LEIBNIZ'S LAW

AND

IDENTITY

LEIBNIZ'S LAW AND IDENTITY

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SCOPE AND CONTENTS:

In this essay I consider various alleged exceptions to the principle of the indiscernibility of identicals -- Leibniz's Law. There are two major difficulties. First, the apparent antinomy that arises when Leibniz's rule combines with the modalities. I argue that there are a number of ways of dealing with this problem and we are not therefore obliged to abandon or modify Leibniz's rule. Second, the unacceptable inference which results when Leibniz's rule is applied in contexts expressing mental attitudes. Here, I show how Leibniz's rule and intentional attitudes combine in a perfectly acceptable way.

I also deal with a number of other minor objections to this rule, from the current literature on the topic, all of which I hope to show present no difficulties. In fine, despite the many apparent counter-examples considered, I hope to show Leibniz's Law, which permits the unrestricted interchange of the terms of an identity sentence, has not been falsified.

CONTENTS

INTRODUCTION

This essay treats some of the problems claimed to be encountered when using Leibniz's Law as a principle of identity.

The formal properties of identity are usually taken to include reflexivity, transitivity, symmetry :-

(i) $(x)(x = x)$

(ii) $(x)(y)(z)(x = y \,\&\, y = z \supset x = z)$

(iii) $(x)(y)(x = y \supset y = x)$

It is generally agreed that at least these properties are integral to the purport of '='. However the notion of identity is not sufficiently characterised by the above conditions. There are many other equivalence relations which are also satisfied by such conditions; for example, congruence, isomorphism, similarity, consanguinity etc. Clearly, what is required to sufficiently characterise identity is a further necessary condition that will distinguish it from all other equivalence relations: a further requirement which will fix identity uniquely. The claim under consideration in this paper is that the principle of "The Indiscernibility of Identicals",[1] attributed to Leibniz,[2] embodies such a condition.

This principle states that "for every x and for every y, if x is identical with y then whatever is true of x is true of y and whatever is true of y is true of x". Thus the proposition that Cicero denounced Cataline is distinguishable from the proposition that Tully denounced Cataline, but they have the same truth-value, because Cicero is identical with Tully. Now from this principle it follows that "given a true statement of identity, one of the two terms may be substituted for the other in any true statement and the result will be true",[3] co-referential expressions are interchangeable *salva veritate* in all contexts. The above rule, the rule for the universal intersubstitutivity of co-referential expressions, is entailed by the principle of the indiscernibility of identicals. This then is the purport of Leibniz's Law.

A formula for the above may be expressed by the following schema :-

$$(x)(y)(x = y . \supset . Fx \equiv Fy)$$

where 'F' is a schematic predicate letter such that 'Fx' can be replaced by any sentential context containing a free occurrence of 'x' and 'Fy' is the result of replacing 'x' in one , some or all of its occurrences in that context by 'y'.

At first blush Leibniz's Law seems to be *a priori* and incontrovertible. How if $a = b$ could there be something true of the object *a* which is untrue of the object *b* ? After all they are the same object. To emphasise the point here, unlike any other equivalence relation, in an identity statement we make a reference to one and only one object. Thus to say that *a* is identical with *b* is to say, in effect, they are one and the same. How then could anything be true of *a* and not be true of *b* ?

There are other compelling reasons why we should be anxious to preserve Leibniz's Law as an "analysis" of identity.

First, as we have seen, this principle, or at least some clear principle, is required to mark-off identity from all other equivalence relations. Leibniz's Law permits the universal interchange of co-referential expressions, distinguishing the identity relation in a way in which transitivity, symmetry and reflexivity (all shared by congruence etc.) do not. Suppose we were to give up Leibniz's Law, in its unqualified form, as defining the identity sign. Clearly any alternative analysis of '=' must at least differentiate the identity relation from every other equivalence relation.

Second, if Leibniz's Law is dropped as a condition of identity, we need to find another principle of comparable generality to justify intersubstitution. I hope to show there isn't one. Let me put the matter another way-- suppose we find that Leibniz's Law, in its unqualified form, is so general that by substituting in accordance with this rule, we are led to countenance unacceptable, or rather, invalid inferences. Consequently we give up Leibniz's Law. Now any alternative analysis of '=' must in some way justify intersubstitution of co-referential terms. Moreover such a rule must be atleast sufficiently general to justify every valid substitution inference given under Leibniz's Law, but not so general that we are led to countenance any invalid inferences. It is extremely difficult to find or formulate such an analysis of identity, (or to suitably amend Leibniz's rule). Some of the attempted emendations are reviewed in the chapters ahead.

Their failure adds to the case for the retention of Leibniz's Law.

Third, together with the reflexivity of identity -- everything is identical with itself -- Leibniz's Law permits the deduction of the other properties of identity mentioned above namely, symmetry and transitivity. Let me quickly show this.[5]

Symmetry is derived in the following way.

For '$\emptyset x$' and '$\emptyset y$' we will substitute '$x = x$' and '$y = x$' respectively.

(i) $x = y \supset (\emptyset x \supset \emptyset y)$ ....Leibniz's Law
(ii) $x = y \supset (x = x \supset y = x)$ ....Substitution on (i) [6]
(iii) $x = x \supset (x = y \supset y = x)$ ....Truth Functional Logic on (ii)
(iv) $x = x$ ....Reflexivity of '='
(v) $x = y \supset y = x$ ....MPP on (iii), (iv)
(vi) $(x)(y)(x = y \supset y = x)$ ....Universal Gen. on (i) - (v)

Transitivity is derived by a similar proof.

For '$\emptyset y$' and '$\emptyset z$' we will substitute '$x = y$' and '$x = z$' respectively.

(i) $y = z \supset (\emptyset y \supset \emptyset z)$ ....Leibniz's Law
(ii) $y = z \supset (x = y \supset x = z)$ ....Substitution on (i)
(iii) $x = y \supset (y = z \supset x = z)$ ....Truth Functional Logic on (ii)
(iv) $(x)(y)(z)(x = y \supset (y = z \supset x = z)$ Universal Gen.on (i) - (iii)

The upshot of the above proofs is that together reflexivity and Leibniz's Law constitute a sufficient basis for the derivation of the conceded properties of the identity sign.

Clearly, in the light of the above, the most powerful argument in defense of Leibniz's Law, as an analysis of identity, would be to show that the alleged counter-examples to this rule, are <u>not</u> in fact counter-examples at all. Let us first consider, by way of clearing the ground, a few of the more obvious misconceptions.

Leibniz's Law evidently is a principle that invites confusion even at the most elementary level. Consider the following :-

From (1) Cicero = Tully

and (2) 'Cicero' is spelled with six letters

follows by Leibniz's Law, the false conclusion

(3) 'Tully' is spelled with six letters.

In fact all that is involved in this example is a confusion between the use and mention of terms. It is not the signs 'Cicero' and 'Tully' which are asserted to be identical when, in line (1), we say that Cicero = Tully but rather the identity statement is about the person Cicero. Following Quine,[7] "the basis of the principle of substitutivity appears quite solid, whatever can be said about the person Cicero should be equally true of the person Tully, this being the same person". The fact is, line (2) is not a statement about a person, we are simply talking about the word 'Cicero' itself. The name occurs there merely as a fragment of a longer name which contains, besides this fragment, the two quotation marks. In other words, the occurrence of 'Cicero' or any expression within the context of quotation marks, is not a purely referential occurrence of that expression and consequently not subject to Leibniz's Law. The application of this rule must be confined to substitutions in <u>use</u> contexts otherwise we shall be saddled with patently unacceptable inferences such as (1) - (3) above.

There are a number of other familiar examples in which it is claimed Leibniz's Law breaks-down but where in fact the alleged difficulty arises out of a more obvious confusion such as, equivocation in the use of a term, substitution on an ill-defined or incomplete statement, or simply the attempt to substitute into statements that are not identity statements. A few examples should suffice to illustrate the point here. Consider the example Linsky uses,[8]

(4) Paul is the Pope

(5) The Pope is the centuries old enemy of Protestantism

(6) Paul is the centuries old enemy of Protestantism.

This example does not demonstrate the failure of Leibniz's Law but is simply an instance of the fallacy of ambiguity. The ambiguity in question is between 'the' as in premiss (4), (i.e. the one and only x who is the Pope), and the abstractive 'the' as in (5) and (6), (i.e.'The Pope' meaning here "all Popes" are the enemy of Protestantism). It is obvious that Linsky obtained (6) from (4) and (5) by confusing 'the Pope' - (abstractive 'the') in (5) with 'the Pope' - (the unique individual x) in (4).[9]

Consider now, as an example of an incomplete statement, the following :-

(7) The composer of the Eroica = The composer of the Missa Solemnis

(8) The composer of the Missa Solemnis was deaf

(9) The composer of the Eroica was deaf.

The conclusion on line (9) is false, but surely the confusion here is easily corrected when we give a more precise formulation of the premiss on line (8). Clearly, (8) should be read as

(8') The composer of the Missa Solemnis was deaf at the time he wrote the Missa Solemnis

from which the conclusion

(9') The composer of the Eroica was deaf at the time he wrote the Missa Solemnis

follows, by Leibniz's Law, in a perfectly straight-forward way.

Finally, we should note here that not all statements of the grammatical form "x is the same F as y" or "x and y are the same F" are identity statements. To put this point another way, the statement "x is the same F as y" does not always analyse into "x is the same as y and x and y are Fs". For instance, "the couch is the same colour as the chair" is not an identity statement, it states that the couch and the chair have a common property; whereas, ofcourse, "Tully is the same man as Cicero" is an identity sentence.

To summarise so far, we cannot prevent ambiguities, incomplete or ill-formed statements, use-mention confusions and so forth, from running into logical difficulties when Leibniz's Law is applied. On the contrary, as I will argue in this paper, the preservation of Leibniz's Law, (viz the preservation of the rule for intersubstitution of co-referential terms, salva veritate), is a condition that any putative identity sentence must satisfy if it is to be a genuine identity sentence.

Even after scrupulously observing the distinctions and avoiding the difficulties already mentioned, Leibniz's rule seems to give rise to some extraordinary consequences. Consider the following paradigm examples wherein the application of Leibniz's Law seems to permit the derivation of a false conclusion from true premisses.

A. The number of planets is nine

It is necessary that nine equals nine

Therefore it is necessary that the number of planets is nine.

(Quine)[10]

B. The author of Marmion is the author of Waverley

George IV believes that the author of Marmion is Scotch

George IV believes that the author of Waverley is Scotch.

(Russell)[11]

C.(i) Suppose Cleopatra's Needle is corroded away by the London fog and is repaired with concrete until eventually, in 1984, none of its original state is left. Cleopatra's Needle in 1884 is the same landmark as Cleopatra's Needle in 1984, but not the same stone. In fact it is no longer stone at all.

This example, due to P.Geach,[12] appears to give us a case where the rule of substitutivity holds under one sortal term (e.g. 'landmark'), but cuts out under another (e.g. 'stone').

(ii) The President of the United States is the Commander-in-Chief of the Armed Services

The President of the United States is given his oath by the Chief Justice

The Commander-in-Chief of the Armed Services is given his oath by the Chief Justice.

This example, due to Linsky,[13] appears to give us a case where where a and b are identical under one sortal concept (e.g. 'man') and both a and b have the property g (e.g. 'official') but a and b are not co-incidental under g. Again, it is claimed, Leibniz's Law does not hold in such cases.

(iii) Consider a theory T, which is a theory about the expressions of a given natural language: let the range of T's quantifiers be token expressions of that language; and suppose the ideology of T be so restricted that in T we cannot give different descriptions of two tokens of the same type-word. Let the statement a = b express an identity relation in the theory T. Now add to T's ideology just one predicate that discriminates between equiform tokens a and b. In the enlarged theory T^1, the identity statement in T is no longer an identity statement. Leibniz's Law does not hold since identity does not confer universal interchange of identicals but rather substitution relative to the definite ideology of any particular theory T.

(Geach)[14]

The examples A - C(iii) above are intended to exemplify cases where it is claimed Leibniz's Law is confronted with insurmountable difficulties: indeed such cases have led many philosophers in recent times to abandon Leibniz's rule, while others have made major distinctions in logic, semantics and ontology, rather than abandon it.[15] As I have said, it seems intuitively quite right that Leibniz's Law should be defended. To make the important point again, to say a and b are identical is to say they are one and the same thing -- how could something be true of a and not be true of b ? Nevertheless any apparent counter-example to Leibniz's rule clearly deserves an adequate explanation.

The question therefore presents itself -- "If we are to preserve Leibniz's Law (and like many others I see no viable alternative), how are we to explain the break-down of this principle in contexts such as the above"? The aim of my essay is to answer this question. The plan of the essay is as follows.

In Chapter II and III, I will outline the difficulties to be encountered by anyone wishing to hold Leibniz's Law without appearing to arbitrarily rule out counter-examples of the form A and B. Chapter II considers modal contexts, (examples of the form A above), which I conclude are not embarrassed by an unrestricted use of Leibniz's Law, provided we are prepared to regard only necessary "name"-identities as genuine identity sentences, since hitherto non-trivial contingent identity statements are, under this interpretation, no longer genuine identity statements: and provided we are willing to accept, or deal with, Quine's objection that modal logic commits us to an ontology which repudiates material objects.

In Chapter III, I consider intentional contexts, (examples of the form B above, in which the main verb is a verb expressing propositional attitude). I hope to show that it is possible, in the cases considered, to resolve the problems confronting Leibniz's Law by arguing that sentences of the form '$(\iota x)(Fx) = (\iota x)(Gx)$' are <u>not</u> genuine identity sentences and therefore are not subject to Leibniz's rule and further, that genuine identity sentences considered here do follow in accordance with Leibniz's Law.

In Chapter IV, I consider two further attempts at denying the unrestricted application of Leibniz's rule, in effect, the attempt to reduce identity to a relation of indiscernibility within specified contexts: the first (§i) relative to sortals, (i.e. examples of the form C(i) and C(ii)), the second (§ii) relative to a prescribed language, (i.e. examples of the form C(iii) above). We also investigate in this Chapter the claim to deny the universal transitivity of Leibniz's Law. I will argue that the objections raised in this Chapter do not present serious difficulties for Leibniz's Law. The fact seems to be neglected by such objections, that an identity statement involves reference to only one object.

So much for the preliminaries. Let us now turn to the first of our allegedly vitiating contexts for Leibniz's Law namely, modal contexts.

CHAPTER TWO

LEIBNIZ'S LAW AND MODALITY

It is easy to prove, from quite straight forward assumptions, that if X and Y are identical, their identity is a necessary one. Consider:- if X is identical with Y, so that whatever is true of X is true of Y, then since it is true that X is necessarily identical with X,[16] it must be true that X is necessarily identical with Y.

Put schematically -

(D) If (1) $X = Y$

and (2) $\Box(X = X)$

then (3) $\Box(X = Y)$

This seems a surprising and counter-intuitive result. To emphasise the point, (D) says that all identities are necessary identities; that is to say, there are no contingent identities. Yet does it not seem only a contingent matter that :-

(4) The man who left his finger print is one and the same individual as the man who robbed the safe.

(5) The tallest man in the room is McLaughlin.

(6) The class of creatures born with a heart is identical with the class of creatures born with a kidney.

(7) Pain occurrences are identical with physiological occurrences of sort g.

(8) The highest paid worker in the factory is the laziest worker in the factory.

Some of these examples may be disputed, but clearly such examples differ from one like

(9) Sir Walter Scott is Sir Walter Scott.

which is necessarily true.

To put the matter simply, it seems quite reasonable to claim identities of the form $a = a$ are necessary whereas identities of the form $a = b$ are by no means, intuitively, necessary identities. Nevertheless, as we have seen from (D) above, this result is forced upon us when we combine Leibniz's rule of universal interchangeability of identicals with the modalities.

In this chapter we will examine three alternative attempts to resolve this problem :-

First, where it is accepted that Leibniz's Law and modal logic are incompatible and Leibniz's Law, in its unrestricted form, is rejected.

Second, the arguments which attempt to show Leibniz's Law is compatible with modal logic.

Third, where it is accepted that Leibniz's Law and modal logic are incompatible and modal logic is rejected

Leibniz's Law is rejected

We might block the derivation (3) above by simply giving up Leibniz's Law as an acceptable principle of identity. This is to say, to obtain the result (3) we are using a strong interchangeability rule; co-referential expressions are interchangeable in all contexts, including modal contexts: so whatever you say here about one of the two identical entities you may say about the other. To make this point more perspicuously another way, we are, in modal contexts, committed to the paradoxical conclusion (3) by substituting into Leibniz's Law. To show this, consider again Leibniz's Law in the form

	(i)	$(x)(y)\{(x = y) \supset (Fx \equiv Fy)\}$	from which we obtain
	(ii)	$(x)(y)\{(x = y) \supset [\Box(x=x) \supset \Box(y=x)]\}$	by subst. $\Box(\ =x)$ for F in (i)
thus	(iii)	$(x)(y)\{\Box(x=x) \supset [(x=y) \supset \Box(y=x)]\}$	Truth Funct.Logic on (ii)
	(iv)	$\Box(a = a) \supset \{(a = b) \supset \Box(b = a)\}$	Univ.elim on (iii)
	(v)	$\Box(a = a)$	Assumed
	(vi)	$(a = b) \supset \Box(b = a)$	MPP (iv), (v)
	(vii)	$(a = b) \supset \Box(a = b)$	Symm. of '=' (vi)

(Incidentally, by employing the same method we can show that all identities are contingent).[17]

Summarily we have shown that from Leibniz's Law

$$(x)(y)\ (x = y. \supset .Fx = Fy)$$

we can derive the antinomy (D) above

$$(a = b) \supset \Box(a = b).$$

Hence we might seek to block the derivation (3) by simply giving up Leibniz's Law, in its unrestricted form, as an acceptable principle of identity. Ofcourse we have no reason to give it up if the paradoxical conclusion which is derived by substituting into Leibniz's Law can also be derived without it. We do not block the counter-intuitive result (3) if we can derive (3) somehow without a rule of universal interchangeability. We may suppose the paradox lies elsewhere. (Though frankly I do not see how this would be done.) And, as we have noted earlier, there are a number of compelling reasons why we should not give up Leibniz's Law too readily as an analysis of identity. Leibniz's Law embodies the property of universal interchangeability which marks identity off from other equivalence relations. If we give up Leibniz's Law we need to find another principle of comparable generality to justify substitution. As far as I can see there isn't one.

To show this we need to consider the arguments that, in one form or another, say we must abandon the conception that Leibniz's Law defines some univocal concept of identity. In short, the claim that we can have two kinds of identity or even degrees of identity -- both of which are invoked in defense of the modalities.

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According to Quine, R.B.Marcus in her 'Identity of Individuals in a Strict Functional Calculas of Second Order', defined two relations of identity between individuals. A weak and a strong identity. The weak one holds between x and y, wherever $(F)(Fx \supset Fy)$: and the strong holds only where $(F)(Fx \mathrel{-\!\!3} Fy)$. Only the strong identity relation is subject to substitutivity valid for all contexts. Hence on this present account, only strong identity is interpretable as "identity" in the usual sense of the term. Now it needs to be stressed that this is not Marcus's position -- nor, contrary to Quine, can I find that she has ever proposed that there be more than one kind of identity. However, in spite of this, it is worth criticising this view since Hintikka, Pap, Rescher, Geach[19] and many others, find such an interpretation of identity, in connection with modal logic, persuasive.

A similar proposal, for example, is implicit in the suggestion from Hintikka that the principle of substitutivity is compatible with modal logic only if x is necessarily identical with y, (Cf. page 25 following). And Rescher, in 'Identity, Substitution, Modality' claims that once we move from "strictly extensional systems of logic" to its application in contingent facts, there is no alternative to accepting modifications to Leibniz's rule. Only in a strictly extensional system, "where extensional contexts alone come into the picture, can we apply Leibniz's Rule in a blanket way, and thus deal without further ado, with one single mode of "identity" ", (p.163, ibid). Outside of strictly extensional systems, we are no longer able to speak of a single identity concept but are led to recognise several distinct, albeit related, identity-type relationships. He suggests that we should adopt a sequence of "degrees of identity"; a sequence of increasingly strong "identity" concepts.

Let me make a central point again here. If a and b are "identical" they are one and the same object. Bearing this in mind, the suggestion that there can be two kinds of identity -- the distinction between a stronger and weaker identity relation -- is quite unintelligible. For instance, if they exhibit a weak form of identity, Venus and the evening star are, in this respect, one and the same object. However, in terms of the strong identity relation, they are two distinct objects ! Similarly, Rescher's notion of increasing degrees of identity is wholly unacceptable. At best it would follow from this proposal that Venus is somewhat but not fully identical with the evening star.

In fine, one wants to know what kind of identity relation is being preferred here that does not entail unrestricted interchange of co-referential terms. As Prof. Wilson says, rather nicely in 'Modality and Identity', "I am like the Boston ladies who _have_ their hats. I have my hat: Leibniz's rule. I do not insist you wear my hat. I just want to know what hat you're wearing", (475). If identity does not entail universal interchangeability, what on earth does it entail ? If identity is not expressed in terms of Leibniz's Law, how are we to express it ?

As a possible answer to this question let us consider briefly Carnap's treatment of the identity relation in 'Meaning and Necessity'. Carnap begins by proposing that every designator within a language "possesses" both extensional and intensional meaning, (i.e., the individual expression 'Venus' refers to both its extension, the planet, and its intension, the individual concept.) Now two designators which have the same extension are said to be factually equivalent, (i.e., 'man' and 'featherless biped' are factually equivalent, if the statement 'An individual is a man if and only if it is a featherless biped' is a factually true statement.) This is to say, identity of extension Carnap calls factual equivalence. Two designators which have the same intension are said to be logically equivalent, (i.e., 'man' and 'rational animal' are logically equivalent, if the statement 'An individual is a man if and only if it is a rational animal' is true on the basis of the meaning or semantic rules of the expressions). This is to say, Carnap calls identity of intensions L-equivalence.[20]

Now two expressions may have identical extensions without having identical intensions. Accordingly, Carnap sets out the conditions underwhich expressions may be interchanged one with another.

> "An expression occurring within a sentence is said to be interchangeable with another expression if the truth-value of the sentence remains unchanged when the first expression is replaced by the second. If, moreover, the intension of the sentence remains unchanged , the two expressions are said to be L-interchangeable. We say that a sentence is extensional with respect to an expression occurring in it or that the expression occurs in the sentence within an extensional contexts, if the expression is interchangeable at this place with every other expression equivalent to it. We say that the sentence is intensional, or that an expression occurs within an intensional context, if the context is not extensional and the expression is L-interchangeable at this place with every other expression L-equivalent to it."
>
> (p.46, ibid)

Carnap goes on to explain that all sentences of his system S_1, which contains only the ordinary connectives and quantifiers but no modal signs, are extensional, whereas sentences of his system S_2, which are of the form 'N(...)', where 'N' is a sign for logical necessity, are intensional.

Thus Carnap formulates two principles of substitutivity for identity, the first for extensional (non-modal) systems such as S_1, where, in effect, Leibniz's Law is modified to stipulate intersubstitutivity only for co-extensive terms, (Cf. his 12.1). The second for modal systems such as S_2, where, in effect, the range of the variables are limited to intensional objects and where L-interchangeability is the condition for intersubstitution of L-equivalent and Necessarily equivalent terms, (Cf. his 12.2)

(Incidentally, as we shall see, Carnap defines a stronger condition than L-interchangeability between intensional expressions for certain contexts -- in belief contexts, he defines a relation he calls "intensional isomorphism" to explicate what is usually understood by "synonymity" or "identical intensional structure", Cf. page 58 ahead .)

So much then for Carnap's proposed analysis of '='. Let us now consider, briefly, its application to the antinomy (1) - (3) which, notice, is the result of a straight-forward application of Leibniz's rule into a modal context. Consider the following example, due to Quine.[21]

(10) The number of planets is nine

(11) It is necessary that nine equals nine

By Leibniz's rule, according to which two expressions naming the same entity may replace one another in any context *salva veritate*, since the expressions 'the number of planets' and '9' are, by hypothesis, the names of the same entity, it follows

(12) It is necessary that the number of planets is nine.

Now with Carnap's method the antinomy (10) - (12) cannot be constructed. For on his analysis first, the two individual expressions in (10) are not the "names" of anything hence, *a fortiori*, they are not the names of the same entity. Moreover, since the intensions of the two expressions, are not L-equivalent, we cannot derive the invalid inference (12).

Initially, Carnap's proposed emendation to Leibniz's Law and solution to the antinomy strike one as very plausible. However there are a number of decisive objections to it, the strongest being objections in principle to his whole semantical method. The main defect here is Carnap's treatment of a semantical system as a system of rules; or rather, his notion that semantical rules are constitutive of the language in question.[22] The nub of difficulty is this. To formulate the conditions underwhich expressions with identical extensions may be interchanged in S_1, (i.e., his 'first principle of interchangeability'), and the conditions underwhich expressions with identical intensions may be L-interchanged in S_2, it has been necessary for Carnap to supply further rules and definitions for the key terms involved in these substitution principles, such as 'true in S_1', 'designates in S_1' etc.

However, before we can understand the more basic rules, upon which the principles of substitutivity depend, we must have an antecedent understanding of the explicandum for which these rules are provided.[23] To treat all such terms as defined terms in S_1, as Carnap does, is notoriously difficult.[24] In fine, while in semantics we require "general" definitions for terms like 'designates', 'truth', 'logical truth' and so on, (and rules specifying their application), such general definitions are not, nor can they be, explications of a language.

There are two further difficulties I want to mention here which confront Carnap's doctrine of intensions or meanings. We noted that Carnap claims that designative expressions have not only extensions but intensions as well. Individual expressions, one-place predicates and sentences, have corresponding to them individual concepts, properties and propositions respectively, these latter being the intensions, or meanings, or logical content, of the expression in question. Moreover, according to Carnap, ..."the semantical rule for a sign has to state primarily its intension; the extension is secondary, in the sense that it can be found, if the intension and the relevant facts are given", (Meaning and Necessity, p.112). As Prof.Wilson asks, "What would it be like to know the intension of, say, 'Chicago' but not to know the extension? And one would want to know the empirical procedures by which one passes from the intension to the extension.... reflection on this difficulty as it stands suggests that there is an atmosphere of science-fiction about the whole theory of individual concepts", ('The Trouble with Meanings', p.5.).

There is a second far more intractable difficulty here, raised by Quine.[25] In Carnap's modal system S_2, or for that matter any other modal language, the variables range over intensions and, since to be is to be the value of a bound variable, the "entities" which are the values of such variables, are not individuals of the concrete world. (We will consider this objection in detail in the following section, page 42 ahead.)

Finally, it is claimed that Leibniz's Law, in its unrestricted form, is perfectly compatible with modal logic anyway, and that the apparent antinomy (1) - (3) is perfectly easily explained. If this can be shown, then, as far as we are concerned, all of Carnap's proposed emendations to Leibniz's rule are beside the point. Let us consider these arguments.

Leibniz's Law and modal logic are compatible

We may simply embrace the conclusion that all identities are necessary. As F.B.Fitch says[26]

> ..."it seems to be a perfectly sound position to hold that every true identity is logically necessary. This is simply the view that an entity is never identical with anything but itself and that of logical necessity it is identical with itself."

Also in her papers 'Extensionality', 'Modalities and Intensional Languages', etc., Ruth Barcan Marcus has argued that in a "strong explicitly intensional language", all identities are necessary. In her system QS4,[27] (which consists of the introduction of quantification in the usual manner and the addition of the axiom $\Diamond(\exists x)A \prec (\exists x)\Diamond A$), the following theorems :-

(13) $(xImy) \equiv (xIy)$ ...i.e., material and strict equivalence are indistinguishable.

(14) $(xIy) \equiv \Box(xIy)$ .i.e., the identity relation holding between x, y is necessary.[28]

Or, to make the same point, in the system QS4 the statement

(15) $(xIy) \ \& \ \Diamond\sim(xIy)$

is a contradiction. Contingent identities are disallowed by (14) above.

Mrs Marcus then argues for a uniform interpretation of the identity relation, in terms of Leibniz's rule, and where Leibniz's rule breaks down in modal contexts, claims we are no longer dealing with a bona fide identity relation. In other words, we are no longer in an "explicitly intensional language". A language is explicit intensional only where it does not equate the identity relation with some weaker form of equivalence. The rule of universal intersubstitutivity holds only for the identity relation -- cases where intersubstitutivity breaks down are taken to exhibit some kind of weaker equivalence relation.[29] i.e., she argues for a distinction between a) identity and b) degrees of stronger and weaker "equivalence" relations -- she proposes to give a uniform meaning to identity, in terms of Leibniz's Law and, for example, talk of 'attributes' and 'classes' as being "equal", not identical.

Needless-to-say, one result of Marcus's proposal is that most statements, hitherto of the form 'a = b', are not now identity statements, but are equivalence statements expressing "functionally equivalent terms". Incidentally, she defines strong functional equality as :-[30]

(16) $(F = G) \supset (x)(Fx \equiv Gx)$ .."where 'F equals G' is like but not synonymous with 'F is identical with G'.."

Clearly what "strong functional equality" amounts to is a two-place predicate, that is symmetrical, reflexive, transitive, and in all other ways behaves like '=', but because, in modal contexts, substitution of mere "functionally equals" breaks down, is not an identity relation.

Let us be clear about this. The identity relation for Marcus must be tautologically true or analytically true, since the only acceptable species of identity statement to satisfy the conditions above will be of the form 'a = a'. Where we claim a = b is a true identity then it must say the same thing as a = a. (This the import of theorem (14) above). Thus if we decide that, say, 'the evening star' and 'the morning star' are interchangeable without antinomy in all contexts, the (apparent) descriptions come to be used as proper names and, presumably, the descriptive content is ignored.

There are a number of major difficulties to be raised here. First, Marcus has stipulated that if x is identical with y then, x is necessarily identical with y, (Cf., theorem (14)).

Many other modal logicians embrace the same conclusion. (For example, in Hintikka's system it is a theorem not that 'if x is identical with y then whatever is true of x is true of y and conversely', but only that 'if x is necessarily identical with y then whatever is true of x is true of y and conversely'). An argument for this might be as follows. Consider the example:-

(17) The number of planets is nine

(18) 9 is necessarily greater than 7

(19) The number of planets is necessarily greater than 7.

Obviously the substitution of the co-referential terms in (17) fails: the conclusion on line (19) is false. Now from the point of view of the modalities, it could be argued, that the reason for this failure is connected with the singular term 'the number of planets', which fails in modal contexts to have the kind of unique reference which is prerequisite for being the substitution-value of a bound variable. Why does the expression 'the number of planets' fail, in modal contexts, to specify a well-defined individual ? Because, while in the actual state-of-affairs it refers to 9, in modal contexts we are also implicitly considering other possible states-of-affairs, in which it refers to larger or smaller numbers. As Hintikka argues in 'The Modes of Modality',

> "This at once suggests an answer to the question as to when a singular term(say a) really specifies a a well-defined individual and therefore qualifies as an admissible substitution value of the bound variables. It does so only if it refers to one and the same individual not only in the actual world, (or, more generally, in whatever possible world we are considering), but also in all the alternative worlds which could have been realised

> instead of it; in other words, if and only if there is an individual to which it refers in all the alternative worlds as well. But referring to it in all these alternatives is tantamount to referring to it necessarily. Hence $(\exists x)N(x = a)$ formulates a necessary and sufficient condition for a term to refer to a well-defined individual".
>
> (p.73)

Summarily, the argument here supports the contention that for Leibniz's Law to be compatible with the modalities, then, for x to be identical with y entails that the identity relation holding between x, y is necessary.[31]

Now one difficulty here is this. If all identities are necessary identities, then examples such as (4) -(8) above, are either necessary or not identity statements. Either way this seems counter-intuitive. Suppose one evening we tag the planet Venus with the proper name 'Evening Star': and at dawn, suppose we tag the same planet with the proper name 'Morning Star'. When we discover we have tagged the same planet, this discovery is empirical and seemingly contingent. (Indeed, it seems an unmistakably "contingent" fact of astronomy that the morning star is identical with the evening star. After all it took empirical investigation in the sixth century B.C. to establish this.) Other examples of the kind of *de facto* identity being appealed to here would include H_2O and water, lightning and a particular sort of atmospheric electrical charge, brain-states and sensation reports, Sir Walter Scott and the author of Waverley and ofcourse examples (4) - (8) above. There is no obvious sense of "necessary" applicable to such identities. The issue here is , can we make sense of attempts to make such examples complicated ways of saying a = a ? Alternatively, are we prepared to concede that such examples are not genuine identity statements; since the identity asserted in each of the examples here does not hold for all possible worlds, a la Hintikka, and is not an analytic claim about the synonymous meaning of terms, as stipulated by Marcus, but rather is a synthetic hypothesis that the referent of two expressions is one and the same object ? An answer to this question is forthcoming but first we need to consider a closely related problem.

As we noted, Marcus stipulates that the identity relation holds only between "name"-identities. In a strong intensional language, for two expressions 'F' and 'G' to be analytically or tautologically equivalent, such that every identity sentence constructed with them is necessarily true, requires that 'F' and 'G' are the "names" of intensions. Similarly, in Hintikka's argument quoted above, there is a case to be found for restricting identity to a relation holding only between singular terms which <u>do</u> have the kind of unique reference, prerequisite for being the substitution-value of a bound variable, in modal contexts. (Although, in fact, Hintikka employs here the unsatisfactory thesis of "strong" identity holding in modal contexts, "weak" identity in non-modal contexts). Now the question posed here is; is there any way in which we might combine Leibniz's Law with the modalities without restricting all genuine identity sentences to name-identities ? Let us consider one possible way of showing this.

In his article 'Modality and Description', A.Smullyan argues that the unrestricted use of modal operators, in connection with statements and matrices embedded in the framework of a logical system such as 'Principia', does not involve a violation of Leibniz's Law.[32] It is Smullyan's contention that modal paradoxes arise, not out of any intrinsic absurdity in the use of modal operators, (as Quine holds), but rather out of the assumption that descriptive phrases are names. He proposes a division of all singular terms into proper names and, overt or covert, descriptions. Proper names, if they name the same object, are always synonymous. Any other occurrence of a (putative) name is to be treated as a description. (Thus if 'Evening Star' and 'Morning Star' are not synonymous in all contexts, they are both to be treated as descriptions.)

Now Smullyan goes on to argue that where singular terms in modal contexts are construed as definite descriptions, (as, say, in example (17) - (19) above), then the inference of paradoxical conclusions from modal premisses may be prevented by restrictions on the scope of these descriptions. In order to show this, he utilises Russell's method of contextually defining descriptive phrases by means of scope expansions.

Consider the following example

(20) Scott is the author of Waverley

in which one of the co-referential expressions is a proper name and the other a definite description. Schematically (20) gives us

(21) $s = (\imath x)(Axw)$

and since

(22) $N(s = s)$

by the unrestricted use of Leibniz's Law we obtain

(23) $N(s = (\imath x)(Axw))$

Clearly, (23) involves us again in the unacceptable antinomy.

Now, as Smullyan points out, when they are expanded in accordance with Russell's definitions, sentences (21) and (22) lead by ordinary logic to

(24) $(\exists x)[(y)(Ayw \equiv y = x)\ \&\ N(s = x)]$

which then, by using scope symbols, contracts to (in broad scope)

(25) $[(\imath x)(Axw)]\ N\{s = (\imath x)(Axw)\}$

rather than to the false sentence (in narrow scope)

(26) $N\{[(\imath x)(Axw)]s = (\imath x)(Axw)\}$

or its expansion

(27) $N[(\exists x)(Ayw \equiv y = x\ \&\ s = x)]$

One apparent advantage to be gained by adopting Smullyan's proposals, is that it appears to offer a way of combining Leibniz's Law with the modalities, without denying that sentences of the form such as (20) are genuine identity sentences. This however looks far from established in the light of Prof.Wilson's subsequent criticism.

Another apparent advantage in Smullyan's proposals, is that it appears to offer a way of combining Leibniz's Law with the modalities without having to introduce intensional objects as the values of variables." This however looks far from established in the light of Quine's subsequent criticism, (Cf., page 42 ff ahead).

In 'Modality and Identity', Prof.Wilson agrees that since the alleged antinomy does not go through when the arguments are expanded a la Russell, in the way Smullyan proposes, it cannot be said to go through in the contracted version (21) - (23) above. ("The fact that it does not go in the contracted version but looks as though it ought to is at worst a paradox whereas if it did go we would have an antinomy".)[33] However, (21) and (23) in their expanded forms are not identity sentences anyway, but rather multiply general sentences. Prof.Wilson then goes on to argue for the Russellian view that we must give up the idea that a definite description, as it occurs in (21) or any other sentence, designates or refers: in the present case, that the description ($\imath$x)(Axw) designates or refers to Scott.

> "Let us consider 'The author of Waverley is Scotch', which expands into 'Precisely one person is an author of Waverley and that person is Scotch'. The latter does not refer to Scott (as Russell insisted all along). If we adopt the reasonable principle that, in general, a sentence containing a defined expression (here 'the author of Waverley') has just those properties its expansion has, we conclude that even the contracted sentence 'The author of Waverley is Scotch' does not refer to Scott, contains nothing that refers to Scott".
>
> (p.472, ibid)

Thus sentences of the form '$s = (\imath x)(Axw)$', where one of the arguments is a description, (and ofcourse sentences of the form '$(\imath x)(Fx) = (\imath x)(Gx)$', where both of the arguments are descriptions), are not genuine identity sentences. They are defined expressions -- this is to say, they are not in primitive notation. Such sentences are not identity sentences but, in fact, are multiply general sentences; (e.g., the defined expression $(\imath x)(Fx) = (\imath x)(Gx)$ expands into the multiply general expression $(\exists y)[y = (\imath x)(Gx) \; \& \; (x)(Fx \equiv x = y)]$), and, as such, do not refer to any specific individual.

Let me put the matter another way. The point here is that descriptions are not "individual terms", and are not to be treated as terms. (Indeed, as Russell has shown, by treating them as terms, which is to say, by applying existential generalisation and instantiation on definite descriptions, we are led to countenance notorious non sequiturs.)[34] Following Russell, Prof. Wilson proposes here, that we keep the restriction of not treating descriptions flanking the identity sign as terms -- not applying generalisation, instantiation, or substitution in accordance with Leibniz's rule -- but rather, we must regard the semantic properties of such a sentence as those properties ascribed to its expansion in multiply general form.

What is accomplished by adopting Prof.Wilson's proposal is this: "where the so-called "antinomy of the name relation" involves definite descriptions we can escape the antinomy without giving up Leibniz's rule... We merely insist (reasonably) that only those inferences are acceptable which are acceptable when recast in primitive notation", (p.473, ibid).

Thus, in answer to our earlier question, if we are to have modal logic, with Leibniz's Law and without antinomies, then, contrary to Smullyan, Prof.Wilson holds there are good grounds for treating only name-identities as genuine identity relations. (Incidentally, Prof.Wilson's argument here seems to me to undermine his "knock-down" argument against intensions. Cf., "the major difficulty" in 'The Trouble with Meanings'.[35] No matter, there are other decisive arguments to be levelled against the doctrine of intensions.)

I have an observation to make here. We are told that s = (ɿx)(Axw) is not a dyadic statement affirming the identity of a pair but rather, it is an (abbreviated) multiply general sentence. Clearly such an interpretation goes against a point made earlier in this paper that, as a matter of fact, an identity assertion was made when George IV learned that Scott is the author of Waverley, or when it was discovered that the morning star is identical with Venus. I think the confusion here is between the "act" of taking two things to be identical and identity *per se*. As we have noted, '=' is a logical expression: we are concerned to give primarily a syntactical account of it and its properties. One such property of the identity sign is that it holds between co-referential terms. I am persuaded by Russell's argument that descriptions are not terms -- they do not, by themselves, refer.

However, even if we accept, as I do, the recommendation that only sentences reducible to name-identities are genuine identity sentences, there are still major difficulties to be over-come before we can combine modal logic with Leibniz's rule, without begging a number of important questions and, moreover, without avoiding the antinomy (1) - (3).

One difficulty, we noted earlier; we may tag the planet Venus with the proper name 'Evening Star', and at the appropriate time of day, we may tag the same planet with the proper name 'Morning Star'. When we discover we have tagged the same planet, this discovery is only contingently true.

Second, as Prof.Wilson has noted in 'Modality and Identity', we can develop the naming antinomy just as easily with proper names as with descriptions, (Cf., page 473, ibid).

Consider :-

(27) Evening Star = Morning Star

(28) N(Evening Star = Evening Star)

(29) N(Evening Star = Morning Star)

J.Myhill, in his article 'An Alternative to the Method of Extension and Intension', avoids the obvious difficulty here by stipulating that no names of a given system will name the same thing, i.e., no two terms name the same individual, (p.302, ibid). As Prof. Wilson notes, this "restriction might seem both evasive and unnecessary". Moreover, it does not help Marcus and most other proponents of modal logic, since they want to defend modal logic for, among other reasons, its utility in "the dissection of most types of empirical statement", (p.77, 'Modalities and Intensional Languages').

But by far the strongest objections to any attempt to reconcile modal logic with Leibniz's rule are raised by Quine. So let us now turn to the last of the three alternatives by which we might block the derivation (1) - (3) -- this is to say, reject modal logic.

Modal logic is rejected

Quine argues convincingly that much of the case for modal logic arises out of a confusion between use and mention. His objection is aimed at modalities in so-called de dicto form, where the operator '□' operates on sentences whereas 'is necessary' ought to be regarded as a predicate of sentences.[37] However Quine admits that modal logic does not "require" confusion of use and mention: in cases where a modality is attributed to an individual or a relation between individuals, (i.e., so-called de re modalities), there is not such a confusion.

In 'Reference and Modality', Quine rejects de re modalities for atleast the following four reasons :-

(a) Leibniz's Law breaks down in sentences expressing de re modalities.

Consider again the argument used here by Quine:

(17) The number of planets is nine

(18) 9 is necessarily greater than 7 .

(19) The number of planets is necessarily greater than 7.

Quine construes (17) as an identity statement. As we have seen, this is not obviously true.

Mrs Marcus would argue that (17) is a "contingent equivalence" and therefore the step from (17) and (18) to (19) is invalid, for the reason that the substitution has not been made on a genuine identity sentence. Prof.Wilson would argue that (17) contains a description, 'the number of planets' and therefore is a multiply general sentence.

An Oxonian might argue "If I count the planets and come up with 9 as the answer, this does not represent a further identification or characterisation of what is referred to by 'the number of planets', unlike say, 'the number of planets is an odd number'. To illustrate the point, consider the sentence

(30) The number of planets is greater than 7

Clearly (30) is most naturally construed as

(31) There are more than 7 planets.

Similarly, we might argue that (17) is more appropriately construed as

(32) There are 9 planets. "

(Cf., B.Rundle, 'Modality and Quantification')

Finally, there is another way we might take (30) that is as

(33) That number, which is the number of planets, namely 9, is greater than 7.

This is to say, if we construe (30) in terms of (33) we are taking the subject of (30) as identifying a certain number. This alternative way of construing (30) arises again in connection with (19) above. Clearly, if (19) is construed as Quine construes it, as saying that there are necessarily more than 7 planets, (19) is false. However, if the subject of (19), 'the number of planets', is used as a referring expression which refers to the number 9, then (19) is true. (It is not necessarily true that the number of planets is 9; but given that this is the case, it is true that 9 is necessarily greater than 7).

Summarily, the point here is that Quine claims that the argument (17) — (19) is invalid since (17) and (18) are true and (19) is false. If (19) is regarded as false, this is because Quine believes that 'the number of planets' does not refer "essentially" to some definite number and so cannot *a fortiori* be replaced by an expression referring to the same number. Hence Leibniz's Law cannot be used to derive (19) from (17) and (18). On the other hand, if (19) is regarded as true — because we take 'the number of planets' to refer essentially to the number 9, then, as far as Quine is concerned, (19) would be validly derived from (17) and (18) in accordance with Leibniz's Law.

(b) Quine also rejects *de re* modalities because we cannot quantify into modal contexts.

Consider the modality

(34) 9 is necessarily greater than 7

this is of the form

(35) $\Box(a > b)$

Quine argues that we cannot existentially quantify into (35) to obtain

(36) $(\exists x)[\Box(x > b)]$

or, in other words, we cannot state

(37) $(\exists x)$(x is necessarily greater than 7)

since, Quine argues, "$(\exists x)$" cannot bind the "x" in "$\Box(x > 7)$".

The problem here can be stated in the following way. As we have seen, Smullyan has shown that if we treat covert names as descriptions and then contextually define all descriptions in accordance with Russellian scope expansions, the principle of substitutivity does not automatically generate paradoxes. But Smullyan has not shown what sense to make of the quantified modal statement -- or rather, how to make sense of quantifying into modal contexts. In fine, the question being asked here by Quine is "just what sense does the expression $(\exists x)[\Box(x > 7)]$[38] make ?". There seem to be atleast two ways of answering Quine's question -- and they both give rise to the same result.

One answer might be that while it is true that a variable inside a modal context (i.e., "x" inside "$\Box(x > 7)$", cannot refer back to a quantifier prior to that context, this is true only if variables range over extensions. If on the other hand, variables range over intensions, say, individual concepts or attributes, they can refer back to the quantifier prior to the modal context. In short, quantification is possible into modal contexts at the expense of "widening" (Quine says narrowing) the range of our variables to include "intensions".

> "Let 'b', 'f', and 'm' mean respectively the class of bipeds, the class of naturally featherless creatures, and the class of men. Then the sentence is true (9) '$fb = m \;\&\; \Diamond fb \neq m$', the non-existence of featherless bipeds other than men being a zoological accident. But, where 'α' is a class variable, the inference from (9) of the sentence '$(\exists\alpha):\alpha = m \;\&\; \Diamond\alpha \neq m$' must be in error, since, having '$\alpha = m$', we could substitute 'm' for 'α' and infer further the false sentence '$\Diamond m \neq m$'. There is no similar objection, however, to the inference from (9) of (10) '$(\exists\phi):(x)(\phi x \equiv x\epsilon m) \;\&\; \Diamond\sim(x)(\phi x \equiv x\epsilon m)$', where ϕ is a variable for attributes; and it would seem that in a logical system containing both modal operators and quantifiers such inferences should be retained".

(A.Church, in his 'Review of 'Notes on Existence & Necessity page 46).

In modal contexts, for a variable to refer back to a quantifier prior to that context, requires that the variable has an intensional range -- a range, for instance, composed of attributes rather than classes, or individual concepts rather than individuals. Now there seems nothing *prima facie* wrong with construing the values of variables as intensions.[39] (Moreover, this move is not embarrassing to most proponents of modal logic. On the contrary, in most cases there would be little point to modal logic if it were not to deal with intensional entities at all.)

However, as Quine points out, the unacceptable consequence of extending the range of variables in this manner would be, for most of us, an overly idealistic ontology. We return to this matter shortly, (Cf., (d) below).

A second possible answer to Quine's question might be along the following lines. By $(\exists x)[\Box(x > 7)]$ we understand "there is an object to be specified in such a way that from the specification it would follow that $\Box(x > 7)$". This specification might be "$x = |\sqrt{x}| + |\sqrt{x}| + |\sqrt{x}| \neq |\sqrt{x}|$", from which it would follow that $\Box(x > 7)$. But is this different from saying that $(x > 7)$ follows logically from "$x = |\sqrt{x}| + |\sqrt{x}| + |\sqrt{x}| \neq |\sqrt{x}|$"? If it is not, then we have not given sense to there being an object satisfying the matrix $\Box(x > 7)$.

Moreover, variables of quantification range over objects <u>not</u> specifications of objects. The problem here is to make sense of there being an object <u>x</u> which is necessarily greater than 7. It seems that the only way in which it might be possible to make sense of this latter notion, i.e., an object that is necessarily greater than 7, is to say that there are some objects, independent of specification, which are 'such-and-such' necessarily. Now does this make sense ? And this is just a way of asking whether the doctrine of "essentialism" is tenable.

(c) Quine rejects <u>de re</u> modalities because they commit us to essentialism.

Quine appears to have conflicting views about the doctrine of essentialism. At one point he says, that "essentialism is indefensible" (in 'Word and Object', p.200), and although it is "abruptly at variance with explaining necessity and analyticity

... the champion of quantified modal logic must settle for essentialism", (in 'Reference and Modality', p.155). But elsewhere he also says that he does not consider that to show modal logic is committed to essentialism is to reduce it to absurdity, (Cf., 'Comments to Marcus's 'Modality and Intensional Languages''). But if commitment to something indefensible does not imply absurdity -- what does ?

As far as our present example is concerned, Quine objects

> "to say in the case of 9, that this number is, of itself and independently of mode of specification, something that necessarily, not contingently, exceeds 7, means adopting a frankly inequalitarian attitude towards various ways of specifying the number. One of the determining traits, the succeeding of 8, is counted as a necessary trait of the number"... Others..."notably its numbering of the planets, are discounted as contingent traits."
>
> "This is how essentialism comes in: the invidious distinction between some traits of an object essential to it (by whatever name), and other traits of it as accidental".
>
> (p.104, ibid).

In fact, Quine's position with respect to the occurrence of terms such as 'necessarily' 'possibly' and so forth, is that modal terms do not attribute a feature to an object but are merely a way of talking about the object. In 'Reference and Modality' he says

> "to be necessarily greater than 7 is not a trait of a number, but depends on the manner of referring to the number".

and again on the same page

> "Being necessarily or possibly thus and so is in general not a trait of the object concerned, but depends on the manner of referring to the object".
>
> (p.148).

Frankly, I do not see we can refuse to accept that the number 9 is necessarily greater than the number 7, no matter how the number 9 is referred to. I can truly say that the number referred to by the English word spelt backwards as an 'e' preceded by an 'n' preceded by an 'i' preceded by an 'n', is necessarily greater than 7. An essential property of the number 9 is that it is greater than 7.[40] To which, no doubt, Quine would display "an appropriate sense of bewilderment", ('Word and Object', P.199).

We cannot let Quine's "sense of bewilderment" with respect to such unrepentant essentialism go by unnoticed. In 'Word and Object' he says

> "Mathmaticians may be conceivably said to be necessarily rational and not necessarily two-legged; and cyclists necessarily two-legged and not necessarily rational. But what of an individual who counts among his eccentricities both mathmatics and cycling. Is this concrete individual necessaily rational and contingently two-legged or *vice versa*. Just in-so-far as we talk referentially of the object, with no special bias towards a background grouping of mathmaticians as against cyclists and *vice versa*, there is no semblance of sense in rating some of his attributes and necessary and others as contingent".
>
> (p.199).

It is not difficult to see how this objection to essentialism could be dealt with. We might argue that while mathmaticians have the accidental property of being two-legged and cyclists have the accidental property of being rational, Quine's "mathmatical cyclist" has the essential properties of being both rational and two-legged. (There is no reason to hold "mathmaticians are rational" and "cyclists are two-legged" as necessary and not to hold "mathmatician-cyclists are rational and two-legged" as necessary

All-in-all, I think essentialism is defensible or, more cautiously, does not present proponents of modal logic with insurmountable difficulties.[41]

(d) Modal logic commits us to an over idealistic ontology

Finally, I think by far the strongest argument Quine has against modal logic, is that, while we may claim to preserve Leibniz's Law in modal languages, it commits us to a very queer ontology in which there are no concrete objects (men, planets, etc.), but rather, there are only corresponding "to each object a multitude of distinguishable entities". For example, there is no such "ball of matter as the so-called planet Venus, but rather atleast three distinct entities: Venus, Evening Star, and Morning Star.", ('Problem of Interpreting Modal Logic', p.47).

In order to appreciate the full force of this charge of idealism,[42] a few words need to be said first about Quine's views on ontology. An expression designates if and only if 'existential generalisation' or 'universal instantiation' with respect to it, is a valid form of inference. It follows that a theory designates all and only those entities which fall within the range of its variables of quantification. The variables range over objects -- not names of objects. (The names of these variables, if they have names are the substituends of variables, not the values). Now if it makes sense at all to speak of the ontology of a theory, it can only be the objects to which the variables range over, this is to say, the values of the theory are the values of its variables of quantification.

How then does all of this commit proponents of modal logic to an idealistic ontology ? As we have seen, Quine is perplexed by the sense to attach to quantification across modalities. Suppose, he says, we accept the following as a partial criterion for existential quantification in modal contexts ;-

(i) "An existential quantification holds if there is a constant whose substitution for the variable would render the matrix true" (p.46, ibid).

A criterion such as this would commit us to an idealistic ontology inthe sense that the variables of quantification would have to range, not over concrete objects but a multitude of distinguishable entities (viz., "concepts") corresponding to each supposed concrete object. To illustrate this point, let us consider the example Quine gives, (p.47, ibid). Using 'C' for 'congruence' to express the relation which Venus, the Evening Star and the Morning Star, (bear to themselves and according to empirical evidence to one another), Quine argues

(38) Morning Star C Evening Star & □(Morning Star C Morning Star)

Therefore, by criterion (i) above

(39) (∃x)(x C Evening Star & □(x C Morning Star)).

But also by criterion (i)

(40) Evening Star C Evening Star & $\sim\Box$(Evening Star C Morning Star)

Thus

(41) $(\exists x)$(x C Evening Star & $\sim\Box$(x C Morning Star)).

Quine continues, "since the matrix quantified in (39) and the matrix quantified in (41) are mutual contraries, the x whose existence is affirmed in (39) and the x whose existence is affirmed in (41) are two objects: so there must be atleast two objects x such that x C Evening Star. If we introduce the term 'Venus' we could infer a third such object in similar fashion", (p.47, ibid).

Thus Quine is led to the conclusion that the contemplated version of modal logic is committed to an ontology which repudiates material objects and leaves only multiplicities in their ~~place~~ -- perhaps "individual concepts" to coin Carnap's phrase, i.e., the-Evening-Star-concept, the-Morning-Star-concept, etc.

Quine allows "that such an adherence to an intensional ontology, with the extrusion of extensional entities altogether from the range of values of variables is indeed an effective way of reconciling quantification and modality. The cases of conflict between quantification and modality depend on extensions as values of variables. However, in the object language where we may unhesitatingly quantify over modalities because extensions have been dropped from among the values of variables, even the individuals of the concrete world have dissappeared, leaving only their concepts behind them."

(Criticism in Carnap's 'Meaning & Necessity,p196.)

Conclusion

It seems clear that, on any view of the matter, when Leibniz's Law and modal logic are brought together, something has to give.

If they are compatible, a la Marcus and Fitch, then all identities are necessary and hitherto contingently significant identities are, under this interpretation, no longer identity sentences. Alternatively, we might attempt to preserve hitherto contingently significant identity sentences, by following Smullyan in arguing that, sentences of the form '$s = (\imath x)(Axw)$' do not in modal contexts lead by Leibniz's rule to antinomies, when we contextually define (overt and covert) descriptions a la Russell, and restrict the scope expansions to inferences which do follow *salva veritate*. However, as Prof.Wilson has argued, this approach is fraught with difficulties. He insists, I think reasonably, that descriptions are not terms of identity sentences: only name-identities are genuine identity sentences. Even so, it is not possible to avoid the "antinomy of the name relation", in the case name-identities, without further, quite major, restrictions. Moreover, as Quine has shown, in any modal language the variables range over intensions only, and this results in a "purified universe in which concrete objects are banished and replaced by pallid concepts".

Ofcourse, we could always follow Quine and give up the modal object language and its operators. No doubt we could survive without the modalities but, I hope to have shown, not all of Quine's arguments are sound arguments for doing so.

Finally, whatever is the outcome of the debate above, I hope to have shown that the argument which says that, because of the antinomies which (allegedly) arise in modal contexts, we do not have an across-the-board justification of Leibniz's Law, is unsatisfactory. We do not need to modify Leibniz's rule here, as Carnap suggests by constructing, in a semantical system, one principle for interchangeability of identicals in extensional contexts and another for intensional contexts. (And, any way., the argument for such principles depends upon unwarranted assumptions about the nature of language). Furthermore, we do not need to give up Leibniz's rule as a characterisation of identity and talk instead of "kinds" or "degrees" of identity, (whatever these might be), as people like Rescher, Hintikka *et.al.*, want to do. I hope to have shown that such proposals are wholly unsatisfactory.

CHAPTER THREE

LEIBNIZ'S LAW AND INTENTIONAL CONTEXTS

Consider the following examples :-

(1) The author of Marmion is the author of Waverley

(2) George IV believes that the author of Marmion is Scotch

(3) George IV believes that the author of Waverley is Scotch.

(4) Cicero = Tully

(5) Tom knows that Cicero denounced Cataline

(6) Tom knows that Tully denounced Cataline

(7) Oedipus's mother is Jocasta

(8) Oedipus wanted to marry Jocasta

(9) Oedipus wanted to marry his mother.

In each of these examples a true premiss, on the second line of each argument, gives rise to a false conclusion, when substituting upon an identity statement. The contexts are characterised by so-called verbs of propositional attitude -- e.g., 'knows' 'believes' 'thinks' 'wonders about' 'expects' 'wants' 'hopes' etc. Since identity statements consist of two referential expressions, each referring to the same object, it is possible that the object is known, believed, wanted etc., under one reference and not under the other. In fine, for any identity a = b it is possible that X knows a and X does not know b., e.g. Oedipus's mother is identical with Jocasta; although it is true that Oedipus wanted to marry Jocasta, it is false that Oedipus wanted to marry his mother.

Obviously these psychological contexts create a problem of intersubstitution for any identity statement so, once again, it seems we are faced with a drastic restriction on Leibniz's Law.

Following the strategy of the previous chapter, we will examine :-

First, an argument which attempts to resolve the difficulty by showing Leibniz's Law and intentional contexts are compatible.

Second, where it is claimed that Leibniz's Law and intentional contexts are incompatible and Leibniz's Law is modified.

Third, where Leibniz's Law and intentional contexts are claimed to be incompatible and sentences expressing intentional attitudes are replaced by certain inscriptional sentences.

<u>Leibniz's Law and intentional contexts are compatible</u>

In his article 'Leibniz's Law in Belief Contexts', R.M.Chisholm argues that Leibniz's Law in the form

(10) "for every x and for every y, if x is identical with y, then whatever is true of x is true of y"

need not involve us in difficulties in belief contexts. Consider the following argument Chisholm uses to show this :-

(11) The author of Marmion is identical with the author of Waverley.

(12) Although it is true George believed that the author of Marmion is Scotch it is false that George believed that the author of Waverley is Scotch.

Chisholm next requires a premiss which will serve as a general statement about the relevance of belief to reality.

(13) For every x, if anyone believes that x has the preoperty F then his believing x is F is something that is true of x; and if he does not believe that x is F, then his not believing x is F is also something that is true of x.

Hence <u>a fortiori</u> we seem to be committed by (10) - (13) to the contradictory conclusion

(14) There exists an x such that George believes that x is Scotch and such that it is false that George believes that x is Scotch.

To solve this problem we must show either that one of the premisses is false or that there is no justification for believing that the premisses commit us to the conclusion. The validity of the argument seems unimpeachable. Furthermore, to suppose (11) or (12) are the source of the difficulty, Chisholm thinks, is to call into question perfectly unproblematic premisses. It seems, either Leibniz's Law on (10), or premiss (13) is responsible for the unacceptable conclusion (14). Use of a principle such as (13) is required, in intentional contexts, in order that we may employ any description in <u>any</u> referential position. To deny (13) would result in the unacceptable thesis that we cannot bear relations to actual things in virtue of our beliefs, (p.245 ibid); we cannot, in intentional contexts, go beyond the circle of our own ideas. It seems then we must deny Leibniz's Law.

According to Chisholm, we may avoid result (14) and retain Leibniz's Law, by employing the distinction of belief sentences which are *in sensu composito* and those *in sensu diviso*,[43] (a distinction which corresponds with one that Quine draws between "notional" and "relational" senses of propositional attitudes.) In belief contexts *in sensu composito*, the subordinate clause is being used to describe the content of belief whereas, in contexts *in sensu diviso*, the subordinate clause refers to the object of belief. Only in the latter case can we quantify into belief contexts. Chisholm also notes, that sentences *in sensu composito* do not imply the corresponding sentences *in sensu diviso*.[44]

He goes on to claim that all sentences expressing propositional attitudes may be paraphrased as a disjunction of *in sensu composito* and *in sensu diviso* statements. For example, we may construe premiss (2) above, as either *in sensu composito*

(2') George believes that there exists just one thing x such that x wrote Marmion and x is Scotch.

which, put schematically, reads

$[Bg]\ (\exists y)\ [(x)(Mx \equiv x = y)\ \&\ Sy]$ where Bg represents 'George believes'

Or, alternatively, we may construe (2) *in sensu diviso*,

(2'') There exists an x such that x wrote Marmion and George believes x to be Scotch.

which, put schematically, reads

$$(\exists y)\ [(x)(Mx \equiv x = y)\ \&\ BgSy]$$

Chisholm argues, if we are to avoid the invalid inference on line (3), then (2) must be regarded as a disjunction of (2') and (2''), i.e. one of the disjuncts being <u>in sensu composito</u> the other <u>in sensu diviso</u>. In other words, he construes (2) as

$$[Bg]\ (\exists y)[(x)(Mx \equiv x = y\ \&\ Sy]\ V\ (\exists y)\ [(x)(Mx \equiv x = y)\ \&\ BgSy]$$

a disjunction, telling us that one or the other of the two possibilities, but not both, obtain.

Let us now proceed to the crux of Chisholm's argument. Using the <u>in sensu composito</u> - <u>in sensu diviso</u> distinction to analyse (12), he suggests that (12) should be paraphrased as follows :-

(a) It is false that: there exists just one thing x such that x wrote Waverley and George believes x is Scotch.

(b) It is false that: George believes that there exists just one thing x such that x wrote Waverley and x is Scotch.

Either

(c) There exists just one thing x such that x wrote Marmion and George believes that x is Scotch

Or

(d) George believes there exists just one thing x such that x wrote Marmion and that x is Scotch.

Formally we may represent the analysis above as

$$\{\sim(\exists y)\ [(x)(Wx \equiv x = y)\ \&\ BgSy]\ \&\ \sim Bg(\exists y)[(x)(Wx \equiv x = y)\ \&\ Sy]\}$$
$$\&\ \{(\exists y)\ [(x)(Mx \equiv x = y)\ \&\ BgSy]\ V\ \ Bg(\exists y)[(x)(Mx \equiv x = y)\ \&\ Sy]\}$$

Summarily, Chisholm's argument for the analysis of (12) in terms of (a) - (d) is that, since statements of the form (2) are paraphrasable by a disjunction of their contexts _in sensu composito_ and _in sensu diviso_ their denials are therefore paraphrasable by a "conjunction" of the corresponding _in sensu composito_ and _in sensu diviso_ contexts, (i.e., -(p v q) = -p & -q DeM.)

Chisholm's claim, we have noted, is that under this interpretation there is no question of deriving the contradictory (14), (viz. There exists an x such that George believes that x is Scotch and it is false that George believes that x is Scotch). He supports this claim with the following arguments.
First suppose (c) is false. Then (d) is true. However (d) is a statement telling us what George believes, that is to say, (d) is _in sensu composito_, whereas the conclusion (14) is _in sensu diviso_. So if (c) is false, (14) does not follow from (10) - (13). What we have to show here is that the conclusion follows from premisses _in sensu diviso_. (Incidentally, we cannot detach (14) by modus ponens of (d) on the antecedents of the conditionals in (13), since the are also _in sensu diviso_).
Suppose now that (c) is true. (c) is _in sensu diviso_.
Now (11) and (c) imply that (a) is false. However, (12) implies that (a) is true. So if (c) is true then (12) is false, (i.e., (c) and (12) are mutually inconsistent). Again the conclusion (14) will not follow from (12). This is to say, (14) cannot be derived from either of the disjuncts in (12), (i.e., if (c) is false then (d), _in sensu composito_, is true and (14), _in sensu_

diviso will not follow and if (c) is true then (12) is false so (14) does not follow. Either way, we cannot derive the unacceptable conclusion (14), (Cf., p.250, ibid).

Chisholm concludes that he has shown that (14) does not follow from (10) - (13), so Leibniz's Law does not break down in belief contexts. The difficulty, he maintains, is not that the argument contains Leibniz's Law as a premiss but rather that there is no way, given the premisses (10) - (13), of deriving (14). The contradictory conclusion is not due to the application of Leibniz's Law but to condoning an inference from statements in sensu composito to statements in sensu diviso. According to Chisholm, this conclusion may be generalised to take care of all other cases where Leibniz's Law, in application to psychological contexts, seems to lead to similar difficulties.[45]

Chisholm's defense of Leibniz's Law, as a criterion of identity belief sentences, is said to be defective for a number of reasons. For example, (in his Review of 'Contributions to Logic and Methodology', p.539), D.Føllesdal objects that Chisholm assumes that Leibniz's Law does not break-down in sensu diviso contexts, in order to make the inference from (11) and (c) to demonstrate the falsity of (12). Føllesdal says that Chisholm is guilty of a petitio here. However, I think he misses Chisholm's point, which is not to derive Leibniz's Law as a conclusion, (i.e., Leibniz's Law is listed as premiss (10), but rather, to show that Leibniz's Law does not fail, since we are not warranted in deriving (14) from (10) - (13). (Incidentally, Føllesdal has a better objection when he questions Chisholm's assumption that belief sentences are "indeterminate" rather than ambiguous as Føllesdal says -- but this too, given Chisholm's premisses, is defensible, p.539, ibid.) Objections might be raised at the details of (my elaboration of) Chisholm's condensed analysis by which he claims to demonstrate that (14) does not follow. However, all of these objections I think miss the major difficulty with Chisholm's putative "solution" to the antinomy.

The point is this. Chisholm's treatment of the difficulties confronting Leibniz's Law in belief contexts is parallel to Smullyan's way of dealing with scope ambiguities, of sentences involving definite descriptions, in modal contexts. Unfortunately, the *ad hoc* character of Chisholm's example and the not-at-all-obvious way he sets about dealing with it, serve only to obscure the issues raised by the simple examples we began with. Let me illustrate Chisholm's position with the simpler version of his example, our (1) - (3) above.

(1) The author of Marmion = The author of Waverley

(2) George believes that the author of Marmion is Scotch

(3) George believes that the author of Waverley is Scotch.

Now we mentioned above, that following Chisholm's analysis, premiss (2) can be interpreted as saying one of two things, either

(i) George believes that there is exactly one author of Marmion and that this person is Scotch.

<u>in sensu composito</u>

or

(ii) There is exactly one author of Marmion and George believes that this person is Scotch

<u>in sensu diviso</u>

Now, for the purpose of explicating Chisholm's position, let us assume that premiss (3) above is <u>in sensu diviso</u>.

i.e. (3') There is exactly one author of Waverley and George believes that this person is Scotch.

The nub of Chisholm's argument is this. If (2) is interpreted in terms of (i), since (i) is <u>in sensu composito</u>, we are not entitled to draw the conclusion on line (3'), (which is <u>in sensu diviso</u>). The example (1) - (3') , under this interpretation is invalid.

On the other hand, if premiss (2) is interpreted in terms of (ii), since (ii) is <u>in sensu diviso</u>, we are entitled to draw the conclusion on line (3').

To spell out the conclusion more fully, according to Chisholm, for Leibniz's Law to be compatible with intentional contexts, arguments of the form (1) - (3) above must be interpreted as follows:-

(2') There is exactly one author of Marmion and George believes that this person is Scotch.

(1) The author of Marmion = The author of Waverley

(3') There is exactly one author of Waverley and George believes that this person is Scotch.

On this interpretation, Leibniz's Law combines quite unproblematically with propositional attitudes.

The foregoing represents fairly, I think, Chisholm's position, that now[46] seems to me to be mistaken.

The trouble is, (3') does not follow by substitution of (2') into (1). For the Left Hand Side of (1) -- 'the author of Marmion', does not occur in (2'), (i.e., only 'the author of Marmion' does). Thus we cannot substitute, in accordance with Leibniz's rule, from (2') into (1).

Notice, premiss (1) is of the form '$(\iota x)(Fx) = (\iota x)(Gx)$'. The difficulties of treating sentences of the form '$(\iota x)(Fx) = (\iota x)(Gx)$' as genuine identity sentences, rather than abbreviations of their expanded multiply general form, viz., $(\exists y)[y = (\iota x)(Gx) \;\&\; (x)(Fx \equiv x = y)]$, seem here, to me, to be decisive. Although, ordinarily, there may be a "derived" rule which permits interchange of $(\iota x)(Fx)$ and $(\iota x)(Gx)$ on the basis

$$(\iota x)(Fx) = (\iota x)(Gx)$$

there will also be limitations on such a derived rule; for instance, ordinary logic will not give the expanded conclusion from the expanded premisses.[47]

To summarise, if we were first inclined to accept the inference from (1) and (2') to the conclusion (3'), (as I was), it is because we regard the inference as warranted by Leibniz's rule; viz., the replacement of 'the author of Marmion' in (2') by the 'the author of Waverley' in (3'), on the basis of the (alleged) identity statement (1). However, even though we have formulated (2') and (3') *in sensu diviso*, a la Chisholm, we cannot derive (3') from (2') substituted on (1). The reason for this, to labour the point, is that (1) is in contracted form, and when expanded into its primitive notation, is *not*, in fact, a genuine identity sentence but is, in fact, a multiply general sentence.

So much for Chisholm's defective analysis and all examples of the form (1) - (3) above, (which by the way includes example (7) - (9), since here again we have, in premiss (7), a multiply general sentence masquerading as an identity sentence). In the conclusion of this chapter, I hope to show how the example (4) - (6), which undoubtedly contains a genuine name-identity, is to be reconciled with Leibniz's rule. However, let us now turn to the arguments which do not accept that Leibniz's rule, in its unqualified form, and intentional contexts are compatible. This is to say, arguments which seek to modify Leibniz's rule in belief contexts while at the same time attempting to guarantee substitutivity *salva veritate* as a condition to be satisfied by co-referential terms, in all genuine identity sentences. The claim, this is to say, that we can have intersubstitutivity of co-referential terms only by employing a stronger criterion than $(x)(y)(x = y. \supset .Fx \equiv Fy)$ in belief contexts.

Leibniz' Law is modified.

Carnap, in 'Meaning and Necessity', (p.53ff), proposes a solution to the (apparent) failure of substitutivity of identicals in intentional contexts, by replacing L-equivalence, by an even stronger criterion of identity namely, "intensional isomorphism". This, speaking generally, involves a part-by-part correspondence within L-equivalent sentences.

Let us begin this section by explaining, briefly, the notion of intensional isomorphism with an example. Carnap considers the expressions '2 + 5' and 'II sum V'. These occur in the language system S_1, in which the expressions 2, 5, II, V are numerical expressions and '+' and 'sum' are numerical operations. Further, we may suppose that according to the semantical rules of S_1, '2' is L-equivalent to 'II', '5' is L-equivalent to 'V', and '+' is L-equivalent to 'sum'.

Thus Carnap argues :-

> "We shall say that the two expressions are intensionally isomorphic or that they have the same intensional structure because not only are they L-equivalent as a whole (i.e., both being L-equivalent to 7) but they consist of three parts in such a way that the corresponding parts are L-equivalent to one another and hence have the same intension".
> (p.56, ibid).

Intensional isomorphism then is to serve as the condition for intersubstitutivity of terms salva veritate -- rather than straight-forward substitution of L-interchangeable terms.

Carnap's motive for using intensional isomorphism, in his analysis of identity with respect to belief contexts, needless to say, is his recognition of the need for a stronger relation than L-equivalence in such contexts. L-equivalent sentences are not L-interchangeable in all belief contexts. Consider again the antinomy that results under the unqualified L-interchangeability of L-equivalent sentences:-

Let D and D' be abbreviations of two logically equivalent sentences, (e.g., 'the number of of inhabitants of Chicago is greater than three million' and ' the number of inhabitants of Chicago is greater than $2^6 \times 3 \times 5^6$ '). Thus

(15) D is L-equivalent to D'.

Now assume D to be a sentence which a person John believes; or, in Carnap's words, "John is disposed to an affirmative response to D". This is to say

(16) John believes that D

Clearly, it follows from the above that if interchangeability of expressions can take place by substitution on the L-equivalent relation holding between them (i.e., L-interchangeability), in Carnap's words, "John is disposed to make an affirmative response to some sentence in some language, which is L-equivalent to D", (p.55) Call this sentence D'. Thus

(17) John believes that D'.

It is easy to see the nub of the inadequacy here. (15) compels us to say that "John believes that D and D'" -- while pre-analytically John may very well be said to believe that D but not that D'. Hence Carnap proposes using the stronger relation of intensional isomorphism.

> "The two sentences must, so to speak be understood in the same way; they must not only be L-equivalent in the whole but consist of L-equivalent parts, and both must be built up out of these parts in the same way"
>
> (p.55, ibid).

Thus, Carnap proposes that only the L-equivalence of each <u>term</u> in the expressions permits their universal interchange in intentional contexts. Hence if

(18) D is intensionally isomorphic with D'

and (19) John believes that D

then (20) John believes that D'

As we noted, Carnap analyses 'belief that D' as a disposition to give an affirmative response to a given sentence D. Thus 'John believes that it will rain tomorrow' analyses into 'John is disposed to give an affirmative response to the sentence 'It will rain tomorrow''. (In fine, linguistic "entities" are the objects of our beliefs or rather, of our intentional attitudes). Carnap is then able to offer the following semantical criteria for intersubstitutivity <u>salva veritate</u> in belief contexts, i.e., for sentences of the form (16) above.

> "There is a sentence $\mathfrak{S}_i$ in a semantical system S' such that (a) $\mathfrak{S}_i$ is intensionally isomorphic to D and (b) John is disposed to an affirmative response to $\mathfrak{S}_i$."
>
> (p.62, ibid)

Curiously, Carnap appears to over-look the possibility that his argument used to show the failure of intersubstitutivity salva veritate of L-equivalent terms, (viz., (15) - (17) above), may be extended to disqualify the inference from (18) and (19) to (20). It seems to me we can reasonably claim that, in a perfectly good sense of belief, while

(21) John believes that 'eye-doctors are eye-doctors'

and that 'eye-doctors are eye-doctors' is intensionally isomorphic with 'eye-doctors are oculists', it is not the case that

(22) John believes that 'eye-doctors are oculists'

(Suppose, for the sake of clarifying the point, John positively disbelieves that 'eye-doctors are oculists').

Now it might be argued that, on Carnap's criteria, John's believing (21) and not (22) is impossible, or rather, Carnap has ruled out, because intensionally isomorphic sentences are to be "understood" in the same way, (Cf., "The two sentences must, so to speak be understood in the same way"..(p.55, ibid)). However, his use of "understood" here is seriously ambiguous. As we have seen, Carnap's treatment of intensional isomorphism depends throughout on wholly semantic criteria. Not upon the psychological reactions of persons to sentences. Hence "being understood in the same way" presumably refers to semantic characteristics. Indeed, if Carnap is referring to psychological reactions here, then he requires independent arguments for his curious treatment of the psychological question -- since to exclude, as Carnap does, the psychological possibility that John may believe (21) and not (22) is a psychological theory, (and a highly improbable one at that). In fine, the limitations which Carnap accepts might prevent John from seeing one sentence is L-equivalent to another, might also prevent him from seeing one sentence is intensionally isomorphic to another. This seems to me to be the crux of Benson Mates objection to Carnap's criterion of identity in belief sentences.

Benson Mates, in his article 'Meaning and Interpretation' argues:-

Let D and D' be abbreviations of two intensionally isomorphic sentences. Then the following are intensionally isomorphic:-

(23) Whoever believes that D believes that D.

and (24) Whoever believes that D believes that D'.

Now the sentence (25) below is true

(25) Nobody doubts that whoever believes that D believes that D.

However, the sentence (26) below is very likely false

(26) Nobody doubts that whoever believes that D believes that D'.

Sentences (25) and (26) are intensionally isomorphic, (a la Carnap), but if (26) is false, as it is likely to be, then the two intensionally isomorphic sentences differ in truth-value.

Mates suggests that his example may invalidate Carnap's original proposal. Carnap, on the other hand, takes Mates's criticism to heart, accepting that his thesis in its present cannot refute Mates point.[48]

H.Putnam, in 'Synonymity and the Analysis of Belief Sentences', tries to meet Mates's objection by tightening the notion of intensional isomorphism such that it includes "identical logical structure" as a further condition to be satisfied by intensionally isomorphic sentences. To illustrate his point, let 'Greek' and 'Hellene' be synonymous expressions. Now consider

(27) All Greeks are Greeks

and (28) All Greeks are Hellenes.

If these two sentences "do not feel synonymous" Putnam argues, "it is because the two differ in logical structure", (p.118, ibid). The sentence (27) has the form 'All F are F' ; while sentence (28) has the form 'All F are G'.

To expand Putnam's point to belief contexts, the sentence

(29) Whoever believes that all Greeks are Greeks believes that all Greeks are Greeks

differs in logical structure from the sentence

(30) Whoever believes that all Greeks are Greeks believes that all Greeks are Hellenes.

Putnam concludes

> "The foregoing consideration leads us to the following modification in the definition of intensional isomorphism (1) Two expressions are intensionally isomorphic if they have the same logical structure and (2) if the corresponding parts are L-equivalent".
> (p.122, ibid)

The upshot of Putnam's emendation is that, if we allow in Mates's examples (23) and (24) above, that sentences D and D' have a different logical structure then they are not, on Putnam's analysis, intensionally isomorphic. Hence we are not disturbed that 'Nobody doubts that (23)' may have a different truth-value from 'Nobody doubts that (24)'

However, even though we may grant Putnam the point that difficulties pertaining to logical structure are met with his revised criterion, he fails to meet the difficulty, which I have suggested is at the root of Mates's objection to Carnap, this is to say, the factual possibility that a person may believe, in a perfectly good sense of "believe", only one of two (or more) intensionally isomorphic expressions. Consider three intensionally isomorphic sentences A, B, C, such that they each have the same logical structure; e.g., let A be 'The snow is white', let B be 'Der Schnee ist Weiss', let C be 'La neige est blanche'. Now following Putnam

(31) Whoever believes that A believes that B

and (32) Whoever believes that B believes that C

The point is, it is perfectly plausible to argue the factual possibility that 'John believes that (31)' whereas 'John does not believe that (32)' (i.e., he positively disbelieves (32).

I think the correct solution to this problem is given by A.Church, in 'Intensional Isomorphism and Identity of Belief', who rebuts Mates's criticism by denying that, in such cases, one of the sentences is really doubted while the other is not.

Church argues that ostensible divergence in belief reponse to intensionally isomorphic pairs, does not in fact hold of intensionally isomorphic sentences but of "equivalent" sentences. Suppose D and D' are intensionally isomorphic sentences, then "'D' is true" is intensionally isomorphic to neither and can be taken to be the real object of doubt where it is claimed that D is believed and D' is doubted. For instance, if in example (30) Hellene is really being used, by the person who gives the counter-example, as a synonym for Greek, then believing that all Greeks are Hellenes just is believing that all Greeks are Greeks. On the other hand, if the synonymous term for 'Greeks', namely 'Hellenes', is being mentioned, we replace the mentioned subsentence 'All Greeks are Hellenes' by "'All Greeks are Hellenes' is true" -- i.e., moving the subsentence into the meta-language, which then yields

(30') Whoever believes that all Greeks are Greeks believes that the sentence 'All Greeks are Hellenes' is true.

This rules out the unacceptable inference obtained where (29) and (30) are supposed to be intensionally isomorphic.

Or to revert to Carnap's example: if a) '2 + 5 = 7' is intensionally isomorphic to b) 'II sum V = VII' and both of these expressions are being used, if John believes a) then John believes b).

This is not to say that Church is uncritical of Carnap's analysis of identity in belief sentences. Indeed Church, in his

'On Carnap's Analysis of Statements of Assertion and Belief', raises a decisive objection to Carnap's thesis. (Prof. Wilson in 'Concept of Language', p.123, gives a clear exposition of this objection, so I will take the liberty of paraphrasing his exposition of Church's argument).

Suppose we have the sentence,

(33) Seneca said that man is a rational animal

Its translation into French is

(34) Seneca dit que l'homme est un animal rational

Now we can give the following paraphrase of (33) on Carnap's proposed analysis

(35) There is a sentence S_1 in a language L such that
(a) S_1 in L is intensionally isomorphic to "man is a rational animal" in English and (b) Seneca wrote S_1 in L.

Similarly, on Carnap's analysis, (34) expands into

(36) Il y a une phrase S_1 dans une langue L de sorte que
(a) S_1 dans L est intentionellement isomorphique par rapport a "l'homme est un animal rational" en Francais at (b) Seneca ecrit S_1 dans L.

The difficulty is that (36) is <u>not</u> a translation of (35). Statement (35) would in fact presumably translate into

(37) Il y a une phrase S_1 dans une langue L de sorte que
(a) S_1 dans L est intentionellement isomorphique par rapport a "Man is a rational animal" en Anglais, et
(b) Seneca ecrit S_1 dans L.

Clearly (37) could not convey to a Frenchman ignorant of English what (35) conveys to an Englishman. The reason for this, as Prof.Wilson notes, is that 'English' is a description and the Frenchman is not familiar with, as the Englishman is, its descriptum. (In the same way, it is not misleading or surprising to an audience familiar with the story, if they are told Oedipus wanted to marry his mother but this is only because they know that Oedipus was unaware that Jocasta, whom he wished to marry, was in fact his mother. Similarly, 'The author of Waverley is Scotch' conveys less to a person who does not know the author of Waverley is Sir Walter Scott than to a person who has that information.)

Because of this apparently overwhelming difficulty with Carnap's rule (or any rule) for intensionally isomorphic sentences, Church goes on to replace intensional isomorphism, as a criterion of identity for belief sentences, by "synonymous isomorphism",('Intensional Isomorphism and Identity of Belief', p.65ff). By synonymous isomorphism, Church proposes that Carnap's requirement of the L-equivalence of terms in intensionally isomorphic expressions, be replaced by the "synonymity" of terms in isomorphic sentences. For two sentences to be synonymously isomorphic, each individual constant and predicator constant in one must be synonymous with its corresponding individual or predicator constant in the other. In addition he requires that steps of the following kinds shall be allowed :-

> "the replacement of an abstraction expression by a synonymous predicator constant; the replacement of a predicator constant by a synonymous abstraction expression; the replacement of an individual description by a synonymous individual constant; the replacement of an individual constant by a synonymous individual description."
>
> (p.67, ibid.)

Summarily, Church claims that synonymous isomorphism, as thus defined for a language S_1, may be extended in a more or less obvious way, to replace Carnap's criterion for identity of belief.

Carnap rejects Church's proposals, (Cf., Fn.48 above), and I think we should. (I do not think Church gives us anything very much different from Carnap or Putnam -- unless there is far more to his notion of 'synonymy' than he indicates in his article ; and, even so, I cannot see why Church supposes that 'synonymous isomorphism' would deal with the translation difficulty, concerning Seneca, we mentioned earlier. By replacing 'synonymously isomorphic' for 'intensionally isomorphic' in the examples (33) - (37) we are still faced with much the same problem , viz., the unavoidable reference to the English language which conveys nothing to a Frenchman.)

There are a number of other major obstacles that stand in the way of Carnap's proposal of intensional isomorphism as the relation permitting intersubstitutivity in belief contexts. As we noted in the previous chapter, such a criterion suffers from the underlaying defects of his claims concerning semantical systems, the main defect being his treatment of language as a system of rules.

Finally, even if we could accept his criterion for identity in belief contexts, there is something decidedly odd with the suggestion that linguistic entities are the objects of our intentional attitudes. Consider again Carnap's formula for translating intentional statements: "John believes that D" is to be translated, in effect, into "John is disposed to an affirmative response to the sentence 'D'". To ensure that intersubstitution is well-behaved in intentional contexts, we mention, rather than use, the sentence referring to the content or object of John's belief.

However, at worst, Carnap's use of quotation marks here, taken literally, would imply falsely that if John believed there are Martians then there is a man who is disposed to an affirmative response to the sentence 'there are Martians'. At best, for Carnap's use of quotation marks to be acceptable, in an analysis of John's belief that there are Martians, would require that the quotation marks indicate, not mention as Carnap thinks, but the use the quoted sentence would have had had it occurred without the quotation marks. Such an oddity is a heavy price to pay for well-behaved intersubstitution -- and this is granting that Carnap's proposals with respect to intensional isomorphism actually work. Clearly, disposition to give an affirmative response to a "sentence" is not a satisfactory characterisation of our intentional attitudes and their objects.

Let us now turn to the view held by Quine, which seeks to retain Leibniz's Law in its unrestricted form and instead of saying that our intentional attitudes have linguistic entities as their objects, a la Carnap, argues that certain sentences which relate people to words may be used to perform all the functions of intentional sentences, without the accompanying antinomy. In fine, the view in which sentences expressing intentional attitudes are rejected in favour of certain inscriptional sentences.

Sentences expressing intentional attitudes are rejected.

First, a little more detailed account of Quine's notion of "referential opacity" is called for. In his 'Three Grades of Modal Involvement' Quine says, "we may speak of a context as referentially opaque when by putting a statement $\emptyset$ into that context we may cause a purely referential occurrence of $\emptyset$ to be not purely referential in the whole context".[49] And in 'Word and Object' we are told

> "An opaque construction is one in which you cannot in general supplant a singular term by a co-designative term (one referring to the same object) without disturbing the truth-value of the containing sentence. In an opaque construction you cannot also in general supplant a general term by a co-extensive term (one true of the same objects), nor a component sentence of the same truth-value, without disturbing the truth of the containing sentence". (p.151)

Accordingly, positions in sentences for which the principle of substitutivity is not a valid mode of inference are referentially opaque: they are sentential positions such that expressions occupying them do not succeed in referring to anything -- although the very same expression will refer in "referentially transparent" (or open) positions.[50] The upshot of this for Quine is that failure of substitutivity does not provide exceptions to Leibniz's Law but rather, provides cases of referential opacity.

An objector might reasonably argue here,[51] "Why can't we say the phrase 'referential opacity' simply baptizes exceptions to Leibniz's Law. All Quine has given is a characterisation of the exceptions to Leibniz's rule, that is, they involve substitution into referentially opaque positions". What is the criterion of the referential opacity of a position ? It is just that the principle of substitutivity, in its unamended form, fails to be a valid mode of inference for that position. I think this criticism is fairly made against Quine's formulation and employment of the referentially transparent-opaque distinction. However, as I hope to show at the conclusion of this chapter, a position not unlike Quine's can be defended successfully to demonstrate the compatibility of name-identities with Leibniz's Law in intentional contexts. But first we have to deal with two difficulties which Quine presents, one of which makes Quine relinquish the transparency of belief.

As we noted earlier, Quine employs a version of the distinction of belief sentences _in sensu composito_, which Quine calls the "notional" context, and belief sentences _in sensu diviso_ which Quine calls the "relational" context. As far as Quine is concerned, all notional contexts are referentially opaque; substitution, if it were permitted into belief sentences interpreted in their notional context, would lead invariably to antinomies. The question presents itself -- "Are belief sentences interpreted in their relational context, where they do refer transparently to the object believed (seemingly), free from antinomy when substituting into them in accordance with Leibniz's rule ?"

Quine recognises that even in this latter case we are led to countenance certain oddities. Take the example

(4) Cicero = Tully

(5) Tom believes that Cicero denounced Cataline

Now by Leibniz's Law, since (4) and (5) are, by hypothesis, in a relational context, we may conclude

(6) Tom believes that Tully denounced Cataline.

However, suppose in this case that we are wrong to conclude (6) since, as a matter of fact, while Tom does believe that Cicero denounced Cataline, also he believes that Tully did _not_.

> "Tully", Tom insists, "did not denounce Cataline. Cicero did". Surely Tom must be acknowledged to believe, in every sense, that Tully did not denounce Cataline and that Cicero did. But still he must be said also to believe, in the referentially transparent sense, that Tully _did_ denounce Cataline. The oddity of the transparent sense of belief is that it has Tom believing that Tully did and did not

> denounce Cataline. This is not yet a self-contradiction on our part or even on Tom's, for a distinction can be reserved between (a) Tom's believing that Tully did and Tully did not denounce Cataline and (b) Tom's believing that Tully did and did not denounce Cataline."
>
> ('Word and Object', p.148)

Quine notes that the oddity is there, and we have to accept it as the price for allowing the transparency of belief sentences, (in relational context), to combine with the unrestricted application of Leibniz's rule.

What are we to make of the oddity Quine refers to ? Clearly, given the three sentences (4) -(6) are referring transparently, (viz., the belief sentences are relational), <u>we</u> must insist that Tom does believe that Tully denounced Cataline. In other words

(4) Cicero = Tully

(5') There exists an x such that x is Cicero and Tom believes that x denounced Cataline

(6') There exists a y such that y is Tully and Tom believes that y denounced Cataline.

How then are we to explain <u>Tom's</u> denial of (6'). It is not simply a matter of Tom being irrational: Quine does not regard Tom's denial of (6') as an explicit contradiction nor is it supposed that Tom believes a contradiction. It is perfectly plausible for Tom to believe that p and to believe that not-p; while at the same time to argue that Tom does not yet believe (p & -p). In other words there is an individual "of whom" Tom (unwittingly) believes contradictory properties. If this is case, and Tom does not know he holds contradictory beliefs (about the same individual), the oddity Quine refers to does not seem to be so stubborn or intractable. The fact that Tom believes Cicero denounced Cataline and does not believe that Tully denounced Cataline, despite the fact that Cicero is identical with Tully, can be remedied, hopefully, by telling Tom that Cicero and Tully are one and the same person. In short Tom can only be convicted of irrationality if he knows that he holds contradictory beliefs, and nothing is indicated in Quine's example that he does; surely then, what is understood by

(38) Tom does not believe that Tully denounced Cataline

(given that premisses (4) - (6) are referentially transparent and Tom is rational), is

(39) Tom does not believe that Tully denounced Cataline because he does not know that Cicero = Tully.

Otherwise, either (4) - (6) are not referentially transparent or Tom is not rational.

Once we are clear about the nature of the oddity involved, I fail to see Quine's example poses a threat to Leibniz's Law, or *vice versa*. I suspect that similar reasons to the above prompted Quine's remark that "this much oddity on the part of belief is tolerable", (p.148, ibid).

However, Quine continues, "more remains that is not". so let us now consider the argument which leads Quine to finally relinquish the transparency of belief.

Quine proposes "where 'p' represents a(true)sentence, let us write 'dp'...as short for the description:

the number x such that ((x = 1) and p) or ((x = 0) and not p)."

(i.e., $dp = (\imath x)\left[(x = 1) \& p) \vee ((x = 0) \& -p)\right].$)

Now if Tom is rational, then

(40) Tom believes a sentence of the form dp = 1 if and only if Tom believes a sentence p.

Hence since

(41) Tom believes that Cicero denounced Cataline

it follows

(42) Tom believes that d(Cicero denounced Cataline) = 1.

Now since 'p' represents a true sentence

(43) dp = d(Cicero denounced Cataline)

And since by Leibniz's Law we are permitted to freely substitute co-referential terms, we get

(44) Tom believes that dp = 1

~~from which it follows, (if Tom is rational),~~

(45) Tom believes that p.

Since 'p' represents any arbitrarily true sentence, the result on (45) is absurd. In sum, it seems to Quine that, from the true premiss 'Tom believes that Cicero denounced Cataline', by the application of Leibniz's Law, we can argue that Tom believes everything which is true. (If he had believed something which is false the same argument could also "prove" that Tom believes everything which is false. Hence Tom ends up believing everything.)

There is an air of fallaciousness in Quine's argument which, though difficult to pin-point exactly, has to do with the claim that we can argue from the transparency of belief that Tom believes everything, (p.149, ibid). Presumably Quine is claiming to move from (40) to (45) by assuming, in each premiss, the referential transparency of belief. Clearly, premisses (42) and (44) are to be construed transparently -- as purely referential occurrences of the sentences, otherwise we could not infer (44) from (42) and (43). But are we entitled then to derive (45) ? This is to say, can we infer (45) from substituting (44) on (40) ? The trouble is (40) -- which is an assumption about Tom's logical acumen -- is by no means obviously referentially transparent. Premiss (40) appears to involve a non-referential occurence of 'dp', where 'dp' occurs in the subordinate clause of the antecedent in the first conditional (of the bi-conditional). The subordinate clause here refers to some number which we have characterised as 'dp' and concerning which Tom believes that it is equal to 1. If (38), or for that matter any of the premisses, are opaque constructions, then Quine's argument is invalid, since Quine seeks to show that from referential premisses alone Tom ends up believing everything.

The upshot of this argument for Quine is that belief statements can occur either transparently or opaquely and, either way, yield antinomies. He then has to face up to the problem of characterising propositional attitudes while preserving Leibniz's rule.

To this end, he introduces a technique not unlike Carnap's, in which instead of speaking of the "object" or "content" of intentional attitudes, proposes that certain sentences which relate people to words or other linguistic entities may be used to perform all the functions of intentional sentences, while ofcourse preserving the unfettered application of Leibniz's rule. What this amounts to is that instead of

(5) Tom believes that Cicero denounced Cataline

we may say

(46) Tom believes-true 'Cicero denounced Cataline'

(46) may be interpreted as a sentence affirming a certain relation to hold between Tom and a linguistic entity or inscription -- but a relation that is true only under the conditions where Tom believes that Cicero denounced Cataline is true.

Though Quine does not discuss it, the application of his inscriptional approach to the antinomy (4) - (6) seems quite clear. In the example (4) - (6), when ordinary statements of propositional attitude combine with Leibniz's rule, we obtain an unacceptable inference. Whereas, using the technique in which we relate Tom only to inscriptions, while we may say

(5') Tom believes-true 'Cicero denounced Cataline'

and (5') is true since Tom does believe Cicero denounced Cataline, and furthermore we may say,

(4) Cicero = Tully

is true, we cannot then derive by Leibniz's rule a statement of the form

(6') Tom believes-true 'Tully denounced Cataline'

since obviously we cannot apply Leibniz's rule in a quotational context.

There is a lot to criticise in Quine's proposals First, I do not think that Quine proves his case that referentially transparent belief sentences inevitably yield antinomies. I have argued that, in the first example Quine gives, the problem is quite simply resolved once we are clear about the nature of the oddity involved. As far as his second example is concerned, I have surmised that Quine's endeavour to show that belief in one one true referentially transparent sentence paradoxically commits us to belief of all true referentially transparent sentences, rests on the interpretation of one of his premisses as non-referential, (viz., in an opaque context), so his argument is invalid.

As far as his proposed notational reform of intentional sentences is concerned, I would argue that the plausibility of Quine's inscriptional approach depends on the assumption that certain semantic sentences are true of certain inscriptions, for example, "the English sentence 'Cicero denounced Cataline' means that Cicero denounced Cataline" This much seems reasonable enough. However, if these semantic sentences are abbreviations for intentional sentences, as they must be, are we not back to the original problem; this is to say, English speaking people use the sentence 'Cicero denounced Cataline' to express the belief that Cicero denounced Cataline. Moreover, since Cicero is Tully, English speaking people use the sentence 'Cicero denounced Cataline' to convey the belief that Tully denounced Cataline -- which, to say the least, is very odd. To put the matter another way, in English 'Cicero denounced Cataline' means that Cicero denounced Cataline. But Cicero is identical with Tully. Thus, in English 'Cicero denounced Cataline' means Tully denounced Cataline -- which is presumably false.[52]

Finally, like Carnap's translations, Quine's notational reform of intentional sentences is dubious, unquestionably awkward and unnecessary since we are able to combine "plain, old-fashioned" intentional sentences with Leibniz's rule, provided we attend carefully to what we are doing.

Conclusion

Clearly, the main problem with belief and other intentional propositions is that they are in general ambiguous, (or indeterminate, it doesn't matter), with respect to reference. Consequently a serious problem attends quantification across such contexts. These problems are highlighted when taken in conjunction with Leibniz's Law.

From Chisholm's (viz., Smullyan's) analysis we have seen the nature of the ambiguity in question. The difference between belief statements in sensu composito , which describe the content of belief, and in sensu diviso which refer to the object of belief. Only in the latter case can one quantify into a belief statement. Only in the latter case do belief sentences transparently refer. Now Chisholm's analysis is defective we found because he failed to recognise that his premiss (11), into which he is substituting, is not a genuine identity sentence. (However his strategy for dealing with the problem is instructive.)

Let us consider again now example (4) - (6).

(4) Cicero = Tully

(5) Tom believes Cicero denounced Cataline

(6) Tom believes Tully denounced Cataline.

Clearly, 'Cicero' and 'Tully' in (4) are individual terms; premiss (4) is a genuine identity statement in which the two terms do transparently refer to the one object

As far as premiss (5) is concerned, we can interpret it as saying either (i) in sensu diviso or (ii) in sensu composito, i.e.,

(i) There is exactly one x such that x is Cicero and Tom believes x denounced Cataline

or

(ii) Tom believes that there is exactly one x such that x is Cicero and x denounced Cataline.

Now when premiss (5) is interpreted in terms of (i) and substituted into (4), we may derive (6) as a valid inference, provided also that (6) is interpreted in sensu diviso.

(6') There is exactly one y such that y is Tully and Tom believes that y denounced Cataline.

On the other hand, under the interpretation of (5) in terms of (ii) we are not warranted in drawing (6) or any conclusion by substitution into (4), _not_ because Leibniz's rule has broken down, but because we have in the one argument muddled a non-referential occurrence of a term, i.e.,'Cicero' as it occurs in (ii), with a _prima facie_ referential occurrence of the term as it occurs in the identity statement, i.e., Cicero = Tully. To make the point again, when premiss (5) is interpreted in terms of (i), the term 'Cicero' refers to the object, rather than the content of belief, namely, the Roman orator. As the occurrence of the same term in the identity statement, _ex hypothesis_ refers to the Roman orator, when (i) is substituted into (4), since the terms refer to the same object, (6') is the only valid inference we can draw. ~~In this case, Leibniz's Law is perfectly compatible with the~~ intentional statement (as formulated above).

Notice I am not simply saying substitution in opaque contexts does not work. To hammer the point home, what I am saying is that since both terms of an identity statement _do_ perform their referential function, (viz., in any identity sentence "co-referential" terms are transparent), we cannot in an argument then take a non-referential occurrence of one or other of the(co-referential) terms and expect to substitute into the identity statement and thereby obtain a result which preserves the truth-value. Substitution in opaque contexts does not work -- but substitution of an opa que contexts into a *prima facie* transparent context doesn't work either. We are not simply baptizing exceptions to Leibniz's Law here. The point is, the occurrence of a name in a belief statement *in sensu composito* is *not* the re-occurrence of a "term" from an identity statement, albeit the same name.

In conclusion, if the truth-value is not preserved in the conclusion of an argument containing a belief statement *in sensu composito* this has nothing to do with the failure of Leibniz's rule but simply indicates the break down of reference; the attempt to interchange a non-referential occurrence of a name with what *ex hypothesis* is a referential occurrence of that name, *qua term*, in an identity statement. I conclude that, so long as we careful, i.e., so long as we ensure that the intentional sentence is *in sensu diviso*, Leibniz's Law and intentional contexts are perfectly compatible -- or perhaps more cautiously, one needs to see better arguments to show that they are not.

CHAPTER FOUR

LEIBNIZ'S LAW AND THE RELATIVIST VIEWS OF IDENTITY

Speaking generally, and in the clearest cases, the doctrine I have called "relativised identity" is the view that there can be in a language more than one identity relation; the relation expressed by '=' is relative, since any ordered pair may stand in this relation relative to one thing but not to another, i.e.,

$a = b \quad \& \quad \sim(Fa \equiv Fb)$

Consequently, adherents of this view argue, satisfaction of Leibniz's Law of unrestricted interchange of co-referential terms _salva veritate_ is _not_ a necessary condition of the truth of the identity statement. On the contrary, whatever properties are integral to the purport of '=' in statements, this cannot include unrestricted interchange of co-designative terms.

Again speaking generally, the incompatibility of this thesis with Leibniz's Law can be shown in the following way :-

(1)	$(x)(y)\{(x = y) \supset (Fx \equiv Fy)\}$	Leibniz's Law
(2)	$(a = b) \supset (Fa \equiv Fb)$	Universal Instantiation (1)
(3)	$\sim(Fa \equiv Fb) \supset \sim(a = b)$	Contraposition
(4)	$\sim(Fa \equiv Fb) \; \& \; (a = b)$	Relativist Thesis
(5)	$\sim(Fa = Fb)$	'&' Elimination (4)
(6)	$\sim(a = b)$	MPP (3), (5)
(7)	$(a = b)$	'&' Elimination (4)
(8)	$\sim\{(x)(y)\;(x = y) \supset (Fx = Fy)\}$	R.A.A. (1) - (7).

In this chapter we will consider three different arguments adduced in support of the Relativist Thesis :-

First, the argument which attempts to show that Leibniz's Law breaks down in most sortal contexts and, consequently, interchangeability holds only under specified (sortal) contexts.

Second, the argument which attempts to show that Leibniz's Law leads to formal paradoxes and, consequently, interchangeability holds only relative to a prescribed language.

Third, the argument which attempts to show that the universal transitivity of Leibniz's Law leads us to hold unacceptable results and, consequently, Leibniz's Law must be abandoned or, at best, modified in the light of these cases.

Leibniz's Law breaks down in sortal contexts. [53]

The thesis I have in mind here may be stated as follows :- "since there are many sortal concepts underwhich a material particular *a* may be individuated, *a* may be identical with some specified material particular *b* under some of the concepts but be distinct from *b* under others." This is to say, *a* can be the same *f* as *b* but not the same *g* as *b*. The Relativist Thesis here says, in other words, that while *a* and *b* are identical under some contexts, they do not coincide under all properties either of them may possess, viz. $(a = b) \,\&\, \sim(Fa \equiv Fb)$. If the substituends for 'F' in Leibniz's Law $(x)(y)(x = y. \supset .Fx \equiv Fy)$ is a sortal expression, then, while F is predicable of x it is often the case that F is not predicable of y.

There are two obvious cases discernable of this present thesis.

(i) where a and b are identical under some sortal concept f and a has the property g which b does not.

And

(ii) where a and b are identical under some sortal concept f and both a and b have the property g but a and b are not coincidental under g.

Let us consider first two examples of type (i) cases (which are suggested by Geach's treatment of the relativisation thesis in 'Reference and Generality').

Giovanni Mon.tini is a famous infant prodigy. Giovanni Mon.tini is the same human being as Pope Paul VI. But Pope Paul VI is not the same infant prodigy, for Pope Paul VI is not an infant, nor the same school boy, bishop, diplomat, cardinal, etc. Now infant, school boy, bishop, diplomat, cardinal and so on, are all sortals and make perfectly good covering concepts. One can count and identify such things and so forth. So this example gives us a case, in appearance at least, where we have

$$(x)(y)\{(x = y) \supset [(f)(fx \equiv fy) \ \& \ -(g)(gx \equiv gy)]\}$$

this is to say, a case where y cuts out under a sortal concept g, for example, 'infant', but can persist through another f, i.e., 'human being'.

To put the matter more perspicuously

(9) Giovanni Mon.tini = Pope Paul VI

(10) Giovanni Mon.tini is an infant prodigy

By Leibniz's rule, we derive Pope Paul VI is an infant prodigy

But (11) Pope Paul VI is not an infant prodigy.

As a second example of type (i) cases, consider the following. Suppose Cleopatra's Needle is corroded away by the London fog and is repaired with concrete until eventually, in 1970, none of its original state is left. Cleopatra's Needle in 1870, is the same landmark as Cleopatra's Needle in 1970, but not the same stone. In fact it is not made of stone at at all, (Cf., Linsky's review of 'Reference and Generality'). This example appears to give us another case where an object is at one time covered by two sortal concepts f and g, i.e., both 'landmark' and 'stone', and continues being the same f but not the same g.

There are many other examples of type (i) cases, (to be found in the history of philosophy especially);[54] which are supposed to show that the rule of unrestricted substitutivity breaks down because a and b are identical under some sortal concepts and not under others. It is enough to say here that, if examples such as the above gain initial plausibility as counter-examples to the rule of unrestricted substitutivity, it is because the individual or entity of which the identity statement is asserted persists through time and Leibniz's Law is in an untensed form. We might seek to avoid the counter-examples by making Leibniz's Law more precise with respect to tenses, by incorporating reference to time. For instance, we might want to say "if x is identical with y, then whatever is true of x at t_1 is true of y at t_1 and conversely". This "solution" is to be avoided however since with it we attempt to significantly associate dates with individual terms and the identity relation whereas, apart from the cases where a date is part of the individual expression itself, e.g., 'The 1812 Overture', dates, times, etc., associate significantly with verbs or predicates only. To illustrate this point, consider

(12) Giovanni Mon.tini, 1900 = Pope Paul VI, 1970

which, as it stands, is unintelligible.

Whereas

(13) Giovanni Mon.tini, who was an infant in 1900 = Pope Paul VI, who is Pope in 1970

is an acceptable assignment of the temporal expressions.

The point is, to say x and y are identical is to say that they are one and the same individual, object, or entity. As far as the identity relation is concerned

Giovanni Mon.tini = Pope Paul VI

and Cleopatra's Needle = Cleopatra's Needle

(and thats the end of the matter).

I think the solution to the first example is absurdly simple. The confusion here is that it applies a (tensed) sortal term, which is true of an individual at a certain stage of his existence, i.e., 'infant', to all the existence of that individual. Clearly, the infant Mon.tini will become the Pope, the Pope was the infant Mon.tini. Thus if the conclusion on line (11) is true, then in premiss (10) we are confusing timeless and tensed expressions within the one predicate, 'is an infant'. If (9) is true, then (10) must be construed as either, a tensed statement

(10') Giovanni Montini was an infant prodigy,

in which case (11) is false since the Pope was an infant prodigy: or, (10) must be construed as tenseless, i.e.,

(10'') Giovanni Montini is (timelessly) an infant prodigy,

in which case (11) is false since the Pope is (timelessly) an infant prodigy.

With the second example, the explanation of why it fails as a counter-example to Leibniz's Law is somewhat more complicated. If it is essential that Cleopatra's Needle designates a "stone" obelisk then when any of the stone has been replaced by concrete we have a different object; hence, when all of the stone has been dissolved by the London fog we have two distinct objects, in which case the example does <u>not</u> describe an identity relation. If however the material is not essential to the designatum of 'Cleopatra's Needle' and the term designates a landmark to be found at a particular position on the Thames embankment then the example does exhibit an identity relation. In this case, 'Cleopatra's Needle' refers to a landmark which has the predicate "is made of stone" appropriate to it in 1870, the predicate "is made of stone and concrete" appropriate to it between 1870 and 1970 and the predicate "is made of concrete" appropriate to it after 1970. (To attempt to ascribe 'is made of stone" after 1970 would involve the same difficulties as attempting to ascribe 'is an infant' to Pope Paul VI in the example (9) - (11).) Notice, if the argument above is correct, then the example presents no difficulties for Leibniz's Law, i.e.,

(i) Cleopatra's Needle = Cleopatra's Needle

(ii) Cleopatra's Needle is made of stone in 1870

(iii) Cleopatra's Needle is made of concrete in 1970,

are three disparate statements.

We mentioned earlier that there is a second form in which the relativist thesis is presented with repect to sortals. These are cases where <u>a</u> and <u>b</u> are identical under some sortal <u>f</u> and both a and b have the property <u>g</u>, but <u>a</u> and <u>b</u> are not coincidental under <u>g</u>.

Linsky, in his article 'Substitutivity' offers an example of our type (ii) cases.

(14) The President of the United States is the Commander-in-Chief of the Armed Forces.

(15) The President of the United States is given his oath by the Chief Justice

(16) The Commander-in-Chief of the Armed Forces is given his oath by the Chief Justice.

(x is identical with y under one sortal concept, (e.g., 'man'), but x and y are not coincidental under another, (e.g., 'official'), although both x and y are instances of this concept.)

In this example it is possible, but hardly to the point, to interpret (16) as true. This is to say, if the President is the Commander-in-Chief _ex officio_, the Commander-in-Chief is sworn in when the President takes office. For (14) - (16) to be a counter-example to Leibniz's Law, (14) and (15) must be true and (16) false. In other words, Linsky requires that we read (16) as saying the Commander in Chief takes an oath which could be called 'the oath of office for the Commander-in-Chief of the Armed Forces', in which case (16) is false.

Now Linsky's difficulty here is fairly easily unmasked. Clearly, the President of the United States is one and the same man as the Commander-in-Chief of the Armed Forces, but though the same man holds both offices of state, they are not the same office. The President and the Commander-in-Chief are the same man but not the same official. Consequently, premiss (15) is elliptical for the more precisely stated

(15') The President of the United States is given the oath of office for the Presidency by the Chief Justice

from which follows

(16') The Commander-in-Chief of the Armed Forces is given the oath of office for the Presidency by the Chief Justice.'

(Cf., example (7) - (9), on page 6 of this essay.)

Incidentally, the trivial juxtaposition of this example is equally easily exposed. Suppose a petitioner asks to see the same official as he saw last time, and the man sees the same official but not the same man, (i.e., official <u>a</u> is succeeded by official <u>b</u>). Clearly, a = b in these circumstances does not ascribe numerical identity to <u>a</u> and <u>b</u> at all.

Geach, in 'Reference and Generality', page 151, gives us another example of type (ii) cases, in his version of the celebrated Heracleitean puzzle. I will paraphrase the argument:-

(17)	Whatever is a river is water	True
(18)	Heracleitus bathed in the same river twice	True
(19)	Heracleitus bathed in the same water twice.	False.

This puzzle is easily dealt with. A mass term, such as "water", does not ordinarily admit qualifications such as "an" or "same". When it is subjected to these qualifications, some special individuating standard is understood from the circumstances. The sentence (19) is regarded as false, if it is regarded as false, because the expression "same water" in this instance, is understood as referring to an aggregate of molecules,say, that surround a man when he bathes. However, under this interpretation 'river' and 'water' in premiss (17) are taken to have different referents, therefore (17) is not an identity statement.

So much for examples of sortal terms which are allegedly counter-examples to Leibniz's Law. I will take up this topic again at the conclusion of this chapter. First, I want to consider a stronger argument against Leibniz's Law due again to P.T.Geach.

Leibniz's Law leads to formal paradoxes.

In his article 'Identity' Geach writes: "I am arguing for the thesis that identity is relative. When one says "x is identical with y" this, I hold, is an incomplete expression; it is short for "x is the same A as y, where "A" represents some count noun understood from the context of utterance".

Geach opposes his theory of relative identity to what he calls the theory of "absolute identity". For Geach, absolute or strict identity is an expression best reserved for identity statements which are true regardless of what theory is under consideration; this is to say, for Geach only absolute identity would confer universal intersubstitutivity and, according to Geach, this is impossible. Let us be clear about this. In challenging absolute identity, he is challenging the principle of unrestricted intersubstitutivity, that the names or other designations of identicals may be substituted for each other salva veritate. As Geach says, "we cannot employ the strong condition imposed on predicates by Leibniz's Law", we cannot use the principle stated in terms "for all properties" or "whatever is true of x is true of y" -- without reference to a particular language, because such unrestrained language will leads us into such notorious paradoxes as Grellings and Richard's. It is then left to the reader to derive these notorious paradoxes.

We can speculate that Geach intended here, a proof along the following lines:-

(i)	$(x)(y)\{(x = y \supset (\emptyset)(\emptyset x \equiv \emptyset y)\}$	where $\emptyset$ is a predicate variable designating properties, Leibniz's Law.
(ii)	$(a = a) \supset (\emptyset)(\emptyset a \equiv \emptyset a)$	Universal Elim. on (i).
(iii)	$(\emptyset)(\emptyset a \equiv \emptyset a) \supset (\emptyset)(\emptyset a \vee -\emptyset a)$	Truth Functional Logic
(iv)	$(a = a) \supset (\emptyset)(\emptyset a \vee -\emptyset a)$	Chain (ii), (iii)
(v)	$(a = a)$	to derive $(\emptyset)(\emptyset a \vee -\emptyset a)$ for all possible theories $a = a$ has to be assumed to be absolute, a la Geach.
(vi)	$(\emptyset)(\emptyset a \vee -\emptyset a)$	MPP. (vi), (v)

In the unrestricted case, this may be read as "all predicates are true or orl false of a". This presumably gives rise to Grelling's paradox when we substitute 'heterological' for a, and 'heterological' as the predicate, (i.e., heterological is either heterological or not. If it is heterological, then it is not, since it is self descriptive: if it is not heterological then it is, since it would be self descriptive.)

Curiously, Geach overlooks the fact that Grelling's paradox is obtainable independently of Leibniz's Law (and for that matter, far more obviously).[55] As we mentioned earlier, (p.14 above), we have no reason to give up Leibniz's Law if the paradoxical conclusion, which is claimed to be derived by substituting into it, can be (more easily) derived without it. We may reasonably suppose that the source of the paradox lies elsewhere. To put the point another way, ofcourse the necessity of avoiding the antinomies will lead us to impose restrictions on the general structure of the language, but these, clearly, are

are not to be thought of as specifically restrictions on Leibniz's Law. But perhaps the above is not Geach's strongest argument in support of the Relativist Thesis.

As we have noted, Geach is arguing that every identity predicate need not preserve interchangeability *salva veritate*, but rather substitutivity relative to the definite ideology of a theory. Following Geach, we will call this theory T. He then illustrates his stronger thesis with the following extended example:-

> "Let the theory T be a theory about the expressions of a given natural language; let the range of T's quantifiers be token expressions of that language; but let the ideology of T be so restricted that in T we cannot give different descriptions for two tokens of the same type-words. Then if a predicable of T, say "Exy" signifies "x is equiform with y", it 56
> will be an I-predicable of the theory T. But now let us add to T's ideology just one predicable that descriminates between equiform tokens; in the enlarged theory T', we can express something that is true of a token a but not true of an equiform token b; so in T' the predicable "Exy" is not an I-predicable any longer."
>
> (pages 5 - 6, ibid).

This then is the nub of Geach's objection to Leibniz's rule. Predicates in a theory T that, in Geach's words, are I-predicable, could always cease to co-refer when further predicates are added to the ideology of the theory. As our language grows richer, as we move from T to T', any given predicate could cease to be an I-predicable and therefore cease to express identity.

An example may help to explain. Consider the following:-

PLANK PLANK

Let us call the word on the left '<u>a</u>' and the word on the right '<u>b</u>'. Now suppose there are no criteria in the theory T for distinguishing token-words. Thus '<u>a</u>' and '<u>b</u>', uttered in T, refer to type-words and it is true that in T, a = b. Now suppose, in the theory T', which contains all the predicates assignable in T, (i.e., T is a proper sub-set of T'), we have the additional criteria for distinguishing between token-words. As a look at the example at the top of this page will show, "a is diverse from b" is true of the improved theory T'. In short, from the diagram above, considered with respect to type-words, a = b ; whereas, considered with respect to token-words, a $\neq$ b. There seems to be a failure of substitutivity here of considerable magnitude. Geach concludes that Leibniz's Law doesn't hold; ~~that identity is relative.~~

Now it seems to me, a a strong line of defense, with respect to the argument above, is open to a defender of Leibniz's Law. Let us begin by noting that by moving from T to T' by adding new predicates to our language T, so that new discernments are now possible, we change the referents of <u>a</u> and <u>b</u>. According to Geach, this reply is unacceptable since, if the referent of a word 'a' in T differs from the referent of the same word in a richer language T', we are committed thereby to the existence of several entities for each word: one for each language in which a further discernment may be made with respect to the referent of that word. In fine, such a solution "generates a baroque Meinongian structure", (which incidentally,Geach notes, "hardly suits Quine's preference for desert landscapes". Geach's attack on Leibniz's Law is directed mainly at Quine's defense of this rule).

Geach's argument here is mistaken. Suppose someone, let us call him Tom, speaks a language so poor in predicates that he cannot distinguish between the type and token instances of the word 'plank' in the example. Suppose then Tom adds to his language predicates that will enable him to draw this distinction. Tom is not thereby committed to the existence of the identity he was committed to prior to the introduction of the type-token distinction. The addition of this, or any new predicates, will require that Tom revises his ontology. The point is, and surely Quine's point is, our commitment is to those values of variables we now hold to be true. Geach's mistake is to presuppose that commitment to an ontology is fixed (somehow), rather than as dynamic as the theory to which new variables are added.

As my final criticism and the strongest criticism against Geach's thesis is, in fact, a criticism against the Relativist doctrine in general, I will hold my fire for the moment, and turn now to the last exponent of the doctrine we will consider, Prior's avowal of a modified Relativist theory and rejection of Leibniz's Law.

<u>The Universal Transitivity of Leibniz's Law breaks down.</u>

A.Prior, in his essay, 'Time, Existence and Identity' states that although many people have of late wanted to reject Leibniz's Law for weak reasons, the possibility of one thing becoming two, as with unicellular biological organism, constitutes a strong case for denying the universal transitivity of identity and thus Leibniz's Law. Take the case of an amoeba, A. Let A divide and form B and C. Now if A really has become B and C and has not simply ceased to exist, then, when someone says "Where is A"?, Prior feels compelled to say "here"! Pointing to B on some occasions and C on others. Thus A is B and also A is C. But it is false that B is C, as they are distinct co-existents. So for Prior transitivity has failed on this occasion, and thus so has Leibniz's Law.

The unacceptable consequences for Leibniz's Law can be shown in the following way:-

(i)	$(x)(y)(x = y . \supset . Fx \equiv Fy)$	Leibniz's Law
(ii)	$(x)(y)(x = y . \supset . Fx \equiv Fy) \supset [(x = y) \supset (x = z \supset y = z)]$	Proved on page 4 above.
(iii)	$(x = y) \supset (x = z \supset y = z)$	MPP. (i), (ii)
(iv)	$(x = y)$	Premiss
(v)	$(x = z \supset y = z)$	MPP. (iii), (iv)
(vi)	$(x = z)$	Premiss
(vii)	$(y = z)$	MPP. (v), (vi)

Prior accepts premisses (iv) and (vi) as justified by the case being one of fission -- as we have seen, he thinks the correct way to describe this phenomena is to say that one thing became two things and is identical to both of them. Since after fission we have two seperate entities, y and z, and two things cannot be identical, the conclusion (vii) must be false. The only premiss (vii) relies on other than (iv) and (vi) is (i) -- this is to say, Leibniz's Law. It follows that Leibniz's Law fails, i.e.,

$$(x)(y)\left[(x = y. \supset .Fx \equiv Fy)\right] \supset \left[(x = y) \supset (x = z \supset y = z)\right]$$

then, by contraposition on the above,

$$\sim\left[(x = y) \supset (x = z \supset y = z)\right] \supset \sim\left[(x)(y)(x = y. \supset .Fx \equiv Fy)\right]$$

Once again, I cannot see that Prior's puzzle presents Leibniz's Law with insurmountable problems. Consider the example with the parent amoeba A, and the offspring amoeba B and C. Clearly A and B do not have the same history; neither does A and C. For B and C correspond to only part of A, not each to all of A. This example seems puzzling mainly because genetic division is able to reproduce exactly similar cell-structure. But this is not to say that A is ever identical with B or C. As Prior notes, it makes no sense to say that two distinct individuals B and C are the same. But neither does it make sense to say A or B are the same, whether A is said to "become" B or not. For example, trees become coal in time but not even Prior seeks to maintain identity here.

Perhaps the best direct objection I have against Prior is that if Prior's analysis is true, and A is B and also C, then if B disappears -- ingested perhaps -- Prior would have to say, "A has ceased to exist and yet remains existing, for C still exists.". I think that one has to say that at the moment of fission A passes away and B and C come to be.

Conclusion

In this chapter, we began by considering the view which rejects unrestricted interchange of identicals because, allegedly, one identity relation can hold for an object while a different identity relation does not. There are examples of sortal expressions which seem to show this but, on closer examination, we found they do not.

In what we called type (i) cases, the main confusion is worth underlining here. It is not a necessary condition of the entity I see today being identical with the entity I saw yesterday that the entity I see today has the same properties, let us say colour, that the entity I saw yesterday had. Leibniz's Law does not absurdly deny the possibility of change. But Leibniz's Law does require that, if they are identical, the entity now has the same colour as the entity I see now has. And it does require that the entity I see today had the same colour as the entity I saw yesterday then had. Certainly, as we have seen, this requires care in the application of Leibniz's rule. But care in the application of the rule seems a small price to pay for the advantages to be gained in retaining Leibniz's Law.

To illustrate what I take to be the main confusion in type (ii) cases, consider the following passages taken from his 'Logic and Knowledge' in which Russell remarks:-

> "Identity is rather puzzling at first sight. When you say "Scott is the author of Waverley", you are tempted to think there are two people one of whom is Scott and the other the author of Waverley, and they happen to be the same. That is obviously absurd but that is the sort of way one is always tempted to deal with identity".
>
> (page 247)

It appears that the proponents of the Relativist Thesis when they argue type (ii) cases have fallen to the temptation. For whereas Russell would explain the statement 'The President of the United States is the Commander-in-Chief of the Armed Forces' as stating that one and only one man instantiates two propositional functions, the fact that the propositions '$\hat{x}$ is a President of the United States' and '$\hat{x}$ is a Commander-in-Chief of the Armed Forces' are different, would seem to lead Geach and Linsky to suppose that we are dealing with two objects.

We then turned to Geach's attack on what he calls "absolute identity", and on Quine's commitment to Leibniz's rule in particular. Geach argues against absolute identity because, he says, it leads to paradoxes. Geach overlooks the fact that the paradoxes in question are obtainable independently of Leibniz's Law. We found his argument that Leibniz's Law commits us to a Meinongian jungle, false. To supply the resources for differentiating between token instances of the same word type, means not simply adding a predicate or two, it means introducing "some sort of spatial co-ordinate system (or something general of the sort) and that means totally revamping the structure of the language. It is no surprise that we find a totally different domain of individuals."

Finally, we noted Prior's claim that Leibniz's rule seriously conflicts with certain biological phenomena. The fact is apparently neglected by Prior, and all advocates of the Relativist's Thesis, that identity involves reference to one and only one object. As we have noted several times already, identity is expressed by those uses of 'is' that one is prepared to expand into 'is the same object as'. As we have asked (tediously) throughout this essay -- if a is the same object as b how could there be something true of the object under a which is untrue of the object under b ? The answer here is simple -- there cannot be. And those who say that in the light of certain examples or considerations, Leibniz's Law should be modified or abandoned, are mistaken -- or more cautiously, I hope to have shown, need better arguments to establish this claim.

CHAPTER FIVE

CONCLUSION

In this paper we have considered various apparent exceptions to Leibniz's Law, expressed in the formula $(x)(y)(x = y . \supset . Fx \equiv Fy)$.

In Chapter I, by way of clearing the ground, we noted some elementary confusions -- ambiguous and incomplete sentences, use mention confusions and so forth -- where it has been claimed Leibniz's Law runs into difficulties. In fact, the boot is on the other foot, Leibniz's Law cannot prevent bad practices from running into even greater difficulties.

In Chapter II, we considered arguments for and against the claim that Leibniz's Law must be modified, or rejected completely, because of the antinomy that occurs when it is combined with *de re* modalities 'necessarily' and 'possibly'. We conceded that provided we are willing to treat the identity relation as a trivial necessary relation holding between name-identities and rule out hitherto contingent identities as, in some way, instances of a weaker equivalence relation, the modalities and Leibniz's Law, in its unrestricted form, are compatible. Moreover, the expressions $\ulcorner \Box(x = y) \urcorner$ and $\ulcorner (\exists x)[\Box(x = y)] \urcorner$ are *prima facie* acceptable; while, at the same time, we cannot ignore Quine's reservation that these lead us to an excessively idealistic ontology. As far as I know Quine's objection has not been satisfactorily answered. We were less impressed by Quine's argument that modal object entails "essentialism"; (I am still not clear what exactly Quine takes the crime involved here to be).

In Chapter III, we considered the problems involved when Leibniz's Law combines with expressions signifying mental attitudes. We noted that the emendations to Leibniz's Law from Carnap, Putnam, Church *et al.*, and the notational reform of intentional sentences proposed by Quine, are for the most part incorrect and anyway unnecessary, (with respect to interchangeability *salva veritate*), since by extending the distinction between contexts *in sensu composito* and those *in sensu diviso*, we can successfully combine Leibniz's Law with epistemic operators. However, we found here that the argument for not accepting descriptions as terms of an identity sentence, is decisive. Sentences of the form '$(\iota x)(Fx) = (\iota x)(Gx)$' and '$s = (\iota x)(Axw)$' are abbreviated multiply general sentences: the abbreviated version must not be treated as having the semantical properties of an identity sentence. The identity relation holds only between "name"-identities. We then argued that, where the context of the intentional sentence is taken as *in sensu composito*, (viz., referring to the content of belief), since the name-identity upon which the substitution is made is referentially transparent, we have no warrant for deriving the antinomous, or any other, conclusion. On the other hand, where the context of the intentional sentence is understood as *in sensu diviso*, (viz., referring to the object of belief), we can derive, by substitution in accordance with Leibniz's rule, a valid conclusion -- (provided of course that this is also taken as *in sensu diviso*).

Finally, in Chapter IV, we considered various examples and arguments, invoked in support of the Relativist Thesis of identity, which have been used to show the impossibility of unrestricted interchange. The claim that Leibniz's Law fails, when we interpret 'F', in the formula $(x)(y)(x = y. \supset .Fx \equiv Fy)$, as the substituend for sortal expressions and, in Geach's argument, where x and y stand for expressions other than names (or descriptions) of individuals, (viz., in T, type-words and in T', token-words) and, Prior's argument that universal substitutivity conflicts with certain biological phenomena. No matter how the Relativist's Thesis is argued, we found no reason for abandoning or modifying Leibniz's Law.

I conclude, that from the arguments considered, it has not been shown that Leibniz's Law fails.

NOTES TO THE TEXT

1. In his article 'Substitutivity', Linsky makes the following useful distinction: "It should be clear that what is under discussion is the principle of the indiscernibility of identicals and not its trivial converse, the identity of indiscernibiles. The latter principle says that if two terms t and t' are interchangeable in every statement salva veritate then ⌜t = t'⌝ is a true statement. The following argument quickly establishes that this is so. If t and t' are interchangeable in every statement salva veritate, they are interchangeable in the true statement ⌜t = t⌝. Therefore, replacing the right-hand t by t' we get ⌜t = t'⌝, salva veritate." (p.140)

To put this matter another way, the identity of indiscernibles lays down conditions which a and b must satisfy in order to be identical. This principle has sometimes been criticised on the grounds that it is logically possible, (though most unlikely), that two distinct things might be exactly similar so that everything true of one would be true of the other, without a being identical with b, (Cf., Wittgenstein, Ramsey.) Be that as it may. The converse is not possible. Even if we do not admit the identity of indiscernibles, I will argue, we must admit the indiscernibility of identicals which states the conditions which follow, given a and b are identical.

2. This principle is in fact the converse of the principle stated by Leibniz's in 'Non Inelegans Specimen Demonstrandi in Abstractis', a fragment of which is published in W & M Kneale's 'The Development of Logic', (p.340). "Terms are the same or coincident which can be substituted one for another wherever we please without altering the truth of any statement". Also Cf., Leibniz's 'Discourse on Metaphysics (IX)', his 'Fourth Paper to Clarke', and 'Monadology(IX)'.

3. 'Reference and Modality' in Quine's 'From a Logical Point of View', Chapter VIII, p.139.

4. I owe the formulation of this schema to Prof.Wilson. My original suggestion was $(x)(y)(x = y \supset (\emptyset)(\emptyset x \equiv \emptyset y)$, which says, in effect, if x and y are the same object they have all their "properties" in common. As Prof.Wilson pointed out to me this involves the controversial assumption that in order to discuss Leibniz's Law we must commit ourselves to properties. (A nominalist would deny that any predicate designates a property. And even if we are committed to realism, it just isn't clear that we should hold that all predicates designate properties or hold that some do not.)

5. Quine, in his 'Comments to R.B.Marcus's Modality & Intensional Languages', gives a neat argument to show if $\emptyset$ and $\not{\lambda}$ both meet the requirements of strong reflexivity and substitutivity

then they are co-extensive. His argument is as follows:- By Leibniz's Law $(x)(y)(\emptyset xy \ \& \ \psi xx \supset \psi xy)$. But by the reflexivity of ψ, we can drop 'ψxx'. So ψ holds where $\emptyset$ holds. By the same argument, with $\emptyset$ and ψ interchanged, $\emptyset$ holds wherever ψ holds.

6. To prove:- $(p \supset (q \supset r)) \supset (q \supset (p \supset r))$

Proof

(1)	$p \supset (q \supset r)$	Premiss
(2)	q	Premiss
(3)	p	Premiss
(4)	$q \supset r$	MPP. (1), (3)
(5)	r	MPP. (2), (4)
(6)	$p \supset r$	C.P. (3) -(5)
(7)	$q \supset (p \supset r)$	C.P. (2) -(6)
(8)	$p \supset (q \supset r)) \supset (q \supset (p \supset r))$	C.P. (1) -(7)

7. Page 139, ibid.

8. Page 143, 'Substitutivity'

9. For a further illustration of this point, compare 'The llama is a woolly beast' -- (x)(x is a llama $\supset$(x is woolly & x is a beast). with 'The llama is eating out of my hand' -- ($\exists$x)(x is a llama & x is eating out of my hand).

10. Cf., pp.139-159, ibid.

11. Cf., p.47ff, 'On Denoting'

12. Cf., Linsky's review of 'Reference and Generality'

13. Cf., Linsky's 'Substitutivity', pp.144 - 145

14. Cf., Geach's 'Identity', pp.3 - 12

15. For instance, Frege developed his theory of sense and reference, Russell his theory of definite descriptions and Quine his theory of opaque and transparent contexts, to be in accord with Leibniz's Law.

16. In our discussion of modal logic we may conveniently limit ourselves to a single modal operator '$\Box$' -- that of necessity. Whatever may be said about necessity, may be said also, with obvious adjustments, about the other modalities, i.e., '$\Diamond$' (possibly), is another modal operator which can be defined in terms of $\sim\Box$.

17. The proof also proceeds by making suitable substitutions into Leibniz's Law. By substituting (=Y) for 'F' in $(x)(y)(x = y. \supset .Fx = Fy)$ we obtain $(x)(y)(x = y. \supset (x = y \supset y = y))$.

Now suppose also that y's being contingently identical with x is also a property of x. Expressing "y is contingently identical" by $\triangle()$ =y and substituting for(=y) in $(x)(y)(x = y \supset (x = y \supset y = y))$ we obtain $(x)(y)(x = y \supset (\triangle(x) = y) \supset \triangle(y) = y))$.
This I hope shows, if x = y and $\triangle(x)$ =y then $\triangle(y)$ =y i.e., that if some identities are contingent then every identity is contingent, Cf., Wiggins 'Identity Statements'.

18. Review of Barcan, Journal of Symbolic Logic 12 (1947), pages 395-396.

19. Cf., Hintikka's 'Modes of Modality' and 'Modality and Quantification'; Pap's 'Semantic and Necessary Truth'; Rescher's 'Identity, Substitutivity, Modality'; Geach's 'Reference and Generality'.

20. We might say that Carnap takes Leibniz's Law as not a sufficient principle of intersubstitutivity for all contexts. I do not think it is correct to say that Carnap does not take Leibniz's Law to be a necessary condition of intersubstitutivity.

21. Cf., pp139-159, ibid.

22. In his book 'Introduction to Semantics', Carnap states that by a semantical system we understand.."a system of rules formulated in the meta-language and referring to an object language." (p.22). Thus it is argued that a 'semantical rule' is a sentence S, of the meta-language, used to define 'sentence in S' 'designates in S' 'analytic in S' and so on. Incidentally, it seems that S is the (meta-meta-linguistic) name of a system of rules. If this is so, how does S appear in the rules ? Is Carnap guilty of confusing use and mention here ?

23. Cf., Quine's 'Two Dogma's of Empiricism', in 'From a Logical Point of View', pp 32-37, esp.33.

24. See the 'Pedagogical Difficulty' in 'Concept of Language' by Prof.Wilson, esp. pages 14-17 & 98.

25. See Quine's criticism of Carnap included in Carnap's 'Meaning and Necessity', pp 196f.

26. 'Attribute and Class', p.551.

27. See 'A Functional Calculas of First Order Based on Strict Implication'.

28. Cf'Modalities and Intensional Languages', pp 303-330 esp.305.

29. In fact this claim is not supported by any clear argument but rather Marcus stipulates this as the purport of 'identity'. However, she also seems to support essentialism.

30. 'Extensionality', page 58.

31. Once again, it is not transparently clear that Marcus would argue in this way, though in her discussion with Quine (in 'Modalities and Intensional Languages'), she gives the impression that she would not be adverse to this kind of essentialism.

32. Ofcourse Smullyan is not primarily concerned to answer the question "Are all genuine identities name-identities"?, but rather, to challenge Quine's rejection of modal logic. However, if his proposals are unacceptable and we still wish to combine Leibniz's Law and the modalities, then there appears to be no alternative to accepting that all genuine identity sentences are name -identities.

33. 'Review of the Philosophy of Rudolph Carnap', p.112.

34. For example, ~(The present king of France is bald)

$(\exists x)(\sim Bx)$

i.e., Something is not bald.

35. In 'The Trouble with Meanings', Prof.Wilson attempts to show that "the whole theory of intensional entities represents not just an ontological extravagence, but a fairly clear absurdity", (p.1). The absurdity is this. "Consider the property blue. It is supposed to be the meaning of the word 'blue'. If 'blue' occurs in an identity sentence, this sentence will be true just in case the other argument also has the property blue as its meaning, just in case, that is, it is strictly synonymous with the word 'blue'. But there is an obvious counter-example,

'The colour of the sky is (identical with) blue'

Here we have a true identity statement whose arguments do not have the same meaning and which is therefore not necessarily true. The property, blue, is not the meaning of 'the colour of the sky' and we are forced to deny that blue is the meaning of 'blue'."

Unfortunately, we are forced to deny no such thing. Since on the account of identity sentences in 'Modality & Identity', 'The colour of the sky is blue', is just not an identity sentence.

36. This is a complete about-turn from my original position. I owe my conversion to Prof.Wilson.

37. Also, ofcourse, Quine objects to Russell's (et al) calling the material conditional 'implies' rather than 'if...then' which, in turn, gave rise to the need for 'strict implication'. It is well known that Quine views this as the paradigm of the use-mention confusion.

38. It is open to defenders of modal logic to argue, as we have noted, that the alleged conflict does not rise in this particular case, since (19) can be regarded in such a way that it is not false; alternatively, and more likely, it could be claimed, (17) is true but not an identity statement.
But either way, this is merely to quarrel with the example and misses the full force of Quine's objection to modal logic.

39. Cf., 'Reference and Modality', page 150-156.

40. More generally, there are a plurality of properties belonging to any given object, I would argue, and some of these properties are essential to it -- the object is what it is by virtue of these properties -- others are contingent.

41. Ofcourse this issue goes far deeper than the present discussion of it; for instance, part of the claim here is that there is something radically wrong with Quine's doctrine of ontology. However, to go any further with this topic here, would take us well beyond Leibniz's Law and the bounds of this essay.

42. See 'From a Logical Point of View', page 181 (sic), for 153.

43. Notice, Chisholm's attempt to reconcile Leibniz's Law with propositional attitudes is very similar to Smullyan's attempt to combine Leibniz's Law with the modalities.

44. The argument for this is obvious. If George believes there is one and only one person who wrote Waverley and who is Scotch, without having any idea who in fact the author of Waverley is, then there is no one person of whom George believes that this individual wrote Waverley and is Scotch. In an analogous way, I know that one and only one person was the first Prime Minister of Canada and that this person was a Canadian, without having any idea who this person was. In fine, (2') does not imply (2'').

45. It is quite usual in this discussion to suppose that a solution to antinomies at the level of belief statements, may be generalised to satisfy all intentional contexts. This strikes me as unduly optimistic, however it is an optimistic assumption I will take full advantage of here.

46. See footnote 36.

47. This point was given to me by Prof.Wilson.

48. Putnam reports that Carnap remains unconvinced by Church's critique, but is, on the other hand, deeply disturbed by Mates's argument. Also, Cf., 'On Belief Sentences: A Reply to Alonzo Church'.

49. 'Three Grades of Modal Involvement' in the'Ways Of Paradox', p.158.

50. Suppose I say, "Oedipus wanted to marry his mother", understood opaquely what I have said is wildly false. But it can be understood transparently and understood in this way, it is true, Oedipus wanted to marry his mother because he wanted to marry Jocasta, the woman who, though he did not know it, was in fact his mother. In sum, understood transparently, Oedipus does not know that the woman he wanted to marry was in fact his mother.

51. This difficulty was brought to my attention by Prof. Wilson.

52. Perhaps part of the problem here is that "means" is badly behaved in such contexts. Prof. Wilson's "signifies" by contrast is well behaved with respect to substitution; Cf., Fn.51.

53. The discussion of the relativist thesis with respect to sortals requires a lengthy analysis of examples. For those with little patience or goodwill for this activity, the results of this section are summarised in the conclusion of this chapter, on page 99ff.

54. See the example in the 'Phaedo', (opening section), where there is the celebrated puzzle of Theseus's ship, which was preserved although, one by one, all of its planks were replaced. Same ship or different ?
Also see Hobbes 'De Corpore' II, 11. (Molesworth ed., p.136). Also see Hume's 'Treatise' I, 4. (Everyman ed., p.244).

55. ~~I overlooked this fact as well in my original draft.~~ No matter, Prof.Wilson came to my aid with the following easy proof.

(i)	ϕa	Premiss
(ii)	ϕa	Reiteration of (i)
(iii)	$\phi a \supset \phi a$	C.P. (i), (ii).
(iv)	$\phi a \lor \sim\phi a$	DeMorgan on (iii)
(v)	$(\phi)(\phi a \lor \sim\phi a)$	Univ.Gen (i) - (iv).

56. Note my 'Exy' for Geach's 'Eξη' -- typographical difficulties.

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