DECENTRALIZED PRICE SYSTEMS

AGENT-BASED MODELS: AN INTRODUCTORY STUDY ON DECENTRALIZED PRICE SYSTEMS

by

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In Chapter 1, we introduce the idea of an Agent-based model(ABM) and review some of the programming concepts that facilitate their implementation. We also discuss how ABMs may offer some help to macro model builders in getting around three key challenges: rich microfoundations, the aggregation problem and agent decision-making. In Chapter 2, we endeavour to develop a fuller understanding of a decentralized price system through the use of ABMs. We review two models in the literature to investigate how decentralized price systems have been incorporated into more elaborate setups. Appealing to the need for simplicity, we construct ABMs to examine the convergence properties of two models that appear in the disequilibrium literature: the Edgeworth model and the Hahn-Fisher model. The Edgeworth model illustrates the need to think more carefully about path-dependence. The implementation of the Hahn-Fisher model raises a question about the convergence properties of this process.

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1.1 Introduction

One way to think about macro phenomena is as emergent properties produced by interactions at the micro level of the individuals who make up the system. There is a burgeoning methodology that endeavours to understand socio-economic systems starting from this perspective. The goal of this paper is to explore this new methodology and connect it to the existing 'disequilibrium' literature with the hope of initiating a fuller description of the mechanism that 'coordinates' the allocation of resources in an economy, the price system. The first section of this paper will provide an explanation of the Agent-based modelling (ABM) and review some of the potential benefits of this methodology. In the next section, the focus will return to the question at hand and will examine why some researchers are uncomfortable about the way that standard 'economic theory' treats the price system as a centralized, equilibrium concept. This paper argues that the standard approach obscures important details about the inner workings of economies and prohibits researchers from being able to answer certain types of questions about the price system. Next, we examine two ABMs which incorporate decentralized price systems in their analysis. However, the complexity of these models makes it difficult to isolate the dynamics we are interested in. So, in the final section we strip away all of the complicating detail and present computational examples of two simple ABMs with decentralized price systems. These simple models draw heavily on the disequilibrium literature. The computational experiments are a good way to better understand the disequilibrium literature and highlight a difficulty in one of the analytic proofs. The paper concludes with a few reflections on the usefulness of ABM and suggests areas for further research. The appendix provides the source code that was used to construct the agents in the two models.

1.2 Agent-based Modelling Methodology

An agent-based model is a computer model of an environment populated with software agents. It is a simple idea. In order to understand a complex system, create a realistic environment and a set of agents who inhabit this environment. The agents are then encoded with methods which describe their behaviours and have common interfaces which allow them to interact with the environment and each other. Agent-based models provide a framework within which the emergence of macro-bevaviour from micro-level interactions can be studied.

A good way to understand ABM modelling is to contrast it with equation-based modelling (EBM) by way of an example. Think of the local pub as a system that one would like to study [1] . What are the aspects of this system that need to be considered? The system can be thought of as an environment and a set of agents who interact through time and space. The environment has certain properties such as the square footage of the bar, the amount of beer on hand and the location of the pool table. The agents have objectives and characteristics which result in behaviour such as drinking beer, playing pool and engaging the attractive sex. The result of all of this activity is a set of *observables*: the volume of beer consumed, the number of pools games played and the amount of phone

[1] The discussion draws heavily on Parunak, Savit and Riolo [27] and the example is inspired by Arthur [2].

numbers exchanged. With this example in mind, it is now time to distinguish ABMs from EBMs.

The most important distinction between an EBM and an ABM is the relationships on which one focuses attention. In an EBM, the focus is on the relationship between the observables. For example, perhaps a researcher is able to identify a positive correlation between the volume of beer consumed and the amount of phone numbers exchanged or perhaps the researcher is interested in the price of a game of pool and exit-rate of the customers. The point is, the EBM methodology endeavours to find a set of equations that accurately express relationships among the *observables.*

In contrast, an ABM focuses on the behaviours of the individuals who comprise the system. A researcher can provide a model of behaviour at the individual level and then allow those agents to interact. For example, perhaps as an individual consumes more beer, they become more likely to give away their phone number. An individual, observing the success of his neighbours may choose to remain at the bar longer as he perceives the probability of receiving a phone number rising. Here, the researcher is more concerned with understanding the micro-level behaviours and interactions which give rise to the observables. Thus, whereas observables are the input in EBMs, ABMs start, not with observables, but with the individual behaviours.

Clearly, both approaches can provide insight into problems that interest economists. While there has been some attempt to introduce 'micro-foundations' into standard macroeconomic models, these attempts have met with fairly sharp critcism.[2] Can ABMs help to reconcile the rich microeconomic behaviour with the seemingly 'stable' aggregate relationships at the macro level? This paper takes a modest view of that question given the author's limited exposure to either type

[2] See Hartley [14] and Kirman [19] for a review of these criticisms.

of approach. However, comments from more seasoned researchers suggest that the ABM methodology may hold some promise for economics and is worthy of further attention,

> ABM is most appropriate for domains characterized by a high degree of localization and distribution and dominated by discrete decisions. EBM is most naturally applied to systems that can be modeled centrally, and in which the dynamics are dominated by physical laws rather than information processing.
>
> - Parunak et al. [27]

In the next section, we examine some of the potential benefits of ABM.

1.3 Potential Benefits of the ABM Methodology

Macroeconomic model building is hard. There are three key issues which have consistently plagued macro model developers. These issues can be categorized as: providing firm microfoundations, resolving the aggregation problem, and developing a plausible model of agent decision-making. This section will review why these concepts are so important to address and will argue that an agent-based framework may provide some additional help in dealing with these issues. Where possible, examples from actual models will be provided.

1.3.1 Microfoundations

The position that 'good' macro models must have sound microeconomic foundations should be taken very seriously. Scarth [33] captures this point when he notes that, "it is utility and production functions that remain invariant to government policy; agents' decision rules do not necessarily remain invariant to shifts in policy. A specific microeconomic base is required to derive how private decision rules may react to major changes in policy." While there is much support for this view, modellers have had a difficult time responding to it. Proposed solutions have themselves come under some rather sharp criticism. The charge is that attempts to incorporate microfoundations, such as in representative agent models, have been marginal at best. The following quote from Hartley [14] makes this point.

> Representative agent models can only be considered micro-foundational models if micro-economics is extremely simple. If, on the other hand, microeconomic theory is rich, involving the complex interactions of heterogeneous agents, representative models are no more or less micro-foundational than the old fashioned Keynesian consumption function.[3]

Other authors, such as Koopmans, have also argued along these lines[4] . If there is truly much value to rich microeconomic underpinnings, then representative agent models are probably of limited use since they do not offer the flexibility to easily incorporate modern microeconomic theory. What is the alternative?

While many attribute the advocacy of microeconomic foundations to the New Classicals, historically, the Austrians have been the most vocal champion of this issue. In fact, for Austrians, the whole notion of microeconomic foundations is a bit of misnomer - for them, everything is microeconomics.

> All of economics is reducible to people. There is no economic fact which can be derived without starting at a study of people's behaviour in a given situation.[5]

Clearly, both the New Classicals and the Austrians believe microfoundations are important. The question becomes then - which approach is most suitable for dealing with this issue? A good description of the two approaches comes from Hartley [14],

[3] Hartley [14], p.170.
[4] See Koopmans [22] for example.
[5] Hartley [14], p.109.

> Austrians provide microfoundations by beginning with the individual and building up. The new classicals are working in the opposite direction. They begin with macroeconomics and attempt to build down.

This paper argues that much can be learned about economic systems using a bottom-up approach such as the ABM methodology.

1.3.2 The Aggregation Problem

The aggregation problem is often ignored in macro models. The simple example below, as described by Hartley [14] suggests that modellers do so at their peril. Take a simple economy with two individuals, Pip and Joe. Their consumption functions are,

Pip	Joe
$C_p = 0.8Y_p$	$C_j = 0.4Y_p$
$Y_p^0 = \$100$	$Y_j^0 = \$100$

Given this setup, a plausible aggregate consumption function might be specified as:

$$\text{Aggregate consumption function C} = 0.6\text{ Y}$$

Adding $Y_p^0 + Y_j^0$ gives aggregate income = \$200, then aggregate consumption will be \$120. It does not matter if we sum across individuals or use the 'well-specified' aggregate consumption function. The problem comes in when we try to look at some sort of policy response in this economy. Suppose that the government decided to redistribute income in this economy. So, for example, take \$50 from Pip and give it to Joe. An analyst looking at the aggregate consumption function would say that consumption would not change. Afterall, aggregate income has not changed. However, looking at the individuals would correctly indicate that consumption would actually change to \$100. The implication is that, while the above model is quite simple, the problems associated with aggregation do not "go away as the world gets more complex; rather, they get worse" (Hartley [14]).

The aggregation problem is not easily dealt with. The so called aggregation conditions are prohibitively strict. Apart from assuming it away, again an ABM approach might be useful. Instead of starting with an aggregate relationship and worrying about the aggregation conditions, one could simply start at the individual level and then just sum up.

A few comments about this are appropriate here. The ABM approach has its difficulties as welll. That is, in order to ever evaluate a model (an important task indeed) one must have incredibly detailed micro-level data. The first response to this criticism is that personalized data is being increasingly gathered through the use of information technology. The other way around the absence of data problem may be to apply the distributions of characteristics about the aggregate population to our 'artificial' population. For example, we may not need to know each individual's income if we know that income is distributed normally across the population with a specific mean.

1.3.3 "Intelligent" agents

Another major difficulty economists have encountered is coming up with a plausible way to model human-decision making. This difficulty is most acute in macroeconomics where the 'game' is messy and the agents face much uncertainty about the payoffs of different strategies. Arthur [2] argues that "perfect, logical, deduction" breaks down when the system becomes complicated. Even if we assume that the entire set of payoffs is known, humans simply do not have the capacity to carry out the complex calculations necessary to determine the optimal strategy.

As he says, while we might be able to figure out a perfectly rational minimax solution to a simple game like "Tic-Tac-Toe", "we do not find rational 'solutions' at the depth of checkers, and certainly not at the modest depths of Chess or Go."

How, then, do humans deal with complicated situations? Arthur [2] argues that "we look for patterns; and we simplify the problem by using these to construct temporary internal models or hypotheses or schmeta to work with. We carry out localized deductions based on our current hypotheses and act on them. And, as feedback from the environment comes in, we may strengthen or weaken our beliefs in our current hypotheses, discarding some when they cease to perform and replacing them as needed with new ones." [6] Thus, the term *inductive rationality.* For example, thinking about tomorrow's price level, an individual might believe that a good model to forecast that variable is to simply take today's price level. If, tomorrow, his forecast turns out to be reasonably accurate, he might choose to retain this model. However, if the model forecasts tomorrow's price level poorly, then he will try another model.

Inductive rationality has two advantages. First, support for this model is found in the psychological literature[7] . This is a claim other models used in economics would be very hesitant about making. Second, a researcher can control the "degree" of intelligence of the agents by specifying the sophistication of the models an agent has at its disposal. A more sophisticated model implies that an agent will learn about his environment more quickly.

The fact that ABM may hold some hope of helping to overcome three of the more formidable challenges facing macro model builders suggests that it is a methodology worth exploring further. In the next section, we describe how to actually implement an ABM. In Chapter 2, an application of ABM is presented

[6] This line of argument is similar to that of Sargent [32].

[7] See Bower and Hilgard [6] and Holland et al. [16] for example.

as a tool to further our understanding of decentralized price systems.

1.4 Implementation of Agent-based Models

There are many different ways to create an agent-based model. The earliest ABMs were produced using dimes and pennies on a ruled sheet of paper and the pieces were moved around by hand [8] . Modern ABMs, however, use a variety of different programming languages and software frameworks to handle their modelling. While too many simulation platforms exist to name here,[9] two of the more popular ones for social scientists are Swarm and RePast. Swarm was originally designed and launched by Chris Langton and released in 1994.[10] In 1999, a Java version of Swarm's higher libraries was introduced. These Java libraries are integrated into the Swarm package through the use of the Java Native Interface (JNI) making the core libraries which are still written in Objective C accessible to its *children* . RePast is an ABM framework much like Swarm except for the fact that it is written entirely in Java. The first version of RePast was released on January 25, 2000 by Social Science Research Computing at the University of Chicago.[11]

Since Java is a more popular language than Objective C with widespread applications and so the additional layers of complication introduced by the JNI made Swarm a less desirable choice. As a result, the models in this paper are produced using RePast.

There are two keys things that need to be understood about Objected-oriented programming which will provide some insight into why this programming

[8] See Schelling [34] pg. 147

[9] See GNU/Linux AI and ALife HowTo at http://www.ibiblio.org/mdw/HOWTO/AI-Alife-HOWTO.html for a fairly comprehensive listing.

[10] For more information about the original and current design teams see http://www.swarm.org/intro-people.html

[11] For more information on RePast see http://repast.sourceforge.net

methodology is well suited for the design and creation of ABMs. The first is the idea of a class. A class has instance variables and methods. The instance variables are the characteristics of the class and the methods are the rules used to modify the instance variables. For example, think of a class called "human". One characteristic of this "human" class is an age (instance variable). Now, we can give this class a method called "birthday". When the birthday method is called, the instance variable age is modified according to the rules dictated by "birthday" (Ie. increment age by one).

The other important element of Object-oriented programming that makes it such a natural methodology for developing ABMs is 'inheritance'. 'Inheritance means that you can define a new class (called the subclass) simply by extending an existing class (called the superclass) in some specific way. For example, in the Hahn-Fisher model presented in Chapter 2, it is natural to think of a "consumer" agent and a "firm" agent, who, in addition to sharing common elements of the agent superclass, each have their own unique instance variables and methods. Figure 1.4 depicts this hierarchy.

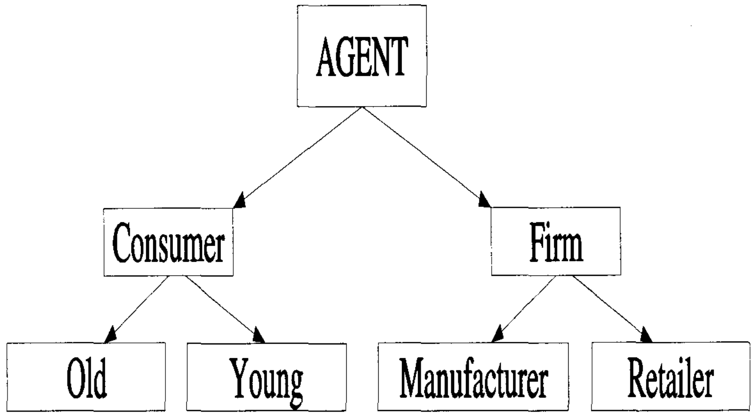

Figure 1.1: A Typical Hierarchy in an ABM

In the next chapter, we will use the ABM methodology to explore decentralized, disequilibrium trading processes by way of computational examples.

Chapter 2

Towards an Understanding of a Decentralized Price System

> First, low inflation is favorable to optimum allocation of "real" resources, that is, labor and physical capital. This is significant, because the closer to optimum is resource allocation, the more the output an economy produces for the same inputs. Thus, optimal allocation of resources is obviously positive over time for living standards. This positive effect on resource allocation results because price signals are more easily and accurately interpreted in a low-inflation environment.
>
> \- Gary H. Stern[1]

While it seems that central bankers[2] put a fair amount of trust in this notion, there does not appear to be considerable evidence either supporting or rejecting such a claim. As Ragan notes, the literature on this topic has not progressed very far and therefore the validity of this claim is "loosely intuitive rather than based on the results from formal models" (Ragan [28], p.21). In order to understand why so few models exist that deal with this issue, it is important to recognize exactly what question is being asked. Ragan states that, if "inflation is to introduce noise into the price system, inflation must carry with it some uncertainty about static and/or intertemporal relative prices" (Ragan [28], p. 22). So, in order to understand the damage that inflation does to the allocative

[1] President of the Federal Reserve Bank of Minneapolis, June 1997.

[2] In addition to the above quote from the Federal Reserve Bank of Minneapolis, similar comments can be found in Ragan [28] published by the Bank of Canada and Hoggarth [15] published at the Bank of England.

efficiency of the price system alluded to by central bankers, one must explain the interaction of inflation with the price system. That is, how does inflation undermine the decisions taken by agents about prices? But, as I argue here, a significant impediment to answering this more difficult question is that we do not have a solid understanding of how a price system emerges in a decentralized economy. This is partly due to the fact that much of standard macro theory treats prices as an equilibrium concept with all agents acting as if prices are given. While this may be a reasonable assumption when doing analytical work, with all agents taking prices as given, this leaves open the question of how these prices are determined in the first place. The goal of this paper is to explore how a price system emerges through the interaction of individuals in the economy.

The first section will provide a description of some of the theoretical issues concerning the price system and dynamic adjustment processes. It will be shown that the centralized nature of these early models with all prices being coordinated through some sort of 'market manager' tend to obscure important details about the actual formation and evolution of prices. The next section will discuss two ABMs in the literature which are representative of how modellers have handled decentralized price systems. These models were designed with specific markets or phenomena in mind and are successful in characterising the behaviour of their target system. However, one problem with these and many other agent-based models is that their complexity makes it difficult to isolate the characteristics of their price systems. That is, two key issues that should be kept in mind is whether a price system is stable and whether or not it converges to a competitive equilibrium. The disequilibrium literature of the 60s and 70s focused on exactly these questions and so, in the final section, the goal is to construct simple agent-based models in order to explore two decentralized trading processes found in that literature: the Edgeworth process and the Fisher-Hahn process. By discarding all of the distracting details of more complex models, it will be easier to understand the stability and convergence properties of our price system. The modest goal is to provide computational examples of these models. The exercise is useful because by operationalizing these models, we can gain a better insight into problems that may not always be adequately identified by a theorist using strictly analytical methods. As it turns out, the construction of the Fisher-Hahn model gives rise to a difficulty in the original proof. Also, these simple models provide a good introduction into the technical aspects of the implementation of an ABM.

2.1 Adjustment Processes: Tatônnement and Trading

It may seem obvious to say that the price of a commodity is determined by supply and demand but the critical point to acknowledge here is that these are merely the channels through which cost and utility operate (Blaug [5], p. 39). So, the price system then is that set of prices which contain information about the underlying costs that suppliers incur bringing goods to market as well as the value that the ultimate consumers of those goods attach to them. The question is, how are these prices determined? For researchers, the usual method is to assume some functional form of individual (or aggregate) demand and supply and then to deduce the *equilibrium* price by setting $Q_i^s = Q_i^d \quad \forall i$. This is the price that prevails in a perfectly competitive market. This assumption is no doubt convenient for researchers and is a logical outcome *in a perfectly competitive market* . All too often, researchers apply some variation of this rule without thinking too deeply about what forces are at work that drive these prices to their equilibrium values. This is a mistake because even the father of this important insight was worried about what mechanisms in the real economy might lead to this 'pure' result. In fact, there is much debate about what type of outcomes this mechanisms produce. Further, as we will see in the Edgeworth Model, the utlimate equilibrium may be significantly affected by the type of mechanism at work. In this section we review the work that has been done on these mechanisms. They can be classified as either a tatônnement process or a trading process[3] The two mechanisms differ in their key assumptions governing trade at non-equilibrium prices. Whereas the tatônement process does not permit trade until the equilibrium price vector has been identified, the trading process does. The next two sections describe these adjustment mechanisms which is then followed by a short discussion on why these early adjustment processes are not suitable for a truly decentralized price system and how one might choose to modify them.

2.1.1 Tatônnement process

The idea behind the tatônnement process is that participants in a market call out prices and then the excess demand function[4] for each commodity is calculated. If the excess demand function is positive, indicating more buyers than sellers at that price then the price of that commodity is increased. Conversely, if the excess demand function is negative, then the price is lowered. The principal characteristic of the tatônnement process is that no trades take place until the set of equilibrium prices are identified[5] . A formal definition of the tatônnement process for a pure exchange economy is given by Takayama [35]. The function f is the excess demand function and define $\hat{p}$, an equilibrium price vector, to be the one such that $f(\hat{p}) = 0$. We then describe the dynamic process by

$$\frac{dp(t)}{dt} = f[p(t)] \quad (= \sum_{i=1}^{m} x_i[p(t)] - \sum_{i=1}^{m} \bar{x}_i) \tag{2.1}$$

[3] Fisher [11] (p.27) proposes the term 'trading' as a more natural way to describe dynamic price adjustment mechanisms which allow trade at non-equilibrium prices (i.e. non-tatônement processes). It is a convention that is adopted in this paper.

[4] $f(x_i) = Q_i^d - Q_i^s$

[5] Walras has said that this type of adjustment mechanism was inspired by the Bourse of Paris which, at Walras' time only allowed trades when demand equalled supply (Daal and Jolink [7], p. 165.

Where

m is the number of individuals in the economy.

$p(t)$ is a vector of prices $[p_1(t), p_2(t), ..., p_n(t)]$

f is the excess demand function.

x_{ij} is individual i's demand for commodity j.

$\bar{x}_{ij}$ is individual i's holdings of commodity j.

2.1.2 Trading process

A trading process is another way of describing how a market clears. As opposed to the tatônnement process where trade takes place only at the equilibrium, here trade occurs at each set of prices on the path towards equilibrium. A set of prices are announced then the agents try to satisfy their plans at that price. However, if the set of prices was not an equilibrium set, then there will be either buyers or sellers who are unable to complete their transactions. That is, if the price was too low for a commodity, then the sellers will not have provided enough of it to satisfy the demand at that price. Nevertheless, transactions are allowed to take place at this non-equilibrium set of prices and then the remaining excess demands indicate which direction the prices should be adjusted. Again, Takayama [35] provides a formal definition of this process in the case of a pure exchange economy. In the trading process, the above dynamic adjustment equation is replaced by

$$\frac{dp_j(t)}{dt} = \sum_{i=1}^{m} x_{ij}[p(t), \bar{x}_1(t), \bar{x}_2(t), ..., \bar{x}_m(t)] - \sum_{i=1}^{m} \bar{x}_{ij}(t), \tag{2.2}$$

$$j = 1, 2, ..., n$$

and

$$\frac{d\bar{x}_{ij}(t)}{dt} = F_{ij}[p(t), \bar{x}_1(t), ..., \bar{x}_m(t)], \tag{2.3}$$

$$i = 1, 2, ..., m;$$

$$j = 1, 2, ..., n$$

m is the number of individuals in the economy.

n is the number of commodities in the economy.

$p_j(t)$ is the price of commodity j.

$p(t)$ is a vector of prices $[p_1(t), p_2(t), ..., p_n(t)]$

x_{ij} is individual i's demand for commodity j.

$\bar{x}_{ij}$ is individual i's holdings of commodity j.

F_{ij} denotes the transaction rules that individuals follow to change their stock of commodities $\bar{x}_{ij}$.

The first equation is similar to the that of the tatônnement process 2.1. The second equation describes the changes in the quantities held by individuals out of equilibrium and that is the unique feature of the trading process.

2.1.3 Towards a more Suitable Adjustment Process

The problem with these two types of dynamic price adjustment processes is that they are centralized in the sense that they seem to describes the behaviour of some sort of 'market manager'. All agents (whether sellers or buyers) take the price set by the 'market manager' and then carry out their plans according to the rules of the market. But who is the 'market manager' and why does he follow this behavioural rule? As Takayama [35] (p.341) points out, "no straightforward explanation such as the profit maximization of producers or the utility maximization of consumers is given." What makes the above processes centralized (and open to the Takayama critique) is that all agents share the same price vector. In a truly decentralized system, each agent would set their own price based on their perception of the underlying fundamentals of the market. In a full information setting, agents may all choose a common price vector so the idea of having individual price setting is just semantics. But, in our models we want to back off the full information assumption because to assume that agents understand the full state of affairs in all markets then calculate market-clearing prices and name the general equilibrium prices as their offer is simply absurd (Fisher [11], p. 6).

When the full information assumption is discarded, a model absolutely requires a truly decentralized, dynamic adjustment process in that it is the decisions made by the agents in the model which affect prices. Furthermore, we need agents to be aware that there is disequilibrium and still act in a rational way. There are two difficult points made here. The first point draws attention to the need for decentralization of the price system in economic models. The second point raises some questions about the notion of rationality.

The following definition by John Rust describes the type of system we are in search of.

> A decentralized system is one which has no identifiable "center" that controls the behaviour or dynamics of the individual agents (ie. processors, particles, consumers, firms, etc.) comprising the system. Instead, control and information processing in decentralized systems is distributed among the agents comprising the system and these agents are autonomous in the sense that their behaviour or laws of motion are governed primarily by their own "objective functions", although their objectives may be affected by messages, competition, or other types of interactions with other agents in the system.
>
> -John Rust [30] (p.30)

The second point, that agents should recognize that they are in disequilibrium and should act rationally, opens up a fairly difficult issue. This point is made forcefully by Fisher [11] (p.11),

> Proper analysis of the disequilibrium behaviour of agents, however, will require some reformulation of the theories of the individual firm and household. This is because the standard equilibrium approach to microeconomics is reflected in these theories. Agents in the standard theory react to given prices and take no account either of the fact that prices may change or the possibility that they may not be able to complete their own transactions [6] . So long as the plans which agents make are compatible, this presents no difficulty; in equilibrium the equilibrium assumptions of agents are fulfilled. If we are to deal with disequilibrium, however, this will not be the case, and we must start at the level of the individual agents.

What would such a consumer or theory of the firm look like? While this question is outside the scope of this paper, it is worth mentioning two areas of work that provide some guidance on where this might take us. The "Intelligent" Agent section in Chapter 1 offers a glimpse of what this new theory would look like. Broadly speaking, the research agenda should be concerned with building models that are populated by agents that "behave like working economists or econometricians"(Sargent [32], p.22).

While these issues are all fairly difficult to answer and this paper makes only modest gains in this direction, the value of ABM as indicated by Kirman and Vriend [20] suggest that it is an important area to begin an exploration of these problems. They argue that,

> if we want to understand the dynamics of interactive market processes, and the emergent properties of the evolving market structures and outcomes, it might pay to analyze explicitly how agents interact with each other, how information spreads through the market, and how adjustment in disequilibrium takes place...a natural way to do this is following an agent-based computational economics approach.

[6] He notes fixed-price, quantity-constrained equilibrium literature has looked at these issues somewhat.

The next section reviews two ABMs which incorporate decentralized, dynamic trading processes. One thing to keep in mind is that ABMs tend to focus on a particular type of market[7] or phenomenon and are generally fairly complicated. The purpose of the next section, therefore, is simply to give a sense of how decentralized price systems are used in more elaborate models. Consequently, the complexity of these models make it difficult to isolate the effect of the pricing rules used by the agents. So, in the final section, we will build two simple ABMs which allow us to work towards a better understanding of a decentralized price system.

2.2 Decentralized, Dynamic Trading Processes

A common feature of ABMs is decentralized, dynamic trading. The framework almost demands it. This can be seen as an advantage or a disadvantage. In an analytical framework, it is most natural to start with perfect competition complete with full information on the part of agents, zero coordination costs and some sort of centralized scheme for market clearing such as tatôonnement. In ABMs, on the other hand, the starting point is imperfect competition with the modeller having to specify which information agents have access to and impose a decentralized system for market-clearing. In light of the difficulty and complexity of the types of systems we study in economics there is probably much to be learned from both approaches. So, since much focus has been on analytical evaluation in the past, we hope to reveal additional understanding with the help of ABMs. In this section we review two ABM models that incorporate decentralized price systems. In the following section we construct two simple ABMs to evaluate and explore price systems as they appear in the analytic disequilibrium literature[8] .

[7] Markets with rich micro-level data or in which experimental work has been conducted.

[8] An excellent introduction to the disequilibrium literature is Fisher [11].

Models in the ABM literature have the following characteristics:

- Decentralized
- Imperfect information
- Heterogeneous agents
- 'Adaptive' behaviour and learning on the part of agents

The first model that we will review is by Howitt and Clower's "The Emergence of Economic Organization" (1998). The goal of their study is to examine how exchange activities are coordinated in a decentralized economy. Uneasy about conventional equilibrium theory's assumption that exchange plans are "coordinated perfectly by an external agent...with no identifiable real-world counterpart", Howitt and Clower construct a model which has trade being coordinated through a set of agents known as 'specialist trading enterprises.' [17]. These can be thought of as commodity-specific firms such as grocery stores coordinating the exchange of groceries or financial intermediaries coordinating the exchange of capital.

The model is made up of many "transactors" who produce one type of commodity and consume another type of commodity. In order to consume anything then, a transactor must trade with another transactor and, given the assumptions of the model, this trade must be coordinated through a firm ("shop"). Initially, the model has no firms which begs the question, how does any trade ever take place? The answer lies in the actions that a typical transactor will go through each period (Howitt and Clower called this period a "week").

Each period, a transactor will perform two actions: entrepreneurship and exchange[9] In the entrepreneurship phase, a transactor has the opportunity to open

[9] Due to the immense detail in Howitt and Clower's model, the brief discussion presented

a firm. That is, the transactor will perform "market research" and based on his results, he may choose to become a firm specializing in particular commodities. As it turns out, firms specialize in only two commodities which happen to be the same as the transactor-entrepreneur's production and consumption commodities. If a transactor does not end up becoming a firm, they then proceed to the exchange phase. Here, a transactor searches for the firms with the most advantageous terms of trade and then attempts to execute an exchange through those firms. An exchange involves a transactor giving a firm a quantity of one good and in return receiving some quantity of another good. To be more precise, each period, a transactor will have produced some amount of his production good. For the sake of argument call it 'labour'. The transactor then approaches a firm and offers his 'labour'. In return the firm gives him some amount of another good. For the sake of argument, let's call it 'money'. Then, the transactor turns around and approaches another firm and offers this 'money'. In return this new firm gives him some amount of yet another good, such as 'groceries'.

So, even though the model is initially comprised of only transactors, it is the entrepreneurship phase that explains the emergence of some firms in the population. Existing firms perform certain actions each period. These include: pricing and exit. Pricing in this model is described as full cost pricing. That is, "motivated by pursuit of gain, but lacking reliable information about the relation of price to profit, the shop posts prices that yield what the owner regards as a normal return on investment" (Howitt and Clower [17], p.12). To understand pricing in this model it is necessary to walk through the actual exchange process. Unlike the example given above where a transactor sold his production good to one firm and bought his consumption good from another, here we will allow both

here can only give a broad treatment to certain features. While an attempt is made to preserve the essence of their work, a reader who is interested should definitely consult the original paper.

exchanges to take place at a single firm. In the language of the example, the firm specializes in 'labour' and 'groceries'. Both types of arrangements are permitted in the Howitt and Clower model.

The firm posts two prices: P_l, the price it is willing to pay for labour and P_g, the price that it is willing to pay for groceries. If a transactor approaches it with 5 units of labour then the firm will buy that labour by giving the transactor $P_l \cdot 5$ units of groceries. (Note that it is possible that some other transactor with 'groceries' for sale may approach the firm. This explains why the firm would also have to post P_g.) The way in which firms adjust prices is somewhat counterintuitive but the logic is correct. If, at the end of a period, the firm has a lot of 'groceries' on hand, then the firm will increase the price at which it is willing to buy 'groceries'. Let y_l be the firms inventories of labour and let y_g be the firm's inventories of groceries[10] . Then, the prices are given by,

$$\begin{aligned} P_l &= (\frac{y_g - costs}{y_l})^+ \\ P_g &= (\frac{y_l - costs}{y_g})^+ \end{aligned}$$

where the notation x^+ denotes the maximum of x and 0.

Thus, the prices are set by the firms in a sensible way. Take P_l for example, if the firm starts to accumulate inventories of y_l, the quantity in the denominator, then the price for that commodity will fall. So, the Howitt and Clower pricing schema is certainly plausible in that firms react to an excess demand function in the way that we would expect. Also, they capture the rich dynamics of a marketplace with firms entering and exiting at a rapid pace in the early stages of development and then when the market matures, the "creative destruction" process seems to stabilize.

[10] The pricing setup is slightly different in Howitt and Clower but the basic idea remains the same. For a full description of their pricing setup see [17] (p.11-12)

The next pricing scheme comes from the ASPEN model, an agent-based microsimulation of the US economy being developed by Sandia National Laboratories (Basu, Pryor and Quint [4]). Aspen is a fairly elaborate simulation model. There are many complexities in the way that the developers have constructed the various interactions amongst the agents in the model. For example, see Figure 2.1[11] . As is obvious from the diagram there are many different agents in this model. While Aspen includes many of the traditional agents that appear in standard macro models, such as households, firms and the fiscal authority, it takes the next logical step and allows for different types of firms and households. As was mentioned earlier, ABMs permit modellers much flexibility in terms of the heterogeneities that they can include in their models. Aspen capitalizes on this benefit and is an example of how one might go about doing this. In the model, they have four different types of firms: food firms, 'Other nondurable' firms, automakers and housing developers. It is possible that each of these different types of firms may respond differently to shocks. This is a clear advantage of an ABM because by not differentiating between them, a model makes the implicit assumption that these differences don't matter. [12] Since the focus of the present paper is on pricing schemes, the reader is referred to the article by Basu, Pryor and Quint [4] for a review of the many other features of the ASPEN model. The method that firms use to set prices in ASPEN is a good example of how agents might 'learn' about their environment. Firms use a *genetic algorithm learning classifier system* GALCS to determine how to change their prices. This system consists of two components: the state that the firm is in and a set of probabilistic behaviours defined for each state. That is the firm calculates which state it is in and then

[11] Taken from (Basu, Pryor and Quint [4]).

[12] Common sense would suggest that these differences are important. Also, some work in the credit channel literature suggest that monetary policy may have differential impacts on small and large firms.

chooses an action from a distribution of actions. The properties of this distribution can change based on the success or failure of the given action. For example, in ASPEN, a firm determines which state it is in based on four "trends": a) whether or not its price has been recently increasing or decreasing, b) whether or not its sales have been increasing or decreasing, c) whether or not its profits have been increasing or decreasing, and d) whether or not its prices are higher or lower than the industry average. Then, GALCS assigns a probability vector $(P^D, P^I, p^C)$to each state, where P^D is the probability that the firm will decrease its price, P^I is the probability it will increase its price, and P^C is the probability it will keep its price constant.

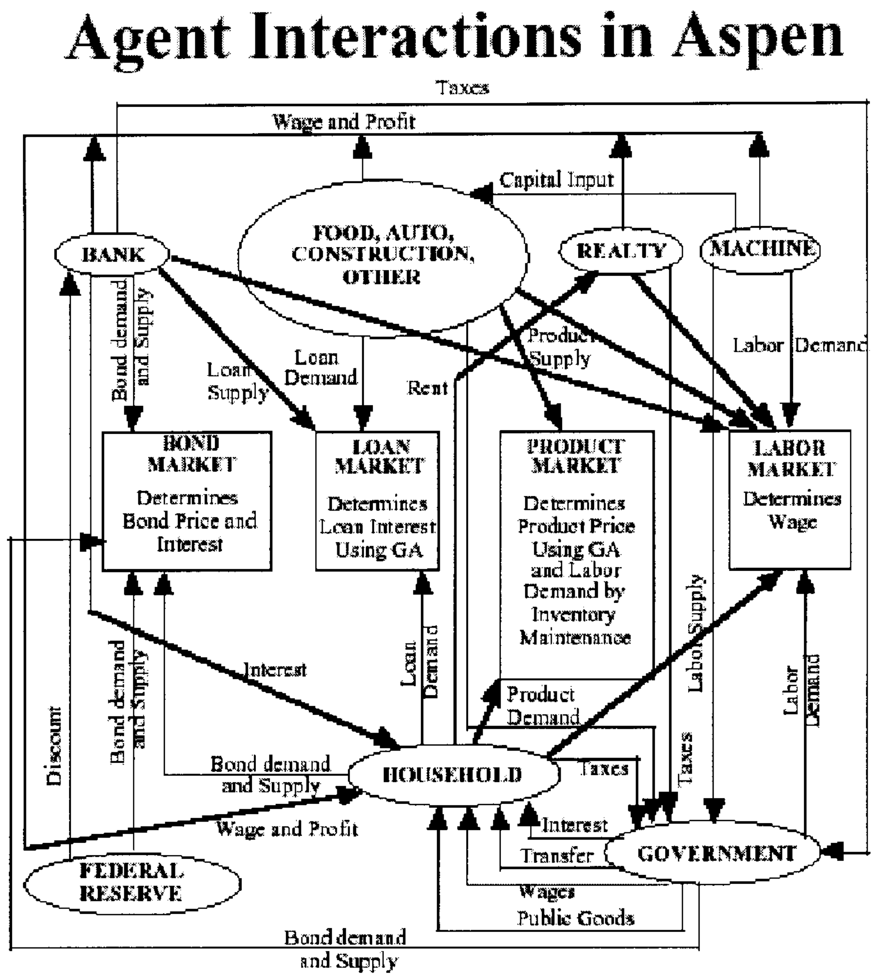

Figure 2.1: The Aspen Model

The expectation here is that when firms learn about their environment, they will make 'good' decisions about how to behave.

Both the Howitt and Clower model and the ASPEN model provide a plausible, decentralized price system. However, it is important to think more carefully about the properties of these price systems. Are they stable? Do they obtain competitive equilibria? The goal of the next section is to start building a framework for answering these types of questions. However, given the complexity of more elaborate models, such as those mentioned above, and the formidable challenge of providing analytical stability proofs, it is necessary to return to simpler models for which these proofs already exist. In the next section, we provide computational examples for two models that are well known in the disequilibrium literature.

2.3 Introduction to the Disequilibrium Models

It is surprising that there has not been more cross-activity between ABMs and the disequilibrium literature. There is a natural fit and many of the papers in these two fields share a strikingly similar motivation. Consequently, the results from the disequilibrium literature in terms of the stability proofs have been largely ignored in the ABM literature. The following computational examples are an exercise in applying the ABM methodology to two disequilibrium trading models. In these models, the only concern is on the price system so we will try to keep them as simple as possible. Since one of the goals is to become more acquainted with the ABM methodology, the models will remain as simple as possible. Despite this simplicity, some interesting results emerge from each of the models. The first model is an application of an Edgeworth trading process. Its behaviour is consistent with what we expect *a priori*, and it highlights the need to think more carefully about "path-dependence". Path-dependence is essentially that property that if agents do not jump instantaneously to equilibrium, then the ultimate equilibrium will be a function of the decisions made in getting there. The second model will be referred to as a Hahn-Fisher trading process as it is Fisher's extension of the work first introduced by Hahn. Trading in the Edgeworth model is the only economic activity permitted. That is, no production or consumption occurs. Much like the introductory general equilibrium models, this model is a pure exchange economy. In the second model, the Fisher-Hahn model, while production and consumption can be thought of as occurring, all commodities are immediately perishable so 'growth' considerations need not be addressed.

2.4 Model I - Edgeworth Model

The Edgeworth barter process consists of successive bilateral trades between two individuals who are randomly matched each period. Trade between individuals is governed by the their preferences and budget constraint. The fundamental assumption with this process is that individuals will participate in trade only if by doing so they can increase their utility. When the process reaches a Pareto optimal point, it cannot move any further by definition; hence it is an equilibrium point. The familiar Edgeworth box (Figure 2.2) captures all of the conditions required for trade in this setup. Person A's preferences are defined by the utility curves which bend towards the lower left corner of the box. An increase in commodity 1 is shown by a movement from left to right along the bottom axis. Likewise, a movement from the bottom of the box to the top represents increasing quantities of commodity 2 for Person A. Person B's preferences are defined by the curves which bend toward the upper right corner of the box. A movement from right to left along the top axis represents an increase in the quantity of commodity 1 for this individual. Movements from top to bottom represent increases in the quantity of commodity 2 for this individual.

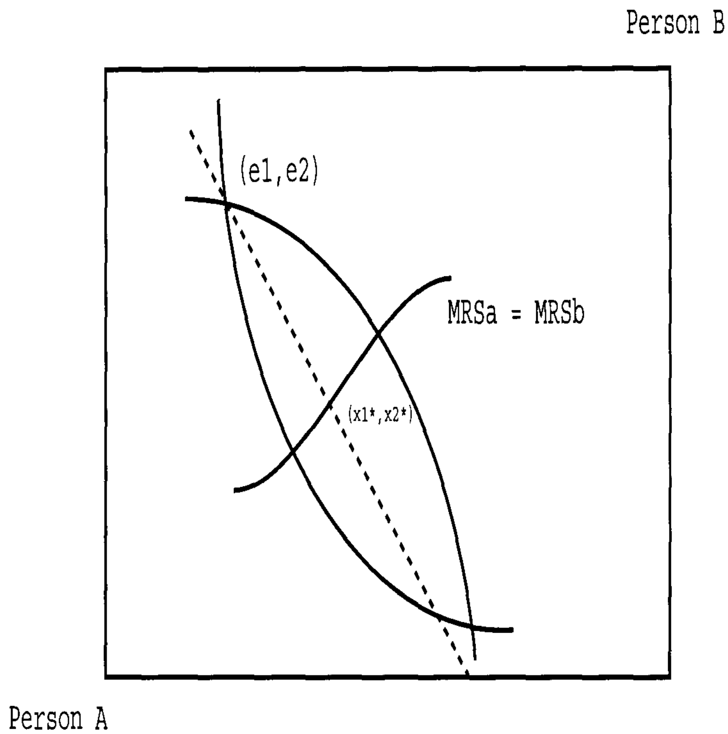

Figure 2.2: The Edgeworth Box

Let u_i be Mr. i's utility function and consider the following constrained maximization problem:

$$Maximize: \quad \sum_{i=1}^{m} \alpha_i u_i(x_i)$$

$$Subject\ to: \quad \sum_{i=1}^{m} x_{ij} = \sum_{i=1}^{m} \bar{x_{ij}},$$

$$\sum_{j=1}^{n} p_j x_{ij} = \sum_{j=1}^{n} p_j \bar{x_{ij}} \quad and \quad u_i(x_i) \geq u_i(\bar{x_i})$$

where $x_i = (x_{i1}, x_{i2}, ..., x_{in})$. Let $\hat{x} = (\hat{x}_1, \hat{x}_2, ..., \hat{x}_m)$ be a solution to this constrained maximization problem. Obviously for each t we have a different $\bar{x_{ij}}(t)$ and hence a different $\hat{x}$. The Edgeworth process moves in the direction of a solution of such a constrained maximum problem; that is,

$$\frac{d\bar{x}_{ij}}{dt} = \hat{x_{ij}}(t) - \bar{x_{ij}}(t), i = 1, ..., m; j = 1, 2, ..., n \tag{2.4}$$

Uzawa [36] (p. 219) shows that Edgeworth's barter process is always globally stable, provided the process has a positive solution starting with an arbitrary positive initial distribution.

The computational example is constructed as a series of bilateral trades between individuals who are randomly matched. The individuals utility functions are defined as Cobb-Douglas, $U = X_1^{\alpha} X_2^{\beta}$. Each individual in the simulation is randomly assigned a value for α and β. Each period a pair of agents are matched up and if there is a utility-imporving exchange available, then the agents will execute the trade. From Figure 2.2, it can be seen that given the endowment point $\bar{X}$, there is a lens that is bound by the two individuals' utility curves through this point. All trades in this lens are utility-improving.

Another concept familiar from micro-economics is that of the contract curve. The curve which is traced out by the condition that $MRS^A = MRS^B$ are the trades that are most efficient in the sense that no further utility-improving trades would be possible between these two agents.[13] There is no reason to assume any trade along the contract curve is more likely than another. One way to think about the 'fairness' of a trade along the contract curve from an agent's perspective is to evaluate it relative to the competitive equilibrium trade. That is, the terms of trade that are derived from an application of the standard competitive equilibrium conditions. While any trade along the contract curve is justifiable, this paper assumed that agents will trade at the 'locally' competitive equilibria[14] . In order to determine the 'local' competitive equilibria, one must determine the Marshallian demand curves for the two individuals involved in the trade.

Calculate Person A's Marshallian demand curves:

$$U^A(x_{1A}, x_{2A}) = x_{1A}^{\alpha} x_{2A}^{\beta}$$

Set up the Lagrangean,

$$\mathcal{L} = x_{1A}^{\alpha} x_{2A}^{\beta} + \lambda_A[e_{1A}P_1 + e_{2A}P_2 - x_{1A}P_1 - x_{2A}P_2]$$
$$\frac{\partial \mathcal{L}}{\partial x_{1A}} = \alpha x_{1A}^{\alpha-1} x_{2A}^{\beta} - \lambda P_1 = 0$$
$$\frac{\partial \mathcal{L}}{\partial x_{2A}} = \beta x_{1A}^{\alpha} x_{2A}^{\beta-1} - \lambda P_2 = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda_A} = e_{1A}P_1 + e_{2A}P_2 - x_{1A}P_1 - x_{2A}P_2 = 0$$

Define: $P = \frac{P_1}{P_2}$ and normalize P_2 at 1. And the Marshallian demand curves for Person A are,

$$x_{1A}* = \frac{\alpha}{\alpha+\beta}(\frac{e_{1A}P + e_{2A}}{P})$$
$$x_{2A}* = \frac{\beta}{\alpha+\beta}(e_{1A}P + e_{2A})$$

[13] Axtell and Epstein's [3] Sugarscape model, while employing Edgeworth trading do not allow their agents to trade along the contract curve.

[14] Models with arbitarily chosen terms of trade were also investigated but it turns out that the non-linearity of agents' preferences make determining the contract curve rather difficult and the results would be qualitatively similar. The difficulty arises due to having to identify the intersection of two non-linear curves: the contract curve and an agent's utility curve.

The same analysis gives us the Marshallian demand curves for person B

$$x_{1B}* = \frac{\gamma}{\gamma+\delta}(\frac{e_{1B}P + e_{2B}}{P})$$
$$x_{2B}* = \frac{\delta}{\gamma+\delta}(e_{1B}P + e_{2B})$$

Competitive General Equilibrium gives,

Excess Demand in market i:

$$E_i = x_{iA} * + x_{iB} * - e_{iA} - e_{iB}$$

For the market for good 2, that is,

$$E_2 = \frac{e_{1A}P + e_{2A}}{1+\alpha/\beta} + \frac{e_{1B}P + e_{2B}}{1+\gamma/\delta} - e_{2A} - e_{2B} = 0$$
$$P[\frac{e_{1A}}{1+\alpha/\beta} + \frac{e_{1B}}{1+\gamma/\delta}] = \frac{e_{2A}}{1+\beta/\alpha} + \frac{e_{2B}}{1+\delta/\gamma}$$

Holds generally as number of agents is increased

$$P = \frac{\sum_{i \in I} \frac{e_{1i}}{1+\alpha_i/\beta_1}}{\sum_{i \in I} \frac{e_{2i}}{1+\beta_i/\alpha_1}}$$

Since agents are trading bilaterlly and, as a result, prices are emerging in a completely decentralized way, there can be many different prices at which a homogeneous commodity trades in any given period. As agents visit more and more neighbours, they will only trade if it is utility-improving. They are, in a sense, learning about their environment. When agents cease trading, it implies that there are no more Pareto improving trades. Since this is the definition of a 'competitive equilibrium', convergence has been achieved. The following two graphs show the evolution of prices over the run of the simulation. In this simple model, the agents are finding the 'competitive equilibria' very quickly as can be seen in Figure 2.4. The mean price of all the trades taking place converges to the 'competitive equilibrium' price denoted by the horizontal line at 1.05 by about the 15th period.

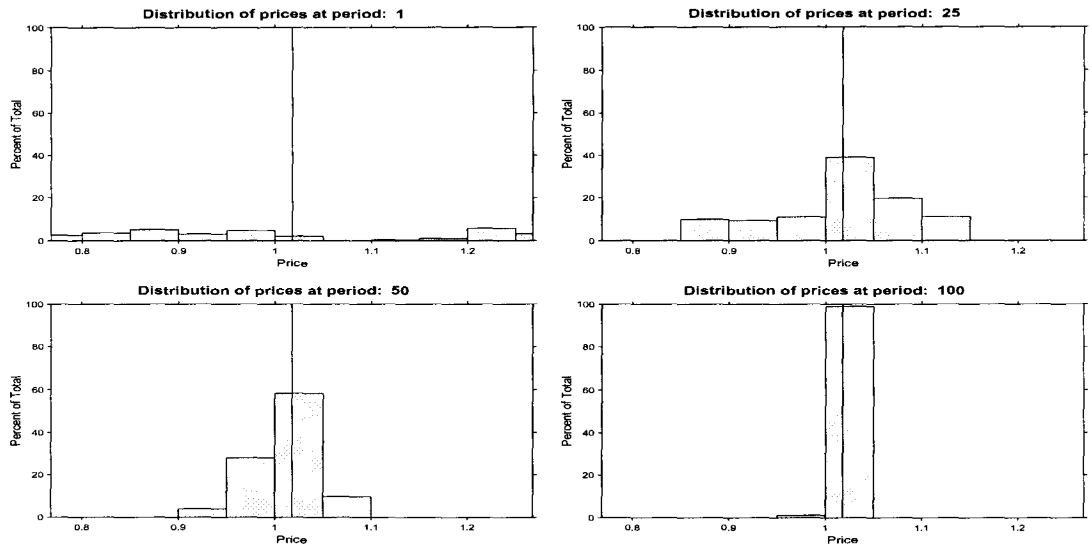

Figure 2.3: Distribution of prices from Edgeworth Model I

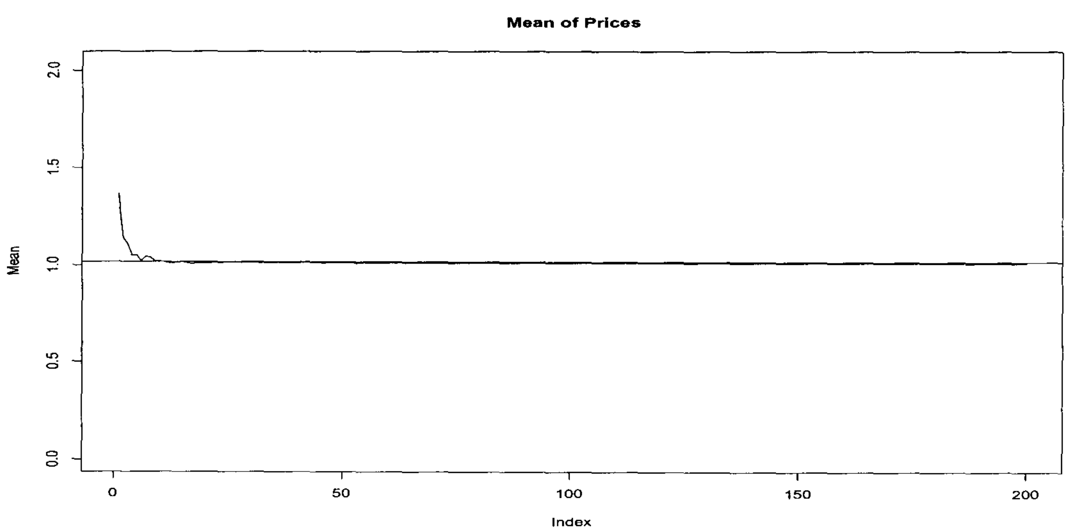

Figure 2.4: Mean of prices from Edgeworth Model I

The following graph, Figure 2.5, shows the prices witnessed by a sample three of the 500 agents in the simulation. It reveals that agents experience different paths to the equilibrium price.

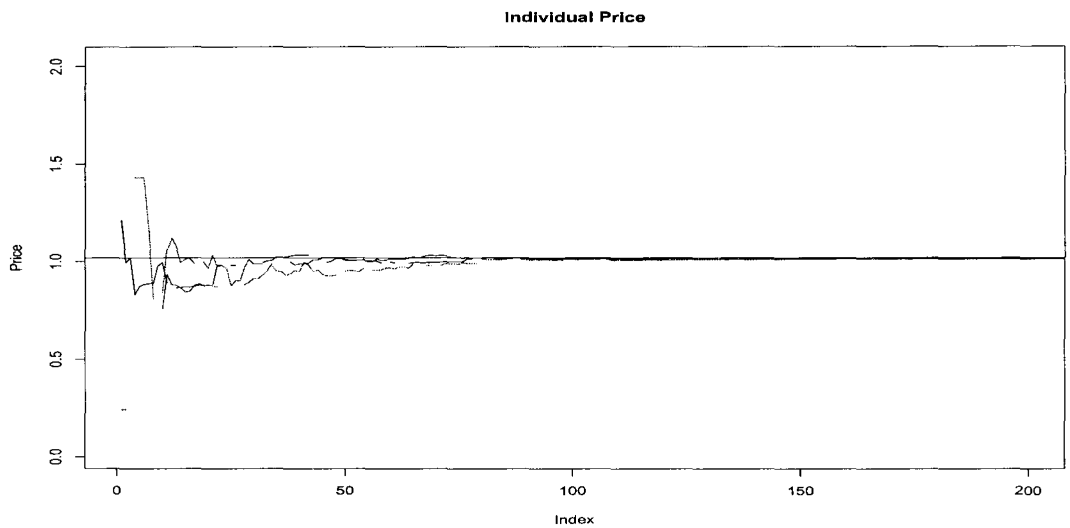

Figure 2.5: Individual prices from Edgeworth Model I

Obviously without empirical work, we cannot say much about the validity of this model. However, the dispersion of prices seems consistent with what we know about real economies. [15] Another aspect of real economies that we can qualitatively explore in our artificial world is its reaction to shocks. Whether these shocks are shifts in the agents preferences (such as would be observed with demographic shifts or simply through changing tastes) or to the supply technologies present in the economy. It may seem a little foolish to talk about these factors in our very simple model but these are necessary first steps towards a more sophisticated model. There is nothing surprising coming from these shocks and they behave as one would anticipate, which is reassuring.

[15] See Lach [23] for an empirical analysis of price dispersion.

Figures 2.6 and 2.7 depict the behaviour of the economy after an income shock. To demonstrate this response, at the 150 period, each agent was endowed with an additional 5 units of commodity 1. This doubled the total supply of commodity 1 and as is obvious from the graphs, the price converges quickly to its new value of 0.5. So, when the total supply of commodity 1 and commodity 2 were virtually identical[16] the price is essentially 1.0. However, once the quantity of commodity 1 is doubled. The price falls to 0.5.

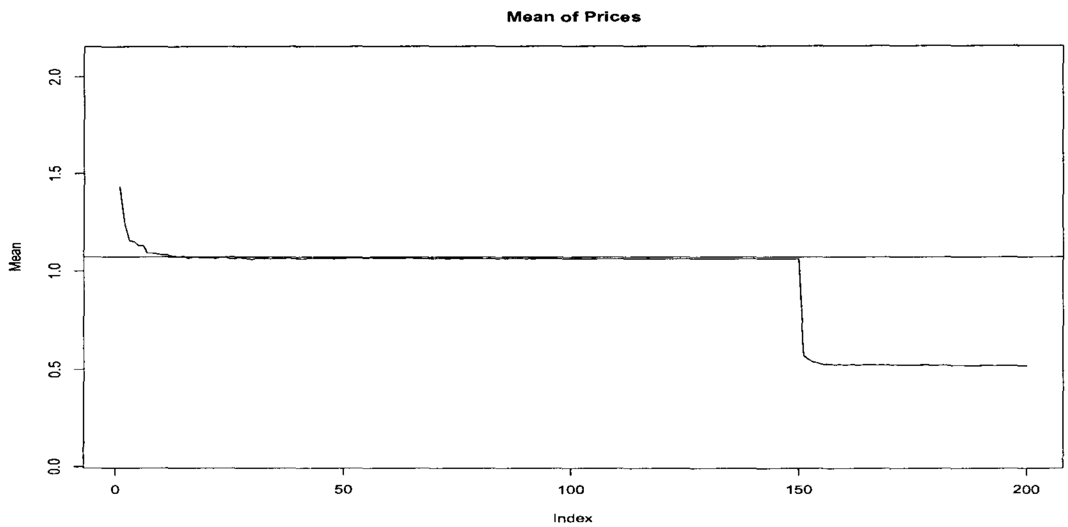

Figure 2.6: Mean of Price from Edgeworth Model I with Shock

[16] Since the original endowments were distributed in a random way, it turns out that there can be slightly more of one commodity or the other. In our model, there was a slightly higher quantity of commodity 2 suggesting that the price should be marginally above 1.0.

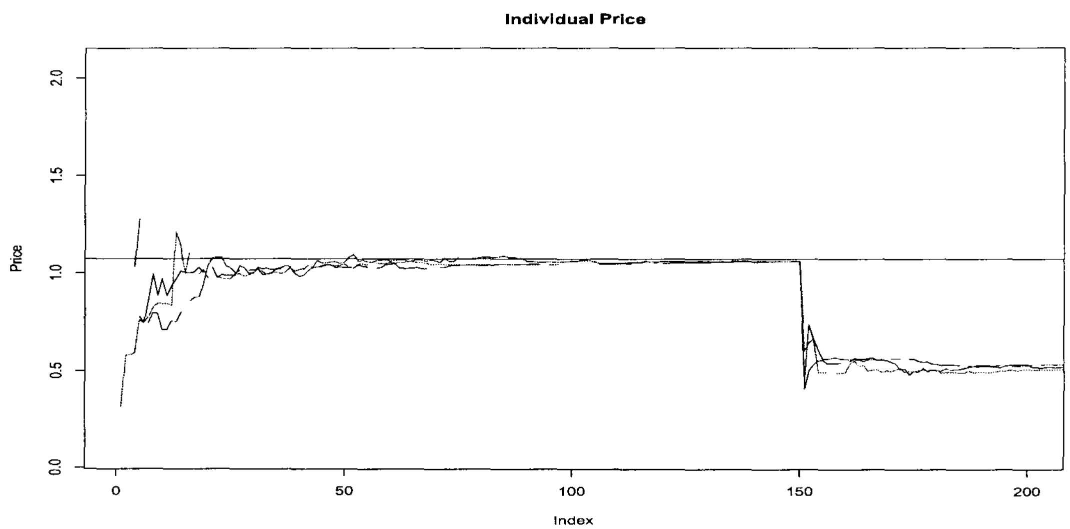

Figure 2.7: Individual Prices from Edgeworth Model I with Shock

Another issue that arises in the context of Edgeworth trading is that of path-dependence. Standard general equilibrium theory assumes that no trade takes place at prices which are different from equilibrium. Thus, we can easily work out what the equilibrium price would be given the preference determined demand and endowment dependent supply of the commodities in our model. However, in our model since trade is not coordinated through some central mechanism, it is possible for agents to exchange commodities before equilibrium is achieved and this can affect the ultimate equilibrium that they arrive at.

For example, when trading is not permitted outside of equilibrium, the price adjustment process can be characterized in the following way,

Individual demand is given by:

$x_i = x_i(p, \bar{x}_i)$

So, market excess demand given by:

$\sum x_i - \sum \bar{x}_i$which can be written as

$\frac{dp(t)}{dt} = f(p, \bar{x_1}, \bar{x_2}, ..., \bar{x_m}).$

that is, without intermediate trading, the individual endowments, $\bar{x}_i$ do not change until $\hat{p}$ is determined. However, if transactions allowed in the process, then, the p's and the $\bar{x}_i$'s change from time to time so that

$$\frac{dp(t)}{dt} = f[p(t), \bar{x}_1(t), \bar{x}_2(t), ..., \bar{x}_m(t)] \tag{2.5}$$

and this has important implications for the ultimate equilibrium that obtains in this model.

So, even the simple Edgeworth trading model produces some interesting dynamics which are qualitatively consitent with the expected behaviour of real economies. However, a shortcoming of this model is that production and consumption considerations are not easily introduced[17] . Another difficulty with this

[17] The reader interested in this subject should review Saldahna [31].

model is raised by Fisher [11].

> While it seems innocuous to assume that individuals will not trade unless they can better themselves by doing so, it is not nearly so simple to assume that trade actually will take place whenever such a situation arises. This is because of the possibility that the only coalitions that can better themselves by mutual trade consists of very large numbers of people. Thus it is possible that there is no mutually advantageous bilateral or trilateral or quadrilateral trade and that the only mutually advantageous trade involves a very complicated swapping of commodities among millions of people. To require, as the Edgeworth Process does, that such a trade must take place is to put very heavy requirements on the dissemination of information and to assume away the costs of coalition formation. (Fisher [11], pg. 30).

But, as Madden [25] demonstrates, this criticism is not fully warranted since he proves that if there's an Edgeworth exchange for some set of agents, then there's an Edgeworth exchange for some pair of agents. That is, Edgeworth exchange requires only bilateral trades. While Fisher's assessment of this aspect of the Edgeworth process was not entirely accurate, he raises a deeper issue about the prohibitive costs of 'do-it-yourself' exchange, a point recognized by Howitt and Clower. This was the rationale in their model where trade occurred only "through the intermediation of firms that establish trading times, affirm the quality of commodities traded, develop procedures to enforce contracts, transfer control of commodities, and so forth" (Howitt and Clower [17], p.6). The model presented in the next section offers some hope in addressing these two difficulties.

2.5 Model II - Hahn-Fisher Model

The formal description of the Hahn process is given in Takayama [35]. The usual setup is a pure exchange economy with trading permitted at non-equilibrium prices. The original Hahn process is centralized in the sense that all agents observe a common price vector and the adjustment of prices is through some 'unspecified' agent. It is worth reviewing since the extension by Fisher to a decentralized model draws heavily on the original work by Hahn. It is based on the assumption that if there is an excess supply of a certain commodity, then all the buyers of this commodity can achieve their desires, and that if there is an excess demand for a certain commodity, then all the sellers of this commodity can achieve their desires. The following relations illustrate this process:

$\frac{dp_j(t)}{dt} = f[\sum_{i=1}^{m} x_{ij}(t) - \sum_{i=1}^{m} \bar{x}_{ij}(t)]$

(1) (Disequilibrium) If $x_{ij}(t) - \bar{x}_{ij}(t) \neq 0$, then sign $[x_{ij}(t) - \bar{x}_{ij}(t)]$ = sign $[\sum_{i=1}^{m} x_{ij}(t) - \sum_{i=1}^{m} \bar{x}_{ij}(t)]$, for all i = 1,2,...,m; j=1,2,...,n.

(2) (Equilibrium) If $[\sum_{i=1}^{m} x_{ij}(t) - \sum_{i=1}^{m} \bar{x}_{ij}(t)] = 0$, then $x_{ij}(t) - \bar{x}_{ij}(t) = 0$, for all i=1,2,...,m.

Basically, the Hahn process states that prices are adjusted according to the sign of the aggregate excess demand function taken with the assumption that markets are sufficiently well organized so that if there is aggregate excess supply, then all buyers can buy (and conversely, if there is aggregate excess demand then all sellers can sell) at non-equilibrium prices. For example, in disequilibrium, if after trade person i finds that he has an excess supply of commodity j (that is, $x_{ij}(t) - \bar{x}_{ij}(t) > 0$), then aggregate excess demand will also be negative. Prices are then adjusted according to the sign of the aggregate excess demand function. Hahn and Negishi [13] have allegedly proved the stability of this process.

The extension by Fisher involves allowing the individuals present in the model to be responsible for adjusting the price vector. In this model, instead of having a common price vector shared by all agents in the economy, prices should be thought of as a matrix with each agent being represented down the rows and commodities being represented across the columns. The original model by Fisher is simple in that all commodities are perishable, so no consideration must be given to inventory concerns. Also, in his original model[18] Fisher focuses on a partial equilibrium setup, that is, he focusses exclusively on one market with a constant number of firms (who always sell) and consumers (who always buy). Fisher admits that the model is not all that sensible but serves merely as a starting point for the discussion of these issues. Sellers behave in the not very sensible way of "setting prices, behaving as though their demand curves were flat, and adjusting their prices according to whether or not they sell out all their supplies. It is not hard to show, given reasonable restrictions on the search behaviour of buyers, that the process can be made to converge to competitive equilibrium" (Fisher [11]). Since the modest goal of this paper is merely to operationalize the models present in the disequilibrium literature such as the Fisher-Hahn model, the next few sections will provide a description the actual coding used to construct these models. The final section will raise a few points about the Fisher-Hahn process which suggest that the competitive equilibrium result may not necessarily obtain and warrants further attention.

The actual coding for the model is given in Appendix C and all attempts were made to be as faithful to the model presented in Fisher [10]. There are two agents: the FisherConsumer and the FisherFirm. At the beginning of each round, consumers are given income and firms produce an amount determined by their supply schedule. Both the income and the amount produced will perish before the beginning of the next round. When a FisherConsumer is called upon to move, he will look around for the firm with the lowest price, and attempt to trade with that firm by reporting how much he would like to buy at the firm's posted price. That amount is determined by the FisherConsumer's demand function. The demand

[18] Fisher developed progressively more sophisticated models over the next couple of decades since this original paper in 1963.

function was of the form:

$$y^d = income/p \tag{2.6}$$

with income varying between different consumers. In certain specifications, if the FisherConsumer visited a firm which had run out of its product, then that consumer could continue looking for the next-best firm. At the end of a FisherConsumer's turn, he would simply wander about in a random way. [19] The number of consumers is fixed at 500 [20] .

Once all FisherConsumers have moved, the FisherFirms must decide how to adjust their posted prices. In this model, FisherFirms all produce along a supply curve of the following form,

$$y^s = 2.5 * p. \tag{2.7}$$

The value 2.5 was chosen in an arbitray way but does not make much difference except to move the equilibrium price around. As specified in Fisher [10], "each firm adjusts its price with the rate of change a monotonic function of its own excess demand." As such, FisherFirms will change their price according to the following equation:

$$price = price * (1 + excessDemand/speedAdj)$$

Where the variable speedAdj is the speed of adjustment and is assigned randomly across the firms. The number of firms is constant at 20.[21]

[19] Clearly, a desirable extension would be to make this movement endogenous.

[20] Except in the last model where the purpose was to isolate the reason why prices were not converging in the expected fashion.

[21] Except in the last model where the purpose was to isolate the reason why prices were not converging in the expected fashion.

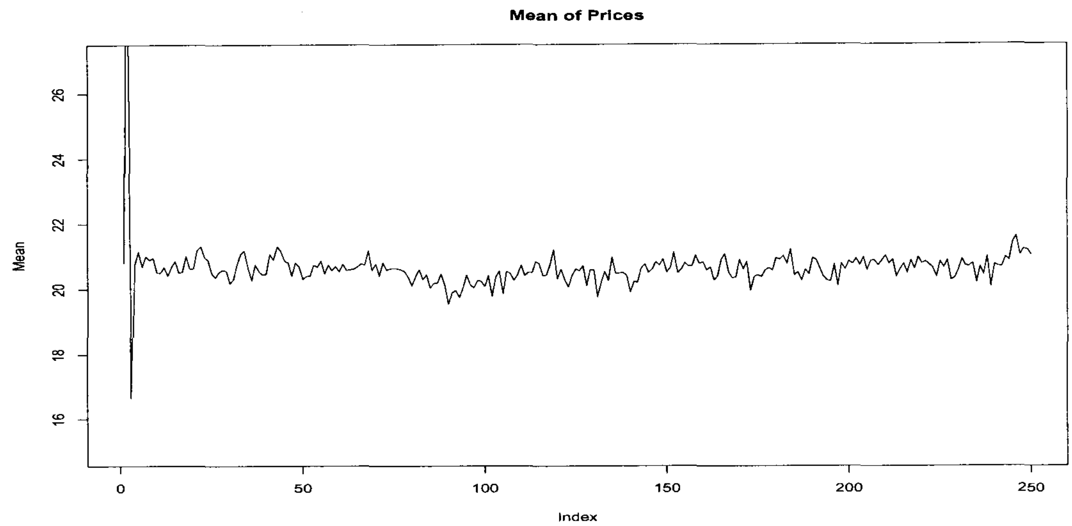

Figure 2.8: Mean of Prices from Hahn-Fisher Model

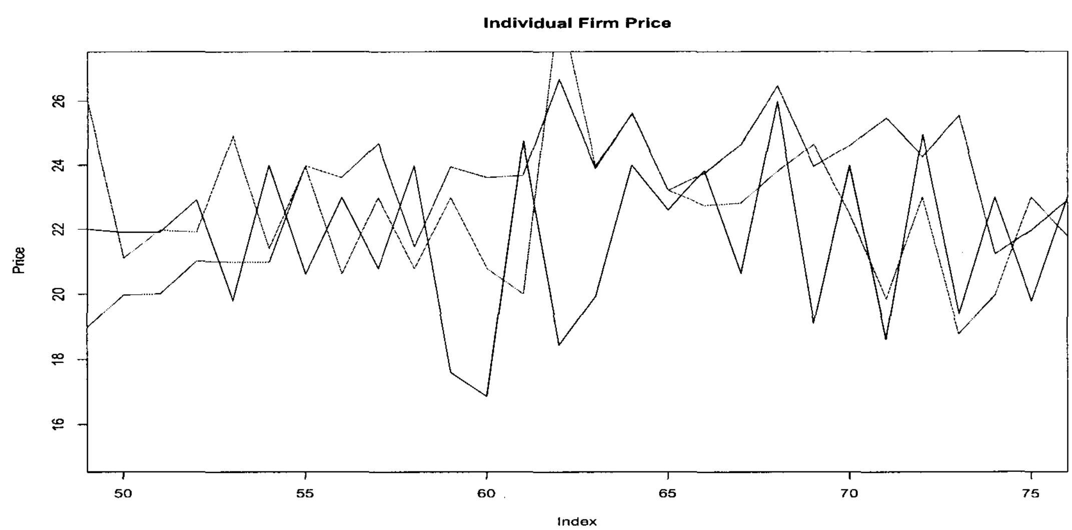

Figure 2.9: Individual Prices observed by Three Firms in the Hahn-Fisher Model

A surprising result in this application of the Fisher-Hahn model is that the price level does not seem to converge to its 'competitive equilibrium' value. Fisher [10] provides an analytic proof suggesting that the price should converge. However, when we actually constructed the model, this did not appear to be the case. In a brief exchange with Dr. Fisher, he suggests that 'discontinuities' in the demand curve facing firms may be the culprit behind this puzzle. In his proof, he assumes that demand curves facing firms are continuous but goes on to say that as consumers switch from a high priced firm to a low priced firm, this will create discontinuities in the demand curve. The following two graphs are an attempt to explain this difficulty.

Figure 2.10 shows the result from a model with 2 firms and 5 consumers. As before, FisherConsumer's consumption is given by

$$y^d = \$100/p$$

and a FisherFirm's supply is given by,

$$y^s = 2.5 * p.$$

Since there are 5 consumers, the aggregate consumption function would be,

$$Y^d = \$500/p$$

and, with 2 firms, the aggregate supply function is.

$$Y^s = 5 * p.$$

This suggests that the competitive equilibrium price would be,

$$Y^d = Y^s$$

$$500/p = 5 * p$$

$$p^2 = 100$$

$$\hat{p} = 10.$$

We can see from Figure 2.10 that prices do not converge to this value. Note also, that path-dependence is not an issue here as the commodities are perishable. Figure 2.11 provides some insight into why the price does not converge. Given the mechanistic way that firms are adjusting their prices, it is possible that they enter into the recurring pattern displayed in the figure. Recall that consumers will go looking for the firm with the lowest price. In order to figure out what is going on, we constructed a model with five consumers and two firms in such a way that the low priced firm will always get all of the consumers (unless the two firms set the same price whereupon they will share demand equally). So, the high priced firm receives no consumers and thus observes excess supply. In the following period, he will decrease his price. The low priced firm, on the other hand, will get all of the demand and so think that there is excess demand for his product. He will increase his price next period. However, what actually happens next period is that the firms just switch roles. That is, the low priced firm becomes the high priced firm and thinks there is now excess supply in the market. And the formerly high priced firm becomes the low priced firm and thinks that there is excess demand in the market. This is what gives rise to the recurring pattern.

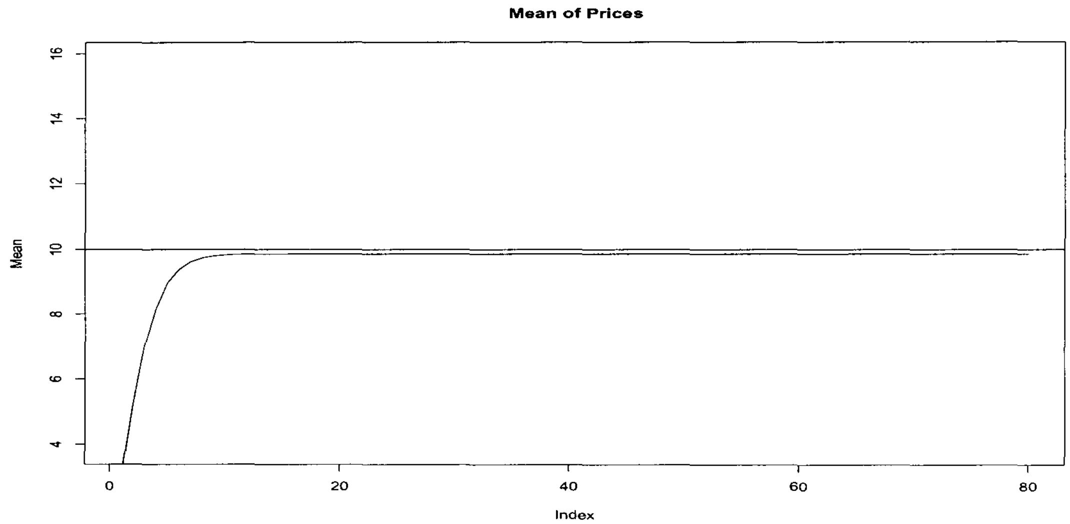

Figure 2.10: Mean of Prices from Hahn-Fisher Model with only Two Firms

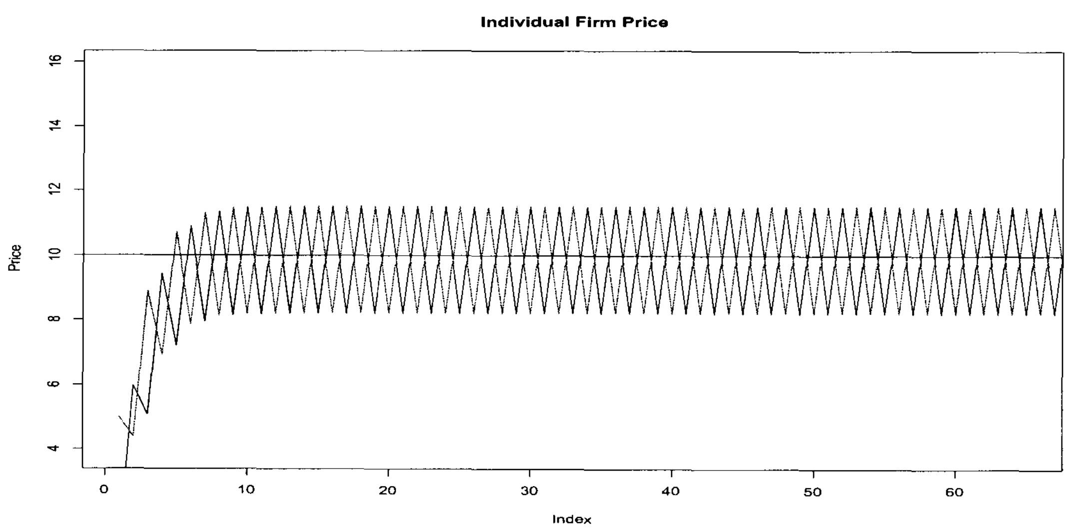

Figure 2.11: Individual Prices from Hahn-Fisher Model with only two firms

While it is true that there is a discontinuity in the demand curve facing the individual firm, it is not clear that it is possible to get around this and still have consumers search for the lowest priced firm. Moreover, the following analysis suggests that it is not the discontinuity which is causing problems for us. Assume that there is only one firm so that there is no opportunity for consumers to switch firms and create discontinuities in the demand curve. In the first panel it is obvious that $\hat{p}$ is the price that clears the market. However, suppose that the firm does not know this market-clearing price and uses the Hahn-Fisher method to try to find it. If the firm sets $p = p_1$ in the first period, it will observe excess supply. This suggests that it will lower its price next period. Let's say that the firm lowers its price to $p = p_2$. Now, the firm observes that there is excess demand in the market and will want to raise its price next period. What can, and does, happen is that it is possible that the firm gets locked into a pattern similar to the one we observed in the two firm case. Before finding the equilibrium price, it finds two prices(one above and one below $\hat{p}$) in which excess demand exactly equals excess supply and then end up oscillating between these two points forever.

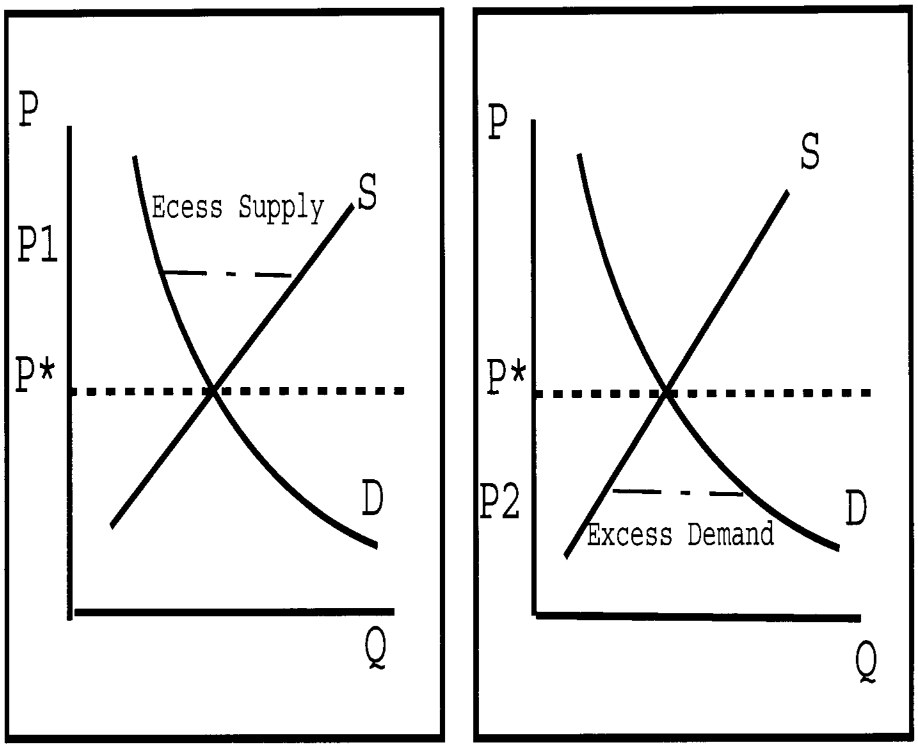

Figure 2.12: The Hahn-Fisher Process does not Necessarily Clear the Market

2.6 Conclusions

In Chapter 1, we introduced the idea of an ABM and reviewed some of the programming concepts which facilitate their implementation. We also discussed some areas where ABMs may have some potential benefit in helping macro model builders get around three key challenges: rich microfoundations, the aggregation problem and agent decision-making. In Chapter 2, we sought a fuller understanding of a decentralized price system through the use of ABMs. We reviewed two models in the literature to investigate how decentralized price systems have been incorporated into more elaborate setups. Appealing to the need for simplicity, we constructed ABMs to examine the convergence properties of two models appearing in the disequilibrium literature: the Edgeworth model and the Hahn-Fisher model. The Edgeworth model illustrates the need to think more carefully about this issue of path-dependence. The implementation of the Hahn-Fisher model gave rise to a question about the convergence properties of this process.

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Appendix A

Java Code for ExchangeAgent

```
    public void trade(){

        neighbors = model.agentGrid.getMooreNeighbors(x,y,
agentVision,agentVision,false);

       if (neighbors.size() > 0 ){
            ExchangeAgent tempAgent = (ExchangeAgent)neighbors.firstElement();

            double[] v = tradePos(this,tempAgent);
            goods[0] = v[0];
            goods[1] = v[1];

            tempAgent.goods[0] = v[2];
            tempAgent.goods[1] = v[3];
       }
   }
```

```
public void randomWalk(){
int newX, newY;

             newY = y + Uniform.staticNextIntFromTo(-1,1);
             newX = x + Uniform.staticNextIntFromTo(-1,1);

// Is there an agent at the new position already? If not, put a
// null at this agent's current position and put this agent at the
// new position.

        if(model.getAgentAt(newX,newY)==null)
            model.moveAgent(this, newX, newY);
        else{

            newY = y + Uniform.staticNextIntFromTo(-1,1);
            newX = x + Uniform.staticNextIntFromTo(-1,1);
            if(model.getAgentAt(newX,newY)==null)
            model.moveAgent(this, newX, newY);
        }
    }

public double utility(ExchangeAgent agent){
        double alpha = agent.alpha;
        double beta = agent.beta;
        double g1 = agent.goods[0];
        double g2 = agent.goods[1];

        double u = Math.pow(g1,alpha)*Math.pow(g2,beta);

        return u;
}

public double MRS(ExchangeAgent agent){
        double alpha = agent.alpha;
        double beta = agent.beta;
        double e1 = agent.goods[0];
        double e2 = agent.goods[1];

        double MRS = (alpha/beta)*(e2/e1);

        return MRS;
}
```

```
public double[] tradePos(ExchangeAgent agent1, ExchangeAgent agent2){

        // a1, b1, etc. defs.
        double[] a = agent1.goods;
        double a1 = a[0];
        double a2 = a[1];
        double alpha = agent1.alpha;
        double beta = agent1.beta;
        double UaBAR = utility(agent1);
        double MRSa = MRS(agent1);

        double[] b = agent2.goods;
        double b1 = b[0];
        double b2 = b[1];
        double gamma = agent2.alpha;
        double delta = agent2.beta;
        double UbBAR = utility(agent2);
        double MRSb = MRS(agent2);

        double e1 = a1+b1;
        double e2 = a2+b2;
        //double lambda = 0.5;//Uniform.staticNextDoubleFromTo(0.4,0.6);
        //double m = -(lambda*MRSa + (1-lambda)*MRSb);  // slope of price line

        double P = -(a2/(1+beta/alpha) + b2/(1+delta/gamma))/(a1/(1+alpha/beta)
+ b1/(1+gamma/delta));
        double A = (delta*alpha)/(gamma*beta);

        // Price line -> X2 = c + mX1; m defined as above
        double c = a2 - P*a1;
        // Two equations: X2 = c + mX1 and X2 = X1*e2/(A*e1 - A*X1 + X1)
        // A is (delta*alpha)/(gamma*beta); Second equation is Contract Curve
        //System.out.println("  m: " + m + "  c: " + c);
        // (m-mA)X1^2 + (mAe1+c-cA-e2)X1 + cAe1 = 0
        // haul out the quadratic formula; soli =
//(- BB +- (BB^2 - 4AACC)^(1/2))/2AA
        double AA = P-P*A;
        double BB = P*A*e1 + c - c*A - e2;
        double CC = c*A*e1;

        double x11,x12,x21,x22;
        if(AA == 0){
            System.out.println("AA is 0");
```

```
            System.out.println("BB is " + BB + " and CC is " + CC);
            x11 = -CC/BB;
            x12 = -CC/BB;
            x21 = c + P*x11;
            x22 = c + P*x12;
        }
        else {
            x11 = (- BB + Math.pow((Math.pow(BB,2) - 4*AA*CC),(0.5)))/(2*AA);
            x12 = (- BB - Math.pow((Math.pow(BB,2) - 4*AA*CC),(0.5)))/(2*AA);
            x21 = c + P*x11;
            x22 = c + P*x12;
        }

        double[] v = new double[4];

        if(x11>0 && x21>0){
            v[0] = x11;
            v[1] = x21;
            v[2] = (e1-x11);
            v[3] = (e2-x21);
        }
        if(x12>0 && x22>0){
            v[0] = x12;
            v[1] = x22;
            v[2] = (e1-x12);
            v[3] = (e2-x22);

        }
        if(x11>0 && x21>0 && x12>0 && x22>0)
            System.out.println("Houston we may have a problem!");

        if(Math.abs(v[0]-a[0])>0){
            model.priceData[(int)(model.getTickCount()-1)]
    [agent1.identity-1] = -P;
            model.priceData[(int)(model.getTickCount()-1)]
    [agent2.identity-1] = -P;
        }

        return v;
}
```

Appendix B

Java Code for FisherConsumer

```
public void trade(){
Vector shops = shops();
        FisherFirm bestFirm = bestFirm(shops);
        double Qd, bought;
        double income = model.consumerIncome;
        if(bestFirm != null) {
            Qd = income/bestFirm.price;
            bought = bestFirm.sales(Qd);
            income = income - bought*bestFirm.price;
            shops.remove(bestFirm);
            bestFirm = bestFirm(shops);
        }
}

public Vector shops(){
Vector shops = new Vector();
        neighbors = model.agentGrid.getMooreNeighbors(x,y,agentVision,
agentVision,false);
        for (Enumeration e = neighbors.elements();
e.hasMoreElements();) {
                FisherAgent tempAgent = (FisherAgent)e.nextElement();
                if(tempAgent.getClass() ==
model.firmList.get(0).getClass())
                    shops.add((FisherFirm)tempAgent);
        }
        return shops;
}
```

```
public FisherFirm bestFirm(Vector shops){
if(shops.size() > 0){
            FisherFirm bestFirm = (FisherFirm)shops.firstElement();
            for (Enumeration e = shops.elements() ;
    e.hasMoreElements();){
                    FisherFirm firm = (FisherFirm)e.nextElement();
                    if(firm.price < bestFirm.price) bestFirm = firm;
            }
            return bestFirm;
        }
        else return null;
}
```

Appendix C

Java Code for FisherFirm

```
public void step(){
        model.priceData[(int)model.getTickCount()-1]
[identity-1] = price;
        price();
        inventory = 2.5*price;
        model.prodData[(int)model.getTickCount()-1]
[identity-1] = excessDemand;
        excessDemand = 0 - inventory;
    }

public double sales(double Q){

        if(inventory >= Q){
            inventory -= Q;
            excessDemand += Q;
            return Q;
        }
        else if(inventory < Q && inventory > 0){
            excessDemand += Q;
            Q = inventory;
            inventory = 0;
            return Q;
        }
        else {
            excessDemand += Q;
            return 0;
        }
    }
public void price(){
        price = price*(1 + excessDemand/speedAdj);
    }
```