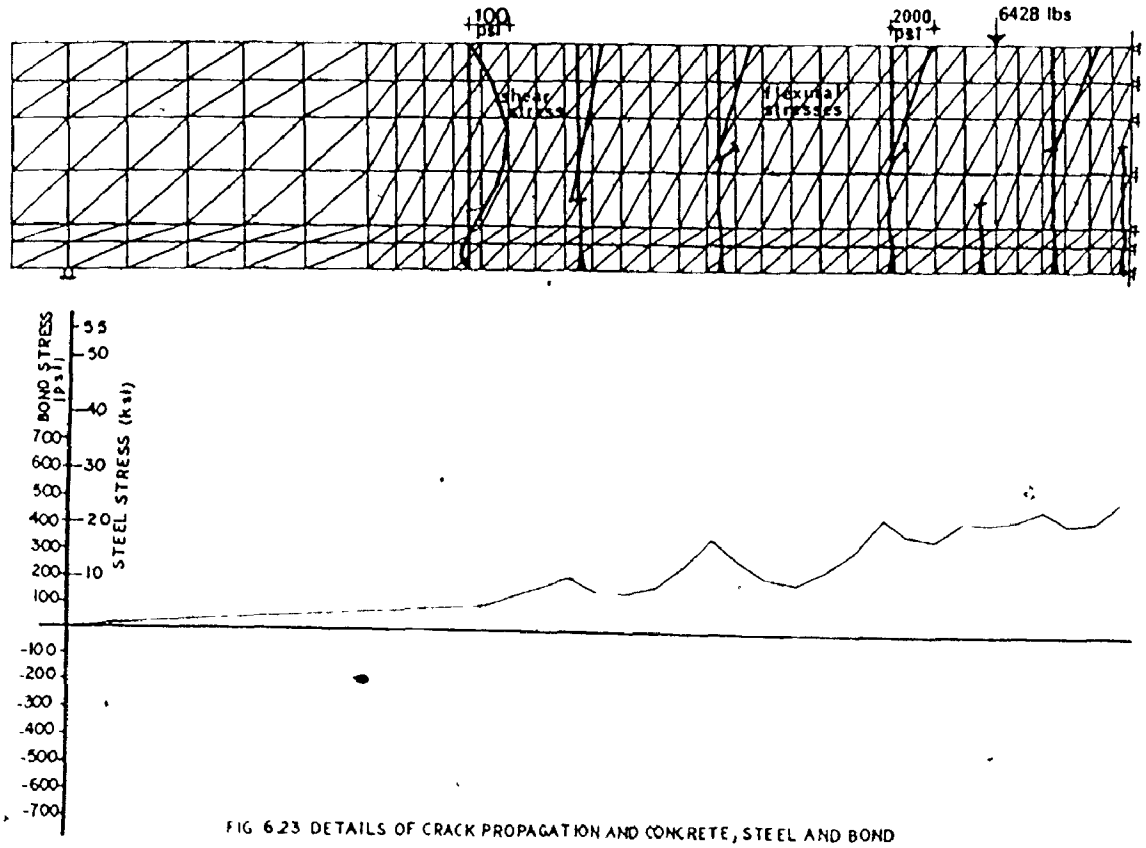


FIG 6.23 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM (7/1)

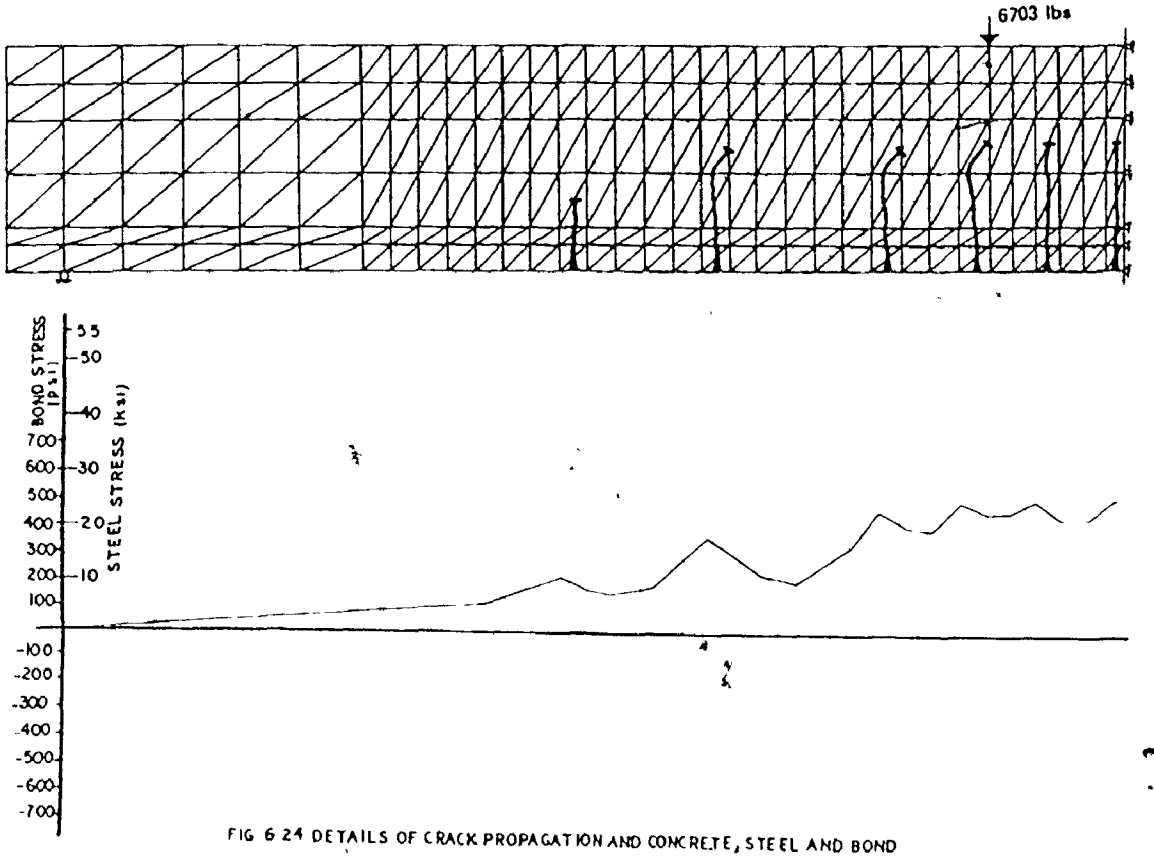


FIG 6.24 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM (7/1)

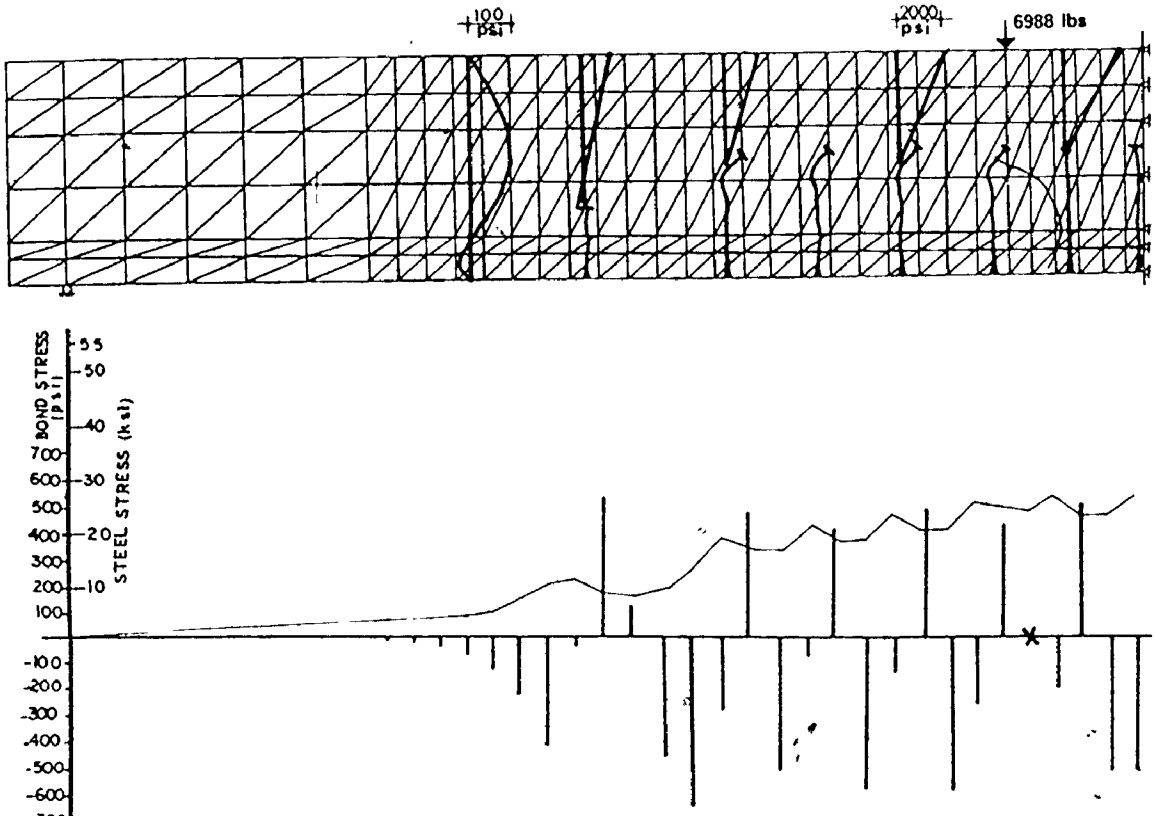


FIG 6 25 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

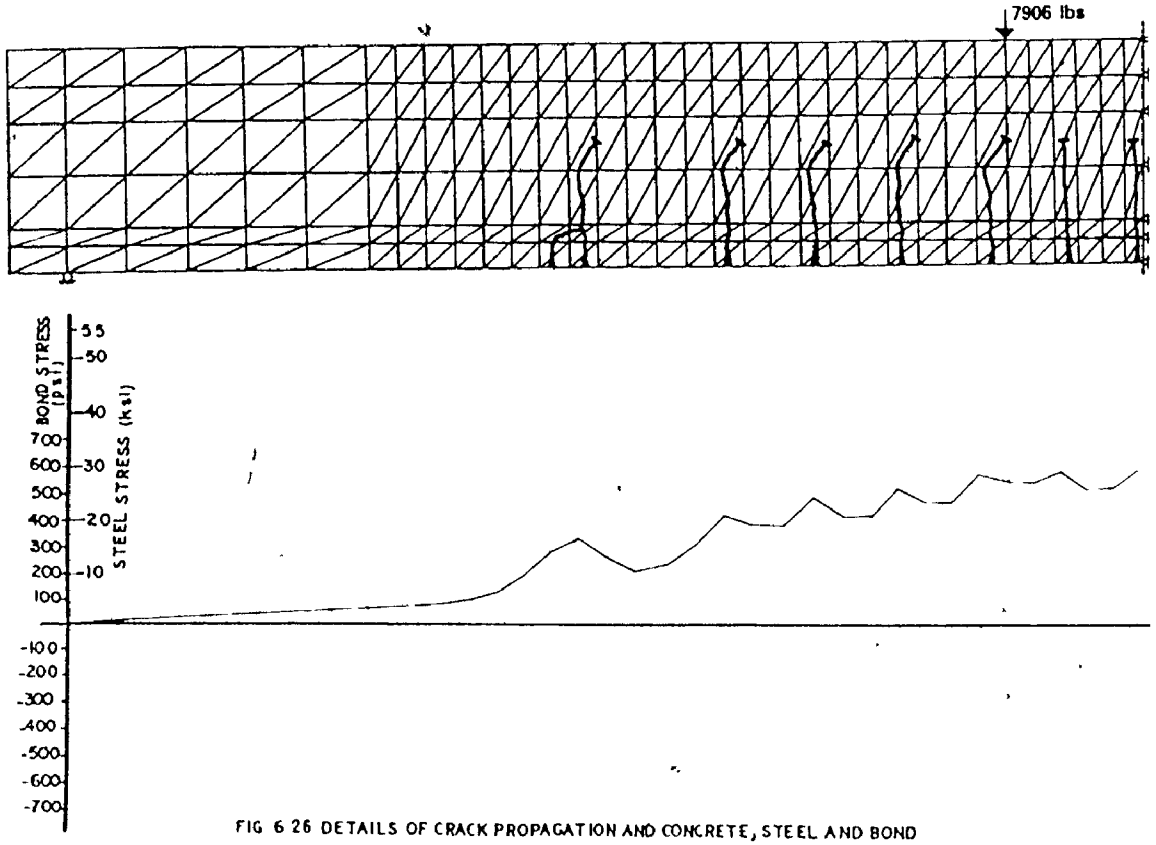


FIG 6 26 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

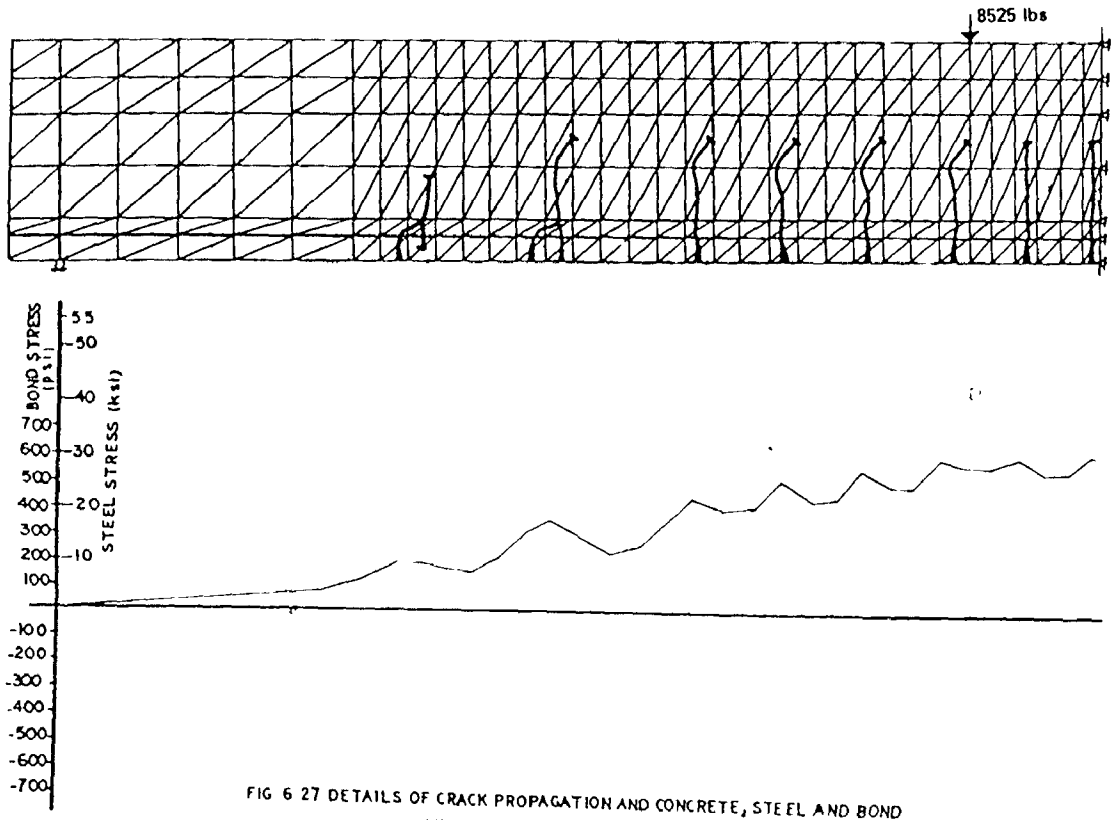


FIG 6 27 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM (7/1)

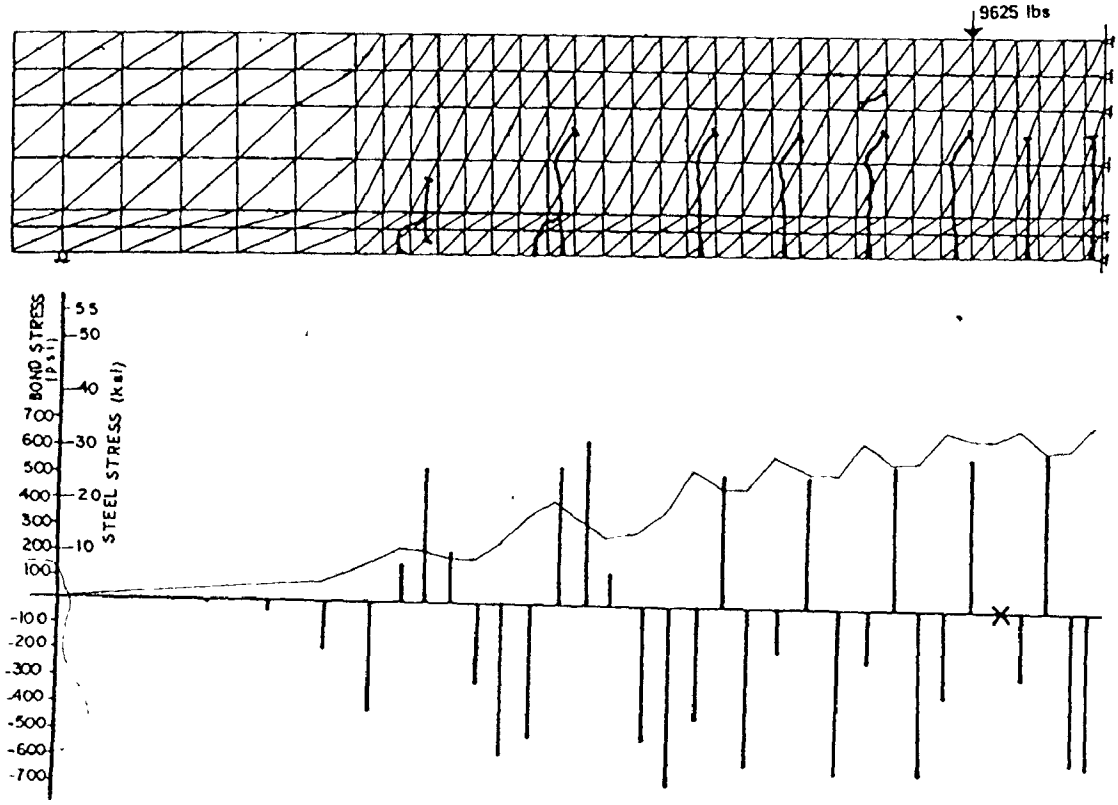


FIG 6 28 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM (7/1)

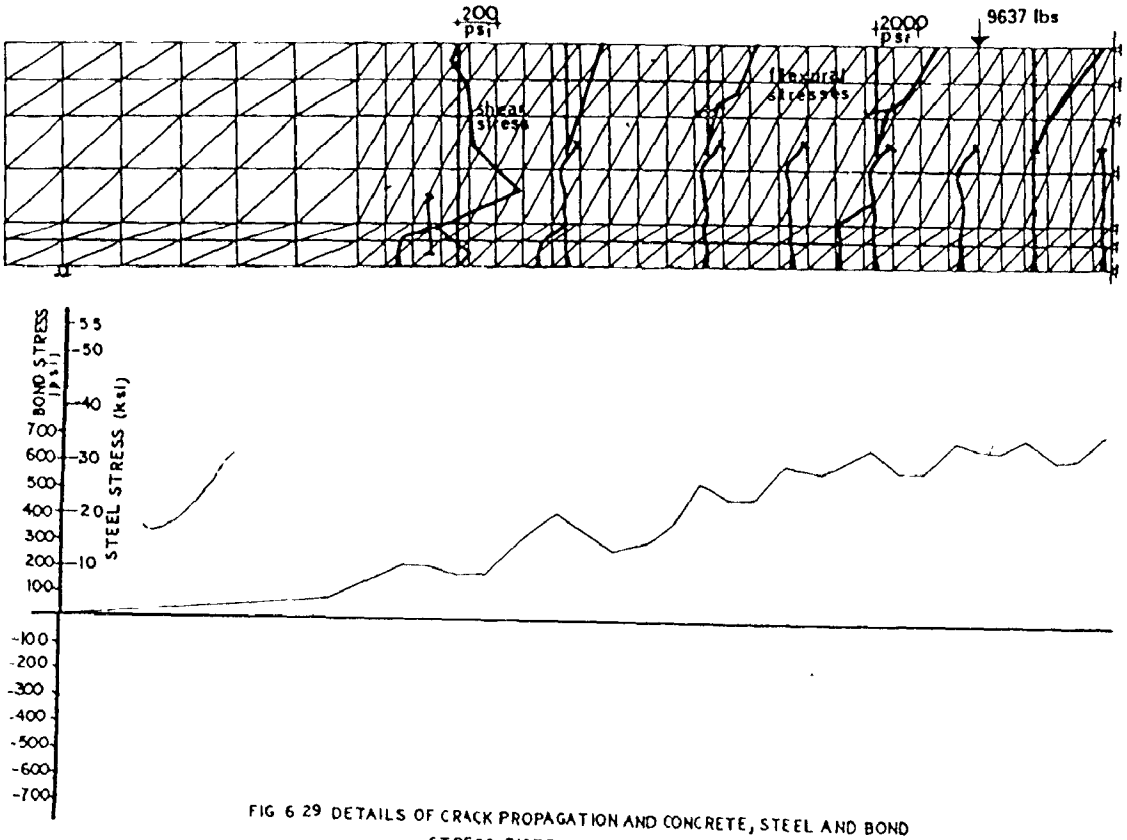


FIG 6 29 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM (7/1)

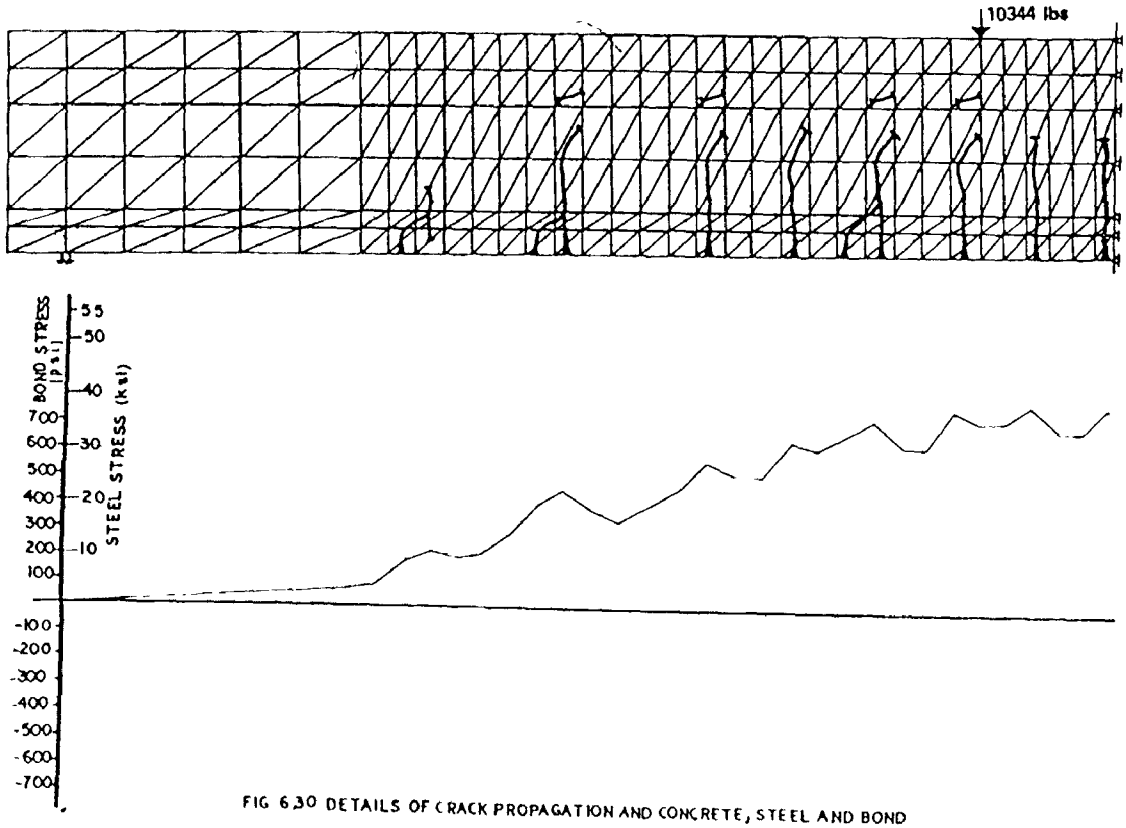


FIG 6.30 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM (7/1)

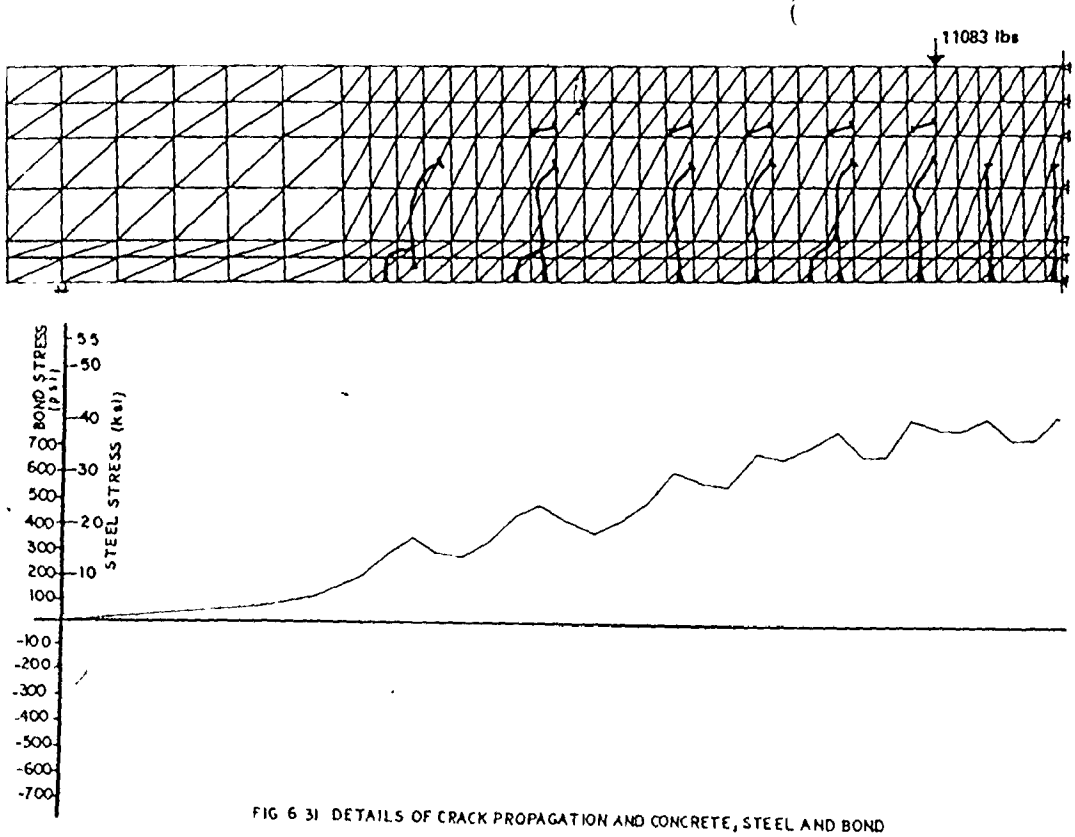


FIG 6 31 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

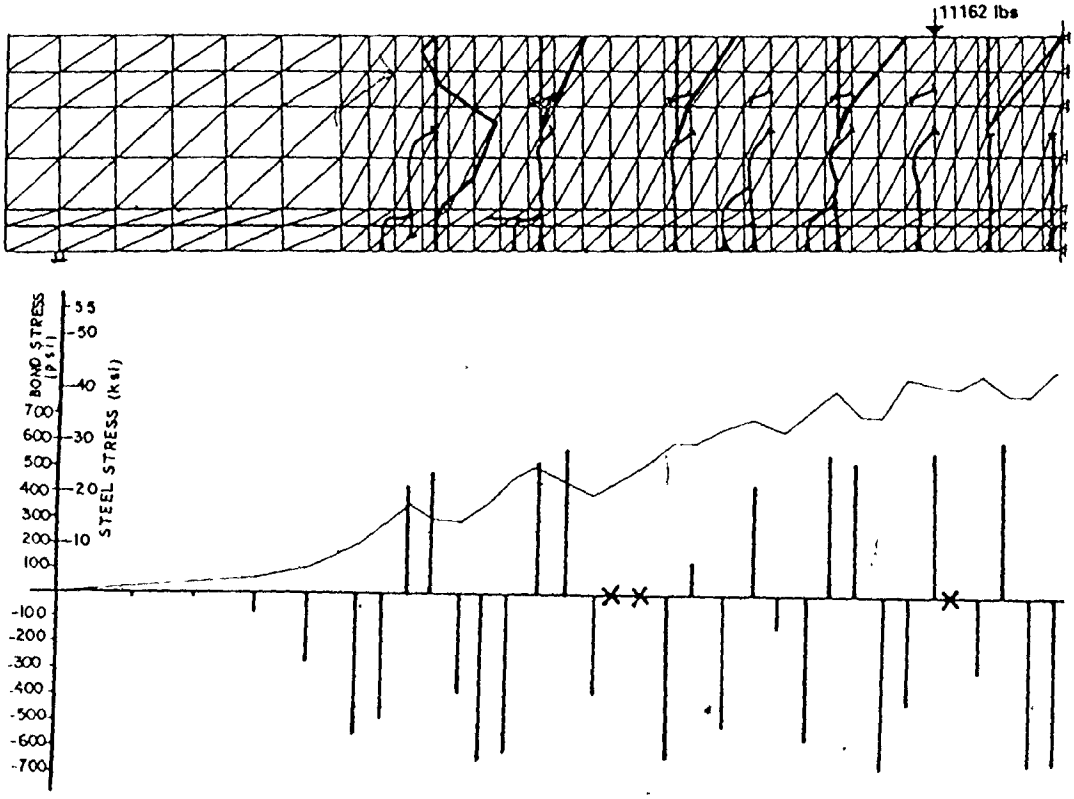


FIG 6 32 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

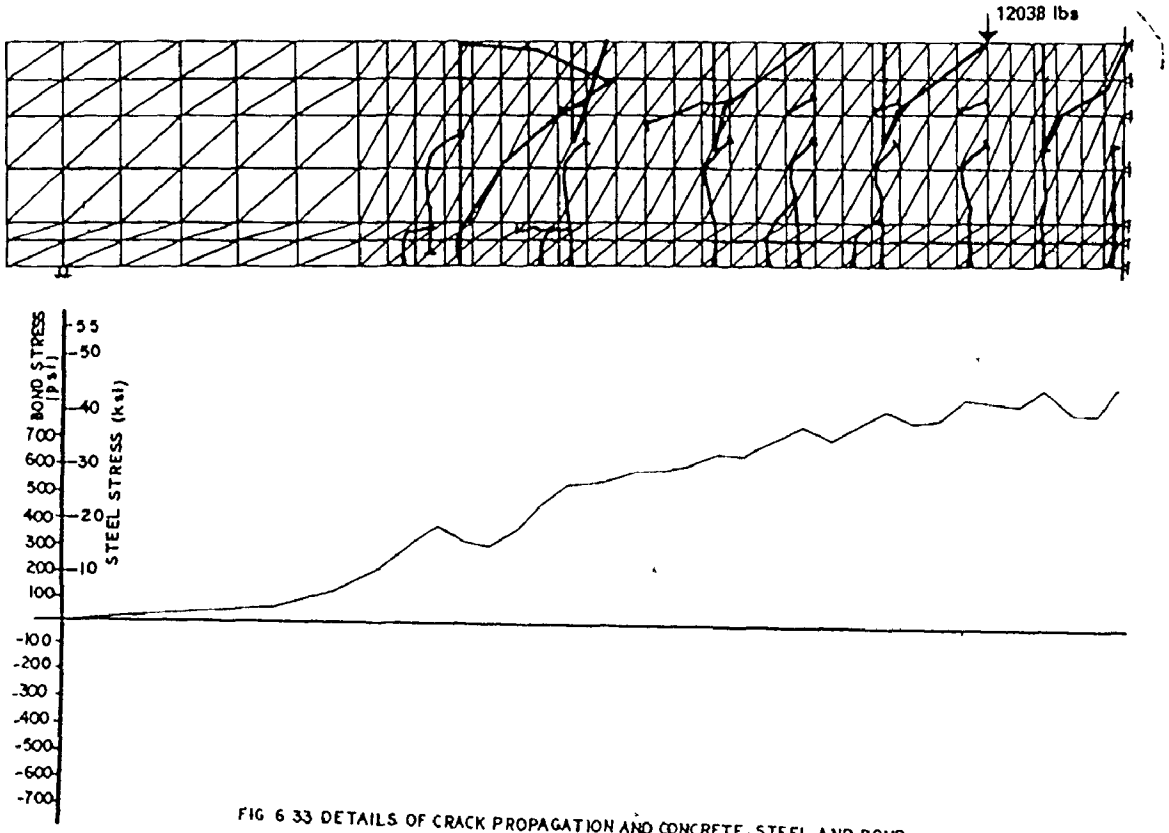


FIG 6 33 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

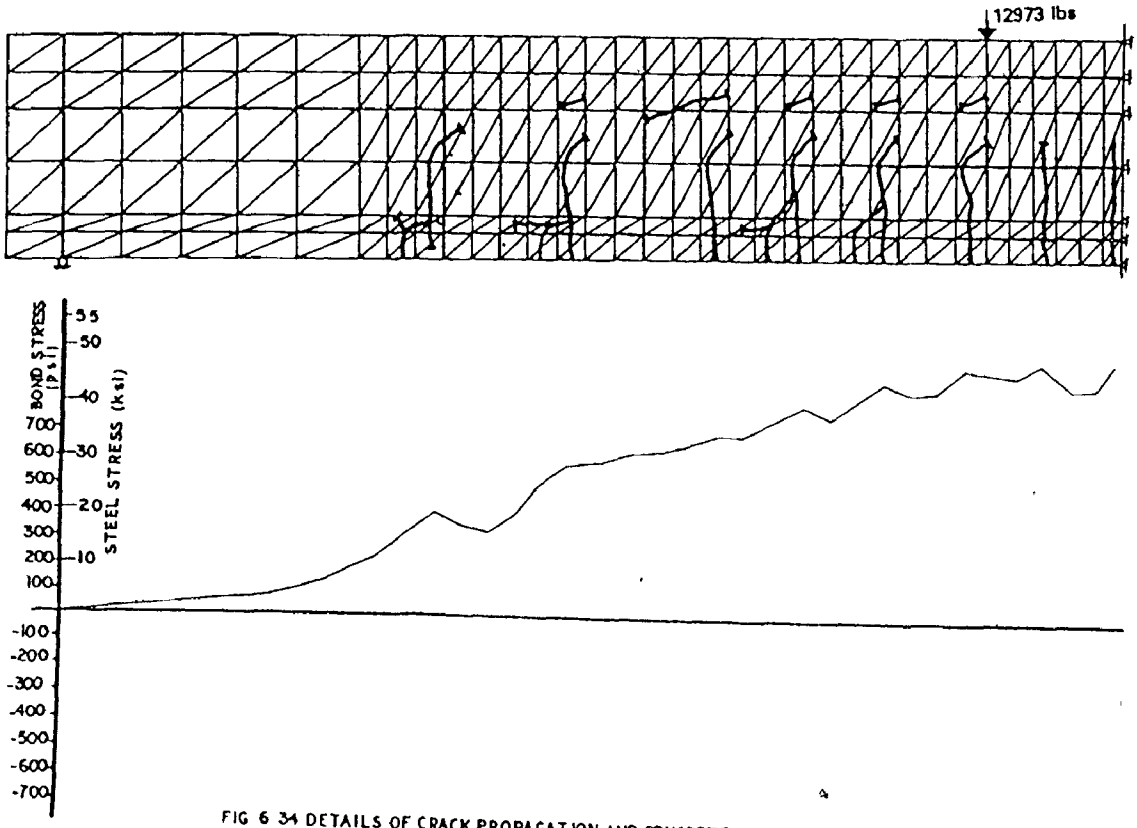


FIG 6 34 DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

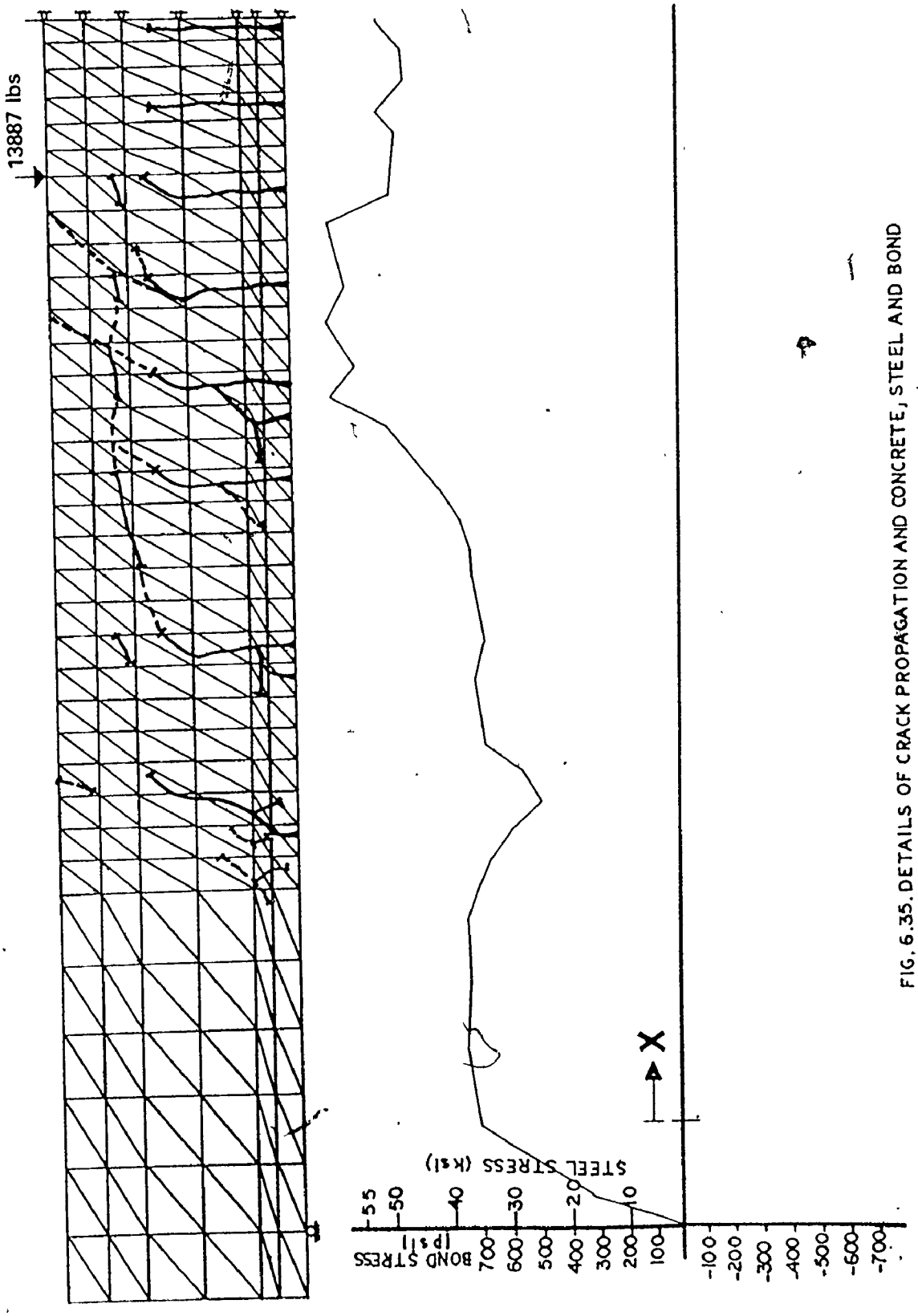


FIG. 6.35. DETAILS OF CRACK PROPAGATION AND CONCRETE, STEEL AND BOND STRESS DISTRIBUTIONS FOR BEAM(7/1)

The steel and the concrete stresses were still well below their failure limits at the iteration prior to the crack formation. Once the diagonal crack began to join with the existing flat cracks, no further stable load level was found and the crack propagated to failure in a manner similar to the sudden failure which is observed in tests. Dowel cracks formed after the diagonal crack formed, but these are not reliable because of the reported instability.

6.6 Conclusion

In this chapter five beams tested by Leonhardt and Walther (28) were described and their results were compared with the analytical results found using the proposed method of analysis. These five beams had different a/d ratios and different bond-slip characteristics.

Load-deflection results and cracking patterns were presented and discussed for all the beams. Concrete and steel stresses, and bond stresses were presented for one beam. The good agreement between the analytical and the experimental results was considered to provide evidence of the validity and the reliability of the analytical method proposed in this investigation.

The results presented and discussed in this chapter provided a rational explanation of the behaviour of some of the most interesting types of failures of simply supported reinforced concrete beams. The crack patterns reported here are believed to reproduce observed phenomena in a fashion which has not been reported previously in the literature.

CHAPTER 7

APPLICATION OF THE FINITE ELEMENT METHOD TO REINFORCED CONCRETE JOINTS

7.1 Introduction

In the previous chapter, the analysis of simply supported reinforced concrete beams were presented and discussed. In this chapter the more complex subject of reinforced concrete joints is investigated. The behaviours of joints with different steel reinforcing details are studied. The McMaster University joint test results are compared with the analytical results obtained from the computerized method of analysis.

Of the six joints tested (see Fig. 4.1), Details (4) and (5) were almost identical, and Detail (6) had the same arrangement but with a higher percentage of steel. Analyses were done for the exact arrangements used in the tests of Details (1), (2), (3), (5) and (6). Also Detail (2) was studied using two different amounts of diagonal stirrup reinforcing steel and additional analyses and discussions were done for Detail (3) using three different end anchorage conditions.

Details of the McMaster University joint testing program were discussed in Chapter 4. In this section a brief summary of the test results is given.

Details (1), (3), (4), (5) and (6) had overall capacities of nearly equal to or exceeding the theoretical section capacities of the intersecting members, whereas Detail (2) had a much lower capacity. Details (4), (5) and (6) which had essentially the same steel arrangements were judged to be superior to the others. These had higher capacities and were easier to fabricate.

Experimental load-deflection and moment-rotation results along with cracking patterns for the tested joints were shown in Figs. 4.2 to 4.11 in Chapter 4. In this chapter, analytical and experimental load-deflection results are presented only for joints (2) and (3) which illustrate representative behaviours.

The load-deformation characteristics used in the analytical method for the joint analyses were previously discussed in Chapters 3, 5 and 6. These characteristics and the equations representing them are the same as those used for the beam analyses discussed in Chapter 6 with the exception being the stress-strain relationship for the steel reinforcing bars. The idealized stress-strain curve for steel reinforcing bars in tension and compression shown in Fig. 3.3 was employed

in the analysis. This curve has a very short flat yield plateau followed by a sloping part representing the actual strain hardening for the steel.

7.2 Failure Modes for Joints

Failure modes for reinforced concrete joints are probably not as well known as they are for beams. However, most of the failure modes discussed for beams in the previous chapter are also applicable to joints.

Flexural failure with its two distinctive modes of excessive yielding of steel in tension and/or crushing of concrete in compression are possible causes of joint failure whether it occurs in the joint or in an intersecting member. Crushing of the concrete in the joint region may not necessarily be considered as flexural failure, in some instances, since this feature can be triggered by some other cause such as excessive cracking of the joint region or diagonal cracks which can reduce the compression zone in one or each of the two intersecting members. Another possible mode of failure of the joint may be diagonal tension failure which is caused by the extension of a type (c) crack as was shown in Figure 4.16. and which was discussed in Chapter 4. Bond failure and/or end anchorage breakdown are common causes of failure in joints. Similarly the yielding of diagonal bars or stirrups may cause the joint to fail immediately.

Distinct failure modes were difficult to identify from the test results. Although characteristic diagonal tension cracks were formed in some of the specimens, this did not mean that the failure mode was necessarily diagonal tension. Some failures, such as a premature yielding of the diagonal stirrups, may cause cracks quite similar to diagonal tension cracks to occur. If one is not aware that the diagonal stirrups have yielded prior to the propagation of the crack, the failure could mistakenly be classified as diagonal tension failure. However, this confusion is avoided with the use of the proposed methods of analysis.

Compressive crushing of the concrete, yielding of the main or the secondary reinforcement and diagonal tension failures are readily detected using the analysis by studying the detailed information on element behaviour at the stages of loading near failure. Bond and/or end anchorage breakdown failures are also fairly easily identified by examining the status of the joint elements which simulate bond-slip and dowel action phenomena.

Of all the possible joint failure modes, flexural failure can be considered as the only desirable type of failure. In fact the ultimate goal of a joint designer should be to ensure that the capacity is controlled by flexure rather than any other type of capacity reducing failure.

7.3 Presentation of Analytical Results and Comparison With Test Results:

7.3.1 General

In this section, the results obtained from the computerized method of analysis described in Chapter 5 are presented and compared where possible with the results of the tests described in Chapter 4 which were conducted by the author at the McMaster University research facilities.

Predicted crack patterns, load-deflection curves, moment-rotation curves and ultimate capacities of the analysed joints are compared with test results. Concrete stress distributions, bond stresses and steel stresses in some sections for a representative joint are also presented for different stages of the loading procedure. Failure modes and failure causes for each joint detail are discussed.

7.3.2 Load-Deflection Results

Load-deflection comparisons are only valid for experimental and analytical results obtained for the same conditions. Details (2) and (3) were analysed using the complete member lengths and the loading closely simulated the testing conditions. This necessitated using a very large number of elements and a relatively coarse mesh. Details (1), (5) and (6) were analysed using different conditions. Nearly one half of the length

of each of the two intersecting members were included in the analyses. This approach was adopted to permit a finer element mesh size with the same number of finite elements as used in the analyses of Details (2) and (3). The finer mesh gives more accurate results and is more reliable for understanding the detailed behaviour of the joint region. However, the use of a finer mesh was not possible for the full member lengths due to computer core storage limitations.

Equivalent bending moments and shearing forces were applied at the ends of reduced member lengths and thus were away from the critical joint region. Hence, this application of equivalent actions in place of the actual applied forces would not have any appreciable effects on the area of interest.

Due to using this analytical procedure for joints (1), (5) and (6), no predicted deflections are available for the extremities of the members. However, moment-rotation relationships which were not affected by this reduction in the intersecting members' lengths are presented for all the joints.

Figures 7.1, 7.2 and 7.3 show the finite element mesh configurations used for Detail (2), Detail (3) and Details (1, 5 and 6) respectively. These figures are presented separately without superimposing the cracking pattern results to avoid crowding at the reduced size shown in this thesis.

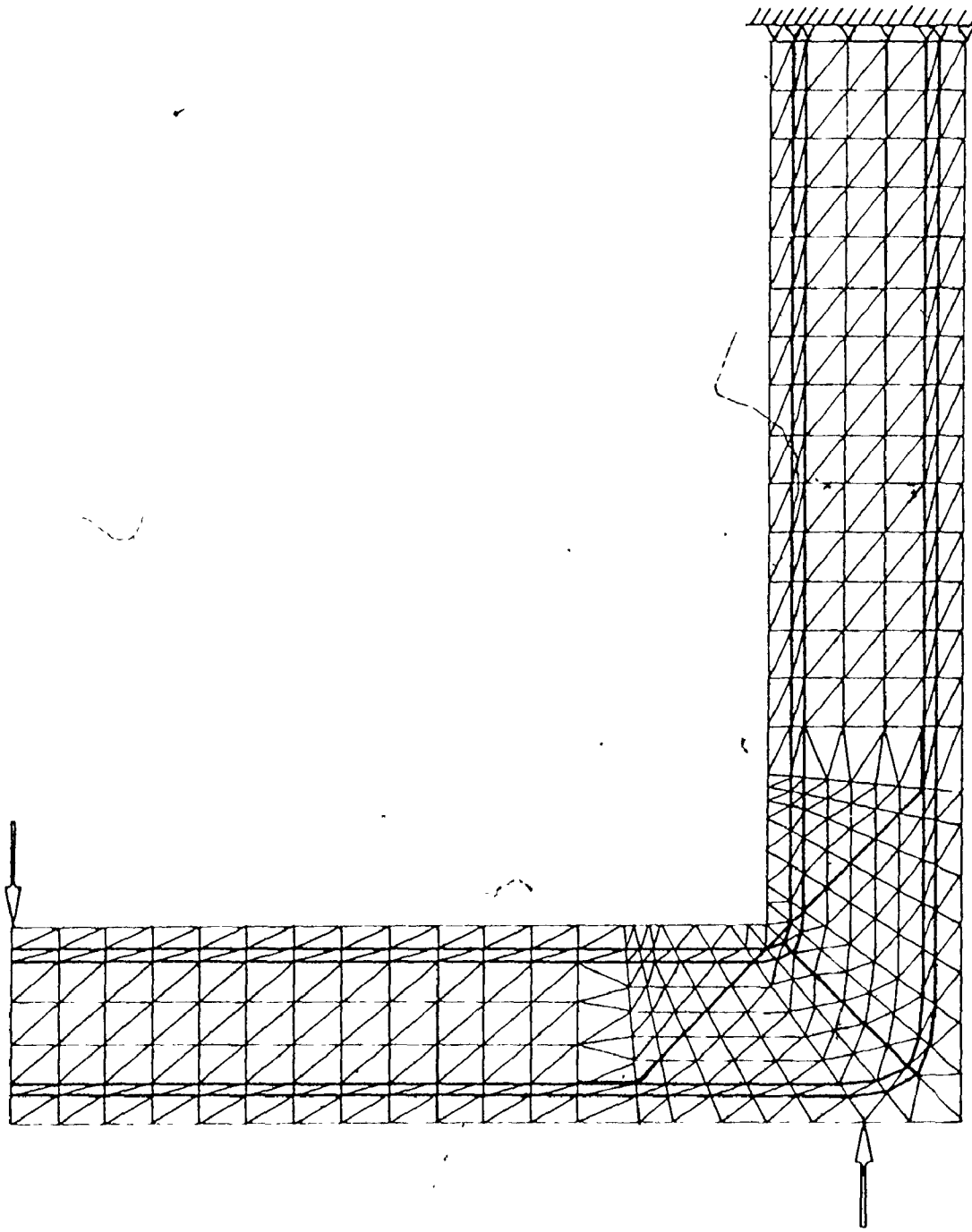


FIG. 7.1. FINITE ELEMENT MESH USED FOR ANALYSIS OF DETAIL (2)

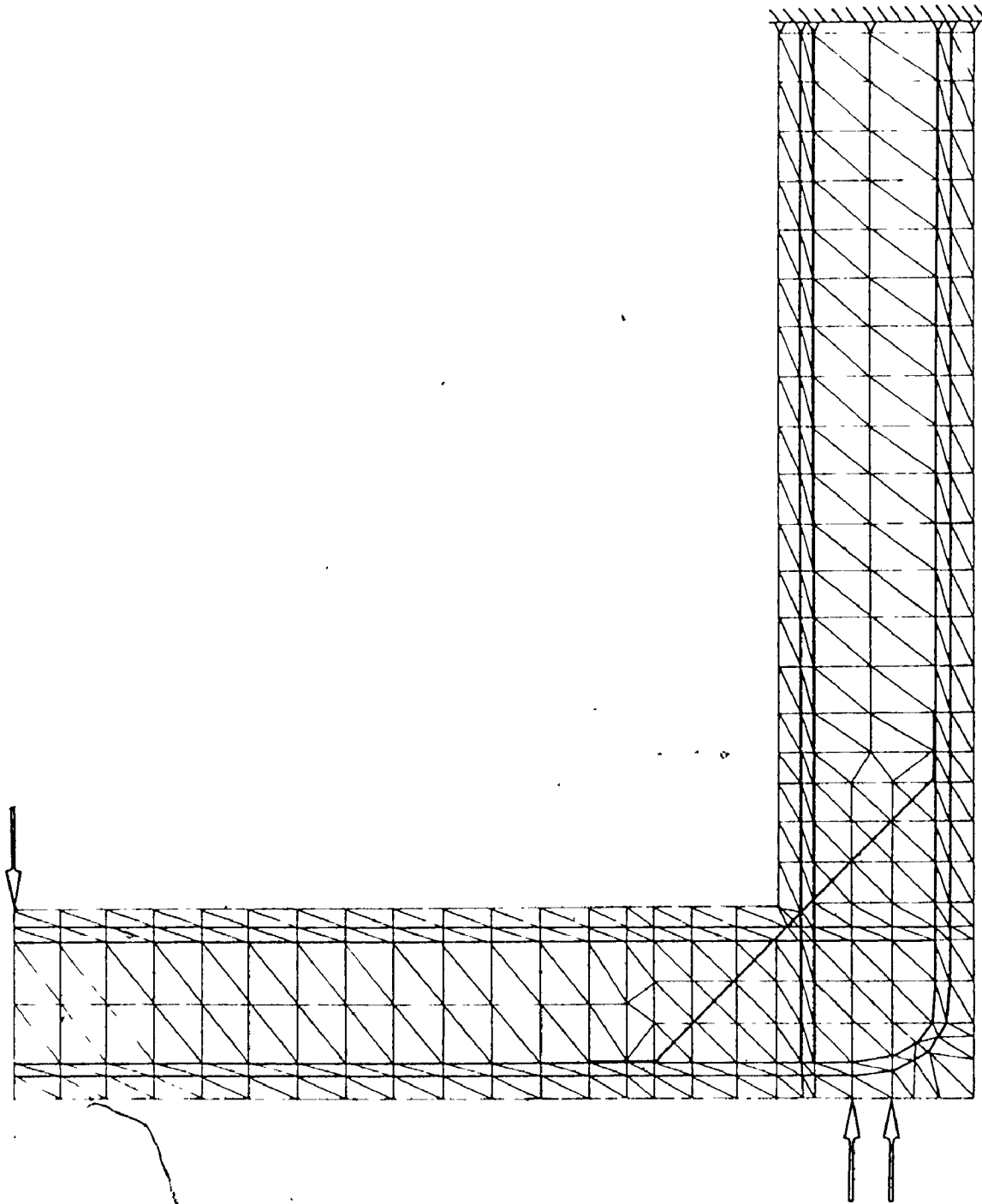


FIG. 7.2. FINITE ELEMENT MESH USED FOR ANALYSIS OF DETAIL (3)

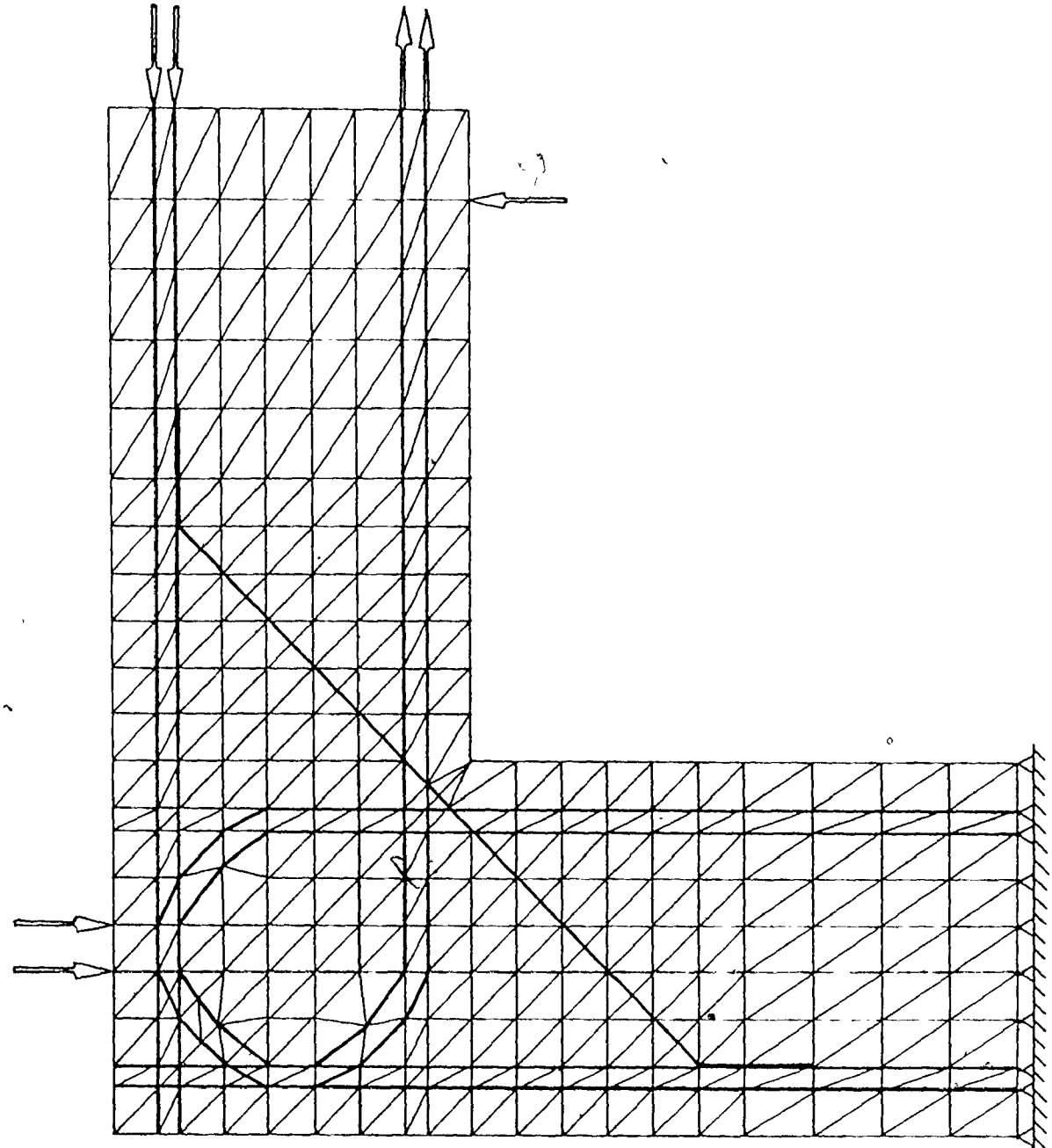


FIG. 7.3. FINITE ELEMENT MESH USED FOR ANALYSIS DETAILS,(1), (5) & (6)

Figure 7.4 shows the load-deflection results obtained experimentally and analytically for Detail (2). It can be easily seen that a very close agreement in the shape (path) of the curves was obtained. However, due to the lower bound nature of the solution and the slight delay of cracking predicted by the analysis for this Detail, the deflection values would be expected to be underestimated for the same load level. In this case computed deflection values ranged between 5% and 30% less. The same detail was analysed twice as can be seen from Fig. 7.4. The analysis of the beam with the same amount of diagonal stirrups used in the experiment showed general agreement with the results obtained experimentally. The failure of the joint was initiated by yielding of the diagonal stirrups at a load 7.5% higher than that obtained experimentally. When the amount of diagonal stirrup steel was increased from 2 No. 3 bars, 2 branch stirrups, (having a ratio of stirrup area to main steel area of $A_{st}/A_s = .5$) to 2 No. 5 bar, 2 branch stirrups (with $A_{s.t}/A_s = 1.40$), the capacity increased from 69% to 87% of the theoretical capacity of the intersecting members. The failure mode changed to a diagonal tension failure which was detected by the analysis. Further discussions concerning failure modes and cracking patterns are presented in following sections.

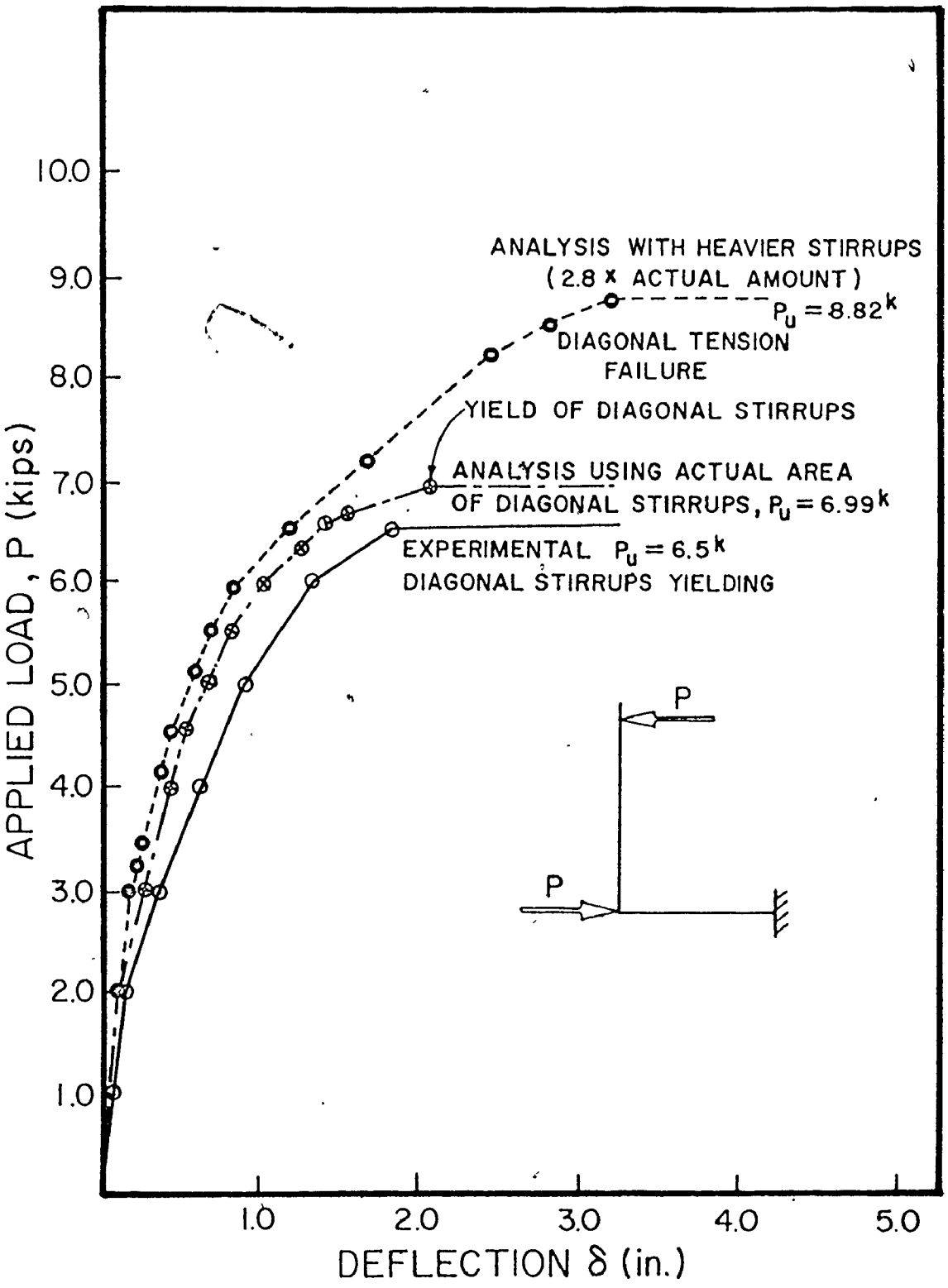


Fig. 7.4 LOAD-DEFLECTION RESULTS FOR DETAIL (2) FROM TEST AND ANALYSIS.

Figure 7.5 contains the load-deflection results for Detail (3). Four curves are shown in Fig. 7.5, where curve D represents the experimental results and A, B and C are the results of the analyses using three different end anchorage conditions for the diagonal bar. Curve A is for the analysis of Detail (3) with an imposed condition of poor end anchorage for the diagonal bar. This condition was created by not extending the diagonal bar parallel to the main reinforcement in the compression zones of each of the two intersecting members. Failure occurred in this case at 92% of the theoretical design capacity. End anchorage breakdown of the diagonal bar occurred at 79% of the theoretical capacity. Redistribution of the forces in the diagonal bar to the strongly welded main steel reinforcing arrangement enabled the joint to carry additional loads up to the failure load when diagonal cracks extended into the compression zone.

The end anchorage of the diagonal bar was then made far stronger by rigidly connecting its ends to the steel reinforcement which would be in compression. The rigid attachment resulted in overstressing the diagonal bar which was forced to carry more load than what it would have if some slip had been allowed. This resulted in yielding of the diagonal bar and consequently failure of the joint because there was no reserve capacity for the redistribution of forces. This was the only case

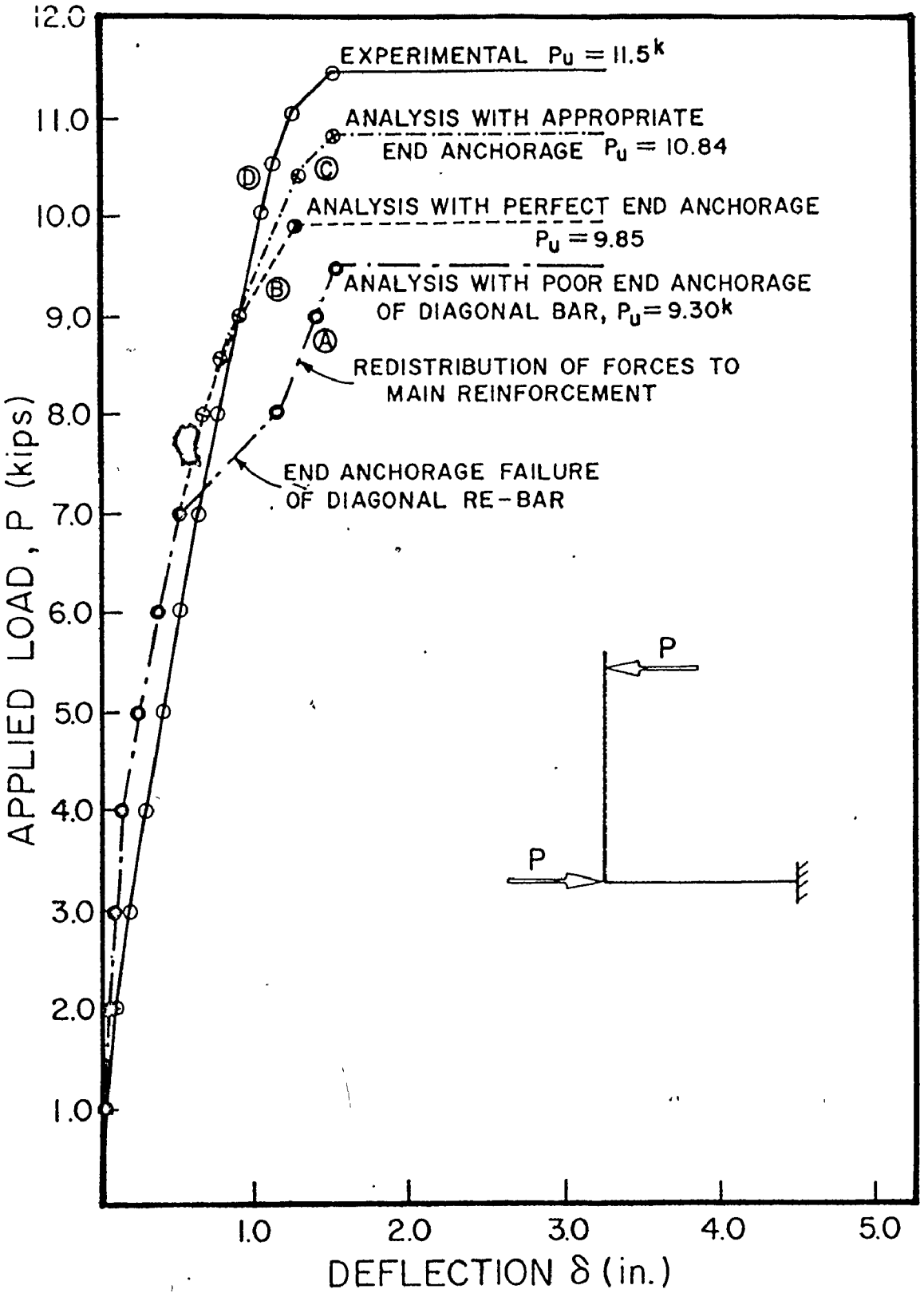


Fig. 7.5 LOAD-DEFLECTION RESULTS FOR DETAIL (3) FROM TEST AND ANALYSIS.

where overstressing of the diagonal bars was observed. In all other cases the diagonal bar stress at failure of the joint ranged from 40% to 65% of its yield strength. The analysis predicted that this arrangement would be able to withstand 97% of the theoretical ultimate capacity of the intersecting members. Curve B shows the load deflection characteristics for this case. The actual end anchorage condition of the diagonal bar was modelled in the analysis by extending the bar ends parallel to the main reinforcement in the compression zones of the intersecting members as shown in the sketch in Fig. 7.8. As illustrated in Fig. 7.5 by curve C, this resulted in an increase in the ultimate capacity. As can be seen from all the curves in Fig. 7.5 the analytical curves cross the experimental one. This may be explained by considering the original underestimation of deformations inherent in the method of analysis, and then the influence of a slight over estimation of the cracking effects which results from reducing the stiffnesses of the cracked element in the direction normal to the crack. The two inaccuracies may tend to balance at a stage of loading which depends among other things on the mesh size chosen for the analysis.

7.3.3 Moment-Rotation Results:

Moment-rotation values reported in this investigation were obtained indirectly from deflection values. The rate of change of the deflection over a short length was adopted as the criterion for the calculation of member rotations both experimentally and analytically. Although a high accuracy for rotations is not guaranteed, they are adequate to provide a reasonable and consistent basis for discussion of the general moment-rotation characteristics of the analysed joints. The results obtained from both the experiments and the analyses are based on the same lengths and deflection locations for calculation of the rotations.

Figures 7.6 to 7.10 illustrate the moment-rotation relationships obtained both experimentally and analytically for all of the analysed joints. It can be seen from these figures that the discrepancies between the rotation results vary considerably from one detail to another. Detail (1) involved the largest discrepancies, while Detail (6) had the least discrepancies. These variations reflect the complexities involved when the several factors which affect joint rotations are considered. A slight delay of a crack growth or the alteration of a crack path or location may affect the rotation values obtained at a particular

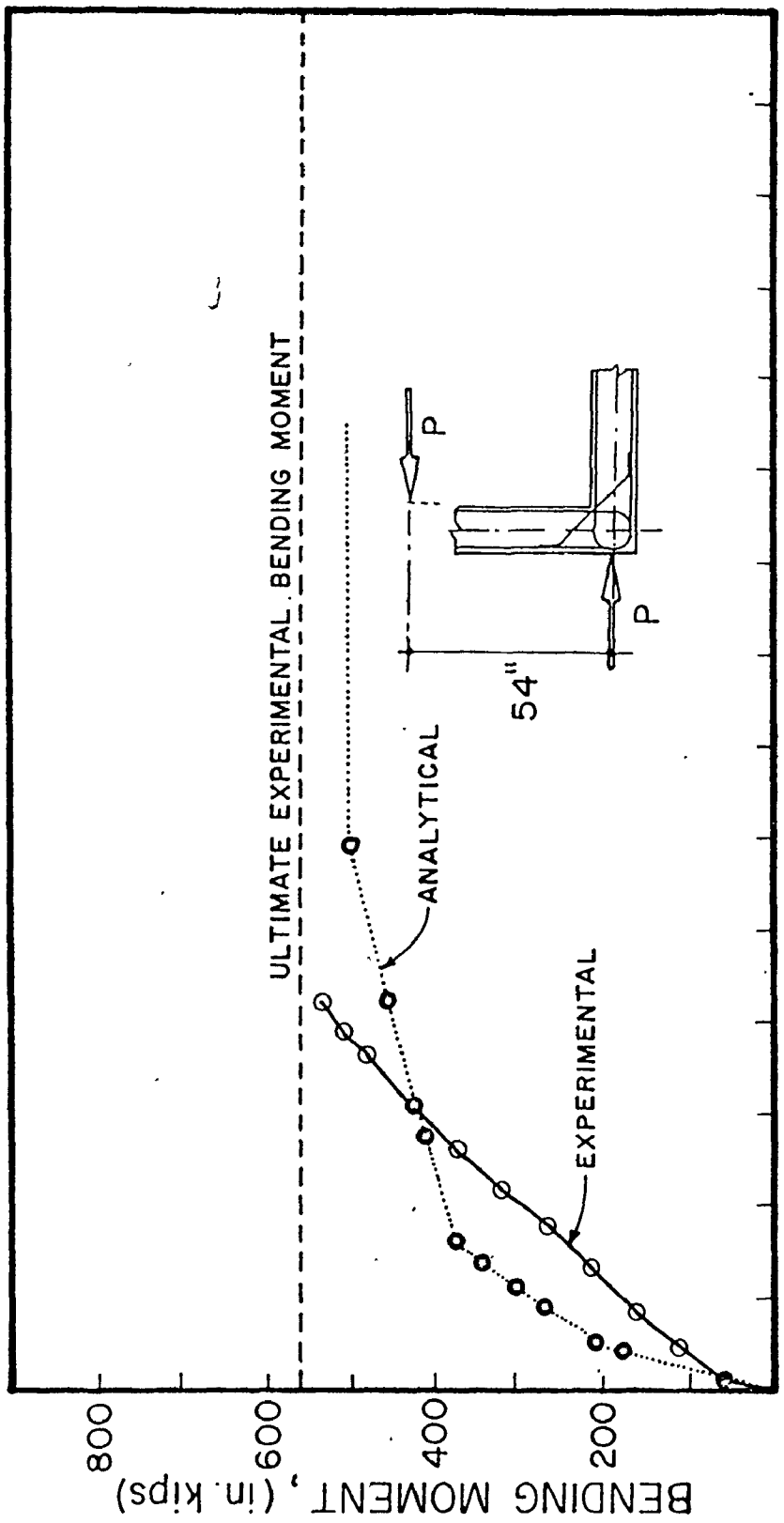


Fig. 7.6 EXPERIMENTAL AND ANALYTICAL MOMENT-ROTATION RESULTS FOR DETAIL(I)

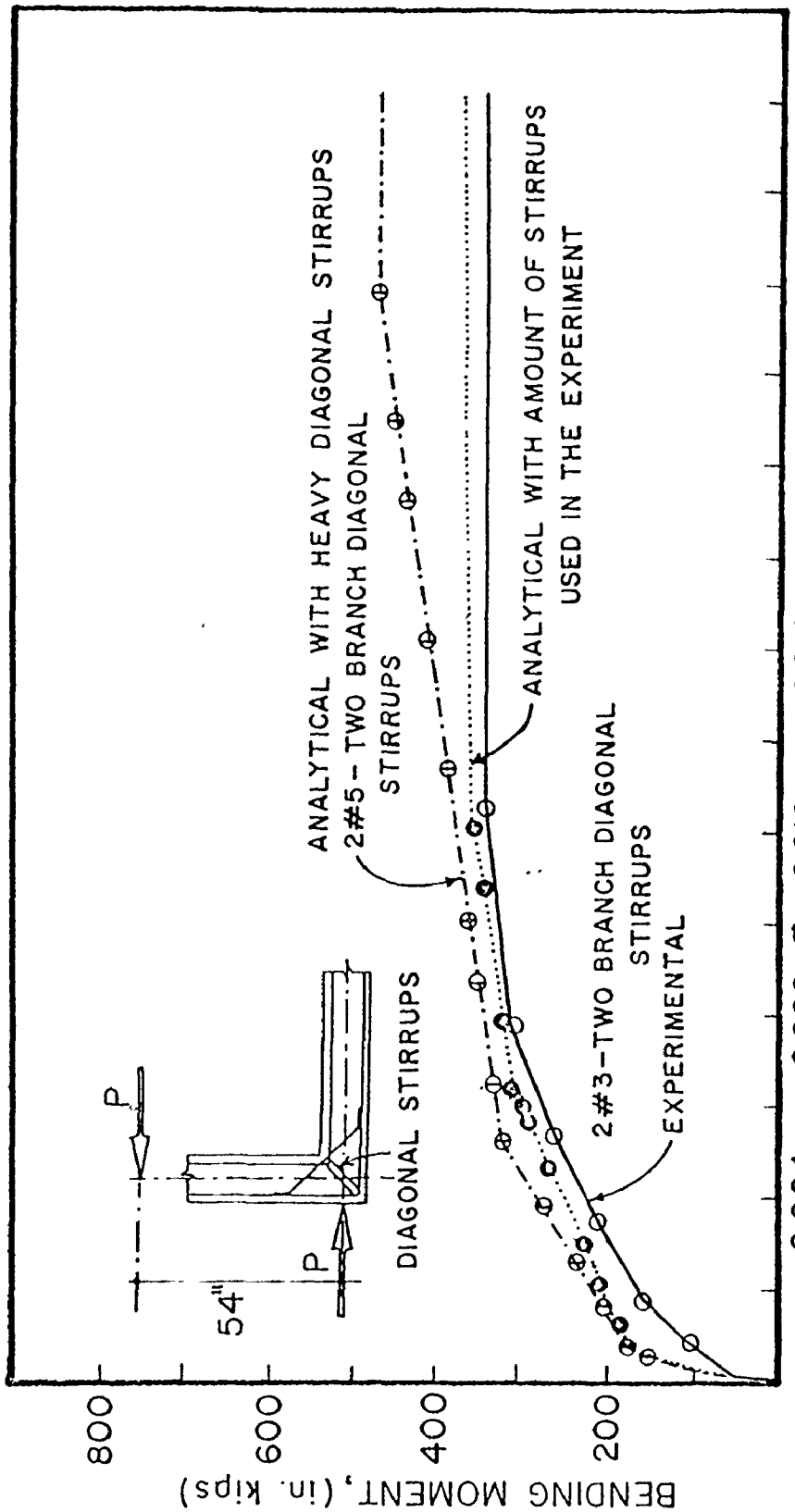


Fig. 7.7 EXPERIMENTAL AND ANALYTICAL MOMENT—ROTATION RESULTS FOR DETAIL (2).

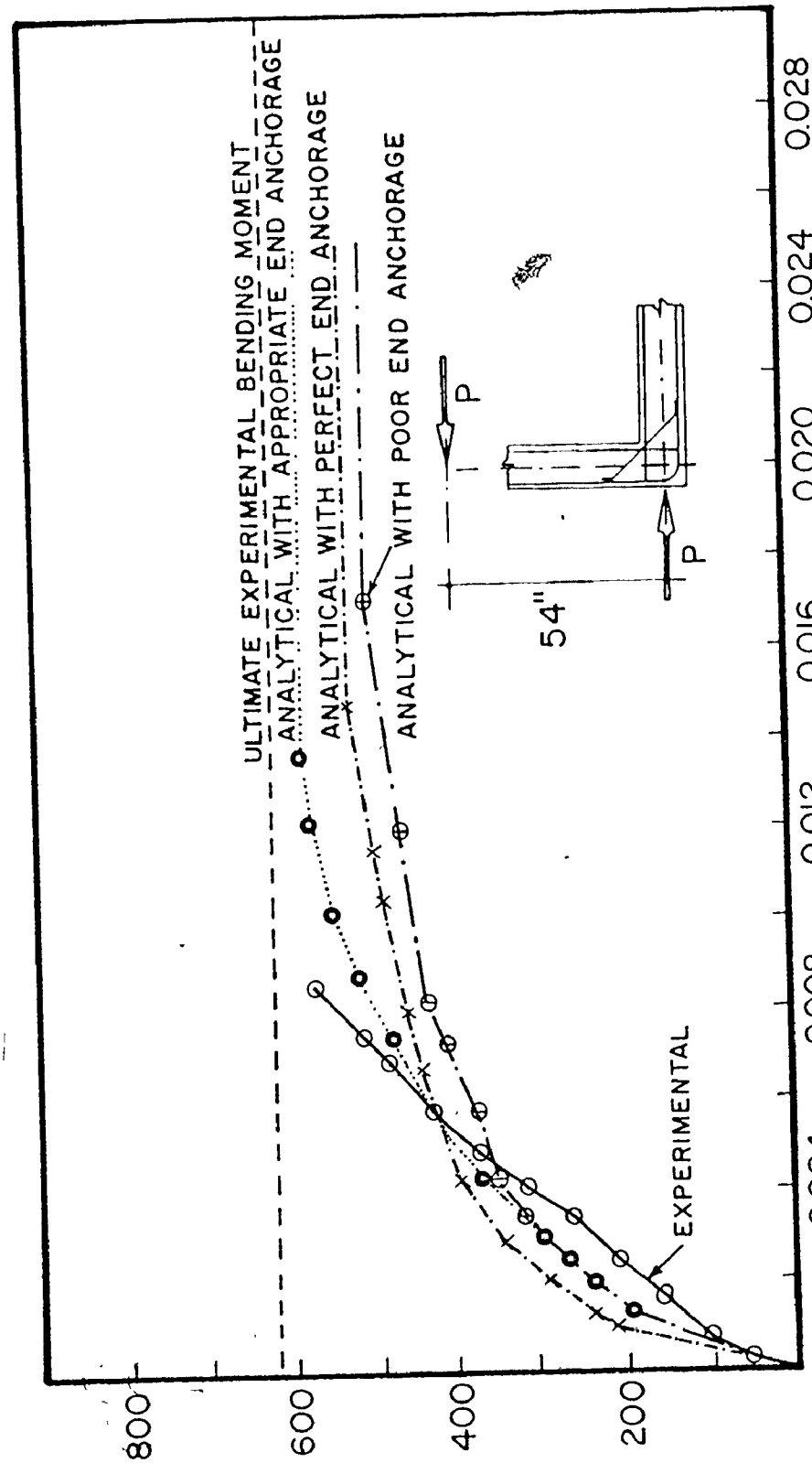


Fig. 7.8 EXPERIMENTAL AND ANALYTICAL MOMENT–ROTATION RESULTS FOR DETAIL (3)

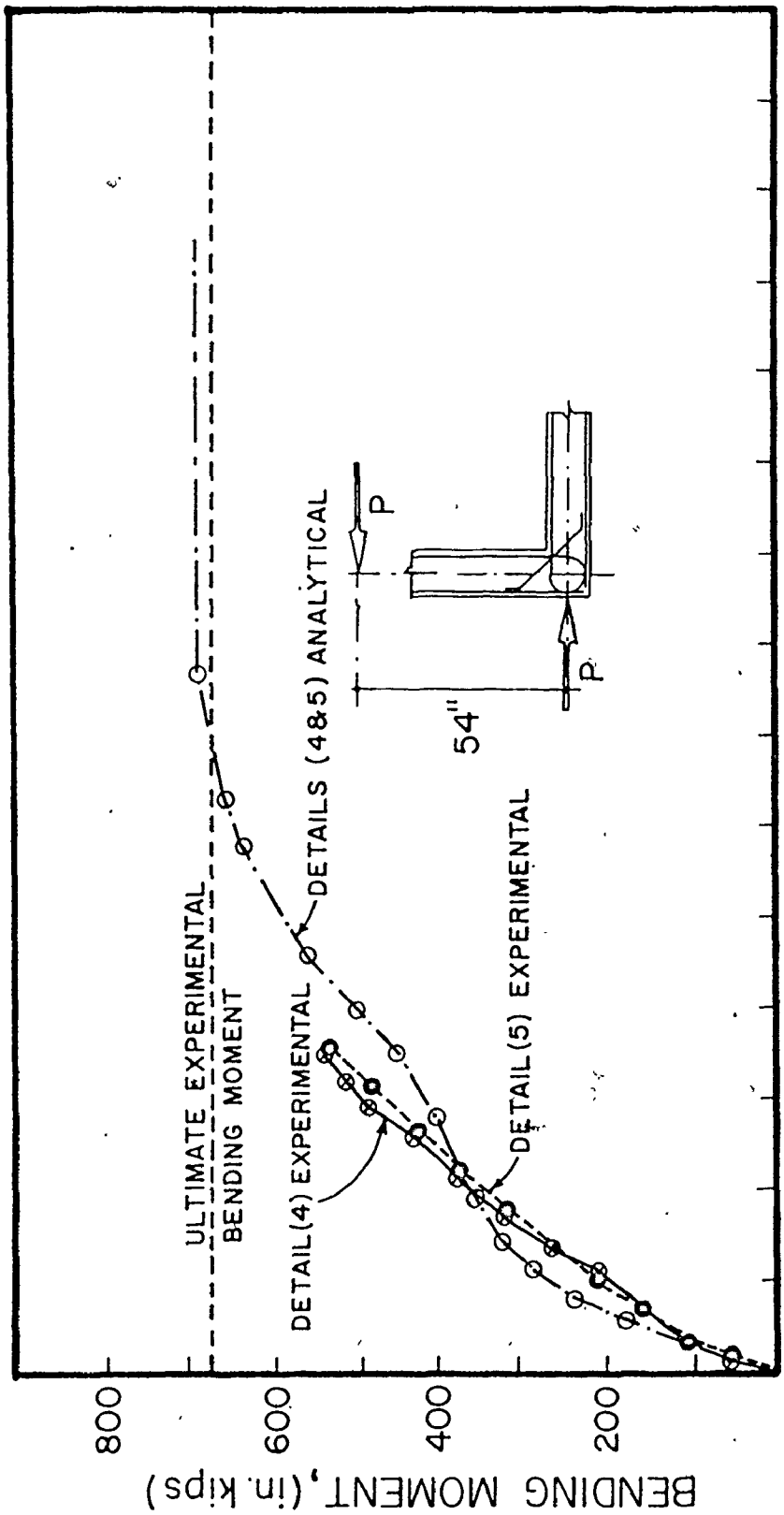


Fig. 7.9 EXPERIMENTAL AND ANALYTICAL RESULTS OF MOMENT—ROTATION FOR DETAILS (4) AND (5).

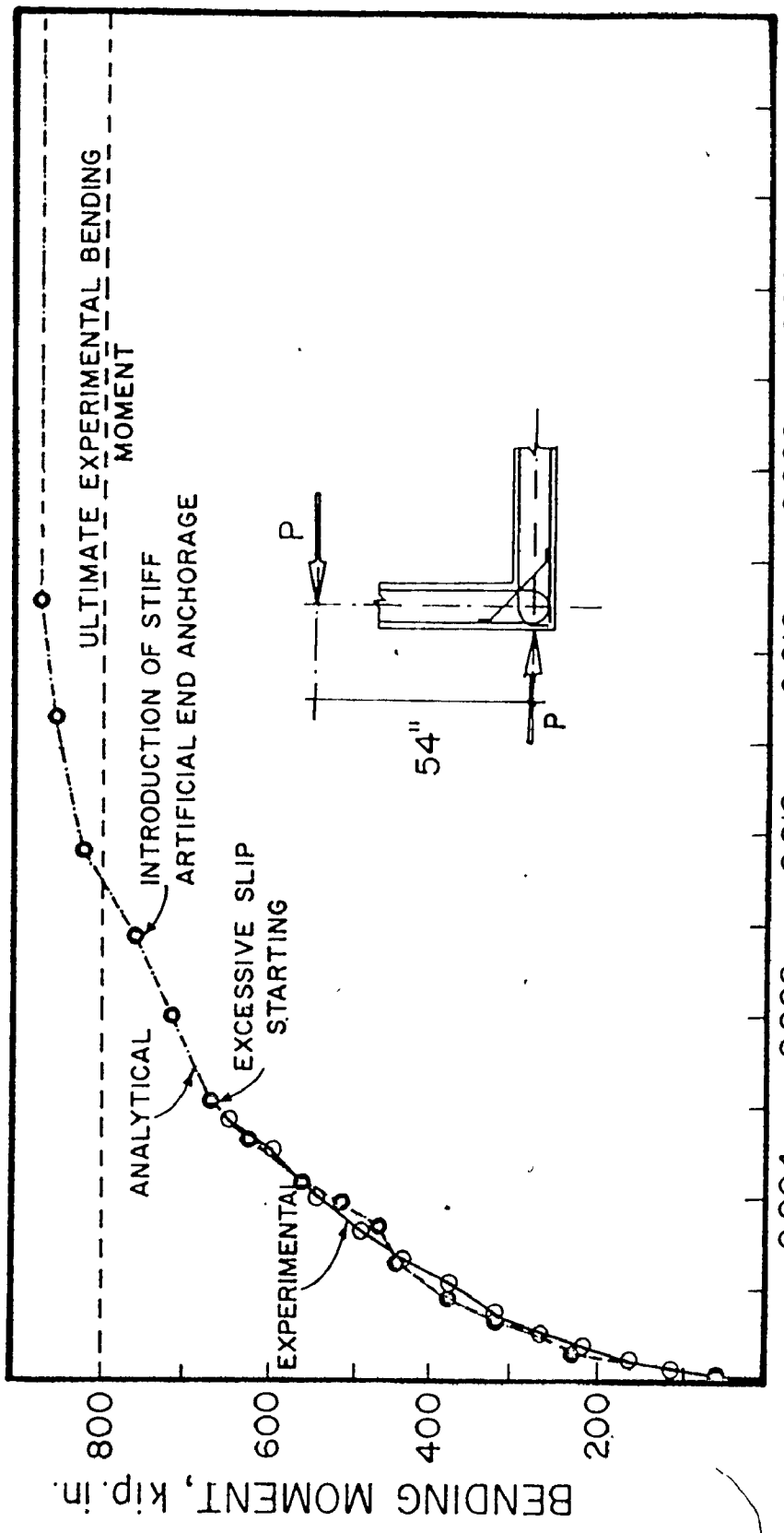


Fig. 7.10 EXPERIMENTAL AND ANALYTICAL MOMENT–ROTATION RESULTS FOR DETAIL (6).

load level. This, however, may not have any appreciable effects on the overall behaviour of the joint. Another possible reason for the observed differences in the moment-rotation curves could be due to inaccuracies involved in modelling the intricate interaction between steel and concrete at their interface. Overestimated or underestimated values of slip between steel and concrete can very easily alter the rotation values, especially in the more highly stressed joint region.

With the exception of the curves in Fig. 7.7, all the remaining analytical curves shown in Figures 7.6 to 7.10 underestimate the rotation values up to a point where the analytical curves cross the experimental ones with the result being generally overestimated deformations. It is suggested that the reasons for this behaviour are mainly the same as those discussed for the deflections. It is also suggested that this typical crossing did not occur for Detail (2), shown in Fig. 7.7 because this detail failed in a premature fashion due to yielding of the diagonal stirrups. This resulted in larger experimental joint deformations at lower loads than were observed for any other detail. The yielding of the diagonal stirrups was predicted by the analysis to start at a load 7.5% higher than that obtained experimentally. This overestimation in the capacity directly resulted in keeping the analytical curve above the experimental one.

7.3.4 Discussion of Behaviour of Each Detail

In this section cracking patterns predicted from the analyses are compared with crack patterns traced from the test specimens. The modes of failure for each analysed joint are also discussed.

Table 7.1 is presented to facilitate an overall comparison between the experimental and analytical results for all the joint details. The predicted first cracking and the failure loads are listed along with the corresponding experimental values. The ratio of the analytical to experimental first cracking load varied from 0.84 to 1.22 for Details (1) to (5) and it was 1.82 for Detail (6). However, the early formation of the corner crack at a load of 2.0 kips, as shown in Fig. 4.11, is likely due to local material weakness in the tested joint. The loads at which subsequent cracks formed were much closer to the predicted values. The ratio between the analytical and experimental ultimate load capacities showed better agreement than that obtained for initial crack formation. The ratios of predicted to experimental ultimate load varied from 0.88 to 1.08.

The scatter is considered to be acceptable and demonstrates the validity of the analysis. This acceptance is based primarily upon the consideration of two aspects. The first being that the variation between test results for similar joints has about the same range of scatter (97).

The second aspect is that the idealization of reinforced concrete as a homogeneous material can by itself account for a large portion of this scatter.

Table (7.1) Beam Cracking and Failure Loads

Joint Detail	First Cracking Load (kips)			Ultimate Failure Load (kips)			
	Exp.	Anal.	av. $\frac{\text{Anal.}}{\text{Exp.}}$	Exp.	Anal.	av. $\frac{\text{Anal.}}{\text{Exp.}}$	$\frac{\text{Anal.}}{\text{Theory}}$
1	4.0	3.35	.84	10.5	9.22	.88	.91
2	3.0	2.84	.95	6.5	6.99	1.08	.69
3	3.0	3.65	1.22	11.5	10.84	.94	1.07
4	3.0	3.34	.95	12.5	11.86	.98	1.17
5	4.0			11.6	12.89*	1.07*	
6	2.0	3.63	1.82	14.75	14.02	.95	.97
					16.22*	1.10*	

* Values are for analyses after main steel anchorage break down and the introduction of strong artificial anchorage.

Table 7.2 shows the analytical crack spacing results in the two intersecting members outside the joint region for all the analysed joints. Average, minimum and maximum crack spacings are given for each joint detail. The mean and standard deviation for the crack spacings are only for the joints indicated which had the same amounts of steel reinforcement.

Table (7.2)

Crack Spacings for Analysed Joint Details

Crack Spacing (inches)			
Joint Detail	Average	Maximum	Minimum
(1)	6.8	8.7	4.4
(2)	6.71	9.6	5.0
(2')	6.92	9.6	5.0
(3)	6.87	9.6	1.8
(3')	9.22	12.4	5.4
(4,5)	7.94	8.8	7.6
(6)	8.04	9.1	7.6
Mean* = 7.08		Standard deviation* = .58	

*For (1), (2), (3), (4&5) only

It can be seen from comparing Tables 4.1 and 7.2, representing the experimentally measured and the analytically predicted crack spacings respectively, that a reasonable agreement is achieved. The analysis tends to overestimate the spacing. However, the experimental and analytical means are close with the analytical being 13% higher than the experimental, which is acceptable, considering the normal range of variation in experimental work. The analytical standard deviation is lower than that obtained from the experimental results which as expected indicates

that the analytical results were more consistent than the experimental data.

Although Detail (1) had the greatest difference between the experimental and analytical capacities, the general trend of the results obtained from the analyses of the different joints remained unchanged from the experimental data. The detail which had the least capacity by testing remained the least when analysed, and the best performing detail also maintained its rank using the analysis. This trend can easily be seen from Table 7.1, in fact this agreement in the general trend along with the tendency of the analysis to generally underestimate the ultimate capacities of the joints, gives confidence and sound basis for comparison between different details.

In the next subsections the behaviour of each joint detail is discussed in more detail.

Table 7.3 lists the loads corresponding to the analytical loading step numbers for the analyses. These loading step numbers are shown on Figures 7.11 to 7.17 and indicate the points to which cracks had propagated at those particular loading steps. The joint detail numbers designated by a prime in Table 7.3 are for special joint conditions having the same general configurations as those for corresponding numbers without

Table (7.3) Loads Corresponding to Load Step Numbers (lbs.)

Load Step No.	Detail (1)	Detail (2)	Detail (2')	Detail (3)	Detail (3')	Details (4,5)	Detail (6)
1	3351	2843	2973	3654	3655	3339	3630
2	3890	2960	3170	4265	4026	3888	3698
3	4006	3034	3185	4315	4269	3914	3912
4	4263	3088	3221	4329	4332	4163	4309
5	4331	3091	3221	4504	4335	4191	4253
6	4562	3157	3224	4516	4495	4610	4628
7	4818	3190	3481	4597	4518	4711	4633
8	4919	3191	3481	4630	4600	4887	4647
9	4960	3192	3483	4707	4635	4907	4929
10	5233	3378	3533	4712	4709	5006	4980
11	5712	3405	3533	4810	4811	5008	4990
12	5761	3541	3765	4890	4890	5569	5120
13	5799	3612	3765	5065	4911	5674	5715
14	6101	3768	3765	5147	5068	5815	6068
15	6362	3921	3765	5343	5145	6023	6232
16	6498	4326	3765	5390	5344	6055	6246
17	6513	4690	3869	5511	5390	6149	6282
18	6829	4864	4174	5555	5408	6198	6302
19	6961	4988	4174	5568	5510	6250	6358
20	7590	5019	4344	5723	5554	6299	7070
21	7873	5520	4344	5830	5660	6719	7224
22	8577	5640	4344	5887	5822	7383	8000
23	9224	5693	4770	5920	5885	7548	8081
24		5932	5138	6010	5918	7637	8232
25		6060	5138	6038	6008	7793	8310
26		6075	5138	6141	6034	8471	8573
27		6310	5138	6234	6196	9293	8718
28		6370	5138	6283	6213	9432	9516
29		6631	5138	6283	6287	9817	9516
30		6990	5283	6285	6406	10533	10282
31			5283	6461	6419	11682	10282

Load Step No.	Detail (2')	Detail (3)	Detail (3')	Details (4,5)	Detail (6)
32	5283	6727	6491	11856	10361
33	5403	6827	6540	12014	11473
34	5403	6958	6627	12363	12416
35	5403	7035	6945	12890	12478
36	5516	7472	7030		13223
37	5516	7582	7377		14018
38	5706	7729	7436		14169
39	5771	8099	7754		14926
40	5796	8213	7978		15359
41	5796	8514	8005		16220
42	5925	8669	8265		
43	5925	8906	8668		
44	6429	9235	8913		
45	6429	9383	9159		
46	6429	9498	9303		
47	6536	9846			
48	6536	10271			
49	6536	10842			
50	6536				
51	6536				
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60	6536				
61	6606				
62	6606				

Load Step No.	Detail (2)	Detail (3)	Detail (3')
63	6608		
64	6608		
65	6755		
66	6755		
67	6755		
68	6755		
69	6755		
70	6755		
71	6755		
72	6755	10842	
73	6755		
74	6755		
75	7075		
76	7075		
77	7075		
78	7075		
79	7075		
80	7075		
81	7075		
82	7075		
83	7075		
84	7144		
85	7144		
86	7144		
87	7144		
88	8217		
89	8217		
90	8443		
91	8443		
92	8443		
93	8443		
94	8819		9303

the primes. These special conditions are discussed in the appropriate subsections dealing with these particular details.

7.3.4.1 Detail (1)

Detail (1) was developed by Nilsson (97) and was tested by the author in the experimental part of this investigation. The experimental results were reported in Chapter 4. This detail, when tested, had an overall satisfactory performance and an ultimate capacity in excess of that required by design of the two intersecting members.

The predicted and experimental cracking patterns at failure for Detail (1) are shown in Fig. 7.11. The numbers shown beside the cracks are those corresponding to the load step numbers listed in Table 7.3.

A comparison between the flexural cracks outside the joint region obtained from testing and the analysis shows very good agreement with respect to the crack lengths and the load levels at the occurrence of such cracks. Crack spacings, however, were overestimated by the analysis as was discussed previously. Crack types (a), (b), (c) and (d) as defined in Fig. 4.16 were observed in both the test and the analysis. Crack type (b) was the first crack to occur in both cases. Cracks of type (c) were

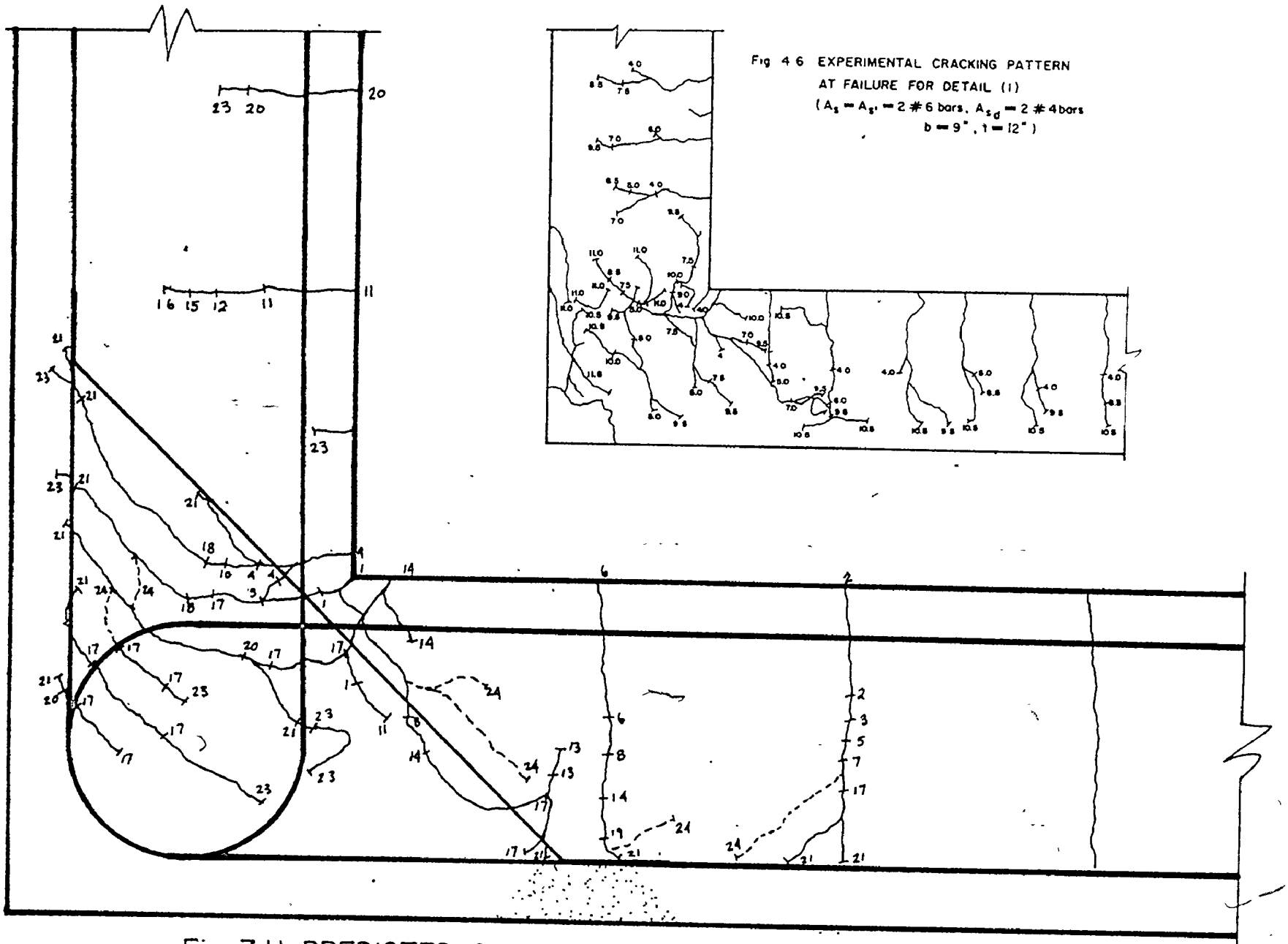

Fig 4 6 EXPERIMENTAL CRACKING PATTERN AT FAILURE FOR DETAIL (1) ($A_s = A_{s'} = 2\#6$ bars, $A_{sd} = 2\#4$ bars, $b = 9"$, $t = 12"$)

Fig. 7.11 PREDICTED CRACKING PATTERN FOR DETAIL (1) DEVELOPED BY NILSSON(97)

observed experimentally to start at a load level of 5.0 kips while this load was predicted to be 6.5 kips. The propagation of types (c) and (d) cracks was the main cause of failure in the analytical model and the test specimen where failure occurred by reduction of the compressive zone of the horizontal member at a distance approximately equal to the depth of the member from the inner face of the joint's vertical member. Several additional cracks of type (c) predicted by the analysis were not observed experimentally. These analytically obtained cracks occurred because of a successive failure of some of the bond elements along the bend of the main reinforcement. These failures may not have occurred in the actual specimen. The bond-slip relationship could not account for the effect of the compression confinement on the inside of the bend. This aspect along with other observations discussed in the next sections leads the author to believe that the failure criterion chosen for bond linkages in this investigation was conservative. This aspect when added to the effect of neglecting the aggregate interlock phenomena may very well be the cause of the general underestimation of ultimate capacities of this detail and the details discussed in the following subsections.

The failure mode of this detail may be classified as shear-compression failure caused by the propagation of

crack types (c) and (d) into the compression zones of the intersecting members. The ultimate capacity of this detail can be considered satisfactory regardless of the fact that the analytical capacity was only 91% of the theoretical ultimate capacity, since the analysis generally underestimates these capacities and since the experimental result was higher than the theoretical requirement.

7.3.4.2 Detail (2)

Detail (2) was developed by Mayfield et.al.(91) and was tested by the author to confirm the results obtained. The experimental results for this detail which were discussed in Chapter 4, indicated very poor overall performance and unacceptable capacity (64% efficiency). In Chapter 4, the great loss in capacity was attributed to the possibility of yielding of the diagonal stirrups. The same cause of failure was obtained analytically. Crack spacings and load levels at crack propagation points obtained from the computer analysis were again in general agreement with the test results. As can be visualized from observation of Figure 7.12, the stirrups did not provide effective confinement for the joint core. This resulted in the early appearance of type (c) cracks which were observed to occur at a later loading stage in all of the other specimens tested.

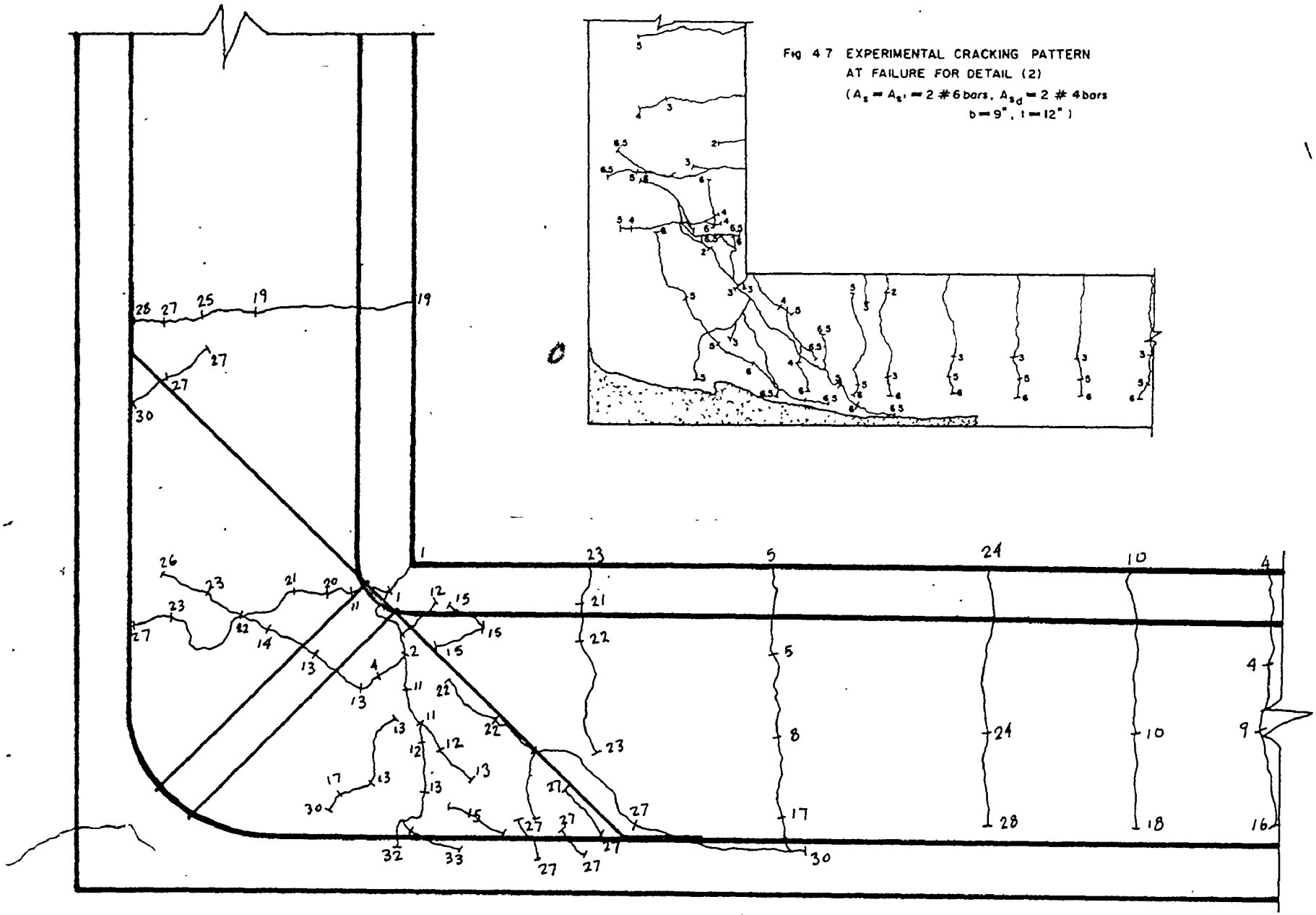

Fig. 7.12 PREDICTED CRACKING PATTERN FOR DETAIL (2), USING THE SAME AMOUNT OF DIAGONAL STIRRUPS AS THAT FOR THE TEST.

To investigate the effect of the amount of diagonal stirrup steel on the overall performance of the joint, another computer analysis was made. The area of diagonal stirrups was increased from 0.44 in^2 to 1.23 in^2 with ratios A_{st}/A_s of 0.5 and 1.4, respectively. As was discussed in Section 7.3.2, the analytical ultimate capacity of the joint increased from 69% to 87% of ultimate theoretical capacity. Fig. 7.13 shows the predicted cracking pattern at failure of the joint specimen. It can be seen that all (a), (b), (c) and (d) types of cracks appeared in this case. Although the diagonal stirrups used in this analysis did not yield at the ultimate capacity of the joint, the joint failed to reach the desired 100% efficiency. This failure was caused by the continuous propagation of crack types (c) and (d) into the compression zones of both intersecting members. The failure modes can be classified as either diagonal tension or shear compression.

The final evidence of failure predicted by the analysis was compressive failure of the concrete elements in the compression zone of the horizontal member. However, this failure occurred only because of the propagation of the diagonal cracks into the compression zone. Diagonal tension failure, with its characteristic sudden extension of the diagonal crakcs to the outermost

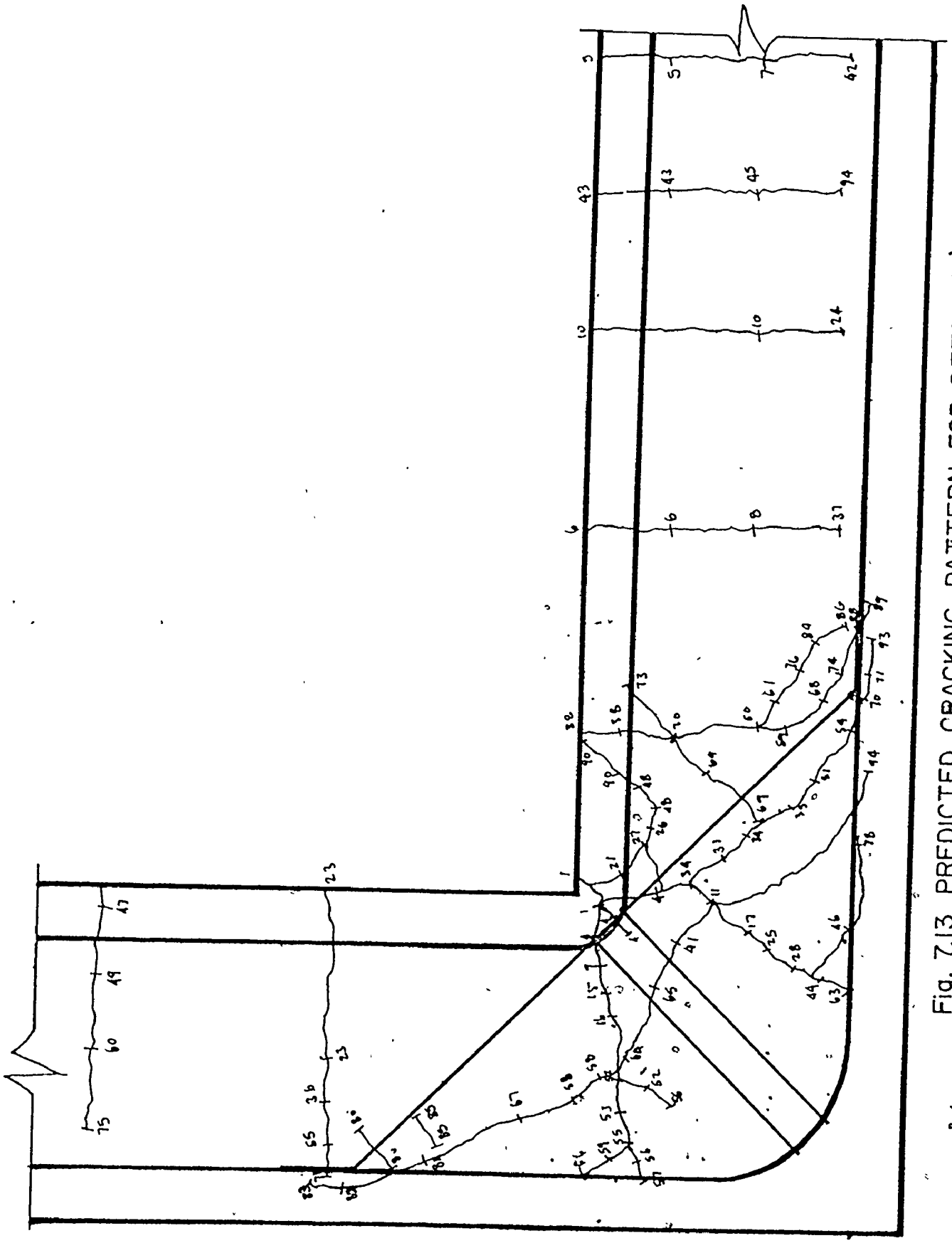

Fig. 7.13 PREDICTED CRACKING PATTERN FOR DETAIL (2'), SAME AS DETAIL (2) BUT USING HEAVIER DIAGONAL STIRRUPS.

compression face, was prevented due to the presence of web reinforcement in both the vertical and horizontal members.

The performance of this joint detail was not satisfactory. However, this conclusion is not in direct contradiction with Mayfield et.al.'s (74,91)results. Although they reported that this method of reinforcing joints is a promising one, the efficiency of the detail dramatically dropped from the range of 99% to 130% for a reinforcement percentage of 0.52% to an efficiency of 38% for a reinforcement percentage of 1.34%. Therefore, the test results of this study and the analytical confirmation of these results should not be surprising. Although the author's results seem to be in direct opposition to Mayfield et.al., it is in effect in opposition to their conclusion and not their test results.

7.3.4.3 Detail (3)

As was discussed in Chapter 4, Detail (3) was the first attempt by the author to overcome the fabrication difficulties encountered with Detail (1)and the apparent deficiencies found in Detail (2). This detail developed 114% of the theoretical ultimate capacity when tested. Figures 7.14 and 7.15 show the predicted cracking patterns for two of the three analysed cases which were described in Section 7.3.2 for Detail (3). These two cases are for the properly and for the inadequately anchored diagonal

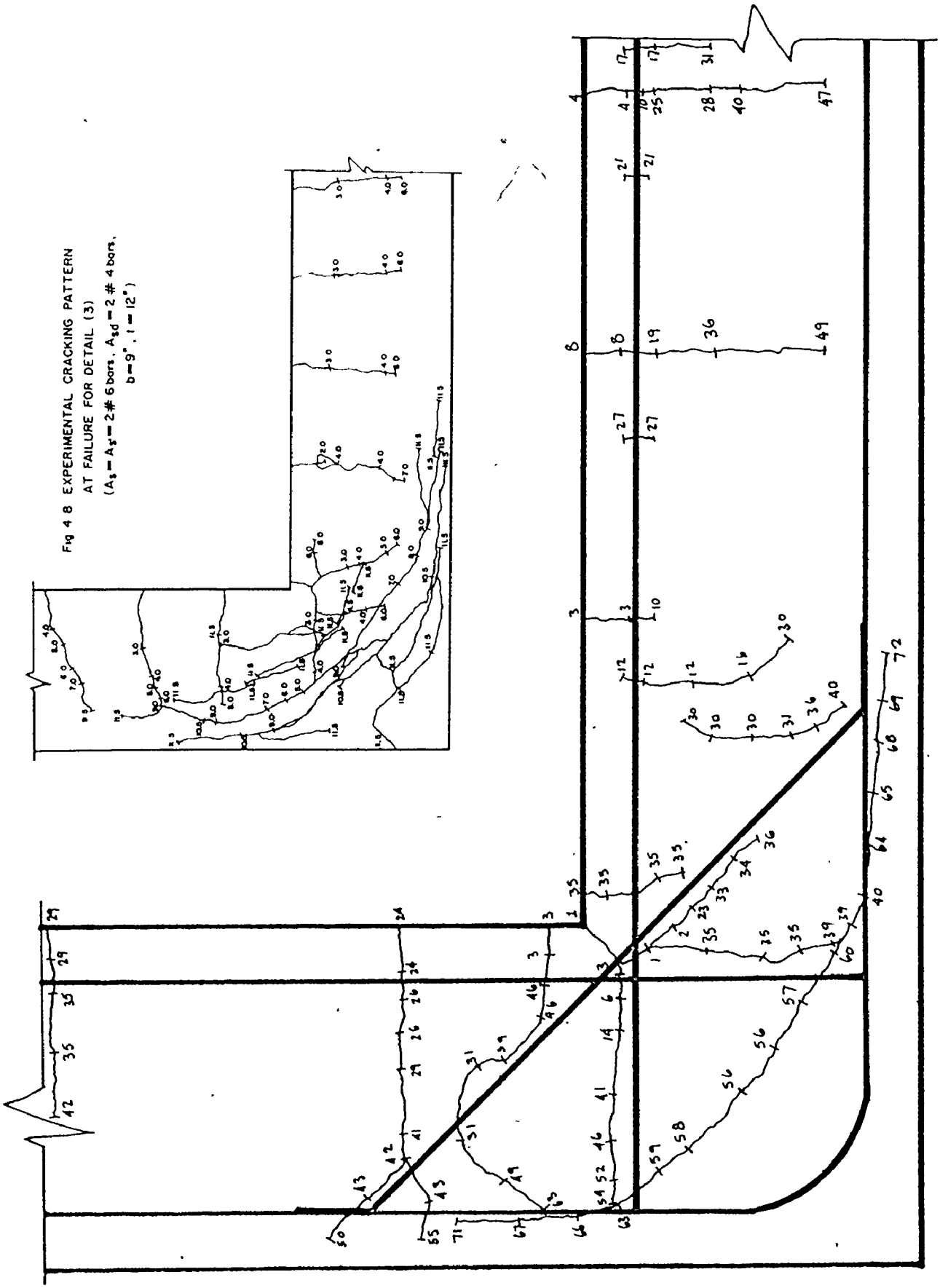

Fig. 7.14 PREDICTED CRACKING PATTERN FOR DETAIL (3), DIAGONAL BAR PROPERLY ANCHORD IN THE COMPRESSION ZONES.

bar respectively. The third case with the diagonal bar rigidly connected to the main steel reinforcement in the compression zone of the intersecting members was not shown because it was predicted to be essentially the same as that shown in Fig. 7.14 up to the load level corresponding to the yielding of the diagonal bar which caused the joint to fail as was discussed in Section 7.3.2.

The failure predicted by the analysis when the diagonal bar was properly anchored, by extending its ends into the compression zone of the intersecting members, was eventually caused by the diagonal cracks types (c) and (d) reducing the compression zones in the two intersecting members. Although this detail developed over 100% efficiency, the ultimate propagation of the diagonal cracks was primarily due to the lack of adequate web reinforcement (stirrups) near the outer face of the joint. It can be seen from Fig. 7.14 that the growth of the diagonal cracks was controlled in a desirable fashion. These cracks in the joint region did not develop until the applied loads reached nearly 93% of its theoretical ultimate capacity.

For the case when poor end anchorage was imposed on the diagonal bar Detail (3'), serious cracking occurred. The excessive cracking which can be seen in Fig. 7.15 was caused by the progressive loss of bond along the diagonal bar. However, when the end anchorage

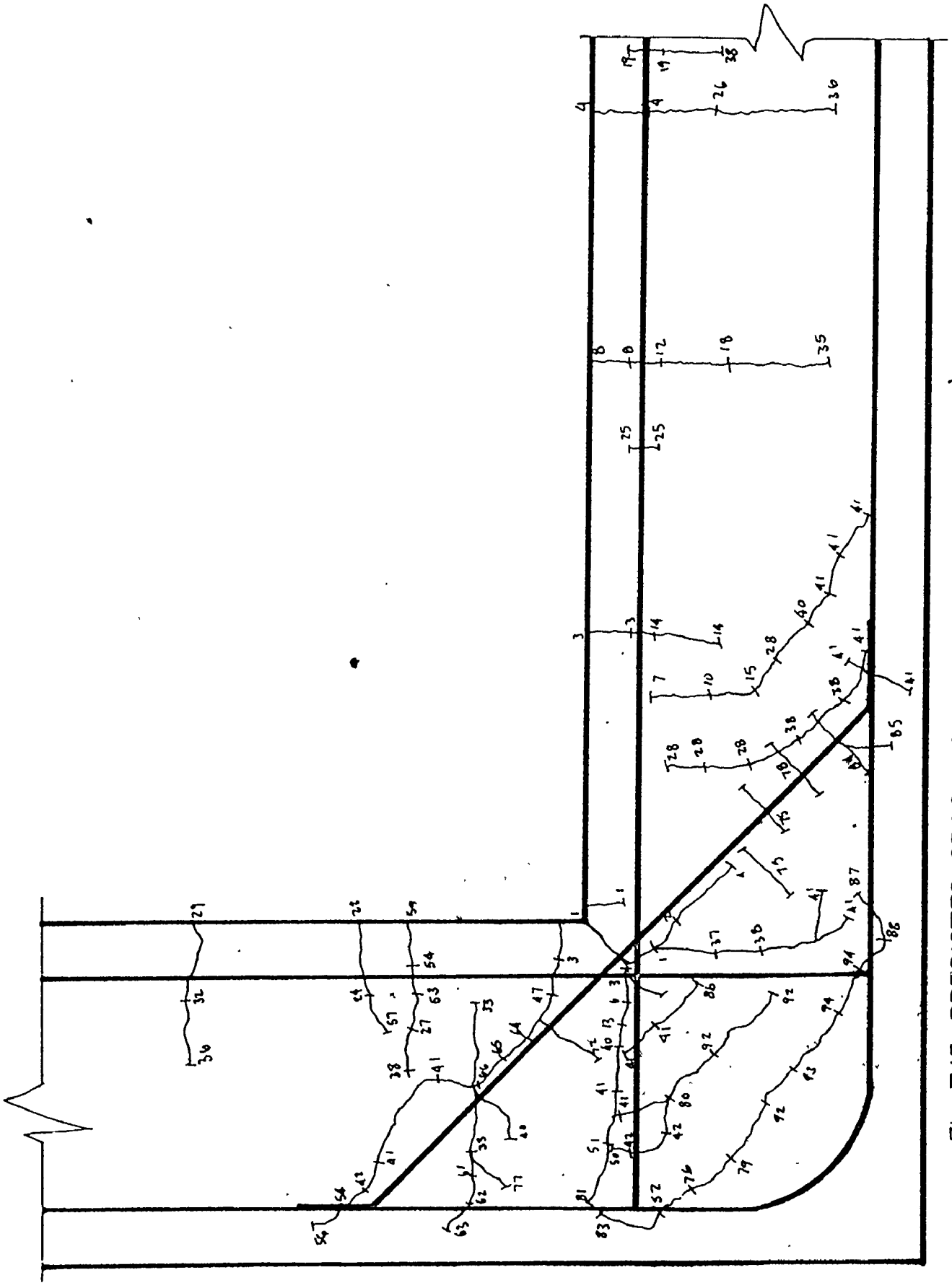

Fig. 7.15 PREDICTED CRACKING PATTERN FOR DETAIL (3'), SAME AS DETAIL (3) BUT WITH POOR ANCHORAGE OF THE DIAGONAL BAR ENDS.

broke down at approximately 79% of the theoretical capacity, the forces carried by the diagonal bar were redistributed to the main reinforcement. After this stage, the cracking occurred at a much faster rate and joint finally failed by diagonal tension at 92% efficiency.

The above discussion illustrates the decisive role of the diagonal bar on the overall behaviour of the joint. It indicates that for the same joint detail not only does the presence of this bar effect the behaviour and the ultimate capacity of the joint, but also that proper anchorage of this bar is extremely important.

It is also evident that a rigid main reinforcement cage such as the welded arrangement used for this detail can be of great help to provide reserve capacity as was shown when the end anchorage of the diagonal bar broke down.

7.3.4.4 Details (4&5)

Details (4&5) had practically identical reinforcing arrangements. As was discussed in Chapter 4 and in Section 7.3.3 of this chapter, these two details exhibited the best overall behaviour of all the joint details studied. The experimental and analytical capacities were both in excess of the theoretical ultimate capacity.

In this subsection not only is the cracking pattern shown in Fig. 7.16 discussed, but also some stress distribution in the steel and concrete along with bond stresses at different locations and loading stages are presented.

Figure 7.16 illustrates the predicted cracking pattern for Details (4&5) at failure of the joint. The ultimate capacity of this detail was predicted to be limited by bond failure (end anchorage breakdown) of the main tensile reinforcement at the joint region. However, to get more information about the behaviour of the joint beyond the initial failure, an ability of introducing strong artificial end anchorage upon bond breakdown was built into the computer program. When this was incorporated in the analysis, the joint was able to develop the yield stresses in the main tension reinforcement.

The analytical results confirmed the results obtained from testing Details (4) and (5). The joint was able to develop the theoretical ultimate capacity without any loss due to diagonal cracking or shear failure. When the anchorage of the main reinforcement was arbitrarily increased, increased capacity was obtained to take advantage of some of the steel strain hardening.

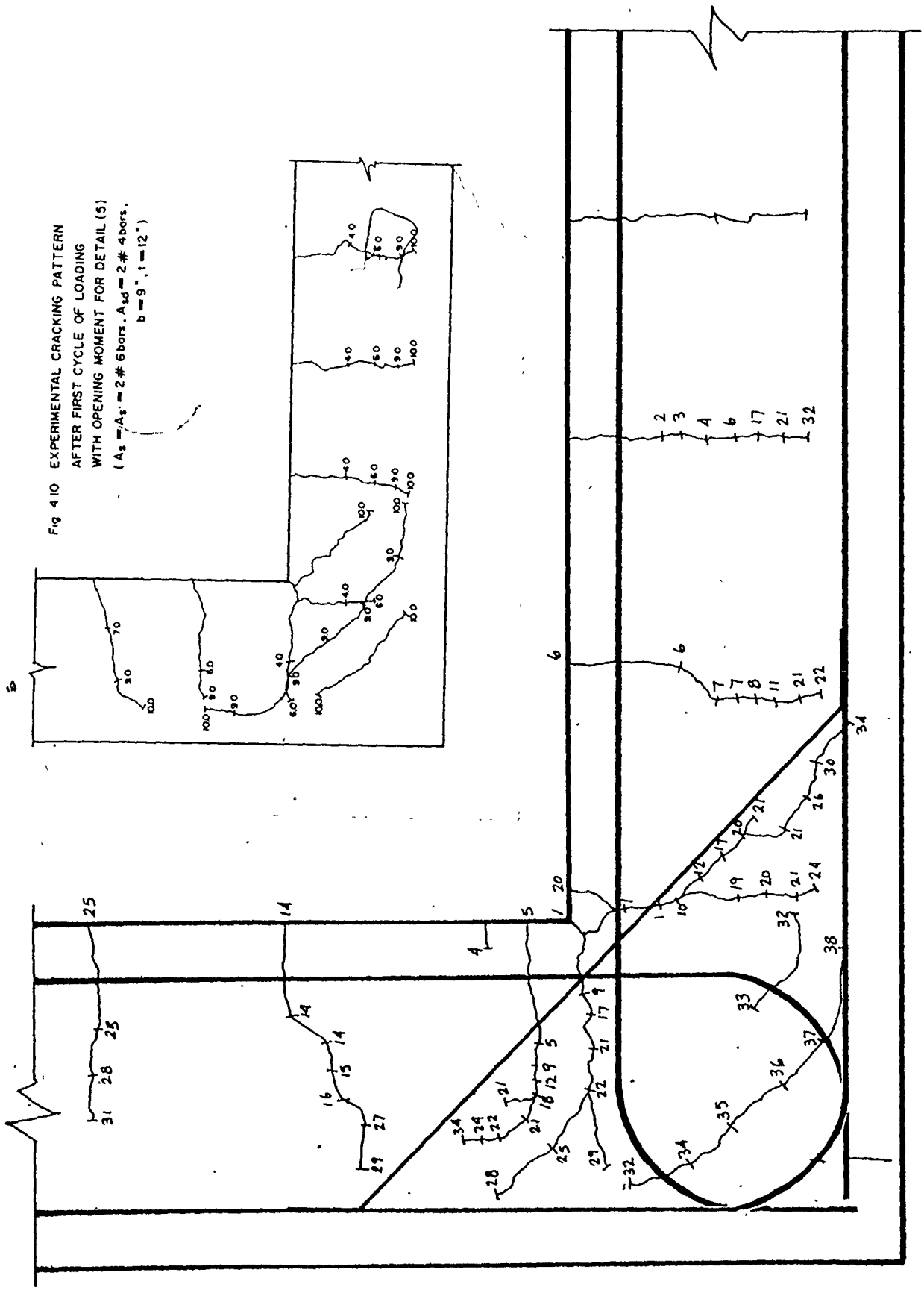

Fig. 7.16 PREDICTED CRACKING PATTERN FOR DETAILS (4 & 5)

Improved anchorage of the main tensile reinforcement may be provided mechanically by welding of end plates to the ends of all of the tension bars. If possible, the inclusion of concrete stubs beyond the joint to complement the anchorage development length can also be helpful (95).

The extension of the tension reinforcement far into the outer corner of the joint seems to be beneficial. This seems to provide the same confinement of the joint's core as that for Detail (1) developed by Nilsson (97) which helps in delaying the diagonal tension cracks. However, excessive bar slippage, especially for heavily reinforced joints, may cause the concrete cover in the joint region over the ends of the tension reinforcing bars to be pushed off (concrete spalling failure). This may be avoided with provision of the mechanical end anchorage or the concrete stubs mentioned above.

From comparison between Figure 7.16 and both Figures 4.9 and 4.10 for joint Details (4) and (5) respectively, it can be seen that the agreement between the tests and the analysis is closer than for any of the details discussed previously. Crack spacing and the extent of crack propagation at certain levels are in very good agreement.

Although comparisons between experimentally measured and analytically computed strains might not

be reliable for a single joint detail because of the limited number of tested specimens, the elastic strains measured before cracking should be fairly consistent for all specimens with different details but with equal percentage of reinforcement.

Figure 7.17 shows the average strains measured for specimen Details (1), (2), (3), (4) and (5) versus the analytical values. In this figure two sets of analytical strains were obtained for the joint region. Since the measured strains were obtained using an 8 inch gauge length, the average strains over 8 inches as well as the localized strains obtained analytically are plotted. It can be seen that the average strains obtained analytically slightly underestimate the experimental values. This is mainly due to the coarse finite element mesh used in the analysis. The localized strains are higher than average, and since there is no experimental counterpart, no direct comparison can be made.

Diagrams of strains found analytically at different sections in the constant moment region of the horizontal member are shown in Fig. 7.18 for different load levels. Figure 7.18 (a), (b) & (c) illustrates the strains at the location of a primary crack (section I-I), at an uncracked section (section II-II) and at the location of a secondary crack (section III-III) respectively.

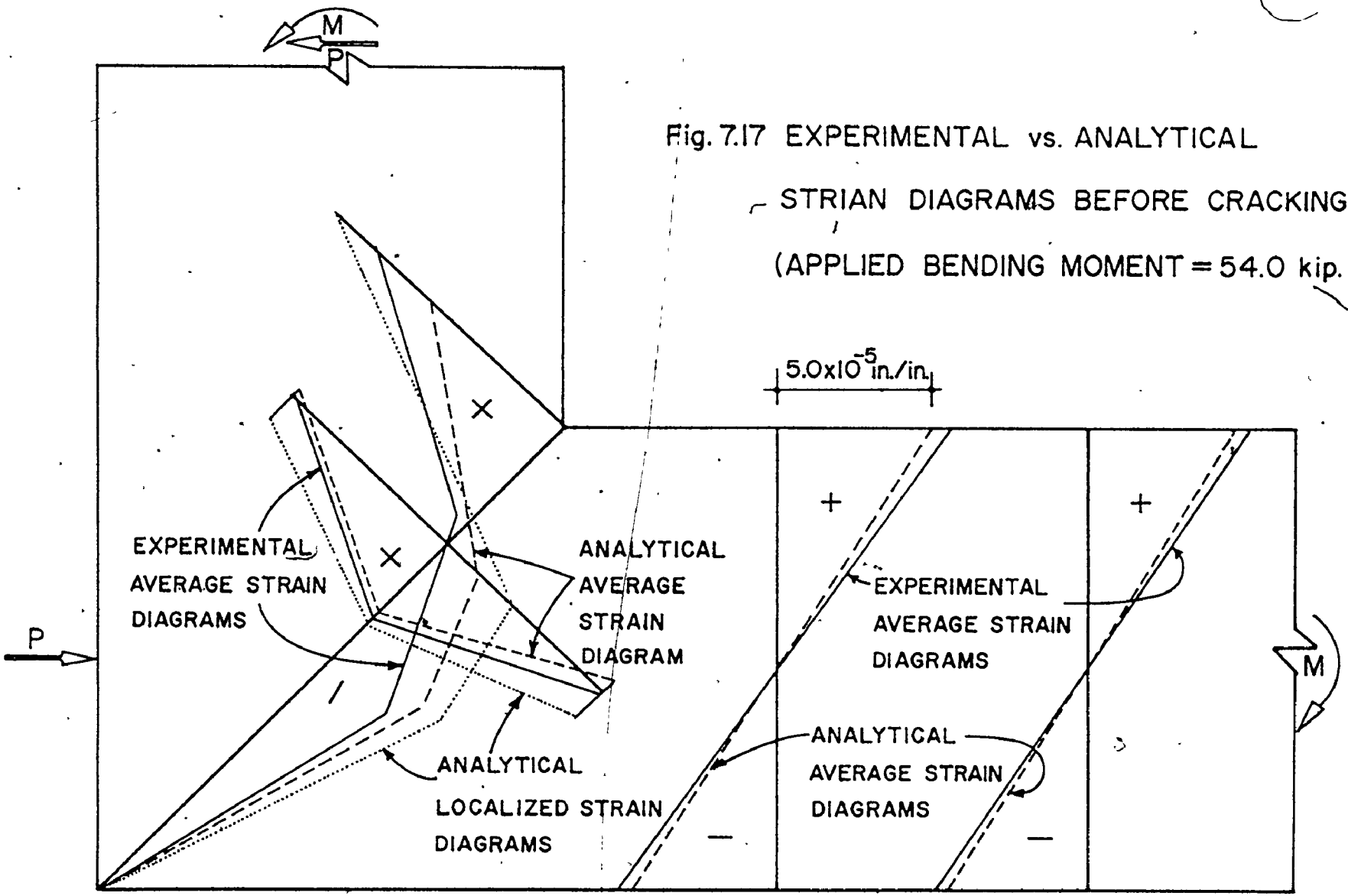

Fig. 7.17 EXPERIMENTAL vs. ANALYTICAL STRIAN DIAGRAMS BEFORE CRACKING. (APPLIED BENDING MOMENT = 54.0 kip. in.

It can be seen from the three diagrams that the strains were linear before cracking. After the occurrence of the first crack at section I-I, the tensile strains tapered off in the vicinity of the crack as can be seen for section II-II. The compressive strains, however, remained very close for sections I-I and II-II. These compressive strains were slightly lower for section III-III and the sections farther from the joint region. Tensile strains increased continuously but in a non-linear fashion in section III-III until a crack occurred.

Stability of crack height was achieved for section I-I at a load level of 8.47 kips, as illustrated in Fig. 7.18(a), when it reached the existing neutral axis. This was not the case for sections II-II or III-III. Section II-II remained uncracked until joint failure occurred. Section III-III was cracked but the crack height was stabilized at an earlier stage than for section I-I. The tensile strains decreased after the further propagation of the two surrounding primary cracks at sections I-I and IV-IV. This decrease in tensile strains (and the tensile stresses) is one of the reasons which causes the fairly realistic crack spacing obtainable using the analytical method presented in this investigation. The primary reasons which cause the concrete tensile strains to decrease in the immediate vicinity of existing cracks are the decreasing stiffnesses of the cracked concrete

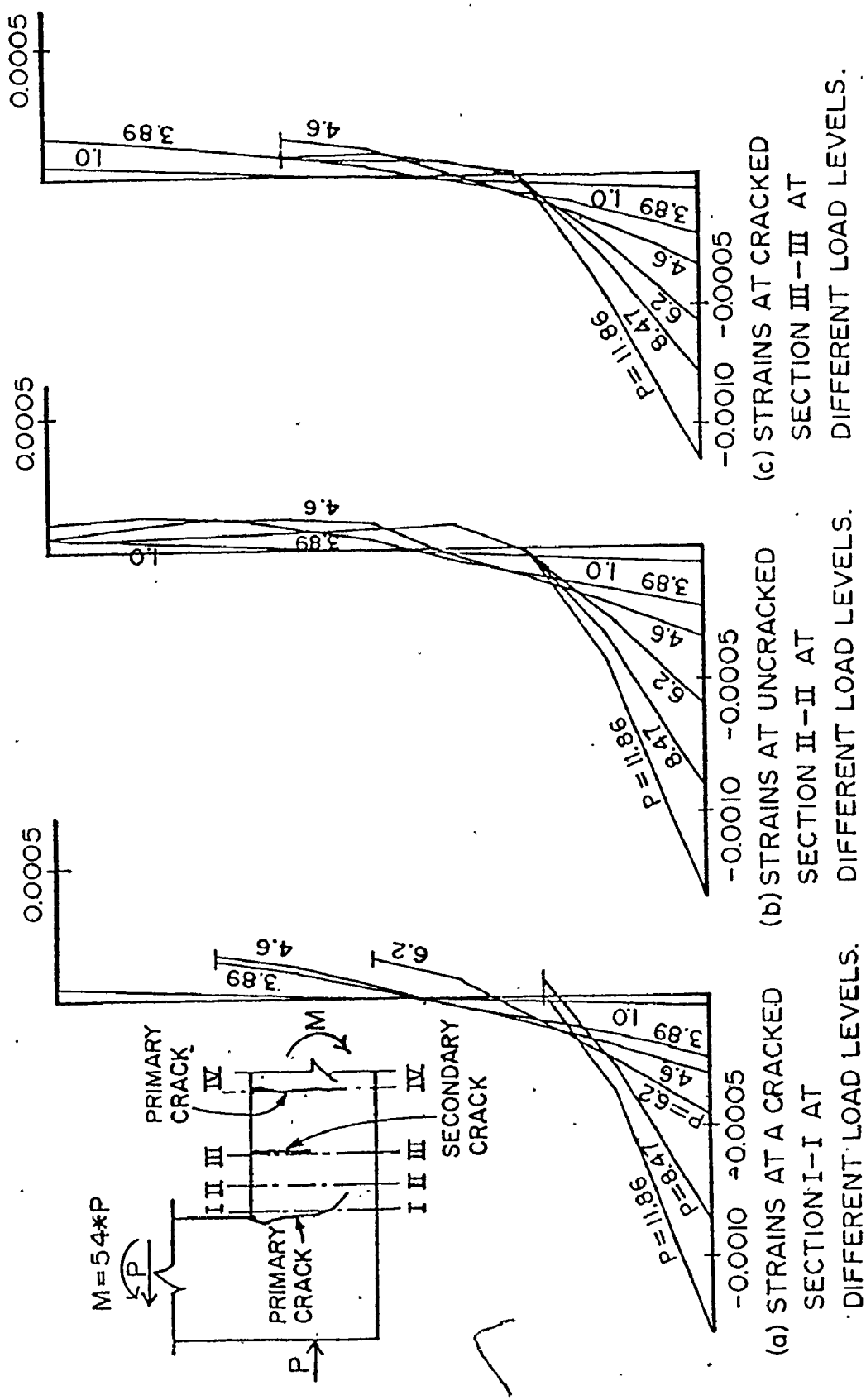

Fig. 7.18 STRAIN DIAGRAMS AT DIFFERENT SECTIONS IN THE CONSTANT MOMENT REGION OF DETAILS (4 & 5).

elements and the destruction of the bond elements adjacent to the faces of the crack. The distance between the cracks is determined by the ability of the bond linkages to accumulate large enough forces to cause the concrete to crack. At a particular stage, and for a certain crack spacing, the forces accumulated by the bond elements should not be able to produce a crack in the concrete and this is when crack spacing stability is achieved.

Figure 7.19 illustrates the analytical distribution of the strains which are normal and tangential to the diagonal line joining the inside and the outside corners of the joint. The strain distributions are plotted for different loading stages of the analysis. The load levels indicated on Figures 7.19(a) and 7.19(b) are the same as those shown in Figure 7.18. It can be visualized from the strain diagrams that tensile strains which cause cracks of types (b), (c) and (d) were present and were able to produce the three crack types at different stages of loading. However, the compressive strains in the normal direction, ε_x, which existed in the joint core were effective in delaying the occurrence of cracks of types (c) and (d) until a later stage of loading (at nearly 80% of the ultimate capacity).

Fig. 7.19 CONCRETE STRAIN DISTRIBUTION IN THE NORMAL AND TANGENT DIRECTIONS TO THE CORNER DIAGONALS OBTAINED FROM THE ANALYSIS OF DETAILS (4 & 5)

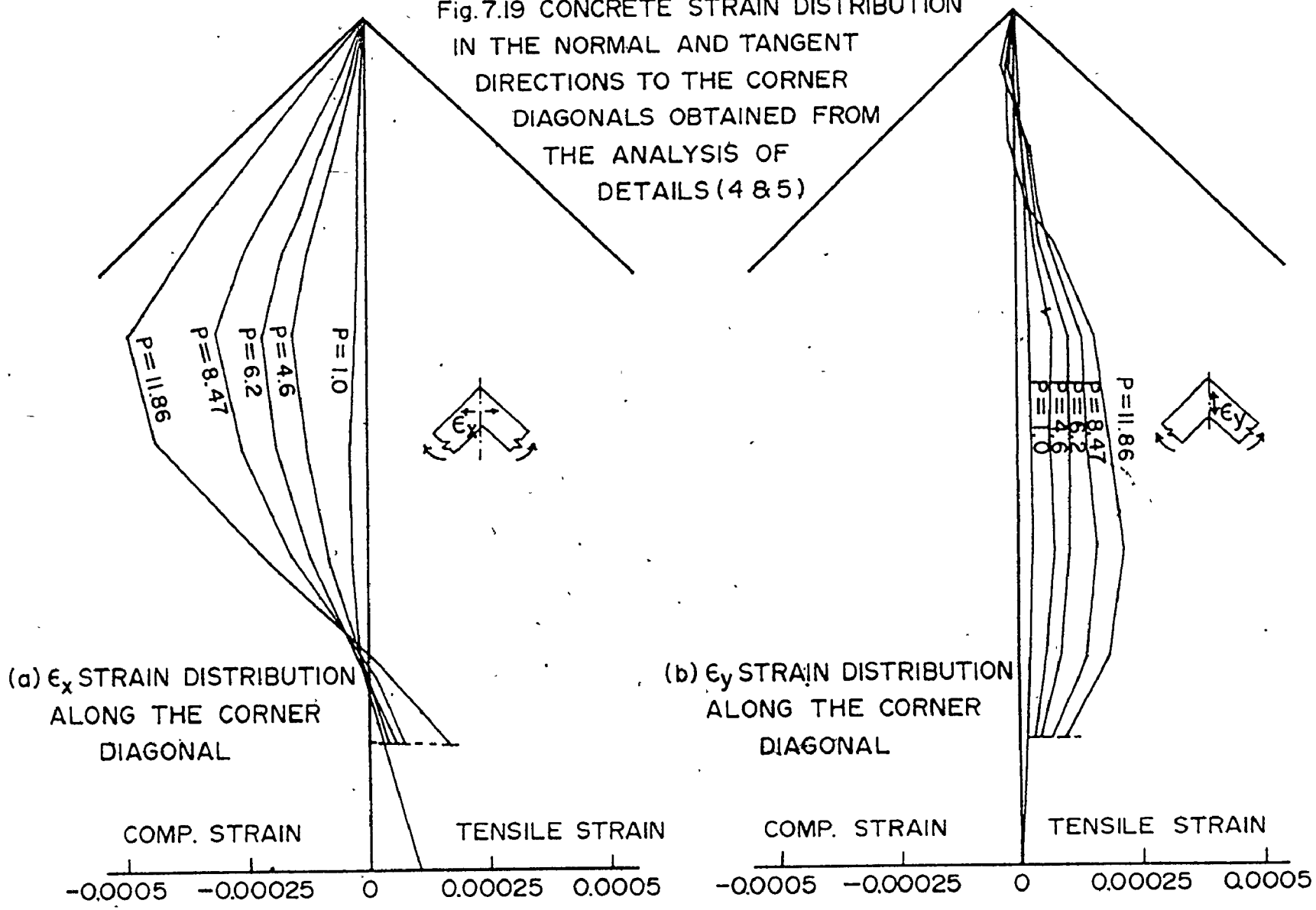

Figure 7.20 demonstrates the history of stress distribution along the main tensile reinforcement. The effects of cracking on stress concentrations in the steel reinforcement, discussed in Chapter 6 for beams, can be seen by observing the correspondence between crack locations and peak steel stresses shown in this figure. While the yielding stresses were obtained for this detail when the artificial end anchorage was introduced after end anchorage breakdown, this was not observed for any of the other details and the other joints failed with steel stresses less than their yield stresses.

To illustrate the adequacy of the amount of steel used for the diagonal bar, steel stresses at three different points as shown in Fig. 7.21 were plotted for different loading stages up to failure of the joint. It can be concluded that the amount of steel used was adequate (50% of the main steel), since the maximum stress in the bar reached at failure of the joint was less than 50% of the yield stress. The compressive stresses in the steel at points (1) and (3) all the way to failure, indicate adequate anchorage.

Bond stresses in some selected bond elements along the main steel reinforcing bars were plotted in Figure 7.22 for different loading stages up to failure.

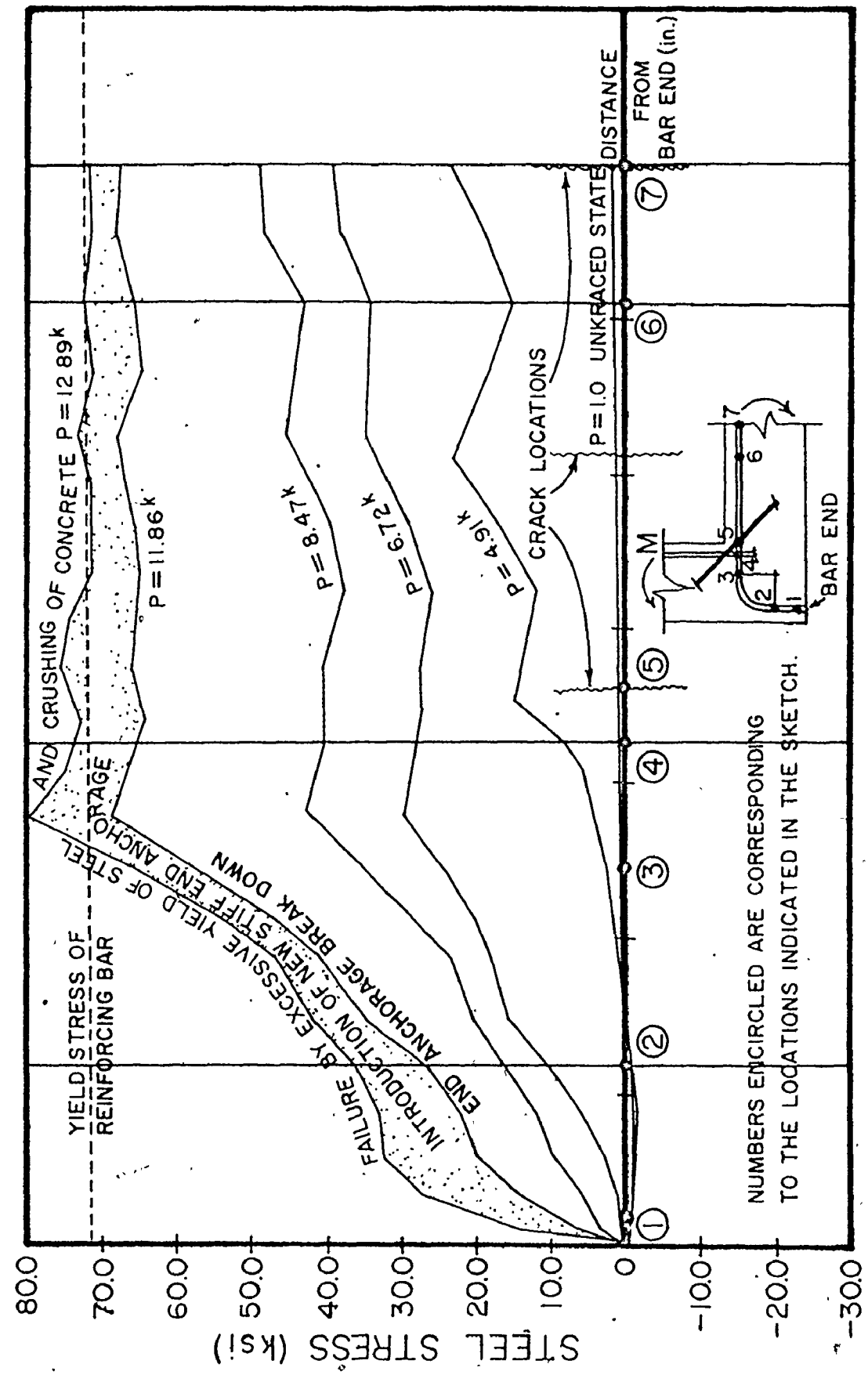

Fig. 7.20 LOCALIZED STEEL STRESSES AT DIFFERENT LOAD LEVELS FROM THE ANALYSIS OF DETAILS (4 & 5).

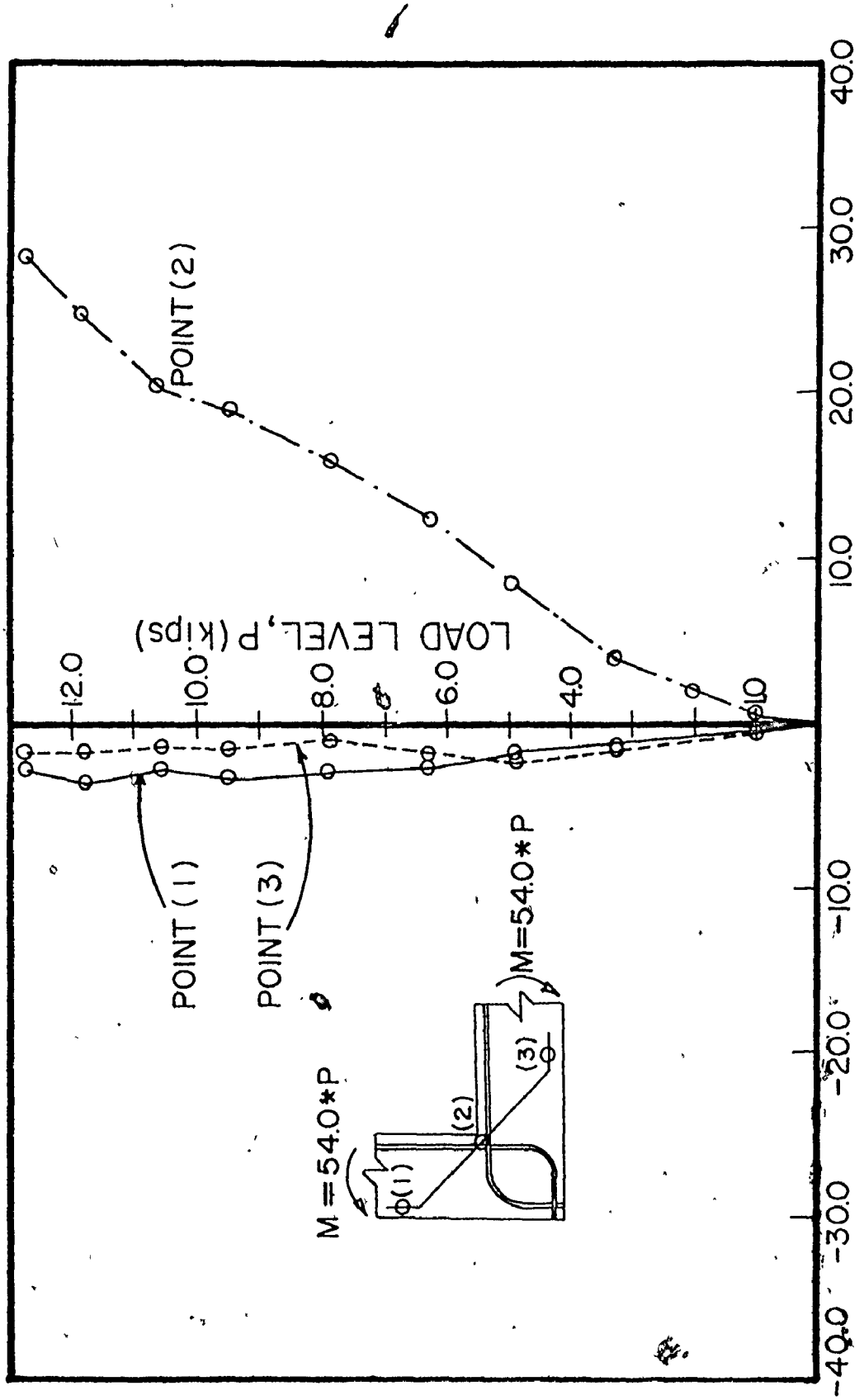

Fig. 7.21 STRESSES IN DIAGONAL BAR AT DIFFERENT LOAD LEVELS FROM THE ANALYSIS OF DETAILS (4 & 5)

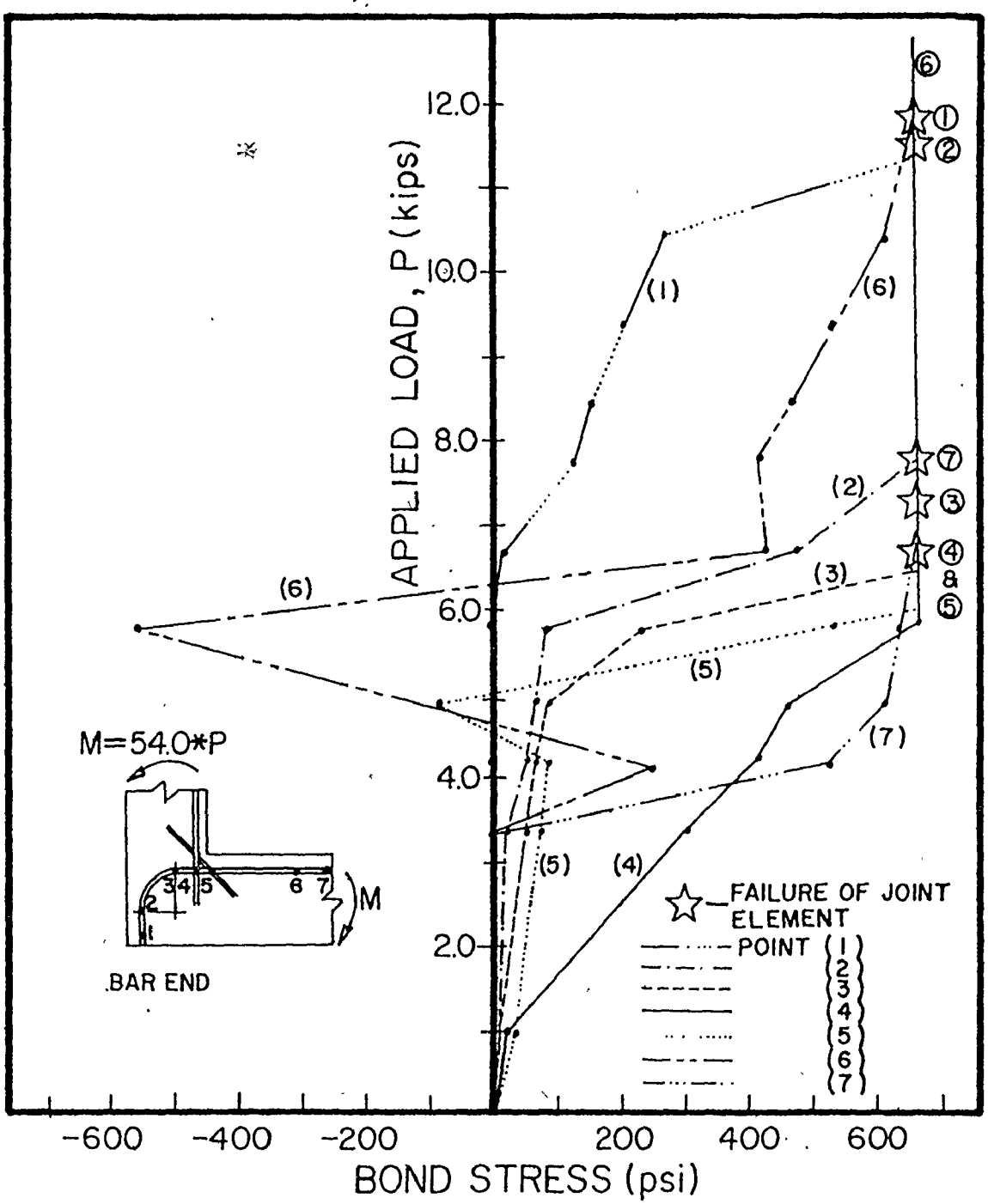

Fig. 7.22 BOND STRESSES AT DIFFERENT LOADING STAGES ALONG THE MAIN REINFORCEMENT FOR DETAILS(4&5)

The progressive bond failures which finally led to end anchorage breakdown can be observed considering the load levels at which the different bond elements have failed. Bond elements at locations 4 and 5 were the first to fail almost simultaneously. The element at position 3 followed shortly thereafter, and the element at position 7 was the next to fail but it did not contribute to the end anchorage problem because it was located in the constant moment region of the specimen. The bond element at position 2 followed these failures at a higher load level. When finally the bond element located at point 1 near the bar end failed at a load level of 11.86 kips, the bar would have slipped with no further bond resistance. This, however, was avoided in the analysis by artificially introducing stiff end anchorages which helped the joint to reach its maximum capacity which was controlled by the yielding of the steel.

7.3.4.5 Detail (6)

Detail (6) was the last of the details tested and analysed in this investigation. As was shown in Table 7.1, this detail failed by end anchorage breakdown at 95% of the theoretical ultimate capacity. The introduction of the artificial end anchorages as was discussed for Details (4 & 5), enabled the joint to develop the yielding stresses in the steel reinforcement.

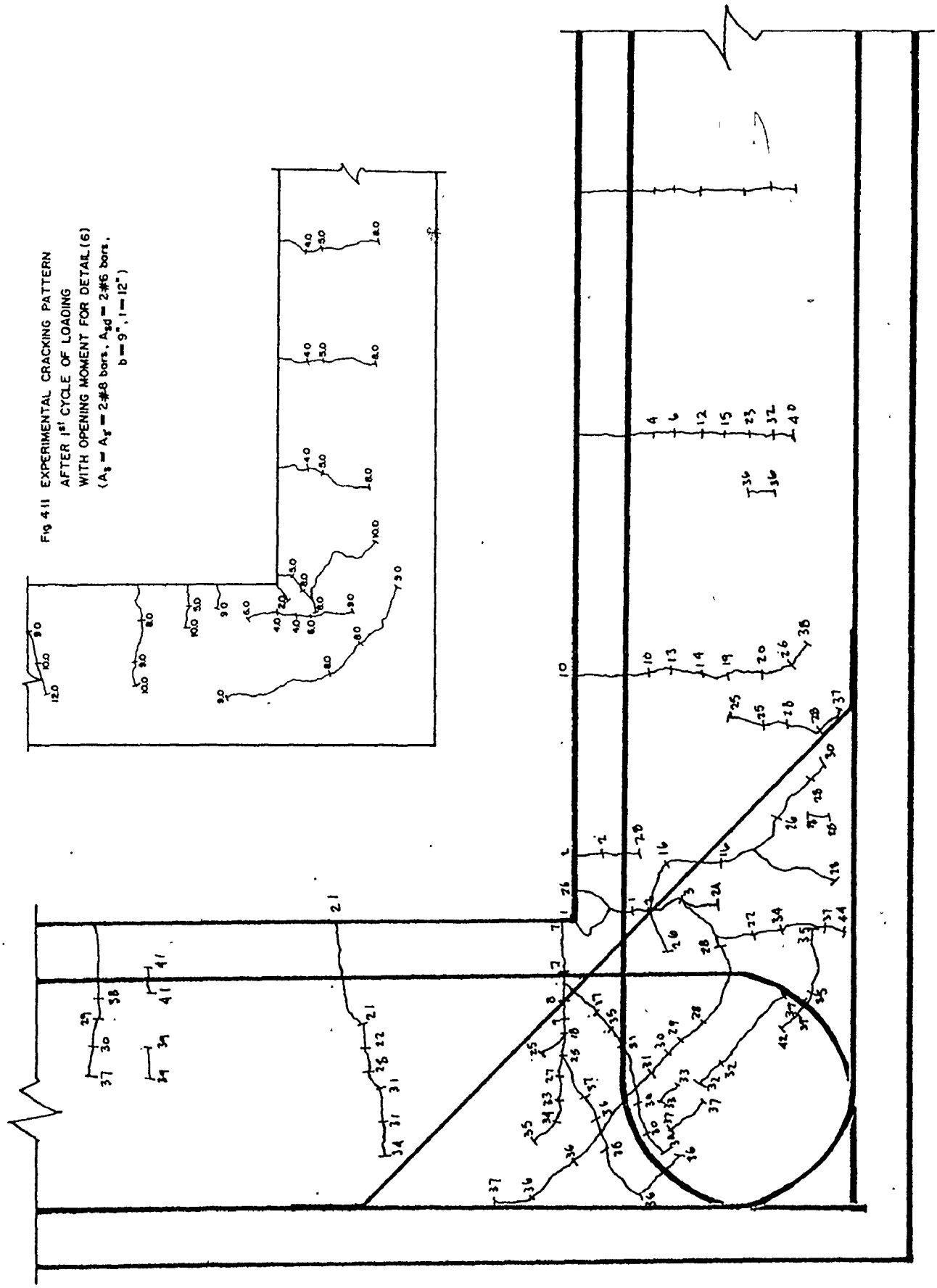

Fig. 7.23 PREDICTED CRACKING PATTERN FOR DETAIL (6)

From the analysis, the general behaviour of this detail was essentially the same as that for Details (4&5) with the exception of the cracking pattern. As shown in Fig. 7.23, more cracks were predicted for this detail than for Details (4&5). This should not be surprising, because of the larger steel and bond forces present which resulted from the higher percentage of reinforcement. The agreement between the predicted cracking pattern shown in Fig. 7.23 and that obtained experimentally is very good.

7.4 Summary

In this chapter the analytical results for joints subjected to positive bending moment were presented and discussed. All of the joints which were tested in the experimental program were analysed using the proposed method of analysis. Additional special cases which investigated the influence of the amount of diagonal stirrups and the degree of anchorage of the diagonal bars were also discussed. Cracking patterns, load-deflection curves and moment-rotation curves were illustrated. For Details (4) and (5), stress and strain distributions in the steel and concrete elements in addition to bond stresses along the reinforcing bars were also presented.

Generally very good agreement between the experimental and analytical results was found. Further

explanations of the behaviour of the joint details, which were not fully understood from the tests, were possible by using the proposed method of analysis. In addition, it was shown that other aspects and influence of variables could be investigated without companion experimental work.

In the next chapter, the summary and conclusions of this study, together with recommendations for future research are presented.

CHAPTER 8

SUMMARY AND CONCLUSIONS

8.1 Introduction

The main purpose of the investigation reported in this dissertation was to provide additional information to aid in understanding reinforced concrete joint behaviour. The specific area of interest was the behaviour of corner joints subjected to positive bending moment tending to open the corner.

Very little analytical information was found in the literature relating to the behaviour of reinforced concrete, beam-column connections. Little attention was given to the theoretical modelling and explanation of the behaviour of such a vital region in any reinforced concrete structure. A relatively few experimental results for different shapes of joints are available in the literature, primarily as a result of two major research programs in Sweden (97) and Britain (74, 91). Special problems in detailing, designing and understanding corner joints have been reported in several publications (74, 91, 97, 103). It was therefore, felt that a contribution to the subject would be made by providing better explanation to help understand the behaviour of corner or knee joints.

A brief summary of the research program along with final conclusions and recommendations for future research are

given in the following sections. More detailed discussions for both the experimental and the analytical parts of this research program have already been presented in the pertinent chapters.

8.2 Summary of the Experimental Investigation

An aim of the experimental investigation was to develop joint details which would perform in a satisfactory fashion. In addition, the results, of the experimental program were to provide data to test the reliability of the mathematical model.

Six tests of joints were performed, where four different details were tested, two of which reproduced arrangements previously reported in the literature. Three of the four details, of which two were developed by the author, were considered satisfactory in terms of attaining joint capacities equal to or exceeding the theoretical capacities, of the intersecting members. One of these details was considered to be superior to all the others. Two more tests were done on this detail; one to confirm the original results and the other to examine the influence of an increased ration of steel reinforcement on the behaviour of the joint. Although the behaviour of these joints under load reversals was not a major aim of this research, the last two tests were subjected to 4 cycles of load reversals before being loaded to failure. The test results indicated satisfactory performance.

8.3 Summary of the Analytical Investigation

A computer program utilizing the finite element technique was developed to study the behaviour of reinforced concrete structures. In the course of developing the program numerous problems were encountered. One of the most challenging problems was the modelling of cracking, so that realistic crack patterns could be predicted. It was found to be necessary to let only one concrete element crack at a time. The element which was stressed the most was the only element which was allowed to crack at any particular cycle of crack stability iterations. Modification of the stiffness of the cracked element was then performed according to the direction of the crack and the new stiffness was entered in the global matrix. This technique allowed for the redistribution of stresses to take place before any other element was allowed to crack. This resulted, in most cases, in relieving some of the elements which were previously overstressed and hence prevented the occurrance of unnecessary cracks.

After crack stability was achieved for a certain load level, the next load increment was automatically determined within the program according to the level of stress in the next most highly stressed element which did not exceed the failure criteria. However, if this increment was larger than a predetermined size of increment, it was then set at the smaller value to avoid divergence due to the non-linear material properties used in this analysis.

The reliability and suitability of the program was verified by comparison with well documented experimental data. The program's accuracy was tested and developed first for simply supported reinforced concrete beams. Compared to joints, they were more easily represented by finite elements much more economical to analyze, and they had the advantage of being more familiar to investigators so that visual and standard mathematical checks would more easily reveal any discrepancies in the analysis.

Comparisons with the beams tested by Leonhardt and Walther (28) were made. The agreement found between the tests and the analyses confirmed the reliability of the method of analysis. Comparisons of data obtained from the analysis with the results of the joint details tested by the author showed very good agreement and provided additional information which assisted in understanding the behaviour of joints.

Analytically predicted crack patterns agreed with the experimental results in a fashion which had not been found previously in the literature.

8.4 Final Conclusions

The analytical results presented in this dissertation documents the behaviour of simply supported reinforced concrete beams subjected to two-point loading. The demonstrated ability to predict cracking patterns and failure modes

which closely resemble those observed for shear and bending type failures in beams is a significant achievement even though the study of reinforced concrete beams was not the central topic of this investigation.

The experimental and analytical results for joints subjected to positive moments provide data in a relatively unexplored area. Considering only the experimental results, the acceptability of previously recommended details and merit of proposed modifications to detailing was examined. It was shown that not all recommended details are adequate. The proposed modified details facilitated fabrication and were capable of developing capacities in excess of that required.

The analytical results reproduced the experimental results with reasonable accuracy and also revealed behavioural characteristics which could not be observed or measured experimentally. The complex interactions in bondslip relationships, cracking, dowel action, and material nonlinearities were incorporated in a manner which could not be aspired to using simpler analytical techniques. In terms of cost and access to relevant information, it is suggested that this research provides a capability for extended studies for futher development of joints. In this way, a wider range of variables than was investigated in this dissertation may be included as well as other ideas for joint details.

The analytical model developed in this investigation was applied to only two components of reinforced concrete

structures.

8.5 Suggestions for Future Research

It is recommended that joints incorporating a wider range of variables be included in future studies. Much more experimental and analytical work will be needed before the problems associated with joint detailing are completely documented. In the course of this investigation, several areas where more research is required were noted. These are listed below:

1. More research on the intricate interaction of steel and concrete is required. Data for bond-slip and dowel action is relatively scarce.
2. The ability to model the response of concrete to biaxial states of stress is required which in turn requires more basic information.
3. More testing of joints is required with the following variables as suggested guide lines:
   a. effects of the unequal dimensions of the intersecting members on joint behaviour,
   b. effect of steel percentage on joint efficiency,
   c. influence of anchorage and development lengths of steel bars in the joint region,
   d. effectiveness of stirrups in the joint region, and
   e. effects of using different material properties.

4. It is suggested that the present finite element program be improved and extended to include the aggregate interlock phenomenon. Also, the presently used method of modifying the stiffness of elements to take cracking effects into considerations can be replaced more accurately by sub-dividing the cracked element into two new elements. Both of these proposals will require extensive additional computer core space and computing time.

It is also suggested that iterations for nonlinear material properties can be included in the analysis when faster generations of computers become available or when the economy permits to do so.

APPENDIX A

The listing of the computer program used in this investigation is given in this appendix along with the following descriptions of the functions of the Main Program and the Subroutines.

The program consists of a main program (MAIN) and seventeen Subroutines. These subroutines are

1. Subroutine (GEOM) computes the areas of elements and the Transformation matrix relating strains and displacements.
2. Subroutine (CNODE) determines code numbers for degrees of freedom, Bandwidth of the global matrix and nodal gravity loading for each element if required.
3. Subroutine (FSM) formulates the stiffness matrix of each individual element (Steel and Concrete).
4. Subroutine (NLOAD) is a subroutine to determine nodal loads including externally applied loads and add gravity loads if desired.
5. Subroutine (CNLP) prints out nodal loads at loaded nodes only.
6. Subroutine (JSTIFF) calcualtes the stiffness

of every joint element in the structure and modifies the necessary joint element stiffnesses to conform with the given bond-slip, dowel action, or aggregate interlock relationships. It also, adds the joint element stiffnesses to the global matrix in the appropriate location.

7. Subroutine (BAND) is a subroutine which utilizes the Choleski technique to solve for nodal displacements in terms of nodal loads.

8. Subroutine (CNDI) prints nodal deflections at prechosen locations only or at all nodes if required.

9. Subroutine (CESS) checks steel and concrete elements against failure criteria and decides if some particular elements need modification to their stiffness. The checks made are that for tension cracking in the concrete, changes in the modulus of elasticity of concrete elements in compression to follow the non-linear stress-strain curve, and for yielding of the main steel bars. Several subroutines are called upon from (CESS) to find the principal strains and stresses in an element and to determine the stresses in a cracked element.

10. Subroutine (SMAX) determines the principal strains and stresses and the direction of potential and existing cracks.

11. Subroutine (JSTRES) is a subroutine which computes the stresses and strains (or displacements), of joint elements parallel and perpendicular to their lengths. The stresses and displacements in the direction parallel to the length of joint element linking concrete and steel represents, the bond stress and slip respectively. The forces and displacements in the direction normal to the length of the elements represent the dowel forces and dowel displacements. In the case of elements joining corresponding concrete nodes the forces and displacements can represent the aggregate interlock phenomena. The failure criteria of these joint elements are also examined in this subroutine and their stiffnesses are modified accordingly for the next iteration.
12. Subroutine (STRSTF) is a subroutine to compute stiffnesses of stirrup elements (bar type elements) and transfer them to another subroutine to have them added to the global stiffness matrix in their appropriate locations.
13. Subroutine (ADSTRP) adds stirrup stiffnesses that were computed in (STRSTF) to the global stiffness matrix.
14. Subroutine (STSTRS) computes stirrup strains and stresses in terms of nodal displacements

and checks the failure criteria for stirrups.

15. Subroutine (FSMCRK) is called upon from the main program to find the modified stiffness of a cracked element. It substitutes the original stiffness of a cracked element with the new stiffness computed using Zienkiewicz's tenchique for stratified materials.
16. Subroutine (STRESS) calculates the stresses and strains of cracked element.
17. Subroutine (SMAER) is a subroutine that finds the stresses and strains in concrete and steel elements and averages the stresses of surrounding elements if required , (68).
18. Program (MAIN) controls and links all of the above mentioned subroutines. It decides on the convergence criteria and performs iterations, if convergence is not satisfied. It also generates the global stiffness matrix with the aid of the calculated individual element stiffness. The program also increases the applied loads when crack stability is achieved, and finally stops the run when completed or when any trouble is identified.

This program requires four disc files for auxiliary storage and uses around 550-700K core storage capacity for corner joints on the IBM-370 computer using double

precision.

The Chart shown in Fig. 5.5 illustrates the organization of the Main Program and the Subroutines.

```
C          P R O G R A M  M A I N                                          IN   10
C      COMPUTER PROGRAM UTILIZING THE FINITE ELEMENT METHOD.               IN   20
C      PREDICTS THE BEHAIOURAL CHARACTERISTICS OF REINFIORCED CONCRETE     IN   30
C      MEMBERS.                                                            IN   40
C      TAKES THE FOLLOWING INTO ACCOUT,                                    IN   50
C            (1) BOND -SLIP RELATIONSHIP                                   IN   60
C            (2) DOWEL ACTION                                              IN   70
C            (3) NON-LINEAR MATERIAL PROPERTIES.                           IN   80
C            (4)  STRAIN HARDENING OF STEEL REINFORCEMENT.                 IN   90
C            (5) CRACKING AND CRACK PROPAGATION EFFECTS.                   IN  100
C            (6) INCLUSION OF STIRRUPS.                                    IN  110
C                                                                          IN  120
C                                                                          IN  130
      IMPLICIT REAL*8(A-H,O-Z)                                             IN  140
      COMMON SM(58000)                                                     IN  150
      DIMENSION EI(20),FF(2),FG(1124),FN(1124),DX(562),DY(562),P(6,6)      IN  160
     1,PT(9),NP(8),R(6,6),S(6,6),UI(20),UNI(20),ND(562,2)                  IN  170
     2,EX(818),EY(818),EXY(818),SEX(818),SEY(818), SEXY(818)               IN  180
     3,          NIJ(3),NNPR(562),F(1124)                                 IN  190
      DIMENSION KK(200),XII(200),YII(200),XJJ(200),YJJ(200),JM(200,4)      IN  200
     $ ,SJ(8,8) ,ENSI(200)                                                 IN  210
     $       ,MFLAG(818),BRA(2),WIDE(818)                                  IN  220
     $         , FACTOR(818,2) ,NEAN(20)                                   IN  230
     $      , CSJ(200),TITLE(10)                                           IN  240
      DIMENSION SKS(4,4),STS(150) ,RAT(150)                                IN  250
      DIMENSION BETA(818),INDC(818)                                        IN  260
     $ KOW(16)                                                             IN  270
      COMMON/BLK1/ENSI                                                     IN  280
      COMMON/BLK2/RAT                                                      IN  290
      COMMON/BLOK3/ DIA1,DIA2,DIA3,N1,N2,N3                                IN  300
      COMMON/BLK7/RATIO,NSWICH,IBM1,NT1,JOB,NCYCLE                         IN  310
      COMMON/BLK9/IPP                                                      IN  320
      READ(5,52) TITLE                                                     IN  330
52    FORMAT(10A8)                                                         IN  340
53    FORMAT(1H1,2X,10A8,/,3X,100('+'),//)                                 IN  350
      WRITE(6,53) TITLE                                                    IN  360
      WRITE(3,53) TITLE                                                    IN  370
      WRITE(6,28)                                                          IN  380
C                                                                          IN  390
C     NN= NO. OF NODAL POINTS                                              IN  400
C     NE= NO. OF FINITE ELEMENTS.                                          IN  410
C     NUE= NO. OF DIFFERENT MATERIALS.                                     IN  420
C     NGI= GRAVITY LOADING IDENIFIER                                       IN  430
C     NLC=NO. OF ALLOWABLE LOAD CASES                                      IN  440
C     KV=ALLOWABLE TOTAL NUMBER OF UNKOWNS.                                IN  450
C     NJE=NO. OF JOINT ELEMENTS                                            IN  460
C     NBOUN= NO. OF NODES HAVING BOUNDARY CONDITIONS OTHER THAN FREE       IN  470
C     NLINE= NO. OF NODAL LINES FOR BEAMS(USED FOR AUTO. DATA GENERATION   IN  480
C     NJPLN= NO. OF NODAL POINTS PER LINE                                  IN  490
C     KOW IS THE NOS. OF NODES THE DEFL. OF WHICH ARE TO BE PRINTED        IN  500
C     NSTR=NUMBER OF STIRRUP ELEMENTS                                      IN  510
C     NIK=NO. OF NODES TO SUPPLY THEIR ORDINATES                           IN  520
C     NCRTCL= THE NODE NO. CONTROLLING DEFLECTION                          IN  530
C     NAXY=1 IF DEFLXION IS IN THE X DIRN.                                 IN  540
C     NAXY=2 IF DEFLXION IS IN THE Y DIRN.                                 IN  550
C     NSWICH IS A PRINTING SWITCH LEAVE BLANK IF STATE IF CRACKED          IN  560
C     ELEMENTS IS NOT DESIRED TO BE PRINTED ,SET TO ANY NO. OTHERWISE.     IN  570
C                                                                          IN  580
      READ (5,3) NN,NE,NUE,NGI,NLC,KV ,NSTR     ,NJE,NBOUN,NLINE,NJPLN    IN  590
     $ ,NIK,NCRTCL,NAXY,NSWICH                                             IN  600
      READ(5,3) (KOW(I),I=1,16)                                            IN  610
      IF(NCRTCL.EQ.0)NCRTCL=NN                                             IN  620
      IF(NAXY.EQ.0) NAXY=1                                                 IN  630
C                                                                          IN  640
C     NANCH= TOTAL NUMBER OF ELEMENTS WELL ANCHORD                         IN  650
C     NEAN = THE NUMBERS OF THE ANCHORD ELEMNTS IN ORDER                   IN  660
C                                                                          IN  670
      READ(5,3) NANCH                                                      IN  680
      READ(5,3) (NEAN(KA),KA=1,NANCH)                                      IN  690
C                                                                          IN  700
C     BRA(I)= WIDTHS OF ELEMENTS CORRESSPONDING TO DIFFERENT MATERIALS.    IN  710
C                                                                          IN  720
      READ(5,5) (BRA(KIM),KIM=1,NUE)                                       IN  730
```

```
C                                                                       IN  740
C     FTENS= ALLOWABLE TENSILE FACTOR FOR THE CONCRETE TO BE MULTIPLIED  IN  750
C     BY CYLINDER CONC STRENGTH**0.5                                    IN  760
C     FPC=CONC CYLINDER STRENGTH                                        IN  770
C     EPSYLD= STEEL YIELDING STRAIN                                     IN  780
C     BNDSLP= INIINITIAL SLOPE OF BOND -SLIP CURVE, IF ZERO DEFAULTS    IN  790
C     TO HOUDE AND MIRZAS RELATIONSHIP                                  IN  800
C     DOWEL = INITIAL DOWEL STIFF. ,SET=0.0 TO DEFAULT TO               IN  810
C     HOUDE AND MIRZA'S                                                 IN  820
C     ALDEFL = MAXIMUM ALLOWABLE DEFL. AFTER WHICH THE PROC. STOPS.     IN  830
C                                                                       IN  840
      READ(5,6) FTENS,FPC ,EPSYLD ,BNDSLP,DOWEL,ALDFL                   IN  850
      IF(BNDSLP.EQ.0.) BNDSLP=1.95E+06                                  IN  860
      IF(ALDFL.EQ.0.0) ALDFL=3.0                                        IN  870
      IF(FTENS.EQ.0.0) FTENS=9.5                                        IN  880
      IF(FTENS.GT.10.0) FTENS=9.6                                       IN  890
      FTENS=FTENS*DSQRT(FPC)                                            IN  900
      DO911  KIKI=1,NJE                                                 IN  910
      KK(KIKI)=0                                                        IN  920
 911  CONTINUE                                                          IN  930
      EC=57620.0*(FPC**0.5)                                             IN  940
      BNET=BRA(1)-BRA(2)                                                IN  950
      IF(DOWEL.EQ.0.0) DOWEL=BNET*EC                                    IN  960
C                                                                       IN  970
C     CALCULATE CONSTANTS FOR CONCRETE STRESS-STRAIN CURVE(SAENZ'S)     IN  980
C                                                                       IN  990
      EPSO=1.0E-05*(FPC**0.25)*(31.5-FPC**0.25)                         IN 1000
      FFC=0.94*FPC                                                      IN 1010
      EPSF=0.0025                                                       IN 1020
      EPSC=-EPSF                                                        IN 1030
      ENOD=FPC/EPSO                                                     IN 1040
      REC=EC/ENOD                                                       IN 1050
      RFC=FPC/FFC                                                       IN 1060
      RSE=EPSF/EPSO                                                     IN 1070
      RCAP=REC*(RFC-1.0)/((RSE-1.0)**2)-1.0/RSE                         IN 1080
      C1=2.0*RCAP-1.0                                                   IN 1090
      C2=RCAP                                                           IN 1100
      C3=RCAP+REC-2.0                                                   IN 1110
      DO 2 I=1,NN                                                       IN 1120
      DO 2 J=1,2                                                        IN 1130
      ND(I,J)=1                                                         IN 1140
2     CONTINUE                                                          IN 1150
C                                                                       IN 1160
C     READ NOS. OF NODAL POINTS  FIXED IN EITHER ONE OF THE TWOO DEGREES IN 1170
C     OF FREEDOM(1=FREE,0=FIXED)                                        IN 1180
C                                                                       IN 1190
      DO 12 KIM =1,NBOUN                                                IN 1200
      READ(5,78) I,ND(I,1),ND(I,2)                                      IN 1210
12    CONTINUE                                                          IN 1220
      DO 13 I=1,NN                                                      IN 1230
      WRITE (6,79) I,ND(I,1),ND(I,2)                                    IN 1240
13    CONTINUE                                                          IN 1250
      WRITE(6,4)                                                        IN 1260
      REWIND  1                                                         IN 1270
      REWIND 2                                                          IN 1280
      REWIND 4                                                          IN 1290
      NT1=1                                                             IN 1300
      NT2=4                                                             IN 1310
      NU = 1                                                            IN 1320
      DO 1 II = 1,NN                                                    IN 1330
      DO 1 JJ = 1,2                                                     IN 1340
      IF(ND(II,JJ).EQ.0) GO TO 1                                        IN 1350
      ND(II,JJ) = NU                                                    IN 1360
      NU = NU + 1                                                       IN 1370
    1 CONTINUE                                                          IN 1380
      NU = NU - 1                                                       IN 1390
C                                                                       IN 1400
C     GENERATE BEAM DATA AUTO. IF REQUIRED.                             IN 1410
C                                                                       IN 1420
      IF(NLINE.EQ.0) GO TO 23                                           IN 1430
      READ(5,6)(XII(J),YII(J),J=1,NLINE)                                IN 1440
      DO 102 J=1,2                                                      IN 1450
      READ(5,6)(FACTOR(I,J),I=1,NJPLN)                                  IN 1460
102                                                                     IN
```

```
      DO 14 J=1,NLINE                                                    IN 1480
      READ (5,3) (IFLAG(I),I=1,NJPLN)                                    IN 1490
      DO 14 I=1,NJPLN                                                    IN 1500
      IF(IFLAG(I).EQ.0) GO TO 14                                         IN 1510
      KX=2*IFLAG(I)-1                                                    IN 1520
      KY=2*IFLAG(I)                                                      IN 1530
      IF(J.GT.13) GO TO 15                                               IN 1540
      SM(KX)=FACTOR(I,1)+XII(J)                                          IN 1550
      SM(KY)=FACTOR(I,2)+YII(J)                                          IN 1560
      GO TO 14                                                           IN 1570
15    SM(KX)=FACTOR(I,2)+XII(J)                                          IN 1580
      SM(KY)=FACTOR(I,1)+YII(J)                                          IN 1590
14    CONTINUE                                                           IN 1600
23    IF(NIK.EQ.0) GO TO 19                                              IN 1610
C                                                                        IN 1620
C     READ NODAL POINT COORDINATES IF NO AUTO. GENERATION WAS USED       IN 1630
C                                                                        IN 1640
      DO 18 IZIB=1,NIK                                                   IN 1650
      READ(5,23000) MFD,XEX,YEX                                          IN 1660
23000 FORMAT(I5,2F10.0)                                                  IN 1670
      KX=2*MFD-1                                                         IN 1680
      KY=2*MFD                                                           IN 1690
      SM(KX)=XEX                                                         IN 1700
      SM(KY)=YEX                                                         IN 1710
18    CONTINUE                                                           IN 1720
19    CONTINUE                                                           IN 1730
      DO 16 L2=1,NJE                                                     IN 1740
      XII(L2)=0.0                                                        IN 1750
      YII(L2)=0.0                                                        IN 1760
16    CONTINUE                                                           IN 1770
      DO 2000 LI=1,NE                                                    IN 1780
      FACTOR(LI,1)=1.0                                                   IN 1790
      FACTOR(LI,2)=1.0                                                   IN 1800
2000  CONTINUE                                                           IN 1810
      DO 104 I=1,NUE                                                     IN 1820
C                                                                        IN 1830
C     READ MATERIAL CONSTANTS FOR CONC. AND STEEL RESP. E,POISSON'S RATI IN 1840
C     O ASSUMED .2 AND .3 RESP.)                                         IN 1850
C                                                                        IN 1860
      READ(5,6)EI(I),UI(I),UNI(I)                                        IN 1870
      EI(I)=1000.0*EI(I)                                                 IN 1880
      EI(1)=EC                                                           IN 1890
104   CONTINUE                                                           IN 1900
      SARR = 0.0                                                         IN 1910
      IPO=0                                                              IN 1920
      IPP=1                                                              IN 1930
      NB = 0                                                             IN 1940
      JC = 0                                                             IN 1950
      NCYCLE=0                                                           IN 1960
      ISEE=1                                                             IN 1970
      NNDX=2                                                             IN 1980
      RATIO=0.0                                                          IN 1990
      LC=0                                                               IN 2000
      IBII=0                                                             IN 2010
      DO 10202 JER =1,NU                                                 IN 2020
      F(JER) =0.0                                                        IN 2030
10202 CONTINUE                                                           IN 2040
      INDEED=ND(NCRTCL,NAXY)                                             IN 2050
C                                                                        IN 2060
C     READ NO. OF EACH ELEM.,I,J,K NODES AND MATERIAL IDENTIFIER(1 FOR   IN 2070
C     CONCRETE, 2 FOR STEEL)                                             IN 2080
C                                                                        IN 2090
      DO 105  J=1,NE                                                     IN 2100
      READ(5,3) NOM,NI,NJ,NK,MET                                         IN 2110
      IF(NOM.NE.J)STOP                                                   IN 2120
      E = EI(MET)                                                        IN 2130
      U = UI(MET)                                                        IN 2140
      UN = UNI(MET)                                                      IN 2150
      XI = SM(2*NI-1)                                                    IN 2160
      YI=SM(2*NI)                                                        IN 2170
      XJ=SM(2*NJ-1)                                                      IN 2180
      YJ=SM(2*NJ)                                                        IN 2190
      XK=SM(2*NK-1)                                                      IN 2200
      YK=SM(2*NK)                                                        IN 2210
```

```
C   * SUBROUTINE GEOM DETERMINES THE ELEMENT AREA AND TRANSPOSE MATRIX P  IN 2220
C   * SUBROUTINE GEOM REQUIRES NO INPUT DATA.                            IN 2230
      CALL GEOM(XI,XJ,XK,YI,YJ,YK,PT,A,ARR)                              IN 2240
      ARW = (UN*ARR)/6.                                                  IN 2250
      ARR = ARR/2.                                                       IN 2260
      SARR = SARR + ARR                                                  IN 2270
      WIDTH=BRA(MET)                                                     IN 2280
      WIDE(J)=WIDTH                                                      IN 2290
      WRITE(6,8) J,MET,NI,NJ,NK,XI,XJ,XK,YI,YJ,YK,E,U,UN,ARR,WIDTH       IN 2300
C     SUBROUTINE CNODE DETERMINES CODE NOS. , BAND WIDTH , AND           IN 2310
C     NODAL GRAVITY LOADING FOR EACH ELEMENT                             IN 2320
      CALL CNODE (J,ND,NP,NB,NU ,FG,NI,NJ,NK,ARW)                        IN 2330
C     SUBROUTINE FSM DETERMINES EACH ELEMENT STIFF MATRIX S.             IN 2340
C     SUBROUTINE FSM REQUIRES NO INPUT DATA.                             IN 2350
      CALL FSM (P,S,R,PT,E,U,ARR ,WIDTH)                                 IN 2360
      WRITE(1) (NP(I),I=1,6),((S(MI,MN),MN=1,6),MI=1,6) ,(PT(K),K=1,9),  IN 2370
     1 A,E,U ,NI,NJ,NK,ARR                                               IN 2380
  105 CONTINUE                                                           IN 2390
      REWIND 1                                                           IN 2400
      IF(NSTR.LE.0) GO TO 2010                                           IN 2410
      SDIAM=0.01                                                         IN 2420
      WRITE(6,65)                                                        IN 2430
65    FORMAT(5X,'STIRRUP INFORMATION',/,5X,20('+'),/,5X,                 IN 2440
     6'  IS   JS    XIS    YIS    XJS    YJS    AREA  LENGTH',/)        IN 2450
      DO 2040 NS=1,NSTR                                                  IN 2460
      RAT(NS)=1.0                                                        IN 2470
2020  FORMAT(3I5,2F10.0)                                                 IN 2480
C                                                                        IN 2490
C     IBX =NO. OF STIRRUP ELEMENT                                        IN 2500
C     IS,JS SHOULD BE CHOSEN SO THAT THE POSTIVE LOCAL MEMBER X-AXIS     IN 2510
C     DOES NOT MAKE MORE THAN 180 DEG WITH THE GLOBAL X-AXIS.            IN 2520
C     SDIA= DIAMETER OF STIRRUP                                          IN 2530
C     BRACH= NO. OF STIRRUP BRANCHES.                                    IN 2540
C                                                                        IN 2550
      READ(5,2020) IBX,IS,JS,SDIA ,BRANCH                                IN 2560
      IF(BRANCH.EQ.0.0) BRANCH=2.0                                       IN 2570
      IF(SDIA.EQ.0.0) SDIA=SDIAM                                         IN 2580
      SDIAM=SDIA                                                         IN 2590
      SAREA=BRANCH*.7854*SDIA**2                                         IN 2600
C                                                                        IN 2610
C     STSTF IS A SUBROUTINE WHICH DETERMINES STIRRUP STIFFNESSES.        IN 2620
C                                                                        IN 2630
      CALL       STRTF(IS,JS,ANG,SAREA,SKS,ALS,EI)                       IN 2640
      NP(1)=ND(IS,1)                                                     IN 2650
      NP(2)=ND(JS,1)                                                     IN 2660
      NP(3)=ND(IS,2)                                                     IN 2670
      NP(4)=ND(JS,2)                                                     IN 2680
      WRITE(2)IS,JS,ANG,SAREA,((SKS(KI,MI),KI=1,4),MI=1,4),              IN 2690
     6 (NP(I),I=1,4),ALS                                                 IN 2700
      MAX=0                                                              IN 2710
      MIN=3000                                                           IN 2720
      DO 402 J=1,4                                                       IN 2730
      IF(NP(J).EQ.0) GO TO 402                                           IN 2740
      IF(NP(J)-MAX)403,403,404                                           IN 2750
404   MAX=NP(J)                                                          IN 2760
403   IF(NP(J)-MIN)405,402,402                                           IN 2770
405   MIN=NP(J)                                                          IN 2780
402   CONTINUE                                                           IN 2790
      NB2=MAX-MIN                                                        IN 2800
      IF(NB2.GT.NB) NB=NB2                                               IN 2810
2040  CONTINUE                                                           IN 2820
      REWIND2                                                            IN 2830
2010  CONTINUE                                                           IN 2840
      IBI=0                                                              IN 2850
      DO 142 KIN=1,NE                                                    IN 2860
      INDC(KIN)=0                                                        IN 2870
142   NFLAG(KIN)=0                                                       IN 2880
      IF(NJE .EQ. 0) GO TO 41                                            IN 2890
C                                                                        IN 2900
C     READ JOINT ELEMENT NOOS. ,I,J,K,L NODAL CODE NUMBERS.              IN 2910
C                                                                        IN 2920
      DO 10300 I=1,NJE                                                   IN 2930
                          ),J=1,4)                                       IN 2940
                                                                         IN 2950
```

```
C                                                                       IN 2960
C     READ THE NUMBERS OF JOINT ELEMENTS JOINING DIFFERENT DIAMETERS    IN 2970
C     OF STEEL,UP TO THREE DIAMS.,                                      IN 2980
C     N(I) IS THE STARTING NO. OF THE JOINT ELEMS.                      IN 2990
C     DIA IS THE BAR DIAM. CORESSPONDING TO THE ABOVE JT.ELEM.NO.       IN 3000
C     CNN IS THE FACTOR FOR DOWEL STIFF. ALWAYS SET TO 1.0 OR ALTER     IN 3010
C     TO STUDY EFFECT OF DOWEL STIFF. ON THE BEHAVIOUR.                 IN 3020
C                                                                       IN 3030
      READ(5,62) N1,N2,N3,DIA1,DIA2,DIA3,CNN                            IN 3040
      CSS1=3.14159*BNDSLP*DIA1                                          IN 3050
      CSS2=3.14159*BNDSLP*DIA2                                          IN 3060
      CSS3=3.14159*BNDSLP*DIA3                                          IN 3070
      CNN=DOWEL*CNN                                                     IN 3080
      CSSF=0.0                                                          IN 3090
      WRITE(6,70)                                                       IN 3100
      WRITE(6,71) ((I,(JM(I,J),J=1,4)),I=1,NJE)                         IN 3110
      WRITE(6,72)CSS1,CNN,CSSF,CSS2,CSS3,N1,N2,N3                       IN 3120
      WRITE(6,73)                                                       IN 3130
C                                                                       IN 3140
C     COMPUTE AND STORE JOINT ELEM. CONSTANTS.                          IN 3150
C                                                                       IN 3160
      DO 63 I=1,NJE                                                     IN 3170
      ENS1(I)=.00                                                       IN 3180
      CSJ(I)=CSS1                                                       IN 3190
      IF(I.GT.N1.AND.I.LE.N2) CSJ(I)=CSS2                               IN 3200
      IF(I.GT.N2.AND.I.LE.N3) CSJ(I)=CSS3                               IN 3210
      II = JM(I,1)                                                      IN 3220
      JJ = JM(I,2)                                                      IN 3230
      IL = JM(I,3)                                                      IN 3240
      JL = JM(I,4)                                                      IN 3250
      XII(I)= SM(2*II-1)                                                IN 3260
      YII(I)= SM(2*II)                                                  IN 3270
      XJJ(I)= SM(2*JJ-1)                                                IN 3280
      YJJ(I)= SM(2*JJ)                                                  IN 3290
      WRITE(6,76) I,XII(I),YII(I),XJJ(I),YJJ(I)                         IN 3300
      NP(1) = ND(II,1)                                                  IN 3310
      NP(2) = ND(JJ,1)                                                  IN 3320
      NP(3) = ND(IL,1)                                                  IN 3330
      NP(4) = ND(JL,1)                                                  IN 3340
      NP(5) = ND(II,2)                                                  IN 3350
      NP(6) = ND(JJ,2)                                                  IN 3360
      NP(7) = ND(IL,2)                                                  IN 3370
      NP(8) = ND(JL,2)                                                  IN 3380
      MAX = 0                                                           IN 3390
      MIN = 3000                                                        IN 3400
      DO 42 J = 1,8                                                     IN 3410
      IF(NP(J) .EQ. 0) GO TO 42                                         IN 3420
      IF(NP(J) - MAX) 43,43,44                                          IN 3430
   44 MAX = NP(J)                                                       IN 3440
   43 IF(NP(J) - MIN) 45,42,42                                          IN 3450
   45 MIN = NP(J)                                                       IN 3460
   42 CONTINUE                                                          IN 3470
      NB1 = MAX - MIN                                                   IN 3480
      IF(NB1 .GT. NB) NB = NB1                                          IN 3490
   63 CONTINUE                                                          IN 3500
    41 CONTINUE                                                         IN 3510
      NB1 = NB + 1                                                      IN 3520
      NV = NB1*NU                                                       IN 3530
      WRITE(6,999) NB,NV                                                IN 3540
      IF(NV.LE.KV) GO TO 122                                            IN 3550
      WRITE (6,17) NV                                                   IN 3560
C                                                                       IN 3570
C     IF NO. OF UNKOWNS IS .GT.ALLOWED STOP EXECUTION.                  IN 3580
C                                                                       IN 3590
      STOP                                                              IN 3600
  122 CONTINUE                                                          IN 3610
      IPX=IPO                                                           IN 3620
      IPO=IPP                                                           IN 3630
      IPP=IPX                                                           IN 3640
      NCYCLE=NCYCLE+1                                                   IN 3650
      IF(NCYCLE.GT.1) NFLAG(JOB)=0                                      IN 3660
      IF(NCYCLE.EQ.1.AND.LC.NE.0) IBM=IBM+1                             IN 3670
      WRITE(6,75) NCYCLE                                                IN 3680
                                                                        IN
```

```
C     GENERATES MAIN STIFFNESS MATRIX                                    IN 3700
C                                                                        IN 3710
      DO 40 JL=1,NV                                                      IN 3720
      SM(JL)=0.0                                                         IN 3730
40    CONTINUE                                                           IN 3740
      DO 20 KJJ = 1,NE                                                   IN 3750
      READ(NT1)(NP(I),I=1,6),((S(MM,MN),MN=1,6),MM=1,6) ,(PT(J),J=1,9),  IN 3760
     1 A,E,U ,NI,NJ,NK,ARR                                               IN 3770
      IF(MFLAG(KJJ).NE.1.OR.INDC(KJJ).NE.0) GO TO 1010                   IN 3780
      BATA=BETA(KJJ)                                                     IN 3790
      WIDTH=WIDE(KJJ)                                                    IN 3800
      E1=E*FACTOR(KJJ,NNDX)                                              IN 3810
C                                                                        IN 3820
C     IF ELEMENT IS CRACKED FIND ITS MODIFIED STIFFNESS.                 IN 3830
C                                                                        IN 3840
      CALL FSHCRK (S,PT,E1,U,ARR,WIDTH,BATA,INDC,KJJ)                    IN 3850
1010  CONTINUE                                                           IN 3860
      WRITE(NT2) (NP(I),I=1,6),((S(MM,MN),MN=1,6),MM=1,6),(PT(J),J=1,9), IN 3870
     1 A,E,U ,NI,NJ,NK,ARR                                               IN 3880
      IF(MFLAG(KJJ).EQ.1.AND.INDC(KJJ).EQ.1) GO TO 149                   IN 3890
      IF(MFLAG(KJJ).EQ.2) FACTOR(KJJ,NNDX)=.001                          IN 3900
      IF(FACTOR(KJJ,NNDX).EQ.1.0) GO TO 47                               IN 3910
      DO 49 JIM=1,6                                                      IN 3920
      DO 49 JEM=1,6                                                      IN 3930
      S(JIM,JEM)=FACTOR(KJJ,NNDX)*S(JIM,JEM)                             IN 3940
49    CONTINUE                                                           IN 3950
      GO TO 47                                                           IN 3960
149   INDC(KJJ)=2                                                        IN 3970
47    DO 7 JJ=1,6                                                        IN 3980
      IF(NP(JJ).EQ.0) GO TO 7                                            IN 3990
      DO 11 II =JJ,6                                                     IN 4000
      IF(NP(II).EQ.0) GO TO 11                                           IN 4010
      IF(NP(JJ)-NP(II))9,10,10                                           IN 4020
10    K=(NP(II)-1)*NB +NP(JJ)                                            IN 4030
      SM(K)=SM(K)+S (JJ,II)                                              IN 4040
      GO TO 11                                                           IN 4050
9     K=(NP(JJ)-1)*NB +NP(II)                                            IN 4060
      SM(K)=SM(K)+S (JJ,II)                                              IN 4070
  11  CONTINUE                                                           IN 4080
7     CONTINUE                                                           IN 4090
   20 CONTINUE                                                           IN 4100
      REWIND NT1                                                         IN 4110
      REWIND NT2                                                         IN 4120
      NTX=NT2                                                            IN 4130
      NT2=NT1                                                            IN 4140
      NT1=NTX                                                            IN 4150
C     ADD STIRRUP STIFFNESSES TO MAIN STIFFNESS MATRIX                   IN 4160
      IF(NSTR.LE.0) GO TO 2050                                           IN 4170
      CALL ADSTRP(NSTR,NB)                                               IN 4180
2050  CONTINUE                                                           IN 4190
      IF(ISEE.EQ.1) GO TO 3100                                           IN 4200
      IF(IBM.GT.0) GOTO 494                                              IN 4210
3100  CONTINUE                                                           IN 4220
      IF(NGI .EQ. 1) GO TO 31                                            IN 4230
C     DETERMINE NOADAL LOADS                                             IN 4240
C     SUBROUTINE NLOAD DETERMINES NOADAL LOADS                           IN 4250
C     SUBROUTINE NLOAD REQUIRES DATA.                                    IN 4260
      IF(RATIO.GT.1.0) GO TO 144                                         IN 4270
      CALL NLOAD(ND,FG,FF,NU,NGI)                                        IN 4280
      GO TO 22                                                           IN 4290
31    CONTINUE                                                           IN 4300
      IF(NLC .EQ. 0) GO TO 22                                            IN 4310
      CALL NLOAD(ND,FG,FF,NN,NGI)                                        IN 4320
   22 CONTINUE                                                           IN 4330
494   CONTINUE                                                           IN 4340
      GO TO 143                                                          IN 4350
144   DO 145 JAS=1,NU                                                    IN 4360
      FG(JAS)=(RATIO-1.0)*F(JAS)                                         IN 4370
145   CONTINUE                                                           IN 4380
143   CONTINUE                                                           IN 4390
      DO 124 JJ = 1,NU                                                   IN 4400
      F(JJ) = F(JJ) + FG(JJ)                                             IN 4410
      IF(NNDX.EQ.1)FH(JJ)=FG(JJ)                                         IN 4420
      IF(NNDX.EQ.2)FN(JJ)=F (JJ)                                         IN 4430
```

```
  124 CONTINUE                                                          IN 4440
      WRITE (6,21)                                                      IN 4450
C     SUBROUTINE CNLP PRINTS OUT NOADAL LOADS                           IN 4460
      CALL CNLP (F,FN,ND,NN,NNPR,IPP)                                   IN 4470
331   IF(NJE .EQ. 0) GO TO 64                                           IN 4480
      CALL       JSTIFF(SJ,JM,KK,ND,XII,XJJ,YII,YJJ,CSJ,CNN,NB,NJE,CSSF IN 4490
     &,NANCH,NEAN)                                                      IN 4500
  333 CONTINUE                                                          IN 4510
  64  CONTINUE                                                          IN 4520
      WRITE(6,24)                                                       IN 4530
C   * SUBROUTINE BAND SOLVES FOR DISPLACEMENTS USING THE CHOLESKI SQUARE IN 4540
C     METHOD.                                                           IN 4550
      DET = 1.E-8                                                       IN 4560
      CALL BAND (SM,FN,NU,NB1, 1 ,DET)                                  IN 4570
      IF(DET)125,126,127                                                IN 4580
125   WRITE(6,25)DET                                                    IN 4590
      GO TO  1003                                                       IN 4600
126   WRITE(6,26)DET                                                    IN 4610
      GO TO  1003                                                       IN 4620
127   CONTINUE                                                          IN 4630
      DFLXN=DABS(FN(INDEED))                                            IN 4640
      IF(DFLXN.GT.ALDFL) STOP                                           IN 4650
      DO 495  JJ=1,NU                                                   IN 4660
      FG(JJ)=0.0                                                        IN 4670
495   CONTINUE                                                          IN 4680
C     SUBROUTINE  CNDI REQUIRES NO INPUT DATA                           IN 4690
C     SUBROUTINE CNDI PRINTS OUT NODAL DEFLECTIONS                      IN 4700
      CALL CNDI(FN,ND,NN,DX,DY,KOW,JC)                                  IN 4710
      IF(NCYCLE.EQ.1) WRITE(3,3000)                                     IN 4720
3000  FORMAT(1X,100('-'))                                               IN 4730
      WRITE(3,75) NCYCLE                                                IN 4740
      IF(IPP.EQ.1) GO TO 707                                            IN 4750
      WRITE(3,29)                                                       IN 4760
707   CONTINUE                                                          IN 4770
C                                                                       IN 4780
C     FIND STRESSES AND STRAINS USING THE AVERAGING TECHNIQUE.          IN 4790
C                                                                       IN 4800
      CALL       SAMER(FN,EX,EY,EXY,SEX,SEY,SEXY,NN,FACTOR,UI,NV      IN 4810
     &     ,NT1,NE,MFLAG,FTENS)                                         IN 4820
C   * SUBROUTINE CESS DETERMINES AND PRINTS OUT THE ELEMENT STRAINS     IN 4830
C     AND STRESSES.                                                     IN 4840
C   * SUBROUTINE CESS REQUIRES NO INPUT DATA.                          IN 4850
      CALL       CESS(FN,EX,EY,EXY,SEX,SEY,SEXY,NE,JC,UI,IPP,FG,ND,    IN 4860
     1 WIDE,NN,IBM,MFLAG,FTENS,EPSC,EPSYLD,C1,C2,C3,EPSO,FACTOR,EC,BETA IN 4870
     &  ,INDC)                                                          IN 4880
      IF (NJE .EQ. 0) GO TO 128                                         IN 4890
      IF(IPP.EQ.1) GO TO 708                                            IN 4900
      WRITE(6,59)                                                       IN 4910
708   CONTINUE                                                          IN 4920
C                                                                       IN 4930
C     FIND BOND AND DOWEL STRESSES AND DISPLACEMENTS IN JT. ELEMS.      IN 4940
C                                                                       IN 4950
      CALL       JSTRES  (JM,DX,DY,XII,YII,XJJ,YJJ,KK,NJE,CSS1,CNN,CSSF IN 4960
     &,CSJ)                                                             IN 4970
  128 CONTINUE                                                          IN 4980
      IF(NSTR.LE.0) GO TO 2060                                          IN 4990
      IF(IPP.EQ.1) GO TO 709                                            IN 5000
      WRITE(6,2070)                                                     IN 5010
709   CONTINUE                                                          IN 5020
C                                                                       IN 5030
C     FIND STRAINS AND STRESSES IN STIRRUPS                             IN 5040
C                                                                       IN 5050
      CALL STSTRS(JC,EI,EPSYLD,FN,NSTR)                                 IN 5060
2060  CONTINUE                                                          IN 5070
      IF(NCYCLE.EQ.1.AND.LC.EQ.0) GO TO 81                              IN 5080
      KAM=IBM-IBMO                                                      IN 5090
      IF(KAM.EQ.1.AND.NCYCLE.EQ.1) GO TO 1004                           IN 5100
      IF(KAM.GT.0) GO TO 1002                                           IN 5110
1004  LC=LC+1                                                           IN 5120
      IF(LC.GE.NLC) STOP                                                IN 5130
C     NNDX=1    IS FOR THE OPTION OF COMBINED INC.+TOTAL                IN 5140
      NCYCLE=0                                                          IN 5150
                                                                        IN
                                                                        IN
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```
      NNDX=2                                                            IN 5180
C     JC=1  IF INC+TOTAL PROCDURE IS REQUIRED.                          IN 5190
      IBMO=IBM                                                          IN 5200
      GO TO 122                                                         IN 5210
1002  IF(NCYCLE.GT.65) STOP                                             IN 5220
      JC=0                                                              IN 5230
81    CONTINUE                                                          IN 5240
      IBMO=IBM                                                          IN 5250
      ISEE=2                                                            IN 5260
      NNDX=2                                                            IN 5270
      IF(IBM.GT.0) GOTO 122                                             IN 5280
      IF(IBM.EQ.0) GO TO 1004                                           IN 5290
3     FORMAT(16I5)                                                      IN 5300
   4  FORMAT(1X,'INPUT DATA DESCRIBING THE FINITE ELEMENTS.'//1X,       IN 5310
     1'    NO.   MET   NI    NJ    NK  XI    XJ    XK    YI    YJ    YK  IN 5320
     2         E       U   UNWT   AREA   WIDTH'/)                       IN 5330
5     FORMAT(12F5.2)                                                    IN 5340
6     FORMAT(8F10.0)                                                    IN 5350
8     FORMAT(1X,5I6,6F6.1,1E12.4,1F6.2,1F6.2,2F6.2)                     IN 5360
17    FORMAT(1X,'SIZE OF THE PROBLEM IS. ',I6,'  PROGRAM CAPACITY EXCEED IN 5370
     1ED.')                                                             IN 5380
21    FORMAT(1X,'            LOAD INCREMENT            TOTAL LOAD'/1X'NODE IN 5390
     1 NO.    X FORCE    Y FORCE    X FORCE    Y FORCE'/)               IN 5400
24    FORMAT(1X,'              DEFLECTION INCREMENT              TOTAL DE IN 5410
     1FLECTION'/1X,'NODE NO.    X DEFLECTION    Y DEFLECTION    X DEFLEC IN 5420
     2TION    Y DEFLECTION'/)                                           IN 5430
25    FORMAT(1X,'MATRIX SM IS NOT POSITIVE DEFINITE.  DET=',E15.7)      IN 5440
26    FORMAT(1X,'DETERMINANT IS ZERO.  DET=',E15.7)                     IN 5450
28    FORMAT(6X,'NODE NUMBER    DEGREES OF FREEDOM',//)                 IN 5460
29    FORMAT(1X,'(FIRST LINE GIVES INCREMENT, SECOND LINE GIVES TOTAL.)' IN 5470
     1/1X,'(THIRD LINE GIVES THE TOTAL CHANGE IN STRESS WHEN DESIRED.) ' IN 5480
     2//4X,'NO.XSTRAIN   YSTRAIN  XYSTRAIN  XSTRESS  YSTRESS  XYSTRESS   IN 5490
     3MAXSTRESS  MINSTRESS  MINSTRAIN  MAX XY STRN DIRN      E     ERROR IN 5500
     4 '/)                                                              IN 5510
  59  FORMAT(1X,'JOINT ELEMENT STRAINS AND STRESSES'/7X,'   N  STRAIN   IN 5520
     1S  STRAIN   N  STRESS   S  STRESS')                               IN 5530
61    FORMAT(12I5)                                                      IN 5540
62    FORMAT(3I5,6F10.0)                                                IN 5550
  70  FORMAT(1X,' NODE NUMBERS OF JOINT ELEMENTS',//,12X,'NI   NJ   NK  IN 5560
     1 NL',//)                                                          IN 5570
  71  FORMAT(1X,5I6)                                                    IN 5580
  72  FORMAT(1X,'JOINT CONSTANTS',//,6X,'CSS1= ',E12.5,6X,'CNN = ',E12.5 IN 5590
     1,6X,'CSSF = ',E12.5,'CSS2=',E12.5,'CSS3=',E12.5,3I5)              IN 5600
  73  FORMAT(//,1X,'COORDINATES OF JOINT NODES',//,9X,'XXI   YII   XJJ  IN 5610
     1 YJJ'/)                                                           IN 5620
75    FORMAT(1X,'ITERATION NUMBER',I5,/,1X,23('-'))                     IN 5630
76    FORMAT(I15,4F10.2)                                                IN 5640
78    FORMAT(3I5)                                                       IN 5650
79    FORMAT(2X,3I10)                                                   IN 5660
 999  FORMAT(//,1X,'HALF BAND WIDTH = ',I6,/,1X,'TOTAL UNKNOWNS = ',I6) IN 5670
1003  WRITE(6,60) NNDX                                                  IN 5680
2070  FORMAT(1X,'STIRRUP STRAINS AND SRESSES',/,1X,29('-'),/,4X,        IN 5690
     6'ST. NO.   STRAIN INC  STRESS INC  TOTAL STRAIN  TOTAL STRESS  ANG IN 5700
     6LE OF INCL.',/)                                                   IN 5710
60    FORMAT(2X,I5)                                                     IN 5720
      STOP                                                              IN 5730
      END                                                               IN 5740
```

```
      SUBROUTINE GEOM(XI,XJ,XK,YI,YJ,YK,PT,A,ARR,J)                     GEO  10
      IMPLICIT REAL*8(A-H,O-Z)                                          GEO  20
      DIMENSIONPT(9)                                                    GEO  30
      A=(XI-XJ)*(YI-YK)-(XI-XK)*(YI-YJ)                                 GEO  40
      ARR= DABS(A)                                                      GEO  50
      PT( 1 )=XJ*YK-XK*YJ                                               GEO  60
      PT( 2 )=XK*YI-XI*YK                                               GEO  70
      PT( 3 )=XI*YJ-XJ*YI                                               GEO  80
      PT( 4 )=YJ-YK                                                     GEO  90
      PT( 5 )=YK-YI                                                     GEO 100
      PT( 6 )=YI-YJ                                                     GEO 110
      PT( 7 )=XK-XJ                                                     GEO 120
      PT( 8 )=XI-XK                                                     GEO 130
      PT( 9 )=XJ-XI                                                     GEO 140
      RETURN                                                            GEO 150
      END                                                               GEO 160
```

```
      SUBROUTINE CNODE(J,ND,NP,NB,NU ,FG,NI,NJ,NK,ARW)                  CNO  10
      IMPLICIT REAL*8(A-H,O-Z)                                          CNO  20
      DIMENSION FG(1)                                                   CNO  30
      DIMENSION ND(562,2),NP(6)                                         CNO  40
      IF(J.NE. 1) GO TO 13                                              CNO  50
      NB = 0                                                            CNO  60
      DO 12 IJ=1,NU                                                     CNO  70
      FG(IJ)=0.0                                                        CNO  80
12    CONTINUE                                                          CNO  90
   13 CONTINUE                                                          CNO 100
      NP(1) = ND(NI,1)                                                  CNO 110
      NP(2) = ND(NJ,1)                                                  CNO 120
      NP(3) = ND(NK,1)                                                  CNO 130
      NP(4) = ND(NI,2)                                                  CNO 140
      NP(5) = ND(NJ,2)                                                  CNO 150
      NP(6) = ND(NK,2)                                                  CNO 160
C     COMPUTE NODAL LOADS DUE TO GRAVITY.                               CNO 170
      IF(ND(NI ,2).EQ.0)GO TO 10                                        CNO 180
      IK=ND(NI ,2)                                                      CNO 190
      FG(IK)=FG(IK)-ARW                                                 CNO 200
10    CONTINUE                                                          CNO 210
      IF(ND(NJ ,2).EQ.0)GO TO 11                                        CNO 220
      IK=ND(NJ ,2)                                                      CNO 230
      FG(IK)=FG(IK)-ARW                                                 CNO 240
11    CONTINUE                                                          CNO 250
      IF(ND(NK ,2).EQ.0)GO TO 25                                        CNO 260
      IK = ND(NK,2)                                                     CNO 270
      FG(IK)=FG(IK)-ARW                                                 CNO 280
25    CONTINUE                                                          CNO 290
      MAX=0                                                             CNO 300
      MIN=3000                                                          CNO 310
      DO 5 KK=1,6                                                       CNO 320
      IF(NP(KK).EQ.0) GO TO 5                                           CNO 330
      IF(NP(KK)-MAX)6,6,7                                               CNO 340
7     MAX=NP(KK)                                                        CNO 350
6     IF(NP(KK)-MIN)8,5,5                                               CNO 360
8     MIN=NP(KK)                                                        CNO 370
5     CONTINUE                                                          CNO 380
      NB1=MAX-MIN                                                       CNO 390
      IF(NB1.GT.NB) NB=NB1                                              CNO 400
      RETURN                                                            CNO 410
      END                                                               CNO 420
```

```
      SUBROUTINE FSM (P,S,R,PT,E,U,ARR,WIDTH)                          FSM  10
      IMPLICIT REAL*8(A-H,O-Z)                                         FSM  20
      DIMENSION P(6,6),S(6,6),R(6,6),PT(9)                             FSM  30
      DO 1 II=1,6                                                      FSM  40
      DO 1 JJ=1,6                                                      FSM  50
      S(II,JJ)=0.0                                                     FSM  60
      P(II,JJ)=0.0                                                     FSM  70
1     CONTINUE                                                         FSM  80
      DO 110 II=1,3                                                    FSM  90
      DO 110 JJ=1,3                                                    FSM 100
      KK=3*(II-1)+JJ                                                   FSM 110
      P(II,JJ)=PT(   KK)                                               FSM 120
      MJJ=II+3                                                         FSM 130
      NJJ=JJ+3                                                         FSM 140
      P(MJJ,NJJ)=P(II,JJ)                                              FSM 150
110   CONTINUE                                                         FSM 160
      EU=E*WIDTH/(1.0-U**2)                                            FSM 170
      S(2,2)=EU                                                        FSM 180
      S(3,3)=EU*(1.0-U)/2.0                                            FSM 190
      S(5,5)=S(3,3)                                                    FSM 200
      S(6,6)=S(2,2)                                                    FSM 210
      S(2,6)=EU*U                                                      FSM 220
      S(6,2)=S(2,6)                                                    FSM 230
      S(3,5)=S(3,3)                                                    FSM 240
      S(5,3)=S(3,3)                                                    FSM 250
      AR=4.*ARR                                                        FSM 260
      DO 4 JJ=1,6                                                      FSM 270
      DO 4 II=1,6                                                      FSM 280
      R(JJ,II)=0.0                                                     FSM 290
      DO 4 KK=1,6                                                      FSM 300
      R(JJ,II)=R(JJ,II)+S(JJ,KK)*P(KK,II)                              FSM 310
4     CONTINUE                                                         FSM 320
      DO 5 JJ=1,6                                                      FSM 330
      DO 5 II=1,6                                                      FSM 340
      S(JJ,II)=0.0                                                     FSM 350
      DO 5 KK=1,6                                                      FSM 360
      S(JJ,II)=S(JJ,II)+P(KK,JJ)*R(KK,II)/AR                           FSM 370
5     CONTINUE                                                         FSM 380
      RETURN                                                           FSM 390
      END                                                              FSM 400
```

```
      SUBROUTINE NLOAD(ND,FC,FF ,NU,NGI)                                NLO  10
      IMPLICIT REAL*8(A-H,O-Z)                                          NLO  20
      DIMENSION FF(2),FC(1),ND(562,2)                                   NLO  30
      IF (NGI .EQ. 1) GO TO 21                                          NLO  40
      DO 20 KK = 1,NU                                                   NLO  50
      FC(KK) = 0.0                                                      NLO  60
   20 CONTINUE                                                          NLO  70
   21 CONTINUE                                                          NLO  80
      READ(5,3) NNL                                                     NLO  90
      DO  2 II = 1,NNL                                                  NLO 100
      READ(5,3) JNU,FF(1),FF(2)                                         NLO 110
      DO 6 JJ=1,2                                                       NLO 120
      IF(ND(JNU,JJ).EQ.0) GO TO 6                                       NLO 130
      IK = ND(JNU,JJ)                                                   NLO 140
      FC(IK) = FC(IK)+FF(JJ)                                            NLO 150
    6 CONTINUE                                                          NLO 160
    2 CONTINUE                                                          NLO 170
3     FORMAT(I15,2F10.0 )                                               NLO 180
      RETURN                                                            NLO 190
      END                                                               NLO 200
```

```
      SUBROUTINE CNLP(F,FN,ND,NN,NNPR,IPP)                              CNL  10
      IMPLICIT REAL*8(A-H,O-Z)                                          CNL  20
      DIMENSION  F(1),FN(1),ND(562,2),NNPR(1)                           CNL  30
      DO 1 JJ=1,NN                                                      CNL  40
      IF(ND(JJ,1).NE.0) GO TO 4                                         CNL  50
      FX1=0.0                                                           CNL  60
      FX2=0.0                                                           CNL  70
      GO TO 5                                                           CNL  80
4     IK=ND(JJ,1)                                                       CNL  90
      FX1=FN(IK)                                                        CNL 100
      FX2=F(IK)                                                         CNL 110
5     CONTINUE                                                          CNL 120
      IF(ND(JJ,2).NE.0) GO TO 6                                         CNL 130
      FY1=0.0                                                           CNL 140
      FY2=0.0                                                           CNL 150
      GO TO 7                                                           CNL 160
6     IK=ND(JJ,2)                                                       CNL 170
      FY1=FN(IK)                                                        CNL 180
      FY2=F(IK)                                                         CNL 190
7     CONTINUE                                                          CNL 200
      IF(FX1.NE.0.0.OR.FY1.NE.0.0) GO TO 3                              CNL 210
      IF(FX2.NE.0.0.OR.FY2.NE.0.0) GO TO 3                              CNL 220
      GO TO 1                                                           CNL 230
3     CONTINUE                                                          CNL 240
      WRITE(6,8) JJ,FX1,FY1,FX2,FY2                                     CNL 250
      WRITE(3,8) JJ,FX1,FY1,FX2,FY2                                     CNL 260
1     CONTINUE                                                          CNL 270
8     FORMAT(1X,I6,4F11.2)                                              CNL 280
      RETURN                                                            CNL 290
      END                                                               CNL 300
```

```
      SUBROUTINE JSTIFF(SJ,JM,KK,ND,XI, XJ ,YI ,YJ ,CSJ,CNN,NB,NJE,CSSF  JST  10
C                                                                        JST  20
C       COMPUTE JOINT ELEMENT STIFFNESSES                                JST  30
C                                                                        JST  40
     8,NANCH,NEAN)                                                       JST  50
      IMPLICIT REAL*8(A-H,O-Z)                                           JST  60
      COMMON SM(58000)                                                   JST  70
      DIMENSION XI(1),XJ(1),YI(1),YJ(1),KK(1),NP(8),SJ(    8,8),JM(200,4 JST  80
     8 ),ND(562,2),NEAN(1),CSJ(1)                                        JST  90
      DO 1 I=1,NJE                                                       JST 100
      CSS=CSJ(I)                                                         JST 110
      DL = ((XJ(I)-XI(I))**2+ (YJ(I)-YI(I))**2)**.5                      JST 120
      A = (XJ(I)-XI(I))/DL                                               JST 130
      B = (YJ(I)-YI(I))/DL                                               JST 140
      CSSB=CSS                                                           JST 150
      CNNB=CNN                                                           JST 160
      IF(KK(I).EQ.2.OR.KK(I).EQ.4) CNNB=0.0001*CNN                       JST 170
      IF(KK(I).GT.2) CSSB=0.0001*CSS                                     JST 180
      A1=(CSSB*A**2+CNNB*B**2)*DL/6.                                     JST 190
      A2=(CSSB*B**2+CNNB*A**2)*DL/6.                                     JST 200
      A3=(CNNB-CSSB)*A*B*DL/6.                                           JST 210
C     ANCORD JOINTS ARE GIVEN HIGH STIFFNESS VALUES                      JST 220
      DO 22 JL=1,NANCH                                                   JST 230
      IF(I.NE.NEAN(JL)) GO TO 22                                         JST 240
      A1=50.*A1                                                          JST 250
      A2=50.*A2                                                          JST 260
      A3=50.*A3                                                          JST 270
22    CONTINUE                                                           JST 280
      SJ(  1,1) = 2.*A1                                                  JST 290
      SJ(  2,1) = A1                                                     JST 300
      SJ(  2,2) = 2.*A1                                                  JST 310
      SJ(  3,1) = -1.*A1                                                 JST 320
      SJ(  3,2) = -2*A1                                                  JST 330
      SJ(  3,3) = 2.*A1                                                  JST 340
      SJ(  4,1) = -2.*A1                                                 JST 350
      SJ(  4,2) = -1.*A1                                                 JST 360
      SJ(  4,3) = A1                                                     JST 370
      SJ(  4,4) = 2.*A1                                                  JST 380
      SJ(  5,1) = -2.*A3                                                 JST 390
      SJ(  5,2) = -1.*A3                                                 JST 400
      SJ(  5,3) = A3                                                     JST 410
      SJ(  5,4) = 2.*A3                                                  JST 420
      SJ(  5,5) = 2.*A2                                                  JST 430
      SJ(  6,1) = -1.*A3                                                 JST 440
      SJ(  6,2) = -2.*A3                                                 JST 450
      SJ(  6,3) = 2.*A3                                                  JST 460
      SJ(  6,4) = A3                                                     JST 470
      SJ(  6,5) = A2                                                     JST 480
      SJ(  6,6) = 2.*A2                                                  JST 490
      SJ(  7,1) = A3                                                     JST 500
      SJ(  7,2) = 2.*A3                                                  JST 510
      SJ(  7,3) = -2.*A3                                                 JST 520
      SJ(  7,4) = -1.*A3                                                 JST 530
      SJ(  7,5) = -1.*A2                                                 JST 540
      SJ(  7,6) = -2.*A2                                                 JST 550
      SJ(  7,7) = 2.*A2                                                  JST 560
      SJ(  8,1) = 2.*A3                                                  JST 570
      SJ(  8,2) = A3                                                     JST 580
      SJ(  8,3) = -1.*A3                                                 JST 590
      SJ(  8,4) = -2.*A3                                                 JST 600
      SJ(  8,5) = -2.*A2                                                 JST 610
      SJ(  8,6) = -1.*A2                                                 JST 620
      SJ(  8,7) = A2                                                     JST 630
      SJ(  8,8) = 2.*A2                                                  JST 640
      DO 20 J = 1,8                                                      JST 650
      IF(J.EQ.8) GO TO 20                                                JST 660
      JJ = J+1                                                           JST 670
      DO 21 K = JJ,8                                                     JST 680
      SJ(  J,K) = SJ(  K,J)                                              JST 690
   21 CONTINUE                                                           JST 700
   20 CONTINUE                                                           JST 710
      II = JH(I,1)                                                       JST 720
      JJ = JH(I,2)                                                       JST 730
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```
      IL = JM(I,3)                                                    JST 740
      JL = JM(I,4)                                                    JST 750
      NP(1) = ND(II,1)                                                JST 760
      NP(2) = ND(JJ,1)                                                JST 770
      NP(3) = ND(IL,1)                                                JST 780
      NP(4) = ND(JL,1)                                                JST 790
      NP(5) = ND(II,2)                                                JST 800
      NP(6) = ND(JJ,2)                                                JST 810
      NP(7) = ND(IL,2)                                                JST 820
      NP(8) = ND(JL,2)                                                JST 830
      DO 4 JK = 1,8                                                   JST 840
      IF(NP(JK).EQ. 0) GO TO 4                                        JST 850
      DO 5 IK = JK,8                                                  JST 860
      IF(NP(IK) .EQ. 0) GO TO 5                                       JST 870
      IF(NP(JK) - NP(IK)) 9,10,10                                     JST 880
   10 K = (NP(IK)-1)*NB + NP(JK)                                      JST 890
      SM(K) = SM(K) + SJ(  JK,IK)                                     JST 900
      GO TO 5                                                         JST 910
    9 K = (NP(JK)-1)*NB + NP(IK)                                      JST 920
      SM(K) = SM(K) + SJ(  JK,IK)                                     JST 930
    5 CONTINUE                                                        JST 940
    4 CONTINUE                                                        JST 950
    1 CONTINUE                                                        JST 960
      RETURN                                                          JST 970
      END                                                             JST 980
```

```
      SUBROUTINE BAND(A,B,N,M,LT,DET)                                   BAN  10
C                                                                       BAN  20
C     SOLVE THE BANDED STIFF. MATRIX FOR DISPLACEMENTS.                 BAN  30
C                                                                       BAN  40
      IMPLICIT REAL*8(A-H,O-Z)                                          BAN  50
      DIMENSION A(2),B(1)                                               BAN  60
      MM=M-1                                                            BAN  70
      NM=N*M                                                            BAN  80
      NM1=NM-MM                                                         BAN  90
      IF (LT.NE.1) GO TO 55                                             BAN 100
      MP=M+1                                                            BAN 110
      KK=2                                                              BAN 120
      FAC=DET                                                           BAN 130
      A(1)=1./DSQRT(A(1))                                               BAN 140
      BIGL=A(1)                                                         BAN 150
      SML=A(1)                                                          BAN 160
      A(2)=A(2)*A(1)                                                    BAN 170
      A(MP)=1./DSQRT(A(MP)-A(2)*A(2))                                   BAN 180
      IF(A(MP).GT.BIGL)BIGL=A(MP)                                       BAN 190
      IF(A(MP).LT.SML)SML=A(MP)                                         BAN 200
      MP=MP+M                                                           BAN 210
      DO 62 J=MP,NM1,M                                                  BAN 220
      JP=J-MM                                                           BAN 230
      MZC=0                                                             BAN 240
      IF(KK.GE.M) GO TO 1                                               BAN 250
      KK=KK+1                                                           BAN 260
      II=1                                                              BAN 270
      JC=1                                                              BAN 280
      GO TO 2                                                           BAN 290
1     KK=KK+M                                                           BAN 300
      II=KK-NM                                                          BAN 310
      JC=KK-NM                                                          BAN 320
2     DO 65 I=KK,JP,MM                                                  BAN 330
      IF(A(I).EQ.0.)GO TO 64                                            BAN 340
      GO TO 66                                                          BAN 350
64    JC=JC+M                                                           BAN 360
65    MZC=MZC+1                                                         BAN 370
      ASUM1=0.                                                          BAN 380
      GO TO 61                                                          BAN 390
66    MMZC=MM*MZC                                                       BAN 400
      II=II+MZC                                                         BAN 410
      KM=KK+MMZC                                                        BAN 420
      A(KM)=A(KM)*A(JC)                                                 BAN 430
      IF(KM.GE.JP)GO TO 6                                               BAN 440
      KJ=KM+MM                                                          BAN 450
      DO 5 I=KJ,JP,MM                                                   BAN 460
      ASUM2=0.                                                          BAN 470
      IM=I-MM                                                           BAN 480
      II=II+1                                                           BAN 490
      KI=II+MMZC                                                        BAN 500
      DO 7 K=KM,IM,MM                                                   BAN 510
      ASUM2=ASUM2+A(KI)*A(K)                                            BAN 520
7     KI=KI+MM                                                          BAN 530
5     A(I)=(A(I)-ASUM2)*A(KI)                                           BAN 540
6     CONTINUE                                                          BAN 550
      ASUM1=0.                                                          BAN 560
      DO 4 K=KM,JP,MM                                                   BAN 570
4     ASUM1=ASUM1+A(K)*A(K)                                             BAN 580
61    S=A(J)-ASUM1                                                      BAN 590
      IF(S.LT.0.)DET=S                                                  BAN 600
      IF(S.EQ.0.)DET=0.                                                 BAN 610
      IF(S.GT.0.)GO TO 63                                               BAN 620
      NROW=(J+MM)/M                                                     BAN 630
      WRITE(6,99) NROW                                                  BAN 640
99    FORMAT(35HOERROR CONDITION ENCOUNTERED IN ROW,I6)                 BAN 650
      RETURN                                                            BAN 660
63    A(J)=1./DSQRT(S)                                                  BAN 670
      IF(A(J).GT.BIGL)  IROW=(J+MM)/M                                   BAN 680
      IF(A(J).GT.BIGL)BIGL=A(J)                                         BAN 690
      IF(A(J).LT.SML)   IROW=(J+MM)/M                                   BAN 700
      IF(A(J).LT.SML)SML=A(J)                                           BAN 710
62    CONTINUE                                                          BAN 720
      IF(SML.LE.FAC*BIGL)GO TO 54                                       BAN 730
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      GO TO 53                                                          BAN 740
54    DET=0.                                                            BAN 750
      ZIG=SML/BIGL                                                      BAN 760
      WRITE(6,100) ZIG,MROW,IROW                                        BAN 770
100   FORMAT(2X,'SMLL/BIGL=',E15.7,2X,'SMLL AT',I6,2X,'BIGL AT',I6)     BAN 780
      RETURN                                                            BAN 790
53    DET=SML/BIGL                                                      BAN 800
55    B(1)=B(1)*A(1)                                                    BAN 810
      KK=1                                                              BAN 820
      K1=1                                                              BAN 830
      J=1                                                               BAN 840
      DO 8 L=2,N                                                        BAN 850
      BSUM1=0.                                                          BAN 860
      LM=L-1                                                            BAN 870
      J=J+M                                                             BAN 880
      IF(KK.GE.M)GO TO 12                                               BAN 890
      KK=KK+1                                                           BAN 900
      GO TO 13                                                          BAN 910
12    KK=KK+M                                                           BAN 920
      K1=K1+1                                                           BAN 930
13    JK=KK                                                             BAN 940
      DO 9 K=K1,LM                                                      BAN 950
      BSUM1=BSUM1+A(JK)*B(K)                                            BAN 960
      JK=JK+NM                                                          BAN 970
9     CONTINUE                                                          BAN 980
8     B(L)=(B(L)-BSUM1)*A(J)                                            BAN 990
      B(N)=B(N)*A(NM1)                                                  BAN1000
      NMM=NM1                                                           BAN1010
      NN=N-1                                                            BAN1020
      ND=N                                                              BAN1030
      DO 10 L=1,NN                                                      BAN1040
      BSUM2=0.                                                          BAN1050
      NL=N-L                                                            BAN1060
      NL1=N-L+1                                                         BAN1070
      NMM=NMM-M                                                         BAN1080
      NJ1=NMM                                                           BAN1090
      IF(L.GE.M)ND=ND-1                                                 BAN1100
      DO 11 K=NL1,ND                                                    BAN1110
      NJ1=NJ1+1                                                         BAN1120
      BSUM2=BSUM2+A(NJ1)*B(K)                                           BAN1130
11    CONTINUE                                                          BAN1140
10    B(NL)=(B(NL)-BSUM2)*A(NMM)                                        BAN1150
      RETURN                                                            BAN1160
      END                                                               BAN1170
```

```
      SUBROUTINE CNDI (FN,ND,NN,DX,DY,KOW,JC)                          CND  10
C                                                                      CND  20
C     PRINTOUT NODAL DISPLACEMENTS.                                    CND  30
C                                                                      CND  40
      IMPLICIT REAL*8(A-H,O-Z)                                         CND  50
      DIMENSION FN(1),DX(1),DY(1),ND(562,2)                            CND  60
      DIMENSION KOW(1)                                                 CND  70
      NEK=1                                                            CND  80
      DO 1 JJ=1,NN                                                     CND  90
      IF(ND(JJ,1).NE.0) GO TO 5                                        CND 100
      DXX=0.0                                                          CND 110
      GO TO 4                                                          CND 120
5     IK=ND(JJ,1)                                                      CND 130
      DXX = FN(IK)                                                     CND 140
4     CONTINUE                                                         CND 150
      IF(ND(JJ,2).NE.0) GO TO 6                                        CND 160
      DYY=0.0                                                          CND 170
      GO TO 9                                                          CND 180
6     IK=ND(JJ,2)                                                      CND 190
      DYY = FN(IK)                                                     CND 200
9     CONTINUE                                                         CND 210
      IF(JC .NE. 0) GO TO 100                                          CND 220
      DX(JJ) = 0.                                                      CND 230
      DY(JJ) = 0.                                                      CND 240
  100 CONTINUE                                                         CND 250
      DX(JJ)=DX(JJ)+DXX                                                CND 260
      DY(JJ)=DY(JJ)+DYY                                                CND 270
      KOWW=KOW(NEK)                                                    CND 280
      IF(JJ.NE.KOWW) GOTO 1                                            CND 290
      NEK=NEK+1                                                        CND 300
      WRITE(6,8) JJ,DXX,DYY,DX(JJ),DY(JJ)                              CND 310
8     FORMAT(1X,I8,4(1PE16.5))                                         CND 320
    1 CONTINUE                                                         CND 330
      RETURN                                                           CND 340
      END                                                              CND 350
```

```
      SUBROUTINE CESS(FN,EX,EY,EXY,SEX,SEY,SEXY,NE,JC,UI,IPP,FC,ND,        CES  10
     1 WIDE,NN,IBM,MFLAG,FTENS,EPSC,EPSYLD,C1,C2,C3,EPSO,FACTOR,EC,BETA    CES  20
     8 ,INDC)                                                              CES  30
      IMPLICIT REAL*8(A-H,O-Z)                                             CES  40
      COMMON SM(58000)                                                     CES  50
      DIMENSION FN(1),EX(1),EY(1),EXY(1),SEX(1),SEY(1),SEXY(1),UI(6)       CES  60
     3,NIJ(3)   ,MFLAG(1)   ,FACTOR(818,2) ,INDC(1)                       CES  70
      DIMENSION  NP(6),PT(9),S(6,6) ,WIDE(1),FC(1),FF(2),BETA(1)           CES  80
      DIMENSION SIXYN(1)                                                   CES  90
      DIMENSION IFLG(100)                                                  CES 100
      COMMON/BLK7/RATIO,NSWICH,IBM1,NT1,JOB,NCYCLE                         CES 110
      XMAX=0.0                                                             CES 120
      ZMAX=0.0                                                             CES 130
      IKOUNT=1                                                             CES 140
      DO 202 JAN=1,100                                                     CES 150
  202 IFLG(JAN)=0                                                          CES 160
      DO 1 KJ=1,NE                                                         CES 170
      IM=0                                                                 CES 180
      J=KJ                                                                 CES 190
      READ(NT1)(NP(I),I=1,6),((S(MM,MN),MN=1,6),MM=1,6),(PT(JKL),JKL=1,9   CES 200
     1  ),A,E,U,NI,NJ,NK,ARR                                               CES 210
C                                                                          CES 220
C     FIND AVERAGE STRESSES AND STRAINS IN ELEMENTS                        CES 230
C                                                                          CES 240
      SEX(J)=(SM(4000+NI)+SM(4000+NJ)+SM(4000+NK))/3.0                     CES 250
  800 CONTINUE                                                             CES 260
      SEY(J)=(SM(5000+NI)+SM(5000+NJ)+SM(5000+NK))/3.0                     CES 270
  801 CONTINUE                                                             CES 280
      SEXY(J)=(SM(6000+NI)+SM(6000+NJ)+SM(6000+NK))/3.0                    CES 290
  802 CONTINUE                                                             CES 300
      IF(MFLAG(J).EQ.1) SEXY(J)=0.0                                        CES 310
      EU=E*FACTOR(J,2)/(1.0-U**2)                                          CES 320
      EX(J)=(SEX(J)-U*SEY(J))/(EU*(1.0-U**2))                              CES 330
  803 CONTINUE                                                             CES 340
      EY(J)=(SEY(J)-U*SEX(J))/(EU*(1.0-U**2))                              CES 350
  804 CONTINUE                                                             CES 360
      EXY(J)=2.0*SEXY(J)/(EU*(1.0-U))                                      CES 370
  805 CONTINUE                                                             CES 380
      IF(U.GT.0.21) GO TO 113                                              CES 390
      E1=E*FACTOR(J,2)                                                     CES 400
  113 CONTINUE                                                             CES 410
      XS=EX(J)                                                             CES 420
      YS=EY(J)                                                             CES 430
      XYS=EXY(J)                                                           CES 440
      IF(JC .NE. 0) GO TO 20                                               CES 450
      EX(J) = 0.                                                           CES 460
      EY(J) = 0.                                                           CES 470
      EXY(J) = 0.                                                          CES 480
   20 CONTINUE                                                             CES 490
      EX(J)=EX(J)+XS                                                       CES 500
      EY(J) = EY(J) + YS                                                   CES 510
      EXY(J)=EXY(J)+XYS                                                    CES 520
      CALL SMAX(XS,YS,XYS,EM1,EM2,EM3,EM4)                                 CES 530
      IF(EM1.GT.0.0.AND.EM2.GT.0.0) GO TO 50                               CES 540
      EM1=EM1                                                              CES 550
      IF(EM2.LT.EM1) EM1=EM2                                               CES 560
   50 CONTINUE                                                             CES 570
      CALL SMAX(EX(J),EY(J),EXY(J),EM1,EM2,EM3,EM4)                        CES 580
      FACTOR(J,1)=1.0                                                      CES 590
      FACTOR(J,2)=1.0                                                      CES 600
      IF(U.GT..21) GO TO 139                                               CES 610
      IF(EM1.GT.0.0.AND.EM2.GT.0.0) GO TO 129                              CES 620
      EM=EM1                                                               CES 630
      IF(EM2.LT.EM1)EM=EM2                                                 CES 640
   51 IF(EM.GT.0.0) GO TO 119                                              CES 650
      IF(U.GT..21.AND.IM.GT.0) GO TO 1                                     CES 660
C                                                                          CES 670
C     FIND NEW TANGENT AND SECANT MODULII FOR CONC.USING SAENZ'S EQN.      CES 680
C                                                                          CES 690
      EMR=DABS(EM/EPSO)                                                    CES 700
      FACT      =(1.0+C1*(EMR**2)-2.0*C2*(EMR**3))/((1.0+C3*EMR-C1*(EMR*   CES 710
     8*2)+C2*(EMR**3))**2)                                                 CES 720
      IF(FACT.LE.0.0) FACT=.01                                             CES 730
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```
      FACTOR(J,1)=FACT                                                  CES 740
      BACT       =1.0/(1.0+C3*EMR-C1*EMR**2+1.0*C2*EMR**3)              CES 750
      FACTOR(J,2) =BACT                                                 CES 760
129   IF(U.LT..21) GO TO 119                                            CES 770
139   AM=DABS(EM1)                                                      CES 780
      BM=DABS(EM2)                                                      CES 790
      CM=AM                                                             CES 800
      IF(BM.GT.CM) CM=BM                                                CES 810
C                                                                       CES 820
C     INCLUDE STRAIN HARDENING EFFECTS FOR STEEL ELEMS.                 CES 830
C                                                                       CES 840
      IF(CM-EPSYLD) 120,120,121                                         CES 850
120   FACTOR(J,1)=1.0                                                   CES 860
      FACTOR(J,2)=1.0                                                   CES 870
      GO TO 119                                                         CES 880
121   FACTOR(J,1)=0.056                                                 CES 890
      CKFM=0.30E+08*EPSYLD                                              CES 900
      CKOC=0.168E+07*(CM-EPSYLD)                                        CES 910
      FACTOR(J,2)=(CKFM+CKOC)/(CM*E)                                    CES 920
119   CONTINUE                                                          CES 930
      IF(IM.GT.0) GO TO 1                                               CES 940
      E=E*FACTOR(J,2)                                                   CES 950
      IF(U.GT..21) E1=E                                                 CES 960
      EROR=(E1-E)/E1                                                    CES 970
      IF(MFLAG(KJ).GT.0) EROR=0.                                        CES 980
      SIX=SEX(J)*E/E1                                                   CES 990
      SIY=SEY(J)*E/E1                                                   CES1000
      SIXY=SEXY(J)*E/E1                                                 CES1010
      IF(MFLAG(J).NE.1) GO TO 300                                       CES1020
      IF(INDC(J).EQ.3 ) GO TO 302                                       CES1030
      BATA=BETA(J)                                                      CES1040
C                                                                       CES1050
C     FOR CRACKED ELEMENTS CALL STRESS TO COMPUTE CORRESSPONDING        CES1060
C     STRESSES AND STRAINS.                                             CES1070
C                                                                       CES1080
      CALL STRESS(XS,YS,XYS,SIX,SIY,SIXY,BATA,E,U)                      CES1090
      GO TO 300                                                         CES1100
302   SIX=0.                                                            CES1110
      SIY=0.                                                            CES1120
      SIXY=0.                                                           CES1130
300   IF(JC .NE. 0) GO TO 30                                            CES1140
      SEX(J) = 0.                                                       CES1150
      SEY(J) = 0.                                                       CES1160
      SEXY(J) = 0.                                                      CES1170
  30  CONTINUE                                                          CES1180
      SEX(J)=SEX(J)+SIX                                                 CES1190
      SEY(J)=SEY(J)+SIY                                                 CES1200
      SEXY(J)=SEXY(J)+SIXY                                              CES1210
C     CALL SMAX(SIX,SIY,SIXY,XM1,XM2,XM3,XM4)                           CES1220
C     WRITE(3,5)KJ,XS,YS,XYS,SIX,SIY,SIXY,XM1,XM2,XM3,XM4               CES1230
      CALL SMAX(SEX(J),SEY(J),SEXY(J),XM1,XM2,XM3,XM4)                  CES1240
      IF(MFLAG(J).EQ.1)  GO TO 301                                      CES1250
      BETA(J)=XM4*.01745329                                             CES1260
301   IF(MFLAG(KJ).EQ.0) GO TO 31                                       CES1270
      IF(NSWICH.EQ.0) GO TO 9                                           CES1280
      WRITE(3,16) MFLAG(KJ)                                             CES1290
31    CONTINUE                                                          CES1300
      IF(IPP.EQ.1) GOTO 707                                             CES1310
      WRITE(3,6) KJ,EX(J),EY(J),EXY(J),SEX(J),SEY(J),SEXY(J),XM1,XM2,XM2 CES1320
     $ ,XM3,XM4,E ,EROR                                                 CES1330
707   CONTINUE                                                          CES1340
C                                                                       CES1350
C     CHECK FAILURE CRITERIA FOR CONCRETE AND STELL ELEMENTS.           CES1360
C                                                                       CES1370
      MER=0                                                             CES1380
C     IF(MFLAG(J).EQ.1.AND.EX(J).LT.0.0.AND.EY(J).LT.0.0)GO TO 91       CES1390
      IF(MFLAG(J).GT.0) GO TO 9                                         CES1400
      IF(U-0.21) 3,3,4                                                  CES1410
3     IF(XM1.GT.FTENS.OR.XM2.GT.FTENS) MER=1                            CES1420
      IF(EM1.LT.-0.003.OR.EM2.LT.-.003) MER=2                           CES1430
      GO TO 8                                                           CES1440
4     IF(EM1  .GT.EPSYLD.OR.EM2  .GT.EPSYLD) MER=3                      CES1450
8     IF(MER.EQ.0) GO TO 22                                             CES1460
      IF(MER-2) 10,11,12                                                CES1470
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C       WRITE(3,13)                                                     CES1480
10      CONTINUE                                                        CES1490
        IFLG(IKOUNT)=J                                                  CES1500
        IKOUNT=IKOUNT+1                                                 CES1510
        IF(IKOUNT.GT.100) GO TO 200                                     CES1520
        GO TO 19                                                        CES1530
11      WRITE(3,14) J,EM1,EM2                                           CES1540
        GO TO 19                                                        CES1550
12      WRITE(3,15) J,EM1,EM2                                           CES1560
19      IBM=IBM+1                                                       CES1570
22      MFLAG(KJ)=MER                                                   CES1580
        IBM1=IBM                                                        CES1590
        GOTO 123                                                        CES1600
91      MFLAG(KJ)=0                                                     CES1610
        WRITE(3,17)                                                     CES1620
        IBM1=IBM1-1                                                     CES1630
C       CALL        ELEMLD(WIDE,PT,SEX,SEY,SEXY,NI,NJ,NK,ND,FC,J      ) CES1640
        GO TO 123                                                       CES1650
9       CONTINUE                                                        CES1660
        IF(MFLAG(J).EQ.1.AND.XM1.GT.FTENS) XM1=0.0                      CES1670
        IF(MFLAG(J).EQ.1.AND.XM2.GT.FTENS) XM2=0.0                      CES1680
        IF(MFLAG(J).EQ.1.AND.EM1.LT.-.0038) GO TO 122                   CES1690
        IF(MFLAG(J).EQ.1.AND.EM2.LT.-.0038) GO TO 122                   CES1700
        GO TO 123                                                       CES1710
122     INDC(J)=3                                                       CES1720
        WRITE(3,13)J,XM1,XM2,XM4                                        CES1730
        IFLG(IKOUNT)=J                                                  CES1740
        IKOUNT=IKOUNT+1                                                 CES1750
        IBM=IBM+1                                                       CES1760
123     CONTINUE                                                        CES1770
        IF(IBM1.GE.NE) STOP                                             CES1780
13      FORMAT(5X,'NEW TENSILE CRACK')                                  CES1790
14      FORMAT(5X,'COMPRESSIVE   FAILURE',I5,2E20.7)                    CES1800
15      FORMAT(5X,'STEEL YIELDED',I5,2E20.7)                            CES1810
16      FORMAT(5X,'ELEMENT FAILED PREVIOUSLY,TYPE OF FAILURE=',I3)      CES1820
17      FORMAT(5X,'CRACK IS CLOSING')                                   CES1830
18      FORMAT(5X,'ELEMENT CRACKED IN TWO DIRECTIONS',I5,2E20.7,2X,F6.1) CES1840
        NIJ(1)=NI                                                       CES1850
        NIJ(2)=NJ                                                       CES1860
        NIJ(3)=NK                                                       CES1870
        IM=1                                                            CES1880
        IF(U.GT..21) GO TO 1                                            CES1890
        FIS=0.7                                                         CES1900
        IF (JC.EQ.0) FIS=0.1                                            CES1910
        EM=EM+FIS*EM1                                                   CES1920
        IF(MFLAG(J).EQ.1) GOTO 149                                      CES1930
        IF(XM1.LE.XMAX) GO TO 159                                       CES1940
        XMAX=XM1                                                        CES1950
        JOB=J                                                           CES1960
        GO TO 159                                                       CES1970
149     CONTINUE                                                        CES1980
        IF(XM1.LT.ZMAX) GO TO 159                                       CES1990
        ZMAX=XM1                                                        CES2000
        JAK=J                                                           CES2010
159     GO TO 51                                                        CES2020
  1     CONTINUE                                                        CES2030
        IF(XMAX.LT.FTENS) GOTO 610                                      CES2040
        WRITE(3,611) XMAX                                               CES2050
611     FORMAT(2X,'ERROR IN MAX TENSILE STRESS IN THE FIELD',F10.5)     CES2060
610     RATIO=FTENS/XMAX                                                CES2070
        MFLAG(JOB)=1                                                    CES2080
        JEB=JOB                                                         CES2090
        WRITE(3,21) JOB,RATIO                                           CES2100
21      FORMAT(2X,'ELEMENT NO.',I5,'IS PREDICTED TO CRACK AT',F5.2,'TIMES CES2110
     $ THE APPLIED LOAD',/,2X,'IF ANOTHER CYCLE OF ITERATION IS REQUIRED CES2120
     6 THIS PREDICTION IS VOID',/)                                      CES2130
        DO 201 JAN=1,100                                                CES2140
        KI=IFLG(JAN)                                                    CES2150
        IF(KI.LE.0) GO TO 201                                           CES2160
        IF(KI.EQ.JAK) GO TO 204                                         CES2170
        WRITE(3,203) KI                                                 CES2180
        MFLAG(KI) =0                                                    CES2190
        IF(INDC(KI).EQ.3) MFLAG(KI)=1                                   CES2200
        IBM1=IBM1-1                                                     CES2210
```

```
      GO TO 201                                                          CES2220
204   CONTINUE                                                           CES2230
      IF(INDC(KI).EQ.3) FACTOR(KI,1)=0.001                               CES2240
      IF(INDC(KI).EQ.3) FACTOR(KI,2)=0.001                               CES2250
      ANGLE =BETA(KI)/.01745329                                          CES2260
      WRITE(3,205) KI,ANGLE                                              CES2270
205   FORMAT(1X,'XXXXX ELEM. NO.',I5,'IS CRACKED NOWXXXXX ANGLE OF CRACK CES2280
     $=',F6.1)                                                           CES2290
203   FORMAT(2X,'CRACK IN ELEMENT NO.',I5,'IS POSTPONED FOR NOW')        CES2300
201   CONTINUE                                                           CES2310
      IBM=IBM1                                                           CES2320
      WRITE(3,609) IBM1                                                  CES2330
609   FORMAT(1X,'TOTAL NUMBER OF CRACKED ELEMENTS=',I5,/,1X,40('-'))     CES2340
      REWIND NT1                                                         CES2350
5     FORMAT(1X,I4,6E12.4,3E13.4,F6.1)                                   CES2360
6     FORMAT(1X,I4,10E10.3,F6.1,E10.3,F10.4)                             CES2370
1681  FORMAT(1X,40X,3E12.4)                                              CES2380
      RETURN                                                             CES2390
200   WRITE(3,612)                                                       CES2400
612   FORMAT(5X,'ERROR--- NO. OF CRKD ELEMS AT THIS CYCLE.GT.100',/)     CES2410
      STOP                                                               CES2420
      END                                                                CES2430
```

```
      SUBROUTINE SMAX(XM,YM,XYM,XM1,XM2,XM3,XM4)                        SMA  10
C                                                                       SMA  20
C     FINDS PRINCIPAL STRAINS OR STRESSES IN ANY ELEMENT.               SMA  30
C                                                                       SMA  40
      IMPLICIT REAL*8(A-H,O-Z)                                          SMA  50
      XM1=.5*((XM+YM)+ DSQRT((XM-YM)**2+4.*XYM**2))                     SMA  60
      XM2=.5*((XM+YM)- DSQRT((XM-YM)**2+4.*XYM**2))                     SMA  70
      XM3=0.5*(XM1-XM2)                                                 SMA  80
      IF( DABS(XYM).GT.0.00001) GO TO 1                                 SMA  90
      XM4=0.00                                                          SMA 100
      XM4=XM4+90.                                                       SMA 110
      RETURN                                                            SMA 120
1     DIR= DATAN2(2.*XYM,XM-YM)                                         SMA 130
      XM4=(0.5/0.0174533)*DIR                                           SMA 140
      XM4=XM4+90.                                                       SMA 150
      RETURN                                                            SMA 160
      END                                                               SMA 170
```

```
      SUBROUTINE JSTRES  (JM,DX,DY,XI,YI,XJ,YJ,KK,NJE,CSS1,CNN,CSSF,CSJ) JST  10
C                                                                         JST  20
C     FINDS JOINT ELEMENT STRESSES AND DISPL.                             JST  30
C                                                                         JST  40
      IMPLICIT REAL*8(A-H,O-Z)                                            JST  50
      DIMENSIONKK(1),JM(200,4),XI(1),XJ(1),YI(1),YJ(1),DX(1),DY(1)        JST  60
     $ ,CSJ(1) ,ENS1(200)                                                 JST  70
      COMMON/BLK1/ENS1                                                    JST  80
      COMMON/BLOK3/ DIA1,DIA2,DIA3,N1,N2,N3                               JST  90
      COMMON/BLK9/IPP                                                     JST 100
      FTENS=600.                                                          JST 110
      FCOMP=-5000.0                                                       JST 120
      DO 2 I=1,NJE                                                        JST 130
      CSS=CSJ(I)                                                          JST 140
      IT = JM(I,4)                                                        JST 150
      JT = JM(I,3)                                                        JST 160
      IB = JM(I,1)                                                        JST 170
      JB = JM(I,2)                                                        JST 180
      DL = ((XJ(I)-XI(I))**2+ (YJ(I)-YI(I))**2)**.5                       JST 190
      A = (XJ(I)-XI(I))/DL                                                JST 200
      B = (YJ(I)-YI(I))/DL                                                JST 210
      UXI     =           DX(IT) - DX(IB)                                 JST 220
      UYI     =           DY(IT) - DY(IB)                                 JST 230
      UXJ     =           DX(JT) - DX(JB)                                 JST 240
      UYJ     =           DY(JT) - DY(JB)                                 JST 250
      UXID    = UXI   *A + UYI   *B                                       JST 260
      UYID    = (-1.)*B*UXI    + UYI   *A                                 JST 270
      UXJD    = UXJ   *A + UYJ   *B                                       JST 280
      UYJD    = (-1.)*B*UXJ    + UYJ   *A                                 JST 290
      ENS = (.5)*(UXID   + UXJD)                                          JST 300
      ENJ = (.5)*(UYID   + UYJD )                                         JST 310
      SS=0.002                                                            JST 320
      CNND=CNN                                                            JST 330
      IF(KK(I).EQ.2.OR.KK(I).EQ.4) CNNB=0.0                               JST 340
      IF(ENJ.LT.0.0.AND.KK(I).EQ.2) KK(I)=1                               JST 350
      IF(ENJ.LT.0.0.AND.KK(I).EQ.4) KK(I)=3                               JST 360
C                                                                         JST 370
C     FIND BOND AND DOWEL DISPLACEMENTS AND STRESSES ALSO CHECK           JST 380
C     FAILURE CRITERIA                                                    JST 390
C                                                                         JST 400
      SLP=DABS(ENS)                                                       JST 410
      IF(SLP.EQ.0.) GO TO 72                                              JST 420
      SIXS=(ENS/DABS(ENS))*(1.95E+06*SLP-2.35E+09*SLP**2+1.39E+12*SLP**3  JST 430
     $ -.33E+15*SLP**4)                                                   JST 440
      SIXS=0.0                                                            JST 450
      SIXN=CNNB*ENJ                                                       JST 460
      IF(KK(I).GT.2) GO TO 70                                             JST 470
      IF(SLP.EQ.0.0) GO TO 72                                             JST 480
      SIXS=(ENS/SLP      )*(1.95E+06*SLP-2.35E+09*SLP**2+1.39E+12*SLP**3  JST 490
     $ -.33E+15*SLP**4)                                                   JST 500
      SIK=DIA1                                                            JST 510
      IF(I.GT.N1.AND.I.LE.N2) SIK=DIA2                                    JST 520
      IF(I.GT.N2.AND.I.LE.N3) SIK=DIA3                                    JST 530
      CSJS=DABS(SIXS/SLP) *SIK*3.14159                                    JST 540
      ERR=(CSS-CSJS)/CSS                                                  JST 550
72    CONTINUE                                                            JST 560
      IF(KK(I).EQ.4) GO TO 71                                             JST 570
      IF(KK(I).GT.1) GO TO 70                                             JST 580
      KK(I)=1                                                             JST 590
      IF(SIXN.GT.FTENS) KK(I)=2                                           JST 600
      IF(SIXN.LT.FCOMP) KK(I)=2                                           JST 610
      IF(SLP.GT.SS) KK(I)=3                                               JST 620
70    IF(KK(I).EQ.2.AND.SLP.GT.SS) KK(I)=4                                JST 630
      IF(KK(I).EQ.3.AND.SIXN.GT.FTENS) KK(I)=4                            JST 640
      IF(KK(I).EQ.3.AND.SIXN.LT.FCOMP) KK(I)=4                            JST 650
      FIS=0.3                                                             JST 660
      SLP=DABS(ENS)+FIS*DABS(ENS-ENS1(I))                                 JST 670
      IF(SLP.GT..002) SLP=.002                                            JST 680
      IF(KK(I).GT.2) GO TO 71                                             JST 690
      CSJ(I)=3.14159*SIK*(1.95E+06-2.35E+09*SLP+1.39E+12*SLP**2-.33E+15   JST 700
     $ *SLP**3)                                                           JST 710
71    CONTINUE                                                            JST 720
      IF(IPP.EQ.1) GO TO 1                                                JST 730
```

```
      KIKI=KK(I)                                                        JST 740
      GO TO (101,102 ,103 ,104) ,KIKI                                   JST 750
101   WRITE(6,11)  I,ENJ,ENS,SIXN,SIXS,ERR                             JST 760
      GO TO 1                                                           JST 770
102   WRITE(6,12)  I,ENJ,ENS,SIXN,SIXS,ERR                             JST 780
      GO TO 1                                                           JST 790
103   WRITE(6,13)  I,ENJ,ENS,SIXN,SIXS                                  JST 800
      GO TO 1                                                           JST 810
104   WRITE(6,14)  I,ENJ,ENS,SIXN,SIXS                                  JST 820
1     ENS1(I)=ENS                                                       JST 830
2     CONTINUE                                                          JST 840
11    FORMAT(1X,I6,4E12.5,1F12.5)                                       JST 850
  12  FORMAT (1X,I6,4E12.5,1F12.5,' JOINT DOWEL FAILURE')              JST 860
  13  FORMAT (1X,I6,4E12.5,' JOINT YIELDED')                           JST 870
14    FORMAT(1X,I6,4E12.5,'JOINT FAILED')                               JST 880
      RETURN                                                            JST 890
      END                                                               JST 900
```

```
      SUBROUTINE STRTF(IS,JS,ANG,SAREA,SKS,ALS,EI)                      STR  10
C                                                                       STR  20
C     COMPUTE STIRRUP STIFF.                                            STR  30
C                                                                       STR  40
      IMPLICIT REAL*8(A-H,O-Z)                                          STR  50
      COMMON SM(58000)                                                  STR  60
      DIMENSION EI(2),SKS(4,4)                                          STR  70
      X=(SM(2*IS-1)-SM(2*JS-1))                                         STR  80
      Y=(SM(2*IS)-SM(2*JS))                                             STR  90
      ALS=DSQRT(X**2+Y**2)                                              STR 100
      STIF=EI(2)*SAREA/ALS                                              STR 110
      DANG=3.1416/2.0                                                   STR 120
      IF(X.EQ.0.) ANG=DANG                                              STR 130
      IF(X.EQ.0.) GO TO 2                                               STR 140
      ANG=DATAN(Y/X)                                                    STR 150
2     CONTINUE                                                          STR 160
      IF(ANG.LT.0.0) ANG=ANG+3.1416                                     STR 170
      CO=DCOS(ANG)                                                      STR 180
      S=DSIN(ANG)                                                       STR 190
      SKS(1,1)=STIF*CO*CO                                               STR 200
      SKS(1,2)=-STIF*CO*CO                                              STR 210
      SKS(1,3)=STIF*CO*S                                                STR 220
      SKS(1,4)=-SKS(1,3)                                                STR 230
      SKS(2,1)=SKS(1,2)                                                 STR 240
      SKS(2,2)=SKS(1,1)                                                 STR 250
      SKS(2,3)=-SKS(1,3)                                                STR 260
      SKS(2,4)=SKS(1,3)                                                 STR 270
      SKS(3,1)=SKS(1,3)                                                 STR 280
      SKS(3,2)=-SKS(1,3)                                                STR 290
      SKS(3,3)=STIF*S*S                                                 STR 300
      SKS(3,4)=-SKS(3,3)                                                STR 310
      SKS(4,1)=-SKS(1,3)                                                STR 320
      SKS(4,2)=SKS(1,3)                                                 STR 330
      SKS(4,3)=-SKS(3,3)                                                STR 340
      SKS(4,4)=SKS(3,3)                                                 STR 350
      XIS=SM(2*IS-1)                                                    STR 360
      XJS=SM(2*JS-1)                                                    STR 370
      YIS=SM(2*IS)                                                      STR 380
      YJS=SM(2*JS)                                                      STR 390
      WRITE(6,1)IS,JS,XIS,YIS,XJS,YJS ,SAREA,ALS                        STR 400
1     FORMAT(5X,2I5,6F7.2)                                              STR 410
      RETURN                                                            STR 420
      END                                                               STR 430
```

```
      SUBROUTINE ADSTRP(NSTR,NB)                                        ADS  10
C                                                                       ADS  20
C     ADD STIRRUP STIFFNESSES TO THE GLOBAL STIFFNESS MATRIX            ADS  30
C                                                                       ADS  40
      IMPLICIT REAL*8(A-H,O-Z)                                          ADS  50
      COMMON SM(58060)                                                  ADS  60
      DIMENSION SKS(4,4) ,NP(4) ,RAT(150)                               ADS  70
      COMMON/BLK2/RAT                                                   ADS  80
      DO 405 IZ=1,NSTR                                                  ADS  90
      READ (2) IS,JS,ANG,SAREA,((SKS(KI,MI),KI=1,4),MI=1,4),           ADS 100
     & (NP(I),I=1,4),ALS                                                ADS 110
      DO 406 IN=1,4                                                     ADS 120
      IF(NP(IN).EQ.0) GO TO 406                                         ADS 130
      DO 407 IM=IN,4                                                    ADS 140
      IF(NP(IM).EQ.0) GO TO 407                                         ADS 150
      IF(NP(IN)-NP(IM)) 408,409,409                                     ADS 160
409   K=(NP(IM)-1)*NB+NP(IN)                                            ADS 170
      SM(K)=SM(K)+SKS(IN,IM)*RAT(IZ)                                    ADS 180
      GO TO 407                                                         ADS 190
408   K=(NP(IN)-1)*NB+NP(IM)                                            ADS 200
      SM(K)=SM(K)+SKS(IN,IM)*RAT(IZ)                                    ADS 210
407   CONTINUE                                                          ADS 220
406   CONTINUE                                                          ADS 230
405   CONTINUE                                                          ADS 240
      REWIND 2                                                          ADS 250
      RETURN                                                            ADS 260
      END                                                               ADS 270
```

```
      SUBROUTINE STSTRS(JC,EI,EPSYLD,FN,NSTR)                          STS  10
C                                                                      STS  20
C     CALCULATE STIRRUP STRAINS AND STRESSES                           STS  30
C                                                                      STS  40
C     FIND STRAINS AND STRESSES IN  STIRRUPS.                          STS  50
      IMPLICIT REAL*8(A-H,O-Z)                                         STS  60
      DIMENSION STS(150),EI(2),FN(1),SKS(4,4),UI(4),NP(4) ,RAT(150)    STS  70
      COMMON/BLK2/RAT                                                  STS  80
      COMMON/BLK9/IPP                                                  STS  90
      DO 1 J=1,NSTR                                                    STS 100
      READ (2)IS,JS,ANG,SAREA,((SKS(KI,MI),KI=1,4),MI=1,4),           STS 110
     C (NP(I),I=1,4),ALS                                               STS 120
      DO 2 K=1,4                                                       STS 130
      UI(K)=0.0                                                        STS 140
      LL=NP(K)                                                         STS 150
      IF(LL.EQ.0) GO TO 2                                              STS 160
      UI(K)=FN(LL)                                                     STS 170
2     CONTINUE                                                         STS 180
      ANGLE=0.0                                                        STS 190
      IF(ANG.GT.1.57081633) ANGLE=3.14159265                           STS 200
      SI=UI(1)*DCOS(ANGLE+ANG)+UI(3)*DSIN(ANGLE+ANG)                   STS 210
      SJ=UI(2)*DCOS(ANGLE+ANG)+UI(4)*DSIN(ANGLE+ANG)                   STS 220
      STRN=(SJ-SI)/ALS                                                 STS 230
      IF(JC.NE.0) GO TO 20                                             STS 240
      STS(J)=0.0                                                       STS 250
20    CONTINUE                                                         STS 260
      IF(ANG.GT.1.57081633) STRN=(SI-SJ)/ALS                           STS 270
      ANG=ANG*57.29578                                                 STS 280
      ANGLE=ANGLE*57.29578                                             STS 290
      STS(J)=STS(J)+STRN                                               STS 300
      STRS=EI(2)*STS(J)                                                STS 310
      STRINC=EI(2)*STRN                                                STS 320
      IF(DABS(STS(J)).GT.EPSYLD) GO TO 22                              STS 330
      IF(IPP.EQ.1) GO TO 1                                             STS 340
      WRITE(6,21) J,STRN,STRINC,STS(J),STRS,ANG                        STS 350
      GO TO 1                                                          STS 360
22    STRINC=0.0                                                       STS 370
      STRS=EPSYLD*EI(2)                                                STS 380
      IF(IPP.EQ.1) GO TO 3                                             STS 390
      WRITE(6,23) J,STRN,STRINC,STS(J),STRS,ANG                        STS 400
3     CONTINUE                                                         STS 410
      RAT(J)=.0001                                                     STS 420
21    FORMAT(1X,I5,6E14.7)                                             STS 430
23    FORMAT(1X,I5,5E14.7,5X,'STIRRUP YIELDED')                        STS 440
1     CONTINUE                                                         STS 450
      REWIND 2                                                         STS 460
      RETURN                                                           STS 470
      END                                                              STS 480
```

```
      SUBROUTINE FSHCRK(S,PT,E,U,ARR,WIDTH,DATA,INDC,KJJ)                    FSH  10
C                                                                            FSH  20
C     FIND MODIFIED STIFFNESSES OF CRACKED ELEMENTS.                         FSH  30
C                                                                            FSH  40
      IMPLICIT REAL*8(A-H,O-Z)                                               FSH  50
      DIMENSION D(3,3),R(6,6),P(6,6),T(3,3),S(6,6),PT(9),INDC(1)             FSH  60
      DO 1 I=1,3                                                             FSH  70
      DO 1 J=1,3                                                             FSH  80
      D(I,J)=0.0                                                             FSH  90
      R(I,J)=0.0                                                             FSH 100
1     CONTINUE                                                               FSH 110
      EU=E*WIDTH/(1.0-U**2)                                                  FSH 120
      D(1,1)=EU                                                              FSH 130
      D(1,2)=EU *U/1000.                                                     FSH 140
      D(2,1)=U*EU/1000.0                                                     FSH 150
      D(2,2)=EU/1000.0                                                       FSH 160
      D(3,3)=EU*(1.-U)/(2.0*1000.0)                                          FSH 170
      C=DCOS(DATA)                                                           FSH 180
      SI=DSIN(DATA)                                                          FSH 190
      T(1,1)=C*C                                                             FSH 200
      T(1,2)=SI*SI                                                           FSH 210
      T(1,3)=-2.0*C*SI                                                       FSH 220
      T(2,1)=T(1,2)                                                          FSH 230
      T(2,2)=T(1,1)                                                          FSH 240
      T(2,3)=-T(1,3)                                                         FSH 250
      T(3,1)=SI*C                                                            FSH 260
      T(3,2)=-T(3,1)                                                         FSH 270
      T(3,3)=T(1,1)-T(1,2)                                                   FSH 280
      DO 2 JJ=1,3                                                            FSH 290
      DO 2 II=1,3                                                            FSH 300
      R(JJ,II)=0.0                                                           FSH 310
      DO 2 KK=1,3                                                            FSH 320
      R(JJ,II)=R(JJ,II)+T(JJ,KK)*D(KK,II)                                    FSH 330
2     CONTINUE                                                               FSH 340
      DO 3 JJ=1,3                                                            FSH 350
      DO 3 II=1,3                                                            FSH 360
      D(JJ,II)=0.0                                                           FSH 370
      DO 3 KK=1,3                                                            FSH 380
      D(JJ,II)=D(JJ,II)+R(JJ,KK)*T(II,KK)                                    FSH 390
3     CONTINUE                                                               FSH 400
      DO 4 II=1,6                                                            FSH 410
      DO 4 JJ=1,6                                                            FSH 420
      S(JJ,II)=0.0                                                           FSH 430
      P(JJ,II)=0.0                                                           FSH 440
4     CONTINUE                                                               FSH 450
      DO 110 II=1,3                                                          FSH 460
      DO 110  JJ=1,3                                                         FSH 470
      KK=3*(II-1)+JJ                                                         FSH 480
      P(II,JJ)=PT(KK)                                                        FSH 490
      MJJ=II+3                                                               FSH 500
      NJJ=JJ+3                                                               FSH 510
      P(MJJ,NJJ)=P(II,JJ)                                                    FSH 520
110   CONTINUE                                                               FSH 530
      S(2,2)=D(1,1)                                                          FSH 540
      S(2,3)=D(1,3)                                                          FSH 550
      S(2,5)=D(1,3)                                                          FSH 560
      S(2,6)=D(1,2)                                                          FSH 570
      S(3,2)=D(3,1)                                                          FSH 580
      S(3,3)=D(3,3)                                                          FSH 590
      S(3,5)=D(3,3)                                                          FSH 600
      S(3,6)=D(3,2)                                                          FSH 610
      S(5,2)=D(3,1)                                                          FSH 620
      S(5,3)=D(3,3)                                                          FSH 630
      S(5,5)=D(3,3)                                                          FSH 640
      S(5,6)=D(3,2)                                                          FSH 650
      S(6,2)=D(2,1)                                                          FSH 660
      S(6,3)=D(2,3)                                                          FSH 670
      S(6,5)=D(2,3)                                                          FSH 680
      S(6,6)=D(2,2)                                                          FSH 690
      AA=4.0*ARR                                                             FSH 700
      DO 5 JJ=1,6                                                            FSH 710
      DO 5 II=1,6                                                            FSH 720
      R(JJ,II)=0.0                                                           FSH 730
```

```
      DO 5 KK=1,6                                                       FSH 740
      R(JJ,II)=R(JJ,II)+S(JJ,KK)*P(KK,II)                               FSH 750
5     CONTINUE                                                          FSH 760
      DO 6 JJ=1,6                                                       FSH 770
      DO 6 II=1,6                                                       FSH 780
      S(JJ,II)=0.0                                                      FSH 790
      DO 6 KK=1,6                                                       FSH 800
      S(JJ,II)=S(JJ,II)+P(KK,JJ)*R(KK,II)/AR                            FSH 810
6     CONTINUE                                                          FSH 820
      INDC(JJ)=1                                                        FSH 830
      RETURN                                                            FSH 840
      END                                                               FSH 850
```

```
      SUBROUTINE STRESS(XS,YS,XYS,SIX,SIY,SIXY,BATA,E,U)                STR  10
C                                                                       STR  20
C     COMPUTE STRAINS IN CRACKED ELEMENTS.                              STR  30
C                                                                       STR  40
      IMPLICIT REAL*8(A-H,O-Z)                                          STR  50
      DIMENSION D(3,3),R(3,3),T(3,3)                                    STR  60
      DO 1 I=1,3                                                        STR  70
      DO 1 J=1,3                                                        STR  80
      D(I,J)=0.0                                                        STR  90
      R(I,J)=0.0                                                        STR 100
1     CONTINUE                                                          STR 110
      EU=E/(1.0-U**2)                                                   STR 120
      D(1,1)=EU                                                         STR 130
      C=DCOS(BATA)                                                      STR 140
      S=DSIN(BATA)                                                      STR 150
      T(1,1)=C*C                                                        STR 160
      T(1,2)=S*S                                                        STR 170
      T(1,3)=-2.0*C*S                                                   STR 180
      T(2,1)=T(1,2)                                                     STR 190
      T(2,2)=T(1,1)                                                     STR 200
      T(2,3)=-T(1,3)                                                    STR 210
      T(3,1)=S*C                                                        STR 220
      T(3,2)=-T(3,1)                                                    STR 230
      T(3,3)=T(1,1)-T(1,2)                                              STR 240
      DO 2 JJ=1,3                                                       STR 250
      DO 2 II=1,3                                                       STR 260
      R(JJ,II)=0.0                                                      STR 270
      DO 2 KK=1,3                                                       STR 280
      R(JJ,II)=R(JJ,II)+T(JJ,KK)*D(KK,II)                              STR 290
2     CONTINUE                                                          STR 300
      DO 3 JJ=1,3                                                       STR 310
      DO 3 II=1,3                                                       STR 320
      D(JJ,II)=0.0                                                      STR 330
      DO 3 KK=1,3                                                       STR 340
      D(JJ,II)=D(JJ,II)+R(JJ,KK)*T(II,KK)                              STR 350
3     CONTINUE                                                          STR 360
      SIX=D(1,1)*XS+D(2,1)*YS+D(3,1)*XYS                                STR 370
      SIY=D(1,2)*XS+D(2,2)*YS+D(3,2)*XYS                                STR 380
      FIY=D(1,3)*XS+D(2,3)*YS+D(3,3)*XYS                                STR 390
      SIXY=FIY                                                          STR 400
      RETURN                                                            STR 410
      END                                                               STR 420
```

```
      SUBROUTINE SAHER(FN,EX,EY,EXY,SEX,SEY,SEXY,NN,FACTOR,UI,NV          SAH  10
     S      ,NT1,NE,IFLAG,FTENS)                                          SAH  20
C                                                                         SAH  30
C     COMPUTE AND STORE NODAL STRESSES AND STRAINS.                       SAH  40
C                                                                         SAH  50
      IMPLICIT REAL*8(A-H,O-Z)                                            SAH  60
      COMMON SH(58000)                                                    SAH  70
      DIMENSION NP(6),S(6,6),PT(9)                                        SAH  80
      DIMENSION FN(1),EX(1),EY(1),EXY(1),SEX(1),SEY(1),SEXY(1),UI(6),     SAH  90
     S II(3),MFLAG(1),FACTOR(318,2)                                       SAH 100
      DIMENSION IO(18)                                                    SAH 110
      DO 1 J=1,7000                                                       SAH 120
      SH(J)=0.0                                                           SAH 130
    1 CONTINUE                                                            SAH 140
      FTONS=1.2*FTENS                                                     SAH 150
      DO 3 J=1,NE                                                         SAH 160
      READ(NT1)(NP(I),I=1,6),((S(MM,MN),MN=1,6),MM=1,6) ,(PT(L),L=1,9),   SAH 170
     1 A,E,U ,NI,NJ,NK,ARR                                                SAH 180
      IF(IFLAG(J).GT.0) ARR=0.0                                           SAH 190
      IF(IFLAG(J).GT.0) U=0.0                                             SAH 200
      DO 2 K=1,6                                                          SAH 210
      UI(K)=0.0                                                           SAH 220
      LL = NP(  K)                                                        SAH 230
      IF(LL.EQ.0) GO TO 2                                                 SAH 240
      UI(K)=FN(LL)                                                        SAH 250
    2 CONTINUE                                                            SAH 260
      XS=(UI(1)*PT(4)+UI(2)*PT(5)+UI(3)*PT(6))/A                          SAH 270
      EX(J)=XS                                                            SAH 280
      YS=(UI(4)*PT(  7)+UI(5)*PT(  8)+UI(6)*PT(  9))/A                    SAH 290
      EY(J)=YS                                                            SAH 300
      XYS=(UI(1)*PT(  7)+UI(2)*PT(  3)+UI(3)*PT(  9)+UI(4)*PT(  4)+UI(5   SAH 310
     1)*PT(  5)+UI(6)*PT(  6))/A                                          SAH 320
      EXY(J)=XYS                                                          SAH 330
      EU=E*FACTOR(J,2)/(1.0-U**2)                                         SAH 340
      SIX=EU*(XS+U   *YS)                                                 SAH 350
      SEX(J)=SIX                                                          SAH 360
      IF(MFLAG(J).EQ.1.AND.SIX.GT.FTONS) SIX=FTONS                        SAH 370
      SIY=EU*(U   *XS+YS)                                                 SAH 380
      SEY(J)=SIY                                                          SAH 390
      IF(MFLAG(J).EQ.1.AND.SIY.GT.FTONS) SIY=FTONS                        SAH 400
      SIXY=EU*(1.0-U   )*XYS/2.0                                          SAH 410
      SEXY(J)=SIXY                                                        SAH 420
      IF(IFLAG(J).GT.0) SIXY=0.0                                          SAH 430
      II(1)=NI                                                            SAH 440
      II(2)=NJ                                                            SAH 450
      II(3)=NK                                                            SAH 460
      DO 10 NMI=1,3                                                       SAH 470
      IEC=4*II(NMI)-3                                                     SAH 480
      SH(IEC)=SH(IEC)+ARR                                                 SAH 490
      SH(IEC+1)=SH(IEC+1)+SIX*ARR                                         SAH 500
      SH(IEC+2)=SH(IEC+2)+SIY*ARR                                         SAH 510
      SH(IEC+3)=SH(IEC+3)+SIXY*ARR                                        SAH 520
   10 CONTINUE                                                            SAH 530
    3 CONTINUE                                                            SAH 540
      DO 20 I=1,NN                                                        SAH 550
   40 IEC=4*I-3                                                           SAH 560
      IF(SH(IEC).EQ.0.0) SH(IEC)=1000.0                                   SAH 570
      SH(4000+I)=SH(IEC+1)/SH(IEC)                                        SAH 580
      SH(5000+I)=SH(IEC+2)/SH(IEC)                                        SAH 590
      SH(6000+I)=SH(IEC+3)/SH(IEC)                                        SAH 600
   20 CONTINUE                                                            SAH 610
      REWIND NT1                                                          SAH 620
      RETURN                                                              SAH 630
      END                                                                 SAH 640
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