A STUDY OF THE ASTROPHYSICALLY IMPORTANT STATES OF $^{31}\text{S}$ VIA THE $^{32}\text{S}(d,t)^{31}\text{S}$ REACTION
A STUDY OF THE ASTROPHYSICALLY IMPORTANT STATES OF

$^{31}\text{S}$ VIA THE $^{32}\text{S}(d,t)^{31}\text{S}$ REACTION

By

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A Study of the Astrophysically Important States of $^{31}\text{S}$ via the $^{32}\text{S}(d,t)^{31}\text{S}$ Reaction

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Professor Alan A. Chen

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Abstract

The astrophysical $^{30}\text{P}(p,\gamma)^{31}\text{S}$ reaction rate is a key quantity used in both classical nova and type I X-ray burst models that predict isotopic abundances produced during nucleosynthesis in the outburst. Currently, uncertainties in $^{31}\text{S}$ structure parameters lead to a variation in the reaction rate by a factor of 20 at nova temperatures ($0.1 < T < 0.4$ GK) causing predicted isotopic abundance ratios in the Si-Ar mass region to vary by factors of up to 4. The $^{30}\text{P}(p,\gamma)^{31}\text{S}$ reaction rate can be determined indirectly by measuring transfer reactions populating excited states in $^{31}\text{S}$. Nuclear structure information for $^{31}\text{S}$ resonant states above the proton threshold of 6131 keV and within the Gamow window that contribute most significantly to the reaction rate can be used to re-evaluate the rate for nova and type I X-ray burst temperatures and reduce current uncertainties.

We have performed an experiment in order to study the level structure of $^{31}\text{S}$ via the $^{32}\text{S}(d,t)^{31}\text{S}$ single-nucleon transfer reaction using the MP tandem accelerator and Q3D magnetic spectrograph at MLL in Munich, Bayern, Germany. Excited states of $^{31}\text{S}$ in the 6-7 MeV region were observed and spin-parity constraints have been suggested.

In this work we describe the experimental setup, data analysis and results for both experiments and provide recommendations for further investigation of the $^{30}\text{P}(p,\gamma)^{31}\text{S}$ astrophysical reaction rate.
To My Mother
&
In the Memory of My Father, William Irvine
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Chapter 1

Introduction

This chapter will explain some of the background knowledge relevant to this specific study in Nuclear Astrophysics as well as provide the scientific motivation behind the interest in $^{31}$S.

1.1 Nucleosynthesis in stellar evolution

Nuclear reactions play an important role in the birth and evolution of the universe. Nuclear reactions are responsible for the chemical elements we see today that were created during the Big-Bang and further formed through nucleosynthesis (production and destruction of nuclear species) by subsequent astrophysical processes. In Nuclear Astrophysics, one of the main objectives is to better understand the structure and evolution of stars which can provide insight into the early days of the universe and how it has chemically evolved over time [1]. Since observing such phenomena directly is impractical due to stellar conditions (ex. temperature, time-scale and location), indirect methods of investigation are employed such as gamma-ray observations. Due to complexity (and the inconvenience of indirect measurements), many astrophysical processes are theoretically constructed using large scale numer-
ical computer models. Many of these models require input quantities that can be directly measured by experimentation. One example of such a quantity is the rate of a specific nuclear reaction involved in the modeled process. This work focuses on experimental quantities needed for reaction rate calculations required for modeling the astrophysical phenomenon known as Classical Novae. An introduction to the basic mechanisms behind the nucleosynthesis of Classical Novae is given in the following subsections.

1.1.1 Birth of a White Dwarf: Hydrogen and Helium burning

Stars are formed when low temperature gas clouds contract to a higher density in the interstellar medium [2]. Initially, the energy generated near the center of the cloud due to the conversion of gravitational energy is radiated away from the star. As the density increases, the energy is no longer completely radiated away and is stored in the interior of the star. As the star contracts further, the gravitational energy causes the stellar interior to heat up. Once the temperature at the core reaches about \(1-2 \times 10^7\) K, fusion of hydrogen nuclei becomes energetically possible providing a new energy source for the star. As hydrogen is converted to helium (hydrogen burning) by the pp-chain reactions, the star is supported from further gravitational collapse by the outward thermal pressure generated from fusion. During this long sequence of the stars life, its size, luminosity and temperature remain unchanged. The so-called main-sequence stars make up the majority of the visible stars in the universe including our own Sun. When all the hydrogen in the core has been converted to helium, the main-sequence ends and the nuclear energy needed to keep the star from contracting further is no longer generated.

As the star contracts further, the core density and temperature increase until the conditions for helium burning are sufficiently met (\(T \sim 0.1–0.4\) GK; \(\rho \sim 10^2–10^5\) g/cm\(^3\)) [3]. The core is now supported against gravitational collapse due to the fusion of helium into carbon and oxygen, known as the triple alpha process. During this process, hydrogen burning still occurs in outer shells surrounding the core. This is the general evolution of a star: As its current nuclear fuel is burned up, it contracts
in order to raise the temperature to burn the next available nuclear fuel. If a star’s mass is too low, it cannot generate the core temperatures required to fuse carbon (the product of helium burning) and an inactive mass of carbon and oxygen will build up at its center. Such stars are known as white dwarfs.

Generally, white dwarfs are composed mostly of carbon and oxygen. However, in some cases the star is massive enough to cause fusion of carbon into neon (and helium), producing a neon-oxygen white dwarf [4]. Trace amounts of magnesium can also be found in white dwarfs originating from stars massive enough to fuse neon with helium into magnesium. The mass of the white dwarf depends on the mass of its progenitor. Masses can vary between $0.16 - 1.33 M_\odot$ in the most extreme cases between low-mass helium white dwarfs to the ultra-massive oxygen-neon white dwarfs [5].

When the star can no longer support itself against gravitational contraction, the density begins to increase causing the core to become degenerate. Degenerate gases arise from the Pauli Exclusion Principle, which says that no two fermions (integer spin particles) can occupy the same quantum state. As the density increases, the gas will resist compression since the electrons can not move into lower occupied orbits (quantum levels). The pressure arising from the electron degeneracy of the core (which is unaffected by temperature unlike thermal pressure) supports the star from further collapse. White dwarfs typically have radii on the order of $1/100$ that of the sun, and is comparable to the Earth’s radius.

1.1.2 Formation of Classical Novae

Classical Novae occur in close binary systems (2 stars in close proximity) consisting of a white dwarf called the primary and a main-sequence star called the donor, whose mass is usually less than the primary’s. When the material on the surface of the donor star (mostly hydrogen and some helium) expands past a region of space in which the material is no longer gravitationally bound to the star (known as the Roche lobe - see figure 1.1), it moves toward the inner lagrangian point and is transferred to the Roche lobe of the primary. In order for the material to cross over beyond the inner lagrangian point into the potential well of the primary, the material

$^{1}M_\odot$ represents the mass of the Sun.
must be pushed from behind by the pressure of the stellar atmosphere surrounding the donor [5]. The material then collects in an accretion disk that surrounds the white dwarf. Some of this accreted matter spirals in and accumulates on the surface of the white dwarf, which is quickly heated due to high surface gravity compression. Eventually, the inner shell of this material becomes degenerate and energetic enough to ignite hydrogen burning via the pp chain, which causes an increase in temperature. The increase in temperature has no effect on the expansion of this layer which is governed by degeneracy pressure and so the temperature continues to increase, fueling more and more fusion reactions. This leads to thermonuclear runaway at the base of the accreted material which eventually causes an explosion.

During thermonuclear runaway, the nuclear burning is dominated by explosive hydrogen burning as a result of breakouts from the CNO cycle\(^2\) that occur at high temperatures \((T \geq 0.1\text{ GK})\) [6]. As a result, heavier nuclear species can be synthesized via \((p,\gamma)\) and \((p,\alpha)\) capture reactions and \(\beta^+\) decays with a typical nucleosynthetic endpoint at calcium [5]. During the explosion, the accreted material on the surface as well as the material synthesized during explosive hydrogen burning is ejected into the interstellar medium. Due to the complexity involved in theoretically predicting these outbursts, analyzing the ejecta can give insight into the dominant

\(^2\)for more information on the CNO cycles, see [1] and [2].
nucleosynthetic pathways occurring during thermonuclear runaway leading up to the explosion.

Nova outbursts eject approximately $10^{-5} - 10^{-4} \text{M}_\odot$ of matter at velocities around $10^6 \text{ m/s}$ . Mixing can also occur between the surface material and the white dwarf core material (Carbon-Oxygen or Oxygen-Neon) which provides more catalyst nuclei for the CNO cycle, thus producing a more energetic explosion. Typical peak temperatures for nova nucleosynthesis occur between 0.1–0.4 GK.

1.2 Motivation for the study of $^{31}\text{S}$

1.2.1 The $^{30}\text{P}(p,\gamma)^{31}\text{S}$ reaction and its importance in astrophysics

The reaction in which $^{30}\text{P}$ captures a proton to form $^{31}\text{S}$ while emitting a gamma ray can be formulaically written as:

$$^{30}\text{P} + p \rightarrow^{31}\text{S} + \gamma,$$

and is an important reaction that occurs in certain astrophysical phenomena. The main focus of this work is the impact of this reaction on nucleosynthesis during classical nova outbursts. As previously discussed, during the explosive hydrogen burning phase leading up to the nova explosion, there are many nuclear reactions that occur. It has been found that the $^{30}\text{P}(p,\gamma)^{31}\text{S}$ reaction serves as a bottleneck for the reaction flow at $A=30$, and therefore its rate directly affects predicted abundances of nuclei in the $30 \leq A \leq 40$ region in nova models [7]. Figure 1.2 gives a better picture of the various reactions leading to this mass region.

The favoured reaction sequences that serve as a passing point to Sulfur and beyond are either $^{30}\text{P}(p,\gamma)^{31}\text{S}(p,\gamma)^{32}\text{Cl}(\beta^+)^{32}\text{S}$ or $^{30}\text{P}(p,\gamma)^{31}\text{S}(\beta^+)^{31}\text{P}(p,\gamma)^{32}\text{S}$ [8]. Other sequences are strongly suppressed as in the case of the $^{29}\text{P}(p,\gamma)^{30}\text{S}$ reaction which $\beta^+$ decays to $^{30}\text{P}$ ($t_{1/2} = 1.18$ s) because of a $(p,\gamma)$-$(\gamma,p)$ equilibrium that occurs between $^{30}\text{S}$ and $^{31}\text{Cl}$ ($Q_{\gamma\gamma} = 284(7)$ keV [9]). Another slow channel is the $^{30}\text{P}(\beta^+)^{30}\text{Si}(p,\gamma)^{31}\text{P}(p,\gamma)^{32}\text{S}$ reaction due to the slow $\beta^+$ decay of $^{30}\text{P}$ ($t_{1/2} = 2.5$
min). Also, since there is only a small initial amount of $^{31}\text{P}$ in the envelope, it must first be synthesized in order for the nucleosynthesis to proceed through $^{31}\text{P}(p,\gamma)^{32}\text{S}$.

$^{30}\text{P}(p,\gamma)^{31}\text{S}$ also plays an important role in linking isotopic abundance ratios of Si found in presolar grains to the nucleosynthetic path in the nova itself. Remnants of stellar processes are sometimes found in fallen meteorites that are broken up and dissolved in acids in laboratories to isolate grains originating from a time before the formation of our solar system and studied via secondary ion mass spectroscopy [10]. Most of the presolar grains found in meteorites are thought to be of supernova origin or from Asymptotic Giant Branch (AGB) stars [11, 12]. However, infrared observations support the formation of SiC, C and O-rich silicate grains originating from classical novae, which could suggest another possible source of the isolated grains. Presolar grains of suspected nova origin extracted from the Murchison meteorite have been found to contain $^{30}\text{Si}/^{28}\text{Si}$ abundance ratios in excess of the solar value by factors of 1.04–2.11 [13, 14]. Early models of ONe nova nucleosynthesis suggested a higher-than-solar value of the $^{30}\text{Si}/^{28}\text{Si}$ ratio, which agreed qualitatively with the experimentally observed ratios and therefore provided the initial diagnostic for the grain origin [15].

The $^{30}\text{P}(p,\gamma)^{31}\text{S}$ reaction is directly related to the production of $^{30}\text{Si}$ since $^{30}\text{P}$ ($\beta^+$) decays to $^{30}\text{Si}$ and therefore, any increase or decrease in the rate of $^{30}\text{P}(p,\gamma)$ should greatly affect the $^{30}\text{Si}$ abundance predicted from nova models. It is therefore necessary to determine the reaction rate of the $^{30}\text{P}(p,\gamma)^{31}\text{S}$ reaction used as input for nova models in order to both fine tune these models to quantitatively agree with observed abundances and to provide good arguments for the identification of future presolar grains.

The reaction rate is also important at higher temperatures as an input to models of the rapid proton capture (rp) process as is the case for type I x-ray bursts (peak $T \approx 1.5\text{ GK}$) [16, 9]. Type I x-ray bursts are similar to Classical Novae in the sense that they occur within a binary system, except with a neutron star in place of the white dwarf. Neutron stars are composed mainly of neutrons and are typically more massive and dense than white dwarfs (mass $\approx 1.4M_\odot$; $\rho \approx 10^{14}\text{ g/cm}^3$) [1]. Nucleosynthesis occurs similarly to that of classical novae leading to a thermonuclear runaway but the peak temperatures are much higher than in classical novae because
of the conversion of a larger amount of gravitational energy. Since neutron stars exert more gravitational force, the material on the surface is not ejected during the explosion but a huge amount of energy is emitted in the form of x-rays, which are observed as a burst. Thus, type I x-ray bursts most likely do not play an important role in contributing to the chemical composition of the universe but are important in determining properties of the neutron star such as its composition, radius and mass. After a burst is executed, a new shell of matter is accreted and the process is repeated. Typical burst times occur over less than 1 minute and repeat after a few hours to a few days.

Figure 1.2: Nuclear reaction pathways leading to higher mass nuclei in the Si-Cl mass region. The solid lines represent the dominant reactions/decays leading to higher mass nuclei while the dashed lines represent slower reactions/decays or reactions/decays that do not lead to the synthesis of nuclei with masses above Sulfur. The half lives of the relevant $\beta^+$ decays are also listed.

### 1.2.2 Reaction rate formalism

The astrophysical reaction rate is an important quantity of interest in Nuclear Astrophysics. It quantitatively describes the change in nuclear abundances due to nuclear interactions within an astrophysical environment and is given in units of
number of reactions per second per unit volume. Here, the general form of the reaction rate as well as the non-resonant and narrow resonance rates will be derived similar to that found in [1] and [2].

Consider a nuclear reaction in a stellar volume of the form \( A + x \rightarrow B + y \) with number density \( n_A \) and \( n_x \) and mass \( m_A \) and \( m_x \) of nuclei \( A \) and \( x \), respectively. According to the theory of nuclear reactions [17], the probability for a single reaction to occur when nuclei \( A \) and \( x \) are within a certain vicinity of each other (related to an area), known as the cross section \( \sigma(v) \), depends on the relative velocity \( v = v_A - v_x \) between \( A \) and \( x \), which is related to the total kinetic energy in the center of mass frame by

\[
E_{cm} = \frac{1}{2} \mu v^2 \tag{1.1}
\]

with the reduced mass defined as \( \mu = m_A m_x / (m_A + m_x) \). If \( A \) is defined as the target nucleus at rest and \( x \) is the projectile then in a time interval \( \Delta t \), \( n_x \) nuclei will encounter \( n_A v \Delta t \) projectiles per unit area along the path of the projectile. The reaction rate can be written as

\[
r = \frac{n_x n_A v \Delta t \sigma(v)}{\Delta t} = n_x n_A v \sigma(v) \tag{1.2}
\]

However, in a stellar environment, the velocity of the nuclei varies significantly and can be described probabilistically by the Maxwell-Boltzmann (MB) distribution

\[
\Phi(v) = 4 \pi v^2 \left( \frac{\mu}{2 \pi k T} \right)^{3/2} \exp \left( -\frac{\mu v^2}{2 k T} \right) \tag{1.3}
\]

where \( k \) is the Boltzmann constant and \( T \) is the temperature of the stellar environment. \( \Phi \) is a probability distribution and should be normalized such that

\[
\int_0^\infty \Phi(v) dv = 1 \tag{1.4}
\]

The reaction rate must be modified in order to take into account the distribution of probable velocities. The reaction rate is averaged over this distribution and thus becomes
\[ r = n_x n_A \int_0^\infty v \Phi(v) \sigma(v) dv \]
\[ = n_x n_A 4\pi \left( \frac{\mu}{2\pi kT} \right)^{3/2} \int_0^\infty v^3 \exp \left( -\frac{\mu v^2}{2kT} \right) \sigma(v) dv \]
\[ = n_x n_A \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E \exp \left( -\frac{E}{kT} \right) \sigma(E) dE \] (1.5)

where Eq. (1.1) was used to transform the integral. Eq. (1.5) is valid if \( A \) and \( x \) are non-identical. The cross section depends on the specific nuclear reaction and therefore some assumptions need to be made in order to evaluate the integral.

First, let us look at the non-resonant reaction rate. Non-resonant reactions have cross sections that vary smoothly with incident kinetic energy. If the incident particle has a high enough kinetic energy such that its de Broglie wavelength is relatively small compared to the size of the target nucleus, it interacts with only a few nucleons on the surface of the target. Such reactions are known as direct reactions. If we assume the cross section is of the form

\[ \sigma(E) = \frac{1}{E} \left( \exp(-2\pi \eta) \right) S(E) \] (1.6)

where the \( 1/E \) term is related to the square of the de Broglie wavelength that describes the energy dependence on the spatial distribution of the interacting nuclei. The exponential term is called the Gamow factor and \( \eta \) is the Sommerfeld parameter given as

\[ 2\pi \eta = 2\pi \frac{Z_A Z_x e^2}{\hbar} \sqrt{\frac{\mu}{E}} = 31.29 Z_A Z_x \sqrt{\frac{\mu}{E}} \] (1.7)

where \( \mu \) is the reduced mass in atomic mass units (amu), \( Z \) is the number of protons and \( E \) is the center of mass energy in keV. The Gamow factor describes the dependence of the cross section due to the Coulomb barrier arising from charged particle interactions and is valid for particle energies far below the Coulomb barrier. As expected, the probability for a particle to tunnel through this barrier increases with particle energy and decays very rapidly for low energies.

The last term \( S(E) \) in the expression for the cross section is called the astrophysical \( S \)-factor and contains the more intrinsic nuclear properties of the specific
reaction, like the structure of the final bound state and the attenuation of the barrier by the nuclear mean field [18]. For non-resonant reactions, the $S$-factor varies smoothly with energy and can be used to extrapolate the cross sections at very low energies comparable to those found in stellar environments that cannot be mimicked in the lab setting. We can re-write Eq. (1.5) using Eqs. (1.6) and (1.7) which yields the expression

$$r = n_A n_x \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \exp \left( \frac{-E}{kT} - \frac{\sqrt{E} G_E}{E} \right) dE$$

(1.8)

where, $E_G$ is the Gamow energy and is given as

$$E_G = \left( -\frac{2\pi Z_A Z_x e^2}{\hbar} \right)^2 \frac{\mu}{2} = 0.978(Z_A Z_x)^2 \mu \text{ (MeV)}$$

(1.9)

The energy dependence in Eq. (1.8) comes from the Maxwell-Boltzmann (MB) factor $e^{-E/kT}$ arising from the velocity distribution in the stellar environment and the Gamow factor, $e^{\sqrt{-E_G/E}}$, arising from the Coulomb barrier [19]. Figure 1.3 shows the product of these two factors producing a new function called the Gamow window. The Gamow window has a peak at $E_0$ defined as the Gamow peak and is found to be

$$E_0 = \left( \frac{kT \sqrt{E_G}}{2} \right)^{2/3} = 1.22(Z_A^2 Z_x^2 \mu T_6^2)^{2/3} \text{ (keV)}$$

(1.10)

where $T_6$ is the temperature in units of MK. The width of the Gamow peak acts as an energy window in which thermonuclear reactions take place for a given stellar temperature. In order to quantify the Gamow peak width, the product of the exponential terms can be approximated by a Gaussian function as

$$\exp \left( -\frac{E}{kT} - \frac{\sqrt{E_G}}{E} \right) \approx \exp \left( -\frac{3E_0}{kT} \right) \exp \left[ -\left( \frac{E - E_0}{\Delta/2} \right)^2 \right]$$

(1.11)

where $\Delta$ is defined as the width of the function at $1/e$ of the peak height given below as

$$\Delta = 4 \sqrt{3} \sqrt{E_0 kT} = 0.749(Z_A^2 Z_x^2 \mu T_6^5)^{1/6} \text{ (keV)}$$

(1.12)
Figure 1.3: (a) shows the product of the Maxwell-Boltzmann factor (dashed line) and the Gamow factor (dashed dotted line) called the Gamow peak (solid line) as a function of energy on a log scale. (b) shows the same Gamow peak on a linear scale (solid line) and the symmetric Gaussian approximation (dashed line) used in Eq. (1.11). The figure is adopted from [1].

and so the effective region in which fusion reactions occur should be between \( E_0 - \Delta/2 \) to \( E_0 + \Delta/2 \).

In the case of some non-resonant reactions, the \( S \)-factor does not change significantly over the Gamow window and can be approximated as a constant \( S(E) = S(E_0) = S(0) = \text{constant} \). However, in most cases, the \( S \)-factor varies less weakly
with energy and can be approximated by a Taylor series expansion around \( E = 0 \) as

\[
S(E) \approx S(0) + S'(0)E + \frac{1}{2}S''(0)E^2 + \cdots \quad (1.13)
\]

Finally, by substituting Eqs. (1.11) and (1.13) into (1.8) and performing the integration, we obtain for the non-resonant reaction rate

\[
r = n_A n_x \times 1.30 \times 10^{-14} (Z_A Z_x \mu)^{1/3} T^{-2/3} S_{\text{eff}}
\]

\[
\times \exp \left[ -4.249 \left( \frac{Z_A^2 Z_x^2 \mu}{T_9} \right)^{1/3} \right] \text{(cm}^{-3}\text{s}^{-1}) \quad (1.14)
\]

where \( S_{\text{eff}} \) is given by

\[
S_{\text{eff}}(E_0) = S(0) \left[ 1 + \frac{5kT}{36E_0} + \frac{S'(0)}{S(0)} \left( E_0 + \frac{35}{36} kT \right) \right.
\]

\[
+ \frac{1}{2} \frac{S''(0)}{S(0)} \left( E_0^2 + \frac{89}{36} E_0 kT \right) \right] \quad (1.15)
\]

Eq. (1.14) dominates the total reaction rate at low stellar temperatures. However, as the temperature of the stellar environment increases (therefore shifting the Gamow window), the \( S \)-factor does not vary so smoothly and contributions from resonant reactions begin to dominate the overall rate.

For resonant reactions, the cross section varies significantly with incident kinetic energy. If the projectile has low enough energy so that its de Broglie wavelength is of the size of the target nucleus, it will temporarily fuse with the target nucleus forming a compound nucleus \( C^* \) [5]. If enough energy is transferred to a nucleon or small group of nucleons within the compound nucleus, it will subsequently de-excite through particle or gamma-ray emission. Contrary to direct reactions, it is a two step process shown below

\[
A + x \rightarrow C^* \rightarrow B + y \quad (1.16)
\]

The excitation energy of the final nucleus \( E_{\text{ex}} \) is greater than the threshold energy or equivalently

\[
E_{\text{ex}} = Q + E_r \quad (1.17)
\]
where $E_r$ is defined as the resonance energy and the reaction Q-value is the difference in the rest energy (or equivalently in the kinetic energy due to energy conservation) between the reactants and products given by the relation

$$Q = (m_A + m_x - m_B - m_y)c^2 = E_B + E_y - E_A - E_x$$  

(1.18)

Another important quantity to consider is the total decay width ($\Gamma$) which is related to the probability for the nucleus to decay. Since the final nucleus is in a state that can decay through both its entrance channel (1) and exit channel (2), the total decay width is a sum of the two individual partial widths $\Gamma = \Gamma_1 + \Gamma_2$. For the purposes of this study, we will focus on isolated and narrow resonances which occur within the temperature region for classical novae as well as type I X-ray bursts. Isolated resonances occur when the spatial separation between resonances is enough to avoid any overlap between their decay widths. A narrow resonance occurs when its total width $\Gamma$ is significantly less than the resonance energy of the state $E_r$. In this case, the cross section can be approximated by the single-level Breit-Wigner formula given as

$$\sigma_{BW}(E) = \frac{\lambda^2}{4\pi} \frac{2J + 1}{(2J_A + 1)(2J_x + 1)} \left(1 + \delta_{Ax}\right) \frac{\Gamma_1 \Gamma_2}{(E - E_r)^2 + \left(\frac{\Gamma}{2}\right)^2}$$  

(1.19)

where $J_A, J_x$ and $J$ are the spins of the target, projectile and resonant state respectively, $E$ is the total energy in the center-of-mass frame, $\lambda$ is the de Broglie wavelength and the factor $(1 + \delta_{Ax})$ accounts for the case when $A$ and $x$ are identical. The relation between the nuclear spins arises from angular momentum conservation and are related by the equations

$$J = J_A \oplus J_x \oplus l$$  

(1.20)

where $\oplus$ represents the vector sum of the quantities and $l$ is the relative orbital angular momentum between the projectile and target nucleus. Parity must also be conserved and follows the relation

$$\pi(J) = \pi(J_A)\pi(J_x)(-1)^l$$  

(1.21)

If a resonance is narrow, then the MB factor does not vary significantly over the
total width and can be approximated as $e^{-E_r/kT}$. Combining this with Eq. (1.19) and substituting into Eq. (1.5), the reaction rate for a single narrow resonance between non-identical particles becomes

$$r = n_A n_x \left( \frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E \sigma_{BW}(E) \exp \left( \frac{-E}{kT} \right) dE$$

$$= n_A n_x \frac{2\hbar^2 \sqrt{2\pi}}{(\mu kT)^{3/2}} \exp \left( \frac{-E_r}{kT} \right) \frac{\Gamma_1 \Gamma_2}{\Gamma} \int_0^\infty \frac{\Gamma/2}{(E - E_r)^2 + \Gamma_2^2} dE$$

$$= n_A n_x \left( \frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 \omega \gamma \exp \left( \frac{-E_r}{kT} \right)$$

(1.22)

where the quantity $\omega \gamma$ is called the resonance strength and is defined to be

$$\omega \gamma = \frac{(2J + 1)}{(2J_A + 1)(2J_x + 1)} \frac{\Gamma_1 \Gamma_2}{\Gamma}$$

(1.23)

If more than one resonance contributes to the cross section, Eq. (1.22) must be summed over all resonances. Thus, the reaction rate for multiple narrow resonances becomes

$$r = n_A n_x \left( \frac{2\pi}{\mu kT} \right)^{3/2} \hbar^2 \sum \omega \gamma_i \exp \left( \frac{-E_i}{kT} \right)$$

(1.24)

Let us turn our attention towards the partial decay widths. For the $^{30}\text{P}(p,\gamma)^{31}\text{S}$ reaction, the partial widths of the entrance and exit channel are $\Gamma_p$ and $\Gamma_\gamma$, respectively. For the case that $\Gamma_p \ll \Gamma_\gamma$ (which occur for resonances up to $E_r \approx 0.5$ MeV), the idea of an effective energy window in which only resonances that fall within that window contribute significantly to the reaction rate is valid. In this case $\Gamma \approx \Gamma_\gamma$ and the resonance strength becomes $\omega \gamma \approx \omega \Gamma_p$ which depends on the charged particle partial width explicitly and the Coulomb barrier penetrability factor implicitly. For higher stellar temperatures in which resonances above $E_r \approx 0.5$ MeV contribute to the reaction rate, the particle partial width dominates the total width and the resonance strength is approximated as $\omega \gamma \approx \omega \Gamma_\gamma$. Thus, the rate is no longer dominated by resonances within the Gamow window since the strength does not depend on the Coulomb barrier penetration probability. In this case, significant contributions to the rate come from lower resonances through the exponential factor $e^{-E_r/kT}$ [1]. The
derivation for the reaction rate due to broad resonances\(^3\) is not included in this work and can be found in [1] and [2]. The total stellar reaction rate is determined by adding the resonant and non-resonant reaction rates for a given temperature.

### 1.2.3 Excited levels of \(^{31}\)S: Reaction rate and uncertainty effects

An important feature to note about Eq. (1.24) is the explicit dependence on the resonance energy and spin of the final nucleus and the implicit dependence on the parity through Eqs. (1.20–1.21). Knowledge of these quantities for each possible contributing resonant state is needed in order to evaluate the astrophysical reaction rate for narrow resonances. Uncertainties in these quantities, especially the resonance energy which appears in the exponent, can greatly affect the rate and therefore the predicted abundances from theoretical stellar models. Using Eqs. (1.10) and (1.11), the resonances in \(^{31}\)S that contribute significantly to the \(^{30}\)P\((p,\gamma)^{31}\)S \((Q = 6130.9(4)\) keV \([20]\)) reaction rate for classical novae \((T_{\text{peak}} = 0.1–0.4\) GK) occur between \(E_r \sim 115–534\) keV corresponding to an excitation energy range between \(E_{\text{ex}} \sim 6246–6665\) keV. Similarly, \(^{31}\)S resonant states between \(E_{\text{ex}} \sim 6390\) keV to well over 7 MeV contribute to the \(^{30}\)P\((p,\gamma)^{31}\)S reaction rate for type I X-ray bursts \((T = 0.4–1.5\) GK).

Many resonant states between 6–7 MeV lack firm spin-parity assignments \((J^\pi\) values\)) and the existence of states that have not been observed cannot yet be excluded. Thus, the \(^{30}\)P\((p,\gamma)^{31}\)S reaction rate is highly uncertain over the temperature region of interest for classical novae and type I X-ray bursts. Figure 1.4 shows an energy level scheme for \(^{31}\)S resonances contributing to the reaction rate between 0.1 and 0.4 GK.

In early models of novae nucleosynthesis such as that found in [7], the \(^{30}\)P\((p,\gamma)^{31}\)S reaction rate was so poorly known that the uncertainty was as large as a factor of 100 up and 100 down, which caused the abundance ratios of isotopes between A=30–37 to vary by factors of \(\sim 2–500\) (500 for \(^{30}\)Si/\(^{28}\)Si) \([7]\). Since 2002, new resonant states have been discovered and more \(J^\pi\) assignments have been determined, which has led to a decrease in the rate uncertainty \([8, 16, 20]\). Also, the use of a Monte

\[^3\]A broad resonance is defined as a resonance in which \(\Gamma/E_r \geq 10\%\) \([2]\).
Figure 1.4: Excited states in $^{31}$S contributing to the $^{30}$P($p,\gamma$)$^{31}$S reaction rate over the temperature region $T=0.1$–$0.4$ GK. The red line indicates the region of resonant states that fall within the Gamow window for temperatures relevant to nova nucleosynthesis. Some levels within this region lack definite spin and parity assignments needed for the determination of the rate. Energies listed are in keV and taken from [16] and [20].

Carlo approximation for calculating rates, which is beyond the scope of this work and is described in detail by [21, 22, 23, 24], has led to a better determination of the uncertainty in the $^{30}$P($p,\gamma$)$^{31}$S reaction rate. In a recent study by Parikh et al. [20] looking at the $^{31}$P($^3$He,$t$)$^{31}$S reaction, it was determined that the current $^{30}$P($p,\gamma$)$^{31}$S reaction rate uncertainty for nova nucleosynthesis leads to a variance in the $^{30}$Si/$^{28}$Si abundance ratio by a factor of 4, and points to unconstrained $^{31}$S $J^\pi$ values for resonances in the region between 6.4–6.6 MeV as the main contribution to the uncertainty.

Unfortunately, the $^{30}$P($p,\gamma$)$^{31}$S reaction cannot be studied directly as there are currently no radioactive $^{30}$P beams available at a high enough intensity to produce significant statistics for analysis. Fortunately there is more than one way to populate resonant states in $^{31}$S using reactions like those studied in [16, 8, 20, 25].
In the present work, resonant states of \(^{31}\)S between 6–7 MeV were populated by the \(^{32}\)S\((d,t)^{31}\)S reaction (\(Q = -8786.2(4)\) keV [26]) in order to determine resonance energies and corresponding \(J^\pi\) values. These quantities can be used to further constrain the \(^{30}\)P\((p,\gamma)^{31}\)S reaction rate and reduce its uncertainty for use as input to stellar models, to further our understanding and classification of many astrophysical processes.
Experimental Approach for the Study of the $^{32}$S($d,t$)$^{31}$S Reaction

This chapter explains the experimental procedure used for the study of excited states in $^{31}$S starting with ion implantation of $^{32}$S targets at The University of Western Ontario in London, Ontario and including the $^{32}$S($d,t$)$^{31}$S reaction performed at The Maier-Leibnitz Laboratorium (MLL) in Munich, Bayern, Germany on two separate occasions in February and July of 2011.

2.1 Ion implantation and target production

Several $^{32}$S targets were made by implanting $^{32}$S ions into natural $^{12}$C foils with the 1.7 MV High Current Tandetron at the Lennard-Hendricks Laboratory located at The University of Western Ontario. The method of ion implantation used to implant the sulfur targets was similar to those used in [27, 28, 29, 30]. Targets prepared using the implantation method exhibit similarities attributed to the precise control of incident beam characteristics which can help to minimize systematic uncertainties associated with non-uniformity [27]. The beam was accelerated by a Tandem Van de Graaf accelerator with a terminal voltage of about 0.5 MV.\(^4\) A Cesium sputter ion source was used to produce $^{32}$S$^-$ ions from a natural iron sulfide target [31]. The $^{32}$S ions are selected out using a magnet which produces a magnetic field $B$ that

\(^4\)See subsection 4.2.2 for more information regarding Tandem accelerators.
discriminates against the desired ions based on mass according to the radius of their path by the equation

\[ r = \frac{mv}{QB} = \frac{p}{QB} \]  

(2.1)

where \( v \), \( Q \) and \( m \) are the velocity, charge and mass of the particle, respectively.

Contrary to most tandem accelerators, no charge stripping was performed. However, the beam particles were accelerated twice because their initial gain in kinetic energy caused by the first acceleration was actually higher than 100 keV and the ions were then decelerated to reach the desired energy. The \(^{32}\)S beam was focused on a layer of \( \sim 40\)-\( \mu \)g/cm\(^2\) of 99% isotopically pure \(^{12}\)C mounted on a glass slide. The carbon slide was placed on a sample holder and held in place by small metal clips. The metal clips were kept in contact with the carbon in order to dissipate the accumulated charge from the carbon slide as it could cause damage to the slide and produce inaccurate readings by the Faraday cage and integrator used to determine the incident beam current. The Fast Faraday cup\(^5\) was dropped in to stop the beam from bombarding the sample during the precise placement and fine tuning of the slide height. The sample chamber was then pumped down to approximately \( 2.2 \times 10^{-7} \) Torr to prevent the incident ions from interacting with any residual gas in the sample chamber. During one of the target implantations, the turbo pump used to pump down the target chamber failed and so there is a possibility that this target may contain some Si or other contaminants due to back flow of the pump oil. This target was not used in the experiment but was put on the target ladder during one of the runs (experiment 1) as a back up (see section 2.2.3 and figure 2.5). The beam spot was simulated by a laser upstream which was used to find where on the sample the beam would be located. In order to fit two implantation sites (targets) on one slide, a separation distance was determined between the beam spot centers. The laser targeting was then turned off and the Faraday cup was removed from the beam pass. A profile was chosen for the implantation that would maximize the amount of Sulphur present in the target based on calculations made by Dr. Lennard and Jack Hendricks, using a program called \textbf{SRIM} which simulates the stopping and ranges of ions in matter. Different doses of beam current at different energies were used to

---

\(^5\)A device used to block the beam located upstream from the target.
maximize the penetration depth and to produce a uniform Gaussian distribution of sulphur (See Figure 2.1). Energies of 100, 90, 80, 70 and 60 keV were used as the incident beam energy with beam currents of 1.3 \( \mu \)A. Input doses were calculated in particles/cm\(^2\), which the in-house software converts to a total charge \( q_c \) in units of C from the relation

\[
q_c = dose \times q \times e \times \text{aperture area} \tag{2.2}
\]

The implanted area is defined by the aperture area which in this case was 1.13 cm\(^2\) (defined by the small aperture setting), and the charge state \( q \) of the beam particles is -1. Table 2.1 shows a summary of the different implantation energies and calculated doses estimated to produce the Gaussian distribution in Figure 2.1.

![Figure 2.1: Number density of \(^{32}\)S ions as a function of depth for an implanted \(^{32}\)S target calculated by SRIM in order to find the optimal dose for each incident beam energy. The dose can be calculated from the area under each curve. Note that the sum of these individual profiles gives a nice gaussian-shaped distribution.](image)

The Faraday cage around the sample collects the charge from the beam current which, is connected to a digital current integrator which puts out a pulse once \( 1 \times 10^{-8} \) C is collected on the cage, adding more charge to the measured current dose. Once the dose reaches the calculated dose, the Faraday cup drops back in to block the beam while the beam is changed to the next desired incident energy. The beam
current in µA is calculated by the integrator which converts the charge/pulse to a current which is displayed as a function of time by the in-house software to help warn the operator in the case of major fluctuations. The sum of the doses in Table 2.1 gives the total dose D (or areal density) which can be converted to an areal thickness density $\Delta x$ (which is referred to as the thickness by those in the ion beam community) by

$$\Delta x = \frac{D \times M}{N_A} \text{ (g/cm}^2\text{)}$$

where $M$ is the molar mass of $^{32}\text{S}$ in g/mol and $N_A$ is Avogadro’s number. Rutherford backscattering (RBS) was performed in order to measure the actual target thickness produced by implantation for two of the targets\(^6\). Targets S1 and S2 were found to be $2.15(9) \times 10^{17}$ atoms/cm\(^2\) and $2.20(9) \times 10^{17}$ atoms/cm\(^2\) respectively or equivalently, $11.4(5)$ µg/cm\(^2\) and $11.7(5)$ µg/cm\(^2\) respectively. Note that the measured target thicknesses do not agree within error of the calculated areal density of $2.01 \times 10^{17}$ atoms/cm\(^2\). Target S1 was used in the July experiment while target S2 was not used at all. The targets used in February 2011 were destroyed before an RBS measurement could be performed and so these target thicknesses are not known and were assumed to be that of target S1 (see the results section in Chapter 4).

Table 2.1: List of doses at different energies for ion implantation of a $^{32}\text{S}$ target.

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Dose (at./cm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>$0.76 \times 10^{17}$</td>
</tr>
<tr>
<td>90</td>
<td>$0.33 \times 10^{17}$</td>
</tr>
<tr>
<td>80</td>
<td>$0.33 \times 10^{17}$</td>
</tr>
<tr>
<td>70</td>
<td>$0.33 \times 10^{17}$</td>
</tr>
<tr>
<td>60</td>
<td>$0.23 \times 10^{17}$</td>
</tr>
</tbody>
</table>

After implantation the targets are ‘floated’, which is a process by which the film is detached from the glass slide. A layer of salt between the slide and the carbon

\(^6\)More information on RBS can be found in [31]. Note that the RBS measurement was performed after the experiment.
foil helps to separate the two when slowly lowered into distilled water. The target is then placed on a metal target frame which helps to support the extremely thin foil. Floating was carried out at the Technical University of Munich where access to target frames with specific geometry needed to fit onto the target ladder in the sample chamber was convenient.

2.2 $^{32}\text{S}(d,t)^{31}\text{S}$ reaction at MLL

Both of the $^{32}\text{S}(d,t)^{31}\text{S}$ experiments were carried out using 24 MeV deuterons. The experimental set up was almost identical in both experiments with the main difference being the angles of the detector in which the reaction products were observed. In the following subsections, a brief overview of each stage involved in the experiment will be given beginning with the ion source and ending with the electronics involved in data acquisition.

2.2.1 $^{2}\text{H}$ Ion source

This section outlines the main source of the unpolarized deuterons ($^{2}\text{H}$ ions) before they are accelerated to the energy of interest. A schematic diagram of the various components and stages of the ion source is shown in Figure 2.2. Molecular deuterium (D$_2$) gas moves into the dissociator by the creation of an atomic jet by adiabatic expansion [32]. A 13.5-MHz radio-frequency (RF) field (or oscillating electromagnetic field) is created within the cold plasma$^7$ of the dissociator by means of a high frequency AC circuit which causes the electrons in the plasma to oscillate in helical orbits [2]. The electrons collide with the deuterium molecules, breaking the bonds to produce neutral atomic deuterium with a dissociation efficiency of about 77 ± 5%. The neutral atomic deuterium then enters an additional pumping stage which is a better vacuum for the atoms to pass through, and helps to minimize the scattering of the beam from residual gas and the charge exchange between atomic and residual molecular deuterium. This additional pumping stage improves the intensity of the beam by approximately 20% and provides atomic beam intensities of

$^7$A plasma is a state of matter similar to a gas in which ionized particles are surrounded by free electrons.
about $5 \times 10^{16}$ atoms/s for the next stage - the Electron Cyclotron Resonance (ECR) ionizer.

The main purpose of the ECR ionizer is to ionize the atomic deuterium to produce $\text{D}^+$ ions by means of electron impact created by a static magnetic field and an oscillating RF field in a plasma. Two pancake coils provide an axial magnetic mirror field which helps confine the plasma and the motion of the electrons. A $2.45$-GHz RF field is applied to the plasma through a waveguide that is insulated from the plasma by a cylindrical pyrex tube and accelerates the electrons in phase with their cyclotron frequency. Additional radial confinement of the plasma is provided by six dipole magnets made of Iron-Neodymium-Boron (FeNdB)-based magnets. The complex motion of the electrons due to these fields ensures that a maximum number of collisions occur between the electrons in the plasma and the deuterium atoms, causing ionization by electron ejection. The efficiency of ionization by electron collision is only a few percent. The positive ion beam is then transported by a system of electrodes which accelerate and decelerate the ions to an energy of $10$ keV by the time they enter the Cs vapour jet target.

In the Cs vapour jet target, a Cs reservoir is heated to $310$ °C and a jet with a pressure of $3$ mbar is formed by vapour expansion through a nozzle directed downwards toward the incoming positive deuteron beam. About $30\%$ of the vapour makes it past the baffle surrounding the nozzle and enters the charge exchange region where pickup of two electrons from the Cs vapour occurs, producing negatively charged deuterium ions. Cs is usually the element of choice to flip the charge of the ion beam because it has a low ionization energy ($\omega_i = 3.89$ eV [2]) and therefore has a tendency to lose electrons easily. The free electrons can be picked up by the positively charged deuterons creating negatively charged $\text{D}^-$ ions. Saturation of the double charge exchange occurs at a Cs areal density of $10^{15}$ atoms/cm$^2$ [33]. The surfaces within the vapour jet target, including the baffle, are kept at $35$ °C which causes the vapour to condense into a liquid phase so that it can be recirculated back into the reservoir. The efficiency of charge flipping by the Cs vapour jet target is about $22\%$. An axial magnetic holding field helps to constrain the charge exchange area and steer the beam towards the next stage of beam production. The $\text{D}^-$ ions leave the vapour jet target stage with energies on the order of a few $100$ keV and
head toward the Tandem accelerator.

![Diagram of deuteron source at MLL](image)

**Figure 2.2:** Schematic diagram of the deuteron source at MLL.

### 2.2.2 Munich M-P tandem Van de Graaff accelerator

In order for the deuterons to reach a kinetic energy sufficient to populate resonance states above 6 MeV in $^{31}$S and to maximize the Coulomb penetrability probability factor, they must be accelerated further. This is accomplished by the MP tandem Van de Graaff accelerator located downstream from the Cs vapour jet target and which has been in operation for over 40 years [34]. Figure 2.3 illustrates a general set up for a tandem accelerator while Figure 2.4 shows the location of the tandem as well as a rough layout of the various components and optics associated with the accelerated ion beam at MLL.

The MP is similar to most Van de Graaff accelerators in that an electrostatically charged terminal located at one end of the device accelerates the charges to the desired energy through a series of electrically insulated tubes kept under high vacuum ($\leq 10^{-6}$ torr) [2]. However, in the case of the tandem accelerator, 2 separate sets of accelerating tubes are contained within one pressure tank with a positive high-voltage terminal at their junction. The terminal is essentially a metallic cylinder with a semispherical dome top that acts as a Faraday cage excluding electric field contributions from the interior. The terminal voltage is determined by the charge $Q$ that is transported to the terminal dome and its capacitance $C$ by the relation $V = Q/C$. If a current ($I_{\text{trans}}$) is supplied to the dome, then the rate of change...
of the terminal voltage is \( \frac{dV}{dt} = \frac{I_{\text{trans}}}{C} \). In order to maintain a stable voltage, the supplied current must remain constant and thus be in equilibrium with charge losses.

Charge loss is attributed to the accelerated ion beam \( (I_{\text{beam}}) \), corona discharge \( (I_{\text{cor}}) \), current leakage from supporting insulators \( (I_{\text{ins}}) \) and current flow through high resistance chains connecting the terminal to ground \( (I_{\text{res}}) \). Thus, the equation given below must hold for a constant terminal voltage.

\[
I_{\text{trans}} = I_{\text{beam}} + I_{\text{cor}} + I_{\text{ins}} + I_{\text{res}}
\]  

(2.4)

In order to minimize charge loss, especially from corona discharge, the terminal is kept in contact with 30%-by-volume SF\(_6\) gas which acts as an electrical insulator. The insulating gas is indeed more effective than dry air and does help with voltage stability; however, a more precise system is needed in order to minimize voltage fluctuations further. This system is briefly described later in this subsection.

Positive charge is delivered to the terminal by three pelletron chains running parallel to each other on each side of the terminal, each supplying 100 \( \mu \text{A} \) of current for a total of 600 \( \mu \text{A} \). The chains consist of cylindrical metal pellets that are charged by induction and joined by pieces of insulating nylon. The chains are attached to pulleys and are driven by a motor which in turn drives two permanent magnet generators supplying power to other terminal components. The negative ions from the Cs vapour jet target are accelerated to an energy equal to the terminal voltage multiplied by the negative charge. The negative ions are then stripped of their electrons due to atomic collisions as they pass through a carbon foil within the terminal, and are accelerated again. When the positive ions emerge from the other end of the accelerator tube, they have an energy equal to \( E_k = eV(1 + q) \) where \( q \) is the charge state of the positive ions. For the case of heavier ion beams, a slit system has been designed to discriminate against unwanted charge states producing beams of differing energies. In the case of deuterons, which have one possible positive charge state of +1, this is not applicable.

After leaving the accelerator, the beam is bent by an analysing magnet which

---

\(^8\)corona discharge is a process in which a current develops from ionization of a gas near a surface at a high potential, creating a plasma which passes charge to areas of lower potential [35].
further defines the energy of the beam. In most tandems, the beam energy is determined by analysing magnets. The B field of the magnet and the corresponding radius of curvature $\rho$ of the particles trajectory in the field determines the energy. However, there are limitations to the absolute accuracy that can be obtained from this method owing to small deviations in the B field and from the physical width of the slits used to accept particles with a given $\rho$. A different method is employed by the MP tandem using time-of-flight information to calculate the energy $E$ and energy spread $\Delta E$ of the beam\textsuperscript{9}. This method has proven to be very successful in determining the beam energy $E$ to a relative uncertainty of $1.5 \times 10^{-5}$ for 20 MeV protons and in determining $\Delta E$ to within 5\%, giving typical values for energy resolution on the order of $\Delta E/E = 2 \times 10^{-4}$.

Furthermore, a control system called the liner stabilization system helps to regulate the beam energy in the case of larger deviations from the nominal (desired) value by providing error signals that are proportional to the energy offset detected by the analysing system and independent of the beam intensity of the tandem. This error signal is then used to modify the terminal potential. The liner stabilization system has proven to be useful in minimizing the fluctuations in terminal voltage from 9 kV fwhm free running terminal voltage to 260 V fwhm with the liner turned on. This corresponds to a reduction factor of about 34 which is significantly greater than pure corona stabilization systems that are used in other tandem accelerators. 90\% of the beam then passes through a collimator and magnification system downstream.

\textsuperscript{9}More information regarding this method is explained in detail in [34].
Figure 2.4: Layout of the MLL facility, showing various components and optics associated with the accelerated ion beam at MLL.

from the tandem located in hall I (see figure 2.4), which defines the beam spot width to be 2 mm detected just upstream from the target [36].

2.2.3 Target interaction and orientation

After leaving the accelerator, the beam then enters the target chamber located just upstream from the Q3D magnetic spectrograph in Hall I (see figure 2.4). The target chamber is a cylindrical metal chamber that houses the target ladder. After the ladder is loaded with target foils, the chamber is sealed and pumped down to a pressure below $5 \times 10^{-6}$ torr in order to minimize interactions of the beam particles and reaction products with any residual gasses. The metal target ladder resembles
an actual ladder and contains five spaces in which different targets can be fastened, including a graphite collimator located in the bottom sixth spot that is used for additional beam focusing. The ladder is useful since the vacuum does not have to be broken each time a target change is required. Figure 2.5 shows the targets used in each experiment and their position on the ladder.

![Targets used in each experiment at MLL, and their position on the target ladder.](image)

Figure 2.5: Targets used in each experiment at MLL, and their position on the target ladder.

The ladder is also free to rotate to an angle $\theta$ in the horizontal plane between the beam and target. This can be beneficial since if the target is rotated to an angle greater than $0^\circ$, the beam will see more of the target and thus more interactions can occur, increasing the statistics for data analysis (provided that the path of the reaction products to the spectrograph is not blocked). Also, reaction products produced in the target traverse a length $\Delta x$ which can be minimized by rotating the target, reducing energy losses due to collisions between the tritons and target particles. Furthermore, the sulfur target can be oriented such that the carbon side of the target is facing the beam. This also helps to minimize energy losses since the lighter deuterons will interact with the thicker, more dense carbon while the heavier tritons produced within the sulphur end of the target experience fewer collisions. For a given spectrograph angle, $\theta_{\text{spec}}$, between the beam and spectrograph, an angle $\theta$ is chosen such that $\theta = \theta_{\text{spec}}$, thus minimizing energy losses. The effective sulfur
thickness $\Delta x_{\text{eff}}$ of the target is increased according to the equation

$$\Delta x_{\text{eff}} = \frac{\Delta x}{\cos \theta}$$  \hspace{1cm} (2.5)

Figure 2.6 shows how the quantities mentioned above are related. These quantities become important for calculations related to the analysis which will be explained in more detail in the following chapter. Table 2.2 gives a summary of each angle between the sulfur target and the beam, and the corresponding effective thickness for each spectrograph angle in each experiment.

Figure 2.6: Schematic illustration of a typical target orientation. The area of acceptance of the spectrograph refers to the region of the spectrograph that can accept reaction products. Note that this figure is not to scale.

Another important quantity of interest is the total charge incident on the target from the deuteron beam. A faraday cup is located at $0^\circ$ behind the target which is connected to a current integrator that determines the amount of charge that has passed through the target. This is a good approximation to the incident charge, as most of the deuterons pass through the target and the charge related to beam-target reactions is therefore negligible. The Faraday cup is a Carbon tube
Table 2.2: A summary of each angle between the sulfur target and the beam and the corresponding effective thickness for each spectrograph angle in each experiment.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Target$^a$</th>
<th>$\theta_{\text{spec}}$ (degrees)</th>
<th>$\theta$ (degrees)</th>
<th>$\Delta x_{\text{eff}}$ ($\mu g/cm^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>25</td>
<td>25</td>
<td>12.6(5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>49</td>
<td>48</td>
<td>17.1(7)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>53.75</td>
<td>53.75</td>
<td>19.4(8)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>58.5</td>
<td>58.5</td>
<td>21.9(9)</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>15</td>
<td>15</td>
<td>11.9(5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>20</td>
<td>12.2(5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25</td>
<td>25</td>
<td>12.6(5)</td>
</tr>
</tbody>
</table>

$^a$see figure 2.5 for target assignment.

2-cm long, with an inner diameter of 10 mm and a wall thickness of 2 mm [37]. This tube sits on a 2-cm thick carbon block which is sufficient to stop the beam and can handle significant high beam intensities and power loads. Carbon is typically used as a beam stop since it has one of the highest melting and sublimation points of all the elements. Two devices called scaler 1 and scaler 3 ‘count’ the output (above some threshold value) from the current integrator associated with the Faraday cup. Scaler 1 is always accumulating the output of the integrator on the Faraday cup while scaler 3 is gated with the data acquisition dead time [37]. This is useful since the difference of scaler 1 - scalar 3 provides the beam normalization for the triton yield in the measured spectrum.

The Faraday cup also provides information on the instantaneous beam current. Typical beam currents within both experiments were found to be in the range of 200 nA to 1 $\mu$A. While the exact value of the instantaneous beam current is not of particular importance for data analysis, it is qualitatively important as a low beam current produces fewer interactions which leads to lower statistics requiring longer experimental runs. Also, a drop in beam current is a good indicator that there could be issues with the beam production/acceleration that need attention.
2.2.4 Q3D magnetic spectrograph

In order to detect the reaction products of interest, a magnetic spectrograph is needed to determine momenta of the charged particles emitted from the reaction. If the rest mass of the particle is also known, then the energy can be determined [38]. Since the energy resolution of the accelerated beam provided by the tandem is on the order of two parts in $10^4$, the desire to have a functioning magnetic spectrograph with this level of stability is obvious [39]. When designing a magnetic spectrograph, important things to consider are the ability of the instrument to spatially separate particles of different momenta using a magnetic field region, so that they can be detected in a position-sensitive detector, while at the same time allowing particles to be accepted into as large a solid angle as possible by the device to avoid excessive run times [40]. This can be difficult as the spectrograph must focus particles with the same momentum emitted into a given solid angle to a spot or line on the detector. Also, aberrations can occur in which the image formed from the object (or the beam spot on the target) by particles of identical momentum is blurred, which needs to be corrected. Furthermore, the ability of the device to disperse groups of particles with small momentum differences is of particular importance as a high dispersion can allow one to effectively ‘zoom in’ on an energy region of interest.

A quantity of interest to spectrograph designers is the resolving power (or resolution) of the device, which is defined as the ability to distinguish two particles of slightly different mass-to-charge ratio (or momentum $p$, equivalently) and is given to first order by the equation

$$\mathcal{R} = \frac{p}{\Delta p} = \frac{DR}{M_H x_0} \quad (2.6)$$

where $D$, $R$, $M_H$ and $x_0$ are the dispersion, nominal orbital radius of the particles path in the magnetic field region, horizontal plane magnification and the spot size on the target respectively. What is important to note from this equation is that the resolution depends directly on the dispersion of the device and inversely on the magnification and target spot size. Also note that the resolving power defined in equation 2.6 is just the inverse of the beam energy resolution defined in subsection 2.2.2 for the tandem.
Another limiting factor for the resolution arises from kinematic broadening, which occurs when particles from a reaction populating a given state in the recoil nucleus appear to have different energies when emitted at slightly different angles. In order to maintain a large solid angle of acceptance, aberrations and kinematic broadening must be corrected by design geometry and specific magnetic field configurations. Typically, spectrographs are designed and optimized using sophisticated computer simulations and ray tracing programs with the constraints mentioned above as parameters. These considerations were taken into account during the design of the Q3D magnetic spectrograph at MLL. The optical layout of the Q3D (or quadrupole and three dipoles) is shown in Figure 2.7. The quadrupole magnet produces a field which focuses the particles in the vertical direction and defocuses in the horizontal direction. The particles are then bent by the first dipole to produce an astigmatic ‘image’ of the target spot in the vertical direction at the multipole element (ME).
This means that the particles produced at the target are focused to a line in the horizontal direction. A second image is produced on the detector in the vertical direction by the fringing fields of dipole 2 (D2) and 3 (D3). The effect of producing intermediate images is to increase the dispersion while at the same time correcting aberrations\(^\text{10}\). The multipole element described in [39] produces a simultaneous dipole, quadrupole, sextupole, octupole and decupole field configuration which helps to correct the kinematic broadening in the horizontal plane. Coupled with the magnetic elements is an electrostatic deflector located between D2 and D3, which provides an electric field perpendicular to the magnetic field from the dipoles and horizontally separates particles with different \(mv/q\) ratios at the detector. The full energy range \(E_{\text{max}}/E_{\text{min}}\) measurable by the Q3D at MLL reported in [39] is 1.21. However, for the experiments described in this work, a ratio of about 1.09 was observed. The high dispersion of the Q3D is important for this work as the states of interest are spread out over the focal plane which is helpful for state identification, peak fitting and yield determination. Another benefit of the Q3D is the rather large maximum solid angle of acceptance of 13.9 msr determined from geometrical parameters including the maximum attainable entrance slit widths of 21×24.5 mm\(^2\) in the horizontal and vertical directions, respectively. Most importantly, the energy resolution attainable by the Q3D is on par with the beam energy resolution provided by the tandem accelerator and is reported to be \(\Delta E/E = 2 \times 10^{-4}\) [41].

The software for the operation of the Q3D requires the reaction of interest and the excited state of interest to be placed at the center of the focal plane as input parameters. The software then calculates the currents required to produce quadrupole, dipoles, multipole and electric fields in order to satisfy the input conditions. Proton NMR, an effect in which certain nuclei in a magnetic field absorb and re-emit electromagnetic energy at a given frequency proportional to the field, is used to measure the magnetic field produced by D2. This allows stabilization and feedback for any needed adjustments to the other pole currents. For example, after the the D2 field stabilizes, the current for the quadrupole is set proportional to the stable field [36].

\(^{10}\text{For more information see [39].}\)


2.2.5 Position sensitive cathode strip detector

The particles that traverse the Q3D must be detected at the focal plane F (see Figure 2.7) by a device that has an equal or better resolution than that provided by the tandem and Q3D in order to exploit this high resolution set-up. The position sensitive cathode strip detector was designed for this purpose and also for the fulfilment of other criteria such as geometrical issues and positioning. A description of the detector and the detection process is described in detail in this section. Figure 2.8 shows a side view of the detector setup.

When a particle enters the detector chamber D, it first passes through the 25-\(\mu\)m-thick Kapton entrance foil into the enclosed detector region filled with isobutane gas at a pressure of 500 mbar [41]. It enters the first proportional counter which is composed of a cathode foil (held at ground) and an anode wire (held at a high positive voltage). The particle then ionizes the gas and creates multiple electron-ion pairs. The electric field provided by the geometry is a radial electric field given by the relation below [42]

\[
E = \frac{1}{r} \frac{V_0}{\ln(b/a)}
\]  

(2.7)

where \(r\), \(b\) and \(a\) are the radial distance from the axis of the anode wire, the distance to the cathode and radius of the anode wire, respectively. Note that the field increases closer to the anode wire and therefore, electrons that are produced near the anode wire will be accelerated, ionizing more gas atoms (referred to as an avalanche). The electron-ion pairs begin to move to the anode and cathode respectively but the positive ions move more slowly than the electrons and so the movement of charges induces a signal (or voltage pulse equivalently) at the anode wire which is proportional to the energy loss of the particle from the initial ionization. For the purposes of the experiments performed at MLL, this energy loss is defined as \(\Delta E1\). The particle then passes through the cathode foil and penetrates another isobutane gas-filled region and then through another cathode foil, both of which are made of 2.9-\(\mu\)m-thick aluminized mylar [43]. The particle then enters the second proportional counter composed of two anode wires in contrast to the first proportional counter. The particle again creates electron ion pairs in the gas due to interactions with the gas molecules and avalanches which occur near the wires.
Figure 2.8: Shown is a vertically sliced side-view of the position sensitive cathode strip detector in which particles enter from the left through the entrance foil. The figure is adopted from [41].

Again, both anode wires produce a signal from this avalanche proportional to the energy loss of the particle. The energy loss measured by both wires is then summed and defined as $\Delta E$. The two wires are also used to center the detector since both should see the same number of incident particles if the detector is centered with respect to the incoming particles. The voltage in eq. 2.7 is the same for each anode wire and is set externally in the control room. Typical voltages for the anode wires used in the experiments were in the range of 1200 to 1360 V, to optimize the energy resolution of the detected particles.

Cathode strips that are located after the anode wires and before the scintillator are also affected by this avalanche. The electrically isolated individual cathode strips are 3.5 mm in width and are separated by a distance of 0.5 mm. 255 strips which each read out an induced charge, cover 890 mm of the 1800-mm curved focal plane of the Q3D. Since the particles cross the focal plane with an angle between 40°-50°, the induced charge by the avalanche usually spans about 6 strips. The cathode strips closest to where the particle passes the anode wires will show a higher induced charge since more electron ion pairs will be generated at this point. The particle position can be identified by a Gaussian fit of the charge distribution induced on
these cathode strips where the peak centroid corresponds to the particle position. The positions of particles with different energy (momenta) on the focal plane is crucial for identification of excited states in $^{31}$S. Figure 2.9 shows the working principle behind this detection method. The particle then travels through a last cathode strip foil into a plastic scintillator where its residual energy $E_{res}$ is deposited and a single output corresponding to this energy is generated. The combination of $E_{res}$, $\Delta E$, $\Delta E_1$ provides a three-fold coincidence for logic triggering for accepted events by the data acquisition system (DAQ)$^{12}$.

The energy information is used for the identification of particle groups and the selection of the particles of interest (tritons). $\Delta E$ can be plotted against $E_{res}$ (banana gate) to identify the tritons as groups of particles from different reactions form a banana shape with protons located at the bottom end of the banana, since they deposit a higher residual energy and experience less energy loss, and heavier alpha particles located at the top for the opposite reason. Also, both $\Delta E_1$ and $\Delta E$ can be plotted on the same graph to identify particle groups in a similar manner (proton gate). The position information is related to the particle’s momentum or energy and is essentially the final spectrum that we see once the correct particle group is identified.

The center of gravity method (whereby a calculation is made based on equations similar to locating the center of gravity of a system of masses) is used to determine the particle position during initial data acquisition. This is less involved compared to the Gaussian fit method illustrated in Figure 2.9 and reduces the dead time, but it also produces systematic effects. These systematic effects show up in the spectrum as periodic minima occurring at intervals equal to the strip length (3.5 mm). Following the initial data acquisition, the charge distribution can then be fitted by a Gaussian function to better determine the particle’s position and remove systematic effects from the center of gravity position determination. The position resolution of the detector comes from the error in the Gaussian function and is reported to be less than 0.5 mm FWHM [41].

Each of the 255 cathode strips is equipped with its own preamplifier connected to a pulse shaper and peak-hold-discriminator [43]. Figure 2.10 shows a drawing of

$^{12}$More information on the DAQ is given in subsection 2.2.6.
Figure 2.9: Top: Top view of the second multiwire proportional counter followed by the plastic scintillator. Electrons produced by a particle near the anode wires create an avalanche which induces positive charge on the cathode strip foil. Bottom: The charge induced on each strip is digitized and plotted as a histogram. A Gaussian distribution (dotted line) is used to calculate the position where the particle has passed the anode wires to within 0.5 mm. The figure is taken from [41].

the single-strip readout electronics. If one of the strips detects a signal above a threshold value, a digital signal is sent to the application-specific integrated circuit (ASIC) which scans the strips for (and only accepts) an event with multiplicity of 3 to 7. What this means is that 3 to 7 neighbouring strips have seen an induced charge above the threshold which is typical for reaction products entering the detector between an angle of 40–50°. If the ASIC accepts an event, the charges are converted to a digital value by one of the 8 analog-to-digital converters (ADCs) and sent to an output buffer. The readout of the position information is triggered by coincidences between signals from the anode wires and scintillator, and excluding the ASIC. Consequently, if the ASIC is busy processing events with the wrong multiplicity or events that do not produce a charge output (i.e., background from neutrons or gamma rays), no position information is read. This dead time associated with the detector electronics must be corrected for during the analysis and is explained in detail in Chapter 3.
Figure 2.10: Schematic layout of the readout electronics of the cathode strip detector. The figure is adopted from [43].

2.2.6 Data acquisition system (DAQ)

The entire experimental setup at MLL is controlled by a data acquisition system called the Multi-Branch System (MBS) and ROOT-based online/offline utility (MARA BOU). The main function of this system is to control the conversion of ana-
log signals of interest produced in the experiment to digital numeric output values that can be interpreted and manipulated by a computer. MARaBOU consists of two main parts called the front-end and back-end [44]. The MBS developed at the GSI research facility in Darmstadt, Germany, composes the front-end which controls data readout, event building and data transport. The back-end provides class libraries (pre-written software routines) for setup, run control, options to produce histograms, data analysis and data storage, and is written using the ROOT analysis software. Figure 2.11 shows how these two systems are related to each other and to the experiment. Figure 2.12 shows the main components of the hardware configuration and how they are connected.

Figure 2.11: Diagram of the tasks performed by the front-end MBS system and back-end ROOT-based analysis system of the DAQ at MLL. The figure is taken from [44].

The devices (or modules) used to initially acquire data from experimentation (e.g. ADCs) are connected and operated under the Computer Automated Measurement and Control (CAMAC) bus system. The CAMAC system was the first system designed for DAQ and control in nuclear and particle physics experiments and was originally put into place in the early 1970s [42]. The CAMAC system is very useful as it allows many modules to be connected simultaneously and for communication between these components [2]. However, as experiments became more complex and
better resolution and higher channel density were required, the need for faster data transfer led to the development of the VERSAmodule eurocard (VME) bus system in the late 1970s. The VME-CAMAC interface (CBV) links the two systems, and data acquired by the CAMAC modules are transferred via a VME system bus connection and written in dual-ported memory (memory that can be both written to and read during the same clock cycle) on a powerful processing PC. A second PC then takes the data written in memory and sends them in formatted blocks to a Linux host. Data readout synchronization is handled by a trigger module.

Figure 2.12: DAQ hardware configuration at MLL. The figure is taken from [44].

The user provides a description of the event structure of the experiment using C++ statements in a ROOT macro, which acts as a set of instructions for both the MBS front-end system controlling the data readout and also for the back-end analyzing program. The statements are then interpreted by ROOT’s C++ interpreter (CINT) which helps to reduce compiling and linking time. An example of a configuration macro is given in Figure 2.13. For the MBS front end, the configuration macro
performs the data readout of the CAMAC and VME systems and formats the events as defined in the macro. For the back-end analysis, the configuration macro provides a set of C++ class definitions and methods that process and organize the incoming data. These tasks include histogram presenting, monitoring and event storage. The user also must add code to the standard Analyze() method which describes the event analysis specific to the given experiment.

The module that links the MBS front-end to the back-end ROOT-based analysis portion is called M_analyze and contains the code needed to run the experiment. M_analyze controls the input of MBS data, the conversion of MBS events to ROOT objects, histogramming and monitoring, event filtering and the storage of event data in ROOT format. It is used to analyze events both online (while data acquisition is occurring) as well as offline. C_analyze provides the graphical user interface (GUI) for the user to control functions provided by M_analyze like mode of operation (online/offline), definition of file and run names, data flow and histogram storage.

Figure 2.13: Example of a configuration macro used during data acquisition at MLL. The macro is used by both the front-end MBS system as well as the back-end ROOT-based analysis portion (see text). Subroutines are highlighted in blue. The figure is taken from [44].

Histograms, fitting functions, filter conditions and graphical cuts are all stored in a memory mapped file which can be accessed by other applications and programs like HistPresent for offline data analysis. HistPresent allows the user to view and ma-
nipulate (re-bin, zoom, add, subtract, divide, etc.) 1, 2 or 3 dimensional histograms. Another important feature of HistPresent is that it allows the user to make ‘cuts’ on a given histogram which act as a filter for data. The user can essentially cut out and plot only those specific events. These filters are known as gates and help to identify the particle group of interest which are then plotted in a position spectrum. In each of the $^{32}\text{S}(d,t)^{31}\text{S}$ experiments performed at MLL, three gates (banana, proton and position gates) were used for particle identification. The method of particle identification for the banana gate is described in subsection 2.2.5. For the proton gate, both $\Delta E_1$ and $\Delta E$ are plotted on the same graph with particle groups of protons to alphas following a more linear trend. The function of the position gate is to cut out any stray particles that appear at the boundaries of the detector height. This method is described in more detail in the following chapter.
In this chapter, an overview of the general data analysis procedures used to study the $^{32}\text{S}(d,t)^{31}\text{S}$ reaction as well as other nuclear transfer reactions is introduced. Analysis techniques described here include particle identification using energy loss information from detectors, spectrograph plotting, peak fitting and energy determination, as well as the distorted wave Born approximation (DWBA) method for assigning spin and parity constraints.

3.1 Particle identification (PID) using energy loss information from detectors

During nuclear physics experiments, there are no detectors or electronics that can directly recognize different nuclear isotopes and so an indirect method must be employed which uses a series of detectors to identify particle groups of interest. The method of particle identification using energy loss signals from various detectors in the experiment is described briefly in Section 2.2. Here, a more formal look at the procedure is explored. Charged particles that pass through a detector interact with the electrons bound to the atoms of the detector material (called the absorber) via the Coulomb interaction [45]. The electrons in the absorber feel an impulse due to the Coulomb force which, depending on the proximity of the interaction, may be
large enough to excite electrons to higher orbital shells within the atom or even to expel the electron completely. The energy transferred to the electron in the absorber atom during a collision is provided by the charged particle and thus each interaction causes a reduction in the velocity of the charged particle. The energy loss per unit length of distance traveled by the charged particle in the absorbing material, $dE/dx$, is called the stopping power and can be described by the Bethe formula given below

$$-\frac{dE}{dx} = \frac{4\pi e^4 Z^2}{m_e v^2} NB$$

with

$$B \equiv z \left[ \ln \frac{2m_e v^2}{I} - \ln \left( 1 - \frac{v^2}{c^2} \right) - \frac{v^2}{c^2} \right]$$

where $v$, $Ze$, $z$, $N$ and $m_e$ are the velocity and charge of the incident charged particle, the atomic number and number density of the absorber atoms, and the electron rest mass respectively. $I$ is a parameter that represents the mean excitation and ionization potential of the absorber material and is usually determined experimentally for each element. What is important to notice about the above equation is that for particles passing through the same thin absorber material $dx$, the energy loss $dE$ (or equivalently $\Delta E$) is proportional to the charge-to-velocity ratio squared of the particle given below [3]

$$\Delta E \propto \frac{Z^2}{v^2}$$

By multiplying the above relationship by the total energy of the particle $E = \frac{1}{2} Mv^2$, we find that

$$\Delta E \times E \propto M \times Z^2$$

and so

$$\Delta E \propto \frac{M Z^2}{E}$$

A plot of $\Delta E$ vs. $E$ should correspond to a hyperbolic curve with different curvatures for different particle groups depending on the value of $MZ^2$, which is 1, 2, 3 and 16 for protons, deuterons, tritons and alpha particles respectively. Figure 3.1 shows a representation of such curves. By measuring the energy loss $\Delta E$ in a thin detec-
tor and the corresponding residual energy deposited into a detector thick enough to completely stop the particle and plotting these quantities in a histogram of $\Delta E$ vs. $E$, different particle groups can be identified clearly.

![Figure 3.1: Representation of a $\Delta E$ vs. $E$ histogram. The different curves correspond to protons (p), deuterons (d), tritons (t) and alpha particles (α) with the shape of the curves determined by the quantity $MZ^2$. This figure is adopted from [3].](image)

For the experiment at MLL, the $\Delta E$ detectors were the multi-wire proportional counters with isobutane gas as the thin absorber material and the plastic scintillator was the thick absorber in which the particles’ residual energy after traversing the detector ($E_{res}$) is deposited. In reality, each particle does not enter the detector perpendicular to or at the same angle and so the proportionality of $\Delta E$ is corrected by a factor of $1/\cos \theta$. The geometry for this situation is shown in Figure 3.2. Consequently, curves in Figure 3.1 become wider bands due to the spread of energy losses for identical particles with different trajectories. Also, the total energy of the reaction products does not vary continuously because of the momentum (or energy) constraints provided by the Q3D and so the bands corresponding to different particle groups become circular ‘blobs’. This method of filtering out groups of unwanted particles is known as the \textit{banana gate} at MLL.
Similarly, when plotting the energy loss from two detectors in a histogram, particle groups appear as circular ‘blobs’ that follow a linear, positively sloped curve with protons, deuterons, tritons and alpha particles located at an increasing distance from the origin. At MLL, this method of particle identification is known as the proton gate. With the use of both the banana and proton gates (as well as the position gate mentioned in section 2.2.6), tritons can be readily identified. Only position information corresponding to these events are plotted in a position spectrum which is used for state and energy determination, as well as for calculating the differential cross section for each state and spectrograph angle used in the Distorted Wave Bron Approximation (DWBA) method of spin-parity determination. An example of a position spectrum for tritons produced from the $^{32}\text{S}(d,t)^{31}\text{S}$ reaction at a beam energy of 25 MeV and a spectrograph angle of 10° is shown in Figure 3.3. What is important to note about this Particle Identification Method (PID) and about Figure 3.3 is that tritons corresponding to different reactions with other target nuclei are present in the spectrum and so further identification must be used to determine target contaminants and any unwanted states appearing in the spectrum. Typically
each experimental run corresponding to an interval of time gets manually sorted offline through the gates mentioned above and individually output and saved as an ASCII data file which is a list of numbers representing the number of counts in a certain channel. Experimental runs with similar parameters (i.e., beam energy, angle, target orientation, etc.) are summed together and are also saved as ASCII files. These ASCII files serve as the position spectrum to be displayed and explored by analysis software which is explained in more detail in section 3.3.

![Figure 3.3: Example of a position spectrum of tritons from the $^{32}\text{S}(d,t)^{31}\text{S}$ reaction at a beam energy of 25 MeV and a spectrograph angle of 10°.](image)

Each peak has a width $\Gamma$ for quantum mechanical reasons, and various contributions from sources throughout the experiment which affect the resolution or equivalently, the ability to distinguish two peaks that are closely spaced. In a ‘perfect’ world one would expect to see a delta function representing each state however; due to these contributions, this is not the case. The contribution to the width of each state due to quantum mechanics ($\Gamma_{QM}$) is derived from the uncertainty principle ($\Delta E \Delta t \geq \hbar / 2$) and is defined as

$$\Gamma_{QM} = \Delta E = \frac{\hbar}{2\tau} = \frac{\hbar \ln 2}{2t_{1/2}}$$  

(3.6)
where \( \tau \) and \( t_{1/2} \) are the lifetime and half-life of the state, respectively. This contribution is typically less than 1 eV for \(^{31}\)S states of interest and is negligible for total \( \Gamma \)'s on the order of 1–10 keV. Contributions to \( \Gamma \) from the experimental apparatus include beam straggling \( \Gamma_{\text{stragg}} \), beam energy spread \( \Gamma_{\text{spread}} \), triton energy loss \( \Gamma_{\text{tel}} \), image of the target spot size \( \Gamma_{\text{tar}} \), kinematic broadening \( \Gamma_{\text{kb}} \) and the intrinsic detector resolution \( \Gamma_{\text{det}} \). Beam straggling and triton energy loss refer to energy loss of the beam and tritons when traversing the target before and after a nuclear reaction occurs. The slight differences in energy affect the final energy of the triton which contributes to peak broadening and a poorer resolution. The beam energy spread is the slight difference in kinetic energy of the beam particles which is defined by the energy resolution of the tandem\(^1\). As discussed in Chapter 2, the resolution is also affected by the magnification of the target spot size (see equation 2.6), by kinematic broadening, and by the uncertainty in the position determination from the detector.

The peak width for a given state is the sum of these contributions added in quadrature and is given as

\[
\Gamma = \sqrt{\Gamma_{\text{stragg}}^2 + \Gamma_{\text{spread}}^2 + \Gamma_{\text{tel}}^2 + \Gamma_{\text{tar}}^2 + \Gamma_{\text{kb}}^2 + \Gamma_{\text{det}}^2 + \Gamma_{\text{QM}}^2}\]  

(3.7)

Note that \( \Gamma \) increases as the spectrograph angle increases due to a larger contribution from \( \Gamma_{\text{tel}} \), since the tritons traverse a larger distance within the target on their way to the spectrograph. Typically, the energy losses, kinematic broadening, and detector resolution tend to dominate Equation 3.7 causing peak broadening and ultimately decreasing the resolution.

### 3.2 Spectrograph plotting

Once the desired reaction products are identified and a position spectrum is produced, another tool is needed in order to identify all possible states that have shown up on the focal plane (or equivalently, in the position spectrum). Such tools are called spectrograph plotters and are used by many experimental nuclear physicists today.

For the purposes of this work, a spectrograph plotting program called Spec-
Plot [46] was used. SpecPlot employs eq. 2.1 to calculate the bending radius \( \rho \) (in eq. 2.1) of each light reaction product with momentum \( p \) and charge \( q \), displaced in a magnetic field \( B \). The momentum \( p \) can be deduced from the total energy \( E \) of the particle by the momentum-energy relation

\[
E^2 = m^2c^4 + p^2c^2
\]  

(3.8)

By substituting the above equation into eq. 2.1, particles with a specific energy \( E \) can be plotted on the focal plane of SpecPlot according to the relation

\[
\rho = \frac{1}{qB} \sqrt{\left(\frac{E}{c}\right)^2 - (mc)^2}
\]  

(3.9)

The energy of each reaction product present on the focal plane corresponds to a state in the heavy recoil nucleus through conservation of energy, which is calculated and plotted by SpecPlot. The light particle energy increases when moving from left to right (increasing \( \rho \)) across the focal plane according to eq. 3.9 and so the energy of these ‘imprints’ corresponding to excited states should therefore decrease when moving in the same direction. Figure 3.4 gives an example of a position spectrum of tritons corresponding to excited states in various recoil nuclei plotted by SpecPlot.

SpecPlot requires up-to-date input files for both nuclear masses and excitation energies of each excited nucleus in order to plot simultaneously states on the focal plane as well as multiple reaction channels on the same plot. The mass input file used is the 2003 nuclear mass list provided by the National Nuclear Data Center (NNDC) [47] and compiled in 2003, containing all the different nuclear species together with their mass excess, binding energy, beta-decay energy, atomic mass and associated uncertainties. The excitation energies for each nuclear species were obtained from the Nuclear Structure and Decay Data (NUDAT) 2 database [48], as well as from [16] and [20] specifically for newly determined excited states of \(^{31}\)S not reported in [48]. The beam energy and species, spectrograph angle, and the identity of the reaction products are also required as input parameters for kinematic corrections\(^2\) to the energy \( E \), and thus affect where tritons corresponding to excited states in \(^{31}\)S appear relative to each other on the focal plane, determined by \( \rho \). Other input

\(^2\)Kinematic corrections are discussed in subsection 3.4.1.
Figure 3.4: Example of a focal plane position spectrum of tritons from various reactions (listed in the leftmost column) plotted with SpecPlot and assuming a beam energy of 24 MeV deuterons at a spectrograph angle of 25°. The spectrum of tritons corresponds to excited states in various recoil nuclei by energy conservation. Energies are listed in MeV.

parameters required are the maximum and minimum bending radii, \( \rho_{\text{max}} \) and \( \rho_{\text{min}} \), which correspond to the maximum and minimum states with energy \( E \) that are present on the focal plane of the position spectrum for a given \( B \) which can be somewhat arbitrary as described below.

Typically, the method used to identify states on the focal plane is to first identify a known state (if the spectrum is easily recognized by the user) or to use a reference state to then try and line up and match all other states in SpecPlot with a corresponding state in the position spectrum. An example of a reference state could be a heavily populated ground state or the state centered on the focal plane by the input parameters for the Q3D with excitation energy \( E_{\text{center}} \). One must keep in mind that since many different magnetic field strengths are used within the Q3D, there is not a single value for the input parameters \( B \) and \( \rho_{\text{center}} \) and therefore nominal values for \( B \), \( \rho_{\text{max}} \), \( \rho_{\text{center}} \) and \( \rho_{\text{min}} \) are chosen such that \( E_{\text{center}} \) appears approximately in the middle of the focal plane which is a good approximation but not exact. Thus,
the peak centroids do not always line up exactly with the lines representing each state in SpecPlot. Each input state has a corresponding uncertainty adding to the fact that it might not be exactly lined up with the centroid observed in the specific spectrum. Also, the Q3D dispersion is not absolutely constant along the focal plane and the focal plane itself can have a slight curvature to it, which can also add to the imprecise correlation between line and centroid. Figure 3.5 shows the method described here for state determination from a position spectrum of excited states in $^{31}$S obtained with a 25-MeV deuteron beam at a spectrograph angle of $10^\circ$.

Figure 3.5: Example of the method of focal plane state determination using SpecPlot. Using the Beam energy, spectrograph angle and nominal magnetic field, states can be identified by trying to match the lines given in SpecPlot representing excited states of different recoil nuclei with peaks observed in the position spectrum. Due to various factors (see text) the states do not always line up exactly. Energies are listed in MeV.

SpecPlot acts as a first order guide for state identification which can help during the formal state and energy determination described in Section 3.3 and in determining any possible target contaminants. SpecPlot is also important during
the initial planning stages for the experiment as it allows the user to determine an optimal beam energy and spectrograph angle in order to obtain a desired spectrum and what possible unwanted contaminant states might be present on the focal plane obstructing a clear view of the state(s) of interest. During the experiment and data acquisition, SpecPlot can provide a warning to the user that something may be amiss, for example if states expected to be on the focal plane are not present.

3.3 Peak fitting and energy determination

Typically, the next step in data analysis is to formally determine the centroid and yield (number of counts within a peak) of peaks in the position spectrum corresponding to excited states in $^{31}$S, which is important for energy determination and spin-parity identification. IGOR PRO [49] is an excellent analysis tool for such a task and provides built-in peak fitting and background subtraction as well as many other functions.

First, the ASCII file of choice obtained during offline data sorting is loaded by IGOR PRO to produce a position spectrum. IGOR PRO provides a built-in smoothing tool which helps to make the spectrum look less jagged and antisymmetric by precomputing smoothing coefficients depending on the smoothing parameters and replacing the spectrum with the convolution of the spectrum and the coefficients. For the purposes of this work, binomial smoothing of order 3 was typically chosen as it employs a Gaussian filter as the smoothing method. An example of a focal plane position spectrum that has been smoothed in IGOR PRO by this method is shown in Figure 3.3. Once the spectrum has been smoothed, peak fitting and background subtraction can begin. IGOR PRO provides a multipeak fitting tool which allows the user to fit a single peak or multiple peaks at the same time. This method can sometimes be difficult as the user must first select the desired channel interval over which fitting will occur and then estimate parameters such as minimum peak width and background level until the desired number of peaks appear in the window. Typically, the function used to fit a peak is Gaussian but in the case that the peak is significantly asymmetric, other functions can be used. The user also has the option to choose a method of background subtraction, which becomes important for deter-
mining the number of counts in a peak. If the background is generally constant and fluctuates evenly about a central value, the program can subtract the background by fitting a straight line through the mean of these fluctuations. However, in the case that the background is not constant, IGOR PRO provides a cubic polynomial function (curved line) which provides a better estimate and therefore a more precise background subtraction. Figure 3.6 shows an example of multipeak fitting with a cubic polynomial background subtraction. During peak fitting, IGOR PRO calculates the channel position of the centroid, area and width of each peak along with the associated errors, which can be displayed in a window and output to other sources.

After the peak centroids have been identified, the energy of each state can be determined using a program called SPANC (SPlitpole Analysis Code) [46] which uses calibration peaks to predict the energy of each state on the focal plane. SPANC requires a description of the targets, reactions used in the experiment, beam energy,
projectile charge state, spectrograph angle, spectrograph B-field and the energy and position for each calibration peak in order to calculate excitation energies [50]. After the peak position and energy information is entered, \textit{SPANC} calculates $\rho$ for each calibration peak. In dealing with energy loss estimation, \textit{SPANC} assumes that the nuclear interaction occurs at a point in the middle of the target interaction layer and so beam energy loss is calculated to this point and then used in the 2-body kinematics calculation. Energy loss is then calculated for the projectile traveling through the remainder of the target at the specified spectrograph angle. The final energy of the projectile is then used in the calculation of $\rho$. In order to then predict unknown peak energies, \textit{SPANC} uses linear regression to estimate $\rho$ as a function of channel using polynomials of order 1 to 4 according to the equation below

$$\rho = a_0 + a_1(\text{Channel} - \text{Channel}[0]) + a_2(\text{Channel} - \text{Channel}[0])^2 + \ldots \quad (3.10)$$

where Channel[0] is the unweighted mean of the channel position of all the calibration peaks. \textit{SPANC} requires at least 3 calibration peaks in order to determine the coefficients in equation 3.10, which are then used together with the channel location (Channel) of the unknown state to estimate $\rho$ and therefore predict the excitation energy $E$ for each state of the recoil nucleus and an associated error which is called the output peak.

Calibration peaks were chosen based on peak height, width, location and uncertainty. Typically, known peaks that were well defined had a low uncertainty, had a width comparable to the average single peak width and were located evenly across the spectrum were used for calibration. Once the first 3 peaks were entered into \textit{SPANC}, the energies of known peaks close to the calibration peaks were determined. If an output peak had an energy that agreed with that reported in the literature and followed the same criteria as mentioned above, it was then input to \textit{SPANC} as another calibration peak. This was repeated until all well defined, known single peaks were exhausted. The energy of each state in the spectrum was then determined and the polynomial order of the fit was chosen based on how well all known peaks in the spectrum agreed with the adopted values. The statistical and systematic uncertainty
in the energy of each resonant state $\delta E_{\text{stat}}$ and $\delta E_{\text{sys}}$ were determined separately. The statistical uncertainty for each state was taken to be the value from the weighted average using the statistical uncertainties from SPANC. The overall systematic uncertainty was estimated by taking half of the greatest difference between separate measurements for each state and then averaging over all states. The statistical and systematic uncertainties were kept separate so as to clearly identify the contributions of each source of error. To find the total uncertainty, $\delta E_{\text{stat}}$ and $\delta E_{\text{sys}}$ can be added in quadrature as given below:

$$\delta E = \sqrt{\delta E_{\text{stat}}^2 + \delta E_{\text{sys}}^2}$$

(3.11)

A second weighted average was performed using several other independent measurements of excited states in $^{31}$S (including this work) and a new adopted value for each state was determined.

3.4 Spin and parity assignments from DWBA analysis

Following state identification and energy determination of the excited nucleus, a procedure known as DWBA can be used in order to identify the total spin $J$ and parity $\pi$ of each state. The procedure involves calculating the differential cross section $\frac{d\sigma}{d\Omega}$ for each state at each spectrograph angle in the center of mass frame and plotting the measured quantities on a graph of a predicted $\frac{d\sigma}{d\Omega}$ distribution for a given $J^\pi$ value. In the following subsections, a detailed description of the procedure is given.

3.4.1 Differential cross section calculations

In order to determine the differential cross section in the lab frame of reference $\left(\frac{d\sigma}{d\Omega}\right)_{\text{lab}}$, one must consider a quantity called the total yield $Y$. In the triton spectrum shown in Figure 3.3, each data point on the curve represents the total number of tritons with a center-of-mass energy $E_{\text{cm}}$ observed within a solid angle covered by the detector. Recall that the tritons observed on the focal plane are related to excited states in the recoil nucleus. Due to various contributions to the total observed width
of each state on the focal plane, tritons related to the same state with energy $E_{ex}$ will have slight differences in their center-of-mass energy $E_{cm}$. The yield $Y_\theta$ for each state at a given spectrograph angle $\theta$ is the sum of all triton events (counts) corresponding to an excited state in the recoil nucleus of energy $E_{ex}$ or equivalently, the area under each peak in the position spectrum. We now must find a way to relate $Y_\theta$ to the differential cross section in the lab frame $\left(\frac{d\sigma}{d\Omega}\right)_{lab}$.

If one considers the cross section $\sigma$ to be an effective area within which a projectile will cause a reaction to proceed, then the effective area $S_{eff}$ of a target with thickness $\Delta x$ containing $n_0$ active target nuclei is [2]

$$S_{eff} = n_0 \sigma$$

(3.12)

By dividing eq. 3.12 by the total area of the target $S$, one obtains the reaction yield $Y$ for a single incident particle

$$Y = \frac{S_{eff}}{S} = \frac{n_0 \sigma}{S}$$

(3.13)

In order to relate the yield per incident particle to the target thickness $\Delta x$, we consider the number of active target nuclei per unit volume $n$ to be

$$n = \frac{n_0}{S \Delta x} = \frac{\rho_d N_A}{A}$$

(3.14)

where $\rho_d$, $N_A$ and $A$ are the density of the active target material, Avogadro’s number and atomic number of the target respectively. By substituting eq. 3.14 into 3.13 and rearranging to solve for $\sigma$, the cross section becomes

$$\sigma = \frac{YA}{\Delta x \rho_d N_A}$$

(3.15)

If $w$ represents the total number of incident particles with charge $qe$ incident on a target, then the total yield for many incident particles $Y_{tot}$ is related to the yield per incident particle $Y$ by the equation

$$Y = \frac{Y_{tot}}{w} = \frac{Y_{tot}qe}{C}$$

(3.16)

where $C$ is the total accumulated charge from the incident beam. By substituting
eq. 3.16 into 3.15 we find the expression for the cross section in terms of the total yield $Y_{tot}$ to be

$$\sigma = \frac{Y_{tot}Aqe}{\Delta x \rho d N_A C} \quad (3.17)$$

In most laboratory set-ups, the detector does not cover the entire $4\pi$ solid angle so the total yield $Y_{tot}$ (and therefore the cross section $\sigma$) cannot be determined directly. However, for a detector at a given angle subtending a solid angle $\Delta\Omega$ and detecting a fraction of the total yield $Y_{\theta}$, we find that the differential cross section as a function of the spectrograph angle is

$$\left(\frac{d\sigma}{d\Omega}\right)_{lab} = \frac{Y_{\theta}Aqe}{\Delta x \rho d N_A C \Delta\Omega} \quad (3.18)$$

In some cases, the detector is so busy processing many incoming events that it might miss an event and so the number of counts in the position spectrum for each state might not be representative of the true quantity. In order to correct for this loss of statistics during the detector ‘dead-time’, the yield $Y_{\theta}$ which is obtained from the area under the peaks in the position spectrum by IGOR PRO, must be multiplied by a dead-time correction factor $D_c$. At MLL, channel 0 of the multichannel analyzer contains counts where the digital part of the detector was busy and not able to give any position information. These events should be included in the yield from the position spectrum because they still triggered the DAQ system from coincidences between the analog $\Delta E$ and $E_{res}$ signals and should therefore be considered as good events. The dead-time correction is estimated from the equation below

$$D_c = \frac{[\text{all counts in spectrum}]}{[\text{all counts in spectrum} - \text{counts in channel 0}]} \quad (3.19)$$

$D_c$ is calculated for each experimental run separately but the position spectra obtained are usually a sum of many individual runs. In order to take this into account, an average can be taken between all $D_c$ values for the individual runs and an associated error ($\delta D_c$) can be determined from the standard deviation from the mean. The equation for determining the differential cross section for a given state at a given spectrograph angle becomes
\[ \left( \frac{d\sigma}{d\Omega} \right)_{\text{lab}} = \frac{D_c Y_\theta A q e}{\Delta x \rho d N_A C \Delta \Omega} \quad (3.20) \]

The error in the differential cross section in the lab frame \( \delta \left( \frac{d\sigma}{d\Omega} \right)_{\text{lab}} \) can be calculated by adding the relative errors of quantities in equation 3.20 considered significant enough to contribute to the total error, together with the dead-time correction factor uncertainty \( \delta D_c \). The rms error is given as

\[ \frac{\delta \left( \frac{d\sigma}{d\Omega} \right)_{\text{lab}}}{\left( \frac{d\sigma}{d\Omega} \right)_{\text{lab}}} = \sqrt{\left( \frac{\delta \Delta x}{\Delta x} \right)^2 + \left( \frac{\delta D_c}{D_c} \right)^2 + \left( \frac{\delta Y_\theta}{Y_\theta} \right)^2} \quad (3.21) \]

In experimental nuclear physics, the observations take place within the rest frame of the laboratory. However, from a theoretical perspective, it is more convenient to deal with quantities in a frame of reference that moves with the center of mass of the two colliding nuclei called the center-of-mass system. Moreover, the computer code used to generate predicted differential cross section distributions calculates this quantity in the center-of-mass coordinate system and therefore the measured values must be converted to this frame of reference. The derivation of the quantities relating the lab and center-of-mass frame is beyond the scope of this work and can be found in [1]. For a spectrograph angle in the lab \( (\theta_{\text{lab}}) \) and center-of-mass \( (\theta_{\text{cm}}) \) frames, the differential cross sections are related by the equation

\[ \left( \frac{d\sigma}{d\Omega} \right)_{\text{cm}} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{lab}} \times \frac{d (\cos \theta_{\text{lab}})}{d (\cos \theta_{\text{cm}})} \quad (3.22) \]

where

\[ \frac{d (\cos \theta_{\text{lab}})}{d (\cos \theta_{\text{cm}})} = \frac{\sqrt{1 - \gamma^2 \sin^2 \theta_{\text{lab}}}}{(\gamma \cos \theta_{\text{lab}} + \sqrt{1 - \gamma^2 \sin^2 \theta_{\text{lab}}})} \quad (3.23) \]

and

\[ \gamma = \sqrt{\frac{m_x m_y E_{\text{beam}}}{m_B (m_y + m_B) Q_{ex} + m_B (m_B + m_y - m_x) E_{\text{beam}}}} \approx \sqrt{\frac{m_x m_y}{m_A m_B} \left( \frac{E_{\text{beam}}}{(1 + m_x/m_A) Q_{ex} + E_{\text{beam}}} \right)} \quad (3.24) \]
where \( m_A, m_x, m_y, m_B \) and \( E_{\text{beam}} \) are the mass of the target nucleus, incident particle, ejectile, recoil nucleus, and the beam energy, respectively. The approximation in eq. 3.20 is obtained by assuming \( m_x + m_A \approx m_y + m_B \). \( Q_{\text{ex}} \) represents the excited state Q-value for the specific reaction and is given as

\[
Q_{\text{ex}} = Q - E_x
\]

where \( Q \) is the ground state reaction Q-value defined in eq. 1.18 and \( E_x \) is the energy of the excited state of the recoil nucleus.

The error in the differential cross section in the center-of-mass frame can be determined explicitly from eqs. 3.22–3.25 by conventional error propagation. However, this is a long and tedious process which can be approximated by the equation given below

\[
\delta \left( \frac{d\sigma}{d\Omega} \right)_{\text{cm}} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{cm}} \times \frac{\delta \left( \frac{d\sigma}{d\Omega} \right)_{\text{lab}}}{\left( \frac{d\sigma}{d\Omega} \right)_{\text{lab}}}
\]

### 3.4.2 DWUCK4 code and DWBA plot generation

Once the experimental values for the differential cross section in the center-of-mass frame \( \left( \frac{d\sigma}{d\Omega} \right)_{\text{cm}} \) have been calculated for each state at each spectrograph angle, a computer code called DWUCK4 [51] provides a predicted distribution for the same quantity to which the experimental values can be compared. DWUCK4 uses the distorted wave Born approximation (DWBA) to simulate a distribution for \( \left( \frac{d\sigma}{d\Omega} \right)_{\text{cm}} \) as a function of \( \theta_{\text{cm}} \) for a given state with spin-parity \( J^\pi \). The experimental quantities can then be plotted on these curves in order to determine if they follow a distribution for a certain \( J^\pi \) assignment. If the experimental data lie on or the uncertainty in the experimental quantities overlaps with a given curve, then the spin and parity intrinsic to the excited nucleus probably is that of the \( J^\pi \) value used by DWUCK4 to simulate the distribution. According to the literature, it is also acceptable for the curve to lie within a few sigma (number of multiples of the error) of the data points. DWUCK4 uses many parameters defined in an input file by the user to calculate these curves including theoretically, and experimentally determined values for the nuclear potential which can usually be found in peer reviewed journal articles and
literature. It also requires orbital angular momentum \((l)\) values as input in order to produce a curve such that eqs. 1.20–1.22 are satisfied for the excited nucleus with a specific \(J^\pi\) configuration. DWUCK4 calculates the differential cross sections in units of \(\text{fm}^2/\text{steradian}\) which is equivalent to \(10^4 \mu\text{b}/\text{steradian}\) (where 1 b \(\equiv 1\) barn \(\equiv 10^{-24}\) cm\(^2\)). The DWUCK4 cross sections are based on nuclear physics models, wave functions, and transition amplitudes and so the actual magnitudes of the calculated cross sections are not to scale of those obtained in the experiment. DWUCK4 can only determine the change in the differential cross sections as a function of angle and thus can only produce the relative variation of this change. As a consequence, the DWUCK4 differential cross sections \(\left(\frac{d\sigma}{d\Omega}\right)_D\) must be scaled to the experimental ones \(\left(\frac{d\sigma}{d\Omega}\right)_E\) such that

\[
\left(\frac{d\sigma}{d\Omega}\right)_E = \left(\frac{d\sigma}{d\Omega}\right)_D \times S \tag{3.27}
\]

where \(S\) is the scaling factor and is related to the spectroscopic factor which describes how well the observed cross sections agree with those predicted by simpler shell model calculations. The DWUCK4 cross section magnitudes are larger than the experimentally determined values and so \(S\) is typically a number between 0 and 1. An example of a DWUCK4 input file that produces the differential cross section as a function of center-of-mass angle for a given state with a given spin-parity is shown in Figures 3.7 and 3.8, respectively.

Figure 3.7: Example of a DWUCK4 input file used to produce a distribution of the differential cross section as a function of center-of-mass angle for the 6835 keV state in \(^{31}\text{S}\) assuming a spin-parity of \(3/2^+\).
Figure 3.8: Differential cross section as a function of center-of-mass angle produced by DWUCK4 for the 6835 keV state of the $^{31}$S recoil nucleus assuming a spin-parity of $3/2^+$. 

If data have already been taken and the experimenter is planning on collecting data at a few more angles, the DWBA curves predicted by DWUCK4 can help during preparation for the next run as it gives an idea of what spectrograph angles are the most important for differentiating between different spin-parity curves (i.e., local minimum and maximum values). Usually the curves diverge from one another at higher angles but the differential cross section always decreases as the spectrograph angle increases which translates to a much longer exposure time at higher angles in order to obtain statistics large enough for analysis. This imposes a constraint on the experiment, and angles have to be chosen wisely in order to maximize statistics as well as the data quality during beam-time.
32\text{S}(d,t)31\text{S} Experimental Results

In this chapter, the main results for the 32\text{S}(d,t)31\text{S} experiments performed at MLL in February and July of 2011 will be presented. First, the results of the energy calibrations will be presented followed by the angular distribution and DWBA curve fitting. A description of the analysis techniques can be found in Chapter 3.

4.1 Resonance energies of 31\text{S}

4.1.1 Calibration

An internal calibration using known 31\text{S} excited states was performed using SPANC to determine the energy of each of the excited states on the focal plane at each angle. A summary for each calibration fit for each angle will be given below starting with the smallest spectrograph lab angle. The calibration energies used were adopted values taken from [20].

- **15° Data (July 2011):** The 6328.6-, 6377.1-, 6543.0-, 6636.9-, 6749.0-, 6833.6-, 6870.4- and 6936.7-keV states of 31\text{S} were used as calibration points and resulted in a fit of polynomial order 3 with a $\chi^2/\nu$ of 2.86 and a p-value of 0.10.

- **20° Data (July 2011):** The 6377.1-, 6543.0-, 6636.9-, 6749.0-, 6833.6-, 6870.4- and 6936.7-keV states of 31\text{S} were used as calibration points and resulted in a fit of polynomial order 2 with a $\chi^2/\nu$ of 0.33 and a p-value of 0.93. A heavily populated state which appears at the very left of the spectrum could be the 6176.3-keV 15\text{O} state from the 16\text{O}(d,t)15\text{O} reaction which suggests that oxidation of the
target may have occurred. Since the state is mostly off the focal plane, the energy of that state was not determined explicitly and the identification of this state was made based on predictions from SpecPlot.

- **25° Data (July 2011):** The 6543.0-, 6636.9-, 6749.0-, 6833.6-, 6870.4- and 6936.7-keV states of $^{31}$S were used as calibration points and resulted in a fit of polynomial order 2 with a $\chi^2/\nu$ of 0.73 and a p-value of 0.70. To the very right of the spectrum, a large peak can be observed and is hypothesized to be the first excited state of $^{11}$C at 2000.0-keV from the reaction $^{12}$C($d$,t)$^{11}$C. This was inferred using SpecPlot since the state was mostly off the focal plane. The presence of this state is to be expected due to the 40 $\mu$g/cm$^2$ $^{12}$C backing of the target.

- **25° Data (Feb. 2011):** The 6636.9-, 6719.9-, 6749.0-, 6870.4- and 6936.7-keV states of $^{31}$S were used as calibration points and resulted in a fit of polynomial order 2 with a $\chi^2/\nu$ of 0.30 and a p-value of 0.90. The first excited state of $^{11}$C also appears to be present in the spectrum at the far right of the focal plane.

- **49° Data (Feb. 2011):** The 6377.1-, 6543.0-, 6636.9-, 6719.9-, 6749.0-, 6870.4- and 6936.7-keV states of $^{31}$S were used as calibration points and resulted in a fit of polynomial order 2 with a $\chi^2/\nu$ of 2.25 and a p-value of 0.11. There appears to be a broad structure in the spectrum adding additional background in the 400 to 1300 channel region. This is attributed most likely to excited states of $^{15}$O at 5183- and 5240.9-keV which, according to SpecPlot, are located precisely in this region. Due to the peak location and the complexity of peak fitting for these states, this conclusion is based on predictions from SpecPlot.

- **53.75° Data (Feb. 2011):** The 6585.8-, 6636.9-, 6719.9-, 6749.0-, 6870.4- and 6936.7-keV states of $^{31}$S were used as calibration points and resulted in a fit of polynomial order 2 with a $\chi^2/\nu$ of 0.05 and a p-value of 1.0. Both the first excited state of $^{11}$C and the excited states of $^{15}$O seen at 49° appear to be present in the spectrum; however, the latter are now shifted to a lower channel region (channel 700 to 100), which is consistent with predictions made by SpecPlot.

- **58.5° Data (Feb. 2011):** The 6585.8-, 6636.9-, 6719.9-, 6749.0-, 6833.6- and 6870.4-keV states of $^{31}$S were used as calibration points and resulted in a fit of polynomial order 3 with a $\chi^2/\nu$ of 0.27 and a p-value of 0.97. The excited states of $^{15}$O and $^{11}$C seen at lower angles appear to have moved off the focal plane, which is
consistent with predictions made by SpecPlot.

4.1.2 Excitation Energies in $^{31}$S

The energies determined in the present work are shown in Table 4.1 along with past independent $^{31}$S measurements for comparison. Our energies were determined by taking a weighted average of each state for each angle. The uncertainties listed in the present work are statistical and an estimated systematic uncertainty of 1 keV should be included additionally for each state. The labeled spectra at each spectrograph angle for the July 2011 and February 2011 experiments are presented in Figures 4.1 and 4.2, respectively.

The energies from the present work listed in Table 4.1 seem to be in good agreement with previous studies including the (now confirmed) doublet at 6395/6405 keV in which the higher energy state had only been observed in [20] and [16].

4.2 Angular distribution analysis and spin-parity assignments

The process of determining the differential cross section in the lab and center of mass frame for each state at a given angle is shown in Section 3.4 of Chapter 3 and will not be repeated here. In this section, the scaling factor for the normalization of the cross section data between the two consecutive experiments and spin-parity assignments based on DWBA plots are presented.

4.2.1 Differential cross section scaling factor

In order to plot both sets of data from the February and July experiments on the same graph, the differential cross sections for each state must be compared using a normalization angle. Theoretically, the differential cross section at a given angle should agree within uncertainty between the two experiments assuming the conditions are the same (beam energy, target nucleus, etc.) and constraints like target thickness are known. 25° was chosen as it was the lowest angle from the
Table 4.1: Nuclear energy levels in units of keV and $J^\pi$ values for $^{31}$S. An additional 1 keV systematic uncertainty should be considered separately for each state in the present work. The final column contains the adopted values for the excitation energies determined from the weighted average of previous studies and present work. Firm spin-parity assignments are shown in parentheses in the final column.

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Figure 4.1: Triton spectra from the $^{32}\text{S}(d,t)^{31}\text{S}$ reaction performed in July 2011. Peaks corresponding to excited states in $^{31}\text{S}$ are labeled with the determined energies in keV.
Figure 4.2: Triton spectra from the $^{32}\text{S}(d,t)^{31}\text{S}$ reaction performed in February of 2011. Peaks corresponding to excited states in $^{31}\text{S}$ are labeled with the determined energies in keV.
February run at which data were taken and so would take the least amount of time for data acquisition during the July run. What was found was that the differential cross sections disagreed at $25^\circ$ between experiments by factors of 1.1–1.74 (from their central values) depending on the state. This could be due to uncertainty in target thickness since the thickness for the target used in February was destroyed accidentally before a Rutherford Backscattering measurement could be made, or possible target contamination in one or both of the targets. Also, the background was high compared to the total number of counts in the spectrum for the July data due to time constraints, which had a significant effect on the cross sections depending on what IGOR PRO determined as the background. Rather than determining a scaling factor for each state and multiplying the differential cross sections for one set of data by these factors, one scaling factor was determined by looking at the entire range of states on the focal plane. Intuitively, this seemed to be a more correct treatment for discrepancies in the cross sections based on target effects or low statistics.

Rather than just comparing the differential cross sections obtained from the Gaussian fits of each state using IGOR PRO, the raw ASCII file was considered in the process. First, all peaks in the July and February $25^\circ$ spectra were fit over the entire range of states using a constant background determined by IGOR PRO. The average background was found to be $0.679 \pm 0.019$ and $4.161 \pm 0.043$ counts/channel for July and February respectively. To find a representation of the total counts from $^{31}\text{S}$ states in each spectrum $T_c$, the total counts over the channel range of states for both spectra were summed, background subtracted, and normalized with respect to the incident beam according to the equation

$$T_c = \frac{D_c \times (T - B \times R)}{C} \quad (4.1)$$

where $D_c$ is the dead-time correction factor, $T$ is the total counts in the spectrum over the channel range $R$, $B$ is the average background for the specific spectrum and $C$ is the total incident charge. The differential cross section scaling factor $S_c$ was calculated according to
\[ S_c = \frac{T_{c,J}}{T_{c,F}} \]

where \( T_{c,J} \) and \( T_{c,F} \) are the representations of the total counts from \(^{31}\text{S}\) states for the July and February 25° spectra respectively. \( S_c \) was found to be \( 1.289 \pm 0.058 \). The uncertainty was determined by varying the quantities \( B \) and \( D_c \) within their range of uncertainty in the calculation for \( S_c \) and choosing the greatest deviation from the central value of 1.289. \( S_c \) represents the scaling factor by which the differential cross sections at each angle from the February experiment were multiplied in order to scale them to the July data. The February data was scaled to the July data because the thickness of the target used in the July run was known and thus the cross sections calculated from that run should be closer to the actual values.

### 4.2.2 Peak fitting techniques for cross section determination of doublet at \( E_x = 6395/6404 \text{ keV} \)

In recent studies and in the present work, it has been observed that a doublet state in \(^{31}\text{S}\) exists near 6400 keV. In the present work, the state above 6400-keV was not completely resolved and its existence was inferred from other experimental evidence. For the 25° (Feb.), 49°, 53.75° and 58.5° spectra, the width of the state near this region appeared to be larger than any of the other known single states on the focal plane by factors of about 1.2–1.8. This suggested that the extra wide state may in fact be two unresolved, closely spaced single states. In order to fit these unresolved peaks, a different technique was employed. First, all known single peaks on the focal plane were fitted using IGOR PRO and their widths determined. An average single state width was determined and the unresolved doublet was then fitted with two peaks with both widths held fixed to this average single state width (excluding 58.5° which did not require any constraints). The peak centroids that these fits produced translated to states with energies at 6404- and 6395-keV which are consistent with previous studies. For the 15° and 20° spectra, the peak width of the doublet was closer to that of a single state and when fitted with a single peak, returned an energy near 6395-keV. This suggested that the differential cross section for the 6404-keV state was fairly low at these lower forward angles. In order to fit
two peaks over this region, more than just the peak width was constrained. For 15°, the width and position of the 6404-keV state was held fixed while only the width of the 6395-keV state was held fixed. For 20°, the width and position of the 6395-keV state was held fixed while the width was held fixed for the 6404-keV state. This method seemed reasonable as the energies had already been obtained from the higher angle data. In both cases, the unconstrained position returned an energy that was consistent with that obtained from the larger angle data and with previous studies. The yields from the fits were then used for the differential cross section determination for both states at each angle.

4.2.3 Optical model potential parameters for DWBA calculations

As mentioned previously, DWUCK4 uses the Distorted Wave Born Approximation (DWBA) for the interactions involved in the reaction in order to predict the differential cross sections for a given spin and parity of the residual nucleus. The distorted waves were calculated by DWUCK4 with an optical potential well for the interactions of the form [52]

\[
V(r) = U_c(r_0c) + V_0f(R_0r, a_r) + i\left(W + W_d\frac{d}{dx_I}\right)f(R_0I, a_I) + V_s\frac{1}{r}\frac{d}{dr}f(R_0s, a_s)i\vec{l} \cdot \vec{s}
\]

(4.3)

The first term in Equation 4.3 is a Coulomb potential of a uniformly charged sphere of radius \(r_0cA^{1/3}\). The second and third terms correspond to a volume Woods-Saxon potential and a surface Woods-Saxon potential, respectively. The fourth term describes a spin-orbit potential of a volume Woods-Saxon form where \(\vec{l}\) is the orbital angular momentum and \(\vec{s}\) is the spin [5]. \(r_0c\) is the reduced charge radius \((R_c = r_0cA^{1/3})\), \(R_0r\), \(R_0I\), \(R_0s\) are the reduced radii of the real, imaginary and spin-orbit potentials respectively and \(a_r\), \(a_I\), \(a_s\) are the diffuseness parameters of the real, imaginary and spin-orbit potentials respectively. \(V_0\), \(W\), \(W_d\) and \(V_s\) are the potential
well depths of the real, imaginary and spin-orbit potentials respectively. The function $f$ is given as

$$f(R_{0k}, a_k) = \frac{1}{1 + exp \left( \frac{r-R_{0k}A^{1/3}}{a_k} \right)}$$  \hspace{1cm} (4.4)$$

where $r$ and $A$ are the nuclear radius and atomic number, respectively. The input potential parameters used by DWUCK4 in the present work have been taken from [52] and are presented in Table 4.2.

### 4.2.4 DWBA plots

Given below are the DWBA plots from the $^{32}$S($d$,t)$^{31}$S experiment performed at MLL for states between (and including) 6377- to 6971-keV. States observed at fewer than three angles were not included in the DWBA analysis. The measured values are plotted together with the theoretical curves derived using DWUCK4. In some cases, more than one spin-parity agreed with the data and so all possible spin-parity values for a given state are plotted on the same graph. The spin-parity constraints obtained from the present work are listed in Table 4.2 in the present work column below each state. A more detailed comparison to the mirror nucleus ($^{31}$P) and previous studies is carried out in the proceeding section.

### 4.3 Comparison with mirror nucleus and other independent experiments

A state-by-state comparison to both the mirror nucleus ($^{31}$P) and to previous independent experimental results is carried out in this section. The adopted $J^\pi$ values listed in Table 4.1 are based on conclusions made from these comparisons. Figure 4.10 shows the level structure of $^{31}$S and $^{31}$P above the $^{30}$P + $p$ threshold of 6130.9(4)-keV [20] with the corresponding mirror assignments inferred from the present work. Finally, the adopted $J^\pi$ values determined in the present work are plotted with previous $J^\pi$ constraints to illustrate any agreement/disagreement between past and present analysis.
• **6377-keV level:** In comparing the spin-parity constraint of $7/2^-$ or $9/2^+$ from the present work to past DWBA analysis done by Parikh et al. [20] ($9/2$) and Jenkins et al. [8] ($9/2^-$), an adopted value of $9/2^+$ is tentatively assigned to this level. The constraint made by Jenkins et al. was based on angular distributions and a mirror assignment corresponding to the 6501-keV state in $^{31}$P. A state at 6233 keV in $^{31}$P with a spin-parity assignment of $3/2^+–9/2^+$ [14] could be the corresponding mirror state with a mirror energy difference of 144-keV. A DWBA plot of both $9/2^+$ and $9/2^-$ is shown in figure 4.11 to illustrate the better agreement to the present adopted value of $9/2^+$. This conclusion is tentative due to the fact that there are only 3 data points available for this state.

• **6395-keV level:** A value of $11/2^-$ is tentatively assigned based on comparison to the value obtained by Jenkins et al. [8] of $11/2^+$. Here again, the parity assignment was made based on a mirror state comparison to the 6454-keV state in $^{31}$P. Looking at $^{31}$P, there is currently no corresponding mirror state with a spin-parity of $11/2^-$. A DWBA plot of both $11/2^+$ and $11/2^-$ is shown in Figure 4.11 to illustrate the better agreement with the present value of $11/2^-$. 

• **6404-keV level:** A value of $11/2^+$ is tentatively assigned to this level. A previous constraint of $3/2^-$, $5/2^-$ or $7/2^+$ based on reaction comparisons and inferences was made by Wrede et al. [14] and does not support the current assignment. A DWBA plot of $11/2^+$ with the Wrede et al. constraints are shown in Figure 4.12 to show the better agreement with the $11/2^+$ curve.

• **6543-keV level:** An adopted value of $9/2^-$ is assigned to this level based on a comparison to the $(7/2, 9/2)$ constraint from Parikh et al. [20]. A DWBA plot of both parity considerations for $7/2$ and $9/2$ are shown in Figure 4.12 to show the better agreement with the $9/2^-$ curve. Currently there is no mirror assignment in the present work for this level.

• **6586-keV level:** An adopted value of $9/2^-$ or $11/2^-$ is tentatively assigned to this level. The constraints made in the present work do not agree with a previous constraint of $7/2$ made by Parikh et al. [20]. A DWBA plot of both parity considerations for $7/2$ together with the present adopted values are shown in Figure 4.13. Currently there is no mirror assignment in the present work for this level.

• **6637-keV level:** An adopted value of $9/2^-$ is assigned to this level based
on a comparison to conclusions made by Parikh et al. [20] (9/2) and Jenkins et al. (9/2−) [8]. The present adopted value agrees with the mirror assignment of the 6793-keV state in $^{31}\text{P}$ made by Jenkins et al [8]. A DWBA plot of both $9/2^+$ and $9/2^−$ is shown in Figure 4.11 to illustrate the better agreement with the present adopted value of $9/2^−$.

- **6720-keV level**: An adopted value of $11/2^−$ is tentatively assigned to this level based on a comparison with a state in the mirror nucleus at 6825-keV corresponding to a mirror energy difference of $-105$-keV. The constraints made in the present work for this level do not agree with a previous constraint of $5/2$ made by Parikh et al. [20]. A DWBA plot of both parity considerations for $5/2$ is shown in Figure 4.14 together with the present adopted value to illustrate the better agreement with the $11/2^−$ curve.

- **6749-keV level**: An adopted value of $9/2^−$ or $11/2^+$ is tentatively assigned to this level. The present adopted values do not agree with the previous constraint of $3/2^+$ made by Parikh et al. [20]. Currently there is no mirror assignment in the present work for this level. A DWBA plot of $3/2^+$, $9/2^−$ and $11/2^−$ is shown in Figure 4.14 to illustrate the better agreement with the present adopted values.

- **6835-keV level**: An adopted value of $11/2^+$ is assigned to this level by comparison to constraints made by Parikh et al. (11/2) [20] and Jenkins et al. (11/2−) [8]. The parity constraint made by Jenkins et al. was based on a mirror assignment corresponding to the 6825-keV state in $^{31}\text{P}$. Currently there is no mirror assignment in the present work for this level. A DWBA plot of both $11/2^+$ and $11/2^−$ is shown in Figure 4.15 to illustrate the better agreement with the present adopted value.

- **6870-keV level**: An adopted value of $11/2^+$ is assigned to this level by comparison to the spin constraint made by Parikh et al. (11/2). Currently there is no mirror assignment in the present work for this level. A DWBA plot of both $11/2^+$ and $11/2^−$ is shown in figure 4.15 to illustrate the better agreement with the present adopted value.

- **6936-keV level**: An adopted value of $3/2^+$ or $5/2^+$ is tentatively assigned to this level by comparison to conclusions made by Kankainen et al. [53] (1/2− 5/2)$^+$. A state at either 7080- (3/2− or $5/2^+)$ or 7084-keV (3/2$^+−7/2^+$) could be the
corresponding mirror state in $^{31}\text{P}$. A DWBA plot of $(1/2^--5/2)^+$ is shown in Figure 4.16 to illustrate the better agreement with the present adopted value.

- **6957-keV level**: An adopted value of $9/2^-$ or $11/2^-$ is tentatively assigned to this level. The constraints made in the present work for this level do not agree with a previous constraint of $1/2^+$ based on angular distributions by Vernotte et al. [54]. Currently there is no mirror assignment in the present work for this level. A DWBA plot of the present adopted values as well as the $1/2^+$ curve is shown in Figure 4.16 to illustrate the better agreement with the present adopted values.

- **6971-keV level**: An adopted value of $11/2^+$ is tentatively assigned to this level based on the present work and the absence of any previous constraints. Currently there is no mirror assignment in the present work for this level.

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Figure 4.10: Level structure of $^{31}\text{S}$ and $^{31}\text{P}$ above the $^{30}\text{P} + p$ threshold of 6130.9(4)-keV [20]. The level energies and adopted spin-parity assignments for $^{31}\text{S}$ are taken from the present work. Level energies and spin-parity assignments for $^{31}\text{P}$ are taken from [14] and [16].
Table 4.2: Optical model parameters for the interactions involved in the $^{32}\text{S}(d,t)^{31}\text{S}$ reaction taken from Tribble et al. [52]. The parameters are used as input by \textbf{DWUCK4} for the angular distribution analysis (DWBA). The well depths are given in units of MeV and the radii are in units of fm. PNLOC represents the non-local correction factor.

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Figure 4.3: Triton angular distributions for resonance states in $^{31}\text{S}$. The square boxes with error bars represent the measured differential cross sections from the $^{32}\text{S}(d,t)^{31}\text{S}$ experiment and the curves represent predicted distributions from DWUCK4. The excitation energy of the state is given at the bottom in units of keV.
Figure 4.4: Triton angular distributions for resonance states in $^{31}$S. The square boxes with error bars represent the measured differential cross sections from the $^{32}$S($d,p$)$^{31}$S experiment and the curves represent predicted distributions from DWUCK4. The excitation energy of the state is given at the bottom in units of keV.
Figure 4.5: Triton angular distributions for resonance states in $^{31}\text{S}$. The square boxes with error bars represent the measured differential cross sections from the $^{32}\text{S}(d,t)^{31}\text{S}$ experiment and the curves represent predicted distributions from DWUCK4. The excitation energy of the state is given at the bottom in units of keV.
Figure 4.6: Triton angular distributions for resonance states in $^{31}\text{S}$. The square boxes with error bars represent the measured differential cross sections from the $^{32}\text{S}(d,t)^{31}\text{S}$ experiment and the curves represent predicted distributions from DWUCK4. The excitation energy of the state is given at the bottom in units of keV.
Figure 4.7: Triton angular distributions for resonance states in $^{31}$S. The square boxes with error bars represent the measured differential cross sections from the $^{32}$S($d,t)^{31}$S experiment and the curves represent predicted distributions from DWUCK4. The excitation energy of the state is given at the bottom in units of keV.
Figure 4.8: Triton angular distributions for resonance states in $^{31}\text{S}$. The square boxes with error bars represent the measured differential cross sections from the $^{32}\text{S}(d,t)^{31}\text{S}$ experiment and the curves represent predicted distributions from DWUCK4. The excitation energy of the state is given at the bottom in units of keV.
Figure 4.9: Triton angular distributions for resonance states in $^{31}\text{S}$. The square boxes with error bars represent the measured differential cross sections from the $^{32}\text{S}(d,t)^{31}\text{S}$ experiment and the curves represent predicted distributions from $\text{DWUCK4}$. The excitation energy of the state is given at the bottom in units of keV.
Figure 4.11: Adopted values in the present work plotted with spin-parity constraints from previous experiments (see section 4.3 in the text for specific details of each state). The square boxes with error bars represent the measured differential cross sections from the $^{32}$S$(d,t)^{31}$S experiment and the curves represent predicted distributions from DWUCK4. The excitation energy of the state is given at the bottom in units of keV.
Figure 4.12: Adopted values in the present work plotted with spin-parity constraints from previous experiments (see section 4.3 in the text for specific details of each state). The square boxes with error bars represent the measured differential cross sections from the $^{32}\text{S}(d,p)^{31}\text{S}$ experiment and the curves represent predicted distributions from DWUCK4. The excitation energy of the state is given at the bottom in units of keV.
Figure 4.13: Adopted values in the present work plotted with spin-parity constraints from previous experiments (see section 4.3 in the text for specific details of each state). The square boxes with error bars represent the measured differential cross sections from the $^{32}\text{S}(d,t)^{31}\text{S}$ experiment and the curves represent predicted distributions from DWUCK4. The excitation energy of the state is given at the bottom in units of keV.
Figure 4.14: Adopted values in the present work plotted with spin-parity constraints from previous experiments (see section 4.3 in the text for specific details of each state). The square boxes with error bars represent the measured differential cross sections from the $^{32}\text{S}(d,\alpha)^{30}\text{S}$ experiment and the curves represent predicted distributions from DWUCK4. The excitation energy of the state is given at the bottom in units of keV.
Figure 4.15: Adopted values in the present work plotted with spin-parity constraints from previous experiments (see section 4.3 in the text for specific details of each state). The square boxes with error bars represent the measured differential cross sections from the $^{32}\text{S}(d,t)^{31}\text{S}$ experiment and the curves represent predicted distributions from DWUCK4. The excitation energy of the state is given at the bottom in units of keV.
Figure 4.16: Adopted values in the present work plotted with spin-parity constraints from previous experiments (see section 4.3 in the text for specific details of each state). The square boxes with error bars represent the measured differential cross sections from the $^{32}\text{S}(d,t)^{31}\text{S}$ experiment and the curves represent predicted distributions from DWUCK4. The excitation energy of the state is given at the bottom in units of keV.
Summary and Outlook

The structure of unbound $^{31}$S states above the $^{30}$P + $p$ threshold of 6130.9(4)-keV is important for determining the $^{30}$P($p,\gamma$)$^{31}$S reaction rate which influences nucleosynthesis in classical novae and type I X-ray bursts.

Past experiments have identified important $^{31}$S level parameters such as resonance energies and spin-parity values for excited states over the temperature region of interest and these constraints have been used to re-evaluate the $^{30}$P($p,\gamma$)$^{31}$S reaction rate [8, 16, 20]. However, many of the spin-parity assignments were constrained using mirror comparisons and educated guesses. Moreover, certain levels simply had no constraints or were constrained to a handful of possible values. In a recent study by Parikh et al [20], it was suggested that the uncertainties in the $J^\pi$ values of the observed 6402-, 6543-, and 6586-keV states could cause the astrophysical $^{30}$P($p,\gamma$)$^{31}$S reaction rate to vary by a factor of up to 20 and could affect isotope yields in the Si-Ar mass region by up to a factor of 4. Parikh et al. also point out the need for additional measurements to confirm assignments from past and present experiments.

We have performed a transfer reaction experiment to study the level structure of $^{31}$S via the $^{32}$S($d,t$)$^{31}$S reaction using the MP tandem and Q3D magnetic spectrograph at MLL in Munich, Bayern, Germany. In contrast to [20], where the $^{31}$P($^3$He,$t$)$^{31}$S reaction was used, we have investigated excited states of $^{31}$S using a single-nucleon transfer reaction. The excellent dispersion and resolution of the Q3D were ideal for this experiment because it allowed us to focus our investigation over a smaller energy region of interest compared to most other spectrographs. This proved
effective as more states on the focal plane could be resolved compared to other $^{31}$S single nucleon transfer studies [16, 14]. Moreover, we have fabricated implanted $^{32}$S targets for the use in our experiments that have the highest $^{32}$S content to date [5] to reduce background contributions and data acquisition time.

In the present work, we have identified 16 previously observed states in the 6 to 7 MeV region and have performed angular distribution analysis for 13 of these states. The energies determined in the present work agree very well within uncertainty to previous studies, with the exception of the 6327-keV state which was only observed at one angle. We have determined the previously unknown spin-parity of the 6404- and 6971-keV states to be 11/2$^+$ and have further constrained the parity ambiguity of the 6543- and 6870-keV states to 9/2$^-$ and 11/2$^+$ respectively. By considering previous experimental constraints and by comparison to the mirror nucleus ($^{31}$P), the 6396-keV state has been further constrained to 3/2$^+$ or 5/2$^+$. We have also found discrepancies between past and present spin-parity assignments for some of the states in the 6-7 MeV energy region from our DWBA analysis.

Although the high spins of the 6404-, 6543- and 6586-keV states determined in the present work might not contribute significantly to the astrophysical $^{30}$P($p,\gamma$)$^{31}$S reaction rate for classical novae, a new rate calculation using a monte carlo method similar to that performed in [20] should still be explored using these new constraints to investigate any changes in the rate and predicted isotopic abundance uncertainties. Since most of the spin-parity assignments determined in the present work either disagree with past experiments or have yet to be confirmed by other independent measurements, it is important for further investigation into the structure of $^{31}$S states above the $^{30}$P + $p$ threshold to be carried out. Such experiments have been performed using Gammasphere at Argonne National Lab in Chicago, Illinois but the results from these measurements have yet to be published. In addition to the conclusions drawn by Parikh et al., the study also emphasizes the need for a radioactive $^{30}$P beam with an intensity greater than $10^6$ pps in order to measure the resonance strength ($\omega\gamma$) directly [20]. In addition to the conclusions made in the present work, we also highlight the importance of the development of a radioactive $^{30}$P beam. Even in the event that all spin-parity values are constrained over the energy region of interest, there are still other poorly known quantities, such as spectroscopic factors.
that are needed to estimate the proton partial decay width ($\Gamma_p$) in the expression for the resonance strength ($\omega\gamma$); and measurements of gamma decay branchings to determine $\Gamma_\gamma$. The accessibility to an intense radioactive $^{30}$P beam would allow one to directly measure the resonance strength of the $^{30}$P($p,\gamma$)$^{31}$S reaction at the relevant astrophysical energies for classical novae and type I X-ray bursts. A proposal for this measurement to be carried out at TRIUMF in Vancouver, British Columbia, has been submitted and approved, and now awaits beam development [55]. In the meantime, this thesis has demonstrated that indirect approaches with stable beams can provide useful information.
Bibliography


