OFF-SHELL EFFECTS IN (d,p) STRIPPING REACTIONS AND THE THREE BODY SYSTEM

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ABSTRACT

The importance of off-shell effects has been investigated in (d,p) stripping reactions and the model triton.

For the stripping reactions a formalism is developed for calculating differential cross-sections and angular distributions assuming a direct process. By taking into account the off-shell background term and its interference several levels in the reaction $^{15}N(d,p)^{16}N$ (unbound) have been interpreted. Contributions to the resonances from other sources are considered by extending the coupled channel theory off the energy shell. In three soluble two-channel models, with square well, δ-function and non-local interactions, the off-shell resonance and bound state behaviour is discussed. The off-shell T-matrix for nuclear reactions is also derived using the R-matrix theory and it is discussed in some numerical detail.

For the model triton the ground state energy for two classes of potentials, a rank two and a partly non-local have been calculated. All these potentials fit the low energy data but in addition they support a continuum bound state (CBS). In each class two sets of potentials, one with a continuum bound state at $\kappa = 5$ fm^{-1} the other at $\kappa = 7$ fm^{-1} have been investigated. The rank-two potentials produce a triton ground state collapse in all cases. The binding increases as the CBS moves to higher energies and decreases when the repulsive form factor becomes stronger.

The partly non-local potentials combine long-range locality with a short range non-locality. Though their non-local parts acting alone produce a collapse no collapse has been found for the total interactions. In fact the binding is almost the same as that of the local potential acting alone.

Ἔχεις μέν ἄλγειν, οἶδα σύμφορον δέ τοι
ὡς ῥᾷστα ταναγκαῖα τοῦ βίου φέρειν .

(Εὐριπίδης, Ἑλένη, χορός, 252)

Well I know thou hast a bitter lot to bear
still 'tis best to bear as lightly as we may
the ills that life is heir to.

(Euripides, Helen, Chorus, 252)

TO BRIGITTE AND GABRIEL AND STELLA

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CHAPTER 1

INTRODUCTION

I.1 THE IMPORTANCE OF OFF-SHELL EFFECTS

Knowledge of the nuclear interaction is steadily increasing in the last few years. The low energy on-shell T-matrix for the two nucleon system now appears to be well known and there exist several potential models that are quantitatively accurate over the elastic energy range. However, in any problem in nuclear physics involving three or more particles knowledge of the on-shell T-matrix, however complete, does not suffice, since off-shell T-matrix elements play an important role. In the past in most cases the solution of such problems has been reduced by force, in other words, by the help of simplifying assumptions, to the solution of effective two body problems. The obvious reason for this procedure is mainly due to the fact that the two body problem can be solved exactly and easily. Gradually appreciation of the importance of realistic off-shell behaviour has become well known, especially after the invention of the Faddeev equations. However most of the work is focussed on the separable interactions due to the simplicity of their application.

Nowadays the three body problem can be formulated rigorously and can be exactly solved even for realistic potentials. It is therefore possible to reinvestigate nuclear problems

to see whether the methods can be improved by taking into account the three body effects. The first part of this work is along this line and deals specifically with the (d,p) reactions. In such a system if we assume that in the initial state there are two particles in a bound state, the deuteron, which is accelerated into a target nucleus, different types of resonances can be expected which are related to the breakup of the deuteron, the formation of the compound nucleus and, if possible, to the scattering of the stripped particle by the target. We are interested in particular in the latter case which is known as (d,p) stripping process to unbound states. The question that we are investigating is how the three body effects are going to influence the resonance behaviour of the system.

On the other hand we can have a three body system consisting of three particles of identical masses. In the low energy region this system could be in a resonance state, n-d, p-d scattering or in a bound state, the model triton. For the latter case a significant amount of work has been done using non-local or partly non-local potentials, however although the on-shell behaviour of these potentials was the same as that of a model local potential, the off-shell behaviour has been substantially different. It is therefore important when using a non-local or a partly non-local potential to determine whether the off-shell behaviour is reasonable. In other words, it is imperative to have criteria with which one will be able to reject all such interactions which though they fit the two-

body system very well, produce unreasonable results in the three-body system due to their different off-shell behaviour. We are addressing this point in the second part of the thesis.

In the next section we are giving a short review and we are outlining the procedures that we are using to investigate the off-shell behaviour of resonances appearing in (d,p) stripping reactions to unbound states. We also indicate in which ways our approach differs from others. In section I.3 we review previous attempts to establish criteria for acceptable non-local interactions and indicate how such criteria can be established. We devote section I.4 to outlining the general plan followed in the thesis.

I.2 CROSS-SECTIONS FOR STRIPPING REACTIONS

Cross-sections for direct stripping reactions to unbound states have been recently subject to a thorough investigation owing to their ability to provide valuable information on the angular momentum, parity and spectroscopic factors of the reaction products.

The theories which are usually employed can be viewed as extensions to the corresponding ones for stripping to bound states (PWBA or DWBA) and are mostly concerned with evaluating the stripping amplitude with a particular accuracy.

For bound state stripping, the used procedure is to consider only the contribution due to the proton-neutron interaction which is the product of the neutron transfer and the

deuteron break-up amplitudes, whereby the integral for the neutron transfer is cut off at a finite radius and it is approximated, either by the value of the integrand at the nuclear radius[1] or by assuming the nuclear wave function outside the nuclear radius to be proportional to spherical Hankel functions[2].

For resonance states, this approach can lead to serious errors since the contribution of the integral beyond the barrier is very important and it is entirely determined by the general requirements of scattering theory.

Huby and Mines[3] have discussed this problem. They used a convergence factor $e^{-\alpha r}$ to evaluate the outer part of the integral at several values of α and then extrapolated numerically to the limit $\alpha \to 0$. Bang and Zimanyi[4] used the one pole approximation near a resonance and chose to employ a Gamow state for the nuclear wave function. A different treatment has been given by Vincent and Fortune[5] who have also discussed the reliability of the extrapolation to $\alpha \to 0$. They have used a scattering solution for the A+n wave function and in addition they consider contour integration to improve the numerical evaluation of the outer part of the integral. In summary, although the above theories tend to give good fits to experiment, there is always a suspicion that these fits might be more indicative of skillful choice of parameters than inherent accuracy of the theory. An improved theory should not only provide reliable analyses of experiment but it should

also provide a more physical understanding of the reaction mechanisms.

The development of an accurate quantum mechanical description of stripping to a resonant state, has been presented by Lipperheide[6] who has derived the stripping amplitude using the graphical technique of Shapiro[7] and has taken into account the reaction kinematics by considering the off-shell extension of the scattering amplitude. In this elegant formalism[8] the cross-section for stripping to resonant states is related to the corresponding measured cross section for the scattering of the stripped particle on the target. The formulae are obtained within the framework of the R-matrix theory using the one pole approximation near an isolated resonance. This approximation which is in general a good one, resides in the assumption that all resonant states of the final nucleus are long lived and energetically far apart. If however two states are close enough, i.e. if the predominant parts of their widths overlap, the above approximation is not satisfactory. The same is true, if background and interference with the background have to be included. The latter can be quite important, when one considers an off-shell extension. Since, like bound states, which are seen as peaks in the energy spectrum of the outgoing protons, resonant states are also seen as peaks, lying on top of a continuum due to the three body breakup, therefore resonant-continuum interference can have an appreciable effect on the cross-section. It is our intention to derive an

exact expression of the scattering amplitude within the above model, thus removing the restriction of an isolated resonance.

Our derivation is based on the eigenfunctions of a Hermitian Hamiltonian[9] which describes the interaction in the internal region and the solutions of the off-shell Schrödinger equation[10] in the external region. We have applied our formalism[11,12] to the first approximation[13] of the transition amplitude and we have shown on the basis of experimental results[14,15,16] that off-shell effects are important in (d,p) stripping processes. In particular using various numerical techniques[17] we show that interference with the off-shell background is more important than the corresponding interference with the on-shell background[18] and one can use this result to interpret resonances which appear in (d,p) and (n,n) processes.

When our results do not entirely agree with the experiment we explain them by considering contributions from other sources[19,20] which are not included in the first order approximation of the amplitude irrespective of whether the latter is expanded around a single pole[21] or is calculated in our way. We test this argument by considering a channel Hamiltonian and its coupled eigensolutions[22] which we use to find an expression for the off-shell T-matrix. Using model calculations[23,26] we show again that off-shell effects are important in reaction processes since they can completely change the resonance behaviour of the system[27].

I .3 THE GROUND STATE COLLAPSE OF THE THREE BODY SYSTEM

The concept of separable interactions has now been widely accepted and a great deal of work has been done which deals with their applications. However it has also become clear, see for example the work of Srivastava[28] and other works cited there, that not only should the separable potentials be fitted to the on-shell data but their off-shell behaviour should also be investigated. This realization has prompted a comparison of the off-shell behaviour by means of the Kowalski-Noyes[29,30] half-shell function $f(p,k)$.

This comparison however is restricted to the off-shell behaviour at some energies and the on-shell behaviour at low energies since the on-shell T-matrix is not known at high energies. It is therefore interesting to investigate these interactions by changing their on-shell properties at higher energies.

The first step in this direction was taken by Beam[31] and Allessandrini[32] who have found that the rank-one separable potential of Tabakin[33] which has a continuum bound state (CBS) produces a ground state collapse (has a deeply bound ground state) in the three-body system. Sofianos et.al.[34] generalized this result by showing that a ground state collapse occurs in the three particle system for separable two-body potentials of rank-one if they have a resonance pole sufficiently close to the real axis. Even more surprising they found that the binding energy of the collapsed state tends to increase when the resonance pole is shifted to higher energies, which indicates that a CBS or a resonance pole near enough to the

real axis is more "dangerous" at very high energies than at lower energies.

However the picture for separable potentials of rank-two was less clear. Sofianos et.al.[34] constructed several separable potentials of rank-two all having the same two nucleon bound state wave function. The rank-one separable potential with the same bound state wave function, i.e. the UPA to all the rank-two interactions, had a resonance pole close to the real axis producing a ground state collapse in the trinucleon system. However with increasing repulsion in the rank-two interactions the ground state collapse soon disappeared again. On the one hand it was found that a ground state collapse could happen for separable potentials of rank-two, but on the other hand it was found not to occur necessarily even if it occurred for the corresponding UPA. More so, the introduction of an additional repulsion moved the resonance pole further away from the real axis and it was not clear if this would not be mainly responsible for the disappearance of the collapse. Similarly inconclusive was the example of a rank-two interaction with a CBS. In this example they were unable to find a collapsed state, finding only weakly bound states in the trinucleon system. However as we shall indicate later, this interaction cannot be regarded as a reasonable one of the type used in two- and three-nucleon calculations on two counts. In the first place it violates the shape independent approxima-

tion for the low energy parameters, a condition known to produce anomalous three-particle binding energy[35,36] and even more its attractive part is of shorter range than the repulsive one, a quite unphysical situation. In view of these ambiguities, it is necessary to investigate this problem to establish clearly whether the presence of a CBS entails a collapse in the three particle ground state. This we do by first considering two sets of rank-two separable interactions in their limiting forms. We construct[37] them with a fixed deuteron wave function and also a fixed CBS at energies of $E_{lab} \approx 2000$ and 4000 Mev respectively. In each set any number of interactions can be generated by changing the parameters of the form factors.

In all cases which we have considered, a ground state collapse still occurs[38] in the three particle system though the binding energy is somewhat reduced when the repulsive form factor in the interaction is made more repulsive. In addition we find that the collapsed state becomes even more tightly bound when the CBS moves to higher energies. This suggests that for this type of interaction the three particle system is not a low energy system any more.

Secondly we consider partly non local potentials which are supposed to provide a better representation of the physical interaction. Here again we restrict ourselves to the limiting forms for simplicity. Since local potentials alone can not produce a CBS and since the total interaction is constructed to

have a CBS at high energy it is difficult to make predictions in this case. However because of the importance of the local potential one is tempted to expect that no collapse will occur. We shall show that this is indeed the case, though we can not definitely exclude the possibility that a collapse will not occur if the CBS moves to very higher energies.

I.4 OUTLINE

In Chapter II we derive the deuteron-nucleus scattering amplitude using the Faddeev equations. We solve the Schrödinger equation off the energy shell in order to relate the differential cross-section to the off-shell stripping amplitude and compare our results with the experiment. We are actually interested in the process

$$1 + (2,3) \rightarrow 2 + (1,3)$$

where the right-hand side finally proceeds to disintegrate

$$(1,3) \rightarrow 1 + 3.$$

If we assume particle 1 to be the target nucleus A and particles (2,3) to be a deuteron d then the above process is called a (d,p) or (d,n) stripping depending on whether particle 2 is a proton or a neutron. In the case in which particle (1,3) remains bound the process is known as stripping to a bound state

as against stripping to an unbound state when (1,3) disintegrates. We should emphasize that this mode of decay is different from the break-up process (1 + (2,3) → 1 + 2 + 3) in which the projectile disintegrates into its constituents in the nuclear and coulomb field of the target A. This is so because in the stripping to unbound states we assume the formation of the intermediate nucleus (A+ one particle), which then proceeds to decay, whereas in the break-up process, the binding effects between target nucleus and stripped particle are neglected.

In chapter III we improve upon this model by including spin and discuss experimental results using differences in on-shell and off-shell scattering. Up to this point we have kept the assumption of a direct reaction mechanism. However in order to be able to describe compound effects we need a formalism able to describe decay in all energetically open channels. We outline such a model in chapter IV. In chapter V we extend the formalism of chapter II through the off-shell T-matrix and we compare the accuracy of this theory with the exact calculations. In chapter VI we investigate off-shell effects in the low energy three body system, the model triton, using separable potentials of rank-two which have continuum bound states. We devote chapter VII to do an investigation similar to that of chapter VII but using more realistic potentials. We give in chapter VIII a summary of our results with our conclusions.

CHAPTER II

OFF-SHELL EFFECTS IN (d,p) STRIPPING

Procedures by which to develop interesting solutions of the three-body model are not obvious. The usual belief that adequate effective potentials can describe the deuteron-nucleaus system rather accurately is too simple. On the other hand exact solutions via the Faddeev equations provide less physical insight since in this model allowance is not made for an additional degree of freedom related to the spectroscopic factors for stripping reactions. However, this difficulty is not essential and due to the importance of the three-body effects in (d,p), (p,d), and similar reactions, the three-body formulation of these cases has some advantage.

II.1 THREE-BODY APPROACH

We adopt here the formalism of Newton[13] to formulate the three-particle scattering problem. This formalism is related to the Faddeev equations but it is more concise and is better suited for our purpose. The main difference is that we choose our final and initial states to be the eigenstates of the first order, rather than the zero order Hamiltonian. The starting point is to realize that for rearrangement collisions we have to distinguish between the initial and final Hamiltonians. For the total Hamiltonian, we have,

[1a] $$H = K_a + V_a = K_b + V_b$$

where $K_a(K_b)$ is the total kinetic energy of the three particles

$(i,j,k = 1,2,3)$ in the initial (final) state respectively and $V_a(V_b)$ is their total initial (final) interaction, viz. $V_a = (\sum_i V_i)_a$. The subscripts a and b refer to the initial and final system and V_i is the force between the two particles labelled by j and k, $j \neq k \neq i$. Since the interactions are additive, the calculation of the scattering amplitudes need not start with the kinetic energy as a zero order Hamiltonian. Suppose the Hamiltonian is written

$$H = H_{oa} + H_a \quad , \quad H_{oa} = K_a + V_a - H_a$$

[1b]

$$H = H_{ob} + H_b \quad , \quad H_{ob} = K_b + V_b - H_b$$

then one may first calculate the states of H_{oa} (H_{ob}) and proceed to calculate the exact quantities corresponding to the full Hamiltonian H from those corresponding to $H_{oa}(H_{ob})$ as a starting point. For the T-matrix elements, we have

[2] $$T_{\beta\alpha} = (\phi_b(E,\beta), H_b \psi^{(+)}(E,\alpha))$$

where H_b is a part of the total potential interaction and $\psi^{(+)}(E,\alpha)$ is the exact scattering eigenstate of the complete Hamiltonian $(K_a + V_a)$ with outgoing boundary conditions:

[3] $$\psi^{(+)}(E,\alpha) = \phi_a(E,\alpha) + g_a^{(+)} H_a \psi^{(+)}(E,\alpha)$$

and

[4] $$g_a^{(+)} = (E - H_{oa} + i\eta)^{-1} .$$

Clearly in our representation ϕ_a and ϕ_b are the eigenstates of the first order Hamiltonians H_{oa} and H_{ob}, where now we have dropped the arguments for convenience.

As an example of the utility of these expressions we consider a deuteron-stripping reaction from a nucleus C. Let the deuteron initially impinge upon C and finally let the neutron be bound in C to form the nucleus F and the proton emerge freely. The full Hamiltonian H, in an obvious notation, is

$$H = K_{DC} + K_{PN} + V_{PN} + V_{CN} + V_{CP} \quad \text{initially}$$

$$H = K_{PF} + K_{NC} + V_{PN} + V_{CN} + V_{CP} \quad \text{finally} \qquad [5a]$$

and we can set

$$H_{oa} = K_{DC} + K_{PN} + V_{PN}, \quad H_a = V_{CN} + V_{CP}$$

$$H_{ob} = K_{PF} + K_{NC} + V_{CN}, \quad H_b = V_{CP} + V_{PN} \qquad [5b]$$

Then the eigenstates of H_{oa} are direct products of the eigenstates of $K_{DC} + (K_{PN} + V_{PN})$ and the eigenstates of H_{ob}, are direct products of the eigenstates $K_{PF} + (K_{NC} + V_{CN})$. If V_{PN} and V_{CN} are known, then the eigenstates of H_{oa} and H_{ob} can be calculated exactly and so can the matrix $T_{\beta\alpha}$ in [2].

It is convenient to rewrite [3] using the identity

[6] $$\frac{1}{A} - \frac{1}{B} = \frac{1}{B}\,(B-A)\,\frac{1}{A} \quad .$$

for $g_a^{(+)}$ and $g^{(+)} = (E-H+i\eta)^{-1}$. One finds

[7] $$\psi^{(+)} = \phi_a + g^{(+)} H_a \phi_a$$

and consequently we have for the transition element [2],

[8] $$T_{\beta\alpha} = (\phi_b, (H_b + H_b g^{(+)} H_a)\,\phi_a) \quad .$$

We now define an operator

[9] $$\hat{T}_{ba} = H_b + H_b g^{(+)} H_a$$

such that $T_{\beta\alpha}$ is a matrix element of $\hat{T}_{ba}$ between the eigenstates of H_{oa} and H_{ob}. Invoking relation [6] for $g^{(+)}$ and $g_a^{(+)}$ it is easily shown that

$$\hat{T}_{ba} = H_b + H_b g_a^{(+)} (H_a + H_a g^{(+)} H_a)$$

or

[10] $$\hat{T}_{ba} = H_b + H_b g_a^{(+)} \hat{T}_{aa} \quad .$$

Equation [10] is very useful. It relates the transition operator for the process a→b to that describing the process a→a and it is particularly suited for elastic (or inelastic scattering) and rearrangement collisions;

[11] $$1 + (2,3) \Rightarrow \begin{cases} 1 + (2,3) \\ 2 + (1,3) \\ 3 + (1;2) \end{cases} \quad .$$

The $\hat{T}$ operators appropriate to the description of the above collisions are respectively $\hat{T}_{11}$, $\hat{T}_{21}$ and $\hat{T}_{31}$. $\hat{T}_{11}$ refers to the scattering of particle (1) by the bound state of (2,3), while $\hat{T}_{21}$ and $\hat{T}_{31}$ refer to the corresponding rearrangements of particles (2) and (3) with particle (1). In terms of $\hat{T}_{11}$, these are

$$\begin{aligned}\hat{T}_{11} &= (V_{12} + V_{13}) + (V_{12} + V_{13})g_1^{(+)}\hat{T}_{11} \\ \hat{T}_{21} &= (V_{23} + V_{21}) + (V_{23} + V_{21})g_1^{(+)}\hat{T}_{11} \\ \hat{T}_{31} &= (V_{31} + V_{32}) + (V_{31} + V_{32})g_1^{(+)}\hat{T}_{11}\end{aligned} \qquad [12]$$

where according to [4] $g_1^{(+)} = (E-K_1 - V_{23} + i\eta)^{-1}$. Formal manipulations are found in Newton[13]. For conciseness we shall only give the steps leading to the final result.

We define the auxiliary operators

$$\tau_{11} = V_{23}g_1^{(+)}\hat{T}_{11}\ ,\ \tau_{21} = V_{13}+V_{13}g_1^{(+)}\hat{T}_{11}\ ,\ \tau_{31} = V_{12}+V_{12}g_1^{(+)}\hat{T}_{11} \qquad [13]$$

in terms of which the original $\hat{T}$'s are expressed as

$$\hat{T}_{11} = \tau_{21}+\tau_{31}\ ,\ \hat{T}_{21} = V_{23}+\tau_{11}+\tau_{31}\ ,\ \hat{T}_{31} = V_{23}+\tau_{11}+\tau_{21}. \qquad [14]$$

After introducing the two body operators in the three body Hilbert space, that is

$$T_1 = V_{23} + V_{23}g_1^{(+)}V_{23} \qquad [15]$$

etc.

then the matrix equation for the τ's is easily found to be

$$\tau = \delta + \delta G^{(+)}\tau \qquad [16]$$

where $G^{(+)}(E) = (E-K_1+i\eta)^{-1}$ is the three body free Green's

operator and

$$[17] \qquad \hat{S} = \begin{pmatrix} 0 & T_1 & T_1 \\ T_2 & 0 & T_2 \\ T_3 & T_3 & 0 \end{pmatrix}$$

For the process 1 => 2, which we are actually interested in, a single iteration of [16] yields,

$$[18] \qquad \hat{T}_{21} = V_{23} + T_3 + T_1 G^{(+)} (T_2 + T_3) + T_3 G^{(+)} T_2 + \ldots .$$

Applying [18] to a deuteron (d), (p = 2, n = 3), impinging onto a nucleus, (C = 1), (C+d) → p+(C+n), and retaining the lower order terms, we have with an obvious notation

$$[19] \qquad \hat{T}_{21} = V_{pn} + t_{Cp} + t_{pn} G^{(+)} (t_{Cn} + t_{Cp}) + t_{Cp} G^{(+)} t_{Cn} ,$$

where matrix elements have to be taken between the eigenstates

$$[20] \qquad \begin{aligned} \phi_a &= |\chi_a, u_a\rangle \\ \phi_b &= |\chi_b, u_b\rangle \end{aligned}$$

with $|\chi_a\rangle$ referring to the spin parts of target and projectile, and $|u_a\rangle = |u_d, \vec{k}_d\rangle$ the product of the internal wave function of the deuteron and an initial plane wave. A similar expression holds for $|u_b\rangle = |u_{nC}, \vec{k}_p\rangle$, where now u_{nC} refers to the neutron-nucleus wave function.

It is easy to give a physical interpretation of the multiple scattering series in [18]. First of all we note that the multiple scattering expansions are rearrangements of the Born series. This can be seen by considering the Born series for the T-matrix

corresponding to the process 1 + (2,3) => 2 + (1,3) and making use of the operator identity[6] for g^{+} and $g_1^{(+)}$. We shall note further that if we write $T_i \approx V_i$ and limit our expansions to the first order in the interaction potentials, we find that [19] gives

[21] $$\hat{T}_{21} \approx V_{pn} + V_{Cp}$$

which is just the first Born approximation for this process.

Usually only the first term in [21] is evaluated for the stripping amplitude and it is referred to as direct stripping. The second term, which describes the nucleus-proton interaction, is accounted for, using distorted rather than plane waves for the outgoing proton. This approximation should work well both for bound - and resonant state, stripping. In the latter case, however, care must be taken since the evaluation of the stripping amplitude includes now the region of space where the compound nucleus forms. This means that other important final states as C+n might exist and therefore competing, compound reactions as well as channel coupling might have to be considered. In this chapter we shall retain the assumption of a direct reaction mechanism but we shall use the full scattering wave function of the C+n system to give a prescription for the stripping amplitude which will be independent of the resonances.

Using [20] and ignoring spins for simplicity, we can write for the transition amplitude in the plane wave Born approximation

[22] $$T_{21} = \langle u_{nC}, \vec{k}_p | V_{pn} | \vec{k}_d, u_d \rangle .$$

Now after eliminating the deuteron potential through partial integration of the Schrödinger equation for the relative motion of the deuteron the above expression leads to,

[23] $$T_{21} = - \frac{t_{B,nC} \cdot t_{d,pn}}{(E_n - \frac{p^2}{2m_n} + i\eta)}$$

where

[24] $$t_{d,pn} = - ((\frac{p_d}{2} - p_n)^2/m_n + \varepsilon_d) \int u_d(\vec{\rho}) e^{-i\vec{\rho}\cdot\vec{\nu}} d\vec{\rho} = G(\vec{\nu})$$

$$\vec{\nu} = (\frac{1}{2}\vec{k}_d - \vec{k}_p)$$

is the amplitude describing the deuteron break-up and

[25] $$t_{B,nC} = (E_n - \frac{p_n^2}{2m_n}) \int u_{nC}(\vec{r}) e^{i\vec{r}\cdot\vec{\mu}} dr$$

$$-\vec{\mu} = \vec{k}_d - \vec{k}_p \frac{m_C}{m_C + m_p}$$

is the fully off-shell scattering amplitude for the transferred neutron with energy E_n and momentum $p_n = \hbar k_n \neq \sqrt{2m_n E_n}$ impinging on the target nucleus. Note that there is a singularity in the energy denomination of [23] and a term $+i\eta$ is required for an outgoing wave. This singularity introduces a phase shift in the scattered wave that modifies its asymptotic form. On the other hand, [25] ensures that no singularity is present in [23] in the case of stripping to a bound state. Also note that in [24] ε_d is the binding energy of the deuteron and we have taken $m_p = m_n = \frac{m_d}{2}$.

Setting $\vec{k}_n \approx \vec{k}_d - \vec{k}_p \frac{m_C}{m_n + m_C}$, which is the condition for forward scattering ($m_C >> m_n$) we get from [25] the half off-shell scattering amplitude and consequently from [23] one finds the

well known relation[6] for stripping to unbound states

$$[26] \quad \left(\frac{d\sigma}{d\Omega_p dE_p}\right) = \frac{2\mu}{(2\pi\hbar)^2}\left(\frac{k_p}{k_s}\right)\left|G\left(\frac{1}{2}\vec{k}_d - \vec{k}_p\right)\right|^2 \mathrm{Im}\, f_c(\vec{k}_n, \vec{k}_n, E_n)$$

where the subscript c, stands for elastic nC scattering and μ is the reduced mass $\frac{\mu_{dA}\mu_{pB}}{\mu_{nA}}$ with B = n+C. The derivation of [26] is given in the Appendix. It will be seen that it is valid only under the assumption that we consider only the states of the nucleus B that can give the pair C+n. This is so because the imaginary part of the off-shell scattering amplitude (appearing in [26]) is related to the total cross-section via the optical theorem and it has been assumed that this mode of decay will dominate the reaction. We shall note here that the nuclear wave function $u_{nC}(r)$ is really still a sum of channel functions, one for each neutron that can be emitted. Only if we were to assume that one and only one partial wave is resonant and the three body scattering amplitude does not depend on other partial waves in the (n,C) system, then the cross-section can be well predicted by the single level theory.

II.2 OFF-SHELL STRIPPING AMPLITUDE

From [26] it is apparent that knowledge of the on-shell scattering amplitude does not suffice to determine the amplitude for stripping into unbound states. On the other hand, the off-shell amplitude is not simply related to its on-shell value. The technique most often used to go off-shell is through separable interactions or through the one pole expansion approximation. We shall give here an explicit solution, within the framework of the R-matrix theory, which although it is easily

obtainable does not appear to have been applied to calculational problems.

We consider the Schrödinger equation off-the-energy shell in the ℓth partial wave

$$[27] \qquad (E-H)\psi_{\ell,q}(E,r) = (E-E')j_\ell(qr)$$

$$E = \frac{\hbar^2}{2m}k^2 \neq E' = \frac{\hbar^2}{2m}q^2$$

and introduce the surface operators

$$[28] \qquad L_b = \frac{\hbar^2}{2ma}\delta(r-a)\left(\frac{d}{dr} - \frac{b}{r}\right)r$$

$$L_L = \frac{\hbar^2}{2ma}\delta(r-a)\left(\frac{d}{dr} - \frac{L_\ell}{r}\right)r$$

where L_ℓ is the logarithmic derivative of an outgoing wave, viz

$$[29a] \quad L_\ell(ka) = \left(\frac{rO'_\ell(kr)}{O_\ell(kr)}\right)_{r=a} = \left(\frac{rI'_\ell(kr)}{I_\ell(kr)}\right)^*_{r=a} = S_\ell(ka) + iP_\ell(ka)$$

with

$$[29b] \quad \begin{aligned} O_\ell(kr) &= G_\ell(kr)+iF_\ell(kr) = -(kr)n_\ell(kr)+i(kr)j_\ell(kr) \\ I_\ell(kr) &= G_\ell(kr)-iF_\ell(kr) = -(kr)n_\ell(kr)-i(kr)j_\ell(kr). \end{aligned}$$

$j_\ell(kr)$ and $n_\ell(kr)$ are the usual spherical Bessel and Neumann functions and b defines the boundary condition of the internal wave function to be specified shortly. We also note the asymptotic solutions

$$[29c] \quad \begin{aligned} F_\ell(r) &\overset{r}{\underset{\infty}{\to}} F_\alpha(r) = \sin\left(kr - \frac{\ell\pi}{2}\right) \\ G_\ell(r) &\overset{r}{\underset{\infty}{\to}} G_\alpha(r) = \cos\left(kr - \frac{\ell\pi}{2}\right) \\ O_\ell(r) &\overset{r}{\underset{\infty}{\to}} O_\alpha(r) = e^{ikr} = I^*_\alpha(r) \end{aligned}$$

In the R-matrix theory, it is customary to divide the configuration space into internal and external regions separated at r=a. The Hermicity of the modified Hamiltonian

[30] $$H = H + L_b$$

is easily verified and we may rewrite [26] as follows

[31] $$(E-H)\psi_{\ell,q}(E,r) = (E-E')j_\ell(qr)+(L_L-L_b)\psi_{\ell,q}(E,r)-L_L\psi_{\ell,q}(E,r)$$

The operators L_b and L_L are introduced in [31] to facilitate the replacement of $\psi_{\ell,q}(E,r)$ by an expansion over a complete set of eigenfunctions $\psi_\lambda(r) = \frac{u_\lambda(r)}{r}$ of the Hamiltonian H with eigenvalues E_λ.

[32] $$H\psi_\lambda(r) = E_\lambda\psi_\lambda(r)$$

[33] $$\psi_{\ell,q}(r) = \sum_\lambda A_\lambda \frac{u_\lambda(r)}{r} \qquad r \leq a$$

[34] $$\psi_{\ell,q}(E,r) = F_\ell(qr)/qr + f_\ell(q,k,E)\frac{O_\ell(kr)}{r} \qquad r \geq a .$$

The right hand side of [31] is simplified by making a suitable choice for the boundary condition b. The particular choice is

[35] $$\left(a\frac{d}{dr}u_\lambda(r)\right)_{r=a} = b\cdot u_\lambda(a)$$

so that

[36] $$\int u_\lambda^2(r)dr = \int \psi_\lambda^2(r)r^2dr = 1$$

we shall also note in passing that

[37] $$L_L \psi_{\ell,q}(E,r) = L_L F_\ell(qr)/qr \qquad r = a$$

which shows the reason for the definition of the operator L_L. Equation [31] can now be reduced to calculable form by inserting [33] and taking overlaps with the basis functions ψ_λ. The resulting equation for $\psi_{\ell,q}(E,a)$ is easily found to be

[38] $$\psi_{\ell,q}(E,a) = (E-E') \int_0^a F_\ell(qr) \sum_\lambda \frac{u_\lambda(r)u_\lambda(r)}{(E-E_\lambda)(qa)}\, dr +$$

$$(b-L_\ell(ka)) \cdot \sum_\lambda \frac{\hbar^2}{2ma} \frac{u_\lambda(a)u_\lambda(a)}{(E-E_\lambda)} \psi_{\ell,q)}(E,a) -$$

$$\left(\frac{d}{da} - \frac{L_\ell(ka)}{a}\right)(F_\ell(qa)/(q) \cdot \sum_\lambda \frac{\hbar^2}{2ma} \frac{u_\lambda(a)u_\lambda(a)}{(E-E_\lambda)}\Big)$$

Recalling the usual definition of the R-function

[39] $$R(a,a) = \frac{\hbar^2}{2ma} \sum_\lambda \frac{u_\lambda(a)u_\lambda(a)}{(E_\lambda-E)} = \sum_\lambda \frac{\gamma_\lambda^2}{(E_\lambda-E)}$$

one finds

[40] $$\psi_{\ell,q}(E,a) = \frac{R(a,a)L - (q)^{-1}(k^2-q^2) \int_0^a F_\ell(ar)R(a,r)dr}{(1-R(a,a)(L_\ell-b))}$$

with

[41] $$L = \left(\frac{d}{da} - \frac{L_\ell}{a}\right)(aF_\ell(qa)/qa) = \frac{(qa)^{-1}}{2i}((L_\ell(\rho)-L_\ell(x))O_\ell(\rho) -$$

$$(L_\ell^*(\rho)-L_\ell(x))I_\ell(\rho) = (qa)^{-1}\,\bar{L}$$

where $\rho = qa$, $x = ka$.

Referring to [39] we shall note in passing that the quantities $\gamma_\lambda^2 = \frac{\hbar^2}{2ma} u_\lambda^2(a)$ are the usual reduced width amplitudes.

From [40] we can easily find A_λ and the fully off-shell amplitude either through

$$[42] \qquad T_\ell(p,q,E) = \int j_\ell(pr)V(r)\psi_{\ell,q}(E,r)r^2dr$$

or using the known relation (Bishop[10]) between the full off-shell and the half-off-shell amplitude. Actually we are interested in the imaginary part of the forward scattering amplitude, Im $f_c(k_n,k_n,E_n)$. By the use of the off-shell optical theorem we have

$$[43] \qquad \mathrm{Im}T(\vec{p},\vec{q},E) = -\frac{1}{(4\pi)}\left(\frac{2m}{\hbar^2}\right)k \sum_\ell (2\ell+1)T^*_\ell(k,p,E)T_\ell(k,q,E)P_\ell(\hat{p}\cdot\hat{q})$$

and consequently we find

$$[44] \qquad \mathrm{Im}f(\vec{q},\vec{q},E) = \frac{1}{4\pi}k\sum_\ell (2\ell+1)\left|f_\ell(k,q,E)\right|^2$$

where $f_\ell(k,q,E)$ is found from [40] and [34] at large a, viz.

$$[45] \qquad f_\ell(k,q,E) = \frac{O_\alpha^{-1}(x)}{q}\left(\frac{+(q^2-k^2)a\int_0^a F_\ell(qr)R(a,r)dr + R(a,a)\bar{L}}{(1-R(L_\ell-b))} - F_\alpha(\rho)\right)$$

with $P_n = \hbar k_n = \hbar q$ the momentum transfer and $k_n \neq k = \sqrt{\frac{2m}{\hbar^2}E}$

In [45] certain special cases may be noted; the half-off-shell hard sphere scattering element $\frac{O_\alpha^{-1}(x)}{q}F_\alpha(\rho)$ or the corresponding on-shell by putting q=k. Also one should note the influence of the first term which is the contribution from within the internal region and it is a typical off-shell effect.

It is more instructive to rewrite [45] in a manner resembling the on-shell collision function. Introducing [29] into [45] one finds:

$$[46]\quad f_\ell(k,q,E) = O_\alpha^{-1}(x)\{\frac{2iA(q)O_\alpha^{-1}(\rho)}{[1-R(L_k-b)]} + \frac{(L_q-L_k)R}{[1-R(L_k-b)]} +$$

$$+ O_\alpha^{-1}(\rho)\ (1 - \frac{(L_q^*-L_k)R}{[1-R(L_k-b)]})I_\alpha(\rho) - 1\}O_\alpha(\rho)/(2iq)$$

where L_q and L_k stands for $L_\ell(\rho)$ and $L_\ell(x)$ respectively, and $A(q)$ is given by

$$[47]\qquad A(q) \equiv (q^2-k^2)a\int_0^a F_\ell(qr)R(a,r)dr.$$

The first two terms inside the curly brackets in [46] are purely due to off-shell effects and they will strongly affect the cross-sections with increasing off-shell distance $s = \frac{p_n^2}{2m_n} - E_n$. They are of course zero for $k_n = k$. The remaining part has the formal structure of the on shell amplitude, where the off-shell effects are manifested through the off-shell penetrability $P_\ell(qa)$ and the shift factor $S_\ell(qa)$. It is apparent that all these three terms will contribute to the pure resonant cross-section, and in addition they will also provide for considerably more distortion through their interference, with increasing s.

II.4 QUALITATIVE PICTURE OF THE STRIPPING PROCESS

In deuteron induced reactions, stripping processes occur in competition with Coulomb excitation, below 3 Mev and with compound reactions, above 30 Mev. In the energy region 3 to 30 Mev however these other two processes can be neglected since they do not contribute substantially* to the total cross-section. When Coulomb forces are neglected (d,p) and (d,n) reactions are equivalent.

In a simple picture we can already see qualitatively some important features of the process. In fig. (1a) the deuteron d impinges on a target A with momentum $\vec{p}_d$. The deuteron is broken up and one of its constituents, say the proton, misses the target. In some cases the neutron will interact (elastically or inelastically) with the target as shown in fig. (1b). In other cases it will be captured by the nucleus to form a bound state, fig. (1c) or it will be instantaneously released (fig. 1(d). In all these processes information about the nucleus B = A+n is obtained. The last mode however is particularly important in extracting information about bound states which are embedded into continuum (CBS). These states appear spaced close to the neutron threshold energy but they can still be resolved experimentally as can be seen from fig. 2.

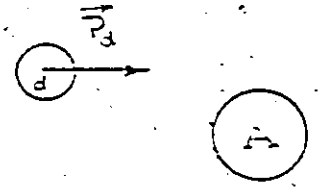
From the conservation laws we have

$$[48] \qquad p_n^2 = (\vec{p}_d - \vec{p}_p)^2 = p_d^2 + p_p^2 - 2p_d p_p \cos\theta$$

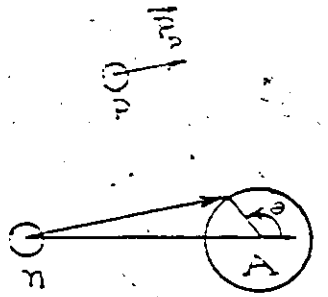
and

$$E_d = E_p + \varepsilon_d + E_n \qquad (49)$$

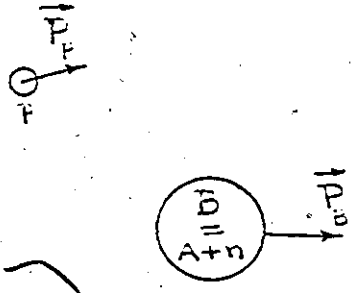
*Page 334 of Einführung in die Grundlagen der Kernphysik, H. von Buttlar, ref. B(2).



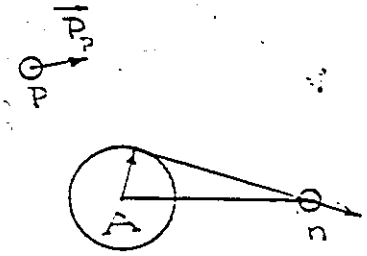
(a) Initial stage: the deuteron d is accelerated on the target A.



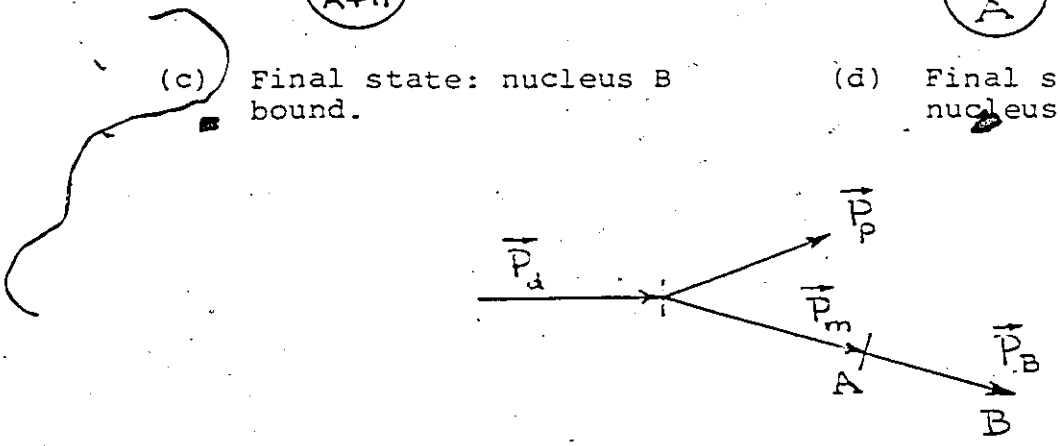
(b) Intermediate state: The deuteron is broken up. The proton p misses the target whereas the neutron n interacts with it.



(c) Final state: nucleus B bound.



(d) Final state not bound, nucleus B decays to A+n.



(e) Momentum picture

Fig. 1 Schematic diagram of a (d,p) stripping process.

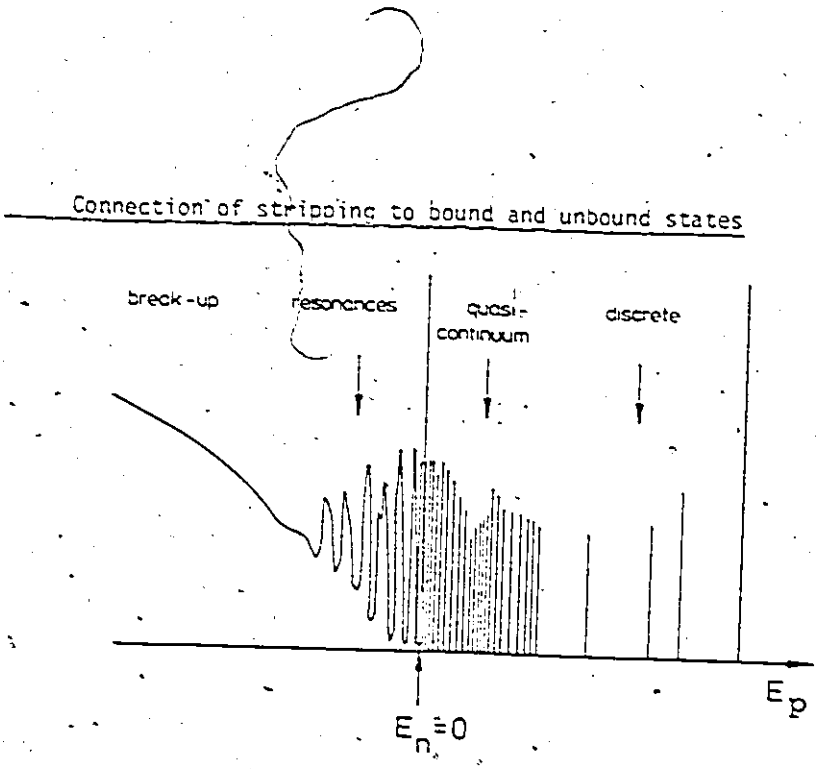


Fig. 2: Schematic view of a spectrum of the (d,p) reaction at a given angle. The threshold for emission of a neutron is denoted by an arrow.

In Fig. 2 a schematic view of the spectrum of a (d,p) stripping reaction is shown. For simplicity of presentation we assume that the transferred particle is a neutron. The discrete well separated states at the high energy end are followed by a region where the level density becomes higher and higher and may not be resolved experimentally any more. This region is called continuum*, yet the levels are still discrete bound states. Above the neutron emission threshold there will be a population of isolated resonances which will go over into an even more structureless continuum at lower energies E_p.

*Proceedings of 1978 INS International Symposium on nuclear reaction mechanism, Fukuoka, Japan. (G. Baur, R. Shyam, F. Rösel and D. Trautmann).

where ε_d is the binding energy of the deuteron. For the energy transferred by the stripped particle we have

[50] $$E_n = E_x + \frac{p_B^2}{2m_B} - Q$$

where E_x is the excitation energy of the target $\frac{p_B^2}{2m_n}$ its recoil and Q is the neutron threshold

[51] $$Q = (-m_B + m_A + m_n) .$$

Using Equations [48] to [51] we get

[52] $$Q - \varepsilon_d - E_x = (1 + \frac{m_p}{m_B})E_p + (\frac{m_d}{m_B} - 1)E_d - 2\frac{\sqrt{m_d m_p}}{m_B}\sqrt{E_d E_p}\cos\theta$$

which shows that if we measure E_p at a certain angle we can get the levels of the nucleus B.

It is easy to show that the neutron has moved off the energy shell by considering the off-shell distance s which is related to the stripping angle via:

[52] $$s = \frac{p_n^2}{2m_n} - E_n .$$

If we define $\frac{\vec{p}_d}{2} - \vec{p}_p = \vec{\nu}$ then we find that, see appendix A

[54] $$s = \varepsilon_d + \frac{\hbar^2}{2\mu_n}\nu^2$$

which shows that s>0.

II.V APPLICATIONS

We have chosen to compute the $^{15}N(d,p)^{16}N$ reaction leading to $J^{\pi} = 1^-$, 1^+ and 1^- states at E_x = 3.519, 4.318 and 4.398 Mev respectively. These states are unbound against neutron decay and they are observed as resonances (Zeitniz[14] in the elastic scattering of neutron by ^{15}N. They occur at neutron energies E_n = 1.095, 1.944 and 2.038 Mev and have been selected for our investigation because of ambiguities arising in their interpretation in earlier works. We have calculated the resonance wave functions $u_\lambda(r)$ in a square well potential with radius r = 2.65 fm (R-matrix radius a = 4.69 fm) and a depth adjusted such that for a given angular momentum ℓ the function $u_\lambda(r)$ has an eigenvalue equal to the energy of the resonance considered. As boundary condition at r = a we have used $b = -\ell$, which defines the eigenvalues $E_{\lambda\ell} = -V + \frac{\hbar^2}{2ma^2}\mu_{\lambda\ell}$, where $\mu_{\lambda\ell}$ is the λ^{th} root of the spherical Bessel function $j_{\ell-1}(\mu_{\lambda\ell}) = 0$.

Clearly with this boundary condition the resonance energies will differ from the eigenvalue $E_{\lambda\ell}$ by a "level shift", but this is immaterial for our present investigation. We shall note here that the use of a nuclear force field with a sharp cut-off at a given radius is very crude and a smoother radial dependence of the potential should be assumed. Indeed our formalism is independent of the choice of the potential form. We have chosen to use a square well however, to facilitate comparison with earlier works.

To account for differences between the actual physical resonance widths and the single particle widths, we have introduced a parameter S to multiply the reduced width amplitudes γ_λ^2 and we have chosen S such that it reproduces the experimental deduced resonance widths in the $^{15}N(n,n)$ system. We remark that in the single level limit the parameter S is the so called "spectroscopic factor" for the reaction.

We have included only six R-matrix levels in our calculations since inclusion of more levels did not provide for any essential variation of our results. We may note in passing that in all cases that we have considered, the variations between the six and the single level limit did not amount to more than 2%, relative to the resonance peak, and one may surmise that the above resonances can be reasonably well described as single particle ones. Figure 3 shows the differential cross section of the $\ell=2$, E_x = 3.519 Mev resonance as a function of the neutron energy E_n for different values at the stripping angle θ_ℓ. Comparison with one of the measured cross sections of Fuchs et al.[15] at θ_ℓ = 15° seems to be good. The angular distribution gives also a good fit to the data but there is a disagreement at very large off-shell distances ($s > 5$). This is partly due to the fact that very large off-shell distances are already contradicted by our assumption of forward scattering ($m_c >> m_n$) which in turn restricts the amount of angular momentum that can be transferred into the target.

We notice that both the single and the six level limit gave satisfactory results and it is thus surprising that Fuchs

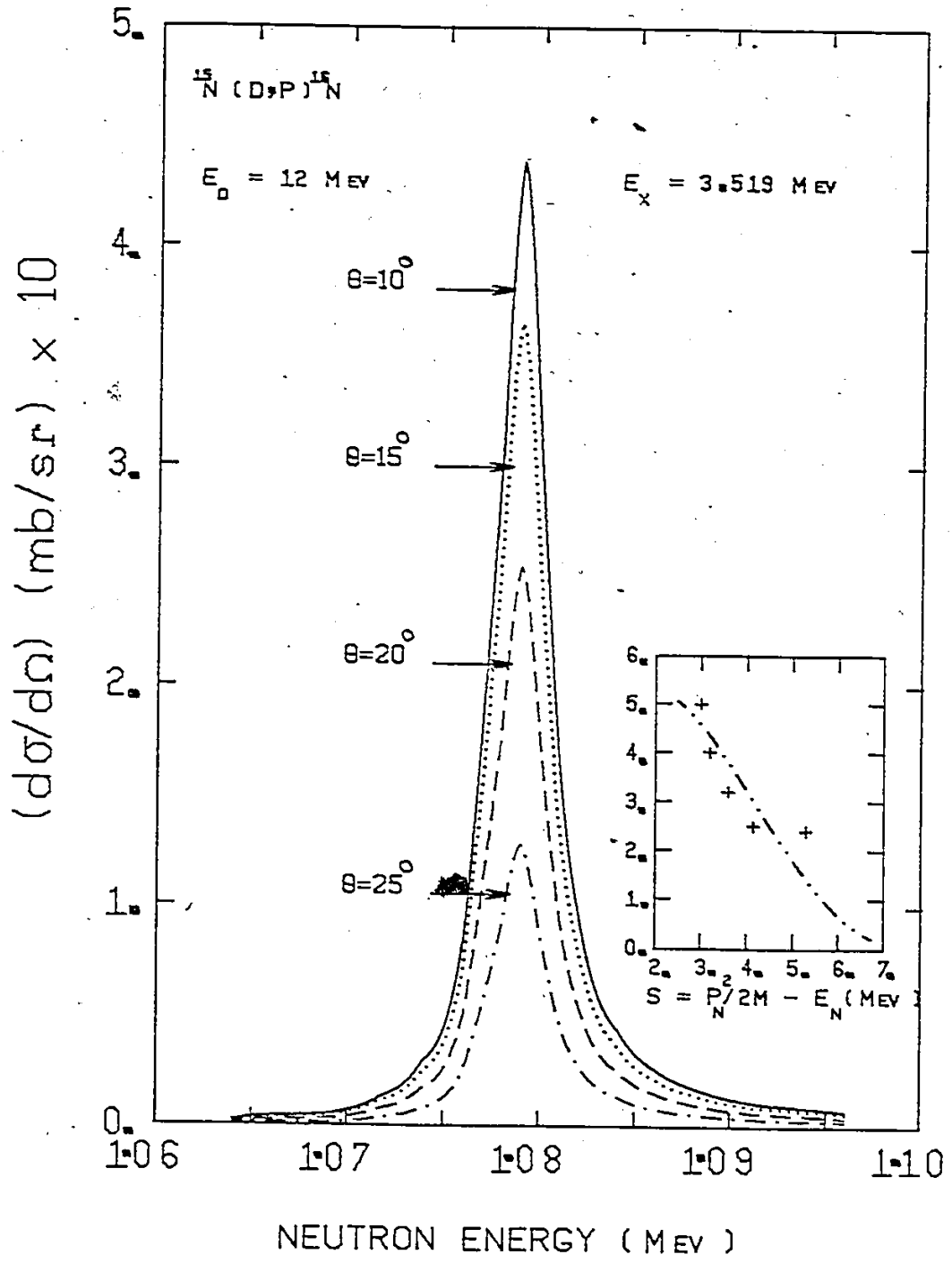


Fig. 3. The differential cross section as a function of the neutron energy E_n at different stripping angles θ. Note that the resonance energy is shifted by 16 eV due to the 'shift factor'. The inset shows the differential cross section as a function of the off-shell distance s. The experimental points are taken from the work of Fuchs et al. (15). The R-matrix parameters for this resonance are r = 2.65, A = 3.2, and b = -ℓ.

et al.[15] reported a less satisfactory single level fit. Since in the single level limit, the two methods differ only in the inclusion of the off-shell background term and evaluation of the radial wave function outside the nuclear interaction, we conclude that the off-shell background term and its interference are quite important in this case.

In passing we note that contrary to the case of the E_x = 5.730 resonance (Möhring and Lipperheide[8]), which becomes symmetric off the energy shell, we see from fig. 4 that the E_x = 4.318 Mev resonance becomes asymmetric as one goes off-shell. Since changes in the resonance shape are entirely attributable to the off-shell effects this is indicative of a very strong dependence on them. In the single level limit it is the imaginary part of the off-shell form factor that accounts for this change in shape and this shows that its inclusion in the calculations can become rather important.

A rather dramatic change of pattern occurs in the $\ell = 0$ E_x = 4.398 Mev resonance as one traverses the gap between $s = 0$ and the smallest value of s in the (d,p) reaction. This resonance appears as a dip in the neutron cross-section but it is not observed in the (d,p) reaction and its interpretation has remained inconclusive so far. It can now be explained qualitatively by considering the contributions of the interference terms both on and off the energy shell. It can be seen from fig. 5 that the on-shell contribution is

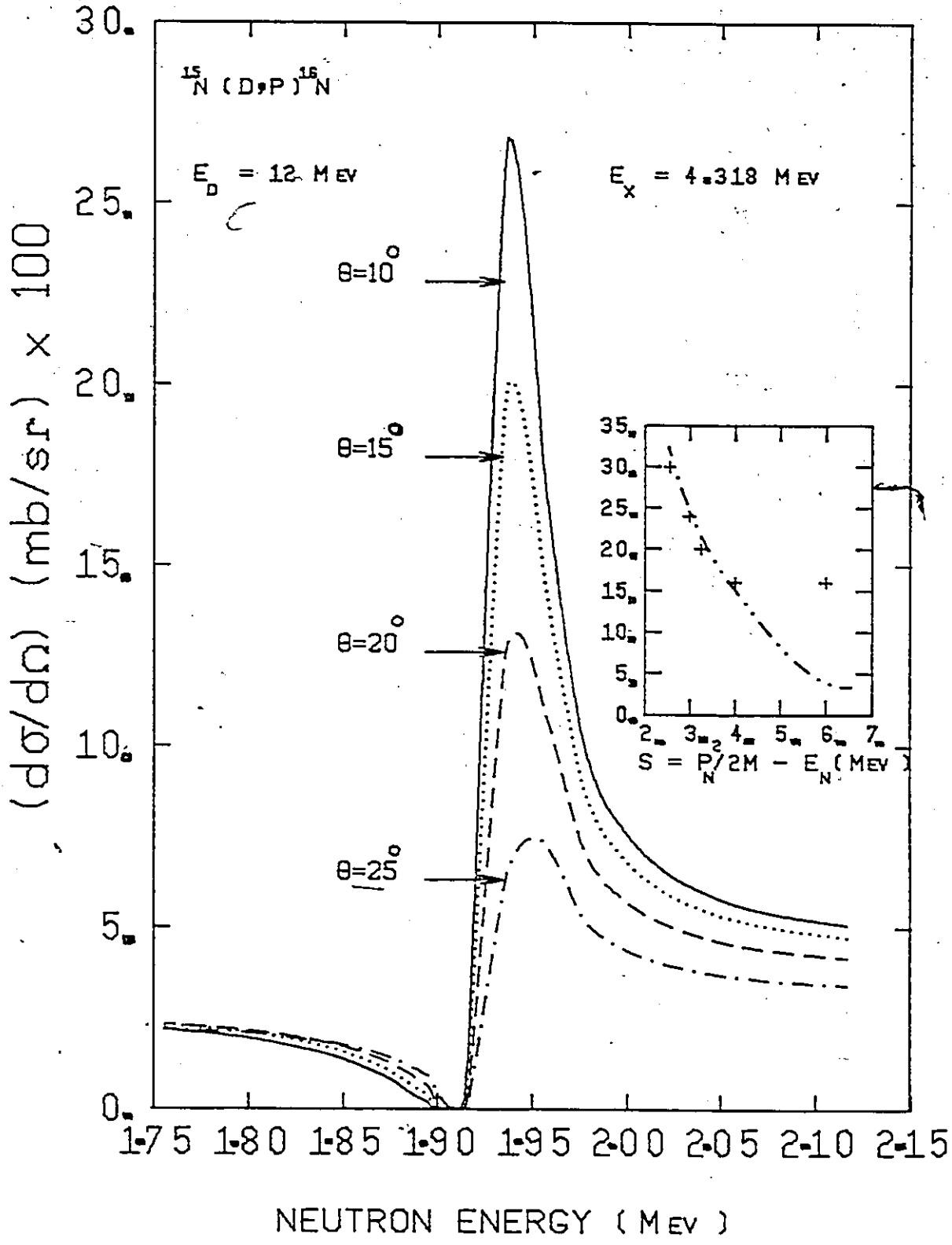


Fig. 4. The differential cross section as a function of the neutron energy E_n at different stripping angles θ. The inset shows the differential cross section as a function of the off-shell distance s. The experimental points are taken from the work of Fuchs et al. (15).

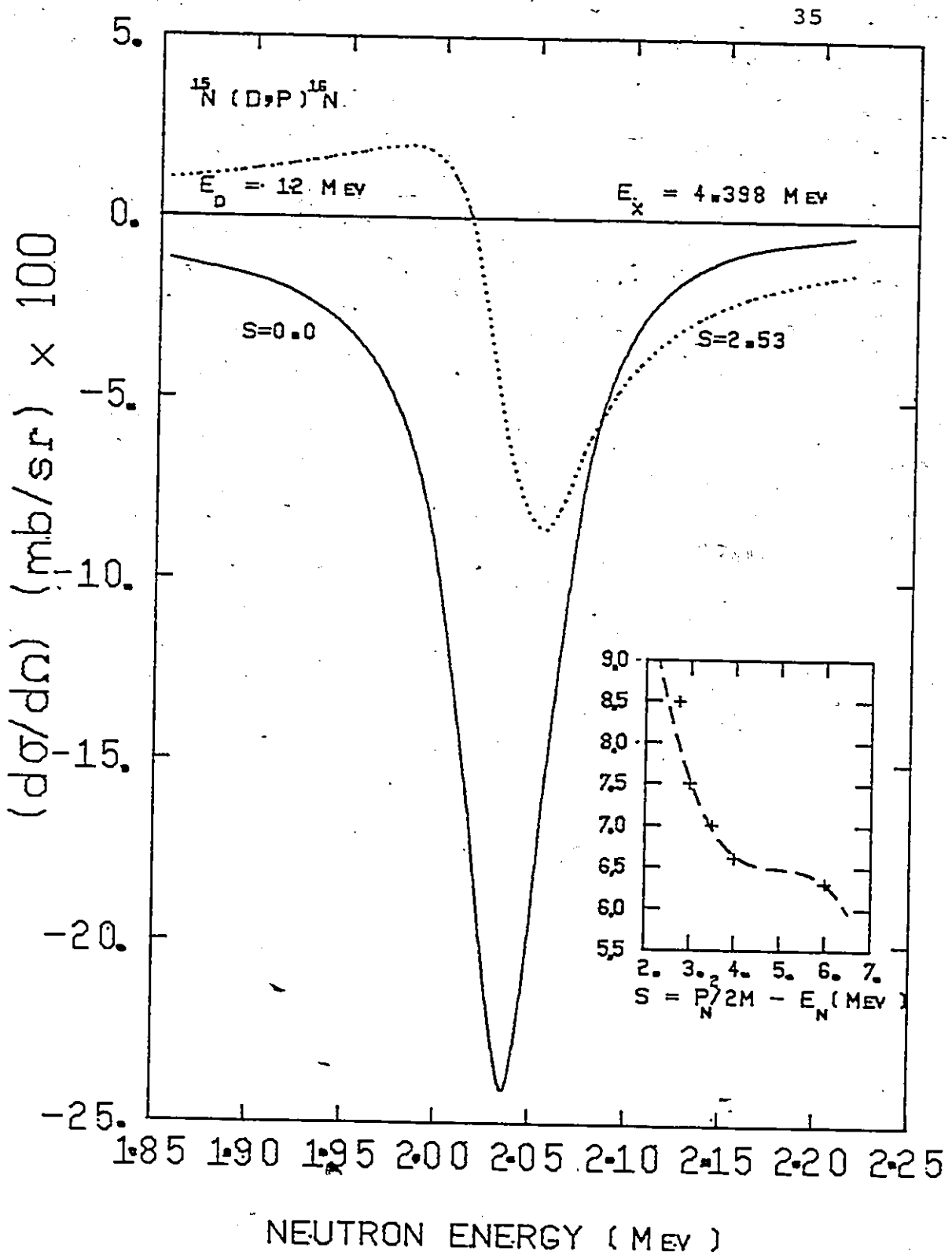


Fig. 5. The differential cross section due to the interference terms only, as a function of the neutron energy E_n. The solid curve is the on-shell contribution normalized to the corresponding dip in the (n,n) cross section and reduced by a factor of 10 to facilitate comparison with the off-shell contribution (dotted graph) at the off-shell distance s=2.53. The inset shows the total differential cross section as a function of s. The experimental points are taken from the work of Fuchs et al. (15) and they are upper limits only.

strongly negative and causes a strong fall-off, of the cross-section at the resonant energy. In contrast, the off-shell contribution is less negative and balances out the pure resonant contribution so that the dip at the (n,n) cross-section disappears in the (d,p) break-up continuum. We should remark here that our calculations show an angular distribution that compares reasonably well with experiment (see also fig. 5).

The above arguments seem also appropriate with respect to the resonance at $E_x = 5.048$ which is the only case of major discrepancy with the analysis of Möhring and Lipperheide[8] Although we were able to obtain an inverted $\ell=2$ resonance in the (n,n) cross-section our calculations exclude an $\ell = 2$ transition, since it shows up as a peak in the (d,p) cross-section. On the other hand with an $\ell = 0$ transition we have obtained some qualitative agreement with the experimental data, but our results are not entirely satisfactory. Skipping ahead, the answer is: maybe this resonance is a mixture of $\ell=0$ and $\ell=2$ waves. In view of this any comments on the possible identification of this resonance are necessarily speculative at this stage. It is possible that inclusion of other coherent processes, for example "proton-transfer"[19] as well as correction for distorted wave effects and a more realistic choice of the potential well, will resolve the ambiguities concerning this resonance, and we shall return to this point in the next chapter.

II.6 CONCLUDING REMARKS

We have presented a general framework for the theoretical description of transfer reactions in the continuum, which is a natural extension of the usual formulation as a three body collision, under the condition of a direct reaction mechanism and within the framework of the R-matrix theory; we have given an exact expression by which one can calculate the differential cross-section including background terms. Our formalism has been applied to a (d,p) stripping reaction and we have shown that apart from verifying earlier results we have found interesting interference effects with the break-up continuum. With these interference effects we were able to resolve some ambiguities regarding levels in the compound system.

Finally we remark that the case of charged-particle-transfer can be treated in the same formalism by simply replacing our in- and out-going wave functions by the Coulomb functions.

CHAPTER III

ASSIGNMENT OF $J^{\pi} = 1^{-}$ FOR THE 5.048 MeV level of ^{16}N BY CONSIDERING THE ON- AND OFF-SHELL BEHAVIOUR

III.1 INTRODUCTION

The spin and parity of the E_x = 5.048 MeV level at neutron energy E_n = 2.732 MeV, in ^{16}N have long been assigned[14] as $J^{\pi} = 1^{-}$. The principal evidence has been derived from n-^{15}N elastic scattering which concluded that the 2.732 MeV ground state transition is a ℓ=0 transition leading to J=1 and negative parity for this state.

The negative parity assignment is also consistent with angular distributions of the $^{14}C(^{3}He,p)$ reaction[16] (tentatively ℓ=3 transfer) and the $^{15}N(d,p)$ reaction (3), where an ℓ=2 component was evident. The results of previous studies of one or two nucleon stripping and pick-up to this state (i.e., ref. (16)) have been ambiguous. A plane wave analysis[8] of $^{15}N(d,p)^{16}N$ (2.732 MeV) gave a tentative ℓ=2 assignment to the stripping pattern. Almost a good fit is also obtained for ℓ=2 in the distorted wave analysis[19] and ℓ=0 is definitely favoured in a later plane wave analysis[11]

The presence of this state has been extremely perplexing because of possible interference with the $d_{3/2}$ single particle resonance, with $J^{\pi} = (1^{-}), (2^{-})$ arising from the coupling of the $p_{1/2}$ proton hole to the $d_{3/2}$ single particle state and merging into the broad symmetric structure centred at about 5.1 MeV.

The calculations of Fuchs et al.[15] suggest that a more accurate estimate of the off-shell to on-shell effect can show

whether this resonance is a case of a mixed ℓ=0, 2 transition. The formalism that we have developed in Chapter II is especially suited for such a study because off-shell effects are calculated exactly and because both (d,p) stripping and elastic scattering can be treated on the same footing. If this state can be reached by a mixed ℓ transition then this should show up in both reactions and the spin values of that state can be determined unambiguously.

III.2 $^{15}N(n,n)^{15}N$ REACTION ANALYSIS

The total cross-section of $n-^{15}N$ up to E_n = 4.5 Mev has been analysed by Zeitnitz et al[14] using the R-matrix description for a single open channel. This formalism has been extensively discussed in the literature, see for example, Lane and Thomas[23] and the following is intended only as a summary of the formulae that we are using here.

The differential cross-sections for collisions of pairs of unpolarized particles are (with c = jsl)

[1] total: $$\sigma_\alpha(\text{tot}) = \frac{2\pi}{k_\alpha^2} \sum_{jsl} g_j \, (\text{Re } T^j_{cc})$$

[2] elastic: $$\sigma_{el} = \frac{\pi}{k_\alpha^2} \sum_{jsl} g_j \, (|T^j_{cc}|^2)$$

where g_j is the spin statistical factor

[3] $$g_j = \frac{(2j+1)}{(2I_1+1)(2I_2+1)}$$

and Re T signifies the real part of the scattering T-matrix. The convention is that of coupling the intrinsic spin I_1 of the incoming particle (neutron) to that of the target (^{15}N) I_2, to make the channel spin $\underset{\sim}{s} = \underset{\sim}{I}_1 + \underset{\sim}{I}_2$ and then form the total angular momentum $\underset{\sim}{j}$ of the system, by coupling $\underset{\sim}{s}$ to the relative orbital angular momentum $\underset{\sim}{\ell}$ of the two particles. If there is no reaction the collision matrix $\underset{\sim}{U} = \underset{\sim}{1}-\underset{\sim}{T}$ is unitary and eqs. (1) and (2) reduce to;

$$[4] \qquad \sigma_\alpha = \sigma_{tot} = \frac{2\pi}{k^2} \sum_{js\ell} g_j |T_{cc}|^2$$

With a single open channel, the R-matrix is a simple function which is related to the T-matrix by

$$[5] \qquad T_{cc} = 1 - \exp(-i2\phi_c)\{1 + 2i \frac{P_\ell R_{cc}}{1 + L_\ell P_\ell R_{cc}}\}$$

In Eq. (5) P_ℓ is the penetration factor, L_ℓ the logarithmic derivative of an outgoing wave for each partial wave ℓ and ϕ_c defines the potential scattering phase-shift. The analysis of Zeitnitz et al[14] was performed by dividing the R-function into an explicit multi-level sum and a background term, and then expanding the background term about the meddian energy E_m of the energy range under consideration. For each resonance a χ^2 minimization technique was used to obtain the best ℓ,j values. Instead of doing this, we retain the R-matrix as a multi-level expansion and we assume fixed values of j, s and ℓ. The selec-

tion rules reduce to $\pi = (-)^{\ell+1}$ and $j = \ell+s$ where the resonance under consideration in ^{16}N has spin J and parity π. In our present example the negative parity assignment restricts the value of s=1 and thus permits the values of $\ell = (j\pm1)$ which for j=1, requires an even ℓ value assignment. Since earlier fits predicted an $\ell=0$ value, it is judged necessary to attempt to improve upon those fits by considering $\ell=2$ contributions. Thus given the results of reference (14), any $\ell=2$ component will unambiguously yield $J^{\pi} = (1^-)$ for this state.

In the following discussion we shall adhere to the PWBA analysis and unless stated otherwise, make use of the data of ref. (11). We expect such a PWBA analysis to give a reasonable fit to the width and the dip of the resonance and we try a linear combination of $\ell=0$ and $\ell=2$ components with a χ^2 minimization routine[17].

The results are shown in fig. (6) The fractional parentage is 35% for $\ell=0$ and 65% for $\ell=2$ and this supports the assignment $J^{\pi} = 1^-$. The difference from the experimental data indicates the inadequacy of the plane wave approximation. It is likely that this behaviour is due to a combination of distortion effects and compound-nucleus effects. We shall note in passing that the $\ell=2$ contribution has been calculated as d-wave interference with the broad $|(1p_{1/2}) \times (1d_{3/2}); 1^-\rangle$ single particle resonance at about 5 MeV. This is done by neglecting the non-resonant scattering cross-section of the partial waves not contributing to the resonance, and assuming that

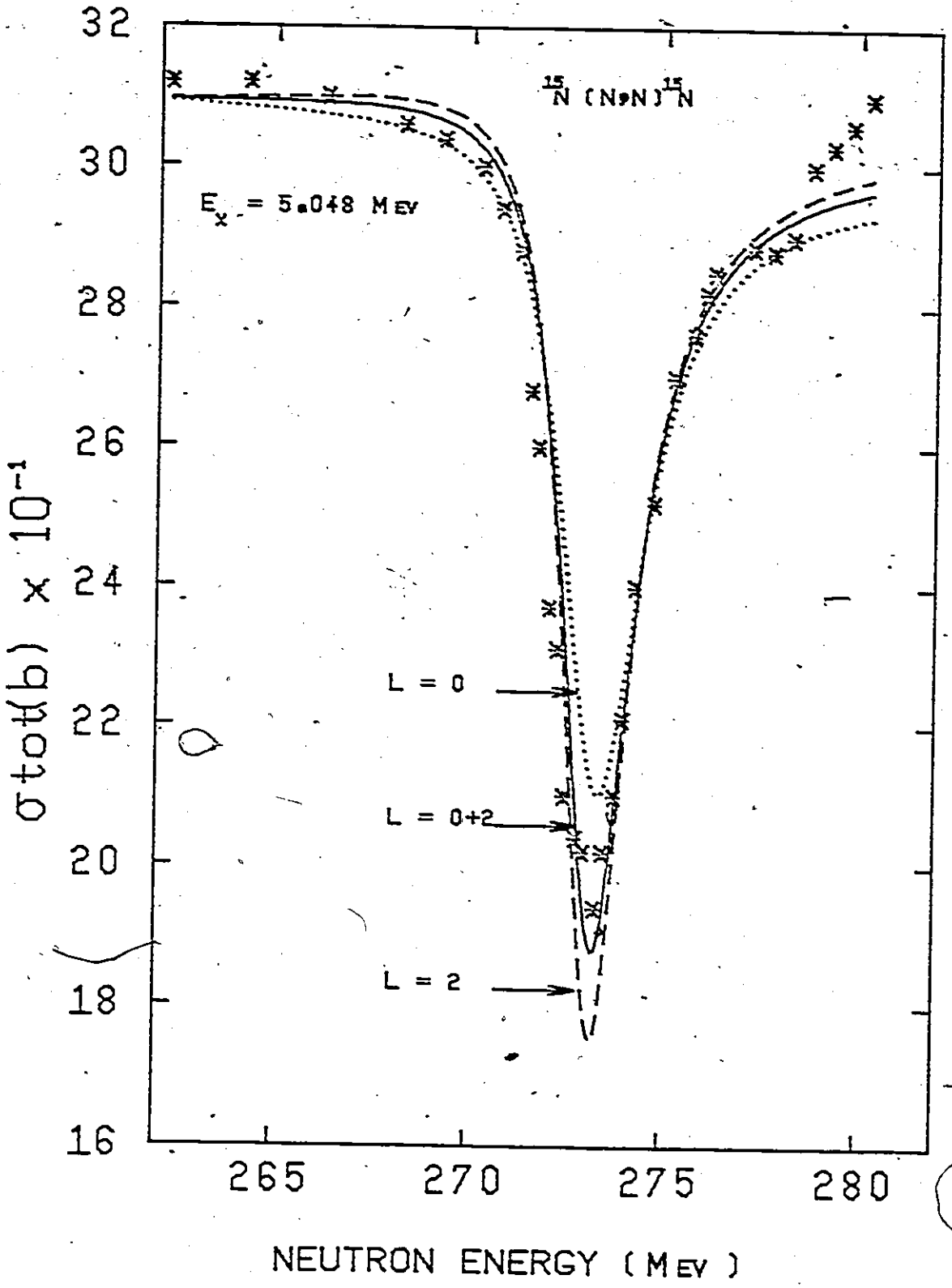


Fig. 6. Total cross section for the reaction $^{15}N(n,n)^{15}N$ leading to the 5.048 MeV state of ^{16}N. The two dashed curves show the ℓ=0 and ℓ=2 contributions. The solid curve is the sum of these two contributions. The experimental points are taken from the works of Fuchs et al. (15).

the cross-section around the resonance is given by

$$\sigma_\ell = \frac{4\pi}{k^2} \frac{(2j+1)}{(2I_1 + 1)(2I_2 + 1)} \sin(\beta_\ell^j + \xi_\ell^j) \qquad [6]$$

where ξ_ℓ^j is the potential scattering phase-shift and β_ℓ^j the resonant scattering phase-shift passing through $\pi/2$ at $E = E_r$. It is obvious that at $\xi_\ell^j = -\frac{\pi}{2}$ there should be a minimum in the cross-section, for which any deviation from zero can be explained by contributions of the other partial waves [18]

III.3 $^{15}N(d,p)^{16}N$ REACTION ANALYSIS

The disqussion of the previous section strongly supports the assignment $J^\pi = (1^-)$ to the 2.732 MeV state. Nevertheless it is important to confirm this assignment through the $^{15}N(d,p)^{16}N$ reaction, since off-shell effects in (d,p) stripping modify the differential cross-section and therefore a fit to the angular distribution of elastic scattering may be different from fits to the stripping patterns.

Inclusion of spins in Eq. (26) of Chapter II is straightward. In the shell model with no configuration mixing a given initial state has only one value of (s,ℓ,j) initially, and the final state has only one s (the same one) and (we assume) one (ℓ,j). In the present case we have coupled the orbital and spin angular momentum of the neutron to a subtotal j, and have then added the spin of ^{15}N: J_T, to obtain the total j. For the differential cross-section in each partial wave (ℓ,j,m) we have

$$[7]\quad \frac{d\sigma}{d\Omega_p dE_p} = \frac{2J+1}{(2J_T+1)(2\pi)^3}\,\frac{(\mu_{dA}\mu_{pB})}{(m_n\mu_{nA})}\,\frac{2n(n+1)}{(1-n)^2}\left(\frac{1}{S} - \frac{1}{S+(n^2-1)\epsilon_d}\sqrt{\epsilon_d}\right)\times$$

$$\frac{\sqrt{E_p E_n}}{E_d}\,|F_{\ell j}(q,k,E)|^2$$

where $F_{\ell j}$ is the off-shell elastic scattering amplitude for the scattering of the neutron by ^{15}N and it is calculated in the same way as in Chapter II. The symbol n=7 is a parameter of the Hulthen wave function.

As in the previous section, we use here a linear combination of the $\ell=0$ and $\ell=2$ components

$$[8]\qquad \sigma(\theta) = \alpha_0\sigma(\theta)_0 + \alpha_2\sigma(\theta)_2$$

It is evident that α_ℓ have to satisfy the condition $\sum_\ell \alpha_\ell = 1$ because the neutron can not be scattered by more than one single particle state simultaneously. To get an inverted shape for the theoretical $\ell = 2$ component we have calculated it, as interference with the potential scattering cross-section σ_{el}^{pot} at the partial wave with the same angular momentum ℓ at the energy $E_n \approx 5.0$ Mev which corresponds to the single particle state $|(1p_{1/2})\times(1d_{3/2});1^-\rangle$ The emphasis here is on the actual mechanics of the calculations for a reasonable physical situation and we don't expect too close an agreement with the experiment. We shall note in passing that for the complete potential scattering cross-section we have:

$$\frac{d\sigma}{d\Omega} = \frac{\lambda^2}{2} \sum_L \sum_{s',s=0,1} P_L(\cos\theta)\delta_{ss'}(2s+1) \sum_{\ell_1,\ell_2=0,1,2} (2\ell_1+1)(2\ell_2+1)$$

[9]

$$\times (\ell_1,\ell_2 00|L0)\sin\xi_{s\,\ell_1}\sin\xi_{s\,\ell_2}(\cos)(\xi_{s\,\ell_1} - \xi_{s\,\ell_2})$$

Equation [9] simplifies for s=1 and $\ell_1 = \ell_2 = 2$ to

$$\frac{d\sigma}{d\Omega} = \frac{3}{4}\lambda^2 \sum_L P_L(\cos\theta) \{(0000|LC)\sin^2\xi_0 + 9.0(1100|L0)\sin^2\xi_1$$

[10]

$$+ 25.0(2200|L0)\sin^2\xi_2)\} .$$

Figure 7 shows the differential cross-section at the resonance energy as a function of the off-shell distance s, which is related to the stripping angle θ through Eqs. [48] and [52] of Chapter II. Since the angular distributions show no structure, the way to fit the experimental data is not obvious. The method that we have used was to fit the ℓ=0 partial wave as well as possible and then adjust the ℓ=2 wave to give the best fit to the data. The fractional parentage is about 50% for each partial wave, which is considerably different than that found in the n-^{16}N scattering. As we have noted previously, however, the off-shell background interferes destructively with an S-wave resonance so that the relative strengths of the partial waves are not to be taken too seriously. Therefore the amount of mixing inferred here can only be considered qualitatively correct. The important result from fig.(7) is that a mixture of ℓ values (ℓ=0 and ℓ=2) is required to fit the (d,p) data and these

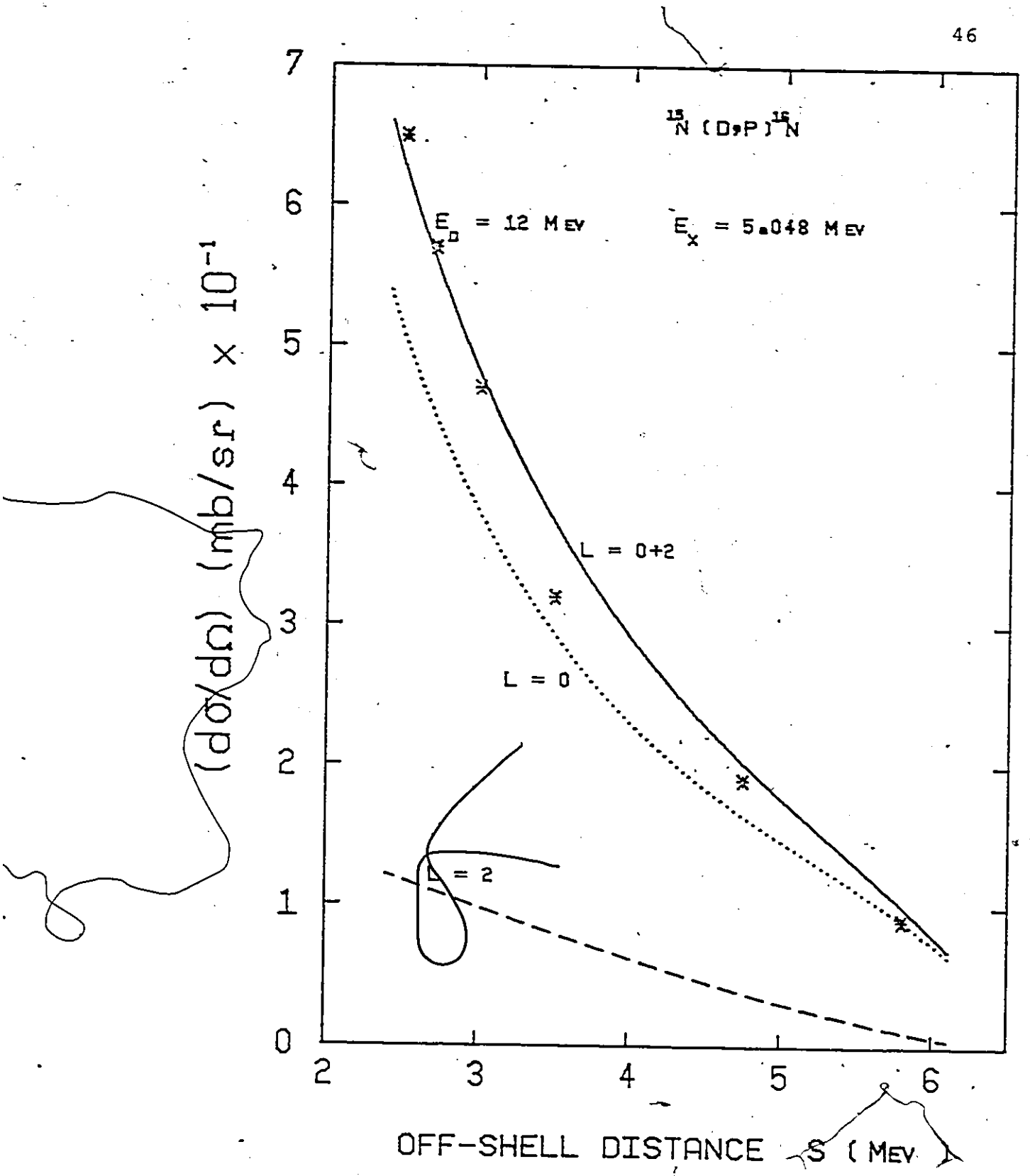


Fig. 7. Angular distributions for the reaction $^{15}N(d,p)^{16}N$ leading to the 5.048 MeV state of ^{16}N. The two dashed curves show the ℓ=0 and ℓ=2 contributions. The solid curve is the sum of these two contributions. The experimental points are taken from the work of Fuchs et al. (15). The R matrix radius for the ℓ=0 resonance is R = 3.1 fm.

considerations in conjunction with the results of the previous section, show that the angular distribution to the 2.732 MeV state can be a superposition of $\ell=0$ and $\ell=2$ waves. Since the presence of any $\ell=0$ contributions determines the spin assignment, the spin and parity of the 2.732 MeV level in ^{16}N can be assigned $J^{\pi} = 1^{-}$. This assignment however is in contradiction with the work of Baxter et al[47] who have assigned $J^{\pi} = 2^{-}$ to that state from measurements on the $^{18}O(d,\alpha)^{16}N$ reaction using a polarized deuteron beam. Detecting the α-particles near 0° they were able to get lower and upper bounds $(\sqrt{2}, \frac{1}{\sqrt{2}})$ for the tensor analyzing power T_{20}. If a measurement of T_{20} falls well away from both these limits then unnatural parity $\pi = (-)^{J+1}$ (not including 0^{-}) can be assigned. In the present case they have obtained unnatural parity and therefore have assigned 2^{-} to this level. We should mention here that a 2^{-} spin requires an $\ell=2$ transition. However since an $\ell=2$ wave does not show up as a dip in the(n,n) cross section the inverted shape of this resonance must be explained as interference with another resonance near this excitation.

III.4 CONCLUDING REMARKS

All the states in ^{16}N up to an excitation energy of 4.3 MeV, have been analyzed previously. The $^{15}N(n,n)^{15}N$ reaction of Zeitnitz et al.[14] predicted a spin and parity as-

signment of the 2.732 MeV state of $J^{\pi} = 1^{-}$ with an $\ell=0$ transfer. The results of Fuchs et al.[15], however, definitely favour an $\ell=2$ transfer with $J^{\pi} = (1^{-}),(2^{-})$. In the present study we have shown that a good fit to the data is obtained by a mixture of $\ell=0$ and $\ell=2$ waves which provide the identification of $J^{\pi} = (1^{-})$ for this state, however the results of Baxter et al[47] may permit a different interpretation.

CHAPTER IV

OFF-SHELL EFFECTS IN COUPLED CHANNELS

IV.1 INTRODUCTION

We have seen in the last two chapters that for deuteron stripping reactions some simple choices for the off-shell extrapolation have proved reasonably successful, for example expressing the transition matrix through its on-shell value around a single and isolated resonance. However, there are difficulties with this procedure: when resonances are broad and overlapping the single pole expansion is violated and this can lead to an unphysical off-shell extension. The same is true of course where inelastic channels are present since bound states in a closed channel are reflected as resonances in the open channels. Furthermore we have seen that off-shell interference can alter the resonance behaviour and in view of the fact that the resonance parameters may depend on the off-shell extrapolation it is important to study the reliability of this extrapolation for (d,p) stripping. One way to do so would be to set up soluble models for the exact three body scattering and test the resonances against their theoretical definitions. Interesting work on these lines for (d,p) and (d,n) stripping to a bound state has been done by Beregi and Lovas[20] who calculated the transition matrix

elements using the Faddev-like equations of Alt, Grassberger and Sandhas and analysed the cross-sections by fitting to the transition matrix a Breit Wigner formula.

Another approach is to apply the method of coupled channels. In the conventional (d,p) stripping to a bound state, many such applications have been made. However, in bound state stripping off-shell effects are not likely to affect the results appreciably since it is assumed that the interaction is taking place at the surface and the contribution of the T-matrix from the interior can be neglected altogether. On the other hand in stripping to an unbound state the wave function in the interior is important since a compound nucleus can be formed which can decay back to the elastic channel or to any other channel that is energetically available. We shall note here that we are observing the outgoing particle only, and therefore our elastic channel is that formed by the target nucleus and the stripped particle and it is always off the energy shell. An example of (d,p) stripping to the unbound state N^{16} decaying back in the elastic channel has been discussed in chapters II and III. In this case the final nucleus N^{16} is unbound to neutron decay and therefore will exclusively decay back to $n\text{-}N^{15}$. Other cases however have been known to exist where other channels can be reached. (For instance the reaction $Li^7(d,n)Be^8$, where Be^8 can decay to (Li^6+d), $(2He^4)$, (Be^7+n)

and the off-shell elastic channel (Li^7+p).

Questions now arise as to what the main effects of the coupling of channels off the energy shell will be and under what conditions the effects will be appreciable. For this purpose we present three soluble models and test the resonance parameters in these models. The details are presented in the following sections.

IV.2 REVIEW OF THE OFF-SHELL ENERGY T-MATRIX

A two-body transition operator T(s) is defined through the integral equation

$$[1] \qquad T(E) = V + V(E-H_o)^{-1}T(E)$$

where H_o is the kinetic energy operator of the relative motion, E is the energy parameter and V is an interaction potential. Defining a wave operator $\Omega(E)$ according to the relation

$$[2] \qquad \Omega(E) = 1 + (E-H_o)^{-1}T(E)$$

it then follows from [1] and [2] that

[3] $$T(E) = V\Omega(E)$$

and

[4] $$(E-H_o-V)\Omega(E) = (E-H_o).$$

Writing [4] in a mixed representation we get

[5] $$(E + \frac{\hbar^2}{2\mu}\nabla^2 - V) \langle \underline{r}|\Omega(E)|\underline{q}\ell m\rangle = (E - Z) \langle \underline{r}|\underline{q}\ell m\rangle$$

where $\langle \underline{r}|\Omega(E)|\underline{q}\ell m\rangle$ means $\psi(E,\underline{q},\underline{r})$, with $\underline{r}$ the space coordinate vector and $\underline{q}$ the wave number. In the following we will restrict ourselves to the case of positive energy parameter so that we can introduce the on and off-shell momenta through the relations

[6] $$k = \sqrt{\frac{2\mu}{\hbar^2}E} \quad \text{and} \quad q = \sqrt{\frac{2\mu}{\hbar^2}Z}$$

and we shall suppress the notation for the spin operators. Equation [5] is the Schrödinger equation off the energy shell. In the many channel case it is a matrix equation

[7] $$(E-H_t-V)|\psi\rangle = (E-H_o)|\phi\rangle .$$

Now, H_t contains an internal part H_I, that gives rise to the different target and projectile states C. Clearly

[8] $$H_t = H + H_I \quad , \quad H_I|C\rangle = \varepsilon_c|C\rangle$$

It is instructive to note here that if only the elastic channel is open, two different excited states can decay to it, and the possibility of coupling still exists. In this case however the eigenvalues ε_c are zero and the momentum numbers are equal in both channels. With

[9] $$|\psi\rangle = \sum_c |C)\psi_c(q_c\ k_c\ r)$$
$$|\phi\rangle = \sum_c |C)\phi_c(q_c\ r)$$

we can rewrite [5] as follows:

[10] $$\sum_c [(k_c^2 - T_\ell)\delta_{cc'} - U_{c'c}]\psi_{cc'} = \sum_c (k_c^2 - q_c^2)\phi_c\delta_{cc'}$$

where we have defined

[11] $$U_{cc'} = \frac{2\mu_c}{\hbar^2} V_{cc'} \quad , \quad k_c^2 = \frac{2\mu_c}{\hbar^2}(E-\varepsilon_c) \quad , \quad T_\ell = \left[-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2}\right] .$$

Physically we have incoming waves in only one channel so that only one term survives in the right hand side of [10]. In fact, in the problem at hand we have off-shell incoming waves only in the elastic channel and we will denote this channel with the subscript 1.

IV.3 SQUARE WELL POTENTIALS

For the sake of simplicity we shall consider here two spinless particles in the relative s-wave of reduced mass μ_1 in channel 1 and reduced mass μ_2 in channel 2. Using natural units $\hbar=c=1$ we can write the coupled Schrödinger equations in the form

$$[12] \qquad [\frac{d^2}{dr^2} + (k_1^2 - U_1)]u_1(r) = U_{12}u_2(r) + (k_1^2-q^2)j_0(qr)$$

$$[13] \qquad [\frac{d^2}{dr^2} + (k_2^2-U_2)]u_2(r) = U_{21}u_1(r) .$$

From [11] we can see that the momentum numbers are related through

$$[14] \qquad k_1^2 = k_2^2 + k_0^2 .$$

where k_0^2 is a measure of the inelastic threshold and it is determined by the masses μ_1 and μ_2. For our purpose we shall use $\mu_1 = 1.0$ Gev, $\mu_2 = 1.01$ Gev which gives

$$[15] \qquad E_{1(\text{threshold})} = \varepsilon_0 = 2(\mu_2-\mu_1) = 20 \text{ Mev}$$

or $k_0^2 = 0.02$ Gev^2. We have defined $U_{ij} = \frac{2m}{\hbar^2} V_{ij}$ and we shall further assume[23] that $V_{21} = V_{12}$ and $U_{21} = U_{12}$.

The solutions of [12] and [13] proceed in a standard way, for $r < a$

[16]
$$u_1(r) = A\sin x_1 r + B\sin x_2 r + C\,\frac{\sin qr}{q}$$
$$u_2(r) = \alpha A\sin x_1 r + \beta B\sin x_2 r$$

and for $r > a$

[17]
$$u_1(r) = \frac{\sin qr}{q} + T_{11}\, e^{ik_1 r}$$
$$u_2(r) = T_{12} e^{ik_2 r} .$$

Inserting [16] in [12] and [13] we get five characteristic equations from which the unknown parameters are easily found to be:

[18]
$$x_1^2 = \tfrac{1}{2}\left(2k_1^2 - k_0^2 - U_2 - U_1 + \left[(k_0^2 + U_2 - U_1)^2 + 4U_{12}^2\right]^{1/2}\right)$$

[19]
$$x_2^2 = \tfrac{1}{2}\left(2k_1^2 - k_0^2 - U_2 - U_1 - \left[(k_0^2 + U_2 - U_1)^2 + 4U_{12}^2\right]^{1/2}\right)$$

[20]
$$\alpha = \frac{U_{12}}{k_2^2 - U_2 - x_1^2} = \frac{k_1^2 - U_1 - x_1^2}{U_{12}}$$

[21]
$$\beta = \frac{U_{12}}{k_2^2 - U_2 - x_2^2} = \frac{k_1^2 - U_1 - x_2^2}{U_{12}}$$

[22]
$$C = \frac{k_1^2 - q^2}{k_1^2 - U_1 - q^2} .$$

We shall note here that the dependence of α and β on k_1, k_2 is an apparent one as can be easily seen by direct substitution of [18] and [19]. Matching the wave functions and their derivatives at $r = a$ and after some straightforward algebra we

get

$$[23]\quad T_{11}=(C-1)e^{-iR_1}\left\{\frac{\sin qa}{q}+\frac{[\beta(e_2-iR_2)-(e_1-iR_1)\alpha](ik_1\frac{\sin qa}{q}-\cos qa)}{D(\rho_1,\rho_2)}\right\}$$

$$[24]\quad T_{12}=(C-1)e^{-iR_2}\frac{\alpha\beta(ik_1\frac{\sin qa}{q}-\cos qa)(e_2-e_1)}{D(\rho_1,\rho_2)}$$

where we have defined:

$$R_i=k_i\alpha\ ,\ \rho_i=x_i a\ ,\ e_i=\rho_i\cot\rho_i$$

$$[25]\quad D(\rho_1,\rho_2)=iR_1(\alpha e_1-\beta e_2)+iR_2(\alpha e_2-\beta e_1)-(\beta-\alpha)(R_1R_2-e_1e_2).$$

We have evaluated the differential elastic and inelastic cross-sections

$$[26]\quad \sigma_{el}(E_1,S)=|T_{11}|^2$$
$$\sigma_{inel}(E_1,S)=|T_{12}|^2$$

for different values of the off-shell distance s, which is defined here by

$$[27]\quad s=Z-E_1$$

and it is related to the stripping angle θ through a well known relation (Eqs. 48 and 52 of Chapter II). The range of the potentials is fixed at (21.24) Gev^{-1} or $\approx$ 4.25 fm and the strengths are so chosen as to reproduce a bound state in the second channel at an energy E_1 = 11.6 Mev or E_2 = -8.4 Mev. They are

$$U_1 \approx U_{12} = 0.1\, U_2 = -0.002 \text{ (Gev)}^2$$

or

$$V_1 = V_{12} = 0.1\, V_2 = -2 \text{ Mev.}$$

Figures 8 and 9 show the differential cross sections in the elastic and inelastic channels for different values of s. The full lines describe the on-shell elastic scattering for s = 0. One immediately notices that the effect of coupling to the second channel is to produce a resonance in the first channel at about the same energy as the energy of the bound state in the second channel. With increasing energy a second resonance will appear in the first channel around 20 Mev. This resonance is related to the inelastic threshold. In fact for our particular choice of parameters this resonance develops at the inelastic threshold ($k_1^2 = k_0^2$) and couples to the two channels in the ratio of strengths of 1 to 2. Let's discuss briefly the three quantities characterising a resonance, namely, the position, the width and the height.

In general the change in the height occurs in such a way as to reproduce the angular stripping pattern of an S-wave. Otherwise in all cases that we have considered in this model, appreciable differences occur in the elastic channel in the other two parameters as one goes off the energy shell. This is somehow surprising since as it can be seen from [23] off-shell effects do not enter the determinant in the expression for the

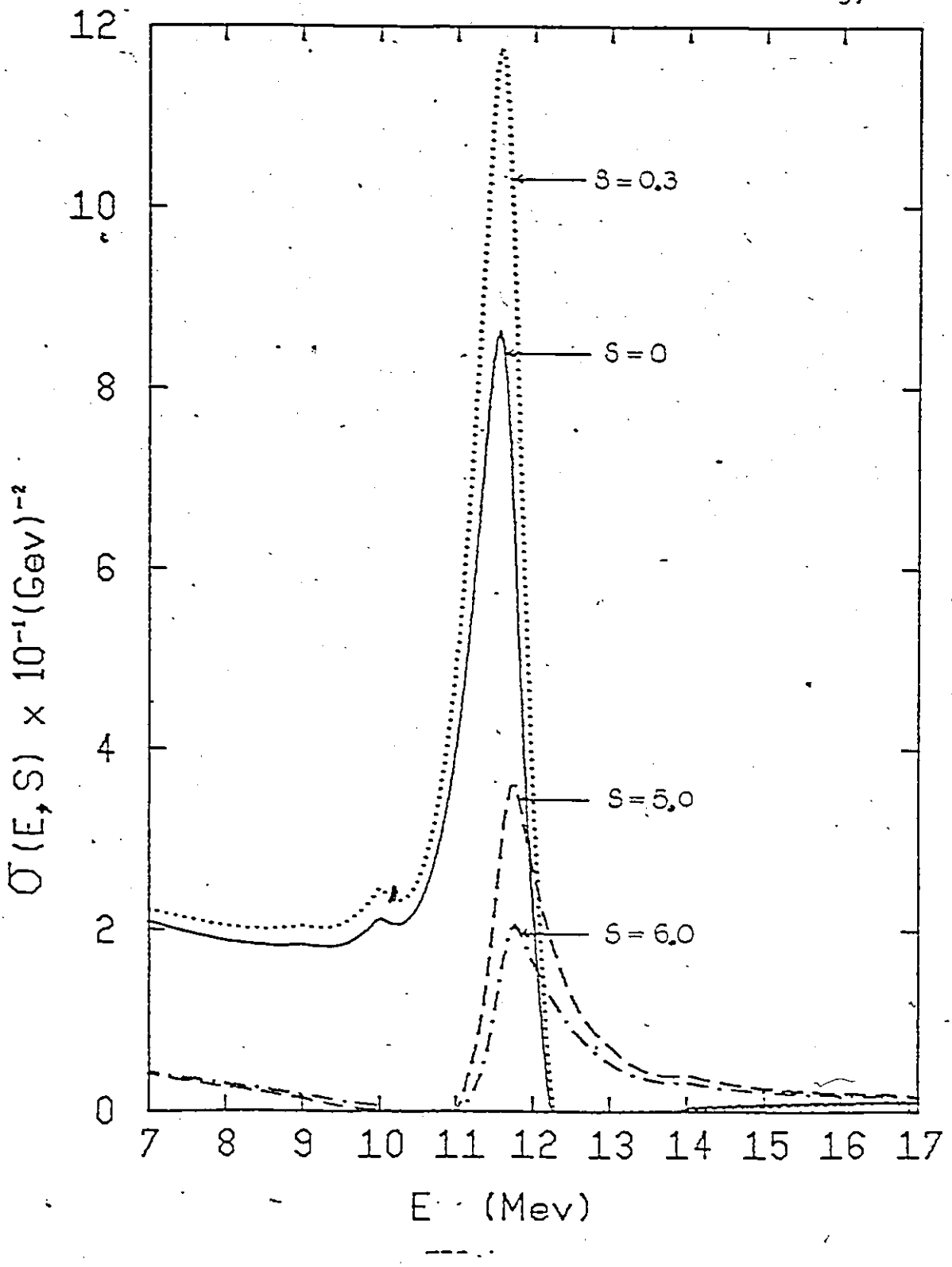


Fig. 8 The differential cross-section in channel 1 as a function of the energy E_1 at different off-shell distances s. We have used h=c=1 and square well potentials with strengths $V_2 = 10$ $V_1 = 10$ $V_{12} = -20$ Mev. The range of all these potentials is set at 21.24 Gev^{-1} and the inelastic threshold is taken at $E_0 = 20$ Mev. The full line is the off-shell contribution. Note that 1 Gev^{-1} = 0.1978 fm.

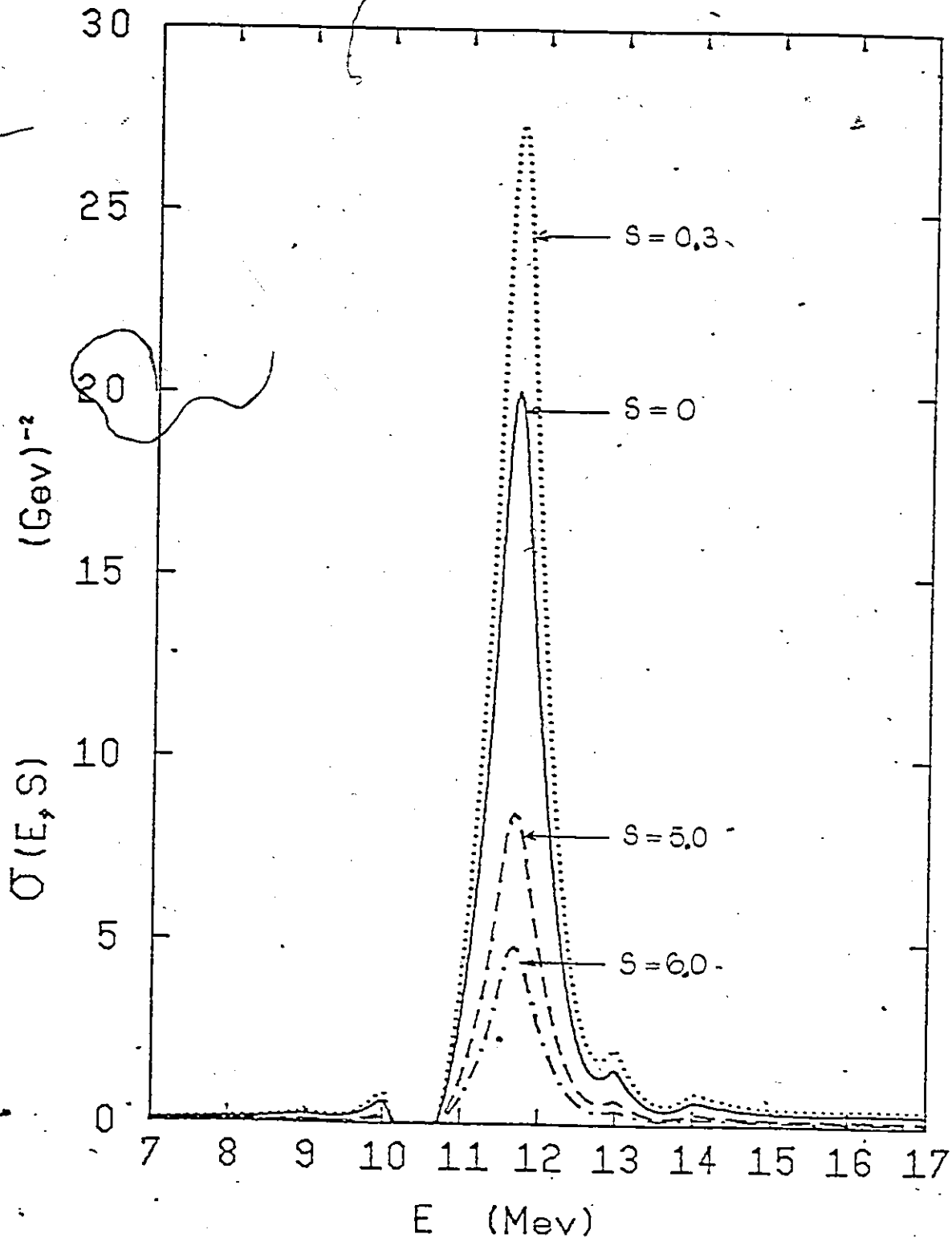


Fig. 9. The differential cross-section in channel 2 as a function of the energy E_1 at different off-shell distances s. For general description see fig. 8.

T-matrix and therefore we would have expected the position and the width to remain unaltered by going off-shell.

The question now arises of the validity of the usual definition of the resonance parameters by locating the zero of the real part of $D(\rho_1,\rho_2)$ for example;

$$[28] \qquad \mathrm{ReD}(E) = 0 \quad , \quad \frac{\Gamma}{2} = \frac{\mathrm{ImD}(E)}{\frac{d}{dE}(\mathrm{ReD}(E))\Big|_{E=E_R}} \; .$$

In considering this question, there are several factors which one immediately notices by looking at the partial wave amplitude $f(E,s) = \frac{N(E,s)}{D(E)}$. (i) The function $N(E,s)$ has a right hand cut, (ii) $N(E,s) \neq 0$ as $|E| \to \infty$ for $s \neq 0$ and (iii) in the inelastic cross-section no changes occur in the shape and the width of the resonance. These suggest that differences in the resonance parameters are not related to the poles of $N(E,s)$ but to the incoming waves which are not present in the inelastic channel.

Furthermore in the elastic channel, the resonance part of the transition matrix element is superimposed on the off-shell background which is a function of the off-shell momentum. Consequently if this background term is small and the resonance is small and isolated, then [28] is expected to be valid. On the other hand if a single particle resonance interferes with its continuum the resonance parameters have to be redefined.

We should emphasize here that it is the destructive interference between background and resonant term that causes

the changes and one should look with extreme caution at the expansion of D(E) around a single pole since it underestimates the importance of the off-shell background. For a complete study of the importance of this term we refer to an excellent paper by Bolle and Kowalski (27). Here we shall reemphasize their argument that if the on-shell amplitude is dominated by a resonance in a certain energy range, we can not expect that the off-shell amplitude is necessarily factorizable for physical parametric energies in the vicinity of the resonance.

However if off-shell effects are important we still can have an off-shell factorized quasi Breit-Wigner form by expanding f(E,s) in the vicinity of a specified off-shell point. This point should be so chosen as to provide the simplest realization of the physical situation. Thus in accord with the analogous situation for on-shell scattering, we conclude that in situations such as the above the resonance parameters should be redefined by means of a partial derivative expansion. Namely, if we define $F(E,s) = D(E)N^*(E,s)$ then we have

$$[29] \qquad f(E,s) = \frac{|N(E,s)|^2}{(E-E_r+\Delta E) + i\frac{\Gamma}{2}}$$

with

$$\Delta E = (s-s_r)\left(\frac{\partial ReF}{\partial S}\Big/\frac{\partial ReF}{\partial E}\right)\Big|_{E_r,s_r}$$

[30]
$$\frac{\Gamma}{2} = \frac{ImF(E,\ )}{\frac{\partial}{\partial E}\,ReF(E,s)}\Big|_{E_r,s_r}$$

and
$$ReF(E_r,s_r) = 0.$$

The example at hand provides substantial justification for the above form. In general the best set of points is that for which $ReD(E_r) \approx 0$ and $ImF(E_r,s_r) \approx 0$. Then [30] is nearly fulfilled and also reduces to [28] at S=0. In the present case, expanding around E_r=11.7 and s_r=3.9 Mev one gets ΔE=200 keV and $\frac{\Gamma}{2}$ = 236 keV against the exact values of 130 keV and 345 keV respectively.

IV.4 δ-FUNCTION POTENTIALS

In the case of δ-function potentials resonances can be produced in the ℓ=0 state for both positive and negative strengths of the potential and the problem of determining the coupling constants has been discussed extensively by Gauthier and Kamal[29]. In this section we want to extend this model off the energy shell and test the resonance parameters.

If we define the Green's functions

[31]
$$g_i(r,r') = -\frac{1}{k_i}\sin k_i r\, e^{-ik_i r'}$$

then the general solutions of the coupled equations [12] and [13] are

$$[32]\quad u_1(r) = \sin k_1 r + g_1(r,a)(U_1 u_1(a)+U_{12}u_1(a)) + (k_1^2-q^2)\int_0^r g_1(r,r')\sin qr'\,dr'$$

$$[33]\quad u_2(r) = g_2(r,a)(U_2 u_2(a)+U_{12}u_1(a)).$$

These solutions asymptotically become,

$$[34]\quad u_1(r) \xrightarrow[\infty]{r} \frac{\sin qr}{q} e^{ik_1 r}\sin k_1 a(U_1 u_1(a)+U_{12}u_2(a))$$

$$[35]\quad u_2(r) \xrightarrow[\infty]{r} -e^{ik_2 r}\sin k_2 a(U_2 u_2(a)+U_{12}u_1(a)).$$

Using [17] we get for the T-matrix

$$[36]\quad T_{11} = -\frac{\sin k_1 a}{k_1}\frac{U_1(1-U_2 g_2(a,a))+U_{12}^2 g_2(a,a)}{D(k_1,k_2)}\frac{\sin qa}{q}$$

$$[37]\quad T_{12} = -\frac{\sin k_2 a}{k_2}\frac{U_{12}}{D(k_1,k_2)}\frac{\sin qa}{q}.$$

From [36] and [37] we expect that the resonance parameters will be unaffected when one goes off-shell. This is so, since the resonance giving a pole in the energy in T_{11} and T_{12} will factorize, independent of the off-shell energy. In other

words, this is a manifestation of the fact that the off-shell T-matrix of a separable non-local potential is separable. On the other hand this will not be true if more than one resonance appears in the same channel, since interference will alter the resonance parameters. We have calculated the elastic and inelastic cross-section as a function of the incoming energy at 3 different values of the off-shell distance s. For simplicity we have taken the parameters of reference[24] with $U_2 = 0$, i.e. no elastic potential in channel 2, $U_{12} = 4$, $U_1 = 16.94$ the elastic threshold $E_0 = 11.56$ Mev and we fixed the interaction radii at $R = 1.01$ $(Gev)^{-1}$. Two resonances are produced in the elastic channel. One at 8.4 Mev and the other at 24 Mev. The first one is usually referred to as a bound state embedded into continuum and corresponds to a bound state ($E_{2B} = -3.16$) in the second channel while the second corresponds to a resonance at 12.44 Mev.

We found no variations of the resonance parameters in the inelastic channel. The situation however is different in the elastic channel where we have two resonances interfering with each other. For better illustration we show in figure 10 the scattering amplitude $|f_{11}(E)|^2 = |kT_{11}(E)|^2$ at 3 different values of the off-shell distance s. It is clear that away from the resonance, off shell effects are small. As one moves close to the resonance energies however the interference effects are obvious. The parameters of the broad resonance change slightly whereas those of the small one change rapidly

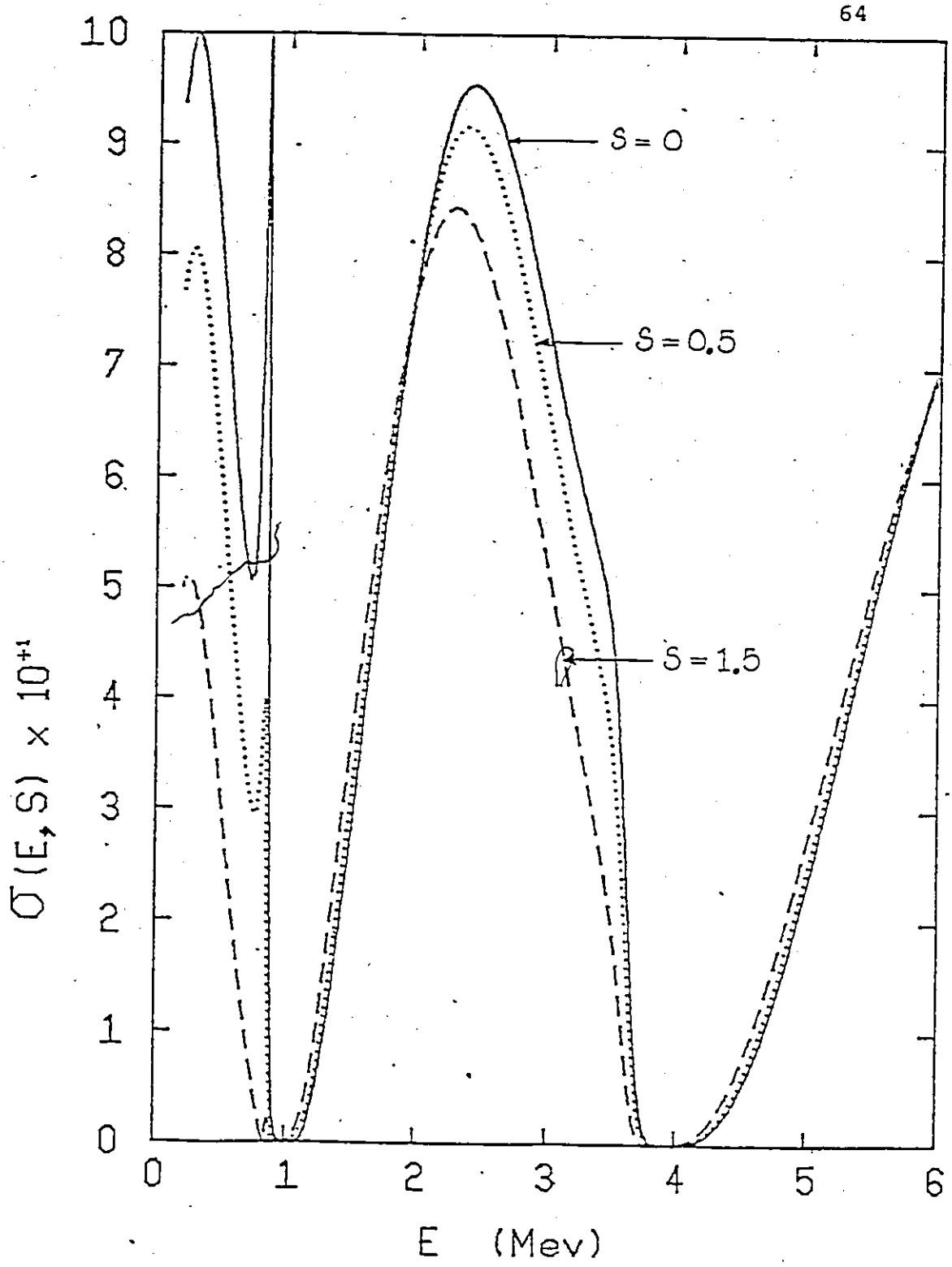


Fig. 10. $\sigma(E,s) = |k_1 T_{11}(E_1,s)|^2$ for the two channel model with delta function potentials as a function of the energy E_1 and the off-shell distance s. The potential strengths are; $U_{12} = 4$ Gev and $U_2 = 0$, $U_1 = 16.94$ both in Gev^2. The interaction radii are fixed at R=1.0 Gev^{-1} and the inelastic threshold is taken at $E_0 = 11.56$ Mev. The full line is the on-shell contribution. Note that for s = 1.5 the resonance at 8.4 Mev is reduced by more than a factor of 200 from its on-shell value. Also note that the energy scale should be multiplied by a factor of 10.

as one goes off-shell. At small off-shell distances the change in shape and width are small and only the size is reduced considerably. For off-shell distances greater than 1.5 Mev a considerable change of pattern occurs; the resonance becomes symmetric and it is reduced by more than a factor of 200 from its on shell value. Furthermore the width is decreased by 0.2 Mev and there is a resonance energy shift of 0.32 Mev towards the inelastic threshold. This shows that in these cases due to the presence of a second resonance, the δ-function potential does not entirely act as a non-local separable force, of rank one, for which a single pole expansion is possible, and therefore an off-shell extrapolation through a Breit-Wigner type formula will not be accurate.

We close this section giving the T-matrix formulae for the more physical case where only the coupling potentials are δ-functions and the other ones are square wells. For simplicity we again assume $U_2 = 0$. The result is:

$$[36] \quad T_{11} = -\frac{\sin k_1 a}{k_1} \frac{U_1 A\rho(q,k_1) - (1-A)U_1\rho(k_1,\kappa_1) - U_{12}^2 g_2(a,a)}{D(k_1)} \cdot \frac{\sin qa}{q}$$

$$[37] \quad T_{12} = \frac{\sin k_2 a}{k_2} \frac{1 - A g_1(a,a) U_1 (\rho(q,k_1) + \rho(k_1,\kappa_1))}{D(k_1)} \frac{\sin qa}{q}$$

where we have defined

$$\rho(x,y) = \frac{x\cot(xa)-y\cot(ya)}{x^2-y^2} \qquad [38]$$

$$\kappa_1^2 = k_1^2 - U_1 \quad , \quad A = \frac{k_1^2-q^2}{\kappa^2-q^2}$$

and

$$D(k_1) = 1 + g_1(a,a)U_1\rho(k_1,\kappa_1) - g_1(a,a)g_2(a,a)U_{12}^2 .$$

It should be noted that all equations reduce to their correct limits on the energy shell.

IV.5 SEPARABLE POTENTIALS OF RANK ONE

Non local potentials of rank one are not commonly used in reaction theory. This is so because potentials of the form $V_{ij} = \gamma_{ij}(v(r)v(r'))$ if they are not energy dependent, do not support more than one bound state for any given value of γ_{ij}. Therefore the expansion of the nuclear wave function in terms of the eigenstates of a separable non-local Hamiltonian of the above form is meaningless except in the case of a single and isolated resonance. In this case the resonance giving a pole in the energy in the reaction amplitude will factorize and since the latter is separable for a separable potential it is possible to relate directly the off-shell value to its on-shell one.

We should note here that with a non local potential, one can obtain a positive energy bound state[20] in one channel which by coupling to another channel becomes a resonance too and therefore a direct off-shell to on-shell relationship might again

be possible. Using the procedure outlined in the preceding section we have for the T-matrix in each partial wave ℓ;

$$[39] \quad T_{11} = -\frac{(I_\ell(k_1)+(k_1^2-q^2)I_\phi(q))[\gamma_{12}^2-\gamma_{11}\gamma_{22})G_{2\ell}+\gamma_{11}]I_\ell(k_1)}{D(k_1)}$$

$$[40] \quad T_{12} = -\frac{(I_\ell(k_1)+(k_1^2-q^2)I_\phi(q))\gamma_{12}I_\ell(k_2)}{D(k_1)}$$

with

$$I_\ell(k_1) = \int_0^\infty rj_\ell(k_1 r)v(r)dr$$

$$[41] \quad G_{i\ell} = \int_0^\infty \int_0^r g_i(r,r')v(r)v(r')drdr'$$

$$I_\phi(q) = \int_0^\infty \int_0^r v(r)g_1(r,r')j_\ell(qr')drdr'$$

and

$$D(k_1) = (1-\gamma_{11}G_{1\ell})(1-\gamma_{22}G_{2\ell})-\gamma_{12}^2G_{1\ell}G_{2\ell}.$$

The fully off the energy shell T-matrix elements are obtained from [39] and [40] by replacing $I_\ell(k_1)$ and $I_\ell(k_2)$ by $I_\ell(p)$. They are symmetric in q and p although not manifestly so in [39] and [40]. This can be easily seen by considering the $\ell=0$ partial wave for which

$$[42] \quad I_\phi(q) = \frac{I(q)-I(k)}{k^2-q^2}.$$

For illustration we consider S-waves only and show some results

of calculations with

[43] $$v(r) = e^{-\alpha r} .$$

We have fixed the interaction parameters $\gamma_{11} = 0.0$; $\gamma_{12} = 2.0$, $\gamma_{22} = -18.0$, $\alpha = 1.0$, and the inelastic threshold at $E_0 = 9$ Mev to produce a negative energy bound state in the second channel. It is seen from figure 11 that the resonance parameters do not change as we go off the energy shell. There exist several reasons for that: i) the T-matrix is separable and the form factors (I(q),I(k)) have no left hand cuts, ii) the resonance in the elastic channel corresponds to a zero of the real part of the determinant. In fact, for the example at hand this resonance corresponds to a negative energy bound state in the uncoupled second channel, and iii) $D(k_1)$ has only one simple zero and no other resonances, thus the condition of a single and isolated resonance is fulfilled.

It is interesting to note here that poles associated with the form factors in the inelastic channel are not reflected in the elastic channel. In our example such a pole appears at an energy $k_{2R}^2 = -\alpha^2$ or $k_1^2 = k_0^2 - \alpha^2$, which is below the inelastic threshold. This pole appears as a resonance in the coupled second channel but does not show up in the elastic one. This is so because it does not exist as a bound state in the uncoupled second channel, since the eigenvalue problem in this channel will restrict the values of the parameter α

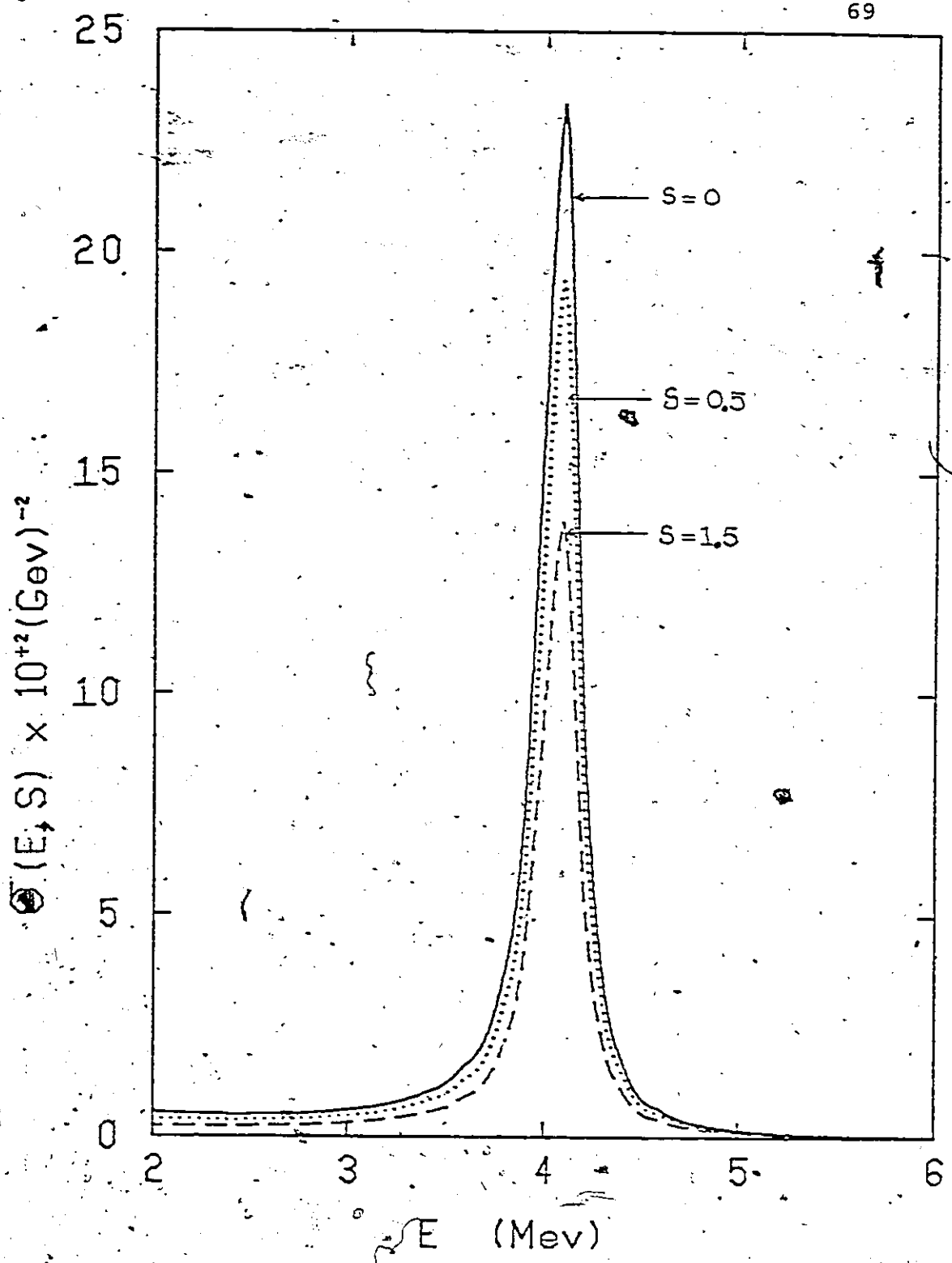


Fig. 11. The differential cross-section $\sigma(E,S)$ for the two channel model with non local potentials of the form $V(r,r') = \gamma_{ij} v(r)v(r')$ as a function of the energy E_1 and the off-shell distance s. The v(r)'s are of the form $e^{-\alpha r}$ with $\alpha = 1.0$ and the coupling constants are $\gamma_{11} = 0$, $\gamma_{12} = 2.0$, $\gamma_{22} = -18.0$, all in Gev^{-2}. The elastic threshold is fixed at $E_0 = 0.9$ Mev.

such that $\alpha > k_{2R}$.

Thus we conclude that only bound states or resonances that exist in the uncoupled inelastic channels will show up as resonances in the elastic one when the interaction between the channels is switched on and that off-shell effects will not have any effect on the resonance parameters if the interaction is non-local separable of rank one. On the other hand, if the separable interaction consists of a sum of terms rather than a single term one expects the off-shell effects to play an important role and we are investigating this.

IV.5 CONCLUDING REMARKS

In this chapter we have presented three exactly soluble models for the study of off-shell resonances through the coupled channel approach. These models provide a practical means for elucidating the bound and resonance properties of a nuclear model Hamiltonian.

Our results show under what conditions the isolation of specific resonance contributions is possible. It is found that all models allow a simple pole expansion. However whereas the separable non-local Hamiltonian of rank one is more suitable to such an expansion, care should be taken when using a square well Hamiltonian, since off-shell effects will have an appreciable effect on the resonance parameters.

CHAPTER V

EXTENSION OF THE OFF-SHELL T-MATRIX TO NUCLEAR REACTION PROBLEMS

V.1 INTRODUCTION

In Chapters II and III we have used the half-off-shell scattering amplitude to explain (d,p) resonances which arise in the ^{16}N nucleus. Our formalism there was based on the internal Green's function of the problem and its associated eigenvalues and eigenfunctions which were taken to be real and to represent the single particle states of the system. However, in practice many resonances appearing in such processes have compound rather than single particle nature. In other words, they are formed or they are decaying via different reaction channels, which can not be accounted for by the single particle treatment. In this chapter we present an extension of our formalism to describe coupled channels phenomena. Our approach is an extension of the off-shell T-matrix in nuclear reactions and uses the R-matrix theory by utilizing the wave functions and energy eigenvalues of the compound nucleus. It is an exact approach in the sense that as the number of compound states used in the calculation is increased, the calculated off-shell T-matrix must become exact. We should note here that a more physical expansion of the R-matrix is possible, for example in terms of eigenfunctions associated with their complex

poles[31]. In the present treatment however we wish to avoid the use of such an expansion since the whole system of eigenfunctions and eigenvalues becomes energy dependent which will present some numerical difficulties.

Before we proceed to our derivation it is probably worthwhile to recall the "channel" concept since there exists some ambiguities in the literature. Lane and Thomas[23] give an illustrative example by considering the reactions of Li^7 with protons

$$Li^7+p \rightarrow Be^{8*} \rightarrow \begin{cases} Li^7+p \text{ (elastic scattering)} \\ Li^7+p \text{ (inelastic scattering)} \\ Be^7+n \\ Li^6+d \\ He^4+He^4 \\ Be^8+\text{photon, etc.} \end{cases}$$

The spin of Li^7 is $I_1 = \frac{3}{2}$ and that of the proton $I_2 = \frac{1}{2}$, therefore the channel spin S can be 1 or 2. Since several incoming orbital angular momentum waves ℓ can contribute to the reaction, this reaction can be initiated by incident waves in several channels c. The formation of the compound nucleus Be^{8*} is possible when the separation distance between Li^7 and p is smaller than the channel radius a_c. Decay of this compound nucleus leads to outgoing waves in all channels for which the relative energy of motion is positive, for example in the present case 6 channels.

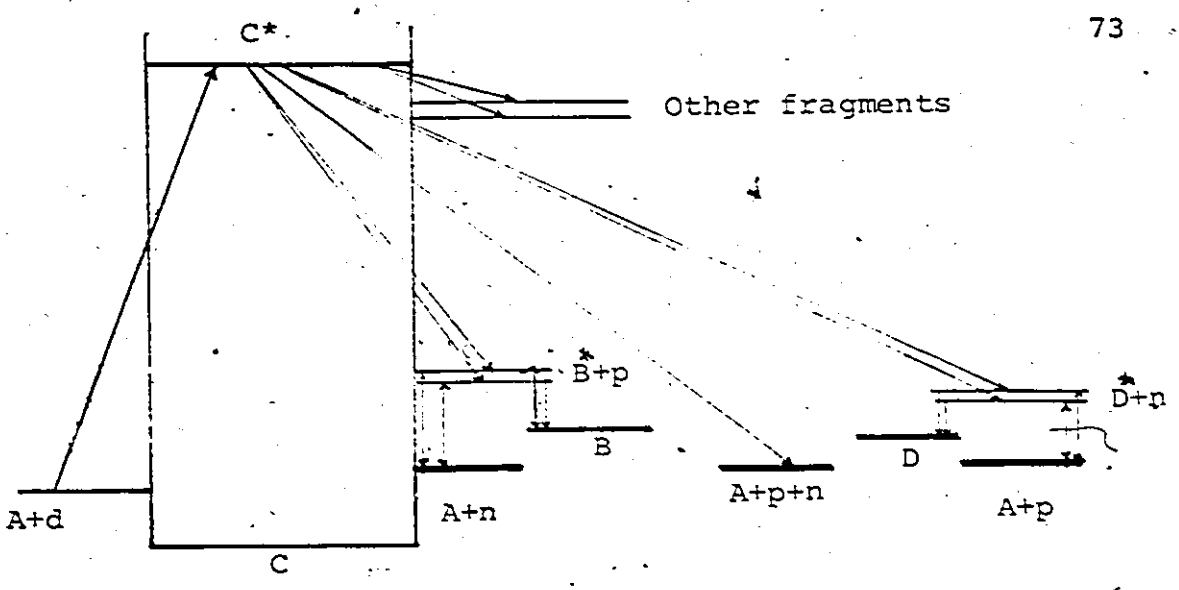


Fig. 12. Schematic picture of the various competing channels in a (d,a) reaction. For simplicity we show only one of the several excited levels of the compound nucleus C*. This level can decay into several channels; for example the bound fragments B/D ((d,p)/(d,n) stripping to bound states) or the target nucleus A+a where a can be a neutron (in this example (d,p) stripping to an unbound state) a proton ((d,n) stripping to unbound state) or both (breakup). In each of these channels several other channels might contribute. For example the double arrows indicate that these levels can also be reached by elastic scattering.

Now consider the decay into Be^7+n. There are several channels that can give this pair which are associated with the excited states of $\overset{*}{Be}^7$. We have seen therefore two different meanings of the channel, the one that is associated with a mass difference and the one that is not. A qualitative picture of the channel concept is given in figure 12.

In section two we formulate the nuclear reaction problem and define our notation. In section three we compare the results of an R-matrix calculation with the coupled channel results of section IV and discuss the effect of the distant levels on the off-shell resonance. We present our conclusions in section four.

V.2 FORMULATION OF THE NUCLEAR REACTION PROBLEM OFF THE ENERGY SHELL

The present derivation is similar in concept to that given in Chapter 4 but is more general in as much as it is extended to a many channel problem.

The Schrödinger equation satisfied within and at the boundary is written as

$$[1] \qquad (E-H)\,|\psi\rangle = (E-H_0)\,|\phi\rangle$$

where $|\psi\rangle$ is the total off-shell wave function and $|\phi\rangle$ represents plane waves. Since in the R-matrix theory the Hamiltonian is truncated at the boundary ($a_c=r_c$), we introduce a Hermitian Hamiltonian

$$[2] \qquad \mathcal{H} = H+L_b$$

where L_b is defined by

$$[3] \qquad L_b = \Sigma_c |c) \frac{\hbar^2}{2m_c a_c} \delta(r_c-a_c)\left(a\frac{d}{dr_c}-\frac{b_c}{a_c}\right) r_c (c|$$

and b_c specifies the boundary condition in channel c. We shall denote by L the operator corresponding to the particular case when b_c is set equal to L_c, the logarithmic derivative of an outgoing wave in channel c. The surface functions $|c)$ are defined in the usual way and if H_I is the internal part of the Hamiltonian giving rise to the different target and projectile states occuring in $|c)$ then,

$$[4] \qquad (H_I-\varepsilon_c)\,|c) = 0.$$

Using equation [2] and defining $\Delta L = L-L_b$ we can rewrite equation [1] as follows:

[5] $$(E-\mathcal{H})|\Psi\rangle = -\Delta L|\Psi\rangle + (E-H_0)|\phi\rangle - L|\Psi\rangle$$

Now we can expand the full wave function $|\Psi\rangle$ in terms of the surface functions $|c)$ and the radial functions $\psi_c(r_c)$ viz:

[6] $$|\Psi\rangle = \Sigma_c |c)\psi_c(r_c) = \Sigma_c |c)\frac{u_c(r_c)}{r_c}$$

and

$$|\phi\rangle = \Sigma_c |c)\phi_c(r_c)$$

Clearly the ψ_c's are the solutions of the coupled equations

[7] $$(T - (E-\varepsilon_c))u_c(r_c) = -\Sigma_{c'} V_{cc'}(r_c)u_{c'}(r_c)$$

and the ϕ_c's are the off-shell incoming waves (Bessel functions). We will restrict ourselves to the case of a positive energy parameter in the following so that we can introduce the on-and-off-shell momenta according to:

[8] $$k_c^2 = \frac{2m_c(E-\varepsilon_c)}{\hbar^2} \quad , \quad q_c^2 = \frac{2m_c}{\hbar^2} z_c$$

In order to be able to use the R-matrix theory we consider another expansion of $|\Psi\rangle$ in terms of the eigenstates $|\lambda\rangle$ of the Hamiltonian $\mathcal{H}$, viz:

[9] $$|\Psi\rangle = \sum_\lambda A_\lambda(k_c, q_c)|\lambda\rangle$$

[10] $$(E_\lambda - \mathcal{H})|\lambda\rangle = 0$$

where the internal states $|\lambda>$ can be expressed in terms of channel states with the help of a suitable set of fixed radial functions $u_{\lambda c}(r_c)$.

$$[11] \qquad |\lambda> = \Sigma_c \frac{u_{\lambda c}(r_c)}{r_c} |c)$$

It is easily seen that $u_{\lambda c}(r_c)$ are the coupled R-matrix states which are solutions of the coupled equations

$$[12] \qquad (T_c - (E_\lambda - \varepsilon_c)) u_{\lambda c} = -\Sigma_{c'} V_{cc'} u_{\lambda c'}$$

They can be found either by exact numerical methods or by expressing them in terms of a complete set of states that are diagonal in the channel indices. The latter is given in Appendix B.

We shall use the convention that a matrix element of the form $<||>$ implies integration over all coordinates whereas elements of the form $(||>$ or $(||)$ imply integration only over the surface variables. With these definitions we have for the channel projections

$$[5] \qquad (c|\Psi> = \psi_c(r_c) = \sum_\lambda A_\lambda (c|\lambda> = \sum_\lambda \frac{u_{\lambda c}(r_c)}{r_c} A_\lambda$$

From equations [5] and [6] we get for the expansion amplitudes

$$[14] \qquad A_\lambda = - \frac{<\lambda|\Delta L|\Psi> - <\lambda|E-H_0-L|\Phi>}{(E_\lambda - E)}$$

In equation [14] we have taken all channels to be open and we have used the relation

[15] $\mathcal{L}|\Psi\rangle = \mathcal{L}|\Phi\rangle$

which is possible since for open channels the external wave function is given by

[16] $|\Psi\rangle = \sum_c |c)(F_c Y_c - O_c \Sigma_{c'} Y_{c'} T_{cc'})$

and the boundary condition operator enables us to get rid of the outgoing wave from the right-hand side of our formal expression for $|\Psi\rangle$ leaving an explicit expression rather than an integral equation. We should note here that for closed channels $\mathcal{L}|\Psi\rangle = 0$ and the term $\mathcal{L}|\Phi\rangle$ in Eq.[14]should be omitted. Equation (14) can be reduced to calculable form by integrating over the radial coordinates. If we multiply the resulting equation by $u_{\lambda c'}(a_{c'})$, sum over λ and recall that

[17] $$\psi_{c'}(a_{c'}) = \sum_\lambda A_\lambda \frac{u_{\lambda c'}(a_{c'})}{a_{c'}}$$

then the resulting algebraic equations can be written in the form

[18] $$\sum_c \{(\delta_{cc'} - R_{cc'}\delta L_c)\psi_c - (k_c^2 - q_c^2) a_c \int_0^a R_{cc'}(r_c, a_{c'})\phi_c(r_c)dr_c\} = 0$$

[19] $$R_{cc'}(a_c, a_{c'}) = \sum_c \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}, \quad \gamma_{\lambda c} = \left(\frac{\hbar^2}{2m_c a_c}\right)^{1/2} u_{\lambda c}(a_c)$$

with

$$\delta L_c = \left(\frac{d}{da_c} - \frac{b_c}{a_c}\right) a_c \psi_c$$

where $R_{cc'}$ is the usual R-matrix and $\gamma_{\lambda c}$ and δL_c are energy-independent reduced width amplitudes and boundary condition parameters, obtained from the channel projection $(c|\lambda>$ of the internal functions. Thus we have succeeded in representing the behaviour of the off-shell scattering wave function ψ_c in the inside region in terms of its properties in the outside region and in terms of the eigenstates and eigenvalues of the Hamiltonian. Equations [18] are exact and they are valid for open and closed channels.

It is interesting to observe that the integral in equation [18] is not only a measure of the off-shell behaviour of the system, but also describes compound effects through the internal function $u_{\lambda c}(r_c)$. This can be seen by noting that in the case of $u_{\lambda c}(r_c)$ being small everywhere except at $r_c = a_c$ we can replace the integral with its integrand. This corresponds physically to a direct reaction in which most of the interaction takes place at the surface and the off-shell effects are manifested through the difference of the s-wave penetrabilities times the incoming wave. On the other hand this simplification can not be made for a channel that comprises compound states since in such a case the wave function can vary rapidly inside the boundary.

Once the solutions to equation [18] have been obtained, the amplitudes A_λ, and hence the complete wave function $|\Psi>$, may be extracted from equations [14] and [9]. On the other hand, equation [18] can be used to construct an expression for the off-shell T-matrix.

The radial motion channel wave function in the outside region has the decomposition

$$[20]\quad \psi_c(r_c) = \left(\frac{m_c}{\hbar_c a_c}\right)^{1/2} \left[F_c(\rho_c)Y_c - O_c(x_c)\sum_{c'} Y_{c'}T_{cc'}\right]\rho_c^{-1/2}$$

with $\rho_c = q_c a_c$ and $x_c = k_c a_c$ and where $F_c(\rho_c)$ and $O_c(x_c)$ are radial wave functions corresponding to incoming and outgoing waves, respectively. It should be noted that the relations between the two sets of solutions with the same argument are;

$$[21]\quad \begin{aligned} I_c(x) &= G_c(x) - iF_c(x) \\ O_c(x) &= G_c(x) + iF_c(x) \end{aligned}$$

where in the absence of Coulomb field the I and O functions are spherical Hankel functions and F and G are the spherical Bessel functions. In what follows we shall drop the arguments of the wave functions when they are evaluated at the boundary and we shall use Italic lettering to indicate dependence on the off-shell momenta. We should also draw attention to the fact that in our derivation the solutions F, in contrast to the solutions O, are defined off-the energy shell and the Wronskian of this pair is not constant but depends on r. Since r dependence of the Wronskian is associated with non local wave functions we see here how the off-shell effects introduce non local effects into the wave function. Another important point is that since the on-shell collision function U(k) is defined as minus the coefficient of the outgoing wave e^{ikr} at infinity corresponding to incoming wave e^{-ikr}, in other words the asymptotic form of the wave function is

[22] $$\psi \sim e^{-ikr} - U(k)e^{ikr}$$

Comparison of Eq. [17] and [19] shows that the usual relation $U = 1-2iT$ between the collision matrix U and the T-matrix is not valid off the energy shell. Inserting Eq. [20] in Eq. [18] we get an exact expression for the T-matrix off the energy shell.

[23] $$T = O^{-1}\rho^{1/2}(1-Rd)^{-1}(1-R(d+\delta))\rho^{-1/2}F+O^{-1}\rho^{1/2}(1-Rd)^{-1}B\rho^{1/2}F$$

where $\delta = L^{in}-L^{out}$ is defined as the difference between the logarithmic derivatives of an incoming and an outgoing wave and the term

[24] $$B_{cc'}=\sum_{\lambda}\frac{\gamma_{\lambda c}\,\gamma_{\lambda c'}}{E_\lambda-E}\,(k_c^2-a_c^2)a_c\int_0^a \frac{u_{\lambda c}(r_c)}{u_{\lambda c}(a_c)}\,\frac{F_c(r_c)}{F_c(a_c)}\,dr_c$$

is a typical off-shell effect. It is useful to define

[25] $$L^{in} = S(\rho) + iP(\rho) = S + iP$$
$$L^{out} = S(x) + iP(x) = S + iP$$

where $S(S)$ and $P(P)$ are diagonal matrices defining the level shift and the penetrability.

A further useful variant of Eq. [2] is obtained by introducing the matrices $F \doteq \frac{1}{2i}(O-I)$ and $\rho(OI)^{-1}$ which is just P. If we define the off-shell hard sphere scattering phase-shift $\Omega = I^{1/2}O^{-1/2}$ and replace T with 2iT then Eq. [20] takes the form

[26] $$T = O^{-1}O[1-\Omega(1+P^{1/2}(1-Rd)^{-1}R\Delta P^{-1/2})\Omega] + 2i\Omega(1-RD)^{-1}BF$$

[27] $$\Delta = \Omega(L-L)\Omega - (L^*-L) \ .$$

It is evident that this last form (Eq. [26]) isolates the parts of T which depend only on the incoming and outgoing states (i.e. P, Ω and $\bar{P}$) from the parts that essentially depend on what happens in the interior region $(1-Rd)^{-1}R$. Note also that the term in the square bracket provides an exact description of both the resonant part and the background part of the transition amplitude, where off-shell effects are manifested through the diagonal matrix

[28] $$\Delta = \Omega[(S-S) + i(P-P]\Omega - [(S-S) - i(P+P)]$$

whereas the last term represents pure off-shell and compound effects.

Equation [26] reveals some other important points. Off-shell effects are important both in the direct and the compound term. Hence the off-shell penetrability P alone can not reproduce the essential features of the off-shell dependence of the amplitude. This is still true, if one considers the direct contribution alone which can be derived by replacing the integral by its value at the channel radius. We shall note further that the part inside the square brackets has the formal structure of the collision matrix U where on shell parameters have been replaced by their off-shell equivalents. Since for the on shell case

the last term is zero and $\Delta = 2iP$ the well known expressions for the on-shell T and U matrix follows:

[29] $$T = 1-\Omega(1+2iP^{1/2}(1-Rd)^{-1}RP^{1/2})\Omega$$
$$T = 1-U .$$

V.3 RESULTS AND DISCUSSION

We now examine the validity of the above theory for the case where exact solutions can be found by simple means. Such exact solutions have been studied in chapter IV.

On the other hand our calculations will simulate the problem of an off-shell nucleon being scattered by the ground state and the first excited state of the target. If we choose the ground state to have J=0 then there is only one entrance channel and one can select the number of exit channels to represent reasonably well the physical situation in the energy interval under consideration.

The wave function in channel c with incoming waves in channel μ is:

[20] $$\psi_c = \phi_c\delta_{\mu c} - O_c T_{\mu c} .$$

Using equation [18] we obtain the coupled equations for the transition matrix elements;

[31] $$\sum_c (\delta_{cc'} - R_{cc'}d_c)O_c T_{\mu c} = (1 - R_{\mu c'}d_\mu)\phi_\mu + B_{\mu c'}\phi_\mu$$

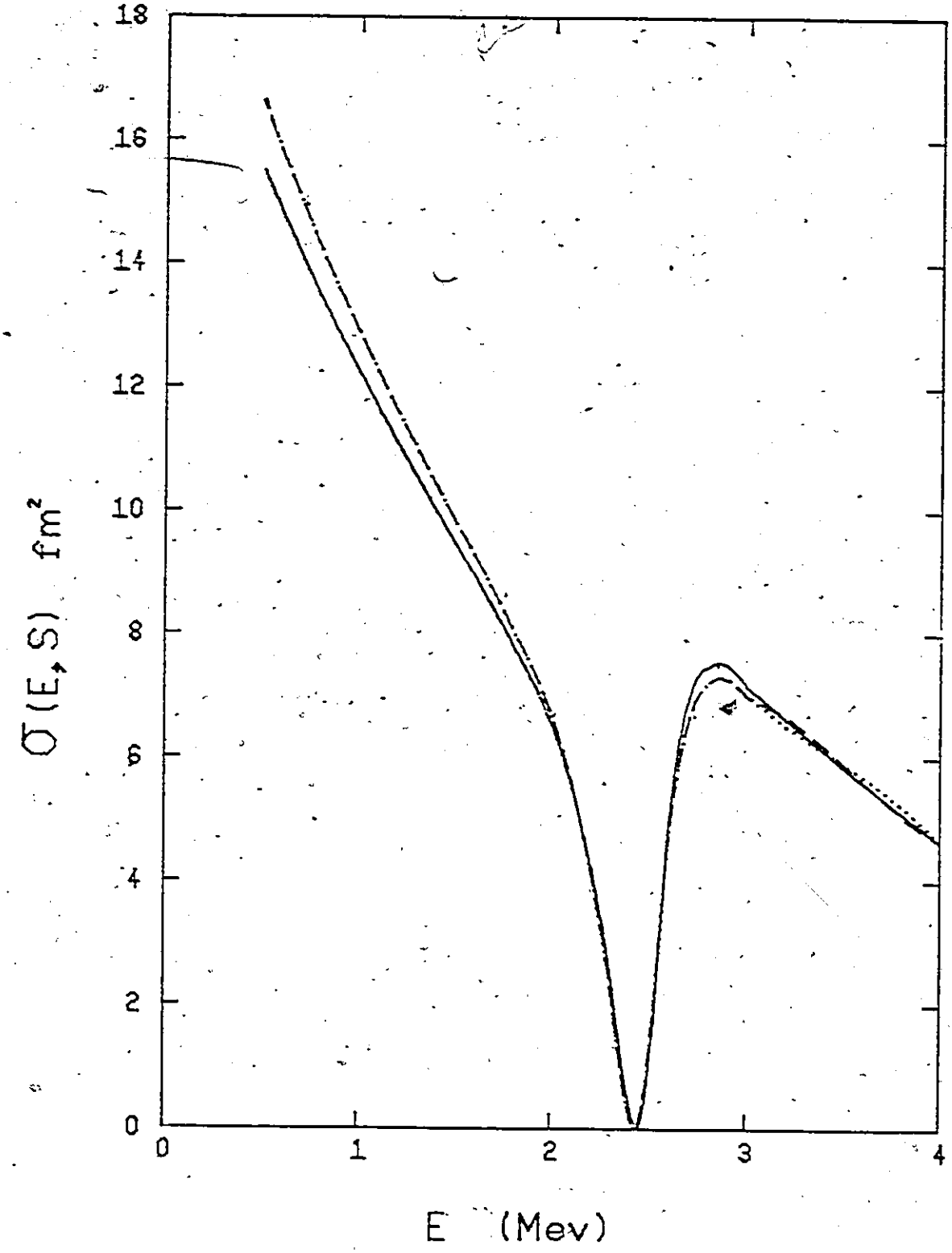


Figure 13. Exact (solid line) and R-matrix calculations (dotted and dashed lines) for the resonance at $E_r = 2.48$ Mev at the off-shell distance s=0. The potential parameters are $V_{12} = V_{21} = 1.072$, $V_{11} = -32.161$, $V_{22} = -39.022$ Mev and $a_1 = a_2 = 6$ fm. The dotted line is for the boundary condition $b_1 = b_2 = 0$ and the dashed line for $b_1 = 0.01$, $b_2 = 0$. Five single particle states are used in both cases. The energy axis for the R-matrix calculations is shifted by $\Delta E_r = 0.068$ Mev.

with $$d_c = L_c^{out} - b_c$$

$$d_\mu = L_c^{in} - b_c .$$

For the set of equations of the above type we take the case of two-channels with $\ell=0$, $a_1 = a_2 = 6$ fm, $V_{11} = -32.161$ Mev, $V_{12} = V_{21} = 1.072$ Mev, $V_{22} = -39.022$ and $\frac{\hbar^2}{2m} = 21.45$ Mev fm^2 (25)
The states that we consider are:

$$|0^+ \times 2s_{1/2}; 1/2^+> \quad \text{and} \quad |1^+ \times 2s_{1/2}; 1/2^+, 3/2^+> .$$

These states arise respectively from the scattering of an s-wave nucleon by the 0^+ ground state and the 1^+ excited stage of the target. We take the 1^+ state to be about 3.5 Mev higher in energy.

With zero coupling these states can not be formed in the compound system and so they will not show up in elastic or inelastic scattering. As we turn on channel coupling they show up as resonances in the elastic cross-section.

Figure 13 shows the $J^\pi = 1/2^+$ resonance in the elastic channel, $|T_{11}/k_1|^2$, for (n,n) scattering. It appears as a dip in the cross-section. The solid line is the exact calculation. The dashed and the dotted lines are indistinguishable from each other and they represent the R-matrix calculations with 5 single particle R-matrix states and the boundary conditions $B_1 = B_2 = 0$ and $B_1 = -0.01$, $B_2 = 0$ respectively. We note that shape, size and width are reproduced to a good degree of accuracy. We should also note further that there is an energy shift of 0.068 MeV in the R-matrix

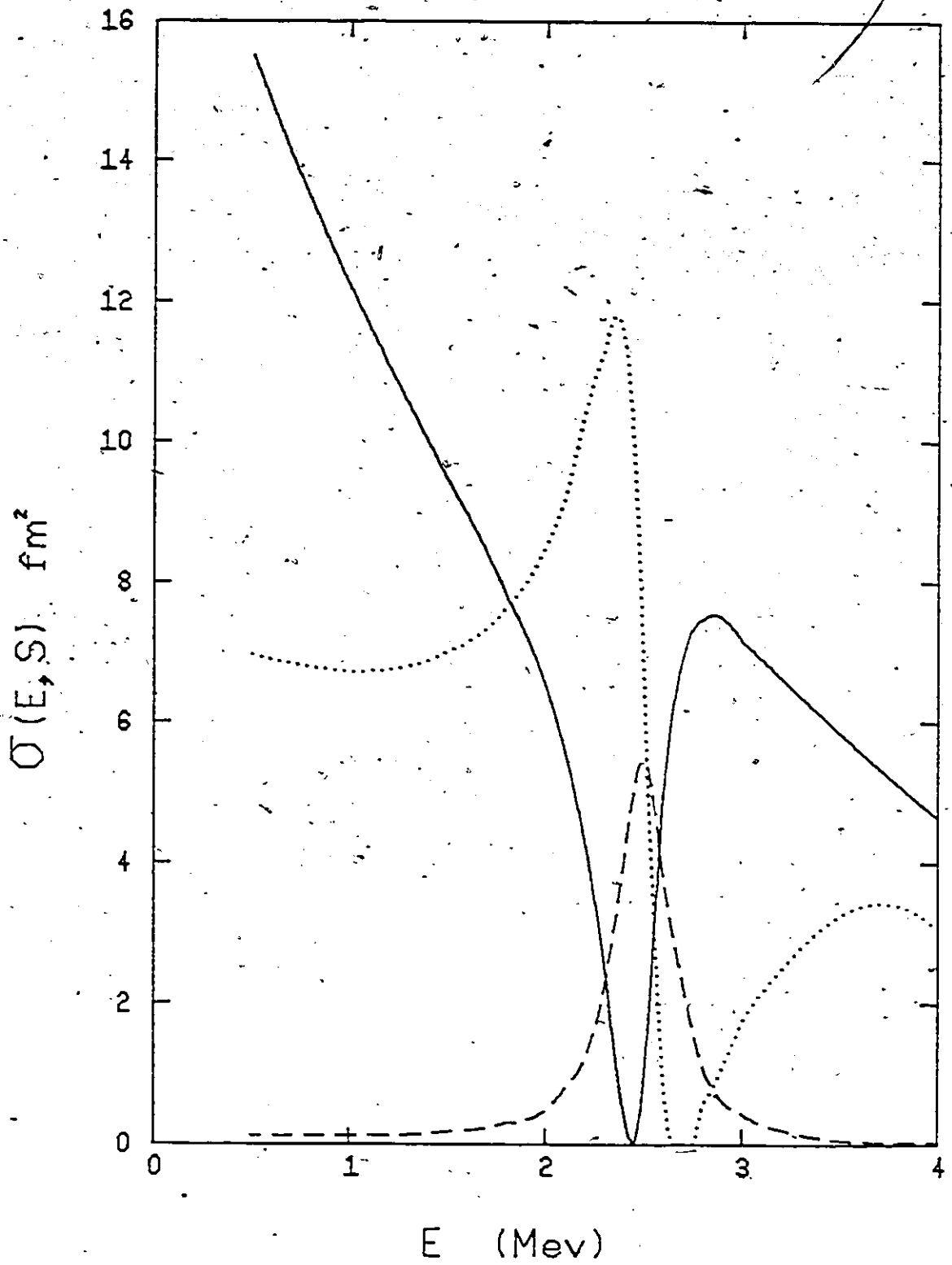


Figure 14: Exact calculation for the resonance of fig. 1 as a function of energy at different off-shell distances. These are s=0 solid lines, s=4.29 dotted line and s=8.58 Mev dashed line. Note how the resonance changes its shape due to the off-shell background at large off-shell distances.

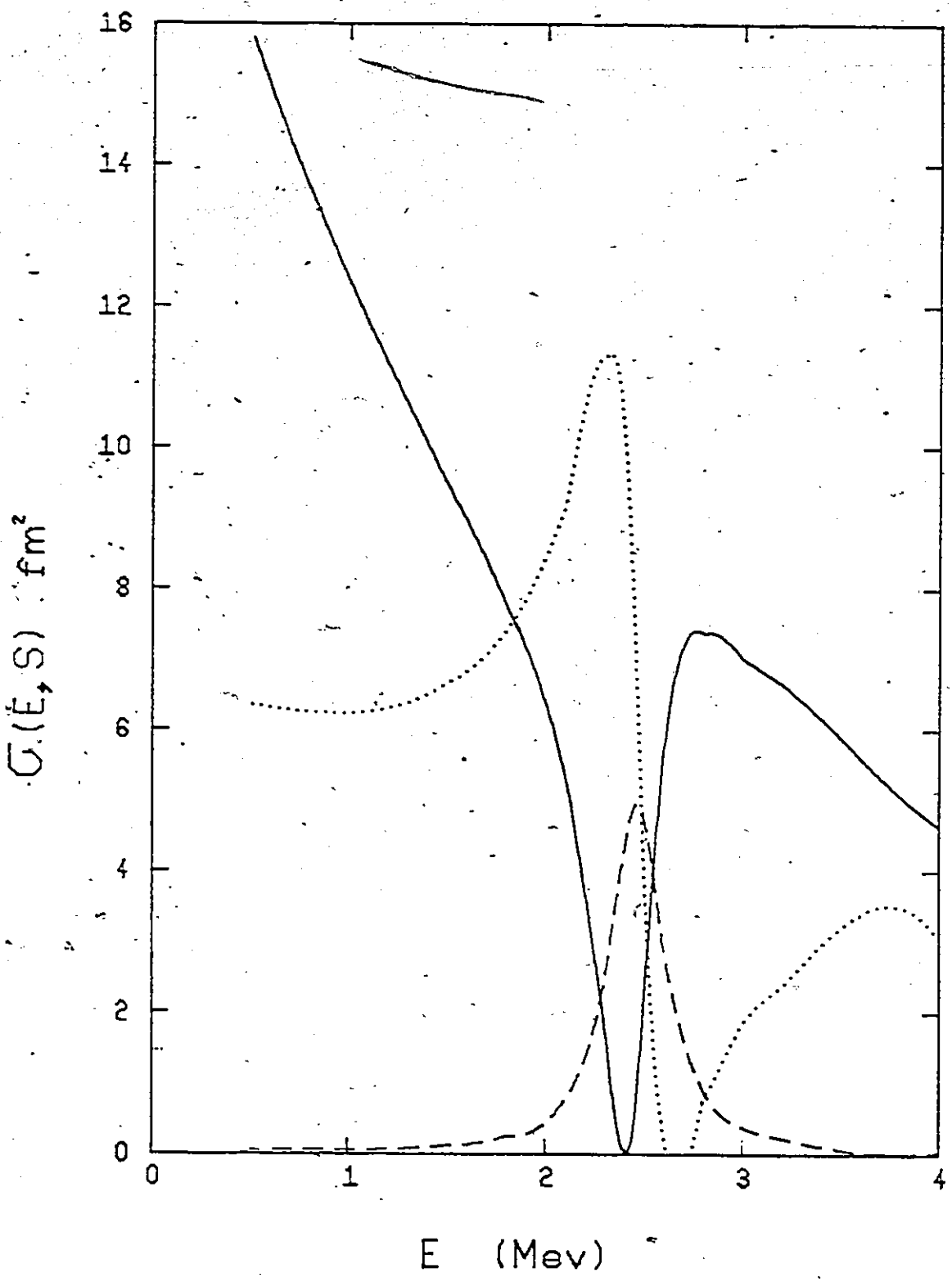


Figure 15. R-matrix calculation for the resonance at $E_r = 2.48$ Mev. We have used five single particle states and the boundary condition $b_1 = b_2 = 0.0$. For general description see fig. 14.

calculations (not shown in the graphs) and we shall return to this point later.

Figure 14 shows the same resonance for different values of the off-shell distance s, Figure 15 is the corresponding R-matrix calculation. The off-shell distance s in the (d,p) stripping reaction is defined as the energy difference between the energy of the stripped particle that is transferred to the target and the energy related to its off-shell momentum, viz. $s = p_n^2/2m_n - E_n$. It is a measure of the stripping angle. We have chosen to present the graphs in this way in order to show how the resonance changes its shape with varying off-shell distance and how well the R-matrix calculations reproduce this change. In the present example it is seen that a resonance in (n,n) scattering can disappear completely or can appear inverted by 180° in the (d,p) cross section.

We have studied this resonance for two reasons; first we have encountered similar types of resonances in the reactions $^{15}N(n,n)$ and $^{15}N(d,p)$. One of these was the $J^\pi = 1^-$ state at 5.048 Mev. There we have explained it as an interference between the $\ell=0$ and $\ell=2$ wave which was also interfering with its background. Secondly here we have a resonance which appears below the threshold energy and the appearance of threshold effects is possible. Since we are below the threshold of the second channel, this channel is closed and its wave function is a decaying exponential $\sim e^{-k_2 r}$. Usually one tries to eliminate the influence

of the closed channels by using a large R-matrix radius a_c. Our formalism of adjusting the boundary condition operator L according to the existing waves in each channel eliminates the closed channels completely but introduces a level shift ΔE_R. In the two-channel case this resonance shift is found by noting that resonances appear at the real zeros of the function

[32] $$D(R_{11},R_{22},R_{12}) = (1-R_{11}d_1)(1-R_{22}d_2)-R_{12}^2 d_1 d_2 \ .$$

Therefore the position of the resonance is given by;

[33a] open channel $$1-R_{11}(\mathrm{Re}d_1) - R_{22}(\mathrm{Re}d_2)+(R_{11}R_{22}-R_{12}^2)\mathrm{Re}(d_1d_2)$$

[33b] closed channel $$1 - \mathrm{Re}(R_{22}d_2) = 0 \ .$$

Assuming the background part R^o_{cc} of the R-matrix to be diagonal in the channel indices, one finds from equation [33] for the resonance shift

[34] $$\Delta E_R \approx |\alpha_{\lambda_1}|^2 R^o_{11}(k_1a_1)^2 + |\alpha_{\lambda_2}|^2(k_2a_2) \ .$$

with

[35] $$\alpha_{\lambda c} = \frac{\bar{\gamma}_{\lambda c}}{(1-R^o_{cc})}$$

where the bar over $\gamma_{\lambda c}$ indicates averaging out over those levels that contribute to the resonance and

[36] $$R^o_{cc} \approx \frac{\tan x_c}{x_c} \ , \quad x_c = \left(\frac{2m}{\hbar^2}(E_R-V_{cc})\right)^{1/2} a_c \ .$$

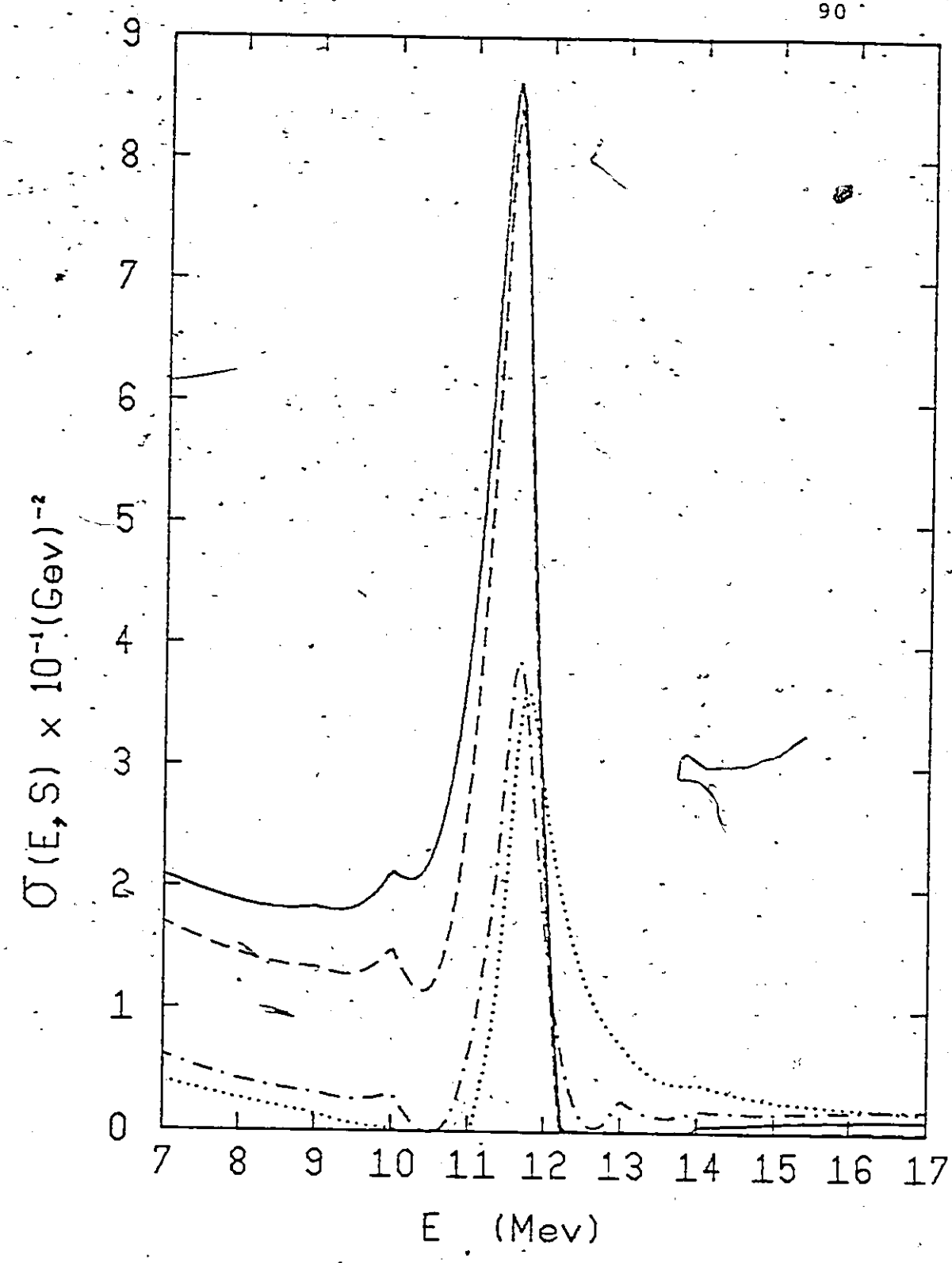


Figure 16. Exact calculations (solid and dotted lines) and R-matrix calculations (dashed and dashed-dotted Lines) for s=0 and s=5 Mev respectively. We have used $b_1 = b_2 = 0$ and five single particle states. The potential parameters are $V_{12} = V_{21} = V_{11} = -2$, $V_{22} = -20$ Mev, and the energy shift $\Delta E_R = 0.28$ Mev.

Equation [34] clearly separates the shift arising from the omission of the background (first term) and that which is due to the second channel being closed. A rough estimate using Eqs. [34] and [35] gives $\Delta E_R = 0.068$ Mev which compares well with the calculated value of 0.06 Mev.

For completeness we apply our formalism to another S-wave resonance which appears as a peak in the elastic cross-section. The potential parameters are $V_{11} = -2$ Mev, $V_{22} = -20$ Mev, $V_{12} = V_{21} = -2$ Mev the radius $a_1 = a_2 = 21.24$ Gev^{-1}, and we take $\hbar = 1$, $\frac{\hbar^2}{2m} = 1$ Gev^{-1} and $\varepsilon_2 = 20$ Mev. Exact and R-matrix calculations are shown in fig. 16.

For the on-shell case we again note that the accuracy is good, though in this case the R-matrix calculations show a smaller width and an energy shift of 0.28 Mev. This narrowing of the width arises from interference between the nearby levels and from closing the second channel. An additional source of error is also the energy dependence of ΔE_R. In practice however this error can be eliminated by choosing $B_c = S_c(E_r)$ which implies $\Delta E_R \approx 0$ at the center of the resonance. The off-shell case shows that the accuracy is also good though it deteriorates somewhat at the tail of the resonance as one moves away off the energy shell. Since most of the contributions to these parts of the resonance come from distant levels, it is obvious that for the present example the influence of the distant levels is quite strong. The policy with regard to the distant levels

has been to take one or two levels below and above the level which is closest to the resonance energy. We see that though this recipe seems to be good for the on-shell case it is not as good in the off-shell case. This is in agreement with our previous observations that the influence of the background on the resonance is stronger off the energy shell. It can be explained by considering the off-shell integral in Eq. [24]. The description of the physical states ψ_c through the compound states $u_{\lambda c}$ and then again through one or a few standing waves (uncoupled states v_{pc}) changes both the slope and the shape of the wave function in the internal region. Therefore the omission of the distant levels will change the absolute value of the integral. These changes are related to the non-diagonal parts of the interaction. They are small when $V_{cc} >> V_{cc'}$ and they become more prounced when $V_{cc} \approx V_{cc'}$.

IV. Conclusions

We have presented a consistent theoretical framework for the description of the off-shell T-matrix which can be used to calculate resonances off the energy shell which appear in stripping processes to unbound states.

This formalism has the advantage of previous approaches, which have dealt with isolated resonances. For example it uses the same parameters to describe the resonance in the elastic scattering and in stripping to unbound states, but it is also extended to a many channel situation. It is based on the real eigenfunction expansion of the coupled Green's function. These eigenfunctions are the coupled R-matrix states $u_{\lambda c}(r_c)$ (compound states) which can be found for example by expressing them in terms of a complete set of states that are diagonal in the channel indices. We have considered two different problems in which compound resonances appear and interfere with each other. Our calculations show that this approach provides a practical means for such problems and that adequate numerical accuracy is easily obtained in the energy region of interest. It will be interesting to apply this formalism to experimental results.

CHAPTER VI

THE BINDING ENERGY OF THE THREE PARTICLE SYSTEM USING NON-LOCAL INTERACTIONS WITH CONTINUUM BOUND STATES

VI.1 INTRODUCTION

The three particle system has been shown[31,32] to undergo a ground state collapse in the case of a rank-one separable interaction[33] which has a continuum bound state (CBS). A ground state collapse has also been found[34] to occur for the case of a rank-one and a rank-two potential having a resonance close to the real axis. Investigations of rank-two potentials having a CBS have as yet not been done, and the purpose of this chapter is to present such a study. To establish clearly whether the presence of a CBS in separable forces of rank two entails a collapse in the three particle ground state, all our interactions were constructed not only with a fixed deuteron wave function, implying a fixed UPA, but also with a fixed CBS energy of $E_{lab} \approx 2000$ and 4000 MeV respectively for the two sets of potentials. The rank two interactions and their UPA therefore have the same CBS. The form factor of the repulsive term in the interactions, is made increasingly repulsive, but although this somewhat reduces the three-particle binding energies a ground state collapse in the three-particle system still occurs in all cases.

In section 2 we discuss the construction of the separable interactions of rank-two and the reasons for our choice of their properties, while the results of our calculations of the ground state of the three-particle system are given in Section 3. Finally these results and their implications are discussed in section 4.

VI.2 SEPARABLE INTERACTIONS OF RANK TWO

Our rank-two potentials acting in an S-state are normalized as follows:

$$[1] \qquad V(k,k') = \frac{1}{2\pi^2}\frac{\hbar^2}{m}\,[-g(k)g(k') + h(k)h(k')].$$

It is important to realize that the decomposition of a separable potential of rank-two (or higher) is not unique as proved by Fiedeldey and Kok[33] and discussed by de Groot[34]. The form factors can be linearly combined, to reexpress the same potential. If we can define $\bar{V}(k,k') = 2\pi^2 \frac{m}{\hbar^2} V(k,k')$ then

$$[2] \qquad \bar{V}(k,k') = -g(k)g(k') + h(k)h(k') = -\bar{g}(k)\bar{g}(k') + \bar{h}(k)\bar{h}(k')$$

with

$$\bar{g}(k) = \frac{1}{\sqrt{1-\alpha^2}}\{g(k) + \alpha h(k)\};$$

and

$$[3] \qquad \bar{h}(k) = \frac{1}{\sqrt{1-\alpha^2}}\{\alpha g(k) + h(k)\}$$

in which α is a real constant, $\alpha^2 < 1$. A limiting potential can be defined as one having the property that

$$[4] \qquad D^{\dagger}(ik_D) = D_1^{\dagger}(ik_D) = 0$$

where $D^{\dagger}$ and $D_1^{\dagger}$ represent the Fredholm determinants of the total and attractive interactions of rank two and one respectively and

$$[5] \qquad E_d = -\frac{\hbar^2}{m} k_D^2$$

is the two-body binding energy. It can be easily proved that any rank two potential can be written in the limiting form which in the presence of a bound state is equivalent to the UPE for that potential. This implies that not only the bound state eigenvalues of the attractive part and the complete potential coincide in the limiting form, but also their bound state wave functions are identical. It is therefore clear that there is no loss of generality if we confine our attention to separable potentials of rank two in the limiting form.

This implies that our repulsive form factor is chosen in such a way that:

$$[6] \qquad \int_0^\infty \frac{g(k)h(k)k^2dk}{k^2 + k_D^2} = 0$$

as was done in ref.(34)

In addition to a fixed deuteron wave function we also impose on our separable interactions additional constraints related to the shape independent approximation, which we shall briefly discuss here.

At low energies, the phase shift is given by the effective range expansion

$$[7] \qquad k\cot\delta(k) = -\frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \ldots$$

which if substituted into

$$[8] \qquad S(k) = \frac{k\cot\delta(k) + ik}{k\cot\delta(k) - ik}$$

and continued to complex values of k, leads to the following result for the bound state pole

$$[9] \qquad k_D = \frac{1 - \sqrt{1 - r_0/a_0}}{r_0} .$$

For limiting potentials the bound state wave function is given by

$$[10] \qquad \psi_B(k) = C_0 \frac{g(k)}{k^2 + k_D^2} .$$

If g(k) only has singularities for $|\mathrm{Im}\ k| > k_D$ we obtain the correct asymptotic behaviour

$$[11] \qquad U(r) \underset{\infty}{\doteq} A_s e^{-k_D r} .$$

With the help of contour integration we obtain

$$[12] \qquad A_s^2 = g^2(ik_D)$$

while in the vicinity of the deuteron pole (i.e. for $k \approx ik_D$) the t-matrix becomes

$$[13] \qquad t_0(p,q,k^2) \approx \frac{g(p)g(q)}{k^2 + k_D^2} .$$

Making an analytical continuation to small positive k-values one finds for the S-matrix

$$[14] \qquad S(k) = \frac{k^2 + k_D^2 - 2ikg^2(k)}{(k+ik_D)(k-ik_D)} .$$

The residue of $S(k)$ at $k = ik_D$ can be obtained from eq. [14] and compared to the result obtainable from eqs. [7] and [8] in the same limit, i.e.

$$\lim_{k \to ik_D} (k-ik_D)S(k) = \frac{-2ik_D}{1-k_D r_0} .$$

This comparison yields

$$[15] \qquad A_s^2 = \frac{2k_D}{1 - r_0 k_D} .$$

This equation gives an explicit relation between the asymptotic normalization constant A_s of the deuteron wave function and the effective range parameter r_0. With the experimental values of r_0 and $k_D = 0.23161$ fm^{-1}, for the nucleon-nucleon interaction in the triplet s-state one finds $A_s = 0.879$. All realistic potential models that include the OPEP tail predict values of A_s very close to this value. de Groot[35] and McGurk[36] showed that form factors which predicted anomalous values of A_s also produced unrealistic binding energies in the triton.

A good fit to A_s was also required by Sofianos et al[34] for their potentials and together with the correct deuteron binding energy ensures that eq. [9] is obeyed within narrow limits, as is also the case for all realistic nucleon-nucleon interactions. It is important to note here that interaction B_4 of ref.(34) does not obey the relation eq. [15]. Instead of the value $A_s = 0.513$ fm$^{-1/2}$ required by the shape independent approximation, which is found to be approximately valid for all their other potentials, one obtains $A_s = 0.327$ for B_4. As we already

pointed out in the introduction this interaction has another highly undesirable feature. Its attractive part is shorter ranged than its repulsion. For these reasons one could disregard the trinucleon binding energy obtained for B_4, which implied that a CBS in the two-nucleon system does not necessarily entail a collapse in the trinucleon system.

Summarizing the constraints on our interactions arising from the bound state, our interactions must obey

$$[16] \qquad D^{+}(ik_D) = D_1^{+}(ik_D) = 1 - \frac{2}{\pi}\int_0^{\infty} \frac{dp\ p^2}{p^2+k_D^2}\, g^2(p) = 0$$

and eqs. [6] and [15].

To obtain the conditions for a CBS we apply the method of the cancellation of the Green's function[37] derived by Krause and Mulligan in coordinate space. We assume a CBS at

$$k^2 = \kappa^2$$

and a corresponding CBS wave function $\phi(r)$ also orthogonal to the repulsive form factor $h(r)$, i.e.

$$[17] \qquad \int_0^{\infty} dr\phi(r)h(r) = 0 .$$

Therefore for an s-state

$$[18] \qquad (\frac{d^2}{dr^2} + \kappa^2)\phi(r) = -g(r)\int_0^{\infty} g(r')\phi(r')dr'.$$

We can only have a cancellation of the Green's function if

[19] $$g(r) = \left(\frac{d^2}{dr^2} + \kappa^2\right)\phi(r) ,$$

while the CBS condition at $k^2 = \kappa^2$ implies that

[20] $$1 = -\int_0^\infty dr g(r)\phi(r) .$$

or equivalently

[21] $$1 + \int_0^\infty dr\phi(r)\left(\frac{d^2}{dr^2} + \kappa^2\right)\phi(r) = 0 .$$

In momentum space we have, due to the orthogonality condition eq. [17], the following CBS conditions

[22] $$D_1^\dagger(\kappa) = D^\dagger(\kappa) = 0$$

where $D_1^\dagger(\kappa)$ and $D^\dagger(\kappa)$ are now complex quantities. We have

[23] $$\mathrm{Re}D_1^\dagger(\kappa) = 1 - \frac{2}{\pi}\int_0^\infty \frac{g^2(p)p^2dp}{p^2 - \kappa^2} = 0$$

$$\mathrm{Im}D_1^\dagger(\kappa) = \kappa g^2(\kappa) = 0 , \text{ or}$$

[24] $$g(\kappa) = 0 .$$

The necessary and sufficient condition for a CBS at $k^2 = \kappa^2$ is $D^\dagger(\kappa) = 0$ and

[25] $$E(\kappa) = 0$$

where

[26] $$E(\kappa) = \frac{D^{\dagger}(\kappa)}{f^{\dagger}(\kappa)}$$

and $f^{\dagger}(\kappa)$ is the Jost function of the nonlocal interaction which differs from $D^{\dagger}(\kappa)$. It can be shown that $f^{\dagger}(k) \neq 0$ for $k > 0$, hence $D^{\dagger}(\kappa) = 0$ implies $E(\kappa) = 0$. Since $E(\kappa_s) = 0$ is associated with a spurious state at $k = \kappa_s$ and $D^{\dagger}(\kappa_c) = 0$ implies a CBS at $k = \kappa_c$ this implies that if there is a CBS it coincides with the spurious s-state. Otherwise a spurious state is associated with a node in the physical scattering solution for $\kappa_s > 0$. This node disappears at scattering energies $k^2 > \kappa_s^2$ except when we have a CBS, i.e. $\kappa_c = \kappa_s$ in which case the node persists at all positive energies and is nearly energy independent.

The orthogonality condition eq. [17] becomes in momentum space

[27] $$\int_0^{\infty} \frac{g(p)h(p)}{p^2 - \kappa^2} p^2 dp = 0 .$$

By imposing this additional condition, similar to the condition for a limiting representation for a rank two potential given by eq. [6], we simplify the construction of a set of potentials of rank two having the same bound state and continuum bound state properties. Our rank-two potentials can be regarded as limiting representations with respect to both the bound state and the CBS, i.e. we have both

[28] $$D^{\dagger}(ik_D) = D_1^{\dagger}(ik_D) = 0$$
$$D^{\dagger}(\kappa) = D_1^{\dagger}(\kappa) = 0 .$$

Unlike the previous investigation by Sofianos et al[34] we now restrict our attention to rank two potentials all having the same bound state and CBS. The additional repulsion only affects the phase shift not the CBS pole. In ref.[34] it also shifted the resonance pole further away from the real axis, making it difficult to draw firm conclusions concerning collapsed states in the three-particle spectrum.

The form factors g(k) and h(k) were all chosen to be sums of Yamaguchi factors:

$$[29] \qquad g(k) = \sum_{i=1}^{N} \frac{\gamma_i}{\beta_i^2+k^2} \approx h(k) = \sum_{i=1}^{N} \frac{\rho_i}{\omega_i^2+k^2} .$$

The parameters γ_i and β_i were constrained to fit the two-body binding energy for an averaged nucleon-nucleon interaction acting in s-states only, i.e. our potentials should be regarded as an average over the triplet and singlet s-state interactions.

$$E_d = 0.431 \text{ MeV} \quad \text{and} \quad A_s^2 = \frac{2k_D}{1-r_0 k_D} = 0.513 \text{ fm}^{-1}$$

and the CBS conditions i.e. eq. [28]. These are given explicitly in appendix C for the special case eq. [29]. We produced two sets of potentials A_i and B_i with a CBS at $\kappa = 5$ fm^{-1} ($\sim$ 2000 MeV) and $\kappa = 7$ fm^{-1} ($\sim$ 4000 MeV) respectively.

The parameters ρ_i and ω_i of the form factor h(k) were constrained by means of the orthogonality conditions for limiting potentials eqs. [6] and [27] (see Appendix C).

The parameters of g(k) and h(k) are given in tables 1 and 2 and the phase shifts for some of the potentials in Figs. 17 (set A) and 18 (set B). All the rank-two potentials produce the same effective range parameters

$$a_0 = 11.06 \text{ fm} \quad \text{and} \quad r_0 = 2.18 \text{ fm}.$$

We plot the deuteron wave functions in Fig. 19 and the form factors h(k) for some potentials in Figs. 20 (set A) and 21 set B). Some of our potentials (A_1 to A_4 and B_1 to B_6) fulfill the additional requirement

$$h(0) = 0.$$

From Figs. 17 to 21 it is seen that with increasing ρ_4, or ρ_3 for the potentials A_5 to A_7 and B_5 to A_7, $|h(k)|$ becomes larger and the phase shifts at lower energies become increasingly more repulsive. Since both $\text{Re}D^{\dagger}(\kappa)$ and $\text{Im}D^{\dagger}(\kappa)$ are zero

$$\tan\delta(\kappa) = \frac{\text{Im}D^{\dagger}(\kappa)}{\text{Re}D^{\dagger}(\kappa)}$$

does not necessarily have to be zero, as happens to be the case for the Tabakin and our rank-one potentials with a CBS. Using the more usual definition[44]

$$\delta_D(k) = - \text{ phase } D^{\dagger}(k) ,$$

the phase shift jumps by an amount of π at $k = \kappa$ in that case, which can be interpreted as a resonance of zero width. However with increasing repulsion the zero of $\delta_D(k)$ shifts to values of $k < \kappa$. A similar phenomenon occurs for the Mongan singlet

Table 1. Parameters of the rank two separable potentials with a continuum bound state at $\kappa = 5$ fm^{-1}

γ	(0.09459	67.80206	-121.09783	-260.53401)
β	(0.75205	8.81122	16.31404	32.0)
ω	(2.0	4.0	8.0	16.0)

	ρ				
A1	-1.65304	15.13734	-46.6007	50	
A2	-3.30608	30.27469	-93.20139	100	$h(0) = 0$
A3	-9.91825	90.82406	-279.60417	300	
A4	-13.22434	121.09875	-372.80556	400	
A5	12.15222	-48.62992	50.		
A6	72.91333	-291.77955	300.		$h(0) \neq 0$
A7	97.2177	-389.0394	400.		

Table 2 Parameters for the rank two separable potentials with a continuum bound state at $\kappa = 7$ fm^{-1}

γ	(0.08971	19.82788	131.22361	-616.72734)
β	(0.75205	8.81122	16.31404	32.0)
ω	(2.0	4.0	8.0	16.0)

ρ

B1	-2.08761	18.11946	-51.57607	50.	
B2	-4.17522	36.23891	-103.15214	100.	
B3	-12.52566	108.71674	-309.45643	300.	h(0) = 0
B4	-16.70088	144.95566	-412.60857	400.	
B5	-41.7522	362.38915	-1031.52143	1000.	
B6	-83.5044	724.7783	-2063.04286	2000.	

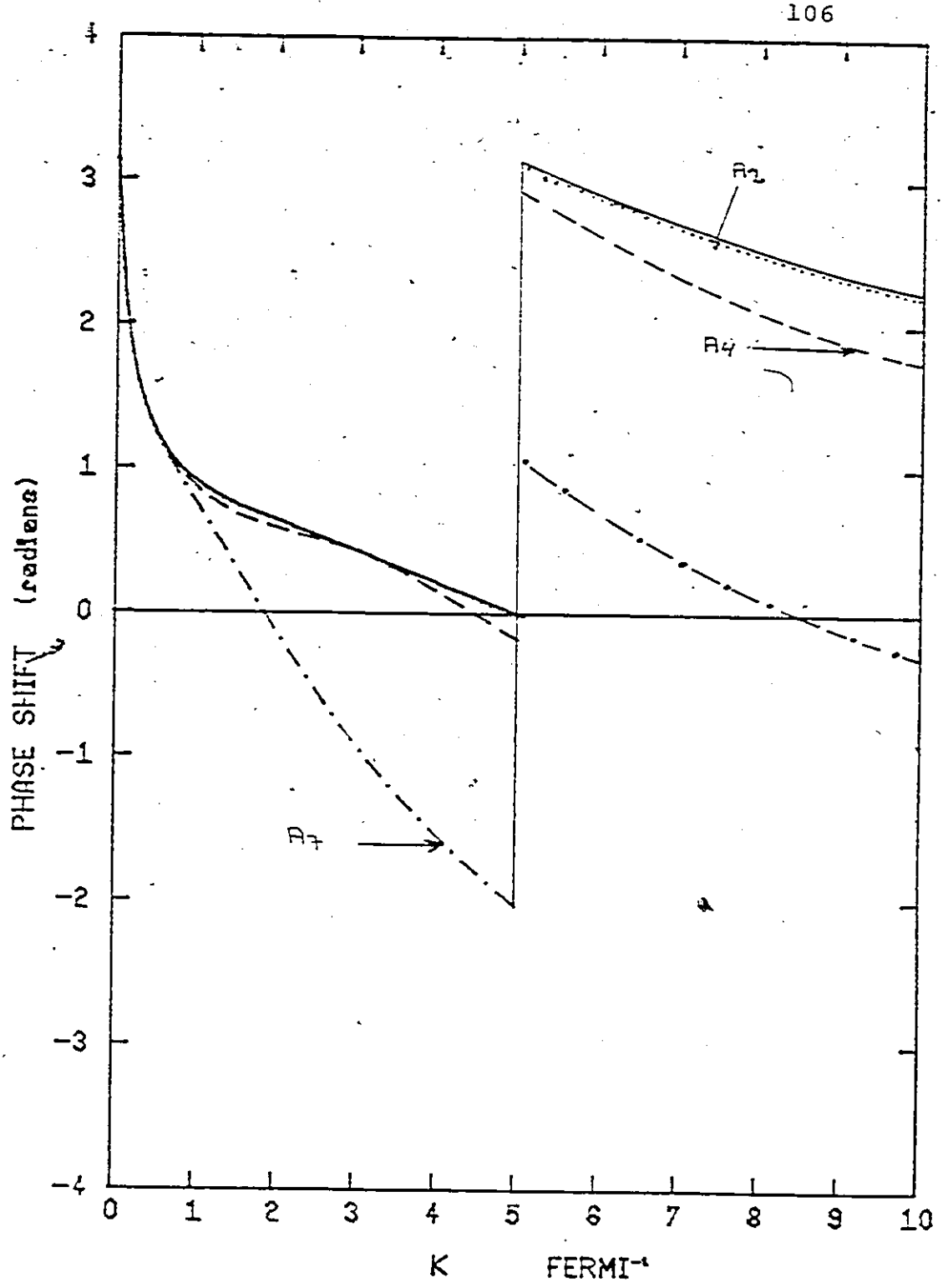


Figure 17. Phase-shift $\delta(k)$ for some potentials of class A ($\kappa = 5$ fm^{-1}) as a function of k. Note that π has been added to the phase-shift at the CBS and that with increasing repulsion the crossing point from positive to negative phase shift moves to lower k, away from the CBS. The solid line is the phase shift for g(k) acting alone.

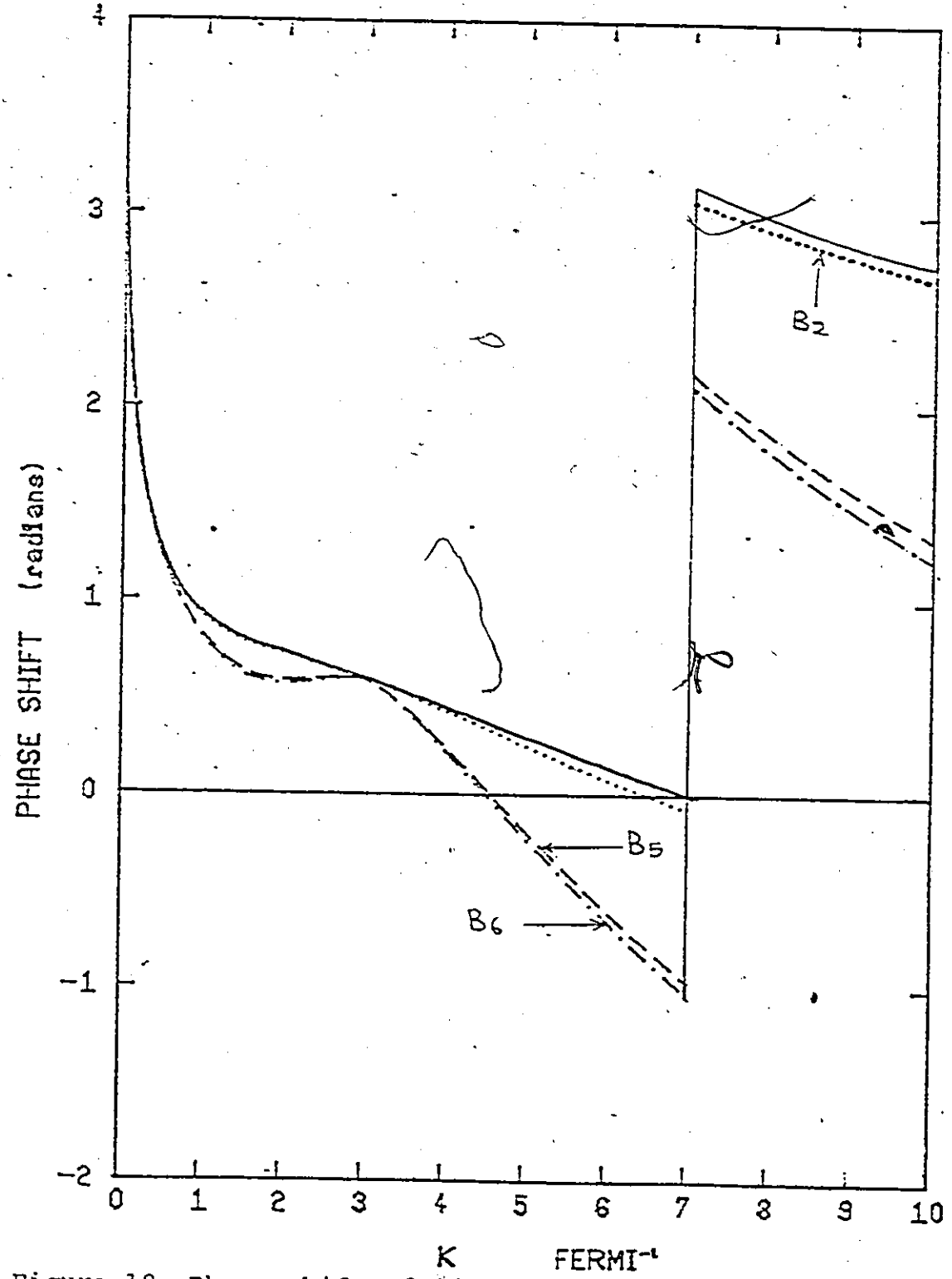


Figure 18. Phase shift of $\delta(k)$ for some potentials of class B ($\kappa = 7$ fm^{-1}) as a function of k. Same remarks apply as for Fig. 17.

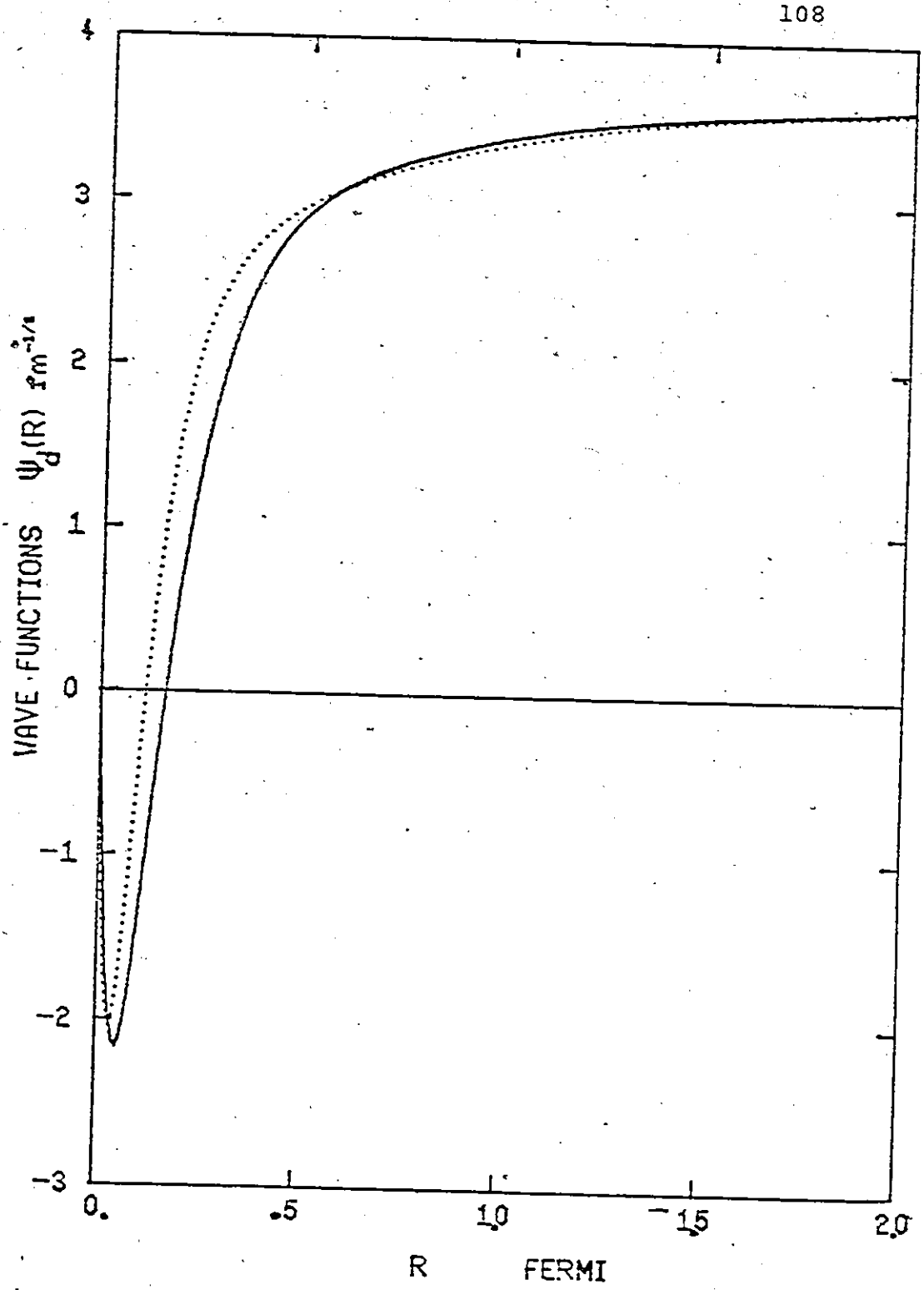


Figure 19. Deuteron wave functions as a function of r. The solid line is for potentials of class A. The dotted line is for potentials of class B.

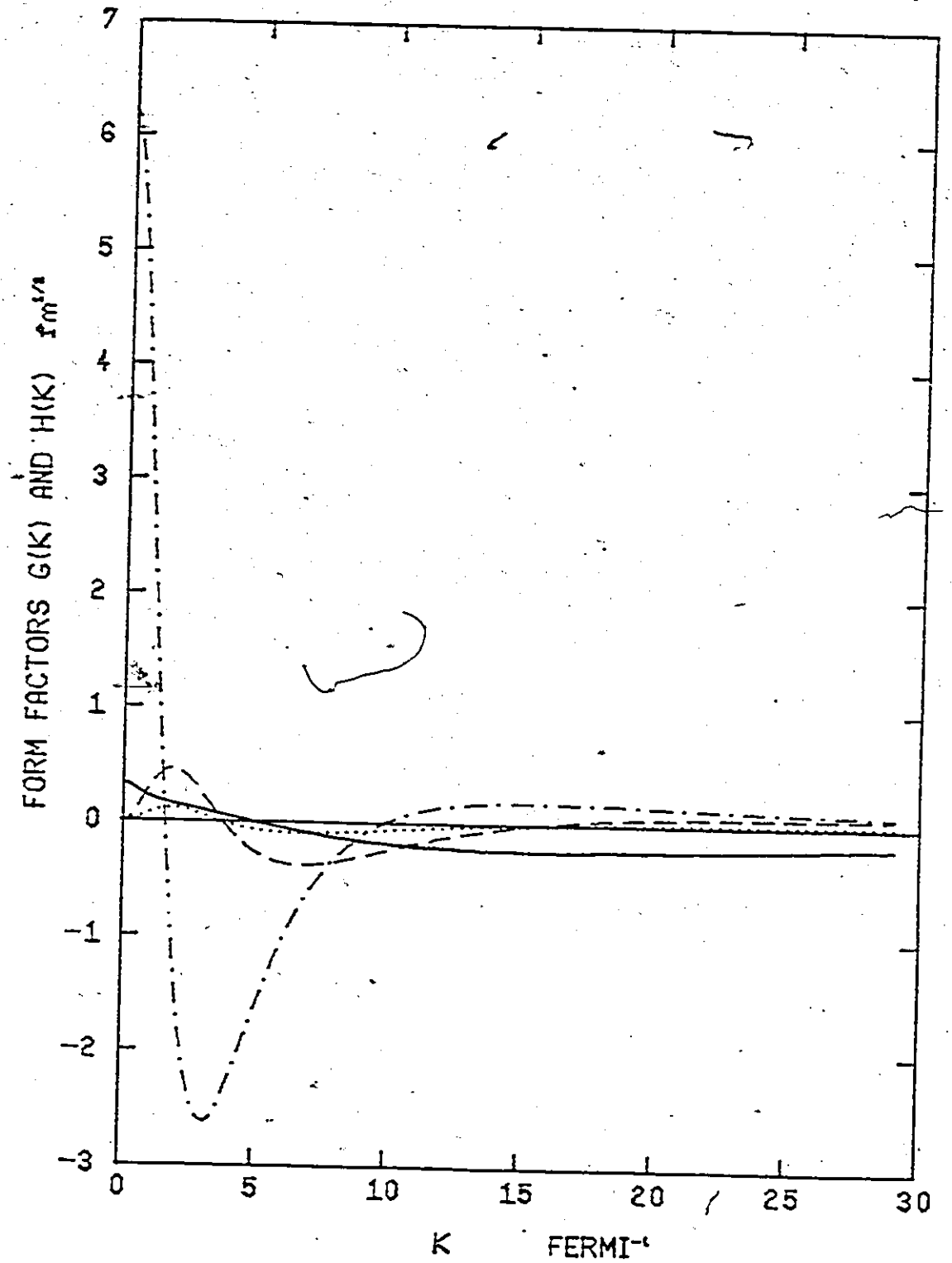


Figure 20. Form factors h(k) for some potentials of class A as a function of k. The solid line is the attractive part g(k). Note that A7 does not fulfill the condition h(0) = 0. The potentials are the same as those in Fig. 17.

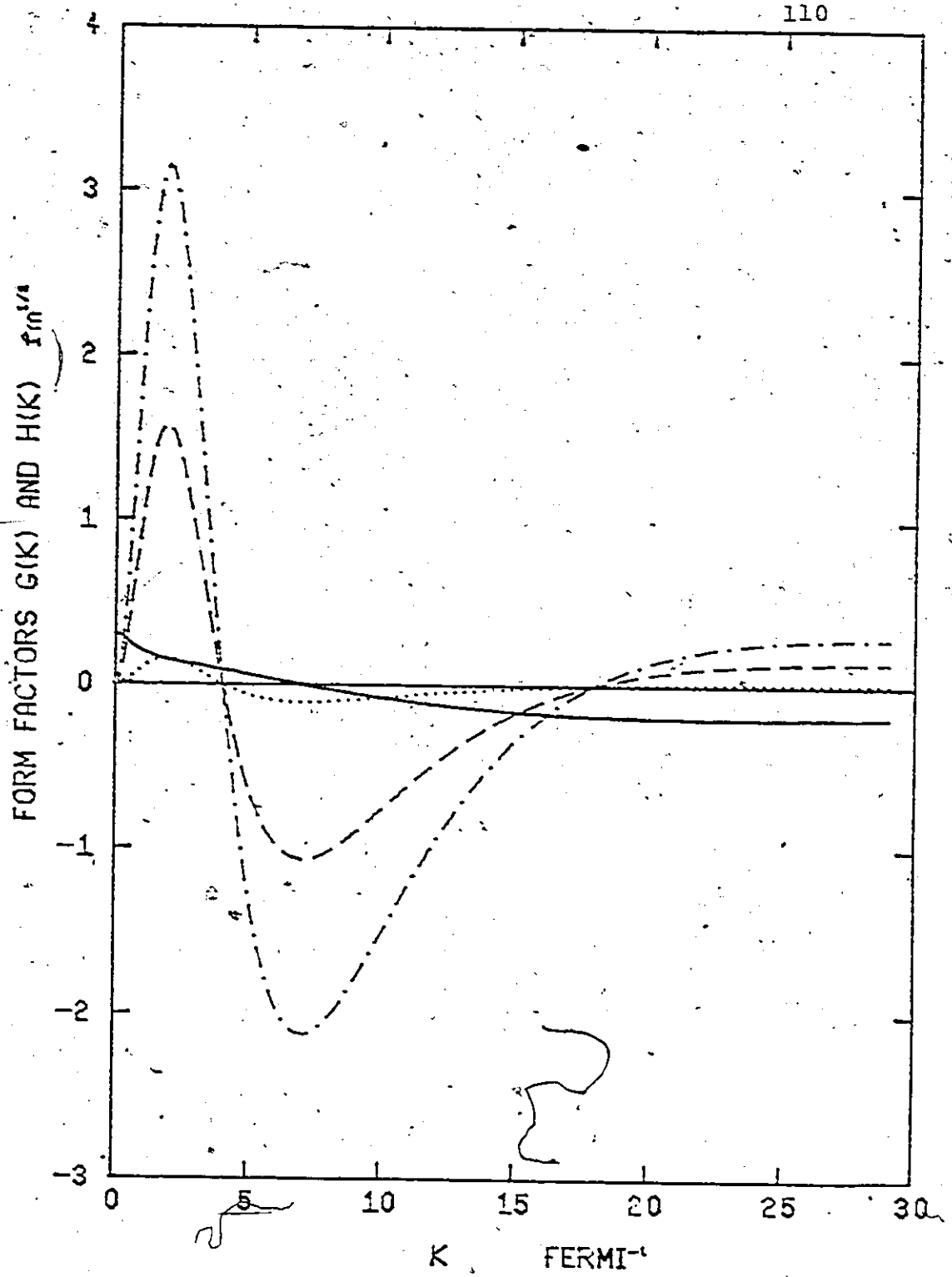


Figure 21. Form factors h(k) for some potentials of class B as a function of k. Note that these form factors fulfill the condition h(0) = 0. The solid line is the attractive part g(k). The potentials are the same as those in Fig. 18.

potential employed by Baghi et al.[44] section VIIC, who employed the alternative definition $\delta_f(k) = -$ phase $f^+(k)$ of the phase shift, which is then continuous at $k = \kappa$. In section 4 we show that the ground state collapse in three- and four-particle systems is closely related to the occurrence of resonances and continuum bound states even for nonlocal interactions of rank higher than one. It seems therefore preferable to employ a definition of $\delta(k)$, namely $\delta_D(k)$ rather than $\delta_f(k)$ which stresses this feature. Sofianos et al[34] found that the three-particle binding energies rapidly increase, for nonlocal interactions, the narrower the resonances become (i.e. the closer to the real axis they move) and reach their maximum at a CBS, which is consistent with their interpretation as a resonance of zero width.

It is interesting to note that the phase shifts for some of the potentials with $\kappa=5$ and $h(0) \neq 0$ become very repulsive for $k<\kappa$ and that it can increase to a value larger than π at the CBS.

VI.4 THREE-PARTICLE BINDING ENERGIES

In this section we confine our attention to a calculation of the ground state energies of the trinucleon system in the so-called model triton or three-boson approximation, with two-body s-state spin-independent interaction, similar to the calculations of Sofianos et al[34]. This is completely sufficient for the purpose of establishing whether a CBS even at very high energies in the two-nucleon system would necessarily be associated with a collapse in the trinucleon system or not, for nonlocal interactions of a more general nature than separable potentials of rank one.

It is not necessary to go into the details of the calculations of the binding energies of the three-particle systems, since these are well known. Having separable form factors of the Yamaguchi type allowed us to calculate the kernels of the coupled integral equations analytically. For further details see Sofianos et al[34].

The binding energies of the ground states of interactions A and B are given in table 3 together with the results for the Beregi potential, a $^3S_1$ potential of rank-one with $g(k) = \frac{\gamma_1}{\beta_1^2+k^2} - \frac{\gamma_2}{\beta_2^2+k^2}$, with $\gamma_1 = 17.399$ fm$^{-3/2}$, $\gamma_2 = 53.683$ fm$^{-3/2}$, $\beta_1 = 2.67$ fm^{-1} and $\beta_2 = 5.34$ fm^{-1} producing a CBS at 259.3 MeV ($\kappa = 2.5$ fm^{-1}) and a deuteron binding energy of -2.225 MeV.

We also calculated the triton binding energy E_t for the $^1S_0$-Mongan potential of rank two with $g(k) = \frac{\gamma_1}{\beta_1^2+k^2}$,

Table 3. Three-nucleon ground state energies

$\kappa = 5$ fm^{-1}			$\kappa = 7$ fm^{-1}		
g(k) Alone	12224.24	MeV	g(k) Alone	24551.147	MeV
A1	12163.8619	MeV	B1	24466.013895	MeV
A2	11985.5036	MeV	B2	24211.91525	MeV
A3	10321.3117	MeV	B3	21703.43753	MeV
A4	9172.1828	MeV	B4	19790.39135	MeV
			B5	8922.4735	MeV
A5	12001.756	MeV	B6	4065.871	MeV
A6	6417.9	MeV	Beregi	3360.328	MeV
A7	4272.34	MeV	Mongan	157.4846	MeV

$h(k) = \frac{\rho_1}{\omega_1^2 + k^2}$ and $\gamma_1 = 10.290$ fm$^{-3/2}$, $\beta_1 = 2.0$ fm^{-1}, $\rho_1 = 22.355$, and $\omega_1 = 4.0$ fm^{-1} with a CBS at 400 MeV. In all cases we found a collapsed three-particle ground state. With increasing repulsion the binding energies E_t of potentials A_1 to A_4 decrease from 12225 MeV to 9172 MeV and from 12225 MeV to 4272 MeV for potentials A_5 to A_7 (with $h(0) \neq 0$). From Figs. 17 and 18 we see that the phase shifts decrease rapidly in the low energy region and that h(k) becomes large compared to g(k) for this set of potentials. The same conclusion applies to the set of potentials B, which has a CBS at $\kappa = 7$ fm^{-1} ($\sim$ 4000 MeV) instead of about 2000 MeV for set A. In that case the binding energies decrease from 24551 MeV to 4066 MeV. In agreement with the results of Sofianos et al[34] for rank one potentials we find that the collapse deepens when the continuum bound state moves to higher and more "unphysical" (at least in terms of the relevance of the potential concept) energies. In the presence of a CBS or resonance close to the real axis, in the two-particle interaction, the three-particle system is clearly not a low energy system any more. It should be noted that the node in the deuteron wave function of the interactions A and B is at a shorter distance for $\kappa = 7$ fm^{-1} than for $\kappa = 5$ fm^{-1} and that its minimum is also less deep in that case. The position of the node and the corresponding minimum of the wave function are often regarded as an indication of the strength of the nonlocality of an interaction. However in this case it is

clear that the interactions of set B produce much more deeply bound states in the three-particle system contrary to what physical intuition would lead one to expect.

For the singlet Mongan[41] interaction we found E_t = 157.5 MeV, which still can be regarded as a collapsed state. It is not surprising that this interaction leads to a much less deeply bound state, since in the first place it does not support a two-particle bound state and secondly it has a CBS at a much lower energy, 400 MeV corresponding to $\kappa = 3.106$ fm^{-1}. As we have seen the collapse deepens when κ becomes larger. For the Beregi[26] potential with $\kappa = 2.5$ fm^{-1} a much more deeply bound state was found with $E_t = 3360$ MeV indicating the importance of the two-body binding energy

We also checked that the potential B_4 of Sofianos et al[34] does indeed only have weakly bound three-particle states. The Hall-Post[40] lower bound was not quite conclusive in this case with a value of $E_L^3 = 184.5$ MeV, but this together with our search for zeros of the three-particle Fredholm determinant indicated the absence of deeply bound state for that interaction. This is the only exception to our general conclusion that a two-particle CBS is associated with three-particle ground state collapse. However as we indicated in section 2 there are good reasons to reject this potential.

VI.4 CONCLUDING REMARKS

We have constructed a large number of increasingly repulsive rank-two potentials, each set with a fixed deuteron wave function, low energy scattering properties and position of the CBS. The CBS states were fixed in the very high energy region at 2000 and 4000 MeV respectively.

The following conclusions can be drawn from the results.

1. A continuum bound state in two particle interactions of rank two which obey the conditions outlined in section VI.2 is in general associated with a collapse in the ground state of the three particle system.
2. If the continuum bound state moves to higher energies the three-particle system tends to become even more tightly bound.
3. Fixing the continuum bound state but increasing the repulsion in the two-particle interaction, results in a considerable decrease in the three-particle binding energy, without affecting the collapse.

Only one exception has been found by Sofianos et al[34], to the general rule that a CBS in the two particle interaction entails a collapse in the three particle ground state. However this separable potential of rank two does have two unacceptable features. In the first place, it does not obey the shape independent approximation, which is valid for all nucleon-nucleon interactions usually employed in the literature. Its breakdown has been shown to be associated with anomalous

results for the triton binding energy[33,35]. It also has an attractive part which is of shorter range than the repulsive part which is even more unphysical. It may be that it is this feature which is mainly responsible for the absence of a collapse. The first conclusion appears to be generally valid for non-local interactions of rank two obeying the shape independent approximation and having a repulsive form of shorter range than the attractive one.

This conclusion appears likely also to hold for any non-local interaction of rank higher than two. However, in most physical cases where non-local two-particle interactions occur they are only partly non-local. One usually has a local potential in the outer region and a non-local one of short range in the inner region. Since local potentials cannot produce a CBS, it may be that even for partly non-local interactions constructed in such a way that a CBS occurs at high energy, no collapse will occur in the three-particle bound state. This question is investigated in the next chapter.

CHAPTER VII

THE BINDING ENERGY OF THE THREE PARTICLE SYSTEM WITH PARTLY NON LOCAL INTERACTIONS

VII.1 INTRODUCTION

We have shown in the preceding chapter that separable potentials of rank 1 and rank 2 which support a continuum bound state in the two-particle system, produce unphysical deeply bound states in the three-particle system. With increasing two-body repulsion the collapsed state becomes less bound but it does not go down enough in order to become a physically acceptable ground state[38]. Ground state collapse in the three-particle system occured even in some cases where the two-body system was unbound as long as it did support a continuum bound state (CBS).

It has also been shown further[34] that the four particle system collapses even sooner than the three-particle one, i.e. when the resonance pole is still further away from the real axis, which is due to the higher density of the four particle system. One could argue that this trend can be extrapolated and therefore it would be interesting to do nuclear matter calculations with nonlocal interactions which support a CBS.

A remarkable and so far unexplained feature of our results is that the collapsed state becomes even more tightly bound when the CBS moves to higher energies. This indicates that

for non-local two-body interactions which support a CBS, or a resonance pole sufficiently close to the real axis, the three- and even more so the four-particle system is not a low energy system any more.

On the other hand in most physical cases where nonlocal two-particle interactions occur, they are only partly non-local. In practice one has a local potential in the outer region and a non-local one only in the inner region. Since local potentials alone can not produce a CBS, one is led to believe that even for partly non-local interactions constructed in such a way that a CBS occurs at high energy, no collapse will occur in the three-particle ground state. This intuitive argument motivated our investigation in the present chapter.

In section 2 we discuss the construction of partly non-local potentials with a short range nonlocality which produces a CBS in the total two-body interaction. In section 3 we present our results for the three-body bound state with these interactions. We devote section 4 to discussing these results and their implications.

VII.2 PARTLY NON-LOCAL AND TWO-BODY INTERACTIONS

As before all our two-body interactions have the same two-body binding energy $E_d = \frac{m}{\hbar^2} k_D^2 = 0.383$ Mev and low energy effective range parameters $a_0 = 11.65$ and $r_0 = 2.27$ fm. They also obey the relation

[1] $$A_s^2 = \frac{2k_D}{1-r_0 k_D}$$

between the asymptotic normalization constant A_s and a_0 and r_0, which was shown[35,36] to eliminate unphysical three-particle re-

sults and which is obeyed by all realistic potential models that include the OPEP tail. To work out our constraints we are employing the method of the cancellation of the Green's function derived by Krause and Mulligan[37] in coordinate space.

For an s-state with a CBS wave function $\phi(r)$ we have

$$[2] \qquad \frac{d^2u(r)}{dr^2} + \kappa^2 u(r) = \int_0^\infty dr'\ V(r,r')u(r')$$

with

$$[3]\ V(r,r') = V_0(r)\delta(r-r') + \lambda_1 \left(\frac{d^2}{dr^2} + \kappa^2\right)\phi(r)\left(\frac{d^2}{dr'^2} + \kappa^2\right)\phi(r') + \lambda_2 f(r) f(r')$$

Formally we can write the CBS wave function

$$u_\kappa(r) = \frac{1}{\left(\frac{d^2}{dr^2} + \kappa^2\right)} V_0(r)u_\kappa(r) + \lambda_1\phi(r) \int_0^r dr' u_\kappa(r')\left(\frac{d^2}{dr'^2} + \kappa^2\right)\phi(r')$$

$$[4] \qquad + \frac{\lambda_2}{\left(\frac{d^2}{dr^2} + \kappa^2\right)} f(r) \int_0^\infty f(r')u_\kappa(r')dr'$$

and since $u_\kappa(r) = \phi(r)$ we obtain the condition for a CBS at

$$k^2 = \kappa^2$$

$$[5a] \qquad \lambda_1 \int_0^\infty dr'\phi(r')\left[\frac{d^2}{dr^2} + \kappa^2\right]\phi(r') = 1$$

and

$$V_0(r)\phi(r) + \lambda_2 f(r) \int_0^\infty f(r')\phi(r')dr' = 0$$

or

[5b] $$f(r) = - C_0^{-1} V_0(r)\phi(r)$$

with

[5c] $$C_0^2 = - \lambda_2 \int_0^\infty V_0(r)\phi^2(r)dr \ .$$

If conditions [5a,c] are fulfilled, the Green's function $(\frac{1}{(\frac{d^2}{dr^2} + \kappa^2)}$ cancels out at $k^2 = \kappa^2$ and the function $\phi(r)$ which decreases exponentially like a bound state wave function is a solution of the Schrödinger equation at that energy. We therefore have a CBS at the energy $E_\kappa = \frac{m}{\hbar^2} \kappa^2$. The parameters λ_i can assume the values

$$\lambda_i = \pm 1 \qquad i = 1 \cdots N$$

For our potentials we have $N = 2$, $\lambda_1 = \lambda_2 = -1$. The choice of $\phi(r)$ is entirely arbitrary provided it obeys the standard bound state conditions $\phi(0) = 0$ and $\phi(\infty) = 0$. In our calculations we have taken

[6] $$\phi(r) = \sum_{i=1}^{2} C_i (e^{-\beta_i r} - e^{-\gamma_i r})$$

and the Malfliet-Tjon potential MTV[42]

[7] $$V_0(r) = \sum_{i=1}^{2} \frac{m}{\hbar^2} V_i \frac{e^{-\alpha_i r}}{r}$$

which has also been employed by Haftel[43] in his investigation of the model triton with partly nonlocal potentials. The CBS wave function $\phi(r)$ at energy E_κ should be of short range, in order to ensure also a short ranged nonlocality. In this work we have selected cases such that $\{\beta_i,\gamma_i\} > \max\{\alpha_i\}$. To simplify our calculations and in order to have a realistic deuteron wave function, i.e. a UPA to the full interaction which is free of any CBS contribution, we impose the following additional conditions on our two-body interaction. Assuming that

$$[8] \qquad \frac{d^2}{dr^2} u_d^{(0)}(r) - k_D^2 u_d^{(0)}(r) = V_0(r) u_d^{(0)}(r)$$

we require that

$$[9a] \qquad \int_0^\infty dr\; g(r) u_d^{(0)}(r) = 0$$

$$[9b] \qquad \int_0^\infty dr\; f(r) u_d^{(0)}(r) = 0$$

with

$$[9c] \qquad g(r) \equiv \left(\frac{d^2}{dr^2} + \kappa^2\right)\phi(r) .$$

Therefore equations [5a,c] and [9a,c] determined different sets of values for the parameters.

$$\{C_i,\beta_i,\gamma_i\} \quad , \quad i = 1,2 .$$

This guarantees that the bound state wave function $u_d(r)$ of the total interaction eq. (3) is identical to the bound state wave function $u_d^{(0)}(r)$ of the local potential alone, i.e.

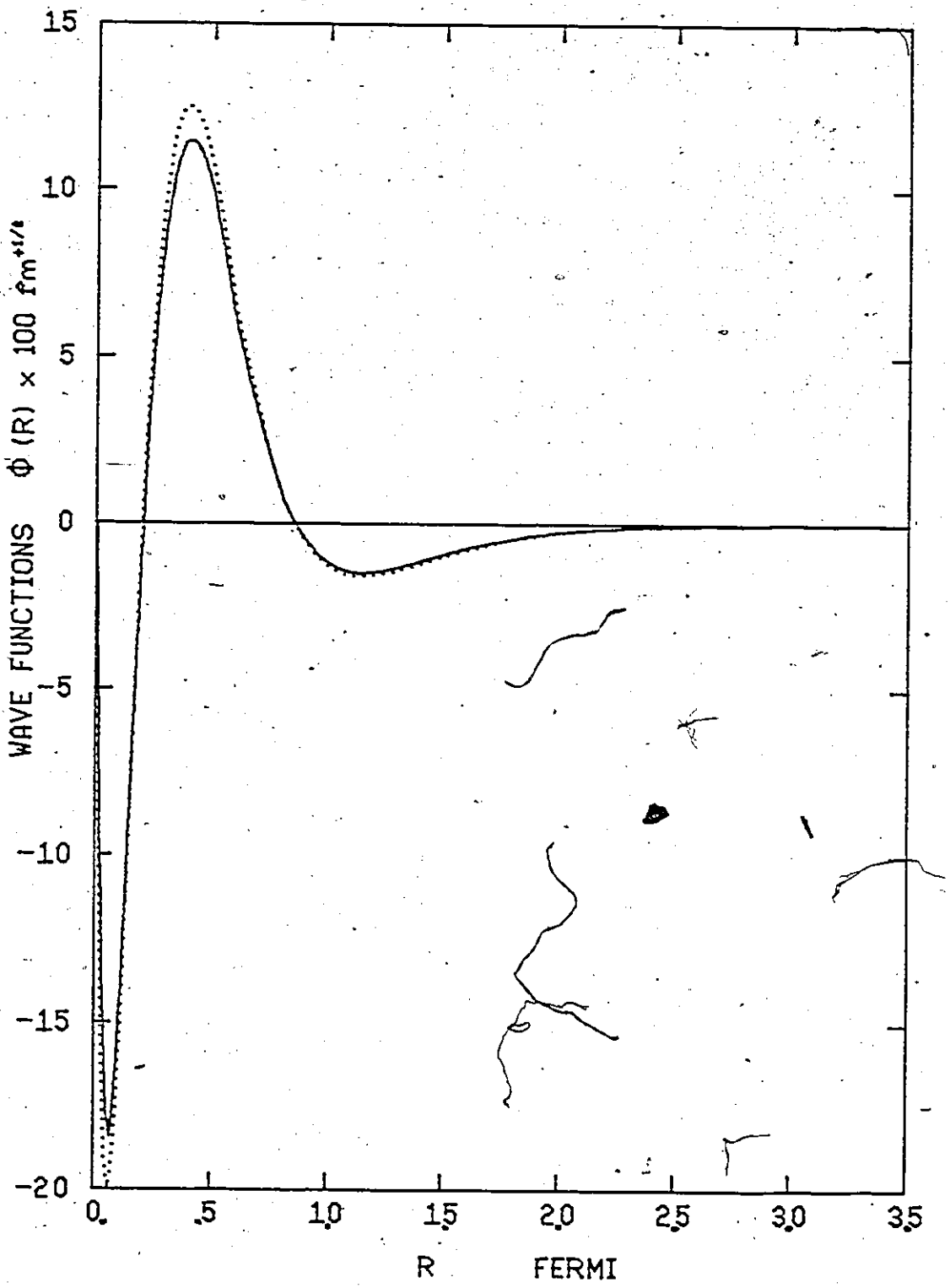


Figure 22. Wave functions $\phi(r)$ as a function of r. Solid line with a CBS at k = 5 fm^{-1}, dotted line with a CBS at k = 7 fm^{-1}.

$u_d^{(0)}(r) = u_d(r)$. It should be noted that our choice of $\phi(r)$, eq. [6] guarantees (because of $\phi(0) = 0$) that the Fourier transform $g(k)$ of $\tilde{g}(r) = (\frac{d^2}{dr^2} + \kappa^2)\phi(r)$ has the property $\tilde{g}(\kappa) = 0$, which is necessary for the presence of a CBS in the total interaction as in the case of rank-one separable potentials[38].

We should finally note that all partly nonlocal potentials which we have constructed have a nodeless ground state wave function although Bagchi et al.[44] have argued that for positive energies a CBS implies a node in the scattering wave function at all energies. We have constructed two classes of potentials, one with a CBS at $\kappa = 5$ and the other with $\kappa = 7$ fm^{-1}. Their parameters are given in Table 4.

III.3 THREE-PARTICLE BINDING ENERGIES

The binding energies of the three particle bound states of the model triton have been calculated by means of the Padé method. A computer code was used[45] which yields a detailed solution for this problem. For more details on this subject we refer to the work of D. Ellis[46]. Fig. 22 shows the CBS wave function $\phi(r)$. It has a very short range, which is necessary to produce also a short ranged non-locality. In fig. 23 we plot the phase-shifts of the total interactions. Note that they jump by π at κ=5 and κ=7 fm^{-1} and they all are identical at low energies as they should be. For comparison we show the phase shifts of the non-local interactions acting alone in Fig. 24. In Table 5, we give the binding energies of the nonlocal parts of the interaction acting alone as well as the binding energies of the partly nonlocal one. As expected the nonlocal parts by themselves produce a three-particle ground state collapse due to the presence of the

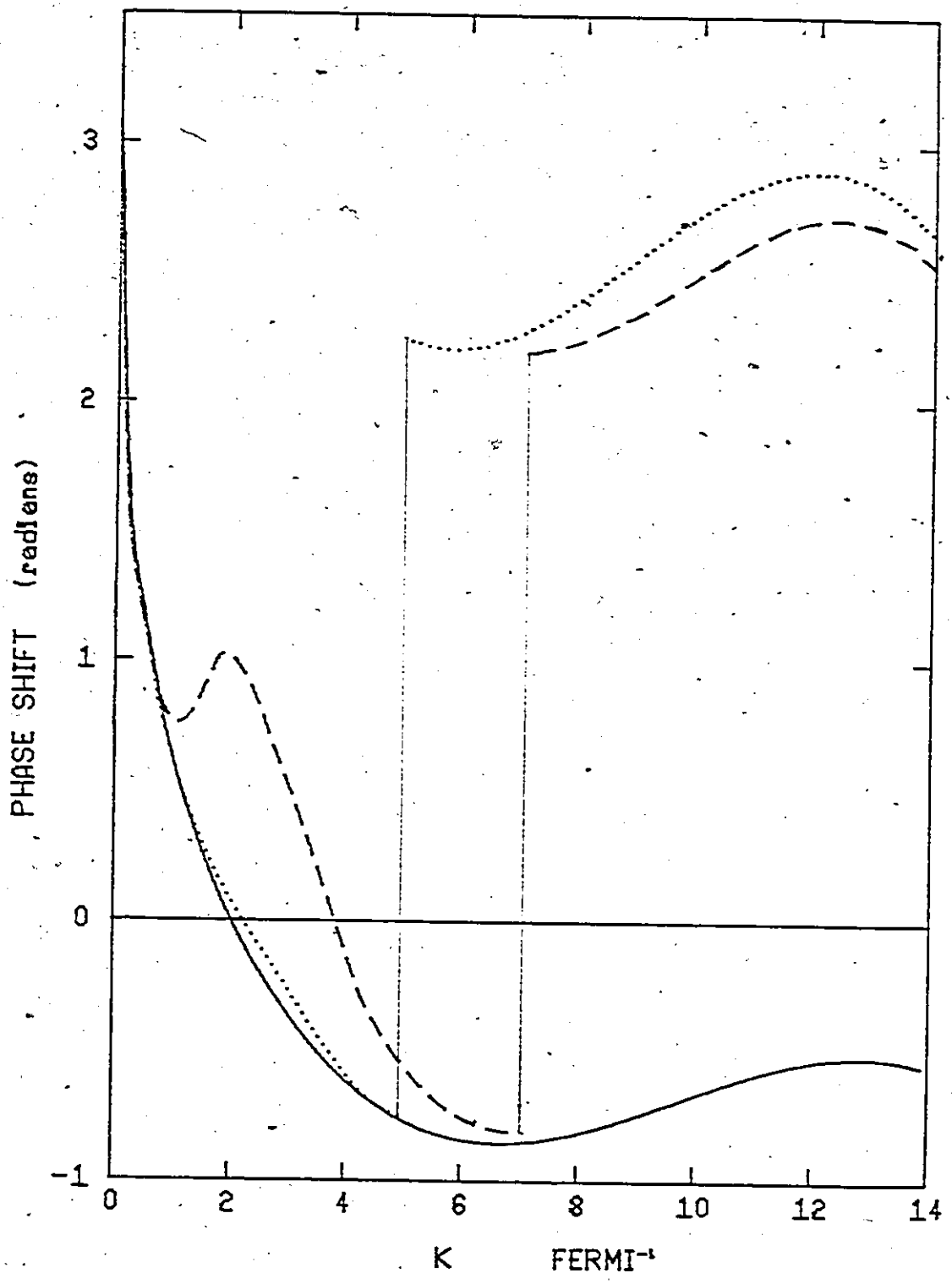


Fig. 23. Two-body phase shifts as a function of k. Solid line is the phase shift for the local potential acting alone, dotted and dashed lines for the total interaction with a CBS at $k = 5$ fm^{-1} and at $k = 7$ fm^{-1} respectively.

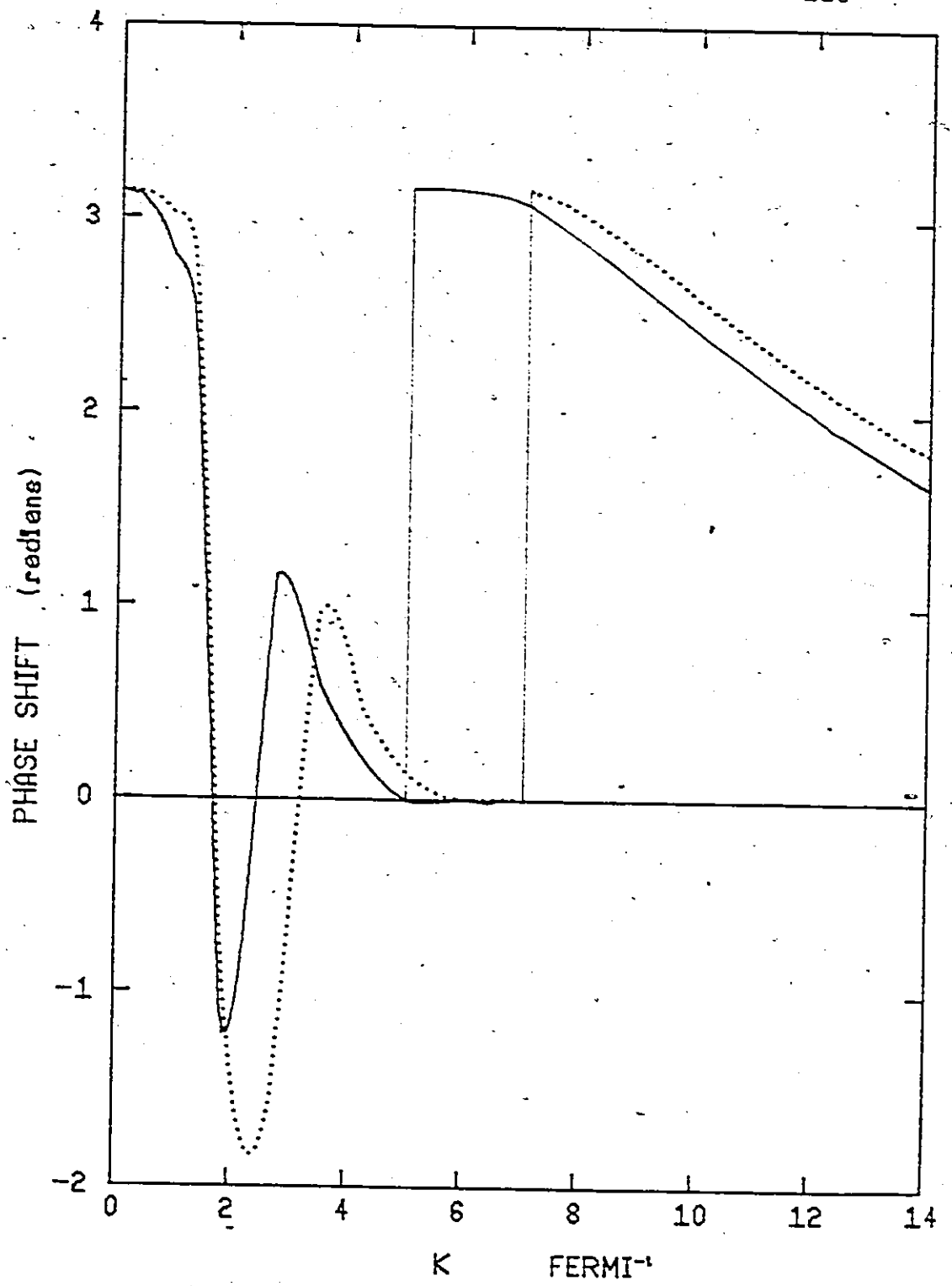


Fig. 24. Two-body phase shifts as a function of k for the two non local potentials (two rank) acting alone. Solid line for a CBS at k = 5, dotted line for a CBS at k = 7.

CBS. However interestingly and maybe somewhat unexpectedly the complete partly nonlocal interactions as a result of their locality in the outer region produce no collapse at all. In fact in both cases the three particle binding energy of the local part acting alone is hardly changed by the addition of the short range nonlocality. The actual variation in E_t is about 0.3 Mev and this is the same result as the one usually obtained, if short range modifications of a given local reference potential are generated by means of unitary transformations. We should note however that usually in such transformations the deuteron wave function is also modified at short distances whereas in our case the deuteron wave function remains fixed. To confirm the fact that our partly nonlocal interactions do not produce three-particle collapse we also calculated the Hall-Post lower bounds which are also given in table 5.

It is rather interesting to note that for our partly nonlocal interactions E_t also increases but only slightly with increasing κ, whereas the lower bound show a very rapid increase with increasing κ (from 5 to 7 fm^{-1}). It is therefore possible that with increasing κ the three particle ground state would collapse even for partly nonlocal potentials, with a very repulsive local core and a very short ranged nonlocality and one should investigate this possibility. On the other hand our results disprove the notion that a CBS at high energies in the two particle interaction necessarily entails a three particle ground state

Table 4

Parameters for the partly nonlocal interactions with CBS at $\kappa = 5$ and $\kappa = 7$ fm^{-1}

	$C_1 = -16.26$	$\beta_1 = 6.233$	$\gamma_1 = 4.658$
$\kappa = 5$	$C_2 = 4.714$	$\beta_2 = 10.45$	$\gamma_2 = 3.556$
	$\lambda_1 = \lambda_2 = -1$,	$C_0 = 0.9642$	
	$\alpha_1 = 1.55$	$V_1 = 674.32/41.47$	
	$\alpha_2 = 3.11$	$V_2 = 1448.44/41.47$	
$\kappa = 7$	$C_1 = -16.54$	$\beta_1 = 4.635$	$\gamma_1 = 4.635$
	$C_2 = 4.758$	$\beta_2 = 10.65$	$\gamma_2 = 3.518$
	$\lambda_1 = \lambda_2 = -1$,	$C_0 = 1.0522$	

Table 5

Binding energies for the two and three particle systems

		Local alone	Rank 1	Rank 2	Total	Lower Bound
$\kappa = 5$	E_D(Mev)	0.383			0.383	8.28
	E_T(Mev)	7.65	10287	10389	7.75	16.56
$\kappa = 7$	E_D(Mev)	0.383			0.383	50.7
	E_T(Mev)	7.65	12122	12240	8.0	101.4

collapse. Such a collapse is clearly dependent on the off-shell behaviour of the two-particle interaction and not only on the presence of a CBS or a narrow resonance at high energies.

4. Conclusions

We have constructed two partly nonlocal interactions with the following properties:

i) Very short range nonlocality consisting of a rank two potential which acts in the core region of a local potential (which has a strongly repulsive core).

ii) The potential is purely local in the outer region. This conforms with the characteristics of many composite particle interactions and with those of the nucleon-nucleon interaction.

iii) A CBS occurs in the total interactions at high energies (in the region where the concept of a potential already loses its meaning in nucleon-nucleon scattering).

iv) The nonlocal part of the partly nonlocal potential by itself produces three particle ground state collapse.

From the results of our calculations of E_T (the binding energy of the ground state of the three-particle system) we conclude that 1) No three-particle ground state collapse occurs for interactions having properties (i) to (iv) (we do not exclude the possibility that with increasing CBS energy the three particle system can collapse even in such cases). However our results serve as counter-examples against the tempting conclusion (from

previous results) that a CBS in the two particle interaction always produces a three particle collapse in the ground state. The requirement that the two particle interaction should not produce a CBS (or narrow resonance) even at very high scattering energies can therefore not be regarded as a constraint which must be imposed on any (even composite) two-particle interaction.

2) Our results identify at least one of the conditions by which three particle ground state collapse can be avoided in spite of the presence of a two particle CBS. However it has not yet been established whether a local outer part of the interaction suffices to avoid a collapse. The repulsive core may also be necessary for this purpose. A negative aspect of our present results is that it becomes even more difficult:

3) To find an explanation for the collapse of the three particle system.

4) To identify those properties of the two particle interaction which insure such a collapse.

The presence of a two particle CBS is only a necessary but not a sufficient condition for the occurrence of the collapse. However, it is clear that for those two-particle interactions which are the physically most interesting ones, i.e. those having a local tail and a repulsive core combined with a short range nonlocality the occurrence of a CBS or a narrow resonance can be quite harmless for the three-particle ground state and presumably therefore also for $N > 3$.

CHAPTER VIII

SUMMARY

This work has dealt with two basic subjects. In the first, we have investigated bound states embedded into continuum, which arise in (d,p) stripping to unbound states. This is done by considering the kinematics of this process which is normally referred to as its off-shell behaviour. The second subject deals with the ground state collapse of the three body system (the model triton). This collapse appears for non-local interactions which support a continuum bound state at high energies. We have discussed our results at length and presented our conclusions consecutively in each chapter. Here we shall summarize the main features again.

Inspection of the cross-sections of a neutron scattered elastically on N^{15} and that of a (d,p) reaction on the same target reveals striking similarities. First, almost all energy levels appear in both spectra, secondly almost all resonances appearing in the (d,p) spectrum are displaced by the same energy shift in comparison to the (n,n) spectrum, and thirdly resonance shape changes appear in (d,p). These observations led Lipperheide[6] to suggest that since the

nucleus N^{16} is unbound to neutron decay the similarities between the (n,n) and (d,p) cross-sections can be explained by considering the stripped neutron to be initially caught by the target and subsequently scattered by it. In fact since the stripped nucleon moves off the energy shell it becomes clear that the missing link between the (d,p) and the (n,n) process is the off-shell scattering amplitude.

Lipperheide and Möhring[8] used this idea to interpret some isolated resonances in N^{16}. We have extended this formalism giving an exact expression of the off-shell scattering amplitude which is able to account for more levels of the resonant state. By computing the $N^{15}(d,p)N^{16}$ reaction leading to $J^{\pi} = 1^-, 1^+$ and 1^- states at the excitation energies 3.519, 4.318, 4.398 respectively we have found reasonable agreement with experiment. In particular we have found that the change of shape and the energy shift are related to interference with the off-shell background, which in some cases can completely eliminate a resonance from the (d,p) spectrum. Such a case for instance is the resonance $J^{\pi} = 1^-$ at $E_x = 4.398$ Mev.

By computing interference and background effects exactly our formalism has the advantage that it can be used for comparison between on-shell and off-shell background and therefore one can make predictions about a spin and parity of the state under consideration. We have used this technique for

example for the resonance $J^{\pi} = 1^{-}$ at $E_x = 2.732$ Mev which we have shown to be a mixture of $\ell=0$ and $\ell=2$.

We have taken care of competing processes by developing a compact formalism in terms of the off-shell T-matrix using the R-matrix theory. We have illustrated the usefulness of this approach by comparing it to a number of exact calculations. Our conclusion is that whereas the on-shell T-matrix can be expanded around a single pole in the case of an isolated resonance, such an expansion is not necessarily adequate for the off-shell T-matrix. The latter depends on the off-shell distance s and an expansion in the vicinity of an off-shell point provides for a better approximation of the physical situation. We may finally note that our formalism can be used for (d,p) and (d,n) reactions by using an appropriate choice of incoming and outgoing waves. It can also be used for any kind of potentials, for instance non-local separable or not. In fact it will be interesting to use a non local non separable potential to investigate the additional off-shell behavior which will arise from the non locality of the potential.

As far as the triton is concerned, it is not so much the off-shell behavior of the potentials which brings about the ground state collapse but their non local form. We have constructed purely non-local potentials which show a collapse and completely equivalent partly non-local which do not.

We can not therefore show conclusively the conditions which entail a collapse. We might summarize our conclusions as follows:

1) For the rank-two separable interactions a continuum bound state entails a collapse in the three body system.

2) The binding becomes stronger when the continuum bound state is moved to high energies.

3) The binding decreases when the repulsive part of the interactions becomes stronger.

4) Our results can be generalized to interactions of higher ranks.

5) For partly non-local interactions which consist of a local part in the outer region and a rank-two form, with a CBS, in the inner region, no collapse occurs despite the fact that the non-local part in itself produces a collapse.

6) We can not conclusively prove at this stage that a collapse will not occur for partly non-local potentials when the CBS is moved to very high energies.

7) The presence of the CBS is a necessary but not a sufficient condition for the occurrence of the collapse.

8) For the physically more interesting interactions, those that have a local tail and a repulsive core combined with a short range non-locality the occurrence of a CBS is quite harmless.

9) Finally to find an explanation for the collapse and to identify the properties of interactions which insure the collapse the following suggestions may be made:

a) Use non-local forms other than Yamaguchi.

b) Use different local forms; this might make the potential less physical but it will be easier to draw comparisons.

c) Investigate the non-local Wronskian for the appearance of spurious states.

d) Relate these potentials to an equivalent local form.

ΤΕΛΟΣ

Die Frage, ob dem menschlichen Denken gegenständliche Wahrheit zukomme, ist keine Frage der Theorie, sondern eine praktische Frage. In der Praxis muss der Mensch die Wahrheit, i.e. Wirklichkeit und Macht, Diesseitigkeit seines Denkens beweisen. Der Streit über die Wirklichkeit oder Nichtwirklichkeit des Denkens, das von der Praxis isoliert ist, ist eine reine scholastische Frage.

(K. Marx, Thesen über Feuerbach)

The question whether objective truth can be attributed to human thinking is not a theoretical question but a practical one. In practice man must prove the truth, that is, reality and power, thissidenis (diesseitigkeit) of his thinking. The dispute over the reality or non reality of thinking, which is isolated from practice, is a purely scholastic question.

(K. Marx, Theses on Feuerbach)

APPENDIX A

THE STRIPPING AMPLITUDE

The differential cross-section for the reaction A(d,p)B is known to be [13]

$$[1] \quad \left(\frac{d\sigma}{d\Omega_p dE_p}\right) = \frac{1}{(2\pi\hbar^2)^2} (u_{dA})^{-1} p_p \mu_{pB} |T_{fi}|^2 \delta(E_f - E_i) .$$

Using (II-22) and the energy conservation relation

$$[2] \quad (E_f - E_i) = W_B + E_p + \varepsilon_d - W_A - E_d$$

we get

$$[3] \quad \left(\frac{d\sigma}{d\Omega_p dE_p}\right) = \frac{1}{(2\pi\hbar^2)^2} (u_{dA})^{-1} p_p \mu_{pB} |G(\tfrac{\vec{k}_d}{2} - \vec{k}_p)|^2 \sum_B \int |T_{B,An}|^2 \delta(E_f - E_i) .$$

Now noting that

$$[4] \quad \sigma_{tot} = \left(\frac{2\pi}{\hbar}\right) (u_{nA})^{-1} \sum_B \int |T_{B,An}|^2 \delta(E_n - (W_B - W_A))$$

and assuming that only one mode of decay dominates the reaction, we can set

$$[5] \quad |t_c(k_n, k_n, E_n)|^2 = \sum_B \int |T_{B,An}|^2 \delta(E_n - (W_B - W_A))$$

$$= \frac{2\hbar^2}{\mu_{nA}} \operatorname{Im} f_c(\vec{k}_n, \vec{k}_n, E_n)$$

and write for the differential cross-section

$$[6] \quad \left(\frac{d\sigma}{d\Omega_p dE_p}\right) = \frac{2}{(2\pi\hbar)^2} \left(\frac{\mu_{dA}\mu_{pB}}{\mu_{nA}}\right) \frac{k_p}{k_d} |G(\tfrac{\vec{k}_d}{2} - \vec{k}_p)|^2 \operatorname{Im} f_c(\vec{k}_n, \vec{k}_n, E_n)$$

where μ stands for reduced mass and, as before $s = \frac{\hbar^2}{2m_n} k_n^2 - E_n \neq 0$.

Using [II-48,54] and [II-44] we can cast A]6] in the form

$$[7] \quad \left(\frac{d\sigma}{d\Omega_p dE_p}\right)_{lab} = \frac{1.0}{(2\pi)^3}\left(\frac{\mu_{dA}\mu_{pB}}{m_n \mu_{nA}}\right)\left(\frac{2n(n+1)}{(1-n)^2}\right)\left(\frac{1}{s} - \frac{1}{s+(n^2-1)\varepsilon_d}\right)^2 \sqrt{\varepsilon_d}\,\sqrt{\frac{E_p E_n}{E_d}} \sum_{\ell} (2\ell+1)\left|f_c(k,k_n,E_n)\right|^2 \quad ; \quad k_n = \sqrt{\frac{2m_n}{h}(E_n+s)}$$

which shows the differential cross-section to be a function of the off-shell distance s and therefore a function of the stripping angle θ. Some other useful relations may also be noted at once by considering the vector diagrams describing the coordinates used in the (d,p) theory.

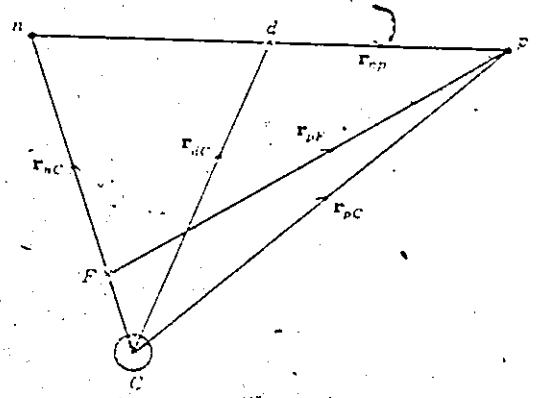


Figure A.1 Vector diagrams describing the coordinates used in the theory of the (d,p) stripping reaction.

Recalling [II.22]

[8] $$T_{fi} = \langle \phi_f | V_{np} | \phi_i \rangle$$

and using

[9] $$|\phi_f\rangle = |\phi_B, \vec{k}_p\rangle \quad , \quad \phi_i = |\phi_d, \vec{k}_d\rangle$$

we get

[10] $$T_{fi} = \int d\vec{r}_{nc} d\vec{r}_{np} \phi_B^*(\vec{r}_{nc}) e^{-i\vec{k}_p \cdot \vec{r}_{pf}} V_{pn} \phi_d(\vec{r}_{np}) e^{i\vec{k}_d \cdot \vec{r}_{dc}} .$$

From the vector diagrams (fig. A1) we get

[11] $$\vec{r}_{pf} = \vec{r}_{np} + \vec{r}_{nc} \frac{m_c}{m_c+m_p} \quad , \quad \frac{m_c}{m_c+m_p} = \frac{\mu_{cp}}{m_n}$$

$$\vec{r}_{dc} = \vec{r}_{nc} + \vec{r}_{pn} \frac{m_p}{m_p+m_n} \quad , \quad \frac{m_p}{m_p+m_n} = \frac{\mu_{pn}}{m_n} .$$

For simplicity of presentation we set $\vec{r}_{np} = \vec{\rho}$ and $\vec{r}_{nc} = \vec{r}$, then Eq. [10] becomes

[12] $$T_{fi} = \int d\vec{r} \phi_B(\vec{r}) e^{i\vec{r} \cdot (\vec{k}_d - \vec{k}_p \frac{\mu_{cn}}{m_n})} \cdot \int d\vec{\rho} \phi_d(\vec{\rho}) V_{np} e^{-i\vec{\rho} \cdot (\vec{k}_p - \vec{k}_d \frac{\mu_{pn}}{m_n})} .$$

Now if we integrate the Schrödinger equation for the internal motion of the deuteron we get for the second integral

[13] $$I = \frac{\hbar^2}{2\mu_{pn}} \left[\left(k_p - k_d \frac{\mu_{pn}}{m_n}\right)^2 + \gamma^2 \right] \int \phi_d(\vec{\rho}) e^{-i\vec{\rho} \cdot (\vec{k}_p - \vec{k}_d \frac{\mu_{pn}}{m_n})} d\vec{\rho}$$

The factor

$$A = \frac{\hbar^2}{2\mu_{pn}} \left[\left(k_p - k_d \frac{\mu_{pn}}{m_n}\right)^2 + \gamma^2\right]$$

can be written

$$[14] \quad A = \frac{\hbar^2}{2\mu_{pn}} k_p^2 + \frac{\hbar^2}{2m_n} k_d^2 \left(\frac{\mu_{pn}}{m_n}\right) + \frac{\hbar^2 k_d^2}{2m_n} + \frac{\hbar^2 k_p^2}{2m_n} - \frac{2k_p k_d}{2m_n} + \varepsilon_d - \frac{\hbar^2 k_p^2}{2m_n} - \frac{\hbar^2 k_d^2}{2m_n}$$

or

$$[15] \quad A = \hbar^2 k_p^2 \left(\frac{1}{2\mu_{pn}} - \frac{1}{2m_n}\right) + \hbar^2 k_d^2 \left(\frac{\mu_{pn}}{2m_n^2} - \frac{1}{2m_n}\right) + \frac{(\hbar k_d - \hbar k_p)^2}{2m_n} + \varepsilon_d$$

$$[16] \quad A = \frac{p_p^2}{2m_p} - \frac{p_d^2}{2(m_p+m_n)} + \frac{(p_d - p_p)^2}{2m_n} + \varepsilon_d.$$

But from the conservation of energy we have

$$[17] \quad \frac{p_p^2}{2(m_p+m_n)} = \varepsilon_d + E_n + \frac{p_p^2}{2m_p}$$

where E_n is the energy of the transferred neutron. Noting that $\vec{P}_d - \vec{P}_p = \vec{P}_n$, the momentum of the transferred neutron, we have from [16] and [17]

$$[18] \quad A = \frac{p_n^2}{2m_n} - E_n \quad , \quad \text{or} \quad A = s.$$

APPENDIX B

A POSSIBLE EXPANSION OF THE COMPOUND STATES

The solutions of Eq. [V-12] can be found either by numerical integration or by introducing a basic set of states $|p\rangle$ corresponding to a model Hamiltonian H_d which is diagonal in the channel indices. In the present case we have

[1] $$H = H_{diag} + H_{coup}$$

[2] $$H_{diag} = H_I + T_c + \langle c|V_{diag}|c\rangle$$

and

[3] $$H_{coup} = \langle c|V_{coup}|c'\rangle = V_c \ .$$

The states $|p\rangle$ are the eigenstates of the equation

[4] $$(H_{diag} + L_b - E_p)|p\rangle = \vec{0}$$

and their radial parts $\langle p|c) = v_{pc}(r_c)/r_c$ are the solutions of the uncoupled equations;

[5] $$(T_c+V_c(r)-(E_p-\varepsilon_c)v_{pc}(r) \quad , \quad T=-\frac{\hbar^2}{2m_c}\left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2}\right)$$

satisfying the boundary condition

[6] $$a_c \frac{d}{dr_c} v_{pc}(r_c) = b_c v_{pc}(r_c)/r_c = a_c \ .$$

They have zero components in all but one channel, so that each one of them is a single particle state only in one of the

channels. The compound states

$$[7] \qquad |\lambda\rangle = \sum_c \frac{1}{r_c} u_{\lambda c}(r_c)\,|c)$$

can be expanded in terms of this complete set viz:

$$[8] \qquad |\lambda\rangle = \sum_p M_{\lambda p}|p\rangle \ .$$

or

$$u_{\lambda c}(r_c) = \sum_p M_{\lambda p} v_{pc}(r) \ .$$

Substituting Eq. [8] into Eq.[V-12] we obtain the matrix equation

$$[9] \qquad M(\tilde{E} + V) = EM$$

where $\tilde{E}$ and E are diagonal matrices with elements E_p and E_λ respectively and V has matrix elements

$$[10] \qquad V_{p'p} = \langle p|V_{coup}|p'\rangle = \int_0^\infty v_{pc}(r)V_{cc'}(r)v_{p'c'}(r)dr.$$

Diagonalizing Eq. [9] one finds the eigenvalues E_λ and the transformations coefficients $M_{p\lambda}$ from which the reduced widths $\gamma_{\lambda c}$ can be found from Eq. [V19] and the R-matrix formed.

APPENDIX C

EXACT REPRESENTATIONS OF THE CBS CONDITIONS

In this appendix we give the constraints obeyed by g(k) and h(k) in analytical form, for the special case of eq. (29).

The condition

$$D^{\dagger}(ik_D) = D_1^{\dagger}(ik_D) = 1 - \frac{2}{\pi}\int_0^{\infty} \frac{g^2(p)p^2dp}{p^2 + k_D^2}$$

can be written as

$$[1] \qquad \sum_i \sum_j \frac{\gamma_i\gamma_j}{(\beta_i+k_D)(\beta_j+k_D)(\beta_i+\beta_j)} = 1$$

while the asymptotic normalization condition for the "deuteron" wave function

$$\int_0^{\infty} dr\, u_d^2(r) = 1 \quad , \quad u_d(r) \underset{\infty}{\sim} A_s e^{-k_D r}$$

becomes

$$[2] \sum_{ij} \sum \frac{\gamma_i\gamma_j}{(\beta_i^2-k_D^2)(\beta_j^2-k_D^2)} \left[1-A_s^2\left(\frac{1}{2k_D} - \frac{1}{\beta_i+k_D} - \frac{1}{\beta_j+k_D} + \frac{1}{\beta_i+\beta_j}\right)\right] = 0$$

The CBS conditions can be given ($\mathrm{Im}D_1^{\dagger}(\kappa) = 0$) as

$$[3] \qquad g(\kappa) = \sum_i \frac{\gamma_i}{\beta_i^2+\kappa^2} = 0$$

and

$$[4] \qquad \sum_i \sum_j \frac{\gamma_i}{\beta_i^2+\kappa^2} \frac{\gamma_j}{\beta_i+\beta_j} = -1$$

which is the analytical form of eq. (20) or (21).

It is easily obtained from eq. (20) if we realize that eq. (29) implies that

$$g(r) = \sum_i \gamma_i e^{-\beta_i r}$$

and therefore (from eq. (19)) that

$$\phi(r) = \sum_i \alpha_i e^{-\beta_i r} = \sum_i \frac{\gamma_i}{\beta_i^2+\kappa^2} e^{-\beta_i r} .$$

(Note that $\phi(0) = 0$ is equivalent to $g(\kappa) = 0$, i.e. to eq. (A3)). The condition $\mathrm{Re}D_1^{\dagger}(\kappa) = 0$ can be analytically expressed as

$$\sum_i \sum_j \frac{\beta_i\beta_j - \kappa^2}{(\beta_i^2+\kappa^2)(\beta_j^2+\kappa^2)(\beta_i+\beta_j)} \gamma_i\gamma_j = 1 . \qquad \text{(A5)}$$

It is easily proved that eqs. (A3) and (A4) are exactly equivalent to eqs. (A3) and (A5), i.e. to $D_1^{\dagger}(\kappa) = 0$.

The form factor $h(k)$ is constrained to obey the orthogonality conditions eqs. (6) and (27) which are given explicitly by eqs. (A6) and (A7) respectively

$$\sum_{ij} \frac{\gamma_i\rho_j}{(\beta_i+k_D)(\omega_j+k_D)(\beta_i+\omega_j)} = 0 \qquad \text{(A6)}$$

and

$$\sum_{ij} \frac{\beta_i\omega_j - \kappa^2}{(\beta_i^2+\kappa^2)(\omega_j^2+\kappa^2)(\beta_i+\omega_j)} \gamma_i\rho_j = 0 . \qquad \text{(A7)}$$

The additional constraint $h(0) = 0$ imposed on some of the potentials gives

$$\sum_{i=1}^{N} \frac{\rho_i}{\omega_i^2} = 0 \qquad \text{(A8)}$$

With N = 4 we can choose ω_1 to ω_4 together with ρ_4 arbitrarily, and then determine ρ_1, ρ_2 and ρ_3 from eqs. (A6) to (A8). By increasing ρ_4 we make our interactions more repulsive.

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