Hume's Dichotomy

by

J.F. Allan

HUME'S DICHOTOMY

HUME'S DICHOTOMY

By

J. F. ALLAN

A Thesis

Submitted to the Faculty of Graduate Studies

in Partial Fulfilment of the Requirements

for the Degree

Master of Arts

McMaster University

May, 1966

MASTER OF ARTS (1966)
(Philosophy)

McMASTER UNIVERSITY
Hamilton, Ontario.

TITLE: Hume's Dichotomy

AUTHOR: J. F. Allan

SUPERVISOR: Dr. James Noxon

NUMBER OF PAGES: vii, 85

Acknowledgements

I am greatly indebted to my tutor, Dr. James Noxon, for much advice, guidance and criticism in the writing of this thesis. I also wish to express my thanks to him for all the assistance given me with Hume, throughout my stay at McMaster.

I am grateful to Dr. S. Najm, who acted as second reader of the manuscript, and made some suggestions.

I also want to thank Dr. D. Odegard, who gave me some assistance in connection with Pap; and also made some suggestions.

I wish to thank also Dr. A. Shalom for his comments; and also Dr. L. Greenspan.

Finally, I thank the Department of Philosophy for the award of a Graduate Teaching Fellowship; and the Province of Ontario for the award of an Ontario Graduate Fellowship.

May, 1966 J. F. Allan

This thesis is dedicated to my
Mother and Father as evidence
that I have not been entirely
idle during my sojourn in Canada.

Analytical Table of Contents.

Introductory

Statement of the purpose of this paper: (i) to show on what grounds the division within the class of "philosophical" relations in Book 1, Part 3, Section I of the *Treatise* and between Relations of Ideas and Matters of Fact in Section 4 of the *First Enquiry*, is made; (ii) to show that the division in both books is a division of propositions into those which are necessary (or, a priori) and those which are contingent (or, a posteriori); and (iii) to show that such a priori propositions are not also "analytic" in the sense of being tautologous, nor are they also synthetic.

The division in the *Treatise*.

The grounds.

By distinguishing his seven "philosophical" relations into two classes, Hume's purpose is to make clear the line of demarcation between knowledge and probability (or, belief); and to show precisely in which spheres knowledge is possible and in which spheres it is not possible.

The four relations which alone 'can be the objects of knowledge' are Resemblance, Contrariety, Degrees in any Quality, and Proportion in Quantity or Number. These relations I refer to as Class I Relations. The remaining three relations, Identity, relations of Time and Place, and Causation, Hume classes under the heading of Probability. These I refer to as Class II Relations.

Notice of the use to which Hume puts Class I Relations in subsequent parts of the *Treatise*.

The way in which the terms "knowledge", "certainty", "intuition", and "demonstration" are used, shown. Hume's criterion of intuitive certainty, I refer to as the direct inspection criterion.

The basic ground of the division between the two classes, I term the "constant/inconstant relations" base: the qualification for membership in the respective classes being that the relations belonging to Class I 'depend entirely on the ideas, which we compare together', these relations are 'invariable' or 'constant' 'as long as our ideas remain the same'; the relations which are to be members of Class II are relations which 'may be changed without any change in the ideas', these relations are 'inconstant' or 'variable'. With respect to Class I, the relations vary with the "ideas"; with respect to Class II, the relations do not vary with the "ideas".

Illustration of Hume's meaning with respect to the constancy of the relations of Class I.

Statement of the grounds of Class I: this class is the class of propositions which are intuitive or demonstrative, the latter consisting in a chain of connected intuitions. The ground of their intuitiveness is the direct inspection criterion; the ground of this criterion itself is that the relations involved in such propositions are constant; and the ground of the constancy of the relation is the make-up or composition of the "ideas" or terms of the proposition.

If such propositions be termed *a priori* (that is, necessary), then it is to be noted that the above criteria are the grounds of such necessity.

Illustration of Hume's meaning with respect to the inconstancy of the relations of Class II.

Two of the relations of Class II, those of identity and those of time and place are a case of 'perception', rather than 'reasoning'. His meaning with respect to this.

Identity, however, whether numerical or qualitative, cannot be in the same position as relations of space and time.

Propositions belonging to Class II being those propositions which are 'neither intuitively nor demonstrably certain', the question, What criterion does Hume employ to test propositions of this class for their lack of intuitive certainty, discussed.

Hume's use of, what I term, the conceivability criterion discussed: examples of his employment of it; the two principles on which it rests.

The way in which this criterion is another way of distinguishing the two classes of propositions, shown.

Discussion of Kemp Smith's tripartite division within the class of "philosophical" relations: the fact that spatio-temporal relations, according to Hume, are directly perceived, leads Kemp Smith to this procedure. I show that these relations do not wreck Hume's division, nor is there any need to set up a third class to accommodate these "animals".

Discussion of Arthur Pap's discussion of Hume's dichotomy: Pap's contention that 'Hume is caught in a dilemma' dealt with.

D. F. Pears' contention that 'the central contention of the first book of the *Treatise* is ...that *a priori* propositions are empty' (i.e. analytic) rejected. (This receives fuller discussion in my discussion of the status of mathematical propositions, for Hume in the *Treatise*).

I The status of mathematical propositions.

In this section, my purpose is to show (i) that, for Hume, mathematical propositions are not "analytic", nor (ii) are they "synthetic a priori" or "synthetic a posteriori", but (iii) that they are *a priori* or necessary, but in so being, they are not also analytic, in the sense of being "tautologous".

Relations involving 'proportions in quantity or number' contrasted with the three relations which 'fall more properly under the province of intuition than demonstration'.

Hume's treatment of geometry in Part 3 (section 1 of Book 1) and in Part 2 (of that book) contrasted with his treatment of algebra and arithmetic.

In view of the account of geometry, both in Part 3 and in Part 2, it might be thought that there is a division _within_ those relations, or propositions involving those relations, which deal with proportions in quantity and number: that algebraic and arithmetical propositions are to be distinguished in _type_ from geometrical propositions. I contend that there is no distinction in _type_ between these two sets of propositions.

Discussion of Atkinson's paper, "Hume on Mathematics": (a) while agreeing with Atkinson that, for Hume, geometrical propositions are not "synthetic a posteriori", and that those of arithmetic and algebra are not "analytic", I reject Atkinson's way of showing this. (b) Atkinson's view that, for Hume, mathematical propositions are "synthetic a priori" rejected: (1) his citations from C. W. Hendel and from Kemp Smith to the effect that 'Hume was sometimes tempted to modify his view in the Kantian direction' examined. Hendel's contention that 'space and time and our ideas of them are produced by the imagination' criticized by W. T. Parry. Parry's criticism taken up by me. Kemp Smith's contentions that 'Hume takes a non-sensationalist view of space and time' and that the 'manner' is not an 'impression' dealt with by me. Annand's objections to Hume's view of space and time as 'compound impressions' answered. Annand's objections to Hume's "coloured points" answered. (2) On the "constant/inconstant relations" base, there can be no "synthetic a priori" propositions. This base ignored by Atkinson.

V _The Division in the First Enquiry_

The way in which the division is made in the _Enquiry_ contrasted with the way in which it is made in the _Treatise_.

The question, What _sort_ of propositions come under the headings of _Relations of Ideas_ and _Matters of Fact_, discussed.

Hume's treatment of mathematics: the evidence reviewed and compared with that of the _Treatise_; Ayer's use of the term "analytic" discussed; note taken of Kemp Smith's use of that term; Kant's use of that term discussed; the question, Does the omission of the distinction between _constant/inconstant_ relations leave open the possibility that mathematical propositions are _synthetic a priori_, discussed.

I. INTRODUCTORY

Hume's distinction, in Section I of Part 3 of Book 1 of the *Treatise*, between those relations which 'depend entirely on the ideas, which we compare together' (T.69)[1] and those relations which 'may be changed without any change in the ideas' (*ibid*), and his distinction between *Relations of Ideas* and *Matters of Fact* in Section 4 of the *First Enquiry*[2] will constitute the subject-matter of this enquiry.

My main purpose, in brief outline, will be (i) to show on that *grounds* the division is made in each of the two books; and (ii) to show that the division both in the *Treatise* and in the *First Enquiry* is a division of propositions into those which are "necessary" (or, *a priori*) and those which are "contingent" (or, *a posteriori*); and (iii) to show that such *a priori* propositions are not "analytic" in the sense of being 'tautologous", nor are they *also* synthetic. The view that, for Hume, all *a priori* propositions are analytic is held, for instance, by D. F. Pears, in his article, *Hume's Empiricism and Modern Empiricism*. Pears writes (p.24)[3]: 'The central contention of the first book of the *Treatise* is...that *a priori* propositions are empty, and that any significant proposition that is not empty must be based on experience.' And the possibility that, for Hume, mathematical

---

[1] All references to *A Treatise of Human Nature* are to the page numbers in the Selby-Bigge edition, Oxford: Clarendon Press, 1888 (1960 reprint). I refer to this as T with the page number following.

[2] I use this brief appellation to refer to *An Enquiry concerning Human Understanding*. All references to that work are to the *sections* in the Selby-Bigge edition: *Hume's Enquiries*, Oxford: Clarendon Press 1902 (second edition). I refer to this as E with the section number following.

[3] *David Hume: A Symposium*. Edited by D. F. Pears. London: MacMillan & Co. Ltd., 1963.

propositions might be *synthetic a priori*, is entertained, for instance, by A. Pap in *Semantics and Necessary Truth*[1], and by R. F. Atkinson, in his paper, *Hume on Mathematics*.[2] Such a possibility, I maintain, is straightaway ruled out; in what way it is ruled out, it is part of the business of this enqiry to show.

I have been unable, in the body of this paper, to take account of Farhang Zabeeh's book, *Hume: Precursor of Modern Empiricism*[3], since it arrived too late for that purpose. Zabeeh refers (p.85) to Hume's divisions, in the *Treatise* and in the *First Enquiry* as the 'analytic-synthetic dichotomy'. This, I regard as a misleading way of talking about Hume's divisions; since, the use of that kind of language and the title of the book itself, imply that Hume held the thesis that all *a priori* propositions are analytic.

Hume explicitly distinguishes between necessary propositions which are "analytic" (in the sense of being tautologous, empty, uninformative) and those which are not analytic in that sense. Hume's usual term for propositions of that type is the term "identical proposition". (In the *First Enquiry* (E.32), he uses the term "tautology".)

None of the propositions which belong to Class I (in the *Treatise*) and to 'Relations of Ideas' (in the *Enquiry*) are, for Hume, 'identical propositions' i.e. analytic, tautologous, empty.

That he does recognize that there are such propositions, I will now show;

---

[1]Part 1, c. 4. New Haven: Yale University Press, 1958.

[2]*Philosophical Quarterly*, 1960, Vol. 10.

[3]The Hague: Martinus Nijhoff, 1960.

and I will also show what he has to say about them. References to this sort of proposition are made at T.50, 127, 200, 248, and E.32. At T.50, he writes: 'In common life 'tis established as a maxim, that the straightest way is always the shortest; which would be as absurd as to say, the shortest way is always the shortest, if our idea of a right line was not different from that of the shortest way betwixt two points.' At.T.127, he writes: 'The likelihood and probability of chances is a superior number of equal chances; and consequently when we say 'tis likely the event will fall on the side, which is superior, rather on the inferior, we do no more than affirm, that where there is a superior number of chances there is actually a superior, and where there is an inferior there is an inferior; which are identical propositions, and of no consequence.' At T.200, he writes: 'As to the principle of individuation; we may observe, that the view of any one object is not sufficient to convey the idea of identity. For in that proposition, _an object is the same with itself_, if the idea expressed by the word, _object_, were no ways distinguished from that meant by _itself_; we really should mean nothing, nor would the proposition contain a predicate and a subject, which however are implied in this affirmation.' (Italics Hume's) At T.248, he writes: '...in saying, that the idea of an infinitely powerful being is connected with that of every effect, which he wills, we really do no more than assert, that a being, whose volition is connected with every effect, is connected with every effect; which is an identical proposition, and gives us no insight into the nature of this power or connection.' And finally at E.32, he writes: 'When a man says, _I have found, in all past instances, such sensible qualities conjoined with such secret powers_: And when he says, _Similar sensible qualities will always be conjoined with similar secret powers_, he is not guilty of a tautology, nor are these propositions in any respect the same.'

Hume does not hold the thesis that all "<u>a priori</u>" propositions are analytic. This, however, does not commit him to holding that there are <u>synthetic a priori</u> propositions. It will be seen in my discussion of the <u>grounds</u> upon which a proposition is necessary for Hume, in what way there can be for him no <u>synthetic a priori</u> propositions. It is my contention that by distinguishing relations into those which are <u>constant</u> and those which are <u>inconstant</u>, the possibility of such a proposition, for Hume, is thereby ruled out.

II. The Division in the Treatise.

Hume, by distinguishing his seven "philosophical"[1] relations into two classes, wants to make clear the line of demarcation between knowledge and probability (or, belief); and to show precisely in which spheres knowledge is possible and in which spheres it is not possible. All that falls short of knowledge, he classes under the heading of probability.[2] Knowledge or certainty, for Hume, is either intuitive or demonstrative; this latter species of evidence consisting in a chain of connected intuitions. The four relations which alone 'can be the objects of knowledge and certainty' (T.70), which 'are the foundation of science' (scientia) (T.73) are Resemblance, Contrariety, Degrees in any Quality, and Proportion in Quantity or Number. These relations I will refer to, as the relations which make up Class I, or simply, Class I Relations.

The remaining three relations, namely Identity, relations of Time and Place, and Causation come under the heading of probability. I will refer to these as Class II Relations.

Before proceeding to an enquiry into the grounds on which Class I is distinguished from Class II, I shall list evidence to show to what purpose Hume puts the relations of Class I. His procedure is to show that if none of these four relations is implied in a certain proposition (under examination),

---

[1] The distinction made by Hume, in sections 4 and 5 of Part 1 of Book 1, between natural and philosophical relations does not, for the present purpose, concern us, But see pp.52-55, where I have discussed this distinction.

[2] Cf. Locke, Essay concerning Human Understanding (Book 4, C.15, S.2ff.) The distinction made within the realm of probability (T.124) is, for the purpose at present, unimportant (Vide. p. 26).

or if the proposition does not involve any of these relations, then that proposition, he concludes, is not intuitively certain. Thus, at T.79, where he is discussing the proposition, 'whatever has a beginning has also a cause of its existence', he remarks that none of his four relations are implied in it, and concludes that 'that proposition is not intuitively certain'. Anyone who is of the opinion that it is, 'must deny these to be the only infallible relations, and must find some other relation of that kind to be implied in it. ....'. At T.463, he makes the same point: 'If you assert, that vice and virtue consist in relations susceptible of certainty and demonstration, you must confine yourself to those four relations, which alone admit of that degree of evidence....' (Italics Hume's). Further (T.464) he regards these four relations as exhausting the content of intuitive or demonstrative knowledge: 'Should it be asserted, that the sense of morality consists in the discovery of some relation, distinct from these (four), and that our enumeration was not complete, when we comprehended all demonstrable relations under four general heads: To this I know not what to reply, till some one be so good as to point out to me this new relation....'.

These relations, then, being the only relations which can be 'the objects of knowledge and certainty' a certainty which is either intuitive or demonstrative, the way in which these terms are being usedhas to be made clear. The terms "knowledge" and "certainty", he uses as the equivalent of each other (T.70, 71, 72); and 'all certainty arises from the comparison of ideas, and from the discovery of such relations as are unalterable' (as synonyms for 'unalterable', he has 'invariable' (T.69) and 'constant' (T.73)) 'so long as the ideas continue the same' (T.79). Thus, it is not his meaning that all comparisons of ideas yield knowledge; only four types of comparisons of ideas (those types which make up Class I) are to be regarded as yielding _knowledge_.

Concerning the term "intuition", he does not give any _precise_ definition of his use of it; he does, however, say something about it, by way of illustration; three of the relations, which belong to Class I, namely, Resemblance, Contrariety, and Degrees in any Quality, says Hume, 'fall more properly under the province of intuition than demonstration' (T.70), since they 'are discoverable at first sight' (_ibid_): 'When any objects[1] _resemble_ each other, the resemblance will at first strike the eye, or rather the mind; and seldom requires a second examination. The case is the same with _contrariety_, and with the _degrees_ of any _quality_. No one can once doubt but existence and non-existence destroy each other, and are perfectly incompatible and contrary. And tho' it be impossible to judge exactly of the degrees of any quality, such as colour, taste, heat, cold, when the difference betwixt them is very small; yet 'tis easy to decide, that any of them is superior or inferior to another, when their difference is considerable. And this decision we always pronounce at first sight, without any enquiry or reasoning.' (_ibid_). In speaking of "proportion in quantity or number", he says that we 'might at one view observe a superiority or inferiority betwixt any numbers or figures; especially where the difference is very great and remarkable.' But 'as to equality or any exact proportion, we can only guess at it from a single consideration; except in very short numbers, or very limited portions of extension; which are comprehended in an instant...' (_ibid_).

---

[1]Following Hume's own procedure, we here make no distinction between the terms "idea" and "object". Prior to Part 4, he uses these terms interchangeably (_Vide_. for example T.69, 71, 157).

The phrases he uses to illustrate his meaning in saying that three of the relations 'fall...under the province of intuition': relations 'discoverable at first sight'; the relation which two "ideas" (or "objects") bear to each other 'at first (i.e. straightaway) strike(s) the eye, or rather the mind'; and 'decisions' concerning these relations 'we always pronounce at first sight, without any enquiry or reasoning'; such relations we can 'observe' 'at one view'; they can be 'comprehended in an instant': show that <u>Intuition</u>, for Hume, is a species of <u>perception</u>; it is a kind of <u>seeing</u> or <u>observation</u>, and as such would be conformable to the use of the word in Latin. N. Kemp Smith terms this, 'apprehension by direct inspection' (p. 355)[1], a relation which can be 'directly intuited' (p. 351) is a relation which can be 'immediately apprehended'. R. W. Church (p. 67)[2] remarks that these relations are 'directly perceived'. A. L. Leroy (p. 76)[3] says that these relations 'se découvrent par perception directe'. And in a footnote to the same page remarks: '...il n'y a qu'à bien considérer les objets présents aux sens pour discerner leur rapport de grandeur. C'est une perception directe...c'est une vue instantanée.' Leroy, then, takes Hume to be using "intuitive certainty" in the sense of direct sense-perception, that is, sense-perception, which admits of no doubt. From now on, I will refer to this criterion of intuitive certainty as the <u>direct inspection</u> criterion.

---

[1]<u>The Philosophy of David Hume</u>. MacMillan & Co., Ltd., London. (1941. Reprint 1964).

[2]<u>Hume's Theory of the Understanding</u>. George Allen & Unwin, Ltd., London. 1935.

[3]<u>David Hume</u>. Presses Universitaires de France, 1953.

I now proceed to the basic ground of the division between the two classes: I will term this the "constant/inconstant relations" base. The qualification for membership in the respective classes, says Hume, is that the relations belonging to Class I 'depend entirely on the ideas, which we compare together' (T.69); and that the relations which are to be members of Class II, are relations which 'may be changed without any change in the ideas' (ibid). Or, as he states it (T.73): 'All kinds of reasoning consist in nothing but a comparison, and a discovery of those relations either constant or inconstant, which two or more objects bear to each other.' At T.69, he states that those relations which belong to Class I are relations which are 'invariable, as long as our idea(s) remain(s) the same.' At T.79, he states it thus: 'All certainty arises from the comparison of ideas, and from the discovery of such relations as are unalterable, so long as the ideas continue the same.' (Italics mine, in both citations). With respect to Class I, briefly what is being maintained is this: if the "ideas" are altered, then the relation is correspondently altered; that is, the relations vary with the "ideas". With respect to Class II: if the relation is altered, the "ideas" are not altered; that is, the relation does not vary with the "ideas".

With respect to Class I, he gives one example to illustrate his meaning: ''tis from the idea of a triangle, that we discover the relation of equality, which its three angles bear to two right ones; and this relation is invariable, as long as our idea remains the same' (T.69). This sort of example would come under the general head of "proportion in quantity and number"; and it is the only example he offers as an illustration of what he means by saying that, in this class, the relations vary with the ideas.

In order to get the general drift of what is here being maintained, I will speak of terms, rather than "ideas" or "objects". Given a term, "A" (say): this term, let us suppose, is <u>composite</u>, that is, it is made up of parts or elements, a_1, a_2.

Now, should one element be omitted, or a new element added, (say), "b", then the term will no longer be designated "A". Now, let us also suppose that these elements (a_1, a_2) have a certain relation to each other; and should the elements be altered in any way, either by omissions or additions, not only is the term itself no longer designated "A", but the relation holding between the elements has also been destroyed.

Let us now take Hume's example of the triangle: the term "triangle" is composed, let us suppose, of (say) two elements, namely, "three angles", and "two right angles", and these two elements have the relation of equality to each other. Should either one or both of these elements be altered in any way, the relation of equality no longer holds. For example, should one of the elements be (say) "four angles", and the other "two right angles", that which was called "triangle" is no longer designated "triangle", and the former relation of equality has also been destroyed. To put the matter in another way: anything made up of elements "four angles" and "two right angles" is no longer termed "triangle".

This, in a very crude form, is what Hume is saying, by saying that the 'relation is invariable, as long as our idea remains the same'.

The other relations which Hume has included as members of Class I (namely, Resemblance, Degrees in quality, and Contrariety) will now be considered. We will begin with Resemblance. Let us take the "proposition", 'A resembles B': the term "A" (proceeding on our supposition) being composite, is made up of elements a_1, a_2, a_3 (say), and the term "B" is made up of elements b_1, b_2, b_3, b_4 (say). So again: alter in any way, either by omissions or additions, any of the elements of "A" or "B", the term "A" itself (or "B") is no longer <u>that</u>

term, and the relation of resemblance holding between A and B has been destroyed.

Similarly, for degrees in any quality; let us take a concrete example: A is lighter in shade than B. If we deepen the colour of A, beyond the saturation-point of B, the relation "lighter than" holding between A and B, has been destroyed. Likewise, if we lighten the colour of B, beyond the saturation-point of A, the relation no longer holds between A and B. And the alterations made to A (or B) disqualify it from being designated "A" (or "B", as the case may be).

Now, concerning Contrariety: the examples which Hume gives of this relation, first, in the section where he lists his "philosophical" relations 'under seven general heads' (Section 5, of Part 1. T.15), are the 'ideas of existence and non-existence'. In this section, with which we are dealing (T.70) he again gives the same examples: 'No one can once doubt but existence and non-existence destroy each other, and are perfectly incompatible and contrary'.

It becomes a little difficult to see just what Hume could mean by an 'idea of existence'; and even more difficult, by an 'idea of non-existence'. In the last section of Part II, namely, Section 6 (T.66-68), where he discusses 'the idea of existence, and of external existence', he argues that there is no idea of existence 'derived from a distinct impression' (T.66)...'...the idea of existence...is the very same with the idea of what we conceive to be existent... whatever we conceive, we conceive to be existent. Any idea we please to form is the idea of a being; and the idea of a being is any idea we please to form'. (T.66-67).

Kemp Smith (p.351)[1] merely remarks: '...Hume's reference to contrariety,

---

[1]*op. cit.*

in the context in which it comes, is extremely bewildering '.

As to what would be an example of contrariety in propositional form, I am not at all clear. Hume (T.19) says 'that 'tis possible for the same thing both to be and not to be' is 'the flattest of all contradictions'. The reference here would seem to be to 'existence' and 'non-existence', the examples which he gives when discussing contrariety. R. F. Atkinson.[1] gives as an example of contrariety in propositional form 'There cannot both be and not be X's'.

The above interpretation, which I have been attempting, is based on the assumption that certain "properties", "characteristics", or "qualities" (call them what you will) are "essential" to a term, in the sense that, without which the term is not the same term. Any omissions or additions to the "properties" of that term thereby disqualifies the term from being designated that term.

With respect to Class I, my purpose has been to show that that class is the class of propositions which are "intuitive" (or, "demonstrative", that is, consisting in a chain of connected intuitions). And the ground of this criterion itself is that the relations involved in such propositions are "constant" or "invariable" or "unalterable" and the ground of this constancy or invariableness is the make-up or the composition of the "ideas" (or, terms of the proposition).

If such propositions be termed "a priori" (that is, "necessary"), then it is to be noted that the above criteria are the grounds of such "necessity". Hume, in the Treatise (Book I) uses the term "a priori" only twice, and both occurrences are on the same page (T.247); and he seems to be using it in the

---

[1] Philosophical Quarterly, Vol. 10, 1960 See below, for my references and discussion of Atkinson's view of Hume on mathematics (section III).

sense that 'to prove' a proposition '_a priori_'[1] is to draw the conclusion 'from the mere consideration of the ideas'.

With respect to Class II: the three relations which make up this class are 'relations of _time_ and _place_' (T.69) or 'situations in time and place' (T.73), _identity_, and _causation_. Relations of this type are relations which 'may be changed without any change in the ideas'. (T.69); or, as he states it (T.73): 'which depend not upon the idea, and may be absent or present even while _that_ remains the same'. He also refers to such relations as being "inconstant" (T.73), in contrast to those of Class I which are "constant", "invariable", or unalterable" (T.73, 69, and 79 respectively). These relations, in other words, do _not_ vary with the "ideas"; if the relation is altered, the "ideas" are not, correspondently, altered. Thus, if A is three feet distant from B, the relation 'three feet distant' may be altered (it may be five, or seventy-five feet, or two hundred miles, distant) without in any way (according to Hume) A and B being altered. That is, the alteration of the relation, makes no difference to A and B. As he states it (T.69): 'the relations of _contiguity_ and _distance_ betwixt two objects may be changed merely by an alteration of their place, without any change on the objects themselves or on their ideas; and the place depends on a hundred different accidents, which cannot be foreseen by the mind.'

With respect to the relation of identity, he says (_ibid_): 'Two objects, tho' perfectly resembling each other, and even appearing in the same place at different times, may be numerically different'. In Section 5 of Part 1, where he first made out his list of seven "philosophical" relations, he remarked that

---

[1]References to this term are more frequent in the First Enquiry. See sections below, dealing with that work.

'this relation (of identity) I here consider as applied in its strictest sense to constant and unchangeable objects'.

With respect to causation, he says (S.B.69): 'the power, by which one object produces another, is never discoverable merely from their idea,...cause and effect are relations, of which we receive information from experience, and not from any abstract reasoning or reflexion.'

By saying that the relation of identity 'may be changed without any change in the ideas (or, objects)', his meaning seems to be that the numerical difference between an object, A, viewed at t_1, and t_2, constitutes the alteration in the relation, as 'applied...to constant and unchangeable objects'. And this difference, or alteration of the relation does not, according to Hume, correspondently make an alteration in the object. And, by saying that the relation of cause and effect between objects is "inconstant", or that such a relation between objects 'may be changed' without any corresponding change in these objects, his meaning is that two objects standing in such a relation are unaffected by that relation in the sense that if the relation did not hold between these two objects, these two objects, designated "cause" and "effect" respectively, would not thereby undergo change or alteration.

Before we proceed, the question, What is meant by saying that there is no 'change' or 'alteration' in the 'ideas' (or, 'objects') here has to be cleared up.

Class I was the class, where the relations varied with the 'ideas'; and by that was meant, that the 'ideas' or 'objects' or terms were constructed or composed in such a way that any alteration or change in the terms disqualified the term from being designated that term; and since the term (or terms) was no longer that term, the relation, which held between the two terms, also broke down.

This class, however, is the class where the construction or composition of the terms, has no bearing on the relation, in the sense that, were there any alteration in the relation holding between the two terms, the terms would not *also* undergo alteration.

Two of the relations of this class, those of *identity* and those of *time* and *place*, are a case of 'perception', rather than of 'reasoning', Hume says (T.73). 'When both the objects are present to the senses along with the relation, we call *this* perception rather than reasoning; nor is there in this case any exercise of the thought, or any action, properly speaking, but a mere passive admission of the impressions thro' the organs of sensation.' (*ibid*). By 'reasoning', he means a comparing or comparison of two 'objects' and the discovery of the relations 'constant' or 'inconstant' which they bear to each other. We may make the comparison when both the objects are present to the senses (a case of *impressions*), or, when neither of the two objects are present (a casa of *ideas*), or, when only *one* of the objects is present to the senses. The latter alternative is more properly termed 'reasoning' (or, inference).

Identity, however, whether numerical or qualitative, cannot be in the same position as relations of space and time; since, in the case of identity, *both* terms of the relation are *not* 'present to the senses along with the relation'; only one of the terms is 'present to the senses', thus, it would be a case of 'reasoning'[1]. Yet, he says (*ibid*) : 'we ought not to receive as reasoning any of the observations we may make concerning *identity*, and the *relations* of *time* and *place*; since in none of them the mind can go beyond what is immediately present to the senses, either to discover the real existence or the relations

---

[1]Hume, however, may be here thinking of the identity relation solely as a case of "ideas"; but why should he?

of objects. 'Tis only causation, which produces such a connexion, as to give us assurance from the existence or action of one object, that 'twas follow'd or preceded by any other other existence or action; nor can the other two relations be ever made use of in reasoning, except so far as they either affect or are affected by it...we readily suppose an object may continue individually the same, tho' several times absent from and present to the senses; and ascribe to it an identity, notwithstanding the interruption of the perception, whenever we conclude, that if we had kept our eye or hand constantly upon it, it would have conveyed an invariable and uninterrupted perception. And this conclusion beyond the impressions of our senses can be founded only on the connexion of cause and effect; nor can we otherwise have any security, that the object is not changed upon us, however much the new object may resemble that which was formerly present to the senses.'

Propositions belonging to Class II are those propositions which are 'neither intuitively nor demonstrably certain' (T. 79). In the case of propositions of Class I, the relation was directly seen to hold, and the ground of this 'seeing' was that the terms being what they are (or, being so constituted), the relation could not but hold, and always hold between such terms. Thus, there was intuitive certainty about the relation's holding between the terms.

In the case of propositions of Class II, the relation is not tied to the terms; with the result that there is no certainty about the relation's holding between the terms. (Just as, if A ties his dog to the gate-post, he knows where the dog is; he may be said to have "intuitive certainty" concerning the whereabouts of his dog; but if the dog is not tied to the gate-post, A has no certainty concerning the animal's whereabouts.)

We come now to the question, What criterion does Hume employ to test propositions of this class for their lack of "intuitive certainty"? The "direct inspection criterion" is ruled out for the reason that the ground of that criterion is not applicable (even although one of the relations of Class II are directly perceived) since it is applicable only where the relation is "constant".

The fact that propositions of the second class are not intuitively or demonstrably certain is stated in an indirect way in Section 2. Propositions belonging to this class yield, what Hume terms "probability"; in contrast to those propositions of Class I which yield "knowledge", this "knowledge" being either intuitive or demonstrative.

Thus, propositions of Class II are, by implication, not intuitively nor demonstrably certain, since they do not yield knowledge. As was noted (vide pp. 5,6), he resorts to the method of showing that, if none of the four relations of Class I are implied in some proposition under examination, that proposition is thereby not intuitively nor demonstrably certain. Thus, at T.78, 79, in examining the proposition 'whatever begins to exist, must have a cause of existence', he remarks that this 'general maxim' is 'supposed to be founded on intuition', but 'if we examine this maxim...we shall discover in it no mark of any...intuitive certainty'. Since 'all certainty arises from the comparison of ideas, and from the discovery of such relations as are unalterable, so long as the ideas continue the same. These relations are (the relations which make up Class I); none of which are implied in this proposition...That proposition therefore is not intuitively certain. At least anyone, who would assert it to be intuitively certain, must deny these to be the only infallible relations, and must find some other relation of that kind to be implied in it; which it will then be time enough to examine.'. He, then, proceeds to put forward another type of argument to prove 'at once, that (this) proposition is neither intuitively nor demonstrably certain. We can never demonstrate the necessity of a cause to every new existence, or new modification of existence, without showing at the same time the impossibility there is, that any thing can ever begin to exist without some productive principle; and where the latter proposition cannot be provided, we must despair of ever being able to prove the former. Now that the latter proposition is utterly incapable of a demonstrative proof, we may satisfy ourselves by considering that as all distinct ideas are separable from each other, and as the ideas of cause and effect are evidently distinct, 'twill be easy for us to <u>conceive</u>[1] any object to be non-existent this moment, and existent the next, without conjoining to it the distinct idea of a cause or productive principle'. This 'separation...of the idea of a cause from that of a beginning of existence, is plainly possible for the imagination; and consequently the actual separation of these objects is so far possible, that it <u>implies no contradition</u> nor absurdity.' (T.79, 80. Italics mine). The argument may be stated thus: Anything possible for the imagination implies no "contradiction"; the separation of the idea of a cause from that of a beginning of existence, is possible (they are two distinct ideas); therefore the separation implies no "contradiction". (Modus Ponens). Now, if this proposition were intuitively certain, such a separation would imply a "contradiction"; it does not imply a contradiction"; therefore, this proposition is not intuitively certain. (Modus Tollens).

This criterion, which I will call the <u>conceivability</u> criterion, is employed by Hume, quite frequently in the <u>Treatise</u> (T.32, 43, 79, 80, 86, 87, 89, 111, 162, 250.).

The above argument is supported by two principles: (i) 'whatever (ideas)

---

[1]Hume uses 'conceive' and 'imagine' synonymously. Other synonymous terms or phrases are: 'form a notion', 'form an idea). <u>Vide</u> especially T.18, 20, 27, 28, 30 (footnote), 32, 38, 39, 41, 43, 51, 53, 54, 55, 66, 67, 72, 120, 150, 162, 186, 201, 625.

are different are distinguishable, and that whatever (ideas) are distinguishable, are separable by the thought and the imagination' (T.18)[1] and (ii) the 'evident principle, that whatever we can imagine is possible' (T.250); or, as he stated it (T.32): 'That whatever the mind clearly conceives includes the idea of possible existence' ; and, whatever is 'plainly possible for the imagination...implies no contradiction' (T.80).

Before I proceed to discuss this criterion, it is to be noted that Hume in his discussion of Class I relations (Section 1, Part 3, Book 1), does not employ the conceivability criterion to show that the propositions involving the four relations are intuitively or demonstratively certain. For that purpose he employed what I termed the direct inspection criterion. It is, however, employed by him to show that Class II propositions are not intuitively nor demonstratively certain; and, by implication, that Class I propositions are intuitively or demonstratively certain. In what way it is related to the constant/inconstant relations base will subsequently be shown.

With respect to this criterion, I shall, first, list examples of Hume's employment of it.

At T.86, 87, where he is discussing 'the inference we draw from cause and effect', he points out that 'there is no object, which implies the existence of any other if we consider these objects in themselves, and never look beyond the ideas which we form of them. Such an inference would amount to knowledge,

---

[1]He regards it as also 'true in the inverse' (that is, converse). And at T.10 he writes: 'Whereever the imagination perceives a difference among ideas, it can easily produce a separation.' At T.36: 'Everything, that is different, is distinguishable; and everything that is distinguishable, may be separated' by the thought and the imagination. At T.40: 'Whereever objects are different, they are distinguishable and separable by the imagination.' He also holds (T.36) that if 'they be not different, they are not distinguishable; and if they be not distinguishable, they cannot be separated.'.

and would imply the absolute contradiction and impossibility of conceiving anything different. But as all distinct ideas are separable, 'tis evident there can be no impossibility of that kind. When we pass from a present impression to the idea of any object, we might possibly have separated the idea from the impression, and have substituted any other idea in its room.' (Italics mine).

At T.89, where he is discussing the 'principle, that instances, of which we have had no experience, must resemble those, of which we have had experience, and that the course of nature continues always uniformly the same' (Italics Hume's), he again points out that such a proposition can be founded only on arguments 'derived either from knowledge or probability.' If there were any demonstrative arguments, says Hume, to prove this proposition, then, it would be 'absolutely impossible' to 'conceive a change in the course of nature.' (Italics mine). But, 'we can at least conceive a change in the course of nature; which sufficiently proves, that such a change is not absolutely impossible. To form a clear idea of anything, is an undeniable argument for its possibility, and is alone a refutation of any pretended demonstration against it.' (Italics mine).

At T.111, discussing the opinion, held by some philosophers, that one 'might immediately infer the motion of one body from the impulse of another, without having recourse to any past observation', he proceeds to give 'an easy proof' that such a view is 'false': 'For if such an inference may be drawn merely from the ideas of body, of motion, and of impulse, it must amount to a demonstration, and must imply the absolute impossibility of any contrary supposition. Every effect, then, beside the communication of motion, implies a formal contradiction: and 'tis impossible not only that it can exist, but also that it can be conceived.' (Italics mine). And since we can form 'a clear and consistent idea' of an 'infinite number of...changes, which we may suppose it to undergo', then it is to be concluded that the inference is not demonstrative.

At T.161: 'Now nothing is more evident, than that the human mind cannot form such an idea of two objects, as to <u>conceive</u> any connexion betwixt them, or comprehend distinctly that power or efficacy, by which they are united. Such a connexion would amount to a demonstration, and would imply the absolute impossibility for the one object not to follow, or to be conceived not to follow upon the other'...'since we can never distinctly <u>conceive</u> how any particular power can possibly reside in any particular object, we deceive ourselves in imagining we can form any such general idea' (S.B.162). (Italics mine).

As was noted above, this criterion rests on two principles, one of which principles is: 'Whatever (ideas) are different are distinguishable, and whatever (ideas) are distinguishable are separable by the thought and the imagination' (T.18); and he also holds that if the "ideas" (or, "objects") 'be not different, they are not distinguishable; and if they be not distinguishable, they cannot be separated'. By saying that the 'ideas' are 'different', it can be seen from the above quoted examples, that his meaning is that of a difference in kind or a qualitative difference, not a numerical difference. The idea that I have of this table at t_1, and the idea that I have of it at t_2 would constitute a difference in number. But the idea of this table, and the idea of this chair are ideas 'different' in kind. ('Difference is of two kinds as opposed either to identity or resemblance. The first is called a difference of <u>number</u>; the other of <u>kind</u>.' (T.15. Italics Hume's.).

The other principle on which it rests is the 'evident principle, <u>that whatever we can imagine, is possible</u>' (T.250). Or, as he states it (T.32): '<u>That whatever the mind clearly conceives includes the idea of possible existence</u>, or in other words, <u>that nothing we imagine is absolutely impossible</u>'. (Italics Hume's). At T.233, he states it thus: 'Whatever is clearly conceived may exist;

and whatever is clearly conceived, after any manner, may exist after the same manner'. Thus at T.32, he says: 'We can form the idea of a golden mountain, and from thence conclude that such a mountain may actually exist. We can form no idea of a mountain without a valley, and therefore regard it as impossible'. And an idea, which is conceivable 'implies no contradiction' (in that idea). (T.32): anything 'conceived by the imagination...implies no contradiction'. At T.29, he uses "impossible" and "contradictory" as equivalent terms. At T.43, he writes: 'Whatever can be conceived by a clear and distinct idea necessarily implies the possibility of existence' and ''Tis in vain to search for a contradiction in any thing that is distinctly conceived by the mind. Did it imply any contradiction, 'tis impossible it could ever by conceived'. At T.80; anything 'plainly possible for the imagination...implies no contradiction nor absurdity'. At T.87, he writes: an inference which amounts to knowledge (or, in other words, any proposition which is intuitively or demonstrably certain) implies 'the absolute contradiction and impossibility of conceiving anything different'. And at T.89: 'to conceive a change in the course of nature...sufficiently proves, that such a change is not absolutely impossible'. And at T.111, he says that a demonstration implies 'the absolute impossibility of any contrary supposition'. At T.95, discussing 'wherein consists the difference betwixt believing and disbelieving any proposition', he remarks that this question is easily answered 'with regard to propositions, that are proved by intuition or demonstration.' Since, 'in that case, the person, who assents, not only <u>conceives</u> the ideas according to the proposition, but is <u>necessarily</u> determined to conceive them in that particular manner, either immediately or by the interposition of other ideas. Whatever is absurd is unintelligible; nor is it possible for the imagination to conceive anything contrary to a demonstration. But as in reasonings...concerning matters of fact, this absolute <u>necessity</u> cannot take place, and the imagination is <u>free to conceive</u> both sides of the question...' (Italics mine).

This criterion, then, is another way of distinguishing the two classes of propositions. With respect to Class I, or to those 'propositions that are provided by intuition or demonstration', one is 'necessarily determined to conceive (the ideas according to the proposition)', that is, in that way in which they occur in the proposition. But with respect to Class II, 'this absolute necessity cannot take place, and the imagination is free to conceive both sides of the question', that is, both the proposition and its opposite.

In what way would this criterion be connected with the constant/inconstant relations base? Class I propositions are those propositions where the relation holding between the two terms is <u>constant</u> or <u>invariable</u>; and the ground of this constancy of the relation is the composition ormake-up of the terms. Thus the relation being <u>tied</u> to the terms, it would be <u>inconceivable</u> for the relation not to hold. Whereas in the case of Class II propositions, the relation is not tied to the terms, thus one can conceive of the relation not holding between these terms.

The fact that spatio-temporal relations, according to Hume, are <u>directly perceived</u>, leads Kemp Smith to make a tripartite division within Hume's seven relations. I am now going to show that these relations do not wreck Hume's divisions, nor is there any need to set up a third class to accommodate these "animals". Kemp Smith writes (p.355)[1]: 'These relations stand by themselves; they cannot be made to fit into either the first or the second group of relations...they agree with the relations of the first group and differ from the relations of identity and

---

[1] op. cit.

causation, in that they can be apprehended with complete immediacy and certainty ...These relations of time and place...fall midway between Hume's two classes. They share in the character of knowledge, in that they are apprehended by direct inspection, and therefore with a certainty which does not permit of doubt. Yet that of which we have this certainty is, like identity and causality, merely matter of fact, i.e. something the opposite of which is always conceivable.'. He proceeds (pp.356,357) to show that 'corresponding to...three types of awareness, Hume would...have distinguished three classes of relations.'. And these three 'modes' or 'types' of awareness are: 'first the immediate awareness through which we apprehend all perceptions, whether passions, sense-perceptions or ideas --a mode of awareness which he accepts as being infallible, and as therefore yielding its own type of <u>de facto</u> certainty and assurance: secondly, the mode of awareness through which, in reflective thinking, we obtain knowledge in the strictest sense of the term -- the propositions which concern content...and the opposites of which are inconceivable: and thirdly, the mode of awareness which he entitles belief...'.

The class of relations which correspond to the <u>first</u> mode of awareness becomes the class into which Kemp Smith puts the relations of time and place. Now, what Kemp Smith is, in effect, saying is this: spatio-temporal relations are "intuitively certain", because 'they are apprehended by direct inspection'; and, because they are <u>so</u> apprehended, they <u>thus</u> belong to Class I of Hume's division. But when Kemp Smith goes on to say that this "certainty" which they do have is 'merely matter of fact, i.e. something the opposite of which is always conceivable',[1] he is saying that these relations <u>also</u> belong to Class II. First, it is to be noted that Kemp Smith uses the term "certainty" <u>both</u> in connection with Class I <u>and</u> Class II, though the certainty belonging to Class II, is 'merely matter of fact'. Hume would not use the term "certainty" as such, in connection with Class II at all; he restricts the term to the relations of

---

[1]What I termed the <u>conceivability</u> criterion.

Class I[1]. But, aside from all this, it would seem, according to Kemp Smith, that in one respect, these relations belong to Class I, and in another respect, they belong to Class II. But these relations exhibit this kind of behavior only by virtue of the application of two types of criteria to them; and also by ignoring the base on which the distinction between the two classes is grounded. Let us see what is the import of these two criteria; and the way in which the basic ground of the distinction is bound up with them. First, the conceivability criterion: A proposition, on this criterion, according to Kemp Smith, would be one 'the opposite of which is always conceivable'; and propositions of this kind belong to Class II. And, by implication, a proposition the 'opposite' of which is inconceivable, would belong to Class I. And the ground of the inconceivability of the opposites of propositions of Class I would be that the relation holding between the terms is constant, or unalterable, 'so long as (the ideas) remain the same' (T.69). And the ground of the conceivability of the opposites of propositions of Class II would be that the relation holding between the terms is inconstant, or variable. Thus, on this criterion, relations of space and time, being inconstant, belong to Class II; they cannot belong to Class I. But Kemp Smith's reason for saying that these relations 'agree with the relations of the first group (Class I), is that they are 'apprehended by direct inspection', thus intuitive. But we need here only ask, What is the ground of this "intuitiveness" of propositions, to see that these relations cannot belong to Class I, on this criterion either. The ground of the intuitiveness of propositions

---

[1]Hume (T.124) distinguishes 'human reason into three kinds, viz. that from knowledge, from proofs, and from probabilities'; but although 'proofs' are 'entirely free from doubt and uncertainty', nevertheless, certainty or knowledge (in Hume's strict sense) belongs only to Class I.

of Class I is the direct inspection criterion. But the ground of this criterion itself is that the relations involved in such propositions are constant or invariable or unalterable; and the ground of this constancy or invariableness is the make-up or the composition of the "ideas" or terms of the proposition. The basic ground of the distinction between the two classes is that some relations are constant, others are inconstant; not that some relations can be directly perceived, others not. And, since relations of time and place are 'such as may be changed without any change in the ideas' (T. 69), thus, inconstant, thus belonging to Class II, there is no need to create a special class for them. They belong in the very class into which Hume has put them. The fundamental ground of the distinction between the two classes is the distinction between those relations which are constant and those which are inconstant. And the fact that these relations are inconstant disqualifies them from membership in the class where the relations are constant. Being able to directly perceive the relation is not what distinguishes the two classes.

In the case of Class I, although we directly perceive that the relation of resemblance holds between A and B, we also see that that relation does always hold so long as A is as it is, and so long as B is as it is.

In the case of Class II, the spatial relation holding between A and B is directly perceived, but we do not also see that this particular relation (of three feet (say)) does always hold, since the relation is not tied to the terms, in the sense that it is by virtue of the terms that that relation holds.

This distinction, made by Hume, between relations which 'depend entirely on the ideas' and relations which 'may be changed without any change in the ideas', is often said to be the distinction between "internal" and "external" relations.

A. Pap (p.73)[1] says: 'What Hume meant by saying that the relation of equality "depends solely upon ideas" is probably...that the equality of 4 and 2+2 <u>follows</u> from the meanings of the terms "4" and "2+2" alone; while, on the other hand, the truth of the judgment of spatial distance cannot be established by merely reflecting on the meanings of "body A" and "body B". Suppose we call relations of the former kind, following a well known philosophical terminology, <u>internal</u>, and relations of the latter kind <u>external</u>. We may then construct a general definition of this distinction, without departing appreciably from the framework of Hume's terminology, as follows: R is an <u>internal relation</u> = a proposition of the form xRy is, if true, <u>necessary</u>, and if false, <u>impossible</u>.

R is an <u>external relation</u> = a proposition of the form xRy is <u>contingent</u>, i.e. if true it is nonetheless logically possible that it should be false, and if false it is nonetheless logically possible that it should be true.

To illustrate, the denial of "2+2 = 4" can be shown to entail the contradiction "$4 \neq 4$", and hence one who denied this statement would attach an unusual meaning either to the term "4" or to the term "2+2". On the other hand, Hume would say that if a house A stands in fact at a distance of one mile from house B, we could nevertheless conceive that this spatial relation between the <u>same</u> objects were different, i.e. without "changing our ideas" of the terms we

---

[1] <u>Semantics and Necessary Truth</u>. New Haven: Yale University Press, 1958.

could suppose that their relation were different.' (Italics Pap's).

Pap's aim is to show (i) that if Hume's division is made in terms of analytic and synthetic propositions, then the division goes to wreck; and (ii) that the division is one between "necessary" and "contingent" propositions. I shall begin with the first of Pap's aims.

On the two pages immediately preceding page 73, Pap deals with Reichenbach's contention((in The Rise of Scientific Philosophy (1951, p.86)) that Hume is maintaining the thesis 'that all knowledge is either analytic or derived from experience'. Pap understands Reichenbach to be using the term "analytic" in the sense of self-explanatory. He then proceeds to take the term "self-explanatory" in the sense of "self-evident", with disastrous results for Reichenbach's interpretation of Hume. Pap now, still working on the analytic/synthetic assumption, proceeds to show that relations which depend 'solely upon ideas' may be taken in the sense of 'following from the meanings of the terms' involved in the propositional form xRy. He next proceeds to take the term "meanings" in the sense of "descriptions", and shows that depending on how we describe A and B (say), the propositional form xRy may, depending on the substitutions, yield either an "analytic" or "contingent" statement. He continues: 'Hume might reply that when he spoke of our "ideas of the objects (judged to be related in a certain way)", he meant the meanings of descriptions in terms of intrinsic properties only' (p.74). He now goes on to show that 'the stipulation... that only intrinsic descriptions may be substituted for x and y in the form xRy would make it impossible to classify "proportions in quantity or number" as either external or internal relations' (ibid). He then concludes by saying: 'What is left of Hume's division after such critical scrutiny, then, is not a division of relations...but simply the division of propositions into necessary and contingent. He might have said rightaway, without detouring over the unsuccessful division of relations into internal and external, that the objects of knowledge are propositions of two radically different sorts, _viz._ necessary and contingent.' (_ibid_) By a "necessary" proposition Pap means one 'which cannot possibly be false' (Glossary p.433). It would, thus, seem that Pap is referring to what I have termed Hume's "conceivability criterion". Finally, operating with this criterion, Pap proceeds to show that 'Hume is caught in a dilemma'.

My purpose now is to investigate these matters. I will first state my own position: it is my contention that Hume's division is a division of propositions into those which are necessary and those which are contingent. The grounds of this necessity I have been showing above.

I will show (i) that Pap is unable to set up the position he wants to attack, namely, the analytic/synthetic dichotomy (in Pap's sense). (Hume is not at all implicated in this, since he never made the division on these terms). And (ii) I propose to get Hume out of Pap's 'dilemma'. In other words, I propose to deal with Pap's objections against the necessary.contingent dichotomy. Pap maintains that 'the equality of 4 and 2+2 _follows_ from the meanings of the terms "4" and "2+2" alone; while...the truth of the judgment of spatial distance cannot be established by merely reflecting on the meanings of "body A" and"body B". Now, the statement "2+2 = 4" would be, according to Pap, an "analytic" statement, since it is 'true by virtue of the meanings of (the) constituent terms'; but a statement about spatial distance, since its 'truth...cannot be established by merely reflecting on the meanings' of the terms, would be not-analytic. But, one has only to ask, in what way is the term "meanings" being used here, for it to be seen that Pap is unable to formulate the very distinction he wishes to discredit. If, by "meanings", is meant "descriptions", then, an "analytic" statement, for Pap, becomes a statement 'true by virtue of the _descriptions_ of (the) constituent terms'; and a statement, which is not-analytic, is one whose 'truth...cannot be established by merely reflecting on the <u>descriptions</u>' of the terms. <u>But</u>, on this view, there cannot be any such things as not-analytic statements, all statements can be shown to be "analytic". Pap (p.74) says that a statement 'about the spatial distance of houses A and B' may be shown, depending on how we make the description, either to turn out "analytic" or "contingent". 'Should we describe', says Pap, 'the house named A as "the house which is exactly one mile south of B", then we obviously obtain an analytically true statement, while if the same object is described in terms of its appearance, the statement is contingent'. But, a description in terms of appearance, I maintain, can be shown to be no less 'obviously...an analytically true statement' thus: 'The white house is one mile south of the blue house', in strict grammatical form is 'The white house is <u>the one which</u> is one mile south of the blue house'. The phrase 'the one which' is adjectival. To what is 'the one which' referring? To the white house. How was the white house described? It was described as 'the house which is white and is one mile south of the blue house'. The statement now reads: 'The house which is white and is one mile south of the blue house is the house which is white and is one mile south of the blue house'. The statement is thus analytic.

But more than this: Pap's way of distinguishing "analytic" statements from statements which are not of that type, leads to the conclusion that no division or dichotomy can be set up. In other words, Pap is unable to set up the target which he wishes to attack.

For, if we term an analytic statement as one which is "true by virtue of meanings" (as Pap does), and all other statements as ones where 'it is not the case that they are "true by virtue of meanings"'; and if we term the former "a", and the latter "-a", it can be seen that the basis of the denial of "a" is "a" itself. In other words, in order to make the denial, the statement must first be "analytic"; that is, all "-a" statements are basically "a" statements --- or, to state it crudely: there can be no "-a" without first "a".

I come now to my second task: to try to get Hume out of Pap's dilemma. Pap, it was seen, concluded that Hume's division is one between "necessary" and "contingent" propositions (p.74)[1]. By a "necessary" proposition, Pap means one 'which cannot possibly be false' (Glossary p.433). Thus, it would seem that Pap is referring to, what I have termed, Hume's "inconceivability criterion". (This criterion Hume himself does not apply to Class I Relations. Nevertheless let us hear Pap). Operating with this criterion, Pap first makes the point that Hume 'identifies...the conceivable with the imaginable' (p.75); this, indeed, is the case.[2] Operating, then, with the term "imaginable", Pap proceeds to 'reconstruct' 'Hume's argument' thus (p.76): first, imaginability entails self-consistency (for Hume); and by saying that 'p is self-consistent', Pap means that 'no dontradiction is formally deducible from p' (p.75. footnote); second, 'not-p is imaginable' is not synonymous with 'not-p is (logically) possible' "yet the former function entails the latter, and the latter is synonymous with 'p is factual (contingent)' (ibid); third, he now "propose(s) to show that Hume is caught in a dilemma. Either 'possible' is meant in the wide sense of 'not self-

---

[1] op. cit.

[2] Vide p.18 , where I listed the evidence for the synonymity of these terms as used by Hume.

contradictory' or in the narrower sense of 'imaginable'. If theformer, then 'not-p is possible' does not entail that p is contingent and if the latter, then imaginability of not-p, being synonymous with the possibility of that state of affairs, cannot be a reason by which the latter could be proved. In other words, if the latter, then Hume's proof of the contingency of p is a petitio principii" (pp.76, 77. Italics Pap's).

I shall deal first with the latter alternative and the charge of question-begging on the part of Hume.

On this criterion: The 'reason' for the imaginability (or, conceivability) of not-p (where p is a proposition) is not that not-p is imaginable or conceivable, but that the relation between the terms is alterable; that the relation can be changed, without any difference being made to the terms of the relation, since (in contrast to ClassI) the relation does not depend on the terms (or, "ideas"). It would be on that basis that the conceivability criterion would work.

Concerning the former alternative: the denial, on the part of Pap, that 'not-p is possible' entails 'p is contingent', if "possible" is meant in the sense of "not self-contradictory". Pap's reason for this denial, is that p may be "synthetic a priori" ("Hume's imaginability criterion of necessity leaves it at least an open question whether a given proposition may not be both synthetic and necessary".) (p.79).

Such a possibility, I maintain, is straightaway ruled out by the fact that Hume's dichotomy or division into two classes is made on the basis of those relations which are "constant" and those which are "inconstant" and that the division on this basis is exhaustive; there can be no third possibility for a relation; it is either "constant" or "inconstant". Furthermore, "synthetic a priori" propositions (if there were such things) translated into Hume's language, would mean propositions which are "intuitive" and "not-intuitive" at the same time.

I will now consider Pap's synthetic a priori contention, in another way. According to Pap, a proposition for Hume is necessary (or, a priori) if one cannot imagine an exception to that proposition; and 'had (Hume) properly distinguished, like Kant, the unimaginable from the logically contradictory, he would have agreed with Kant that (the proposition 'Not more than 3 straight lines can intersect at right angles in one point') is synthetic a priori' (p.77).[1] Pap assumes, first, that all contingent propositions are, for Hume, 'empirical generalizations'. He next proceeds to consider the above proposition, saying that it is not an empirical generalization, any more than the Law of Excluded Middle is such a generalization. Thus the proposition is not contingent, therefore it must be necessary. Up till now, Pap has been making use of the necessary/contingent dichotomy. Now Hume, according to Pap, would have classed this proposition as "necessary", since one cannot imagine an exception to it. And if this proposition is necessary and if it is not an empirical generalization, then, says Pap, it must be synthetic a priori. Pap's assumption, it would seem, is that if a proposition is necessary and if it is also not an empirical generalization, then it must be synthetic a priori. In other words, Pap seems to be maintaining that a necessary proposition is ipso facto a synthetic a priori proposition. This does seem very queer. All that this argument seems to prove is that the proposition is necessary i.e. that it is not contingent.

---

[1] op. cit.

The question now to be answered is: What sort of proposition is it for Hume? The proposition 'Space is 3-dimensional' would, for Hume, be in the same category as 'The straight line is the shortest distance between two points', which Hume, at T.49, 50, points out is not "analytic" in the sense of "tautologous". The proposition 'Not more than three straight lines can intersect at right angles in one point' would, for Hume, be seen to hold good in the same way as one can see that this shade of blue is lighter than that shade of blue. It is this 'seeing', this 'direct perception' which is the mark of an intuitive or necessary proposition, for Hume (as I have abundantly shown in the previous pages of this paper).

D. F. Pears, in his article <u>Hume's Empiricism and Modern Empiricism</u> (p.24)[1] writes: 'The central contention of the first book of the <u>Treatise</u> is, as Professor Ayer says, that <u>a priori</u> propositions are empty, and that any significant proposition that is not empty must be based on experience.' (Pears adds a footnote: 'Cf. Enquiry...IV, pt.i.') (<u>Where</u> Ayer said this, he doesn't say; and at the time of writing I have been unable to discover where in Ayer this reference to the <u>Treatise</u> is).[2]

By an "empty" proposition, Pears means an "analytic" proposition (<u>vide ibid</u>, sentence preceding quotation). Pears, then, is ascribing to the <u>Treatise</u> the view that all <u>a priori</u> propositions are analytic, that is, tautologous, that is, empty.

---

[1]<u>Vide</u> p.1

[2]It may be that Pears <u>means</u> that Ayer's thesis is that 'all <u>a priori</u> propositions are empty...', and that he (Pears) is ascribing that view to Hume.

Now, although Hume _does_ hold that Class I propositions are _a priori_, in his sense of that term, he certainly does _not_ hold that they are _also_ "analytic", in Pears' sense.

Since this shall be discussed more fully, in the next section, where I discuss the status of mathematical propositions for Hume, I shall here content myself with saying that there is not a single scrap of evidence in the _Treatise_ to _support_ such a view. (Where there is no evidence, there is no view). But, there _is_ evidence from Hume _himself_ that he regarded tautologies (Hume preferred to call them "identical propositions") as being 'of no consequence' (_vide_ T.50, T.127, 200, 248) Thus at T.49, 50, he rejects the mathematician's 'definition of a right line' as 'the shortest way betwixt two points' on the ground of the absurdity of it thus being tautologous. (also quoted by Pap _op. cit._ p.70).

III. THE STATUS OF MATHEMATICAL PROPOSITIONS IN THE TREATISE

In this section, my purpose will be to show (i) that, for Hume, mathematical propositions are not analytic in the sense of being tautologous, nor (ii) are they synthetic a priori or synthetic a posteriori, but (iii) that they are a priori or necessary, in Hume's sense of that term.

* * * * * * * * *

Three of the relations (or, propositions involving these relations) of Class I, namely, resemblance, contrariety, and degrees in quality 'fall more properly under the province of intuition than demonstration' (T.70). In the case of 'degrees of any quality', it may be difficult to pronounce at first sight that x is lighter in shade than y, 'when the difference betwixt them is very small', but, when the difference between them (two colours, say) is 'considerable', it is 'easy to decide', their superiority or inferiority to each other.

Concerning 'proportions of quantity or number', we may, Hume says, 'proceed after the same manner', and 'observe' 'at one view' 'a superiority or inferiority betwixt any numbers, or figures; especially where the difference is very great and remarkable' (ibid). But, 'as to equality or any exact proportion, we can only guess at it from a single consideration; except in very short numbers, or very limited portions of extension; which are comprehended in an instant, and where we perceive an impossibility of falling into any considerable error.' (ibid) Where this cannot be done, 'we must settle the proportions with some liberty, or proceed in a more artificial manner' (ibid. Italics Hume's). In part II, of Book I (Treatise), he explains this 'artificial manner' more fully (T.47): ''Tis evident, that the eye, or rather the mind is often able at one view to determine the proportions of bodies, and pronounce them equal to, or greater or less than each other, without examining or comparing the number of their minute parts' (his coloured mathematical points. Vide T.40)[1]. These judgements, he continues, 'are not only common, but in many cases certain and infallible. When the measure of a yard and that of a foot are presented, the mind can no more question, that the first is longer than the second, than it can doubt of those principles, which are the most clear and self-evident'.

This, however, he qualifies: 'But tho' its (the mind's) decisions concerning these proportions be sometimes infallible, they are not always so;...We frequently correct our first opinion by a review and reflection; and pronounce those objects to be equal, which at first we esteemed unequal; and regard an object as less, tho' before it appeared greater than another. Nor is this the only correction, which these judgments of our senses undergo; but we often discover our error by a juxta-position of the objects; or where that is impracticable, by the use of some common and invariable measure, which being successively applied to each, informs us of their different proportions. And even this correction is susceptible of a new correction, and of different degrees of exactness, according to the nature of the instrument by which we measure the bodies, and the care which we employ in the comparison.' (ibid).

Further (T.48): '...sound reason convinces us that there are bodies vastly more minute than those, which appear to the senses[2]...(thus) we clearly perceive,

---

[1]Between the 'absurdity' of the system of mathematical points, and the 'absurdity' of the system of physical points, there is, says Hume, 'evidently a medium, viz. the bestowing a colour or solidity on these points'.

[2]How exactly this squares with his contention that 'all the perceptions of the human mind resolve themselves into...Impressions and Ideas' (T.1), he never says; and I can't say either.

that we are not possessed of any instrument or art of measuring, which can secure us from all error and uncertainty. We are sensible, that the addition or removal of one of these minute parts, is not discernible either in the appearance or measuring; and as we imagine, that two figures, which were equal before, cannot be equal after this removal or addition, we therefore suppose some imaginary standard of equality, by which the appearances and measuring are exactly corrected, and the figures reduced entirely to that proportion.' (Italics Hume's). By calling the standard "imaginary", or "a mere fiction of the mind" (ibid), his meaning is that since our 'very idea of equality is that of such a particular appearance corrected by juxta-position or a common measure, the notion of any correction beyond what we have instruments and art to make' is thus fictitious. (Italics mine).

Thus, when he speaks of settling the proportions in quantity or number after an "artificial manner" (T.70), he is referring to measurement-standards; but any standard of measurement employed for this purpose is subject to the qualifications which I have just been quoting in extenso.

Under the heading of proportion in quantity or number, he comprises geometry, algebra and arithmetic. His treatment of these, I shall discuss in that order. First, geometry: by such a subject,[1] he means (T.70) 'the art, by which we fix the proportions of figures'. This 'art' lacks 'perfect precision and exactness' (T.71); furthermore, it can never attain such qualities, for the reason that its axioms ('first principles') are 'drawn from the general appearance of the objects' (ibid); and since these 'original and fundamental principles are derived merely from appearances' (ibid), it can never reach 'a full certainty'.

---

[1]I use this vague word in order to avoid calling geometry a "science". Hume uses the term "art", and puts it in italics. At T.71, he states that algebra and arithmetic are 'the only sciences' which have 'a perfect exactness and certainty' (Italics mine).

In this section (section 1 of Part 3), he gives one illustration of his meaning: 'Our ideas seem to give a perfect assurance, that no two right (straight) lines can have a common segments; but if we consider these ideas, we shall find, that they always suppose a sensible inclination of the two lines, and that where the angle they form is extremely small, we have no standard of a right line so precise, as to assure us of the truth of this proposition'. And concludes by saying that ''tis the same case with most of the primary decisions of the mathematics' (ibid) -- that is, 'with geometry', he must mean. (Vide infra concerning Algebra and Arithmetic.)

I shall first list (and, in doing so, I will, for the most part, use Hume's own words, without troubling with quotation marks) what he says in Part 3, concerning geometry; I shall, then, refer to his fuller treatment of the subject in Part 2. He remarks (T.70) that if the difference between any figures is very great, we might observe at one view a superiority or inferiority between them. (For example, we would not have any difficulty in seeing at one glance that a triangle of altitude 1" is smaller than a triangle of altitude 6"). But, as to equality or exact proportion, we can, from a single consideration, only guess at it; very limited portions of extension, however, can be (he says) comprehended in an instant, without our falling into any considerable error. In those cases where the proportions can not be settled by sight, recourse is made to measurement. He denies to geometry the title of a perfect and infallible science, for the following reason: its first principles are drawn from the general appearance of objects; and when we take into consideration the incalculable minuteness of which nature is susceptible, the appearance of the object can not give us any assurance or security that our calculation is precise and exact. Nevertheless, he maintains, although it can never attain to a full precision and exactness, because its fundamental principles are derived merely from appearances, these very principles which are the foundation of the subject, by their very simplicity, cannot lead us into any considerable error: When the eye determines that only one straight line can be drawn between two given points, and that straight lines can not concur; such principles as these, although derived from appearances, and thereby incapable of perfect precision and exactness, save the subject from any large amount of error. (T.72).

In Part 2 (T.45), he writes: 'When geometry decides anything concerning the proportions of quantity, we ought not to look for the utmost precision and exactness. None of its proofs extend so far. It takes the dimensions and proportions of figures justly; but roughly, and with some liberty. Its errors are never considerable; nor would it err at all, did it not aspire to such an absolute perfection.' (Italics Hume's).

In deciding the equality, superiority, or inferiority of lines (or, surfaces) to each other, the standard employed is neither that of an enumeration of the points, which make up the line (or, surface), nor that of congruity. The former standard is 'entirely useless, and that it never is from such a comparison we determine objects to be equal or unequal with respect to each other. For as the points, which enter into the composition of any line or surface, whether perceived by the sight or touch, are so minute and so confounded with each other, that 'tis utterly impossible for the mind to compute their number, such a computation will never afford us a standard, by which we may judge of proportions' (T.45. Italics in text). The latter standard, being based on the 'equality of the number of the points', is likewise 'useless'. (T.46, 47). 'The only useful notion of equality, or inequality, is derived from the whole united appearance and the comparison of particular objects' (Appendix T.637).[1] ''Tis evident, that the eye,

---

[1] Prepared by Hume, for insertion at T.47.

or rather the mind is often able at one view to determine the proportions of bodies, and pronounce them equal to, or greater or less than each other, without examining or comparing the number of their minute parts. Such judgments are not only common, but in many cases certain and infallible. When the measure of a yard and that of a foot are presented, the mind can no more question, that the first is longer than the second, than it can doubt of those principles, which are the most clear and self-evident' (T.47).[1]

At T.50, 51, he writes: 'the ideas which are most essential to geometry, viz. those of equality and inequality, of a right line and a plain surface, are far from being exact and determinate, according to our common method of conceiving them.' (That is, from the 'united' or 'general appearance' of the objects). 'Not only we are incapable of telling, if the case be in any degree doubtful, when such particular figures are equal; when such a line is a right one, and such a surface a plain one; but we can form no idea of that proportion, or of these figures, which is firm and invariable. Our appeal is still to the weak and fallible judgment, which we make from the appearance of the objects, and correct by a compass or common measure; and if we join the supposition of any farther correction, 'tis of such-a-one as is either useless or imaginary. In vain should we have recourse to the common topic, and employ the supposition of a deity, whose omnipotence may enable him to form a perfect geometrical figure, and describe a right line without any curve or inflextion. As the ultimate standard of these figures is derived from nothing but the senses and imagination, 'tis absurd to talk of any perfection beyond what these faculties can judge of; since the true perfection of anything consists in its conformity to its standard.' (Italics mine).

---

[1] Vide p. 38 for Hume's qualification to the above remarks.

In the case of algebra and arithmetic, by contrast, we do have a '<u>precise</u>' standard, 'by which we can judge of the equality and proportion of numbers; and according as they correspond or not to that standard, we determine their relations, without any possibility of error' (T.71). Two numbers are said to be equal, when the units, of which each number is composed, are equal: 'When two numbers are so combined, as that the one has always an unite answering to every unite of the other, we pronounce them equal; and 'tis for want of such a standard of equality in extension, that geometry can scarce be esteemed a perfect and infallible science.' (<u>ibid</u>).

I come now to the question of the <u>status</u> of mathematical propositions, or propositions involving relations of proportions in quantity and number. Propositions involving such relations belong to Class I, the class consisting of propositions which yield <u>knowledge</u> or <u>certainty</u>; a certainty which is either intuitive or demonstrative.

In view of Hume's account of geometry, both in Part 3 and in Part 2, it might be thought that there is a division <u>within</u> those relations which deal with proportions in quantity and number: that algebraic and arithmetical propositions are to be distinguished in <u>type</u> from geometrical propositions. It might be said, for example, that, for Hume (in the <u>Treatise</u>) geometrical propositions are "empirical", or, "synthetic", or, "synthetic a posteriori", and that algebraic and arithmetical propositions are "analytic" (to use terminology not itself used by Hume). The view that propositions involving the four relations which make up Class I, of which proportions in quantity and number, i.e. geometrical, algebraic and arithmetical propositions, are one of that number, are "analytic" (in the sense <u>specified</u> by Pap and Pears, I have dealt with above)
I propose to show that, for Hume, there is <u>no</u> such division <u>within</u> those relations

which deal with proportions in quantity and number; that geometrical propositions are <u>not</u> to be distinguished in <u>type</u> from algebraic and arithmetical propositions. The only difference between geometrical propositions <u>and</u> algebraic and arithmetical propositions is that, in the case of the former, perfect precision is lacking, in the case of the latter, we do have perfect precision. The 'standard, by which we...judge of the equality and proportion of numbers' (namely, the procedure of marking off the units, of which the number if composed, one by one) is 'entirely useless' as far as the proportions of figures are concerned; no enumeration or marking off of points is possible, due to the incalculable 'minuteness of which nature is susceptible.'

We have to proceed merely from the 'general' or 'united appearance' of objects. This, however, is <u>not</u> a distinction in <u>type</u> between the two sets of propositions. Geometrical propositions are "intuitive" in the same way that algebraic and arithmetical propositions are "intuitive": ''Tis evident, that the eye, or rather the mind is often able <u>at one view</u> to determine the <u>proportions</u> of bodies, and pronounce them equal to, or greater or less than each other, without examining or comparing the number of their minute parts. ...When the measure of a yard and that of a foot are presented, the mind can no more question, that the first is longer than the second, than it can doubt of those principles, which are the most clear and self-evident.' (T.47, Italics mine.).

The grounds of this "intuitiveness", I have shown above (p. 12); here I will only emphasize that the relation 'greater than' (say) holding between two <u>figures</u> can be '<u>observed</u>' (T.70) in the <u>same</u> way as the same relation holding between two <u>numbers</u>. Thus, the fact that 'perfect precision' is lacking in geometry, does not entail the fact that geometrical propositions are not "intuitive".

These propositions are "intuitive": neither geometrical nor algebraic and arithmetical propositions are "analytic"; nor are geometrical propositions empirical, synthetic, or synthetic a posteriori. An empirical proposition, for Hume, is one where the relation holding between the two terms is "inconstant", or "variable", or, "alterable"; geometrical propositions are one of that number where the relation is "constant", or, "invariable", or, "unalterable".

If one wants to term these "intuitive" propositions "necessary" or "a priori" (in Hume's sense; that is, in the sense that they are <u>intuitively</u> certain), I raise no objection to that, so long as it is understood in <u>what</u> sense the terms "necessary" or "a priori" are being used.

R. F. Atkinson, in his paper <u>Hume on Mathematics</u>[1] inclines towards the view that, for Hume, mathematical propositions are "synthetic a priori". He writes (p.127): '(Hume) came closer in the <u>Treatise</u> to regarding mathematical propositions as synthetic necessary than analytic truths; and that it is at least disputable whether a significantly different view was taken in the <u>Enquiry</u>.' Before offering his evidence in support of that view, he wishes first 'to dispose of the view which a quick reading of the <u>Treatise</u> might suggest that Hume there held geometrical propositions to be synthetic a posteriori and those of arithmetic and algebra to be analytic.' (ibid). In dealing with this paper, I shall begin by stating Atkinson's reasons for maintaining that geometrical propositions are <u>not</u> "synthetic a posteriori", and that those of arithmetic and algebra are <u>not</u> "analytic". While I <u>do</u> maintain that, for Hume, geometrical propositions are not <u>synthetic a posteriori</u> and that algebraic and arithmetical propositions are not <u>analytic</u>, I disagree with Atkinson's way of showing this. Secondly, I shall deal with Atkinson's view that mathematical propositions are "synthetic a priori".

---

[1]<u>Philosophical Quarterly</u>, 1960, Vol. 10.

Atkinson is quite clear about the *fact* that Hume did not '*explicitly* pose the question, Analytic or Synthetic?' (p.127, Italics mine); his whole paper, however, seems to be haunted by this ghost. For, after offering evidence (he quotes T.49, 50) to show 'that Hume does not regard geometrical propositions as analytic' (p.131), he writes: 'Hume, then, in effect regarded geometrical propositions as *synthetic*. But, in view of his radical empiricism, must it not be inferred from this that he also regarded them as a posteriori?' (ibid. Italics mine). Atkinson's answer is: No. But, before producing evidence in support of his answer, he interprets Leroy as holding that for Hume geometrical propositions (or, axioms) are "synthetic a posteriori". (*vide* footnote 10, p. 131). Such an interpretation of Leroy is, I think, wrong. Indeed, it would commit Leroy to holding that, for Hume, algebraic and arithmetical propositions are *likewise* "synthetic a posteriori". For, what Leroy says about these two sets of mathematical propositions (geometrical *and* algebraic and arithmetical) is the same for each. Leroy[1] writes: 'Dans le cas des *relations d'idées*, il n'y a qu'à bien considérer les objets présents aux sens pour discerner leur rapport de grandeur. *C'est une perception, directe ou indirecte*; *c'est une vue instantanée*. *Chaque pas en avant dans une démonstation se fait par une vue de ce genre*. Sans doute, c'était cet empirisme géométrique que Lord Stanhope condamnait. Mais Hume est formel; la géométrie porte sur des apparences sensibles, même si ces apparences sont les plus évidentes et les moins trompeuses.' (p.76, footnote 4). And speaking of arithmetical propositions (pp.77, 78), he writes: 'L'arithmétique arrive à plus de certitude, parce qu'elle dispose d'un critère précis de l'égalite de deux nombres. Elle fait se correspondre une à une les unités qui constituent les deux nombres. On peut

---

[1] Op. cit.

donc établir de longues chaînes de raisonnement sans crainte d'erreur...Mais, ici encore, c'est la perception directe d'une égalite, ou d'une équivalence, qui assure la rectitude d'un raisonnement et la certitude des résultats'. (Italics mine in both quotations).

It is this 'perception directe', this 'vue instantanée' which is the mark of an "intuitive" or necessary or a priori proposition for Hume.

The evidence produced by Atkinson to show that geometrical propositions are not "synthetic a posteriori", is the same evidence which leads him to conclude that such propositions are "synthetic a priori". He writes (pp.131, 132): '...the following passage shows (that Hume) was prepared to accord the highest necessity rating to a patently synthetic proposition -- and, as for Kant, so for Hume necessity is always a mark of the a priori: "'Tis evident, that the eye...self-evident"' (p.47)[1]' Before discussing this, the way in which Atkinson shows algebraic and arithmetical propositions not to be analytic, will be discussed. Atkinson takes an "analytic" proposition to be one the negation of which is a formal contradiction ('analytic in the sense of having formally contradictory negations' (p.128)). He remarks that, although Hume does use the term "contradiction", 'it does not appear...that he confines the term "contradiction" to the sense of "formal contradiction" '(p.128); 'Hume's usual practice is to use "contradictory" as a simple synonym for "inconceivable", and indeed to use the latter as if it were synonymous with "unimaginable" '(p.129), And, says Atkinson, '(Hume) certainly holds that (the negations of mathematical propositions, or of "relations

---

[1]The reference is to the page number in the Selby-Bigge edition. (Quoted in full by me on p. 41).

of ideas" propositions generally) are inconceivable so long as the ideas compared together remain the same' (p.128. Italics mine); but this, as Atkinson says, does not involve a formal contradiction. Atkinson now proceeds to the remaining three relations (or, propositions involving these relations): of contrariety, he says: 'Any proposition Hume would regard as asserting contrariety (it has to be remembered that the only ideas in themselves contrary are existence and non-existence), say, "there cannot both be and not be X's" would of course be analytic, for it cannot be denied without formal contradiction. But the case is different with propositions asserting resemblance or degrees in quality. It is unfortunate that Hume gives no explicit examples, so that one cannot be sure that he would include under this head so patently synthetic a proposition as "Tom is more like Dick than he is like Harry", but he could surely not repudiate the following examples: "Blue is more like green than it is like scarlet"[1]..." Black differs from white" ...and "Ice is colder than steam". Of such propositions it can at the very least be said that they are not obviously analytic-they have close affinities with such notorious contenders for the role of synthetic necessary truth as "Nothing can be red and green all over". And it is completely clear that Hume does not think that the truth of such propositions follows from the definitions of their terms. He rather thinks that they are "seen" to be true when the objects in question are presented in the sense of observed or imagined. He even takes this view of a proposition asserting a proportion in quantity- "A yard measure is longer than a foot measure"- which could very plausibly be held

[1] Atkinson inserts in parenthesis the sources of these examples.

to be true by definition (p.47)[1]. It would, of course, be grotesque to read into these passages a definite contention that mathematical propositions are synthetic necessary truths. But it is surely clear that this possibility is not even by implication excluded ...'(p.129, 130). And on page 132, he writes: '...the main burden of this paper is that it is misleading to classify Hume's views by reference to distinctions which he did not himself make. But, if this is to be done at all, the least objectionable way to do it seems to me to be to regard Hume as holding, like Kant, that the propositions of geometry are synthetic a priori i.e. necessarily true but to be established not by the analsis of concepts but by an appeal to intuition. 'I will also quote Atkinson's remaining words (as far as the Treatise is dealt with by him), since I propose to deal with this contention that mathematical propositions, for Hume, are "synthetic a priori"; and also with the following: 'Kant', says Atkinson, 'is, however, undoubtedly the more through and self-consistent in working out his view. He saw that if geometrical propositions were a priori then the intuition in question must be pure intuition, and that space must be a pure (form of) intuition.

...Hume's "official" view of space and time is...diametrically opposed to Kant's but there are nonetheless not infrequent indications in the Treatise that Hume was sometimes tempted to modify his view in the Kantian direction. This has been noticed by several of Hume's commentators. For instance, C. W. Hendel in his Studies in the Philosophy of David Hume (Princeton, 1925. Chap. V) and by Kemp Smith, who writes: "Since the only impressions which (Hume) has allowed are impressions lacking in any element of extension or duration, the spatial and

---

[1]The reference is to the page number in the Selby-Bigge edition.

temporal features so undeniably apprehended by the vulgar consciousness have to be treated as non-empirical, and therefore, by implication, as being a priori"[1] (The Philosophy of David Hume. 1941, Reprint 1949, p.548, and cf. pp.288-9)' (p.132). Before I deal with the "synthetic a priori" contention, the first business to clear up is this business of "intuition", and the supposed temptation, on the part of Hume, 'to modify his view in the Kantian direction'. One of Hume's commentators, C. W. Hendel, in the new edition (1963) of his Studies, no longer holds the position referred to by Atkinson;[2] chapter V of the original edition, he omitted entirely from the new edition (1963); substituting in its stead "Appendix III, On Space and Time: Correction of Former Errors." The criticisms levelled by W. T. Parry[3] against Hendel's view of Hume on Space and Time are conceded in that Appendix by Hendel. The gist of Parry's criticism being: 'space and time relations are directly perceived' (Studies, p.502). Hendel had tried 'to show that Hume suggests that space and time and our ideas of them are produced by the imagination'; that it had been suggested, according to Hendel, by Hume, 'that the "manner" in which we perceive objects in space and time represents our mental disposition' (Studies, p.500). In other words, what Hendel had been maintaining is this: Hume had classed contiguity in space and time as a "natural" relation, and since all natural relations are 'the effect of operations of the imagination' (the words are Hendel's, op. cit. p.501), contiguity in space and time is 'the effect of operations of the imagination'. Parry makes his point by citing T.73: 'All kinds of reasoning consist

---

[1]This quotation continues (it is not continued by Atkinson): 'For though Hume does not himself draw this conclusion, his use of the phrase "manner of appearance" amounts to a virtual admission of it."

[2]It is to be noted: Atkinson's paper was published in 1960, and thus the new edition of Hendel's Studies was not available for his consultation. Whether Atkinson himself would agree with Hendel's 'correction of former errors', I do not know.

[3]Vide: Studies (New York: The Bobbs-Merrill Co. Inc., 1963).

in nothing but a comparison, and a discovery of those relations, either constant or inconstant, which two or more objects bear to each other...When both the objects are present to the senses along with the relation, we call this perception rather than reasoning; nor is there in this case any exercise of the thought, or any action, properly speaking, but a mere passive admission of the impressions thro' the organs of sensation. According to this way of thinking, we ought not to receive as reasoning any of the observations we may make concerning identity, and the relations of time and place; since in none of them the mind can go beyond what is immediately present to the senses...' Parry comments: 'From this it appears that two objects in spatio-temporal relations may be both "present to the senses along with the relation", and that this is a case of perception, without any action of the mind; hence without any action of the imagination. So I hold that, for Hume, the relations of space and time may more properly be said to be perceived than produced by the imagination (Studies, p.502. Italics in text). Hendel comments: 'Parry's contention is entirely justified: space and time relations are directly perceived' (ibid). Parry continues: '...as to Mr. Hendel's contention that, since the ideas of space and time are complex, Hume should explain "the formation of such ideas by the principle of an imagination operating according to its native tendencies." Hume never intimates that all complex ideas are formed by imagination' (ibid).

Up till now, I have been concerned only with making Hendel's position clear. I want now to take up some of Parry's points. His citation of T.73, to show that, for Hume, spatio-temporal relations are 'directly perceived', does not solve the problem (as it stands),[1] it merely accentuates it.

---

[1]Since we do not have the whole text of Parry's paper (only selections of it from Hendel), I do not know in what detail the problem has been dealt with by Parry.

When Parry maintains that Hume does not maintain that '<u>all</u> complex ideas are formed by the imagination', he must be referring to Hume's statement (T.13): 'Amongst the effects of this union or association of ideas, there are none more remarkable, than those complex ideas, which are the common subjects of our thoughts and reasoning, and <u>generally</u> arise from some principle of union among our simple ideas' (Italics mine). In other words, it would seem that Hume is here maintaining that not all, only some complex ideas are the 'effects' of 'association'. But, at the beginning of this section (4), Hume had been employed about the task of explaining why it is that, 'as all simple ideas may be separated by the imagination, and may be united again in what form it pleases', 'the same simple ideas...fall regularly into complex ideas (as they commonly do)' (T.10), and had given his explanation in terms of a 'uniting principle' or 'bond of union' or 'association' or 'attraction' among the simple ideas: 'nothing would be more unaccountable than the operations of that faculty, were it not guided by some universal principles, which render it, in some measure, uniform with itself in all times and places' (<u>ibid</u>). The simple ideas unite together by <u>attraction</u>, thus forming complex ideas or wholes; just as a house (a whole, a complex idea) is made up of bricks (simple ideas) and cement (the attraction or uniting principle)[1]. It may, however, be objected that Hume's use of the word "commonly", again suggests that not all complex ideas are <u>so</u> constructed. It seems, then, by no means <u>precisely</u> clear whether <u>all</u> complex ideas are 'effects' of 'association'. But,

[1] I am making use of the illustration given by J. A. Passmore in <u>Hume's Intentions</u> (p.106), Cambridge University Press, 1952. Hume in the <u>Abstract</u> speaks of the 'cement of the universe'.

what _is_ precisely clear is that Hume subdivides complex ideas into _relations_, Modes and substances (T.13). Now, if relations are that species of complex ideas which _are_ the 'effects' of 'association', and, if by this is meant that relations are 'produced by the imagination, (i.e. by a 'uniting principle' among simple ideas which 'guides' the imagination---Hendel must mean) then Parry's point that _not_ _all_ complex ideas are 'effects' of 'association' is beside the point. But, if relations are _complex_ ideas, and complex ideas are produced by a 'uniting principle' among the simple ideas, then, when Hume maintains that this 'uniting principle' or 'association' 'arises from' the relations of resemblance, contiguity in time or place, and cause and effect (T.11), he is saying that the 'uniting principle' arises from these three relations: ''Tis plain, that in the course of our thinking, and that in the constant revolution of our ideas, our imagination runs easily from one idea to any other that _resembles_ it, and that this quality alone is to the fancy a sufficient bond and association. 'Tis likewise evident, that as the senses, in changing their objects, are necessitated to change them regularly, and take them as they lie _contiguous_ to each other, the imagination must by long custom acquire the same method of thinking, and run along the parts of space and time in conceiving its objects. As to the connexion, that is made by the relation of _cause_ _and_ _effect_, ...there is no relation, which produces a stronger connexion in the fancy, and makes one idea more readily recall another, than (that relation) betwixt their objects' (_ibid_. Italics Hume's).

Thus, it would seem that there is an evident circularity: relations give rise to the uniting principle among simple ideas, and the uniting principle gives rise to relations.[1]

---

[1]Cf. M. R. Annand, _The Monist_, 1930 Vol. XL, pp.585, 586: 'On the one hand, association is treated by (Hume) as that which gives rise to ideas of relation, and, on the other, ideas of relation are treated by him as the features that give rise to association.'.

But, Hume distinguishes relations into two kinds, those which he terms "natural" (T.15, 170), and those which he terms "philosophical" (T.14): 'The word Relation is commonly used in two senses considerably different from each other. Either for that quality, by which two ideas are connected together in the imagination, and the one naturally introduces the other (by attraction); or for that particular circumstance, in which, even upon the arbitrary union of two ideas in the fancy, we may think proper to compare them.' (T.13). And those relations which give rise to an 'attraction' or 'uniting principle' among the simple ideas, are what Hume terms "natural" relations. And when Hume says that the term relation, in the sense of "natural" relation, 'is always the sense, in which we use the word', 'in common language', 'and 'tis only in philosophy, that we extend it to mean any particular subject of comparison', it would seem that he is restricting the use of the word to the 'common language' use. Thus, when he subdivides complex ideas into relations, modes and substances, the relations to which he is here referring, it might seem, are those "natural" relations. In the passage cited above by Parry, spatio-temporal relations there are being discussed by Hume as a "philosophical" relation (whether Parry himself is aware of this, I do not know, since we do not have the text of the whole paper; only selections of it from Hendel). Thus, if Hume is restricting the term "relation" to "natural" relations, and if the spatio-temporal relation here being discussed is a "philosophical" relation (as it is, in the above passage), then, Hendel's contention that it is a product of association, is justifiably criticised by Parry.[1]

---

[1]Kemp Smith seems to understand Hume as 'limit(ing) the term "relation" to "natural" relation' (p.252, op. cit). He writes (pp.251, 252): 'Common language, (Hume) suggests, allows resemblance to be a relation only when some perception, itself already in the mind, calls up some other idea owing to the resemblance between them; it does not properly allow of its being described as a relation when discovered through the comparison of two ideas already both present to the mind.' He further writes (pp.250, 251): '...in tracing the manifold effects, i.e. products, of association, Hume gives first place...to complex ideas, as distinguished into relations, modes

(Continuation of footnote from page 50)

and substances. Here he is following the classification of complex ideas given by Locke in the first three editions of the *Essay*. Presumably Hume's copy of the *Essay* was in one of these editions. In the fourth edition of the *Essay* Locke had inserted a passage in which he shows appreciation of the fact that the really fundamental distinction is not between simplicity and complexity, but between primary or original and secondary or derivative. All non-primary ideas are based on primary ideas; but it is not merely by an act of mechanical assembling or combining that they arise out of them. In the case...of ideas of relations, an act of the mind is required: *bringing two ideas, whether simple of complex, together, and setting them one by another, so as to take a view of them at once, without uniting them into one; by which it gets all its ideas of relations* (*Essay*, Book II, ch. 12). Ideas of relations...not being due to a process of compounding, are not properly describably as complex...ideas, and are not therefore explicable merely by means of the mechanism of association. As being "philosophical" relations, they are not a sub-species of complex ideas, but distinguishable from and co-ordinate with them. Hume's adoption of Locke's first and cruder method of classification is what has made possible for him his attempted restriction of the term "relations" to what he entitles the "natural" relations...'.

Moreover, it seems to me that Hendel has been seriously misled by Hume's use of the term "manner". Hendel curiously says (p.441)[1]: 'the use of the term "manner"...does not seem "makeshift"[2] if one bears in mind Hume's classical education...for "manners" like "customs" harks back to the Latin root of "morals". If...Hume came into philosophy through the "gateway of morals"[3], what could be more natural than the use of "manner" to signify man's way in the case of perceiving...'. What, it is to be asked, has "manner" to do with "manners" (mores) or "customs"!

It is perfectly clear from these remarks of Hendel's that Hendel is regarding the spatio-temporal relation as a "natural" relation. Hume never maintained that all relations of space and time are "natural", only contiguity in place or time was listed as one of the three "natural" relations. Moreover, in the context with which we are dealing with that relation, Hume is speaking of philosophical relations.

As noted, Atkinson cites Kemp Smith (pp.548, and 288-9)[4]. Since the point of Kemp Smith's comments there, is that the 'manner of appearance' (T.34) is not an 'impression', it would seem that this interpretation of Hume by Kemp Smith, leads Hume, according to Atkinson, into the Kantian position. The point made in the two passages from Kemp Smith (one of which is quoted by Atkinson, the other of which is not) is the same. The second passage (pp.288-9) has a direct

---

[1]Op. cit.

[2]Kemp Smith's term (op. cit. p.289) for Hume's way of talking about the ideas of space and time.

[3]Kemp Smith's phrase, and also his belief.

[4]Op. cit.

reference to geometry and I shall quote it (it is not quoted by Atkinson)[1]; and immediately following the quotation, I shall refer to Hume. Kemp Smith writes: 'Hume's treatment of geometry as being an inexact science has customarily been regarded as prescribed for him by his sensationalism. This...is not really a tenable interpretation. Hume takes a non-sensationalist view of space and time. He has refused to adopt the easy line of treating extensity and duration as disclosed in simple impressions. (Had he done so, he would have had to allow that the impressions, however simple, are, at least in thought, divisible, and he would not therefore have been able to employ the arguments upon which he has relied in refuting the hypothesis of infinite divisibility)'

I take issue with this, and with the contention that the 'manner' is not an 'impression'. I will begin with the latter contention.

Let us hear Hume (Part II, Section 3, pp.33, 34); he begins by invoking his principle: 'No discovery could have been made more happily for deciding all controversies concerning ideas, than that impressions always take the precedency of them, and that every idea, with which the imagination is furnished, first makes its appearance in a correspondent impression...Let us apply this principle, in order to discover farther the nature of our ideas of space and time. Upon opening my eyes, and turning them to the surrounding objects, I perceive many visible bodies; and upon shutting them again, and considering the distance betwixt these bodies, I acquire the idea of extension. As every idea is derived from some impression, which is exactly similar to it, the impressions similar to this idea of extension, must either be some sensations derived from the sight, or some internal

---

[1]For the passage from Kemp Smith quoted by Atkinson, Vide supra p.49.

impressions arising from these sensations. Our internal impressions are our passions, emotions, desires and aversions; none of which, I believe, will ever be asserted to be the model, from which the idea of space is derived. There remains therefore nothing but the senses, which can convey to us this original impression. Now what impression do our senses here convey to us? This is the principal question, and decides without appeal concerning the nature of the idea. The table before me is alone sufficient by its view to give me the idea of extension. This idea, then, is borrowed from, and represents some impression, which this moment appears to the senses. But my senses convey to me only the impressions of coloured points, disposed in a certain manner. If the eye is sensible of anything farther, I desire it may be pointed out to me. But if it be impossible to show anything further, we may conclude with certainty, that the idea of extension is nothing but a copy of these coloured points, and of the manner of their appearance.'

And, since 'time' is also a 'manner', I am going to quote Hume on that too (both passages may throw the proper light on this whole problem).

He begins by invoking the principle, namely: 'Whatever objects are different are distinguishable, and that whatever objects are distinguishable are separable by the thought and the imagination.'[1] The first full formulation of this principle is at T.18 (which I have just quoted). At T..10, we have the first indication of it (so far as I can see): 'Where-ever the imagination perceives a difference among ideas, it can easily produce a separation.' The formulation of it at the

---

[1]It is also 'true in the inverse', i.e. 'converse'.

passage, I am now going to quote (T.36) is as follows: 'Everything, that is different, is distinguishable; and everything, that is distinguishable, may be separated' (by the thought and the imagination).

He writes (T.36, 37): 'In order to know whether any objects, which are joined in impression, be separable in idea, we need only consider, if they be different from each other; in which case, 'tis plain they may be conceived apart.' ...(he then quotes the principle, I cited above)...'if on the contrary they be not different, they are not distinguishable; and if they be not distinguishable, they cannot be separated. But this is precisely the case with respect to time, compared with our successive perceptions. The idea of time is not derived from a particular impression mixed up with others, and plainly distinguishable from them; but arises altogether from the manner, in which impressions appear to the mind, without making one of the number. Five notes played on a flute give us the impression and idea of time; tho' time be not a sixth impression, which presents itself to the hearing or any other of the senses. Nor is it a sixth impression, which the mind by reflection finds in itself ...it (the mind) only takes notice of the <u>manner</u>, in which the different sounds make their appearance; and that it may afterwards consider without considering these particular sounds, but may conjoin it with any other objects. The ideas of some objects it certainly must have, nor is it possible for it without these ideas ever to arrive at any conception of time; which since it appears not as any primary distinct impression, can plainly be nothing but different ideas, or impressions, or objects disposed in a certain manner, that is, succeeding each other.'

From the first quotation, it can be seen that 'the idea of extension is nothing but a copy of these coloured points, and of the manner of their appearance'.

From the second quotation, it can be seen that 'the idea of time is not derived from a particular impression mixed up with others,and plainly distinguishable from them; but arises entirely from the manner, in which impressions appear to the mind, without making one of the number...(it is) nothing but different ideas, or impressions, or objects disposed in a certain manner, that is, succeeding each other'.

With respect to extension, he is maintaining that the idea of it, is a copy of the 'coloured points' and of their arrangement; or, to express this in a more accurate way, it is a copy of the arrangement of the 'coloured points'; that is, a copy of the positions of the 'coloured points' (coloured points or just one coloured point must have position, that is, arrangement) and when Hume speaks of 'manner', he means 'arrangement', that is, 'positions'. Now, these 'positions' or 'arrangement' of the points, is a complex impression, an impression which is 'copied' in idea.

With respect to time: 'the manner in which the different sounds make their appearance', is not 'any primary distinct impression, (it) can plainly be nothing but different ideas, or impressions...disposed in a certain manner, that is, succeeding each other'. Again, the disposition or arrangement is a complex impression, an impression which is copied in idea. And complex impressions are a collection of simple impressions. Hume (at T.38) refers to the 'impression' which represents (that is, copies) extension, as a 'compound impression', that is, a complex impression. He writes: 'That compound impression, which represents extension, consists of several lesser impressions, that are indivisible to the eye or feeling, and may be called impressions of atoms or corpuscles endowed with colour and solidity. But this is not all. 'Tis not only requisite, that these atoms should be coloured or tangible, in order to discover themselves to our senses; 'tis also necessary we should preserve the idea of their colour or tangibility in order to comprehend them by our imagination. There is nothing but the idea of their colour or tangibility which can render them conceivable by the mind. Upon the removal of the ideas of these sensible qualities, they are utterly annihilated to the thought or imagination.' (T.38, 39).

What I have been showing is that the 'manner' is a complex impression, or a 'compound impression', as Hume calls it.[1]

But, now we run into two problems. The problems are these: (i) what is the exact content of Hume's fundamental principle?; and (ii) in what way exactly does he employ it? (And by that I mean this: when Hume asks for the 'impression' to be pointed out, it is to be asked, what kind of impression? Simple or Complex? These problems confronted me in a previous paper[2]; here I will merely say that even if the impression is complex, the matter stands thus: although there are some complex impressions which are not exactly copied in idea (T.3: 'I observe, that many of our complex ideas never had impressions, that corresponded to them, and that many of our complex impressions never are exactly copied in ideas...I have seen Paris (complex impression); but shall I affirm I can form such an idea of that city, as will perfectly represent all its streets and houses in their real and just proportions?'), the fact that the idea is not an exact representation of the complex impression, does not thereby mean that it is no genuine idea; it is genuine (as genuine as the idea of Paris), it is an idea which is made up out of a mass of simple impressions, albeit that they (the simple impressions) are not all exactly

---

[1] Contrast this with Kant: 'That in the appearance which corresponds to sensation I term its matter; but that which so determines the manifold of appearance that it allows of being ordered in certain relations, I term the form of appearance. That in which alone the sensations can be posited and ordered in a certain form, cannot itself be sensation; and therefore, while the matter of all appearance is given to us a posteriori only, its form must lie ready for the sensations a priori in the mind, and so must allow of being considered apart from all sensation.' (pp.65,66). Critique of Pure Reason, trans. by N. Kemp Smith. New York: St. Martin's Press, 1965, Italics in text

[2] "Idea" in the Empiricist Programme of Locke, Berkeley and Hume. (Written in a course conducted by my tutor, Dr. James Noxon).

represented or copied in the idea which we have.

M. R. Annand (An "Examination of Hume's Theory of Relations", p.588)[1] correctly (that is, from the point of view of the evidence) remarks that Hume 'describes extension as "a compound impression"...', but Annand goes on to raise some objections to this. I shall quote what Annand says, since I propose to deal with it. Annand writes: '...unless these "lesser impressions" '(vide the quotation from Hume's text on page 59) 'can be regarded as present together, they must follow one another, and thus precede the "compound impression". On the latter supposition, extension would consist of parts none of which could be present at the same time, and all of which must cease to be present before extension itself could come into being. But the former supposition is precluded by what Hume inculcates in regard to time. It is true he does not assert in so many words that all visual impressions must be successive, but he <u>does</u> assert that "the impressions of touch," which along with those of sight he had represented as constituting the "compound impression" of extension, "change every moment upon us." And after having made out extension to be a compound of coexistent impressions, he proceeds to speak of the idea of time as derived "from the succession of our perceptions <u>of every kind</u>, ideas as well as impressions". The parts of time cannot, he urges, be coexistent; and seeing that "time itself is nothing but different ideas and impressions succeeding each other," it would follow that "the parts of time" are those "perceptions of every kind" from which the idea of time is derived. If, then, <u>all</u> impressions, as parts of time, are successive, how can some impressions, as parts of space, be coexistent?' (pp.588, 589). (Italic's Annand's).

---

[1]<u>The Monist</u>, 1930, Vol. XL, pp.581-597.

Hume's reply would be: the 'compound impression' as a whole, is a co-existent; it is one 'perception'. Time is derived from a succession of these co-existents. As Hume puts it at T.237: '(the compound impression) is co-temporary in (its) appearance in the mind.' (Italics mine).

Concerning Kemp Smith's second point, namely that had Hume treated extensity and duration as being 'disclosed in simple impressions', 'he would have had to allow that the impressions, however simple, are, at least in thought, divisible, and he would not therefore have been able to employ the arguments upon which he has relied in refuting the hypothesis of infinite divisibility': first, I have pointed out that extension (and time) are, for Hume, complex impressions or 'compound' impressions, and these complex impressions are a collection of simple impressions. Secondly, although the simple impressions are 'at least in thought divisible', nevertheless this process of division is not infinite, according to Hume. He writes (T.27): ''Tis...certain, that the imagination reaches a minimum, and may raise up to itself an idea, of which it cannot conceive any sub-division, and which cannot be diminished without a total annihilation.' And again (ibid): ''Tis the same case with the impressions of the senses as with the ideas of the imagination. Put a spot of ink upon paper, fix your eye upon that spot, and retire to such a distance, that at last you lose sight of it; 'tis plain, that the moment before it vanished the image or impression was perfectly indivisible...' Again at T.32: ''Tis certain we have an idea of extension...(and) that this idea, as conceived by the imagination, tho' divisible into parts or inferior ideas, is not infinitely divisible, nor consists of an infinite number of parts: For that exceeds the comprehension of our limited capacities. Here then is an idea of extension, which consists of parts or inferior ideas, that are perfectly indivisible...'. At T.39, he writes: 'The capacity of the mind is not infinite; consequently no idea of extension (or duration) consists of an infinite number of parts or inferior ideas, but of a finite number, and these simple and indivisible...'.

M. R. Annand (op. cit.) makes the same kind of objection as that of Kemp Smith noted above. Annand, in the context of Hume on lines, surfaces and points, writes: '...when he (Hume) comes to explain what is meant in geometry by a line, surface, or solid, his failure to do so is...apparent. Each of these is, he avers, a complex of "coloured points", so that one line is equal in length to another if it comprises the same number of "points"' (I interrupt here in order to make Hume's position clear; I have remarked above, p.41, that Hume regarded this standard of equality as 'entirely useless, and that it never is from such a comparison we determine objects to be equal or unequal with respect to each other (T.45. Italics Hume's)) 'But these "points" are so minute and so confused with one another that there is no possibility of counting them. It is, indeed, difficult to treat such a statement seriously. If a line be a collection of "coloured points", it can only be made up of coloured surfaces lying side by side, and separated, therefore, from one another. How could these constitute a line, or even a continuous surface? Not only so. Each "coloured point", or surface, would be divisible into parts, and these again into smaller parts, and so on indefinitely' (pp.591, 592).

The short answer to the latter statement is, that for Hume, the case is not so (as I have shown above). Hume, by maintaining that 'the capacity of the mind is limited' (T.26), a fact which is 'universally allowed' and which is 'evident from the plainest observation and experience' struck at the root of the contentions of the devotees of the doctrine of infinite divisibility.

Regarding the other objections, on the part of Annand, that on Hume's idea of a line, there could not be any such things as lines, Hume, it has to be admitted, nowhere makes clear how "coloured points" lying contiguous to each other, can make up a continuous line. This objection, however, is based on the assumption that a line is that which is <u>not</u> made up of points, but is that which is a <u>continuum</u>. Hume, in reply, might reject such an idea of a line.

I have been concerned with showing that the view (mentioned by Atkinson) that there is some back-sliding, on the part of Hume, into the Kantian position, does not square with the <u>evidence</u> from Hume himself: space and time relations are 'directly perceived'; and the 'manner' of the 'appearance' of the 'coloured points' is a 'compound impression'.

As was seen above, (p. 45) Atkinson did not restrict the "synthetic a priori" interpretation <u>merely</u> to geometrical propositions, he included also the propositions of algebra and arithmetic. Such an interpretation of Hume just won't work, and it won't work for this very short reason: Atkinson (like Pap) seems to have ignored the very ground of the division, the "constant"/"inconstant" relations base and <u>its</u> grounds. To talk of a "synthetic a priori" proposition is to talk of a proposition, where the relation holding between the two terms is at one and the same time "constant" and "inconstant". To talk of a <u>synthetic a priori</u> proposition would be to talk of a proposition, where the relation holding between the two terms of the proposition would, at one and the same time, be "constant" and "inconstant". For, a 'synthetic' proposition, for Hume, would be one where the relation holding between the two terms of the proposition is "inconstant" or "alterable" 'without any change in the ideas' (T.69). That is, the relation does not vary with the "ideas". And an 'a priori' proposition, for Hume, would be one where the relation is "constant" or "invariable" or "unalterable" (T.73, 69, 79 respectively). The ground of this constancy or invariableness of the relation being the make-up or composition of the "ideas" or terms of the proposition. These relations 'depend entirely on the ideas' (T.69), ie the relation varies with the "ideas" (any change in the ideas makes for a corresponding change in the relation which holds between the ideas).

IV. The Division in the First Enquiry

In the Treatise (Book 1, Part 3, Section 1), Hume had said: 'All kinds of reasoning consist in nothing but a comparison, and a discovery of those relations, either constant or inconstant, which two or more objects bear to each other' (T.73). Those relations, whether "constant" or "inconstant", were classed under the heading of "philosophical" relations, of which he said there were seven in number. He divided these seven into two classes, one of which contained four of these seven "philosophical" relations; and these four he termed "constant" or "invariable" or "unalterable", since they 'depend entirely on the ideas, which we compare together' (T.69). The other of the two classes was made up of the remaining three relations. These relations were "inconstant" or "variable", since they were 'such as may be changed without any change in the ideas' (ibid).

In the First Enquiry, Hume says: 'All the objects of human reason or enquiry may naturally be divided into two kinds,...Relations of Ideas, and Matters of Fact. Of the first kind are the sciences of Geometry, Algebra, and Arithmetic; and in short, every affirmation which is either intuitively or demonstratively certain. That the square of the hypothenuse is equal to the square of the two sides, is a proposition which expresses a relation between these figures. That three times five is equal to the half of thirty, expresses a relation between these numbers. Propositions of this kind are discoverable by the mere operation of thought, without dependence on what is anywhere existent in the universe. Though there never were a circle or triangle in nature, the truths demonstrated by Euclid would for ever retain their certainty and evidence' (E.20).[1]

---

[1]All references to the First Enquiry are to the section numbers in the Selby-Bigge edition; referred to as E with the section number following.

Concerning Matters of Fact, he says: '(they) are not ascertained in the same manner; nor is our evidence of their truth,...of a like nature with the foregoing. The contrary of every matter of fact is still possible; because it can never imply a contradiction, and is conceived by the mind with the same facility and distinctness, as if ever so conformable to reality. That the sun will not rise to-morrow is no less intelligible a proposition, and implies no more contradiction than the affirmation, that it will rise. We should in vain...attempt to demonstrate its falsehood. Were it demonstratively false, it would imply a contradiction, and could never be distinctly conceived by the mind' (E.21).

Before proceeding to discuss this division in the First Enquiry, it is to be noted, that there is no mention made of the term "philosophical" relation, nor is there any list of these relations given; and thus no division of the seven philosophical relations into two classes, of four and three members respectively. Nor is there any mention of the distinction, (which is at the basis of the division into two classes (in the Treatise)) between those relation which are "constant" and those which are "inconstant". Nor is there any mention here of what I termed the "direct inspection" criterion. Concerning geometry, no reference is made to the point on which he insisted in the Treatise that it lacks 'perfect precision and exactness'; but this defect, which was noted in the Treatise, did not, however, debar it from being classed under the heading of certainty or knowledge, this certainty being either intuitive or demonstrative, the latter consisting in a chain of intuitions. As Hume stated in the Treatise (T.45): 'When geometry decides anything concerning the proportions of quantity, we ought not to look for the utmost precision and exactness. None of its proofs extend so far. It takes the dimensions and proportions of figures justly; but roughly, and with some liberty. Its errors are never considerable; nor would it err at all, did it not aspire to such an absolute perfection'. Nevertheless, geometrical propositions in the Treatise, are still intuitive, since the relation involved is constant. I now want to discuss this division in the Enquiry in more detail.

The two classes in the Enquiry are now termed Relations of Ideas and Matters of Fact. In the Treatise, when he first introduced the division between what he termed knowledge and probability (in Sections 1 and 2 respectively, of Part 3 of Book 1) he spoke of those relations which 'depend entirely on the ideas' (such relations being constant or invariable); and those relations which 'may be changed without any change in the ideas' (such relations being inconstant or variable).

Towards the end of Book 2 (T.413), he speaks of 'the abstract relations of our ideas' and 'those relations of objects, of which experience only gives us information': 'The understanding exerts itself after two different ways, as it judges from demonstration or probability; as it regards the abstract relations of our ideas, or those relations of objects, of which experience only gives us information'. At the beginning of Book 3, Section 1 (T.458), the expressions 'the real relations of ideas' and 'real existence and matter of fact' are employed. In the same section (T.463), he writes :'the operations of human understanding divide themselves into two kinds, the comparing of ideas, and the inferring of matter of fact'. And there specific mention is made of the four relations listed in Book 1. At T.466, he says that 'Reason or science is nothing but the comparing of ideas, and the discovery of their relations'.

The expressions 'Relations of Ideas' and 'matters of fact' are thus quite common, especially in Book 3.

What is of note is that when we turn to the division in the Enquiry, all reference to constant and inconstant relations has been dropped. He now merely speaks of relations of ideas. Under the heading, Relations of Ideas, he lists the 'sciences of Geometry, Algebra, and Arithmetic' (Italics mine) and 'every affirmation which is either intuitively or demonstratively certain'. With respect to geometry, I will, for the present, merely note that in the Treatise (Section 1 of Part3), he had referred to it as an 'art', reserving the title of 'science' to algebra and arithmetic. The status of geometry will be dealt with subsequently.

Since no list here has been given corresponding to the list of four relations in the Treatise, Hume's assertion that under the heading of Relations of Ideas is to be included 'every affirmation which is either intuitively or demonstratively certain', is, apart from mathematics, somewhat vague. What relations, apart from those of proportion in quantity and number, are to be included under this heading? It may be, of course, that the reference to resemblance, contrariety, and degrees in quality is implied when he speaks of 'every affirmation which is...intuitively...certain'.[1] In the Treatise, these relations fell 'more properly under the province of intuition than demonstration' (T.70). But, since no list of these relations is here given, it is quite unclear whether the reference is to the three above-mentioned relations. In the last section but one of the Enquiry (E.131), he writes: 'It seems to me, that the only objects of the abstract science or of

---

[1]This seems to be N. Kemp Smith's view, in his paper The Naturalism of Hume (Mind. N.S. Vol. 14, 1905), since he collates the First Enquiry (E.20, 30) with the Treatise (Section 1, of Part 3, of Book 1).

demonstration are quantity and number, and that all attempts to extend this more perfect species of knowledge beyond these bounds are mere sophistry and illusion...the sciences of quantity and number...may safely, I think, be pronounced the only proper objects of knowledge and demonstration'. He continues in the final section: 'All other enquiries of men regard only matter of fact and existence'. And in the final paragraph, he concludes that all <u>but</u> 'abstract reasoning concerning quantity and number' and 'experimental reasoning concerning matter of fact and existence' is 'nothing but sophistry and illusion'. It would seem, then, that proportion in quantity and number are the only relations which he will allow under the heading <u>Relations of Ideas</u>.

It may be said that here (E.131) he is speaking merely of 'theobjects of demonstration', and since in the <u>Treatise</u>, proportion in quantity and number, for the most part, came under the province of demonstration, the question remains open whether the three above-mentioned relations would come under the heading of 'affirmations which are intuitively certain' mentioned at E.20.

Now, although the three relations which, in the <u>Treatise</u>, fell 'under the province of intuition', are not mentioned in the <u>Enquiry</u>, he does speak of affirmations which are intuitively certain; but, since he has given no examples, it is unclear what <u>sort</u> of propositions, he would regard as 'intuitively certain'. At E.29, he speaks of the connexion between two propositions, under examination, as not being 'intuitive', since 'there is required a medium, which may enable the mind to draw such an inference'. Upon what ground a proposition is intuitively certain in the <u>Enquiry</u> is not explicit. There is no reference to the principle of direct inspection; which principle or criterion, in the Treatise, had its basis in the constancy of the relation holding between the two terms of the proposition. But since the distinction between constant and inconstant relations is omitted in the Enquiry, it is uncear whether or not it is on that criterion that a proposition is 'intuitively certain'.

It would seem, however, that his general practice is to sink intuitive propositions under the general heading of 'demonstrative reasoning'. At E.30, he writes: 'All reasonings may be divided into two kinds, namely, demonstrative reasoning, or that concerning relations of ideas, and moral reasoning, or that concerning matter of fact and existence'. In Book 3 of the Treatise, he refers to the four relations which make up Class I, as 'demonstrable relations' (T.464); even although three of these in Book 1, came under 'the province of intuition'. And at E.131, he uses the term 'knowledge' and 'demonstration' equivalently; and in the Treatise, the term 'knowledge' or 'certainty' was used in the sense of 'intuitive' or 'demonstrative' certainty'.

In the Enquiry, a 'demonstrative' proposition is one which 'implies no contradiction' (E.30); 'whatever is intelligible, and can be distinctly conceived, implies no contradiction' (ibid). The criterion or test of a demonstrative proposition is that its 'negation' (E.132) is inconceivable: 'Every (demonstrative) proposition, which is not true, is...confused and unintelligible. That the cube root of 64 is equal to the half of 10, is a false proposition, and can never be distinctly conceived' (ibid). (In the Treatise, the conceivability criterion was not employed by Hume, with reference to those propositions which make up Class I). A 'demonstrative' proposition is one the 'negation' of which is <u>inconceivable</u>, he maintains. A proposition, on the other hand, which asserts a <u>matter of fact</u>, is one the negation of which is <u>conceivable</u>. The terms "inconceivable" and "impossible", he uses as the equivalent of each other, and by "impossible" he means 'implies a contradiction'. Thus a proposition which is <u>inconceivable</u>, is one which <u>implies a contradiction</u> (in the ideas) (<u>Vide.</u>E. 21,30,132). The assumption being, it would seem, that the mind is not capable of conceiving a contradiction. This assumption itself is worthy of examination. I cannot, however, in this paper, enter into that matter.

This leads to the question, Upon what ground is a proposition 'conceivable' or 'inconceivable'? We are faced with this problem not only in the Enquiry, but in the Treatise too.. For, if in the Treatise, Hume does not employ this criterion with the constant or inconstant relations as its basis, then the whole ground of the "conceivability" criterion is most unclear. And, in the Enquiry, all reference to the constancy and inconstancy of relations has been omitted.

I come now to the question, What sort of propositions come under the heading of Relations of Ideas and Matters of Fact. Under the former appellation belongs, Hume says, 'every affirmation which is either intuitively or demonstratively certain'; under the latter, propositions, which are not intuitively or demonstratively certain. To say that the former type of propositions are concerned merely with ideas, whereas the latter are not, is not very helpful; since, in Hume's terminology, all are "ideas". Frequently, he refers to 'reasonings' which are 'a priori' (E.23, 24, 25, 27, 36n, 60n, 89n, 132, 132n.) and those which are experiential; or, 'demonstrative' or 'moral' ('probable') reasoning. Thus, propositions which embody the former and the latter species of reasoning could be termed 'a priori' and experiential propositions respectively. Once in the Second Enquiry[1], he contrasts the pairs, 'a priori' and 'a posteriori' (E.188). Now, although all this is so, it should be noted in what way these terms are being used. For Hume, an 'a priori' proposition is one which is intuitively or demonstratively certain; an experiential proposition is one which is not so. Upon what ground propositions are intuitively

---

[1]I use this brief appellation to refer to 'An Enquiry concerning the Principles of Morals. The reference is to the section numbers in the Selby-Bigge edition (cited p. 1).

or demonstratively certain or not, is extremely unclear, since all reference to the constancy or inconstancy of the relation is here omitted. As I have shown, it is on that ground alone, that the distinction in the Treatise between the two types of propositions is made. If the Enquiry is a mere abridgement of the earlier work, is it to be supposed that for a statement of the ground, reference has to be made tohis fuller treatment of the subject in the Treatise? I think not; and for this reason: in the Enquiry, although he speaks of 'affirmations' which are intuitively certain (E.20), and of inferences not being 'intuitive' (E.29, 32), in the case of the latter references, 'intuitive', in the sense that one just sees that B (say) follows from A, his general practice, in conformity with his assertion (E.131) that 'the only proper objects of knowledge and demonstration' are proportion in quantity and number, is to speak merely of demonstrative or 'a priori' propositions, the 'negations' of which propositions are 'inconceivable', and being inconceivable, they 'imply a contradiction'.

It is maintained by some writers that Hume held the view that all a priori propositions are 'empty' (that is, 'analytic', that is, 'tautologous'). D. F. Pears[1] did not merely confine this view to the Enquiry, but stated that such a view was 'the central contention of the first book of the Treatise.' With respect to the Treatise, I have shown that such a view is not at all applicable. Is such a view applicable to the Enquiry? The answering of this question involves a discussion of Hume's treatment of mathematics, since it is his view (E.131) that proportion in

---

[1] Vide. p. 1, for the reference.

quantity and number are 'the only proper objects of knowledge and demonstration'. In other words, he restricts the field of the a priori to mathematics. And, what he has said concerning mathematics in the Enquiry is indeed precious little. His references to mathematics are confined to the following section: E.20, 27, 48, 124, 124n, 125, 125n, 131.

At E.20, he writes: '(Mathematical) propositions...are discoverable by the mere operation of thought, without dependence on what is anywhere existent in the universe. Though there never were a circle or triangle in nature, the truths demonstrated by Euclid would for ever retain their certainty and evidence'. With regard to geometry, contrasting what is here said with what he had maintained in the Treatise, it can be seen there is no reference here to the lack of 'perfect precision and exactness' involved in the 'ideas' with which geometry deals. Nor is there any reference to the point on which he remarked in the Treatise (Section 1 of Part 3, T.72, 73) concerning the application of his fundamental principle to our reasonings in mathematics.

But, further on in the Enquiry, his view regarding the 'ideas' in geometry is identical with the statement of the matter at T.72, 73 (Treatise); at E. 48 he writes: 'The great advantage of the mathematical sciences...consists in this, that the ideas...being sensible, are always clear and determinate, the smallest distinction between them is immediately perceptible...An oval is never mistaken for a circle, nor a hyperbola for an ellipsis...If any term be defined in geometry, the mind readily, of itself, substitutes, on all occasions, the definition for the term defined: Or even when no definition is employed, the object itself may be presented to the senses, and by that means be steadily and clearly apprehended.'

Again, at E.124, 124n, and 125, the position which he had maintained in Part 2 of the Treatise, is here taken up, concerning the infinite divisibility of extension (and of time or duration), and mathematical points' Concerning 'the paradoxical conclusions of geometry or the science of quantity' (E.125), he writes: 'It seems to me not impossible to avoid these absurdities and contradictions, if it be admitted, that there is no such thing as abstract or general ideas, properly speaking; but that all general ideas are, in reality, particular ones, attached to a general term, which recalls, upon occasion, other particular ones, that resemble, in certain circumstances, the idea, present to the mind...If this be admitted...it follows that all the ideas of quantity, upon which mathematicians reason, are nothing but particular, and such as are suggested by the senses and imagination, and consequently, cannot be infinitely divisible...' (footnote, *ibid*).

And at E.27, he remarks on the use of geometry to assist in the applications of the laws of natural philosophy, 'by giving us the just dimensions of all the parts and figures which can enter into any species of machine; but...the discovery of the law itself is owing merely to experience...'.

And finally (E.131), he distinguishes between a mathematical proposition and one in which it is merely a matter of defining the terms of which it is composed, to be convinced of its truth. 'That *the square of the hypothenuse is equal to the squares of the other two sides*, cannot be known, let the terms be ever so exactly defined, without a train of reasoning and enquiry. But to convince us of this proposition, *that where there is no property, there can be no injustice*,[1] it is only necessary to define the terms, and explain injustice to be a violation of property. This proposition is, indeed, nothing but a more imperfect definition.

[1] *Vide*. Locke, *Essay*, Book IV, C.3, Section 18.

It is the same case with all those pretended syllogistical reasonings, which may be found in every other branch of learning, except the sciences of quantity and number; and these may safely be pronounced the only proper objects of knowledge and demonstration.'.

These, then, being all the references to mathematics, it can be seen that, apart from the statement at S.B.20, Hume's view of geometry, is, in substance, that of the Treatise. It is also clear that he does not regard geometrical propositions as a mere matter of definitions (E.131); nor are arithmetical and algebraic propositions so regarded by him, since his remarks there concern not merely geometry, but 'the sciences of quantity and number' (ibid. Italics mine). His statement at E.20, is in direct opposition to the subsequent references to geometry in the Enquiry, and in light of the other references, I just do not know what to make of that statement. Would this isolated statement be evidence sufficient that Hume regarded geometrical propositions as 'analytic' or 'empty' or 'tautologous'? As was noted above (p.6), Hume employs the terms 'a priori' reasoning and 'demonstrative' reasoning synonymously. D. F. Pears[1] is of the opinion that, for Hume, all a priori propositions are 'empty', that is, 'analytic', a view which he ascribed both to the Treatise and the Enquiry; with respect to the latter he referred the reader to E.20. But, first, we must see in what way the terms "analytic", "empty" and "tautologous" are being used by those who use them. Pears mentions Ayer (vide p. 35) where I quoted Pears' words). A. J. Ayer (p.31)[2] writes: 'Like Hume, I divide all genuine propositions into two classes:

---

[1] Vide. p.1

[2] Language, Truth and Logic. New York: Dover Publications, Inc., (2nd edition, 1946).

those which, in his terminology, concern 'relations of ideas', and those which concern 'matters of fact'. The former class comprises the a priori propositions of logic and pure mathematics, and these I allow to be necessary and certain only because they are analytic. That is, I maintain that the reason why these propositions cannot be confuted in experience is that they do not make any assertion about the empirical world, but simply record our determination to use symbols in a certain fashion". (Italics mine). It is not completely clear to me in this passage whether Ayer is ascribing his view to Hume. In a later passage (p.150), however, he quotes E.30: 'It implies no contradiction that the course of nature may change, and that an object, seemingly like those which we have experienced, may be attended with different or contrary effects...whatever is intelligible, and can be distinctly conceived, implies no contradiction, and can never be proved false by any demonstrative argument or abstract reasoning a priori', and comments: 'Here Hume is supporting our contention that...propositions which cannot be denied without self-contradiction are analytic'. In his introduction to Logical Positivism (p.10)[1], Ayer writes: 'Like (Hume), (logical positivists) divided significant propositions into two classes; formal propositions, like those of logic or pure mathematics, which they held to be tautological...and factual propositions, of which it was required that they should be empirically verifiable'. Ayer explains the use of 'tautological' thus: 'a statement (which) agrees with every truth distribution... it is true in any circumstance whatsoever' (p.11) and further remarks that the point about tautologies is that they do not 'say anything about the world' (p.12).

---

[1]Logical Positivism. Edit. by A. J. Ayer. The Free Press, Glencoe, 1959.

From these passages, it can be seen that the terms 'analytic' and 'tautological' are being used synonymously by Ayer: analytic or tautological propositions 'do not make any assertion about the empirical world'...they do not 'say anything about the world'; furthermore, they 'cannot be denied without self-contradiction'.

Hume's statement at E.20 may lend support to the view that mathematical propositions do not'say anything about the world', since Hume there writes: 'Propositions of this kind are discoverable by the mere operation of thought, without dependence on what is anywhere existent in the universe. Though there never were a circle or triangle in nature, the truths demonstrated by Euclid would for ever retain their certainty and evidence.' But it is this very statement which is at variance with his other statements in the Enquiry about geometry.

An analytic proposition is also one 'which cannot be denied without self-contradiction' but in the passage quoted by Ayer, Hume merely says: '...whatever is intelligible, and can be distincly conceived, implies no contradiction...' (E.30) or whatever is 'unintelligible...can never be distinctly conceived' (E.132). Hume nowhere says that the 'negations' (his word) of mathematical propositions are self-contradictory. All that he says is 'that the cube root of 64 is equal to the half of 10, is a false proposition, and can never be distinctly conceived'.

N. Kemp Smith, in his paper The Naturalism of Hume,[1] uses the term "analytic" to describe propositions the opposite of which is impossible to conceive (pp.156, 157), and in that sense, mathematical propositions in the Enquiry may be termed "analytic".

---

[1] Vide. p. 68

Remarking on 'the two very distinct meanings which he (Hume) ascribes to the term "reason",', he quotes the First Enquiry (E.30): "All reasonings may be divided into two kinds, namely, demonstrative reasoning, or that concerning relations of ideas, and moral reasoning, or that concerning matter of fact and existence". He continues: 'The first kind of reasoning is analytic. Since the relations discovered are involved in the ideas compared, being such as cannot be changed without change in the ideas, their truth is guaranteed by the law of non-contradiction. The relations thus revealed are those of resemblance, contrariety, degrees in quality, and proportion in quantity or number; and as the mathematical sciences of geometry, algebra and arithmetic, involve only such relations, they are rendered possible by such discursive analytical thinking. "That three times five is equal to the half of thirty, expresses a relation between these numbers. Propositions of this kind are discoverable by the mere operation of thought, without dependence on what is anywhere existent in the universe. Though there never were a circle or triangle in nature, the truths demonstrated by Euclid would for ever retain their certainty and evidence"' (First Enquiry, E.20). Kemp Smith continues: 'This logical necessity, which consists in the impossibility of conceiving the opposite, is the sole form of rational necessity known to us, and it supplies a standard in the light of which we are enabled to detect its complete absence from all our knowledge of matters of fact.'

But, for Hume, mathematical propositions are not analytic in Kant's sense of that term, as I will show. Kant defines an 'analytic judgment' thus (p.48)[1]

---

[1] Critique of Pure Reason, trans. by N. Kemp Smith. (Edition cited).

'...the predicate B belongs to the subject A, as something which is (covertly) contained in this concept A...(such a judgment) can also be entitled explicative, (since it adds) nothing through the predicate to the concept of the subject, but merely (breaks) it up intothose constituent concepts that have all along been thought in it, although confusedly...'; and such a judgment 'rest(s) on the principle of contradiction' (p.54)

For Hume, at E.131, contrasts the propositions, the square of the hypothenuse is equal to the squares of the other two sides and where there is no property, there can be no injustice, remarking that the former 'cannot be known, let the terms be ever so exactly defined, without a train of reasoning and enquiry', whereas in the case of the latter, 'it is only necessary to define the terms, and explain injustice to be a violation of property' to be convinced of it. Are mathematical propositions, then, for Hume in the Enquiry, in Kant's terminology "synthetic a priori"? Such a view, I maintained, was ruled out, in so far as the Treatise, was concerned, for the reason that there was no third possibility for the relation holding between the two terms of the proposition, it was either "constant" or "inconstant". In the Enquiry, however, the distinction between "constant" and "inconstant" relations has been omitted. Does this omission, then, leave open the possibility that mathematical propositions are "synthetic a priori" in the Enquiry?

But, first, let us see what Kant means by such a judgment or proposition. A "synthetic judgment" is one where 'the predicate B' 'lies outside the concept A, although it does indeed stand in connection with it...(such a judgment may be termed) ampliative (because it) add(s) to the concept of the subject a predicate which has not been in any wise thought in it, and which no analysis could possibly extract from it.' (p.48)[1]. Thus, the proposition 'the straight line between two

---

[1] op. cit.

points is the shortest' (the example also used by Hume (T.49, 50) to show that such a proposition is not a tautology) 'is a synthetic proposition. For my concept of <u>straight</u> contains nothing of quantity, but only of quality. The concept of the shortest is wholly an addition, and cannot be derived, through any process of analysis, from the concept of the straight line.' (p.53)[1]. It is also '<u>a priori</u>' : '...mathematical propositions, strictly so called, are always judgments <u>a priori</u>, not empirical, because they carry with them necessity, which cannot be derived from experience ' (p.52)[2]. When Kant says (discussing the proposition 7 + 5 = 12) 'the concept of 12 is by no means already thought in merely thinking this union of 7 and 5; and I may analyse my concept of such a possible sum as long as I please, still I shall never find the 12 in it. We have to go outside these concepts, and call in the aid of intuition which corresponds to one of them, our five fingers, for instance, or...five points, adding to the concept of 7, unit by unit, the five given in intuition...(and) with the aid of the hand see the number 12 come into being.' (p.53)[3]; and when Hume says (E.131): 'That <u>the square of the hypothenuse is equal to the squares of the other two sides</u>, cannot be known, let the terms be ever so exactly defined, without a train of reasoning and enquiry', it may be thought that, were he acquainted with Kant's terminology, such a proposition he would have termed "synthetic a priori".

Such a view, in so far as the <u>Treatise</u> was concerned was straightaway ruled out, for the reason that such a proposition would be one where the relation

---

[1] <u>Op</u>. <u>cit</u>.

[2] <u>Op</u>. <u>cit</u>.

[3] <u>Op</u>. <u>cit</u>.

is both "inconstant" and "constant" at the same time; or, to state it in another way, the proposition would be both intuitive and not-intuitive at the same time. Now, since the division in the First Enquiry is not made on the basis of the constancy and inconstancy of the relations, but solely on the basis of the conceivability criterion, it might seem that Pap's view[1] that 'Hume's imaginability criterion of necessity leaves it at least an open question whether a given proposition may not be both synthetic and necessary' (p.79), and that 'Hume's proof of the contingency of p (where p is a proposition) is a petitio principii' (p.77)[2], although ruled out by me, in so far as the Treatise was concerned, is now applicable to the First Enquiry. This question, then, has to be investigated. First, this will be said. The distinction in the Enquiry, (as in the Treatise) is, on the face of it, a distinction between propositions which are "a priori" (or, necessary) and those which are "a posteriori" (or, contingent). And as A. Flew remarks (p.64)[3] in his notice of Reichenbach's contention that Hume '"arrives at the result that all knowledge is either analytic or derived from experience..."' 'If analytic and synthetic are to be employed...as mere synonyms for necessary and contingent, or for a priori and a posteriori...In such an usage to speak of a synthetic a priori would indeed be obviously contradictory.' Flew continues (ibid): 'But if the words are not used simply in this uneconomical way it may be more doubtful whether it is really correct to attribute to Hume the view that mathematics contains no synthetic elements.'. That is, 'synthetic' in Kant's

---

[1] Vide. p. 33

[2] Op. cit.

[3] Hume's Philosophy of Belief: A Study of his First 'Enquiry'. London: Routledge & Kegan Paul, 1961.

sense, Flew means, (He cites E.131).[1]

R. F. Atkinson[2], discussing Hume's view of mathematics in the Enquiry writes: 'It appears to me that the treatment of mathematics in the Enquiry is simply a shortened and simplified version of that in the Treatise.' (p.133); and Atkinson's view of mathematical propositions in the Treatise was that such propositions are not analytic, but that they may be synthetic a priori. I discussed this view of Atkinson's, in connection with the Treatise, and if Hume's view in the Enquiry is a 'shortened' and 'simplified' version of that in the Treatise', then, my remarks concerning Atkinson's view in that earlier work, apply, according to Atkinson's view of the two works, likewise to the Enquiry. But, such a view on the part of Atkinson, implies that, for Hume in the Enquiry, the base on which the distinction between Relations of Ideas and Matters of Fact rests, is the constant/inconstant relations base. Yet Hume nowhere mentions this base. He, in so far as his words go, has abandoned it, in the Enquiry. The base in the latter work now, on which the distinction rests is the conceivability principle or criterion. And if that criterion is bound up with the constant/inconstant relations base, then, there can be no such things as synthetic a priori propositions. But, if theconceivability principle is in no way connected with that base, then, perhaps there can be, for Hume in the Enquiry, such propositions; this conceivability criterion, then, has to be investigated.

First, I will restate in what way the conceivability criterion would

---

[1] Vide. p. 75 where the reference is quoted by me.

[2] Article cited by me (p. 45).

operate on the constant/inconstant relations base. _Second_, I shall try to see on what ground that criterion can operate, devoid of the constant/inconstant relations base. On the latter base, the 'negations of propositions of 'Relations of Ideas' would be _inconceivable_, on the ground that the relation is 'constant'; whereas the 'negations' of 'matter of fact' propositions would be _conceivable_ on the ground that the relation is 'inconstant' or 'variable'.

As was noted in Section II of this paper, the _conceivability_ criterion is grounded on two principles; the denial of a tie or connection among ideas or the separability of ideas 'by the thought and the imagination'; and 'that nothing we imagine is absolutely impossible' or 'whatever the mind clearly conceives includes the idea of possible existence' (T.32). And since 'there is no matter of fact which we believe so firmly that we cannot conceive the contrary' (E.39), and since we are able to 'conceive the contrary' of a 'matter of fact' proposition, it thus lacks necessity. Thus this criterion is another way of distinguishing propositions which are _necessary_ and those which are not necessary. And to say that a proposition is _synthetic a priori_ would be to say, on Hume's view, that the proposition is 'conceivable' and 'inconceivable' at the same time. Since, if a necessary or _a priori_ proposition is one the 'negation' of which is _inconceivable_ (_vide_. E.30,132), and if a synthetic or 'matter of fact' proposition, for Hume, is one the 'negation' of which is _conceivable_ (_vide_.E.21), then a _synthetic a priori_ proposition would be one where the 'negation' is both _conceivable and inconceivable_ at the same time.

So, I contend, that there can be, on Hume's view, no such things as _synthetic a priori_ propositions either on the constant/inconstant relations base or on the conceivability base, devoid of the constant/inconstant relations base.

BIBLIOGRAPHY

(Those books or papers which are marked with an asterisk are those from which points have been discussed by me, in the text of this paper.)

*Annand, M. R. "An examination of Hume's Theory of Relations", The Monist, Vol. XL, 1930.

*Atkinson, R. F. "Hume on Mathematics", The Philosophical Quarterly, Vol. 10, 1960.

*Ayer, A. J. Language, Truth and Logic. 2nd edition. New York: Dover Publications Inc.

*-------------. ed. Logical Positivism. The Free Press of Glencoe, 1959.

*Church, R. W. Hume's Theory of the Understanding. London: George Allen & Unwin Ltd. 1935.

*Flew, Antony. Hume's Philosophy of Belief: A Study of his First 'Inquiry', London: Routledge & Kegan Paul, 1961.

*Hendel, C. W. Studies in the Philosophy of David Hume. New York: The Bobbs-Merrill Co. Inc., 1963.

-------------. Studies in the Philosophy of David Hume. Princeton, 1925.

*Hume, David. A Treatise of Human Nature (edit. by L. A. Selby-Bigge). Oxford: Clarendon Press, 1960.

*--------------. An Enquiry concerning the Human Understanding. (edit. by L. A. Selby-Bigge). Oxford: Clarendon Press, 1902.

*Kant, Immanuel. A Critique of Pure Reason. (trans. by N. Kemp Smith). New York: St. Martin's Press, 1965.

Laird, John. Hume's Philosophy of Human Nature. London: Methuen & Co. Ltd., 1932.

*Leroy, André-Louis. David Hume. Presses Universitaires de France, 1953.

*Locke, John. An Essay concerning Human Understanding. London: Ward, Locke & Co., Lt

Maund, C. Hume's Theory of Knowledge. Lodon, 1937.

Moore, G. E. Some Main Problems of Philosophy. (FF. 5, 6). London: George Allen & Unwin Ltd., 1962.

----------. Philosophical Studies. London: Routledge & Kegan Paul, Ltd., 1965.

*Pap, Arthur. Semantics and Necessary Truth. New Haven: Yale University Press, 1958.

*Passmore, John A. Hume's Intentions. Cambridge University Press, 1952, (This book was kindly lent to me by my tutor, Dr. James Noxon).

*Pears, D. F. (ed.). David Hume: A Symposium. London: Macmillan & Co. Ltd., 1963.

Price, H. H. "The Permanent Significance of Hume's Philosophy", Philosophy, Vol. 15, 1940.

*Smith, N. Kemp. The Philosophy of David Hume. London: MacMillan & Co. Ltd., 1964.

*-----------. "The Naturalism of Hume", Mind. N.S. Vol. 14, 1905.

Zabeeh, Farhang. Hume: Precursor of Modern Empiricism. The Hague: Martinus Nijhoff, 1960.